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ARTICLE *in* INDUSTRIAL & ENGINEERING CHEMISTRY RESEARCH · MAY 2015

Impact Factor: 2.59 · DOI: 10.1021/acs.iecr.5b00290

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Efficient Lagrangian Decomposition Approach for Solving Refinery Production Scheduling Problems Involving Operational Transitions of Mode Switching

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ABSTRACT: Due to the complexity of the production process, short-term scheduling for refineries is one of the most challenging problems. To address large-scale industrial refinery scheduling problems, a spatial Lagrangian decomposition approach is proposed to decompose the whole problem into several production processing subproblems and one blending and delivery subproblem. Some auxiliary constraints are added in the subproblems to accelerate the convergence of Lagrange multipliers. An initialization scheme of Lagrange multipliers, a hybrid method to update the Lagrange multipliers, and a heuristic algorithm to find feasible solutions are also designed. Computational results on three cases with different lengths of time horizons and different numbers of orders show that the proposed Lagrangian scheme is effective and efficient.

1. INTRODUCTION

In the oil refinery industry, production scheduling is an essential tool to allocate resources and increase profit. Short-term scheduling of refinery operations is one of the most challenging problems due to the complexity of the production process. The research area of refinery scheduling has received considerable attention. There have been several papers of scheduling models and approaches published in the past two decades. Dimitriadis et al.[1] provided the necessary theoretical basis for the exploitation of partial resource equivalence, which allows large resource−task networks (RTNs) to be reduced to smaller but completely equivalent ones. Such reductions are particularly significant in problems involving many sequence-dependent changeovers. Based on both discrete and continuous time representations, Pinto et al.[2,3] presented a nonlinear planning model for refinery production and formulated scheduling problems in oil refineries as mixed-integer optimization models. Göthe-Lundgren et al.[4] described a production planning and scheduling problem in an oil refinery company. The aim of the scheduling is to decide which mode of operation to use in each processing unit at each point in time, in order to satisfy the demand while minimizing the production cost and taking storage capacities into account. Jia and Ierapetritou[5,6] developed a comprehensive mathematical programming model for the scheduling of oil refinery operations. The overall problem is decomposed spatially into three domains: the crude oil unloading and blending, the production unit operations, and the product blending and delivery. The subproblems are modeled using the continuous-time representation and solved separately. Luo and Rong[7] presented a hierarchical approach for short-term scheduling problems in refineries. The approach has two decision levels: the optimization model at the upper level and the heuristics and rules adopted in the simulation system at the lower level. They also introduced the iteration procedure between the upper and the lower levels. Wu and Ierapetritou[8] presented a simultaneous solution of production planning and scheduling problem through a hierarchical framework. The planning problem aggregates orders in the planning period and considers uncertainty utilizing a multistage stochastic programming formulation. The production for the current stage is provided to the scheduling problem, which is solved using a continuous-time formulation. An iterative framework is developed to converge the planning and scheduling results before solving for the next period. Mouret et al.[9] introduced a problem integrating the two main optimization problems appearing in the oil refining industry: refinery planning and crude-oil operations scheduling. The refinery planning problem is regarded as a flow sheet optimization problem with multiple periods. The refinery system is assumed to operate in a steady state. Shah and Ierapetritou[10] presented a comprehensive integrated optimization model for the scheduling problem of production units and end-product blending problem. The model is based on continuous-time formulation and incorporates quantity, quality, and logistics decisions related to real-life refinery operations. Lima et al.[11] addressed the long-term scheduling of a real-world multiproduct single stage continuous process for manufacturing glass. This process features long minimum run lengths, and sequence dependent changeovers of the order of days, with high transition costs. Three different rolling-horizon algorithms based on different models and time aggregation techniques have also been developed. Cao et al.[12] developed a rolling-horizon optimal control strategy to solve the online scheduling problem for a real-world refinery diesel production based on a data-driven model. A mixed-integer nonlinear programming (MINLP) scheduling model considering the implementation of nonlinear blending quality relations and quantity conservation principles

Received: January 22, 2015
Revised: April 23, 2015
Accepted: May 28, 2015
Published: May 28, 2015

DOI: 10.1021/acs.iecr.5b00290
Ind. Eng. Chem. Res. 2015, 54, 6508−6526

is presented. Some excellent reviews have also been published. Shah et al.[13] presented an extensive literature review of methodologies for addressing scheduling, planning, and supply chain management of oil refinery operations. Joly[14] published a paper to clarify the central role of refinery planning and scheduling activities in the Petrobras refining business. Major past and present results are outlined, and corporate long-term strategies to deal with present and future challenges are presented. This research is valuable and meaningful for the development of refinery scheduling.

For refinery production scheduling problems, it is crucial to implement the operation modes for production units. Therefore, the advanced control of production units has to be taken into account in the modeling procedure. Lv et al.[15] proposed a new predictive control scheme by using the split ratio of distillate flow rate to that of bottoms as an essential controlled variable. Correspondingly, a new strategy of integrated control and online optimization is developed on a high-purity distillation process. With the strategy, the process achieves its steady state quickly. On the basis of the implementation of unitwide predictive control, finite numbers of optimal operating modes of the production units can be obtained. Shi et al.[16] proposed a refinery scheduling model which is discrete-time and mixed-integer linear programming (MILP). The operation modes in the model are defined based on real-world operations implemented by the special unitwide predictive control. Also, the transitions of mode switching are involved to formulate the dynamic nature of production more accurately.

In the scheduling model presented by Shi et al.,[16] operation modes and transitions of mode switching are defined for each production unit. It needs a lot of binary variables to present the operation state of each unit at each time point in the scheduling model. As the number of scheduling time intervals increases, it usually leads to a very large-scale combinatorial problem which needs more computational effort to solve. Lagrangian decomposition method is a choice to solve a large-scale scheduling problem, which has been successfully applied to large-scale mathematical programming problems. Temporal and spatial decomposition[17] can be adopted according to the problem structure. Lagrangian decomposition has been used successfully in the steel-making process,[18] production planning, and scheduling integration,[19] the supply chain of an electric motor,[20] and shale-gas systems.[21] Neiro and Pinto[22] presented two Lagrangian decomposition strategies applied to multiperiod planning of petroleum refineries under uncertainty. Shah et al.[23] presented a novel decomposition strategy for solving large-scale refinery scheduling problems. The original problem is decomposed at intermediate storage tanks such that inlet and outlet streams of the tank belong to the different subsystems. Mouret et al.[9] proposed a Lagrangian decomposition approach applied to the integration of refinery planning and crude-oil scheduling problems. Some classical approaches to solve the dual problem have been proposed, such as the subgradient method,[24,25] the cutting plane method,[26,27] and the box-step method.[28] To apply a Lagrangian decomposition approach successfully, the convergence of Lagrange multipliers and solving of subproblems are challenging tasks. Some excellent papers[29−33] have been published on this.

In this paper, an efficient Lagrangian decomposition approach is presented for refinery production scheduling problems proposed by Shi et al.[16] The binary variables which describe the operation modes and transitions in the model are redefined to get rid of the linearization, and the size of the primal model can be reduced. The whole problem is decomposed into several production processing subproblems and one blending and delivery subproblem. The mass balance constraints for intermediate oils are dualized. Some auxiliary constraints are added in the subproblems to make the solutions close to the feasible solutions of the primal problem. Considering that the Lagrange multipliers can be interpreted in an economic sense, they are initialized by the average production cost. A hybrid method to update the Lagrange multipliers is adopted to accelerate the convergence of Lagrange multipliers. To obtain feasible solutions, a feasibility scheme is presented. The computational results show that the Lagrangian decomposition algorithm is effective and efficient. The advantage of the proposed method is especially apparent for large-scale problems.

This paper is organized as follows. In section 2, the discrete-time scheduling model is reformulated by redefining the binary variables which describe the operation modes and transitions. The auxiliary variables and constraints are no longer needed, and the size of the primal model can be reduced. In section 3, the Lagrangian decomposition scheme is proposed. The methods for decomposing the problem, initializing and updating the Lagrange multipliers, solving the subproblems, and obtaining the feasible solutions are discussed. Three cases are solved and discussed in section 4. Finally, conclusions and future work are presented in section 5.

2. DISCRETE-TIME SCHEDULING MODEL REFORMULATION

The overall system of a typical refinery is depicted in Figure 1.[16] It can be decomposed into three stages: crude oil feeding, product processing, and blending and delivery.

The crude oil is fed from crude oil tanks. The amount of crude oil is sufficient. The production units include the atmospheric distillation unit (ATM), the vacuum distillation unit (VDU), the fluid catalytic cracking unit (FCCU), the hydrogen desulfurization unit (HDS), the etherification unit (ETH), the catalytic reformer unit (RF), the methyl *tert*-butyl ether unit (MTBE unit), and the hydrorefining units (HTU1 and HTU2). The operation modes for production units are shown in Table 8 of Appendix A. The yields and operation costs of processing units are shown in Tables 9−13 of Appendix B. There are eight types of product oil in the blending and delivery stage, including five types of gasoline and three types of diesel. The product oil is stored in the oil tanks. The objective function is to minimize the total cost of production and penalties.

The primal model from Shi et al.[16] is named as P_0. The detail of the model P_0 are as follows.

2.1. Primal Model P_0.

The constraints of model P_0 can be classified into three sets. The first set contains constraints for transitions between different operation modes. The second set describes the production constraints involving mass balance, capacity, blending, and delivery. The third set contains the auxiliary constraints for the linearization.

2.1.1. Transition Constraints.

$$\sum_{m} y_{u,m,t} = 1, \quad \forall\, u \in U,\, t \in T \tag{1}$$

$$x_{u,m,m',t} \le y_{u,m',t}, \quad \forall\, u \in U,\, t \ge 2,\, m \in M_u,\, m' \in M_u \tag{2}$$

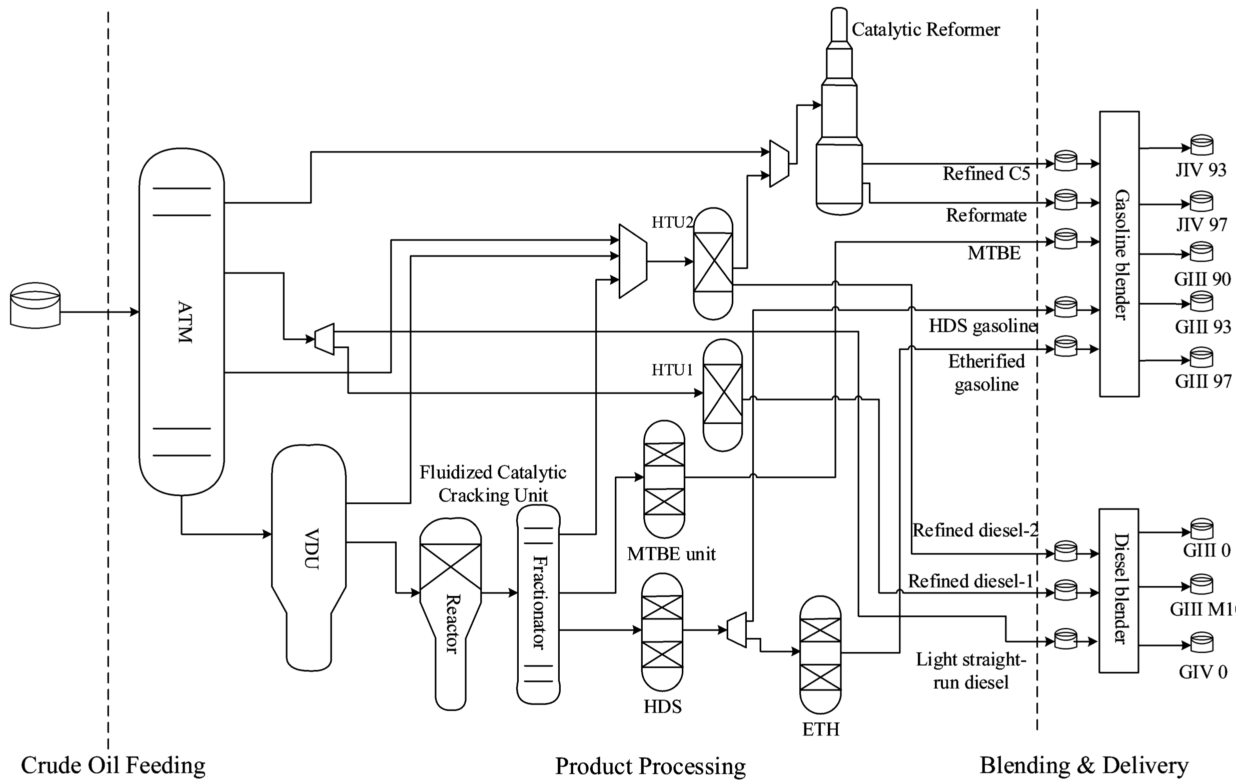

Figure 1. Simplified flow sheet of refinery system.

$$x_{u,m,m',t} \leq y_{u,m,t-\mathrm{TT}_{u,m,m'}}, \quad \forall\ u \in U,\ t > \mathrm{TT}_{u,m,m'},\ m \in M_u,\ m' \in M_u \tag{3}$$

$$x_{u,m,m',t} \leq y_{u,m,1}, \quad \forall\ u \in U,\ 2 \leq t \leq \mathrm{TT}_{u,m,m'},\ m \in M_u,\ m' \in M_u \tag{4}$$

$$x_{u,m,m',1} = 0, \quad \forall\ u \in U,\ m \in M_u,\ m' \in M_u \tag{5}$$

$$x_{u,m,m,t} = 0, \quad \forall\ u \in U,\ m \in M_u,\ t \in T \tag{6}$$

$$\sum_m \sum_{m'} x_{u,m,m',T_{\max}} = 0, \quad \forall\ u \in U,\ m \in M_u,\ m' \in M_u \tag{7}$$

$$\mathrm{TT}_{u,m',m}(y_{u,m,t-1} + y_{u,m',t} - 1) \leq \sum_{t'=t}^{t+\mathrm{TT}_{u,m',m}-1} x_{u,m,m',t'},$$
$$\forall\ u \in U,\ m \in M_u,\ m' \in M_u,\ t \geq 2 \tag{8}$$

$$y_{\mathrm{FCCU},m,t} = y_{\mathrm{HDS},m,t}, \quad m \in M_{\mathrm{FCCU}}, \quad M_{\mathrm{FCCU}} = M_{\mathrm{HDS}} \tag{9}$$

$$y_{\mathrm{HDS},m,t} = y_{\mathrm{ETH},m,t}, \quad m \in M_{\mathrm{HDS}}, \quad M_{\mathrm{HDS}} = M_{\mathrm{ETH}} \tag{10}$$

2.1.2. Production Constraints.

$$\mathrm{QO}_{u,\mathrm{oi},t} = \sum_m \sum_{m'} \mathrm{xQI}_{u,m,m',t}\mathrm{tYield}_{u,\mathrm{oi},m,m'} + \sum_{m'} \mathrm{xyQI}_{u,m',t}\mathrm{Yield}_{u,\mathrm{oi},m'}, \quad \forall\ u \in U,\ \mathrm{oi} \in \mathrm{OI},\ t \in T \tag{11}$$

$$\sum_{u_1} \mathrm{QO}_{u_1,\mathrm{om},t} = \sum_{u_2} \mathrm{QI}_{u_2,t},$$
$$\forall\ \mathrm{om} \in \mathrm{OM},\ t \in T,\ u_1 \in U_{\mathrm{om}}^{\mathrm{out}},\ u_2 \in U_{\mathrm{om}}^{\mathrm{in}} \tag{12}$$

$$\sum_u \mathrm{QO}_{u,\mathrm{oc},t} = \mathrm{QI}_{\mathrm{oc},t}, \quad \forall\ \mathrm{oc} \in \mathrm{OC},\ t \in T,\ u \in U_{\mathrm{oc}}^{\mathrm{out}} \tag{13}$$

$$\mathrm{INV}_{\mathrm{oc},1} = \mathrm{INV}_{\mathrm{oc,ini}} + \mathrm{QI}_{\mathrm{oc},1} - \mathrm{QO}_{\mathrm{oc},1}, \quad \forall\ \mathrm{oc} \in \mathrm{OC} \tag{14}$$

$$\mathrm{INV}_{o,1} = \mathrm{INV}_{o,\mathrm{ini}} + \mathrm{QI}_{o,1} - \sum_l D_{l,o,1}, \quad \forall\ o \in O \tag{15}$$

$$\mathrm{INV}_{\mathrm{oc},t} = \mathrm{INV}_{\mathrm{oc},t-1} + \mathrm{QI}_{\mathrm{oc},t} - \mathrm{QO}_{\mathrm{oc},t},$$
$$\forall\ \mathrm{oc} \in \mathrm{OC},\ t \geq 2 \tag{16}$$

$$\mathrm{INV}_{o,t} = \mathrm{INV}_{o,t-1} + \mathrm{QI}_{o,t} - \sum_l D_{l,o,t}, \quad \forall\ o \in O,\ t \geq 2 \tag{17}$$

$$\sum_{\mathrm{oc}} Q_{\mathrm{oc},o,t} = \mathrm{QI}_{o,t}, \quad \forall\ o \in O,\ t \in T \tag{18}$$

$$\sum_o Q_{\mathrm{oc},o,t} = \mathrm{QO}_{\mathrm{oc},t}, \quad \forall\ \mathrm{oc} \in \mathrm{OC},\ t \in T \tag{19}$$

$$\mathrm{QI}_u^{\min} \leq \mathrm{QI}_{u,t} \leq \mathrm{QI}_u^{\max}, \quad \forall\ u \in U,\ t \in T \tag{20}$$

$$\mathrm{INV}_{\mathrm{oc}}^{\min} \leq \mathrm{INV}_{\mathrm{oc},t} \leq \mathrm{INV}_{\mathrm{oc}}^{\max}, \quad \forall\ \mathrm{oc} \in \mathrm{OC},\ t \in T \tag{21}$$

DOI: 10.1021/acs.iecr.5b00290
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$$\mathrm{INV}_o^{\min} \le \mathrm{INV}_{o,t} \le \mathrm{INV}_o^{\max}, \quad \forall\, o \in O, t \in T \tag{22}$$

$$r_{\mathrm{oc},o}^{\min} \sum_{\mathrm{oc}'} Q_{\mathrm{oc}',o,t} \le Q_{\mathrm{oc},o,t} \le r_{\mathrm{oc},o}^{\max} \sum_{\mathrm{oc}'} Q_{\mathrm{oc}',o,t}, \quad \forall\, \mathrm{oc} \in \mathrm{OC}, o \in O, t \in T \tag{23}$$

$$\mathrm{PRO}_{o,p}^{\min} \sum_{\mathrm{oc}} Q_{\mathrm{oc},o,t} \le \sum_{\mathrm{oc}} \mathrm{PRO}_{\mathrm{oc},p} Q_{\mathrm{oc},o,t} \le \mathrm{PRO}_{o,p}^{\max} \sum_{\mathrm{oc}} Q_{\mathrm{oc},o,t}, \quad \forall\, o \in O, p \in P, t \in T \tag{24}$$

$$D_{l,o,t} \ge 0, \quad \forall\, l \in L, o \in O, t \in T \tag{25}$$

$$\sum_{t=1}^{T_{l1}-1} D_{l,o,t} = 0, \quad \forall\, l \in L, o \in O \tag{26}$$

$$\sum_{t=T_{l2}+1}^{T_{\max}} D_{l,o,t} = 0, \quad \forall\, l \in L, o \in O \tag{27}$$

$$\sum_t D_{l,o,t} \le R_{l,o}, \quad \forall\, l \in L, o \in O \tag{28}$$

2.1.3. Auxiliary Constraints.

$$\mathrm{xQI}_{u,m,m',t} + \mathrm{xQI1}_{u,m,m',t} = \mathrm{QI}_{u,t}, \quad \forall\, u \in U, m \in M_u, m' \in M_u, t \in T \tag{29}$$

$$\mathrm{xQI}_{u,m,m',t} \le x_{u,m,m',t} \mathrm{QI}_{u,t}^{\max}, \quad \forall\, u \in U, m \in M_u, m' \in M_u, t \in T \tag{30}$$

$$\mathrm{xQI1}_{u,m,m',t} \le (1 - x_{u,m,m',t}) \mathrm{QI}_{u,t}^{\max}, \quad \forall\, u \in U, m \in M_u, m' \in M_u, t \in T \tag{31}$$

$$\mathrm{xQI}_{u,m,m',t} \ge 0, \quad \forall\, u \in U, m \in M_u, m' \in M_u, t \in T \tag{32}$$

$$\mathrm{xQI1}_{u,m,m',t} \ge 0, \quad \forall\, u \in U, m \in M_u, m' \in M_u, t \in T \tag{33}$$

$$\mathrm{xy}_{u,m',t} \le y_{u,m',t}, \quad \forall\, u \in U, m' \in M_u, t \in T \tag{34}$$

$$\mathrm{xy}_{u,m',t} \le 1 - \sum_m x_{u,m,m',t}, \quad \forall\, u \in U, m' \in M_u, t \in T \tag{35}$$

$$\mathrm{xy}_{u,m',t} \ge y_{u,m',t} + \left(1 - \sum_m x_{u,m,m',t}\right) - 1, \quad \forall\, u \in U, m' \in M_u, t \in T \tag{36}$$

$$\mathrm{xy}_{u,m',t} \ge 0, \quad \forall\, u \in U, m' \in M_u, t \in T \tag{37}$$

$$\mathrm{xyQI}_{u,m',t} + \mathrm{xyQI1}_{u,m',t} = \mathrm{QI}_{u,t}, \quad \forall\, u \in U, m' \in M_u, t \in T \tag{38}$$

$$\mathrm{xyQI}_{u,m',t} \le \mathrm{xy}_{u,m',t} \mathrm{QI}_{\mathrm{u,t}}^{\max}, \quad \forall\, u \in U, m' \in M_u, t \in T \tag{39}$$

$$\mathrm{xyQI1}_{u,m',t} \le (1 - \mathrm{xy}_{u,m',t}) \mathrm{QI}_{\mathrm{u,t}}^{\max}, \quad \forall\, u \in U, m' \in M_u, t \in T \tag{40}$$

$$\mathrm{xyQI}_{u,m',t} \ge 0, \quad \forall\, u \in U, m' \in M_u, t \in T \tag{41}$$

$$\mathrm{xyQI1}_{u,m',t} \ge 0, \quad \forall\, u \in U, m' \in M_u, t \in T \tag{42}$$

2.1.4. Objective Function. The objective function of the scheduling problem is to minimize the cost of production and material storage and penalties for stockout.

$$\begin{aligned} \min f_0 = \min \sum_T \Big(& \mathrm{QI}_{\mathrm{ATM},t} \mathrm{OPC} \\ & + \sum_u \sum_m \sum_{m'} \mathrm{xQI}_{u,m,m',t} \mathrm{tOpCost}_{u,m,m'} \\ & + \sum_u \sum_{m'} \mathrm{xyQI}_{u,m',t} \mathrm{OpCost}_{u,m'} \Big) \\ & + \sum_t \alpha \left(\sum_o \mathrm{INV}_{o,t} + \sum_{\mathrm{oc}} \mathrm{INV}_{\mathrm{oc},t} \right) \\ & + \sum_l \sum_0 \beta_l \left(R_{l,o} - \sum_t D_{l,o,t} \right) \end{aligned}$$

The mixed-integer linear programming model is as follows:

P_0:

$$\min f_0$$

subject to (s.t.) constraints 1−42.

In the model P_0, if $y_{u,m,t}$ is equal to 1, unit u is in the transition of mode switching from the previous operation mode to operation mode m or the steady state of operation mode m. The decision variables $x_{u,m,m',t}$ denote whether unit u is in the transition from operation mode m to m' during time interval t. Based on these definitions, some bilinear and trilinear terms are introduced in the objective function and constraints. To linearize these terms, additional binary variables $\mathrm{xy}_{u,m',t}$, additional continuous variables $\mathrm{xQI1}_{u,m,m',t}$, $\mathrm{xQI1}_{u,m,m',t}$, $\mathrm{xyQI}_{u,m',t}$, and $\mathrm{xyQI1}_{u,m',t}$, and auxiliary constraints 28−41 are added to the model P_0.

2.2. Reformulated Model *P*. In this section, the binary variables which describe the operation modes and transitions in the model are redefined to get rid of the auxiliary variables and constraints. The size of the scheduling model can be reduced.

$z_{u,m,t}$ is used to represent whether unit u is in the steady state of operation mode m during time interval t, and $x_{u,m,m',t}$ is used to represent whether unit u is in the transition from operation mode m to m' during time interval t. If $z_{u,m,t}$ is equal to 1, it means that unit u is in the steady state of operation mode m during time interval t. If $x_{u,m,m',t}$ is equal to 1, it means that unit u is in the transition from m to m' during time interval t. Figure 2 is an illustration of the operation mode expression.

The length of each transition in the figure is two time intervals. The values of $z_{u,m,t}$ and $x_{u,m,m',t}$ are as follows.

$$z_{u,\mathrm{A},t} = \begin{cases} 1, & t = 9, 10 \\ 0, & \text{otherwise} \end{cases} \qquad z_{u,\mathrm{B},t} = \begin{cases} 1, & t = 1, 2 \\ 0, & \text{otherwise} \end{cases}$$

$$z_{u,\mathrm{C},t} = \begin{cases} 1, & t = 5, 6 \\ 0, & \text{otherwise} \end{cases}$$

DOI: 10.1021/acs.iecr.5b00290
Ind. Eng. Chem. Res. 2015, 54, 6508−6526

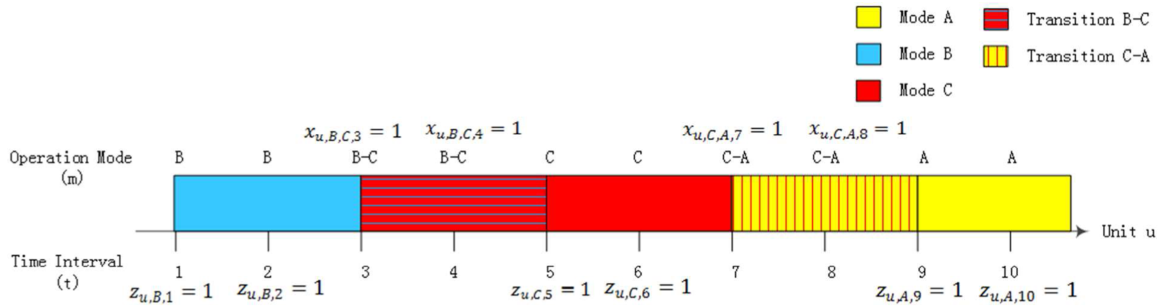

Figure 2. Illustration of the operation mode expression in the reformulated model P.

$$x_{u,\mathrm{B},\mathrm{C},t} = \begin{cases} 1, & t = 3, 4 \\ 0, & \text{otherwise} \end{cases} \qquad x_{u,\mathrm{C},\mathrm{A},t} = \begin{cases} 1, & t = 7, 8 \\ 0, & \text{otherwise} \end{cases}$$

$$x_{u,\mathrm{B},\mathrm{C},3} = 1$$

Furthermore, $\mathrm{sQI}_{u,m,t}$ and $\mathrm{tQI}_{u,m,m',t}$ are used to represent the input amount of unit u during time interval t in the steady state of operation mode m or the transition from operation mode m to m', respectively. The additional binary variables, additional continuous variables, and auxiliary constraints are no longer needed.

The transition constraints 1–4 and 8–10, the production constraints 11 and 20, and the objective function are modified as follows.

2.2.1. Transition Constraints. At each time interval, each unit can only be in one type of steady state or transition. Therefore, constraint 1 in model P_0 can be modified as follows.

$$\sum_{m,m'} x_{u,m,m',t} + \sum_{m} z_{u,m,t} = 1, \quad \forall\, u \in U, t \in T \tag{43}$$

The total duration of transitions for each unit has an upper bound.

$$\sum_{m,m'} x_{u,m,m',t} \le N_u, \quad \forall\, u \in U, t \in T \tag{44}$$

where N_u indicates the maximum number of time intervals for unit u in transitions in the scheduling horizon. The level of N_u is determined by the actual production experience.

Each unit cannot be in different steady states at two adjacent time intervals. As the definition of transitions, if the operation mode of one unit changes, there must be a transition between two steady states.

$$z_{u,m,t} + z_{u,m',t+1} \le 1, \quad \forall\, u \in U, t < T_{\max}, m \ne m' \tag{45}$$

Constraints 2–4 in model P_0 can be modified as follows. The relationships between $x_{u,m,m',t}$ and $z_{u,m,t}$ are as follows. If $x_{u,m,m',t}$ is equal to 1, it means that unit u is in the transition from operation mode m to mode m' during time interval t. Therefore, there must have been corresponding $z_{u,m,t'}$ and $z_{u,m',t'}$ which are equal to 1. $\mathrm{TT}_{u,m,m'}$ is used to indicate the length of the transition from operation mode m to m' of unit u.

$$\sum_{t'=t-\mathrm{TT}_{u,m,m'}}^{t-1} z_{u,m,t'} \ge x_{u,m,m',t}, \quad \forall\, u \in U, t > \mathrm{TT}_{u,m,m'}, m \ne m' \tag{46}$$

$$\sum_{t' \le t-1} z_{u,m,t'} \ge x_{u,m,m',t}, \quad \forall\, u \in U, 2 \le t \le \mathrm{TT}_{u,m,m'}, m \ne m' \tag{47}$$

$$\sum_{t'=t+1}^{t+\mathrm{TT}_{u,m,m'}} z_{u,m',t'} \ge x_{u,m,m',t}, \quad \forall\, u \in U, t < T_{\max} - \mathrm{TT}_{u,m,m'}, m \ne m' \tag{48}$$

$$\sum_{t' \ge t+1} z_{u,m',t'} \ge x_{u,m,m',t}, \quad \forall\, u \in U, T_{\max} - \mathrm{TT}_{u,m,m'} \le t \le T_{\max} - 1, m \ne m' \tag{49}$$

There cannot be new mode switching before a transition ends. If both $z_{u,m,t-1}$ and $z_{u,m',t+\mathrm{TT}_{u,m,m'}}$ are equal to 1, there is a transition of unit u from operation mode m to mode m' during time intervals t to $t + \mathrm{TT}_{u,m,m'} - 1$. Constraint 8 in model P_0 can be modified as follows.

$$\mathrm{TT}_{u,m,m'}(z_{u,m,t-1} + z_{u,m',t+\mathrm{TT}_{u,m,m'}} - 1) \le \sum_{t'=t}^{t+\mathrm{TT}_{u,m,m'}-1} x_{u,m,m',t'}, \quad \forall\, u \in U, m \in M_u, m' \in M_u, t \le T_{\max} - \mathrm{TT}_{u,m,m'} \tag{50}$$

Constraints 9 and 10 in model P_0 can be modified as follows.

$$z_{\mathrm{FCCU},m,t} = z_{\mathrm{HDS},m,t}, \quad m \in M_{\mathrm{FCCU}}, \quad M_{\mathrm{FCCU}} = M_{\mathrm{HDS}} \tag{51}$$

$$z_{\mathrm{HDS},m,t} = z_{\mathrm{ETH},m,t}, \quad m \in M_{\mathrm{HDS}}, \quad M_{\mathrm{HDS}} = M_{\mathrm{ETH}} \tag{52}$$

2.2.2. Production Constraints. The variables $\mathrm{sQI}_{u,m,t}$ and $\mathrm{tQI}_{u,m,m',t}$ are used to represent the input amount of unit u during time interval t in the steady state of operation mode m or the transition from operation mode m to m' respectively.

Mass balance constraints for the outflow ports of units include the following. Constraint 11 in model P_0 can be modified as follows. The term $\sum_m \sum_{m'} \mathrm{tQI}_{u,m,m',t}\,\mathrm{tYield}_{u,oi,m,m'}$ expresses the flows in the transitions, and the term $\sum_m \mathrm{sQI}_{u,m,t}\mathrm{Yield}_{u,oi,m}$ expresses the flows in the steady states.

DOI: 10.1021/acs.iecr.5b00290
Ind. Eng. Chem. Res. 2015, 54, 6508–6526

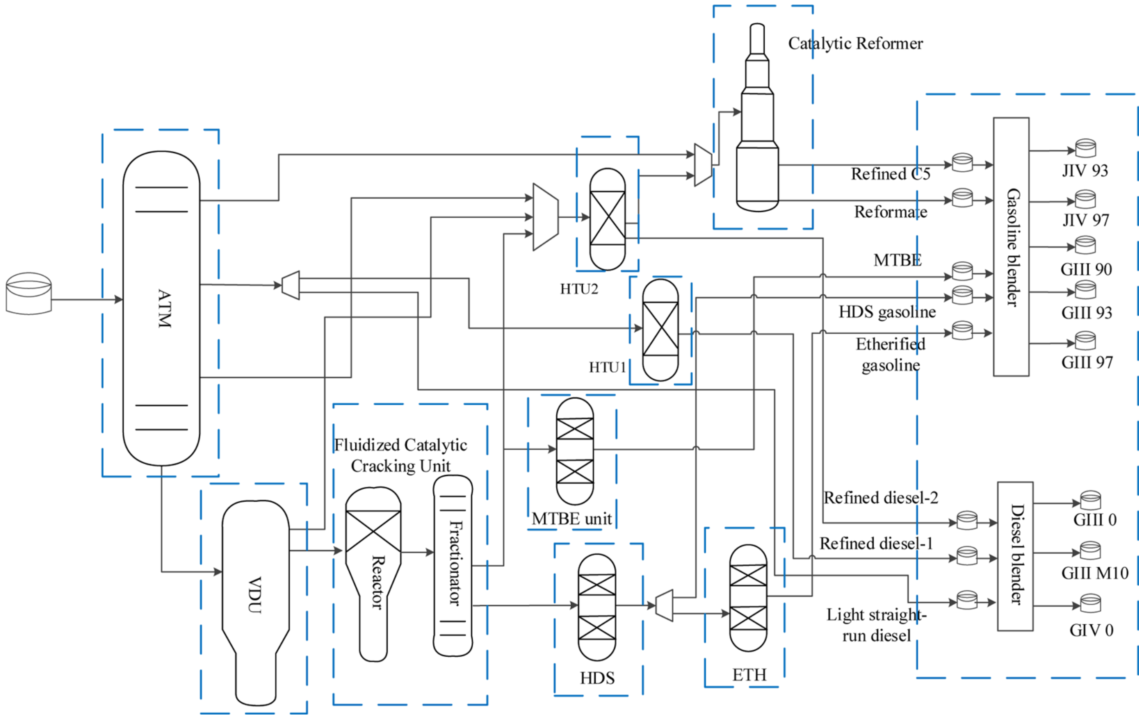

Figure 3. Partitions of the production system.

$$\mathrm{QO}_{u,\mathrm{oi},t} = \sum_{m}\sum_{m'} \mathrm{tQI}_{u,m,m',t}\mathrm{tYield}_{u,\mathrm{oi},m,m'} + \sum_{m} \mathrm{sQI}_{u,m,t}\mathrm{Yield}_{u,\mathrm{oi},m}, \quad \forall\ u \in U,\ \mathrm{oi} \in \mathrm{OI},\ t \in T \tag{53}$$

$$\mathrm{QI}_{u,t} = \sum_{m}\sum_{m'} \mathrm{tQI}_{u,m,m',t} + \sum_{m} \mathrm{sQI}_{u,m,t}, \quad \forall\ u \in U,\ t \in T \tag{54}$$

The input amounts of unit u during time interval t either in the steady states or in the transitions must lie between the lower and upper bounds. Constraint 20 in model P_0 can be modified as follows.

$$z_{u,m,t}\mathrm{QI}_u^{\min} \le \mathrm{sQI}_{u,m,t} \le y_{u,m,t}\mathrm{QI}_u^{\max}, \quad \forall\ u \in U,\ m \in M_u,\ t \in T \tag{55}$$

$$x_{u,m,m',t}\mathrm{QI}_u^{\min} \le \mathrm{tQI}_{u,m,m',t} \le x_{u,m,m',t}\mathrm{QI}_u^{\max}, \quad \forall\ u \in U,\ m \in M_u,\ m' \in M,\ t \in T \tag{56}$$

2.2.3. Objective Function. The objective function is modified as follows:

$$\min f = \min \sum_{t} \Big(\mathrm{QI}_{\mathrm{ATM},t}\mathrm{OPC} + \sum_{u}\sum_{m}\sum_{m'} \mathrm{tQI}_{u,m,m',t}\mathrm{tOpCost}_{u,m,m'} + \sum_{u}\sum_{m} \mathrm{QI}_{u,m,t}\mathrm{OpCost}_{u,m}\Big) + \sum_{t} \alpha\Big(\sum_{o} \mathrm{INV}_{o,t} + \sum_{\mathrm{oc}} \mathrm{INV}_{\mathrm{oc},t}\Big) + \sum_{l}\sum_{o} \beta_l\Big(R_{l,o} - \sum_{t} D_{l,o,t}\Big)$$

The reformulated model P is as follows.

P:

$$\min f$$

s.t. constraints 5−7, 12−19, 21−28, and 43−56.

3. LAGRANGIAN DECOMPOSITION SCHEME

3.1. Lagrangian Decomposition Steps. In the reformulated model P, the steady states and transitions of mode switching are defined for each production unit. It needs a lot of binary variables to present the operation states of each unit at each time point. As the number of scheduling time intervals increases, it usually leads to a very large-scale combinatorial problem which is difficult to solve.

The overall refinery production system in Figure 1 can be separated as three stages: crude oil feeding, production processing, and blending and delivery of product oils. As mentioned above, in this scheduling problem, the crude oil is fed from crude oil tanks and the amount is sufficient. Therefore, the subproblems are obtained from the production processing stage and the blending and delivery stage. The whole production system is decomposed into 10 parts: ATM, VDU, FCCU, HDS, ETH, MTBE, HTU1, HTU2, RF, and blending and delivery (B&D). The first nine subproblems are production processing subproblems. Each of them contains a processing unit. The tenth subproblem is about blending and delivery.

The partitions of the production system are shown in Figure 3.

To decompose the whole problem, the mass balance constraints for the intermediate oils need to be dualized. The intermediate oils contain the material oils for processing units and the component oils for blending. The material oils are named by the corresponding units. For example, the material oil iFCCU is the material oil of FCCU.

DOI: 10.1021/acs.iecr.5b00290
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In the reformulated model P, the mass balance constraints for the material oils were written as

$$\sum_{u_1} \mathrm{QO}_{u_1,\mathrm{om},t} = \sum_{u_2} \mathrm{QI}_{u_2,t}, \quad \forall\ \mathrm{om} \in \mathrm{OM},\ t \in T,\ u_1 \in U_{\mathrm{om}}^{\mathrm{out}},\ u_2 \in U_{\mathrm{om}}^{\mathrm{in}} \tag{12}$$

where OM indicates sets of the material oils.

It can be separated as follows.

$$\mathrm{QO}_{\mathrm{ATM,iVDU},t} = \mathrm{QI}_{\mathrm{VDU},t}, \quad \forall\ t \in T \tag{57}$$

$$\mathrm{QO}_{\mathrm{VDU,iFCCU},t} = \mathrm{QI}_{\mathrm{FCCU},t}, \quad \forall\ t \in T \tag{58}$$

$$\mathrm{QO}_{\mathrm{FCCU,iHDS},t} = \mathrm{QI}_{\mathrm{HDS},t}, \quad \forall\ t \in T \tag{59}$$

$$\mathrm{QO}_{\mathrm{HDS,iETH},t} = \mathrm{QI}_{\mathrm{ETH},t}, \quad \forall\ t \in T \tag{60}$$

$$\mathrm{QO}_{\mathrm{FCCU,iMTBE},t} = \mathrm{QI}_{\mathrm{MTBE},t}, \quad \forall\ t \in T \tag{61}$$

$$\mathrm{QO}_{\mathrm{ATM,iHTU1},t} = \mathrm{QI}_{\mathrm{HTU1},t}, \quad \forall\ t \in T \tag{62}$$

$$\mathrm{QO}_{\mathrm{ATM,iHTU2},t} + \mathrm{QO}_{\mathrm{VDU,iHTU2},t} + \mathrm{QO}_{\mathrm{FCCU,iHTU2},t} = \mathrm{QI}_{\mathrm{HTU2},t}, \quad \forall\ t \in T \tag{63}$$

$$\mathrm{QO}_{\mathrm{ATM,iRF},t} + \mathrm{QO}_{\mathrm{HTU2,iRF},t} = \mathrm{QI}_{\mathrm{RF},t}, \quad \forall\ t \in T \tag{64}$$

The mass balance constraints for the component oils were written as

$$\sum_{u} \mathrm{QO}_{u,\mathrm{oc},t} = \mathrm{QI}_{\mathrm{oc},t}, \quad \forall\ \mathrm{oc} \in \mathrm{OC},\ t \in T,\ u \in U_{\mathrm{oc}}^{\mathrm{out}} \tag{13}$$

where OC indicates sets of component oils.

According to the decomposition scheme above, by dualizing the mass constraints for the material oils 57–64, the mass constraints for the component oils 13, and the transition constraints 51 and 52, the reformulated model P is decomposed into 10 subproblems. In the nine production processing subproblems, the decision variables are the operation states and input amounts of units at each time point. In the blending and delivery subproblem, the decision variables are the amount and type of component oils used in blending at each time point and the amount and type of product oils being stored or delivered at each time point.

The Lagrangian function is as follows.

$$\begin{aligned} L = {} & \sum_t \Big(\mathrm{QI}_{\mathrm{ATM},t}\mathrm{OPC} + \sum_u \sum_m \sum_{m'} \mathrm{tQI}_{u,m,m',t}\mathrm{tOpCost}_{u,m,m'} \\ & + \sum_u \sum_m \mathrm{sQI}_{u,m,t}\mathrm{OpCost}_{u,m}\Big) \\ & + \sum_t \alpha\Big(\sum_o \mathrm{INV}_{o,t} + \sum_{\mathrm{oc}} \mathrm{INV}_{\mathrm{oc},t}\Big) \\ & + \sum_l \sum_o \beta_l\Big(R_{l,o} - \sum_t D_{l,o,t}\Big) \\ & + \sum_t \lambda_{\mathrm{iVDU},t}(\mathrm{QI}_{\mathrm{VDU},t} - \mathrm{QO}_{\mathrm{ATM,iVDU},t}) \\ & + \sum_t \lambda_{\mathrm{iFCCU},t}(\mathrm{QI}_{\mathrm{FCCU},t} - \mathrm{QO}_{\mathrm{VDU,iFCCU},t}) \\ & + \sum_t \lambda_{\mathrm{iHDS},t}(\mathrm{QI}_{\mathrm{HDS},t} - \mathrm{QO}_{\mathrm{FCCU,iHDS},t}) \\ & + \sum_t \lambda_{\mathrm{iETH},t}(\mathrm{QI}_{\mathrm{ETH},t} - \mathrm{QO}_{\mathrm{HDS,iETH},t}) \\ & + \sum_t \lambda_{\mathrm{iMTBE},t}(\mathrm{QI}_{\mathrm{MTBE},t} - \mathrm{QO}_{\mathrm{FCCU,iMTBE},t}) \\ & + \sum_t \lambda_{\mathrm{iHTU1},t}(\mathrm{QI}_{\mathrm{HTU1},t} - \mathrm{QO}_{\mathrm{ATM,iHTU1},t}) \\ & + \sum_t \lambda_{\mathrm{iHTU2},t}(\mathrm{QI}_{\mathrm{HTU2},t} - \mathrm{QO}_{\mathrm{ATM,iHTU2},t} - \mathrm{QO}_{\mathrm{VDU,iHTU2},t} \\ & - \mathrm{QO}_{\mathrm{FCCU,iHTU2},t}) + \sum_t \lambda_{\mathrm{iRF},t}(\mathrm{QI}_{\mathrm{RF},t} - \mathrm{QO}_{\mathrm{ATM,iRF},t} \\ & - \mathrm{QO}_{\mathrm{HTU2,iRF},t}) + \sum_t \lambda_{\mathrm{oc},t}\Big(\mathrm{QI}_{\mathrm{oc},t} - \sum_u \mathrm{QO}_{u,\mathrm{oc},t}\Big) \\ & + \sum_{m,t} \lambda_{\mathrm{FH},m,t}(z_{\mathrm{HDS},m,t} - z_{\mathrm{FCCU},m,t}) \\ & + \sum_{m,t} \lambda_{\mathrm{HE},m,t}(z_{\mathrm{ETH},m,t} - z_{\mathrm{HDS},m,t}) \end{aligned} \tag{65}$$

It can be separated as

$$\begin{aligned} L_{\mathrm{ATM}} = {} & \sum_t \Big(\mathrm{QI}_{\mathrm{ATM},t}\mathrm{OPC} \\ & + \sum_m \sum_{m'} \mathrm{tQI}_{\mathrm{ATM},m,m',t}\mathrm{tOpCost}_{\mathrm{ATM},m,m'} \\ & + \sum_m \mathrm{sQI}_{\mathrm{ATM},m,t}\mathrm{OpCost}_{\mathrm{ATM},m}\Big) \\ & - \sum_t \lambda_{\mathrm{iVDU},t}\mathrm{QO}_{\mathrm{ATM,iVDU},t} - \sum_t \lambda_{\mathrm{iHTU1},t}\mathrm{QO}_{\mathrm{ATM,iHTU1},t} \\ & - \sum_t \lambda_{\mathrm{iHTU2},t}\mathrm{QO}_{\mathrm{ATM,iHTU2},t} - \sum_t \lambda_{\mathrm{iRF},t}\mathrm{QO}_{\mathrm{ATM,iRF},t} \\ & - \sum_{\mathrm{oc},t} \lambda_{\mathrm{oc},t}\mathrm{QO}_{\mathrm{ATM,oc},t} \end{aligned} \tag{66}$$

$$\begin{aligned} L_{\mathrm{VDU}} = {} & \sum_t \Big(\sum_m \sum_{m'} \mathrm{tQI}_{\mathrm{VDU},m,m',t}\mathrm{tOpCost}_{\mathrm{VDU},m,m'} \\ & + \sum_m \mathrm{sQI}_{\mathrm{VDU},m,t}\mathrm{OpCost}_{\mathrm{VDU},m}\Big) + \sum_t \lambda_{\mathrm{iVDU},t}\mathrm{QI}_{\mathrm{VDU},t} \\ & - \sum_t \lambda_{\mathrm{iFCCU},t}\mathrm{QO}_{\mathrm{VDU,iFCCU},t} \\ & - \sum_t \lambda_{\mathrm{iHTU2},t}\mathrm{QO}_{\mathrm{VDU,iHTU2},t} \end{aligned} \tag{67}$$

DOI: 10.1021/acs.iecr.5b00290
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$$L_{\mathrm{FCCU}} = \sum_{t}\Big(\sum_{m}\sum_{m'} \mathrm{tQI}_{\mathrm{FCCU},m,m',t}\mathrm{tOpCost}_{\mathrm{FCCU},m,m'} + \sum_{m} \mathrm{sQI}_{\mathrm{FCCU},m,t}\mathrm{OpCost}_{\mathrm{FCCU},m}\Big) + \sum_{t} \lambda_{\mathrm{iFCCU},t} Q\,\mathrm{QI}_{\mathrm{FCCU},t} - \sum_{t} \lambda_{\mathrm{iHDS},t}\mathrm{QO}_{\mathrm{FCCU,iHDS},t} - \sum_{t} \lambda_{\mathrm{iMTBE},t}\mathrm{QO}_{\mathrm{FCCU,iMTBE},t} - \sum_{t} \lambda_{\mathrm{iHTU2},t}\mathrm{QO}_{\mathrm{FCCU,iHTU2},t} - \sum_{m,t} \lambda_{\mathrm{FH},m,t} z_{\mathrm{FCCU},m,t} \tag{68}$$

$$L_{\mathrm{HDS}} = \sum_{t}\Big(\sum_{m}\sum_{m'} \mathrm{tQI}_{\mathrm{HDS},m,m',t}\mathrm{tOpcost}_{\mathrm{HDS},m,m'} + \sum_{m} \mathrm{QI}_{\mathrm{HDS},m,t}\mathrm{OpCost}_{\mathrm{HDS},m}\Big) + \sum_{t} \lambda_{\mathrm{iHDS},t}\mathrm{QI}_{\mathrm{HDS},t} - \sum_{t} \lambda_{\mathrm{iETH},t}\mathrm{QO}_{\mathrm{HDS,iETH},t} - \sum_{\mathrm{oc},t} \lambda_{\mathrm{oc},t}\mathrm{QO}_{\mathrm{HDS,oc},t} + \sum_{m,t} \lambda_{\mathrm{FH},m,t} z_{\mathrm{HDS},m,t} - \sum_{m,t} \lambda_{\mathrm{HE},m,t} z_{\mathrm{HDS},m,t} \tag{69}$$

$$L_{\mathrm{ETH}} = \sum_{t}\Big(\sum_{m}\sum_{m'} \mathrm{tQI}_{\mathrm{ETH},m,m',t}\mathrm{tOpCost}_{\mathrm{ETH},m,m'} + \sum_{m} \mathrm{QI}_{\mathrm{ETH},m,t}\mathrm{OpCost}_{\mathrm{ETH},m}\Big) + \sum_{t} \lambda_{\mathrm{iETH},t}\mathrm{QI}_{\mathrm{ETH},t} - \sum_{\mathrm{oc},t} \lambda_{\mathrm{oc},t}\mathrm{QO}_{\mathrm{ETH,oc},t} + \sum_{m,t} \lambda_{\mathrm{HE},m,t} z_{\mathrm{ETH},m,t} \tag{70}$$

$$L_{\mathrm{MTBE}} = \sum_{t} \mathrm{QI}_{\mathrm{MTBE},t}\mathrm{OpCost}_{\mathrm{MTBE}} + \sum_{t} \lambda_{\mathrm{iMTBE},t}\mathrm{QI}_{\mathrm{MTBE},t} - \sum_{\mathrm{oc},t} \lambda_{\mathrm{oc},t}\mathrm{QO}_{\mathrm{MTBE,oc},t} \tag{71}$$

$$L_{\mathrm{HTU1}} = \sum_{t}\Big(\sum_{m}\sum_{m'} \mathrm{tQI}_{\mathrm{HTU1},m,m',t}\mathrm{tOpCost}_{\mathrm{HTU1},m,m'} + \sum_{m} \mathrm{QI}_{\mathrm{HTU1},m,t}\mathrm{OpCost}_{\mathrm{HTU1},m}\Big) + \sum_{t} \lambda_{\mathrm{iHTU1},t}\mathrm{QI}_{\mathrm{HTU1},t} - \sum_{\mathrm{oc},t} \lambda_{\mathrm{oc},t}\mathrm{QO}_{\mathrm{HTU1,oc},t} \tag{72}$$

$$L_{\mathrm{HTU2}} = \sum_{t}\Big(\sum_{m}\sum_{m'} \mathrm{tQI}_{\mathrm{HTU2},m,m',t}\mathrm{tOpCost}_{\mathrm{HTU2},m,m'} + \sum_{m} \mathrm{QI}_{\mathrm{HTU2},m,t}\mathrm{OpCost}_{\mathrm{HTU2},m}\Big) + \sum_{t} \lambda_{\mathrm{iHTU2},t}\mathrm{QI}_{\mathrm{HTU2},t} - \sum_{t} \lambda_{\mathrm{iRF},t}\mathrm{QO}_{\mathrm{HTU2,iRF},t} - \sum_{\mathrm{oc},t} \lambda_{\mathrm{oc},t}\mathrm{QO}_{\mathrm{HTU2,oc},t} \tag{73}$$

$$L_{\mathrm{RF}} = \sum_{t} \mathrm{QI}_{\mathrm{RF},t}\mathrm{OpCost}_{\mathrm{RF}} + \sum_{t} \lambda_{\mathrm{iRF},t}\mathrm{QI}_{\mathrm{RF},t} - \sum_{\mathrm{oc},t} \lambda_{\mathrm{oc},t}\mathrm{QO}_{\mathrm{RF,oc},t} \tag{74}$$

$$L_{\mathrm{B\&D}} = \sum_{t} \alpha\Big(\sum_{o} \mathrm{INV}_{o,t} + \sum_{\mathrm{oc}} \mathrm{INV}_{\mathrm{oc},t}\Big) + \sum_{l}\sum_{o} \beta_l\Big(R_{l,o} - \sum_{t} D_{l,o,t}\Big) + \sum_{\mathrm{oc},t} \lambda_{\mathrm{oc},t}\mathrm{QI}_{\mathrm{oc},t} \tag{75}$$

The relaxed Lagrangian problem is denoted as $P_{\mathrm{R}}(\lambda)$. The production processing subproblems are denoted as P_{ATM}, P_{VDU}, P_{FCCU}, P_{HDS}, P_{ETH}, P_{MTBE}, P_{HTU1}, P_{HTU2} and P_{RF}. The blending and delivery subproblem is denoted as $P_{\mathrm{B\&D}}$. The details are as follows.

$P_{\mathrm{R}}(\lambda)$: min L
s.t. constraints 5−7, 14−19, 21−28, 43−50, and 53−56.
P_{ATM}: min L_{ATM}
s.t. constraints 5−7, 43−50, and 53−56; u = ATM.
P_{VDU}: min L_{VDU}s.t. constraints 5−7, 43−50, and 53−56; u = VDU.
s.t. constraints 5−7, 43−50, and 53−56; u = FCCU.
P_{HDS}: min L_{HDS}
s.t. constraints 5−7, 43−50, and 53−56; u = HDS.
P_{ETH}: min L_{ETH}
s.t. constraints 5−7, 43−50, and 53−56; u = ETH.
P_{MTBE}: min L_{MTBE}
s.t. constraints 53−56; u = MTBE.
P_{HTU1}: min L_{HTU1}
s.t. constraints 5−7, 43−50, and 53−56; u = HTU1.
P_{HTU2}: min L_{HTU2}
s.t. constraints 5−7, 43−50, and 53−56; u = HTU2.
P_{RF}: min L_{RF}
s.t. constraints 53−56; u = RF.
$P_{\mathrm{B\&D}}$: min $L_{\mathrm{B\&D}}$
s.t. constraints 14−19 and 21−28.
The dual problem is denoted as P_{D}.
P_{D}: $\min_{\lambda}(P_{\mathrm{R}}(\lambda))$

The scheme of the Lagrangian decomposition algorithm is shown in Figure 4. The main challenges in the scheme are the initialization and update of Lagrange multipliers, the solving method of subproblems, and the feasibility scheme of the primal problem.

3.2. Initialization of Multipliers. In this paper, the mass balance constraints of the intermediate oils are relaxed and the whole scheduling problem is decomposed into 10 subproblems. The intermediate oils can be seen as the products for the production stage and the raw material for the consume stage. The Lagrange multipliers can be seen as prices of the oils.

For a given type of component oil, if the consuming amount in the blending and delivery part is larger than the production amount, that is, $\sum_u \mathrm{QO}_{u,\mathrm{oc},t} < \sum_u \mathrm{QI}_{u,\mathrm{oc},t}$, the corresponding value of the Lagrangian multiplier $\lambda_{\mathrm{oc},t}$ will increase in the updating step to increase the incentive to produce this type of component oil and decrease the incentive to select this component oil for blending. On the other hand, if $\sum_u \mathrm{QO}_{u,\mathrm{oc},t} > \sum_u \mathrm{QI}_{u,\mathrm{oc},t}$, the corresponding value of the Lagrangian multiplier $\lambda_{\mathrm{oc},t}$ will decrease in the updating step to decrease the incentive to produce this type of component oil and increase the

DOI: 10.1021/acs.iecr.5b00290
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incentive to select this component oil for blending. At proper prices, the consuming amount $\sum_u QO_{u,oc,t}$ and the production amount $\sum_u QO_{u,oc,t}$ will reach a balance. The Lagrange multipliers can be estimated by calculating the production costs of component oils.

$$\text{outCost}_u = (\text{inCost}_u + \text{OpCost}_u)/\text{Yield}_u, \quad \forall\ u \in U \tag{76}$$

where inCost_u indicates the estimated cost of input material of unit u and outCost_u indicates the estimated price of output product of unit u.

For a given unit u, the production cost of its output oil can be calculated as the sum of the production cost of its input oil and the operation cost divided by the yield of the output oil. If unit u has several operation modes, the operation cost and the yield may be different in different operation modes. OpCost_u and Yield_u in constraint 76 are calculated as the average of the operation costs and the yields of different modes respectively to estimate the price of output product of unit u. The estimated prices are taken as the initial Lagrange multipliers.

Figures 5 and 6 show two toy examples to illustrate the calculation. In Figure 5, the three units are connected in series. Each unit has two operation modes. From Figure 5 we can see that the input material of unit B is the output of unit A and the input material of unit C is the output of unit B. The output amount of each unit depends on its input amount and current operation mode.

According to constraint 17, the input material of unit B can be calculated by

$$\text{inCost}_B = \text{outCost}_A = (\text{inCost}_A + \text{OpCost}_A)/\text{Yield}_A$$

where OpCost_A and Yield_A are estimated in the average sense by the two operation modes of unit A. The input material of unit C can be calculated in the same way by

$$\text{inCost}_C = \text{outCost}_B = (\text{inCost}_B + \text{OpCost}_B)/\text{Yield}_B$$

From Figure 6 we can see that the input material of unit C consists of the output flows from unit A and unit B. The percentage of the output flow from unit A is p_A. The output amount of each unit depends on its input amount and current operation mode. The proportion of the output flows from unit A and unit B has a range by the production of upstream units. An average proportion is taken to estimate the cost of input material of unit C.

According to constraint 17, the input material of unit C can be calculated by

$$\begin{aligned}\text{inCost}_C &= \text{outCost}_A p_A + \text{outCost}_B(1 - p_A) \\ &= [(\text{inCost}_A + \text{OpCost}_A)/\text{Yield}_A]p_A \\ &\quad + [(\text{inCost}_B + \text{OpCost}_B)/\text{Yield}_B](1 - p_A)\end{aligned}$$

3.3. Update of Multipliers.

Some classical approaches to solve the dual problem have been proposed, such as the subgradient method,[24,25] the cutting plane method,[26,27] and the box-step method.[28] Mouret et al.[9] proposed a hybrid method which was based on the three concepts to update the Lagrange multipliers. In this paper, a new hybrid method which added exponentially weighted subgradients[30] is used in the updating to get faster convergence. At iteration IT + 1, the Lagrange multipliers are updating to the solutions of the following problem $\hat{P}_D^{IT+1}$.

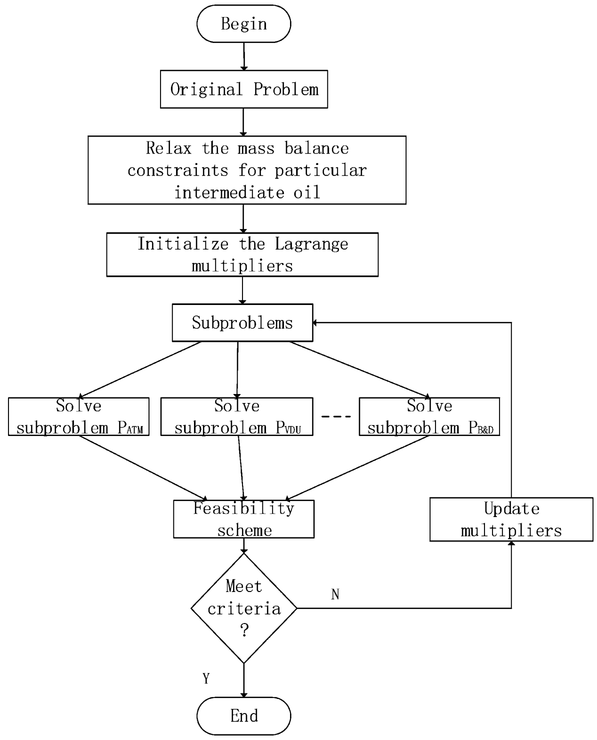

Figure 4. Steps of the Lagrangian decomposition algorithm.

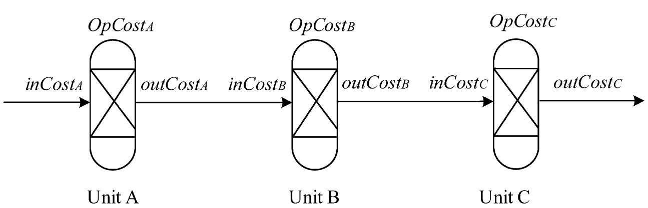

Figure 5. Simplified structure of toy example I.

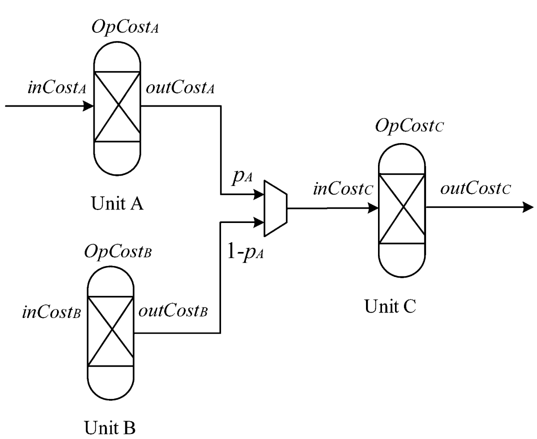

Figure 6. Simplified structure of toy example II.

DOI: 10.1021/acs.iecr.5b00290
Ind. Eng. Chem. Res. 2015, 54, 6508−6526

$\hat{P}_{D}^{IT+1}$:

min η

s.t.

$$\eta \leq \sum_{t} (QI_{ATM,t}^{it}OPC + \sum_{u}\sum_{m}\sum_{m'} tQI_{u,m,m',t}^{it}tOpCost_{u,m,m'} + \sum_{u}\sum_{m} sQI_{u,m,t}^{it}OpCost_{u,m}) + \sum_{t}\alpha(\sum_{o} INV_{o,t}^{it} + \sum_{oc} INV_{oc,t}^{it}) + \sum_{l}\sum_{o}\beta_{l}(R_{l,o} - \sum_{t} D_{l,o,t}^{it}) + \sum_{t}\lambda_{iVDU,t}(QI_{VDU,t}^{it} - QO_{ATM,iVDU,t}^{it}) + \sum_{t}\lambda_{iFCCU,t}(QI_{FCCU,t}^{it} - QO_{VDU,iFCCU,t}^{it}) + \sum_{t}\lambda_{iHDS,t}(QI_{HDS,t}^{it} - QO_{FCCU,iHDS,t}^{it}) + \sum_{t}\lambda_{iETH,t}(QI_{ETH,t}^{it} - QO_{HDS,iETH,t}^{it}) + \sum_{t}\lambda_{iMTBE,t}(QI_{MTBE,t}^{it} - QO_{FCCU,iMTBE,t}^{it}) + \sum_{t}\lambda_{iHTU1,t}(QI_{HTU1,t}^{it} - QO_{ATM,iHTU1,t}^{it}) + it\sum_{t}\lambda_{iHTU2,t}(QI_{HTU2,t}^{it} - QO_{ATM,iHTU2,t}^{it} - QO_{VDU,iHTU2,t}^{it} - QO_{FCCU,iHTU2,t}^{it}) + \sum_{t}\lambda_{iRF,t}(QI_{RF,t}^{it} - QO_{ATM,iRF,t}^{it} - QO_{HTU2,iRF,t}^{it}) + \sum_{t}\lambda_{oc,t}(QI_{oc,t}^{it} - \sum_{u} QO_{u,oc,t}^{it}) + \sum_{m,t}\lambda_{FH,m,t}(z_{HDS,m,t}^{it} - z_{FCCU,m,t}^{it}) + \sum_{m,t}\lambda_{HE,m,t}(Z_{ETH,m,t}^{it} - Z_{HDS,m,t}^{it}), \quad it = 1, ..., IT \tag{77}$$

$$\lambda_{iVDU,t} = \lambda_{iVDU,t}^{IT} + \mu(f^{up} - f^{low})\frac{g^{IT}}{||g^{IT}||^2} + \delta \tag{78}$$

$$\lambda_{iFCCU,t} = \lambda_{iFCCU,t}^{IT} + \mu(f^{up} - f^{low})\frac{g^{IT}}{||g^{IT}||^2} + \delta \tag{79}$$

$$\lambda_{iHDS,t} = \lambda_{iHDS,t}^{IT} + \mu(f^{up} - f^{low})\frac{g^{IT}}{||g^{IT}||^2} + \delta \tag{80}$$

$$\lambda_{iETH,t} = \lambda_{iETH,t}^{IT} + \mu(f^{up} - f^{low})\frac{g^{IT}}{||g^{IT}||^2} + \delta \tag{81}$$

$$\lambda_{iMTBE,t} = \lambda_{iMTBE,t}^{IT} + \mu(f^{up} - f^{low})\frac{g^{IT}}{||g^{IT}||^2} + \delta \tag{82}$$

$$\lambda_{iHTU1,t} = \lambda_{iHTU1,t}^{IT} + \mu(f^{up} - f^{low})\frac{g^{IT}}{||g^{IT}||^2} + \delta \tag{83}$$

$$\lambda_{iHTU2,t} = \lambda_{iHTU2,t}^{IT} + \mu(f^{up} - f^{low})\frac{g^{IT}}{||g^{IT}||^2} + \delta \tag{84}$$

$$\lambda_{iRF,t} = \lambda_{iRF,t}^{IT} + \mu(f^{up} - f^{low})\frac{g^{IT}}{||g^{IT}||^2} + \delta \tag{85}$$

$$\lambda_{oc,t} = \lambda_{oc,t}^{IT} + \mu(f^{up} - f^{low})\frac{g^{IT}}{||g^{IT}||^2} + \delta \tag{86}$$

$$\lambda_{FH,m,t} = \lambda_{FH,m,t}^{IT} + \mu(f^{up} - f^{low})\frac{g^{IT}}{||g^{IT}||^2} + \delta \tag{87}$$

$$\lambda_{HE,m,t} = \lambda_{HE,m,t}^{IT} + \mu(f^{up} - f^{low})\frac{g^{IT}}{||g^{IT}||^2} + \delta \tag{88}$$

$$g^{IT+1} = (1 - \gamma)g^{IT} + \gamma g^{IT+1'} \tag{89}$$

$$t \in T, \quad \mu \in [-\infty, \bar{\mu}], \quad \delta \in [-\bar{\delta}, \bar{\delta}], \quad \gamma \in [0, 1]$$

The term $g^{IT+1'}$ is the subgradient calculated from the solutions of subproblems. g^{IT+1} is the subgradient used to update Lagrange multipliers at iteration IT + 1. The superscript IT denotes the iteration. f^{low} and f^{up} indicate the lower bound and the upper bound of the reformulated problem P. $\bar{\mu}$ and $\bar{\delta}$ are given parameters.

3.4. Solutions of Subproblems. The subproblems decomposed from the primal problem P are presented in section 3.1. The efficiency of solving subproblems is a critical factor in the Lagrange decomposition scheme.

In the primal problem, there are no lower and upper bound constraints for the component oils in the blending and delivery stage. The input amounts of component oils for blending are constrained by the mass balance constraint 13. The optimal values of the input amounts of component oils in the primal problem are usually away from 0 or their maximal values due to the order required. However, when constraint 13 is dualized to decompose the primal problem, the subproblems are all linear programming problems. If the blending and delivery subproblem is solved directly, the input amounts of component oils for blending are 0 or their maximal values determined by the blending capacity which are far from the optimal solutions of the primal problem. In fact, the input amounts of component oils are constrained further by the yields of units from the production process.

$$QI_{oc}^{min} \leq \sum_{u} QI_{u,oc,t} \leq QI_{oc}^{max}, \quad \forall\, oc \in OC, t \in T \tag{90}$$

Adding the auxiliary constraint 90 to the blending and delivery subproblem would not influence the feasibility and optimality of the original problem. Thus, the blending and delivery subproblem can be solved more efficiently.

The new blending and delivery subproblem $P'_{B\&D}$ is as follows:

$P'_{B\&D}$:

min $L_{B\&D}$

s.t. constraints 14–19, 21–28, and 90.

DOI: 10.1021/acs.iecr.5b00290
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Table 1. Sizes of Cases

			model P_0			reformulated model P		
case	no. time intervals	no. orders	no. constraints	no. variables	no. binary variables	no. constraints	no. variables	no. binary variables
1	18	5	16 252	14 966	1424	10 725	9 836	1064
2	24	3	21 600	19 540	1928	14 285	12 700	1448
3	24	5	21 664	19 940	1928	14 349	13 100	1448

Table 2. Orders of Product Oils (Times and Quantities) for Case 1

			gasoline (tons)					diesel (tons)		
order	T_{l1}	T_{l2}	JIV93	JIV97	GIII90	GIII93	GIII97	GIII0	GIIIM10	GIV0
1	1	6	150	530	0	250	1050	0	1050	2000
2	3	8	250	250	130	0	650	400	800	1200
3	7	12	250	250	0	200	400	600	650	2000
4	9	14	150	550	200	0	550	550	950	1050
5	13	18	150	400	0	150	900	250	1300	1300

Table 3. Statistics of Solutions for Case 1

name	obj value (¥)	CPU time (s)	iteration	gap in CPLEX (%)	dual gap (%)
model P_0	174,135,850	10800	–	28.52	–
reformulated model P	172,979,797	543.5	–	0	–
Lagrangian dec algorithm	172,979,797	90.0	13	0	4.81
reformulated model P	173,411,190	90.2	–	1.69	–

3.5. Feasibility Scheme. With the current value of Lagrange multipliers, the subproblems can be solved. Generally, the solutions of subproblems are not feasible for the primal problem P by violating the dualized constraints. In this paper, this means that the production amount of a given type of the component oils and the material oils is not equal to the consuming amount in the other part of the subproblems. Therefore, a feasibility scheme is needed.

With good Lagrange multipliers, it is believed that the values of binary variables in the solutions of subproblems are close to the optimal values. Thus, a heuristic algorithm is introduced to obtain solutions that satisfy all constraints of the primal problem P, including the mass balance constraints for material oils and component oils. In the heuristic algorithm, some binary variables $z_{u,m,t}$ and $x_{u,m,m',t}$ are fixed according to the solutions of subproblems at iteration IT. It can be described as constraints 91 and 92.

$$z_{u,m,t} = z^{\mathrm{IT}}_{u,m,t}, \quad \forall\ u \in U^{\mathrm{IT}},\ m \in M_u,\ t \in T \tag{91}$$

$$x_{u,m,m',t} = x^{\mathrm{IT}}_{u,m,m',t}, \quad \forall\ u \in U^{\mathrm{IT}},\ m \neq m',\ t \in T \tag{92}$$

U^{IT} denotes the set of units in which the fixed binary variables are fixed in the iteration IT. Each unit will be turned in some iterations. $z^{\mathrm{IT}}_{u,m,t}$ and $x^{\mathrm{IT}}_{u,m,m',t}$ are obtained by the solutions of subproblems in the iteration "IT".

Then, solve the feasible problem P_{fea} which is described as follows.

P_{fea}:

$$\min f$$

s.t. constraints 5–7, 12–19, 21–28, 43–56, 91, and 92.

The solution of P_{fea} is a feasible solution of the reformulated problem P.

3.6. Stopping Criterion. The stopping criterion of the Lagrangian decomposition algorithm is based on the dual gap. The dual gap must be less or equal to the given value ε.

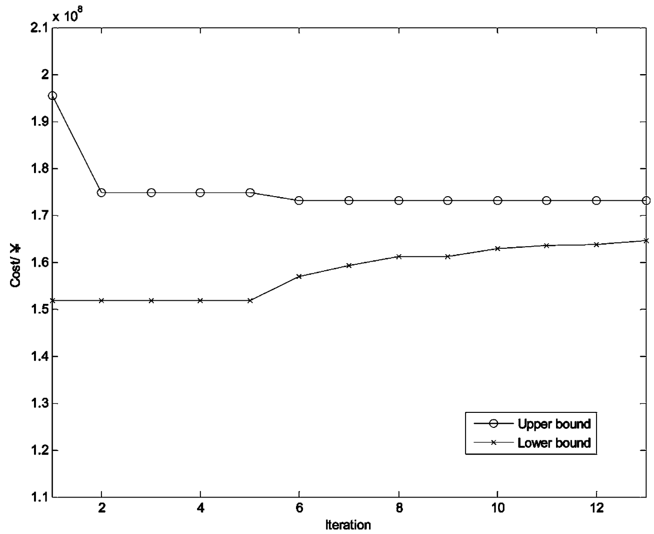

Figure 7. Convergences of lower bound and upper bound for case 1.

$$\mathrm{gap} \le \varepsilon \tag{93}$$

The dual gap is calculated by the lower bound f^{low} and the upper bound f^{up} of the reformulated problem P.

$$\mathrm{gap} = \frac{f^{\mathrm{up}} - f^{\mathrm{low}}}{f^{\mathrm{up}}} \tag{94}$$

3.7. Complete Lagrangian Decomposition Algorithm. In summary, the Lagrangian decomposition algorithm is as follows.

Step 1. Decompose the reformulated model P into 10 subproblems: P_{ATM}, P_{VDU}, P_{FCCU}, P_{HDS}, P_{ETH}, P_{MTBE}, P_{HTU1}, P_{HTU2}, P_{RF}, and $P'_{\mathrm{B\&D}}$.

Step 2. Define the feasible problem P_{fea}.

Step 3. Initialize the Lagrange multipliers by constraint 76.

Step 4. Solve subproblems. Obtain the dual objective value by summarizing the objective values of the subproblems. Update the lower bound f^{low} of the reformulated problem P.

Step 5. Solve the feasible problem P_{fea}. Calculate the objective value of the original problem by the solution of P_{fea}. Update the upper bound f^{up} of the reformulated problem P.

Step 6. Calculate the dual gap with constraint 94. If the stopping criterion is satisfied, stop the algorithm. Otherwise, update the Lagrange multipliers by solving $\hat{P}_D^{\mathrm{IT}+1}$ and go to step 4.

DOI: 10.1021/acs.iecr.5b00290
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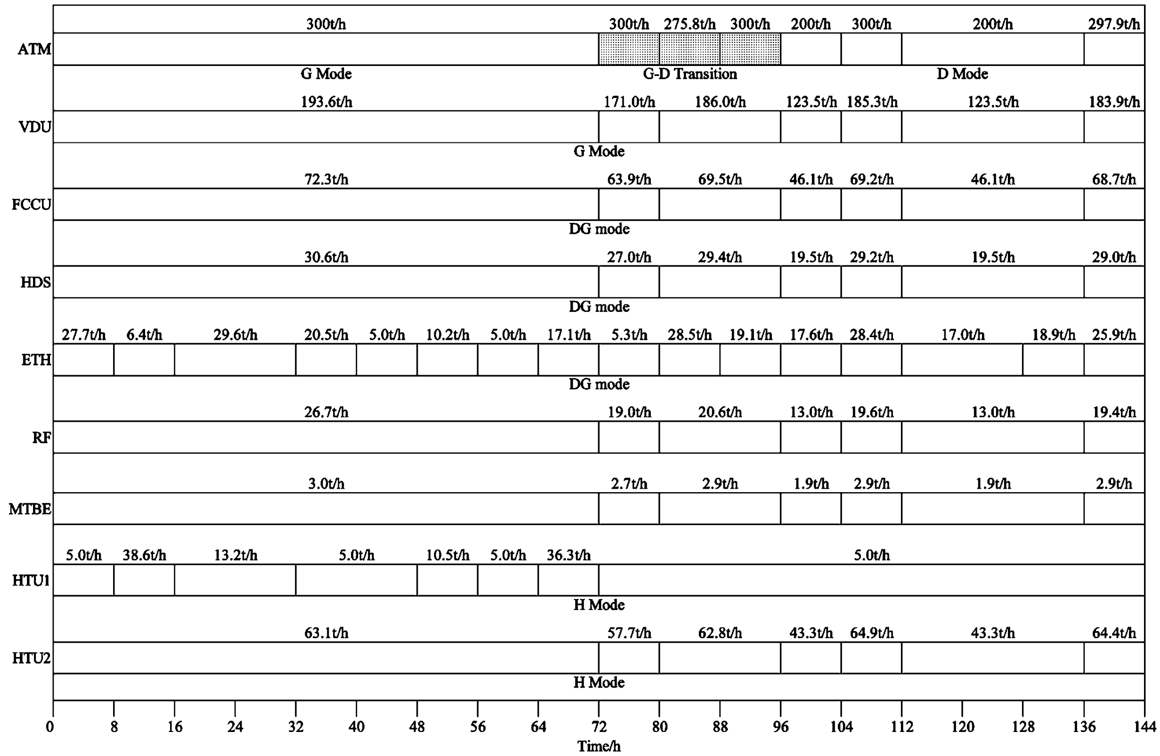

Figure 8. Gantt chart obtained by the Lagrangian decomposition algorithm for case 1.

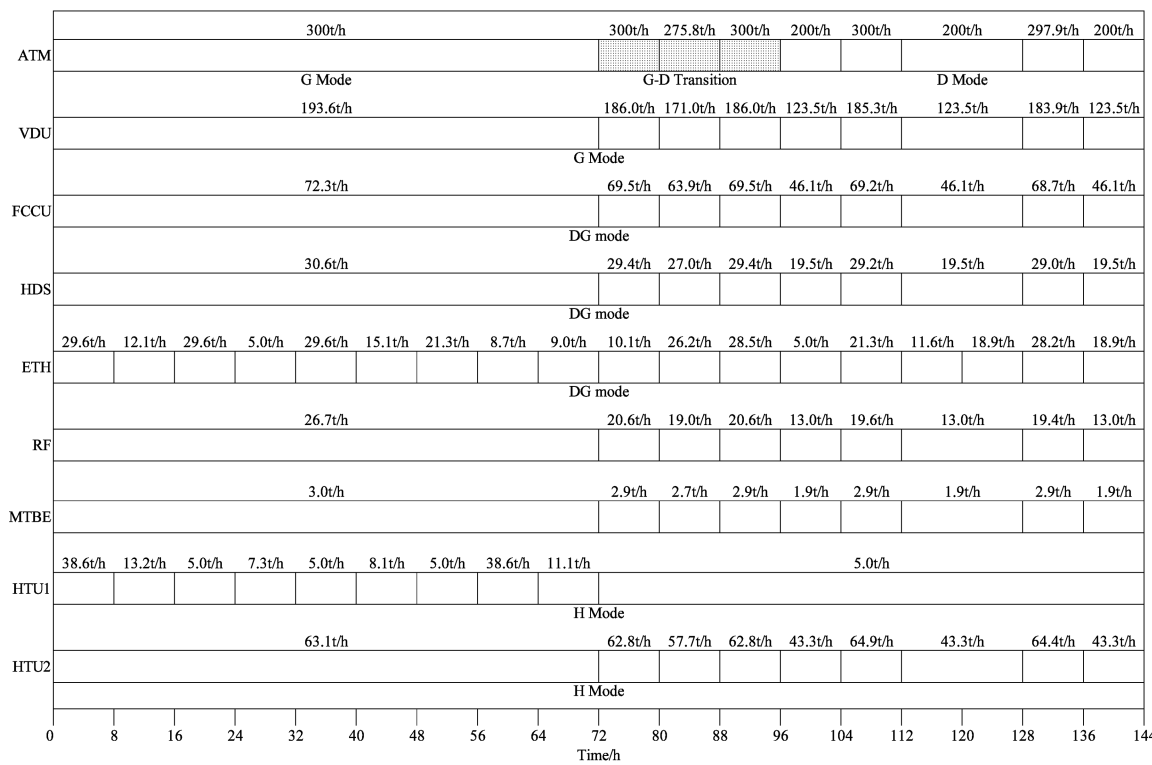

Figure 9. Gantt chart of the optimal solution obtained by the reformulated model P for case 1.

4. RESULTS

In this section, three cases which are based on the flow sheet shown in Figure 1 are generated to test the proposed Lagrangian decomposition algorithm. The sizes of the three cases are shown in Table 1. Case 1 and case 3 have the same number of orders, while case 2 and case 3 have the same number of time intervals. The cases are solved by the model P_0, the reformulated model P, and the Lagrangian decomposition algorithm, respectively. The scheduling horizons are discretized uniformly with time intervals of 8 h. In the updating of Lagrange multipliers, $\overline{\mu}$ is set to 1. $\overline{\delta}$ is set to 300. γ is set to 0.6. In the stopping criteria, the dual gap ε is set to 5%. All the cases

DOI: 10.1021/acs.iecr.5b00290
Ind. Eng. Chem. Res. 2015, 54, 6508−6526

Table 4. Orders of Product Oils (Times and Quantities) for Case 2

			gasoline (tons)					diesel (tons)		
order	T_{l1}	T_{l2}	JIV93	JIV97	GIII90	GIII93	GIII97	GIII0	GIIIM10	GIV0
1	1	8	300	1200	0	200	1200	600	2500	3000
2	9	16	500	500	450	400	2000	900	1800	4000
3	17	24	500	1000	0	200	1500	900	2000	3000

Table 5. Statistics of Solutions for Case 2

name	obj value (¥)	CPU time (s)	iteration	gap in CPLEX (%)	dual gap (%)
model P_0	233,807,800	10800	–	29.15	–
reformulated model P	231,455,181	1208.0	–	0	–
Lagrangian dec algorithm	231,516,945	139.4	16	0	4.95
reformulated model P	232,070,977	139.6	–	1.23	–

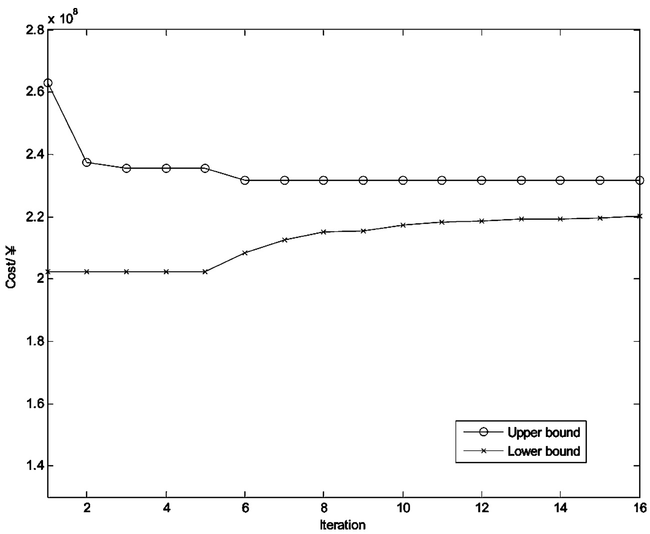

Figure 10. Convergences of lower bound and upper bound for case 2.

are executed by CPLEX in GAMS 24.2.2 using a CPU Intel Xeon E5-2609 v2 at 2.5 GHz with RAM 32.0 GB. The time limit for solver is set to 10 800 CPU s.

The key component concentration ranges of the product oils are shown in Tables 14 and 15 in Appendix C. The parameters of the scheduling model are shown in Table 16 in Appendix D.

4.1. Case 1. The orders of the product oils for case 1 are in Table 2. T_{l1} means the start time interval for delivery of order l. T_{l2} means the due time interval for delivery of order l. The statistics of solutions are shown in Table 3.

The iteration details of the Lagrangian decomposition algorithm are shown in Figure 7.

The production schedules of units obtained by the Lagrangian decomposition algorithm and the reformulated model P for case 1 are shown in Figures 8 and 9. In the Gantt charts, the duration of processing is represented as a rectangle. The rates of input flow and operation mode are denoted above and below the rectangles, respectively. The transitions are denoted as gray rectangles.

As shown in Table 1, through the reformulation, the binary variables can be reduced by about 25.3%. From Table 3 we can see that, compared with the model P_0, the reformulated model P can get the optimal solution more effectively. The optimal objective value can be found in 543.5 s by solving the reformulated model P, while the solution obtained by the model P_0 still has a gap of 0.67% with the optimal value after 3 h. By using the Lagrangian decomposition algorithm, the optimal solution can be found in 13 iterations. The CPU time consumed by the Lagrangian decomposition algorithm to meet the stopping criterion is about 1/6 of that by the reformulated model P. It can also be seen that the solution obtained by using the Lagrangian decomposition algorithm is better than that by the reformulated model P under the same CPU time.

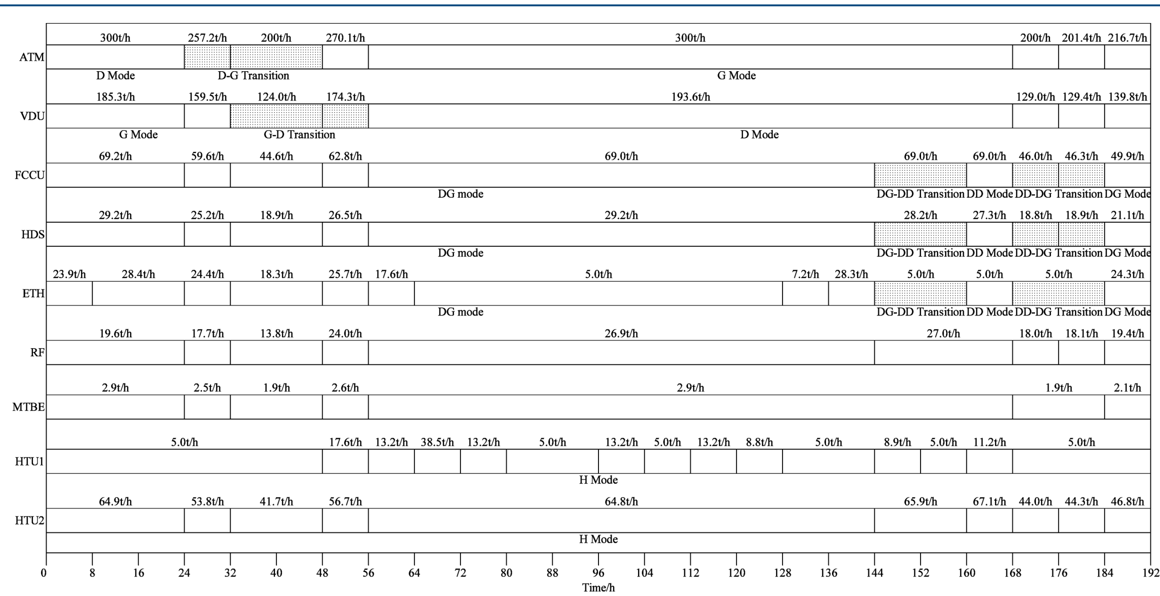

Figure 11. Gantt chart obtained by the Lagrangian decomposition algorithm for case 2.

DOI: 10.1021/acs.iecr.5b00290
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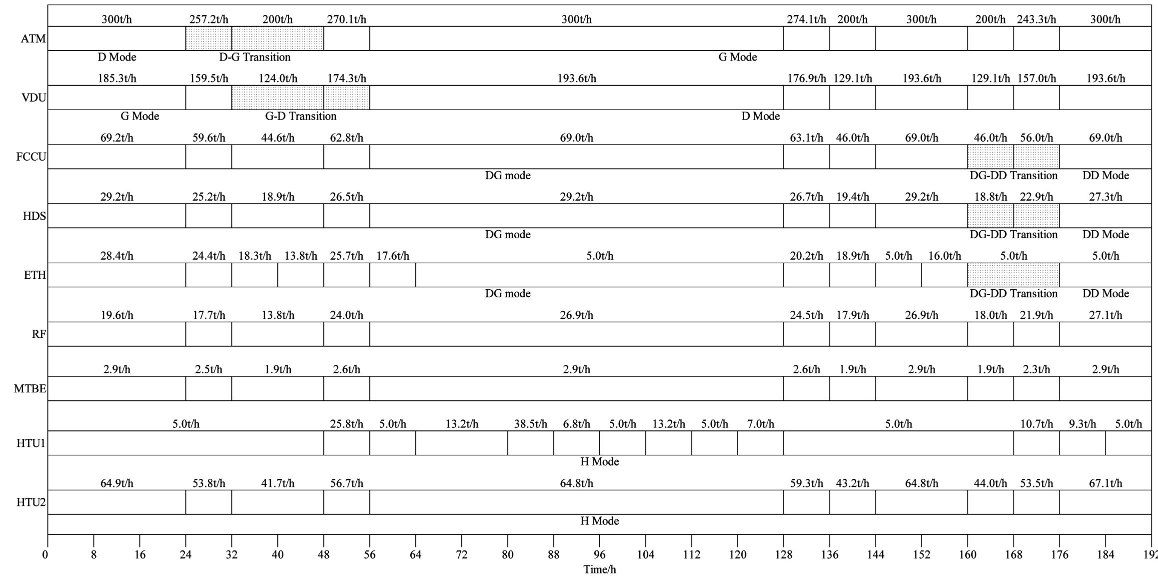

Figure 12. Gantt chart of the optimal solution obtained by the reformulated model P for case 2.

Table 6. Orders of Product Oils (Times and Quantities) for Case 3

			gasoline (tons)					diesel (tons)		
order	T_{l1}	T_{l1}	JIV93	JIV97	GIII90	GIII93	GIII97	GIII0	GIIIM10	GIV0
1	1	6	200	500	0	400	1000	600	1000	1500
2	5	10	400	300	100	0	1150	400	1650	1500
3	10	15	400	500	200	150	500	300	800	3000
4	15	20	200	600	150	0	850	500	1400	2000
5	19	24	100	800	0	250	1200	600	1500	2000

Table 7. Statistics of Solutions for Case 3

name	obj value (¥)	CPU time (s)	iteration	gap in CPLEX (%)	dual gap (%)
model P_0	232,656,290	10800	–	28.80	–
reformulated model P	230,632,826	2176.9	–	0	–
Lagrangian dec algorithm	230,632,826	111.6	12	0	4.84
reformulated model P	232,372,621	111.8	–	2.18	–

4.2. Case 2. The orders of the product oils for case 2 are in Table 4. The statistics of solutions are shown in Table 5.

The iteration details of the Lagrangian decomposition algorithm are shown in Figure 10.

The production schedules of units obtained by the Lagrangian decomposition algorithm and the reformulated model P for case 2 are shown in Figures 11 and 12.

From Table 1 and Table 5 we can see that the comparison among the model P_0, the reformulated model P, and the Lagrangian decomposition algorithm is similar to that in case 1. Through the reformulation, the binary variables can be reduced by about 24.9%. The optimal objective value can be found in 1208.0 s by solving the reformulated model P, while the solution obtained by the model P_0 still has a gap of 1.02% with the optimal value after 3 h. The gap between the solution obtained by the Lagrangian decomposition algorithm and the optimal solution is 0.03%, and it can be said that it is a near-optimal solution. The CPU time consumed by the Lagrangian decomposition algorithm to meet the stopping criterion is about 1/9 of that by the reformulated model P. It also can be seen that the solution obtained by using the Lagrangian decomposition algorithm is better than that by the reformulated model P under the same CPU time.

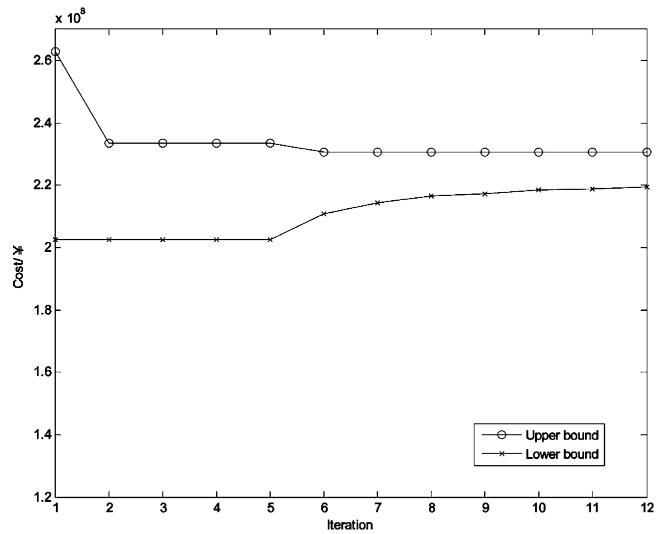

Figure 13. Convergences of lower bound and upper bound for case 3.

The numbers of binary variables and constraints of the reformulated model P in case 2 are about 40% more than those in case 1. Also, the solving time of the reformulated model P for case 2 is about 2 times the solving time for case 1. On the other hand, the CPU time consumed by the Lagrangian decomposition algorithm to meet the stopping criteria for case 2 is about 55% more than that for case 1. It shows that, as the scheduling time intervals increase, the solving time of the primal problem increases significantly, while the solving time of the Lagrangian decomposition algorithm increases more slowly.

DOI: 10.1021/acs.iecr.5b00290
Ind. Eng. Chem. Res. 2015, 54, 6508–6526

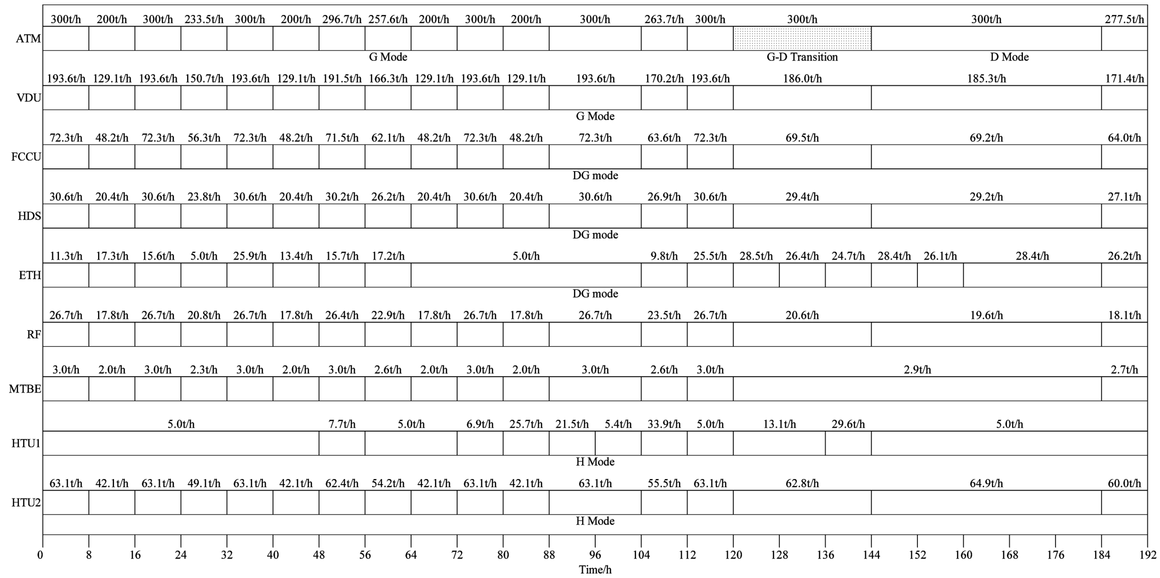

Figure 14. Gantt chart obtained by the Lagrangian decomposition algorithm for case 3.

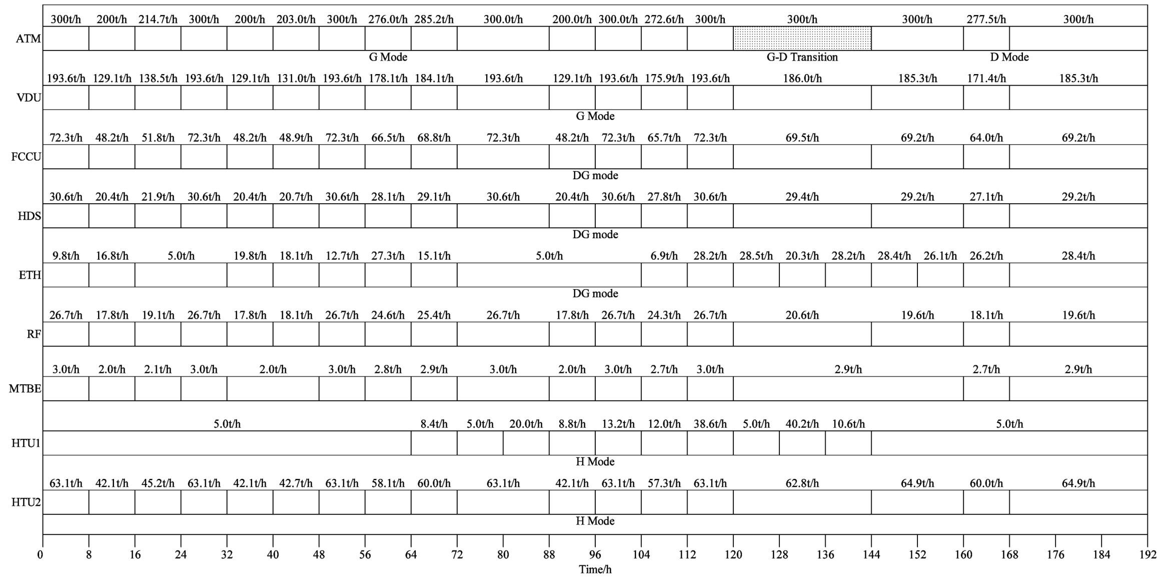

Figure 15. Gantt chart of the optimal solution obtained by the reformulated model P for case 3.

4.3. Case 3. The orders of the product oils for case 3 are in Table 6. The statistics of solutions are shown in Table 7.

The iteration details of the Lagrangian decomposition algorithm are shown in Figure 13.

The production schedules of units obtained by the Lagrangian decomposition algorithm and the reformulated model P for case 3 are shown in Figures 14 and 15.

From Table 1 and Table 7 we can see that the comparison among the model P_0, the reformulated model P, and the Lagrangian decomposition algorithm is similar those in case 1 and case 2. Through the reformulation, the binary variables can be reduced by about 24.9%. The optimal objective value can be found in 2176.9 s by solving the reformulated model P, while the solution obtained by the model P_0 still has a gap of 0.88% with the optimal value after 3 h. With the use of the Lagrangian decomposition algorithm, the optimal solution can be found in 12 iterations. The CPU time consumed by the reformulated model P is about 20 times that by the Lagrangian decomposition algorithm to meet the stopping criterion. It also can be seen that the solution obtained by using the Lagrangian decomposition algorithm is better than that by the reformulated model P under the same CPU time.

The number of binary variables of the reformulated model P in case 3 is the same as that in case 2 because the number of time intervals are same. The number of constraints in case 3 is a little more. The solving time of the reformulated model P for case 3 is about 1.8 times the solving time for case 2. On the other hand, the CPU time consumed by the Lagrangian decomposition algorithm to meet the stopping criterion for case 3 is about 20% less than that for case 2 due to fewer iterations.

4.4. Summary. To summarize the above three cases, it can be concluded that the reformulated model P can get the solutions by less computational effort than the primal model P_0. However, the solving time of the reformulated model P is still increased corresponding to the number of time intervals. This

DOI: 10.1021/acs.iecr.5b00290
Ind. Eng. Chem. Res. 2015, 54, 6508−6526

means that for large time horizons the computational effort for the optimal solutions of scheduling problems will become unrealistic. The proposed Lagrangian decomposition algorithm can obtain optimal or near-optimal solutions more efficiently. The convergences of lower and upper bounds perform similarly on different scale cases. Compared with case 1, case 3 has the same number of orders and the number of time intervals increases. As a consequence, the numbers of binary variables and constraints of the reformulated model P in case 3 are about 40% more than those in case 1. The solving time of the reformulated model P for case 3 is about 4 times the solving time for case 1, while the CPU time consumed by the Lagrangian decomposition algorithm increases by about 24%. Compared with case 2, case 3 has the same number of time intervals and the number of orders increases. The numbers of binary variables and constraints of the reformulated model P are almost the same. The solving time of the reformulated model P for case 3 is about 1.8 times the solving time for case 2. The CPU time consumed by the Lagrangian decomposition algorithm decreases about 20% less than that for case 2 due to less iterations. As the problem scale increases, especially the number of time intervals increases, the reformulated model P leads to a very large-scale combinatorial problem, and the advantage of the Lagrangian decomposition algorithm becomes more apparent. The stopping criterion of dual gap is set to 5%, which is enough for the algorithm to find optimal or near-optimal solutions.

5. CONCLUSION

In this paper, the reformulated primal problem P is decomposed by the mass balance constraints of intermediate oils and 10 subproblems are obtained. Initial Lagrange multipliers are estimated in an economic sense and a hybrid method to update the Lagrange multipliers is used in the updating to get faster convergence. To find feasible solutions, a heuristic algorithm is also designed. The computational results show that the Lagrangian decomposition algorithm is effective and efficient. The advantage of the proposed algorithm is especially apparent for large-scale problems.

For this kind of problem, the Lagrange multipliers can be interpreted in an economic sense and initial values can be estimated. By the initialization, the solutions can be close to the optimal one and the Lagrange multipliers can usually get a faster convergence. The efficiency of solving subproblems is another challenge for the Lagrange decomposition scheme. From this paper, we have two conclusions. First, although there are some methods to linearize the bilinear or trilinear terms, it will introduce additional binary variables in the model. The number of binary variables is a big factor of the solving time. Therefore, the bilinear or trilinear terms should be avoided if at all possible when modeling. Second, after dualizing some constraints of the primal problem, the obtained subproblems may lose some information. This means that the feasible field of some variables is expanded to somewhere the solution is infeasible for the primal problem. If some auxiliary constraints can be constructed for the subproblems, the feasible field will be constrained, and then the subproblems can be solved more efficiently.

■ APPENDIX A. OPERATION MODES FOR PRODUCTION UNITS

Table 8 shows the operation modes (G, D, DG, GD, DD, GG, H, M) for the production units ATM, VDU, FCCU, ETH, HDS, HTU1, and HTU2.

■ APPENDIX B. YIELDS AND OPERATION COSTS OF PROCESSING UNITS

Tables 9, 10, 11, 12, and 13 include the yields and operation costs for ATM, VDU, FCCU, HTS, ETH, HTU1, HTU2, RF,

Table 8. Operation Modes for Production Units

	unit						
	ATM	VDU	FCCU	ETH	HDS	HTU1	HTU2
mode	G	G	GG	GG	GG	M	M
	D	D	GD	GD	GD	H	H
			DG	DG	DG		
			DD	DD	DD		

Table 9. Yields and Operation Costs of ATM and VDU

	ATM				VDU		
mode	SRD (%)	LD (%)	HD (%)	AR (%)	VHD (%)	VR (%)	oper cost (kgoe/t)
G	7.008	15.349	8.109	64.534	11.286	37.352	11
D	4.576	19.403	9.267	61.754	12.580	35.652	11.5

Table 10. Yields and Operation Costs of FCCU

	FCCU			
mode	gasoline (%)	diesel (%)	rich gas (%)	oper cost (kgoe/t)
GG	45.664	22.216	5.02	58
GD	42.583	23.104	5.01	57
DG	42.261	23.418	4.15	56.5
DD	39.580	26.683	4.13	56

Table 11. Yields and Operation Costs of HDS and ETH

	HDS		ETH	
mode	gasoline (%)	oper cost (kgoe/t)	gasoline (%)	oper cost (kgoe/t)
GG	86.2	27.18	93.2	47.6
GD	79	28.98	90	49.56
DG	97	24.48	98.1	44.66
DD	88	26.73	94.1	47.11

Table 12. Yields and Operation Costs of HTU1 and HTU2

	HTU1		HTU2		
mode	diesel (%)	oper cost (kgoe/t)	diesel (%)	naphtha (%)	oper cost (kgoe/t)
H	99.4	9	89	9	11
M	99.4	8	93	4	10

Table 13. Yields and Operation Costs of RF and MTBE

RF			MTBE	
gasoline (%)	C5 (%)	oper cost (kgoe/t)	MTBE (%)	oper cost (kgoe/t)
90	10	83	120	13.84

DOI: 10.1021/acs.iecr.5b00290
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and MTBE. Operation costs are given in kilograms of oil equivalent per ton; 1 kgoe is equal to 10 000 kcal.

APPENDIX C. KEY COMPONENT CONCENTRATION RANGES OF PRODUCT OIL

Tables 14 and 15 list property values that must be met for various gasoline and diesel fuels. In Table 14, RON is the research octane number, and in Table 15, CN is the cetane number.

Table 14. Requirements for Property Values of Gasoline

gasoline	property	
	RON	sulfur content (%)
JIV93	≥93	≤0.005
JIV97	≥97	≤0.006
GIII90	≥90	≤0.015
GIII93	≥93	≤0.015
GIII97	≥97	≤0.015

Table 15. Requirements for Property Values of Diesel

diesel	property		
	CN	sulfur content (%)	condensation point factor
GIII0	≥49	≤0.035	≤1.6184
GIIIM10	≥49	≤0.035	≤1.1995
GIV0	≥51	≤0.01	≤1.6184

APPENDIX D. PARAMETERS OF THE SCHEDULING MODEL IN RESULTS

Table 16 includes the parameters of the scheduling model as discussed in section 4.

Table 16. Parameters of the Scheduling Model

parameter	value
QI_{ATM}^{min}	200 t/h
QI_{ATM}^{max}	300 t/h
QI_{ETH}^{min}	5 t/h
QI_{HTU1}^{min}	5 t/h
OPC	3,882 ¥/t
α	500
β_l	30000
$r_{MTBE,o}^{max}$	0.1
$TT_{ATM,m,m'}$, $TT_{VDU,m,m'}$	3 time intervals
$TT_{FCCU,m,m'}$, $TT_{HDS,m,m'}$, $TT_{ETH,m,m'}$	2 time intervals
$TT_{HTU1,m,m'}$, $TT_{HTU2,m,m'}$	1 time interval

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Notes

The authors declare no competing financial interest.

ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China (No. 61273039, No. 21276137).

NOTATION

Sets/Indices

U = set of production units with index u
U_{om}^{out} = set of production units producing material oil om
U_{om}^{in} = set of production units consuming material oil om
U_{oc}^{out} = set of production units producing component oil oc
U^{IT} = set of units in which fixed binary variables are fixed in iteration IT in Lagrangian decomposition algorithm
M_u = set of operation modes for unit u with index m and m'
T = set of time intervals being scheduled with index t
OI = set of intermediate oil with index oi
OM = set of material oil with index om
OC = set of component oil for blending with index oc
O = set of product oil with index o
L = set of orders with index l
P = set of properties with index p
it, IT = iteration

Binary Variables

$x_{u,m,m',t}$ = 1 if unit u is in the transition from operation mode m to m' during time interval t
$y_{u,m,t}$ = 1 if unit u is in the transition of mode switching from the previous operation mode to operation mode m or steady state of operation mode m
$z_{u,m,t}$ = 1 if unit u is in steady state of operation mode m during time interval t

Continuous Variables

$INV_{oc,t}$ = inventory of component oil oc at the end of time interval t
$INV_{o,t}$ = inventory of product oil o at the end of time interval t
$PRO_{o,p,t}$ = value of property p for product oil o during time interval t
$QI_{u,t}$ = input flow of unit u during time interval t
$sQI_{u,m,t}$ = input amount of unit u during time interval t in steady state of operation mode m
$tQI_{u,m,m',t}$ = input amount of unit u during time interval t in transition from operation mode m to m'
$QO_{u,oi,t}$ = output flow of intermediate oil oi of unit u during time interval t
$QI_{o,t}$ = input flow of product oil o during time interval t
$QI_{oc,t}$ = input flow of component oil oc during time interval t
$QO_{u,oc,t}$ = output amount of component oil oc from unit u during time interval t
$QO_{u,om,t}$ = output amount of material oil om from unit u during time interval t
$D_{l,o,t}$ = delivery of product oil o for order l during time interval t
$Q_{oc,o,t}$ = blending flow from component oil oc to product oil o during time interval t
$inCost_u$ = estimated cost of input material of unit u
$outCost_u$ = estimated price of output product of unit u
$\lambda_{oc,t}$ = Lagrangian multiplier of component oil oc during time interval t
$\lambda_{iVDU,t}$ = Lagrangian multiplier of input oil of VDU during time interval t
$\lambda_{iFCCU,t}$ = Lagrangian multiplier of input oil of FCCU during time interval t
$\lambda_{iHDS,t}$ = Lagrangian multiplier of input oil of HDS during time interval t
$\lambda_{iETH,t}$ = Lagrangian multiplier of input oil of ETH during time interval t
$\lambda_{iMTBE,t}$ = Lagrangian multiplier of input oil of MTBE during time interval t
$\lambda_{iHTU1,t}$ = Lagrangian multiplier of input oil of HTU1 during time interval t

DOI: 10.1021/acs.iecr.5b00290
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$\lambda_{\mathrm{iHTU2},t}$ = Lagrangian multiplier of input oil of HTU2 during time interval t
$\lambda_{\mathrm{iRF},t}$ = Lagrangian multiplier of input oil of RF during time interval t
$\lambda_{\mathrm{FH},m,t}$ = Lagrangian multiplier of operation mode m of FCCU and HDS during time interval t
$\lambda_{\mathrm{HE},m,t}$ = Lagrangian multiplier of operation mode m of HDS and ETH during time interval t
f^{up} = upper bound of original problem P
f^{low} = lower bound of original problem P
g^{IT} = subgradient used to update Lagrange multipliers at iteration IT
$g^{\mathrm{IT}'}$ = subgradient calculated from solutions of subproblems at iteration IT′

Auxiliary Binary Variable

$\mathrm{xy}_{u,m',t}$ = product of $y_{u,m',t}$ and $(1 - \sum_m x_{u,m,m',t})$

Auxiliary Continuous Variables

$\mathrm{xQI}_{u,m,m',t}$ = product of $x_{u,m,m',t}$ and $\mathrm{QI}_{u,t}$
$\mathrm{xQI1}_{u,m,m',t}$ = auxiliary variable for linearization
$\mathrm{xyQI}_{u,m',t}$ = product of $\mathrm{xy}_{u,m',t}$ and $\mathrm{QI}_{u,t}$
$\mathrm{xyQI1}_{u,m',t}$ = auxiliary variable for linearization

Parameters

α = inventory cost of component oil and product oil per period
β_l = penalty for stockout of order l per ton
$\mathrm{INV}_{\mathrm{oc}}^{\mathrm{min}}$ = minimum storage capacity of component oil oc
$\mathrm{INV}_{\mathrm{oc}}^{\mathrm{max}}$ = maximum storage capacity of component oil oc
$\mathrm{INV}_{o}^{\mathrm{min}}$ = minimum storage capacity of product oil o
$\mathrm{INV}_{o}^{\mathrm{max}}$ = maximum storage capacity of product oil o
$\mathrm{INV}_{\mathrm{oc,ini}}$ = initial storage of component oil oc
$\mathrm{INV}_{o,\mathrm{ini}}$ = initial storage of product oil o
OPC = price of crude oil c
$\mathrm{OpCost}_{u,m}$ = operational cost of unit u in steady state of operation mode m
$\mathrm{tOpCost}_{u,m,m'}$ = operational cost of unit u in transition from operation mode m to m'
$\mathrm{PRO}_{o,p}^{\mathrm{min}}$ = minimum value of property p for product oil o
$\mathrm{PRO}_{o,p}^{\mathrm{max}}$ = maximum value of property p for product oil o
$\mathrm{PRO}_{\mathrm{oc},p}$ = value of property p for component oil oc
$R_{l,o}$ = demand for product oil o of order l
$\mathrm{TT}_{u,m,m'}$ = time of transition from operation mode m to m' of unit u
T_{l1} = start time interval for delivery of order l
T_{l2} = due time interval for delivery of order l
Yield_u = yield of unit u in the average sense
$\mathrm{Yield}_{u,oi,m}$ = yield of intermediate oil oi of unit u in steady state of operation mode m
$\mathrm{tYield}_{u,oi,m,m'}$ = yield of intermediate oil oi of unit u in transition from operation mode m to m'
$\mathrm{QI}_{u}^{\mathrm{min}}$ = minimum value of input flow of unit u
$\mathrm{QI}_{u}^{\mathrm{max}}$ = maximum value of input flow of unit u
$\mathrm{QI}_{\mathrm{oc}}^{\mathrm{min}}$ = minimum value of component oil oc
$\mathrm{QI}_{\mathrm{oc}}^{\mathrm{max}}$ = maximum value of component oil oc
$r_{\mathrm{oc},o}^{\mathrm{min}}$ = minimum ratio of component oil oc in blending product oil o
$r_{\mathrm{oc},o}^{\mathrm{max}}$ = maximum ratio of component oil oc in blending product oil o
T_{max} = maximum number of time intervals in scheduling horizon
N_u = maximum duration of transitions for unit u in primal problem

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DOI: 10.1021/acs.iecr.5b00290
Ind. Eng. Chem. Res. 2015, 54, 6508−6526