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Optimal Design and Layout of Industrial Facilities: An Application to Multipurpose Batch Plants

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A mixed integer linear programming (MILP) formulation for the optimal design and layout of generic industrial facilities was proposed in part I of this work (see preceding paper). The applicability of the model is explored in this paper, through a study of the simultaneous design and layout of a batch multipurpose plants. Using as a basis the design model proposed by Barbosa-Póvoa and Macchietto (*Comput. Chem. Eng.* **1994**, *18*, 1013−1042), detailed design and layout aspects are studied in a single-level model. The resulting model determines *simultaneously* the optimal plant topology and layout over a two-dimensional space. In addition, the optimal plant operating conditions, such as schedule and resources consumption and production, are also obtained. Again, the problem is formulated as a mixed integer linear problem (MILP) in which binary variables are introduced to characterize operational and topological choices. The applicability of the proposed model is demonstrated through the solution of a set of representative examples.

1. Introduction

In part I of this work,[2] the simultaneous design and layout of general industrial facilities was proposed. The application of this approach to the detailed design of multipurpose batch plants is now presented.

The design of batch processing facilities involves a large number of interacting decisions. This is the case, in particular, for multipurpose batch plants because of the nature of such facilities (i.e., sharing of resources), where design and operating aspects must be considered simultaneously with the design problem. Also, layout issues concerning the spatial allocation of equipment items and their interconnections often appears as an important part of the design problem. Traditionally, these aspects have been considered a posteriori once the main plant design task has been completed. However, the interactions between layout and the remainder of the design decisions are often quite strong, and therefore, a simultaneous approach is desirable.

The design problem of batch multipurpose plants is addressed in the literature with a variable level of detail, but no layout aspects have yet been considered.

Papageorgaki and Reklaitis[3] addressed the general batch plant design problem in which the main equipment was selected allowing a flexible unit-to-task allocation. Plant operation over several campaigns was studied, and the optimal allocation of products to campaigns and of the campaign lengths was determined. Later, Barbosa-Póvoa and Macchietto[1] studied a single-campaign problem in which different products could be produced. Plant topology was considered, and the design problem led to the choice of both the main equipment items and the associated network of connections. Using this model, a more generic model was proposed by Barbosa-Póvoa[4] in which product sequencing, operational restrictions, and cleaning needs, as well as cleaning-in-place (CIP) design circuits, were considered. Simple layout issues were studied, and the need for a more in-depth investigation of this issue was identified.

More recently, Lin and Floudas[5] proposed a continuous model for the design of multipurpose batch plants in which MILP and MINLP formulations were developed.

The layout problem within batch process facilities has also been studied by several authors. Layout has mainly been treated as a stand-alone problem to be solved once the main equipment items and their interconnections have been determined. Georgiadis et al.[6] used a space discretization technique to consider the allocation of equipment items to floors and the detailed layout of each floor. Also, Papageorgiou and Rotstein[7] proposed a mathematical programming model for determining the optimal process plant layout. Their model employed a continuous representation of space and took account of many important features of the plant layout problem. Finally, a simultaneous approach to plant design, scheduling, and layout was adopted by Realff et al.[8] for the case of pipeless batch plants.

In this paper, the variable layout formulation presented in part I of this series[2] is combined with the detailed design model of multipurpose batch plants proposed by Barbosa-Póvoa and Macchietto.[1] Thus, the design and layout of batch plants are solved in a systematic way. A previous work by the same authors[9] highlighted this problem and presented some results. This work is now generalized on the basis of a more

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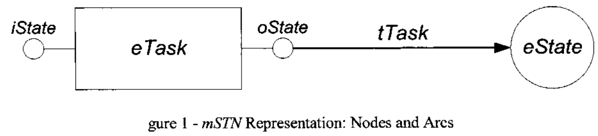

Figure 1. mSTN representation: Nodes and arcs.

detailed model. It determines *simultaneously* the optimal plant topology (i.e., the choice of the plant equipment and the associated connections) and the optimal layout (i.e., the arrangement of processing equipment storage vessels and their interconnecting pipework over a two-dimensional space), as well as the optimal plant operation (schedule and resources consumption and production).

The interactions arising from the operating conditions, the suitability and availability of equipment, the various layout constraints, and the presence of equipment items of various sizes are taken into account in one single level.

The problem is formulated as a mixed integer linear problem (MILP) in which binary variables are introduced to characterize operational and topological choices.

The applicability of the proposed model is illustrated through the solution of a set of representative examples.

2. Problem Representation

The maximal state–task network (mSTN) as defined by Barbosa-Póvoa[4] is used to represent the batch design problem.

The process recipes are defined through a state–task network (STN) graph representation,[10] and the plant is characterized through a normal flowsheet, that is, an equipment network of vessels, processing units, and possible connections. The STN represents the precedence structure of the product networks in which materials are defined as state nodes and the operations transforming input material states into output material states are described through task nodes. Additionally, material proportions, processing times, and utility requirements for each task must be defined.

Having the process recipes along with the plant description (equipment characteristics and plant structure), the maximal STN (mSTN) automatically combines the operations and equipment network by performing the mapping between the two. This mapping is defined by (1) the suitability of each unit in the equipment network to carry out processing tasks and to store material states (2) the suitability of connections to transport material states, and the resources (equipment, utilities, operators, etc.) required by each task.

The resulting mSTN is characterized by different types of nodes and arcs (see Figure 1): eTask i/g nodes representing the processing tasks i that can be performed in unit g; eState s/g nodes representing materials in state s that can be stored in unit g; iState/oState nodes representing the origin/location of the material entering/leaving the eTask defined as zero-capacity states; and, finally, transfer tasks (tTask arc) are introduced between the input/output points defined through these latter states whenever a link exists between them.

The main advantage of this representation is that it unambiguously and explicitly represents the location of all material states within the plant as well as the allocations of processing, storage, and transfer tasks

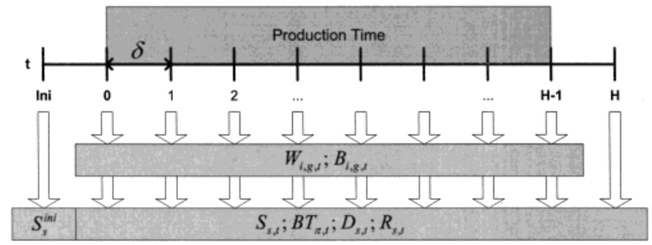

Figure 2. Process events and planning horizon.

that are potentially necessary and structurally feasible within the problem under study.

3. Problem Statement and Characteristics

The simultaneous design and layout of batch processing facilities can be stated as follows:

Given

- the STN descriptions of product recipes with associated parameters and resource requirements (equipment, utilities, etc.);
- a plant superstructure (network of possible units and connections) with associated capacities and suitabilities;
- production demands, a time horizon (H), operation mode, and availability profiles of all resources;
- equipment item geometrical shapes and sizes over a two-dimensional area;
- input and output points locations within each equipment space;
- safety and operability minimum and maximum distances between equipment items;
- space and equipment allocation limitations; and
- capital costs for units and connections

Determine

- the optimal operations schedule—sizes, allocation and timing of all batches, storage and transfers;
- the optimal plant configuration—equipment network and sizes;
- the optimal plant equipment arrangement—coordinates and orientation;
- the optimal associated connectivity structure—inputs and outputs.

so as to optimize a given economic objective function, namely, the minimization of the capital cost of the plant, while fulfilling all of the process/plant constraints.

The developed model allows for the description of general processing networks described by multiproduct and multipurpose plants.

The mathematical formulation (see section 4) is based on a discretization of time such that the planning horizon H is divided into a number of elementary steps of fixed length. All process events are allowed to occur only at the interval boundaries (vertical arrows in Figure 2) and not between them. For instance, the transfer of material by transfer task π at the beginning of period t, in an amount represented by the continuous variable $BT_{\pi,t}$, can occur only at any time boundary between 0 and H. On the other hand, the variables $W_{i,g,t}$ and $B_{i,g,t}$, describing the allocation and the batch of task i in unit g at time t, respectively, are defined only for any time boundary between 0 and $H - p_i$. Parameter p_i sets the duration of task i, and in this way, no tasks are allowed to finish past the allowed time horizon, H. For simplicity, in a processing time equal to unity is assumed in Figure 2 and variables $W_{i,g,t}$ and $B_{i,g,t}$ are therefore defined until $H - 1$.

Design and operation decisions are represented by continuous variables (batch sizes, equipment capacities, amounts of material, etc.), and discrete choices by binary variables (equipment existence, task allocations to equipment and time, etc.).

Equipment items (units, dedicated storage, and connections) are selected optimally from the defined plant superstructure while operation is optimized so as to satisfy all constraints.

For each product, requirements are defined as fixed or variable within ranges. Demands (and supplies) can be associated with specific equipment units.

Equipment in discrete size range(s); mixed storage policies; shared intermediate material; material merging, splitting, and recycling; in-phase and out-of-phase operation in any combination; and instantaneous transfers are all allowed.

Also, a single-campaign structure with a fixed product slate is assumed for nonperiodic operation.

The plant is defined within a two-dimensional continuous space where the equipment items to be installed in the available space are described by rectangular or irregular shapes as characterized in part I of this work.[2] Rectilinear distances are assumed, providing a more realistic estimate of the piping costs as opposed to direct connections.

Multiple equipment connectivity inputs and outputs are considered, as well as space limitations.

Finally, the objective function is defined in terms of the capital costs of the plant. This accounts for the units and connections costs, a function of the selected equipment capacity, material, and suitability. For the connections, the final piping length (rectilinear distance within the plant) is also considered.

4. Mathematical Formulation

The model for the simultaneous design and layout of batch processing facilities includes the mathematical formulation presented in part I of this work,[2] along with the mathematical formulation defined in this paper.

The following formulation is adapted from the design with nonperiodic operation defined in Barbosa-Póvoa,[4] using the indices, sets, parameters, and associated variables defined on the basis of the above model characteristics and presented in the Nomenclature section at the end of this paper. Also included from part I of this paper[2] are the global indices g (i.e., j in Barbosa-Póvoa[4]), c, and oi; the set OI_c; and the binary variables E_g and E_c, which represent the installation of unit g and connection c, respectively.

Using these definitions and variables, the mathematical model characterizing the simultaneous two-dimensional design and layout of batch processing facilities is presented below.

Objective Function. In this paper, the minimization of the capital cost, including fixed and variable costs for equipment units and connections, is assumed for objective function (OF)

$$\min \sum_{(c,oi,oi')|(oi,oi')\in \mathrm{OI}_c} (\mathrm{CC}_c^0 E_c + \mathrm{CC}_c^2 D_{\mathrm{oi,oi'}}) + \sum_g (\mathrm{CC}_g^0 E_g + \mathrm{CC}_g^1 V_g) \quad (1)$$

Other OFs could alternatively be applied, such as the maximization of the plant profit for given products demands and prices, as defined in Barbosa-Póvoa.[4]

Constraints. Along with the constraints defined in part I of this work,[2] a number of additional constraints must be introduced. These include the existence, capacity, and batch size of the processing units, as well as their storage, connectivity, mass balance, and production requirement constraints.

Processing Unit Existence Constraints. The assignment of units to processing tasks is established by two basic rules: (1) At any time, each equipment unit is either idle or processing a single task. (2) Tasks cannot be pre-empted once they have been started.

$$\sum_{i\in I_g} \sum_{t'=t-p_i+1\geq 0}^{t\leq H-p_i} W_{i,g,t'} \leq E_g \quad \forall g,\ t|g \in \mathrm{PU};\ t = 0, ..., H-1 \quad (2)$$

If a suitable task $W_{i,g,t}$ is processed in unit g, then equipment g is installed ($E_g = 1$). Otherwise, if the equipment unit is not installed ($E_g = 0$), no tasks can be processed in this unit. The sum over period t' guarantees the continuity of task i. The sum over all tasks i excludes the possibility for other potential processing tasks to occur in unit g over that same time period.

Capacity and Batch Size Constraints. The capacity of each equipment unit is directly related to the amount of material (batch size) processed in that unit. If no processing is occurring, the batch size is 0. Otherwise, it must be between the maximum ($\phi_{i,g}^{\max} V_g W_{i,g,t}$) and the minimum ($\phi_{i,g}^{\min} V_g W_{i,g,t}$) available unit capacity. The bilinear term can be reformulated into a linear form, according to Shah,[11] using the following constraints

$$\phi_{i,g}^{\min} V_g - V_g^{\max}(1 - W_{i,g,t}) \leq B_{i,g,t} \quad \forall i,\ g,\ t|g \in K_i;\ t = 0, ..., H - p_i \quad (3)$$

$$B_{i,g,t} \leq \phi_{i,g}^{\max} V_g \quad \forall i,\ g,\ t|g \in K_i;\ t = 0, ..., H - p_i \quad (4)$$

These constraints ensure that the batch size $B_{i,g,t}$ is defined within the available capacity, if equipment unit g is chosen to process task i at any time t. Furthermore, if a suitable task is not processed in unit g at any time t ($W_{i,g,t}$= 0), then the batch size, $B_{i,g,t}$, is forced to be 0 by the constraint

$$B_{i,g,t} \leq V_g^{\max} W_{i,g,t} \quad \forall i,\ g,\ t|g \in K_i;\ t = 0, ..., H - p_i \quad (5)$$

In addition, the capacity V_g is defined within the range of available capacities for unit g according to the constraints

$$V_g^{\min} E_g \leq V_g \quad \forall g \quad (6)$$

$$V_g \leq V_g^{\max} E_g \quad \forall g \quad (7)$$

Storage Constraints. Dedicated storage vessels can be used to store materials. For these cases, we have

$$S_s^{\mathrm{ini}} \leq V_g \phi_{s,g} \quad \forall s,\ g|g \in K_s \quad (8)$$

$$S_{s,t} \leq V_g \phi_{s,g} \quad \forall s,\ g,\ t|g \in K_s;\ t = 0, ..., H \quad (9)$$

If the amount of material in eState s at the beginning of period t ($S_{s,t}$) is greater than 0, then the associated capacity V_g takes the corresponding positive value weighted by the size conversion factor $\phi_{s,g}$. The case in which multipurpose vessels are used to store material is modeled as described above for the processing units.

To avoid the use of a storage vessel as a simple passage point, with no material in storage during the entire planning horizon, and to prevent the inclusion by default of such vessels in the plant structure, the following constraint must be present

$$\sum_{t}^{H} \sum_{\pi\in\Pi_g^{\mathrm{sink}}} \mathrm{BT}_{\pi,t} - V_g^{\max} E_g \le 0 \quad \forall g|g \in \mathrm{DSV} \quad (10)$$

Connectivity Constraints. Each transfer task π between two connection points has only one associated state of material. However, each connection might be suitable for the transfer of different kinds of materials. Although the plant superstructure can include many kinds of possible transfer tasks, the minimization of the plant cost will typically result in the installation of a limited set of connections. The amount of material $\mathrm{BT}_{\pi,t}$ transported by transfer task π at the beginning of period t is constrained by the available capacity BT_c of its associated connection c, weighted by the size conversion factor $\phi_{\pi,c}$

$$\mathrm{BT}_{\pi,t} \le \mathrm{BT}_c \phi_{\pi,c} \quad \forall c, \pi, t|\pi \in I_c;\ t = 0, ..., H \quad (11)$$

On the other hand, the capacity BT_c of each connection must fall between predefined capacity lower and upper bounds, namely

$$\mathrm{BT}_c^{\min} E_c \le \mathrm{BT}_c \quad \forall c \quad (12)$$

$$\mathrm{BT}_c \le \mathrm{BT}_c^{\max} E_c \quad \forall c \quad (13)$$

Mass Balances Constraints. Mass balance equations are applied between materials delivered and received. Thus, the amount of material in a state during any time period must equal the amount of the same material being produced, consumed, and transferred by all incident tasks, along with the amount of material of the state existing in the previous time period.

For the eStates

$$S_{s,0} = S_s^{\mathrm{ini}} + \sum_{\pi\in\Pi_s^{\mathrm{out}}} \mathrm{BT}_{\pi,0} - \sum_{\pi\in\Pi_s^{\mathrm{in}}} \mathrm{BT}_{\pi,0} - D_{s,0} + R_{s,0} \qquad \forall s|s \in e^{\mathrm{state}} \quad (14)$$

$$S_{s,t} = S_{s,t-1} + \sum_{\pi\in\Pi_s^{\mathrm{out}}} \mathrm{BT}_{\pi,t} - \sum_{\pi\in\Pi_s^{\mathrm{in}}} \mathrm{BT}_{\pi,t} - D_{s,t} + R_{s,t} \qquad \forall s, t|s \in e^{\mathrm{state}};\ t = 1, ..., H \quad (15)$$

For the iStates

$$0 = -\sum_{i\in T_s^{\mathrm{in}}} \sum_{g\in K_i} \rho_{i,s}^{\mathrm{in}} B_{i,g,t-p_{i,s}^{\mathrm{lag}}\le H-p_i} + \sum_{\pi\in\Pi_s^{\mathrm{out}}} \mathrm{BT}_{\pi,t} \qquad \forall s, t|s \in i^{\mathrm{state}};\ t = 0, ..., H \quad (16)$$

For the oStates

$$0 = \sum_{i\in T_s^{\mathrm{out}}} \sum_{g\in K_i} \rho_{i,s}^{\mathrm{out}} B_{i,g,t-p_{i,s}^{\mathrm{proc}}\le H-p_i} - \sum_{\pi\in\Pi_s^{\mathrm{in}}} \mathrm{BT}_{\pi,t} \qquad \forall s, t|s \in o^{\mathrm{state}};\ t = 0, ..., H \quad (17)$$

Production Requirement Constraints. For each product, the sum of all materials generated from the same STN product state (s^{STNProd}) stored or delivered in any unit in the plant at the end of the production horizon (H) can float between predefined minimum ($Q_{s\mathrm{STNProd}}^{\min}$) and maximum ($Q_{s\mathrm{STNProd}}^{\max}$) production requirements

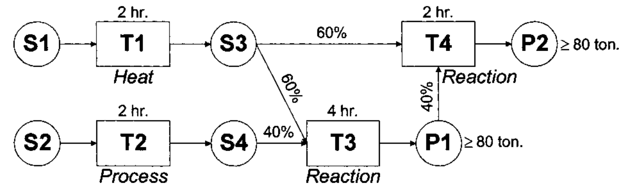

Figure 3. Process recipe.

$$Q_{s\mathrm{STNProd}}^{\min} \le \sum_{s\in s^{\mathrm{STNProd}}} (S_{s,H} - S_{s\mathrm{STNProd}}^{\mathrm{ini}} + D_{s,H}) \qquad \forall s^{\mathrm{STNProd}}|Q_{s\mathrm{STNProd}}^{\max} > 0 \quad (18)$$

$$\sum_{s\in s^{\mathrm{STNProd}}} (S_{s,H} - S_{s\mathrm{STNProd}}^{\mathrm{ini}} + D_{s,H}) \le Q_{s\mathrm{STNProd}}^{\max} \qquad \forall s^{\mathrm{STNProd}}|Q_{s\mathrm{STNProd}}^{\max} > 0 \quad (19)$$

For each intermediate delivery ($D_{s,t}$), the same idea is applied, resulting in the following lower and upper bounds

$$D_{s\mathrm{STN},t}^{\min} \le \sum_{s\in S_s\mathrm{STN}} D_{s,t} \quad \forall s^{\mathrm{STN}}, t|D_{s\mathrm{STN},t}^{\max} > 0;\ t = 0, ..., H \quad (20)$$

$$\sum_{s\in S_s\mathrm{STN}} D_{s,t} \le D_{s\mathrm{STN},t}^{\max} \quad \forall s^{\mathrm{STN}}, t|t = 0, ..., H \quad (21)$$

Finally, for each receipt of material, we have

$$R_{s\mathrm{STN},t}^{\min} \le \sum_{s\in S_s\mathrm{STN}} R_{s,t} \quad \forall s^{\mathrm{STN}}, t|R_{s\mathrm{STN},t}^{\max} > 0;\ t = 0, ..., H \quad (22)$$

$$\sum_{s\in S_s\mathrm{STN}} R_{s,t} \le R_{s\mathrm{STN},t}^{\max} \quad \forall s^{\mathrm{STN}}, t|t = 0, ..., H \quad (23)$$

The combination of the above production-requirement constraints enables the definition of arbitrary time profiles of product deliveries and raw material receipts to/from individual locations, as well as the global production amounts.

In conclusion, the objective function in eq 1, along with an appropriate selection of constraints (eqs 2−34 of part I[2] and eqs 2−32 of part II), define the MILP model for the simultaneous layout and design of multipurpose batch processing plants.

5. Examples

The general algebraic modeling system (GAMS[12]) was used, coupled with the CPLEX optimization package (version 6.5). All of the problems were solved on a Pentium II 450-MHz computer.

Example 1. A case study based on one of the examples proposed by Barbosa-Póvoa and Macchietto[1] and also discussed in Barbosa-Póvoa et al.[9] is presented to illustrate the model applicability.

A plant is designed to produce two different products (P1 and P2) through the following process recipe (see Figure 3): (1) for task T1, heat raw material S1 for 2 h to produce the unstable intermediate S3; (2) for task T2, process raw material S2 for 2 h to form the intermediate S4; (3) for task T3, mix intermediate

Table 1. Equipment Characteristics, Cases 2 and 3

case	unit	suitability	capacity (tonne)	cost (10^3 cu)	elements	dimensions α/β	in	$\Delta x_{oi}/\Delta y_{oi}$	out	$\Delta x_{oi}/\Delta y_{oi}$
2	1a	T1/T2	70	14		8/8	OI1	−3/−0.5	OI14	3/−0.5
							OI2	−3/−1.5	OI15	3/−1.5
3	1a	T1/T2	70	14	1a1	4/8	OI1	−1/−0.5		
							OI2	−1/−1.5		
					1a2	4/4			OI14	1/1.5
									OI15	1/1.5
2	1b	T1/T2	70	15		6/4	OI3	−3/1.5	OI16	3/−1.5
							OI4	−3/0	OI17	3/0.5
3	1b	T1/T2	70	15	1b1	4/8	OI3	−1.5/0.5		
							OI4	−1.5/−1		
					1b2	4/4			OI16	1.5/−1.5
									OI17	1.5/0.5
2/3	1c	T1	70	18		6/2	OI5	−3/0	OI18	3/0
2/3	2a	T3/T4	120	40		5/5	OI6	−2.5/−1		
							OI7	0/−2.5	OI19	2.5/1
							OI8	−2.5/2		
							OI9	2.5/0		
							OI10	−2.5/1.5	OI20	2.5/−2
2/3	V1	store S1	unlimited	1		6/3			OI21	3/1
									OI22	3/−1
									OI23	3/0.5
2/3	V2	store S2	unlimited	13		6/3			OI24	3/1
									OI25	3/0.5
2/3	V4	store S4	50			5/1	OI11	−2.5/0	OI26	0/0.5
2/3	V5	store P1	unlimited	1		4/3	OI12	−2/0	OI27	−2/−1
2/3	V6	store P2	unlimited	1		4/3	OI13	−2/0		

material S3 with material S4 in a ratio of 60:40, and let them react for 4 h to form product P1; and (4) for task T4, mix S3 with P1 in a ratio of 60:40, and let them react for 2 h to form product P2.

The production requirements of this plant are 80 tonne for each material P1 and P2 over a time horizon *H* of 8 h. Three different cases are solved in which it is illustrated how layout restrictions such as space availability might influence the final plant design and associated costs.

Case 1. The first case is the stand-alone design problem, as proposed by Barbosa-Póvoa and Macchietto,[1] without layout considerations. The solution provides the final plant topology and associated schedule for an objective function defined as the sum of all units installed cost, considering only parameter CC_g^0 in eq 1, with all other parameters taken as zero.

Case 2. In case 2, the design and layout problems are solved simultaneously as presented in Barbosa-Póvoa et al.,[9] but using the generalized model proposed in this paper. The solution provides the optimal plant topology, layout, and schedule for an objective function defined as the total cost of units and connectivity, considering only parameters CC_g^0 and CC_c^2 in the objective function in eq 1, with all other parameters taken as zero. A limited availability area of 21 × 6 m^2 is considered (see Table 1).

Case 3. To show the model generality and, in particular, the possibility of modeling irregular equipment shapes, case 2 is again solved, but this time considering units 1a and 1b with irregular shapes (see Table 1).

The plant superstructure used is shown in Figure 4, and the equipment characteristics are given in Tables 1−3, where units 1a, 1b, 1c, 2a, and V4 have fixed capacities. Connections, assuming unlimited capacity, associated suitabilities, and costs are listed in Table 4. Vessels V1 and V2 initially have 200 and 100 tonne each of S1 and S2, respectively.

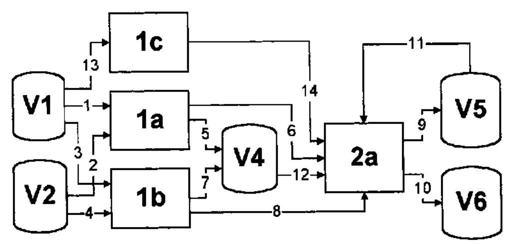

Figure 4. Plant superstructure topology.

Table 2. Links for Equipments with Irregular Forms

element *j*	element *j*′	Δx	Δy
Elem_1a1	Elem_1a2	4	−2
Elem_1b1	Elem_1b2	3	−1

Table 3. Connections and Associated Costs[a]

connection	suitability	output	input
C1	S1	OI21	OI1
C2	S2	OI24	OI2
C3	S1	OI22	OI3
C4	S2	OI25	OI4
C5	S4	OI15	OI11
C6	S3/S4	OI14	OI8
C7	S4	OI16	OI11
C8	S3/S4	OI17	OI6
C9	S5	OI19	OI12
C10	S6	OI20	OI13
C11	S5	OI27	OI9
C12	S4	OI26	OI7
C13	S1	OI23	OI5
C14	S3	OI18	OI10

[a] Connection costs = 500 cu/m (currency units per meter).

Figure 5 shows the mSTN representation and the relevant information concerning eStates, iStates, oStates, eTasks, tTasks (π), and associated connections *c*, as well as sets T_s^{in}, T_s^{out}, Π_s^{in}, and Π_s^{out}.

Production requirements bounds $Q_{sSTNProd}^{min}$, $Q_{sSTNProd}^{max}$, $D_{sSTN,t}^{min}$, $D_{sSTN,t}^{max}$, $R_{sSTN,t}^{min}$, and $R_{sSTN,t}^{max}$ are all assumed to be

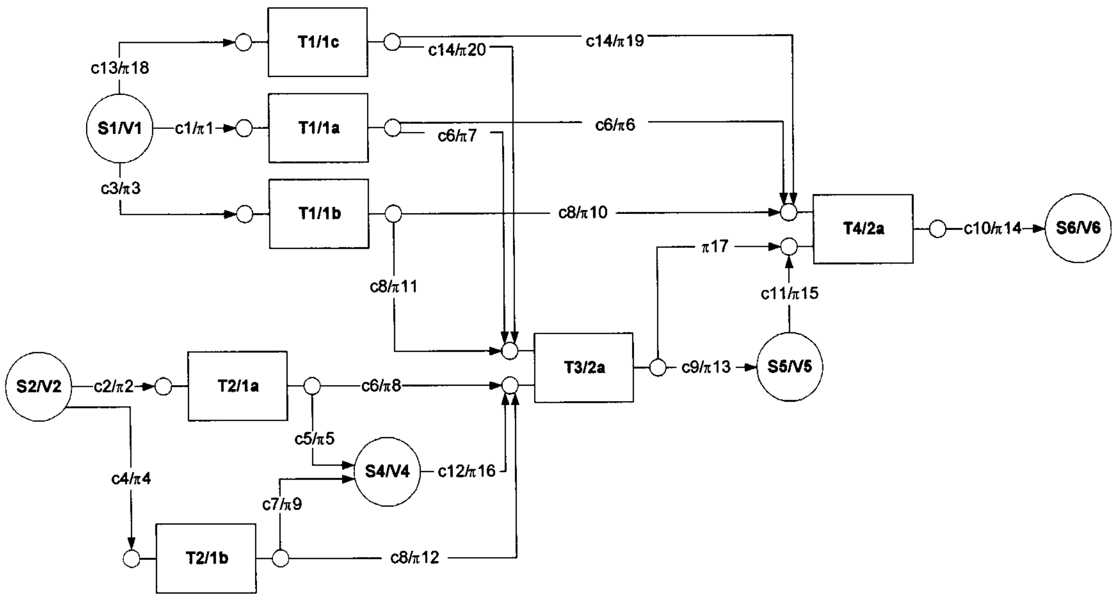

Figure 5. mSTN representation.

Table 4. Problem Statistics[a]

case	OF	CPU s	nodes	iterations	NIV	NV	NC
1	73 000	0.17	6	28	79	481	756
2	77 000	2.48	326	3854	196	758	1105
3	77 000	4.23	412	4894	230	800	1189

[a] OF = objective function, NIV/NV = number of integer/total variables, NC = number of constraints.

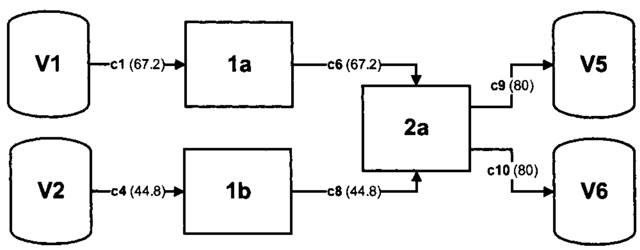

Figure 6. Optimal plant topology, case 1.

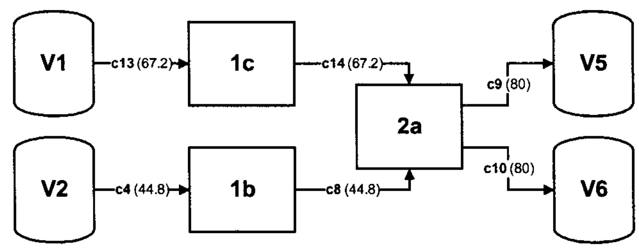

Figure 7. Optimal plant topology, cases 2 and 3.

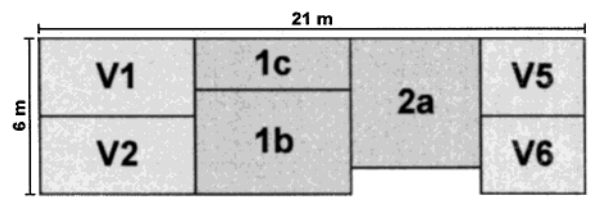

Figure 8. Optimal plant layout, case 2.

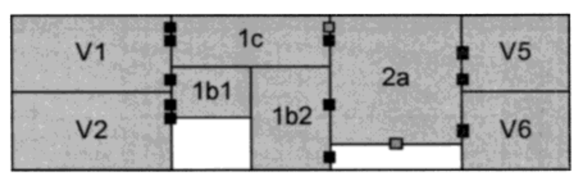

Figure 9. Optimal plant layout, case 3.

null, with the exception of product STN states S5 (product P1) and S6 (product P2), as stated before. These states have the following minimum and maximum production requirements: $Q_{S5}^{\min} = Q_{S6}^{\min} = Q_{S5}^{\max} = Q_{S6}^{\max} = 80$. All utilization factors $\phi_{i,g}^{\max}$ are equal to 1 and $\phi_{i,g}^{\min}$ to 0. All size factors $\phi_{s,g}$ and $\phi_{\pi,c}$ equal unity.

The results obtained showed that, for case 1, the final plant is characterized by the choice of processing units 1a, 1b, and 2a, along with vessels V1, V2, V5, and V6 (see Figure 6). This corresponds to a capital cost of 73 000 cu (where cu denotes currency unit). These units and related capacities (V_g), along with the associated connections and respective capacities (BT_c), are depicted in Figure 6.

For case 2, because of space restrictions, the plant topology differs from case 1, where unit 1a was chosen in place of unit 1c (Figure 7), resulting in a more costly plant (77 000 vs 73 000 cu). The associated plant layout is shown in Figure 8.

The difference in optimal plant topology between cases 1 and 2 is explained by the space availability restrictions. In this context, unit 1a, which has higher space requirements (8/8) when compared with unit 1c (6/2), cannot be installed in the final plant because of the restricted total space availability (21×6 m^2). Indeed, when trying to optimize the layout of the plant topology obtained from case 1 with the space restrictions defined in cases 2 and 3, an infeasible problem is obtained. This shows the importance of considering design and layout aspects simultaneously in the design of a plant.

When comparing the final plant layouts for cases 3 and 2 (Figures 8 and 9), different space occupations are obtained, with no effect on the final objective function (see Table 4). This translates into a more accurate representation of the equipment space occupation, allowing for a better management of the plant space (see space occupied by equipment unit 1b).

In operational terms, for case 1, unit 1a is performing task T1, unit 1b is performing task T2, unit 2a is a multitask unit performing tasks T3 and T4, and no

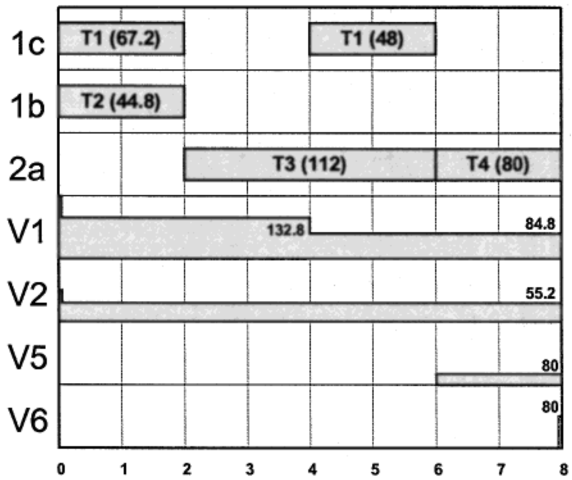

Figure 10. Optimal plant scheduling, cases 2 and 3.

intermediate storage is chosen (not depicted). For cases 2 and 3 (see Figure 10), in which aspects of operation, design, and layout are treated simultaneously, the results obtained are different from those obtained in case 1. In these cases, the processing units chosen are 1b, 1c, and 2a, together with tanks V1, V2, V5, and V6.

Some computer statistics for the final results are shown in Table 4 (both cases with 1% margin of optimality).

Cases 1 and 2 are solved quite efficiently, although it is worth noting that case 1 does not cover layout. For case 3, the presence of irregular equipment units did not significantly increase the size of the model, and therefore, a very similar solution time was obtained when compared to case 2 (4.23 vs 2.48 CPU s; see Table 4).

Example 2. Based on one of the examples proposed by Barbosa-Póvoa,[4] a new example is presented. A plant must be designed to produce two different products (P1 and P2) through the following process recipe (see Figure 11): (1) for task TT1, heat feed F1 for 1 h to produce intermediate I1; (2) for task TT2, mix feed F2 and feed F3 in a ratio of 50:50, and let them react for 2 h to form 40% of intermediate I2; (3) for task TT3, mix I1 (hot F1) and intermediate I2 in a ratio of 40:60, and let them react for 2 h to form I3 and product 1 (state P1) in a ratio of 60:40 (natural separation); (4) for task TT4, mix feed F3 and intermediate I3 in a ratio of 20:80, and let them react for 1 h to form the impure material represented by state I4; and (5) for task TT5, distill impure material I4 to separate product 2 (state P2) after 1 h, and recover pure intermediate I3, which is recycled in the ratio of 90:10, after 2 h.

The design and layout problems are solved simultaneously. The solution provides the final plant topology, layout, and schedule for an objective function defined as the sum of the cost of units (fixed and capacity-dependent) and the cost of connections (fixed and distance-dependent), i.e., the objective function in eq 1. The minimum production requirements of this plant are 20 and 10 tonne for materials P1 and P2, respectively, over a time horizon H of 12 h. A limited availability of 20×20 m^2 is considered for the area.

The equipment characteristics are given in Table 5, where capacities are given in tonnes and costs in 10^3 currency units (cu). Connections, assuming unlimited capacity, associated suitability and costs are listed in Table 6. Vessels VV1, VV2, and VV3 initially have 50 tonne of F1, 100 tonne of F2, and 100 tonne of F3, respectively.

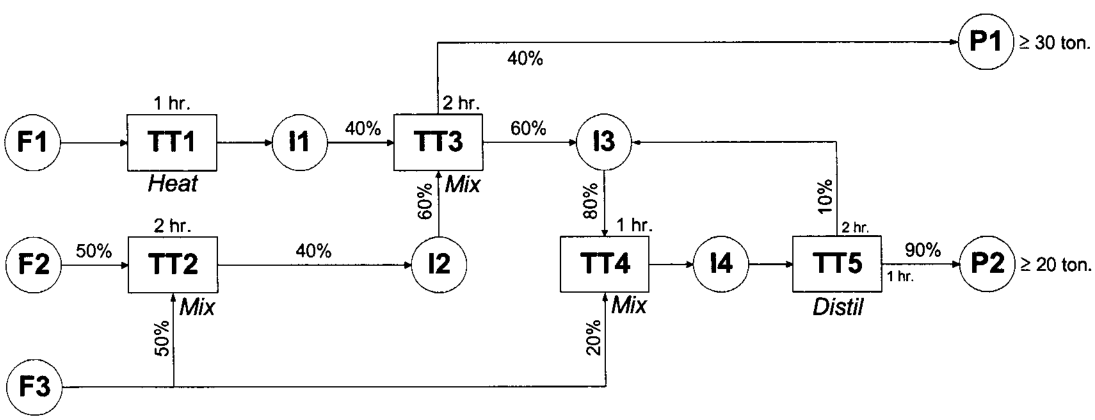

Figure 11. Process recipe.

Table 5. Equipment Characteristics

unit	suitability	capacity min/max	costs fixed/variable	dimensions α/β	in	$\Delta x_{oi}/\Delta y_{oi}$	out	$\Delta x_{oi}/\Delta y_{oi}$
H	TT1	20/50	10/0.02	5/3	OI1	−2.5/0	OI11	2.5/0
R1	TT2, TT3, TT4	50/70	15/0.05	7/4	OI2	−3.5/−2	OI12	3.5/2
R2	TT2, TT3, TT4	50/70	12/0	8/3	OI3	−4/1.5	OI13	4/1.5
St	TT5	50/80	15/0.03	6/4	OI4	0/−2	OI14	−3/2
VV1	store F1	unlimited	0	4/2			OI15	1/0
VV2	store F2	unlimited	0	7/3			OI16	3.5/0
VV3	store F3	unlimited	0	5/3			OI17	2.5/1.5
VV4	store I1	10/30	3/0.01	4/2	OI5	−2/−1	OI18	2/1
VV5	store I3	10/70	1/0.01	4/2	OI6	−2/0	OI19	2/0
VV6	store I2	10/60	1.5/0.01	6/3	OI7	−3/1.5	OI20	3/1.5
VV7	store I4	50/100	2/0.02	5/4	OI8	−2.5/2	OI21	0/−2
V8	store P1	unlimited	0	8/4	OI9	4/1		
V9	store P2	unlimited	0	7/2	OI10	3.5/0		

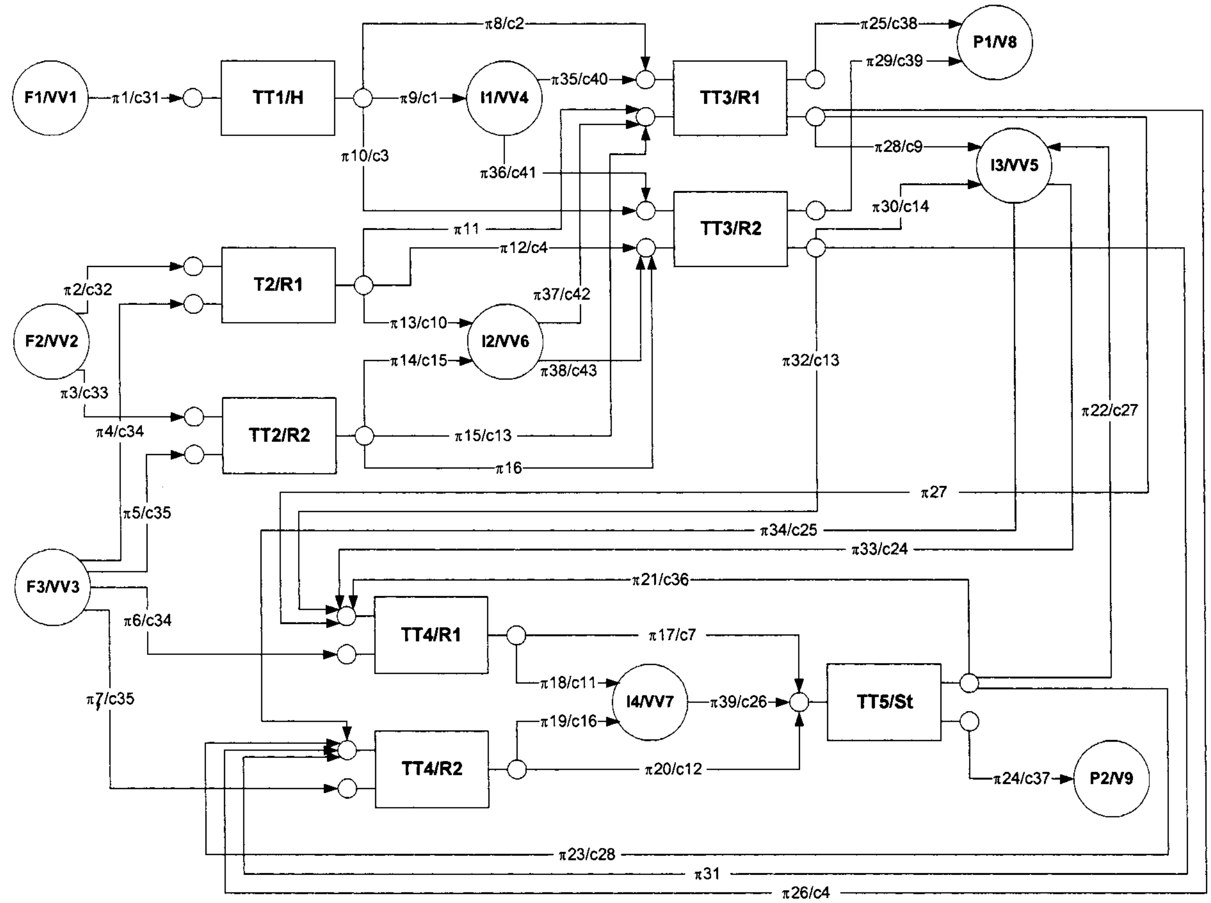

Figure 12. mSTN representation.

Table 6. Connections and Associated Costs

connection	cost	suitability	out	in
1	5/0.02	I1	OI11	OI5
2	7/0.03	I1	OI11	OI2
3	7/0.03	I1	OI11	OI3
4	7/0.03	I2, I3	OI12	OI3
7	3/0.015	I4	OI12	OI4
9	5/0.02	I3	OI12	OI6
10	5/0.02	I2	OI12	OI7
11	5/0.02	I4	OI12	OI8
12	7/0.03	I4	OI13	OI4
13	7/0.02	I2, I3	OI13	OI2
14	5/0.02	I3	OI13	OI6
15	5/0.02	I2	OI13	OI7
16	5/0.02	I4	OI13	OI8
24	2/0.01	I3	OI19	OI2
25	2/0.01	I3	OI19	OI3
26	5/0.02	I4	OI21	OI4
27	5/0.02	I3	OI14	OI6
28	7/0.03	I3	OI14	OI3
31	0	F1	OI15	OI1
32	0	F2	OI16	OI2
33	0	F2	OI16	OI3
34	0	F3	OI17	OI2
35	0	F3	OI17	OI3
36	0	I3	OI14	OI2
37	0	P2	OI14	OI10
38	0	P1	OI12	OI9
39	0	P1	OI13	OI9
40	0	I1	OI18	OI2
41	0	I1	OI18	OI3
42	0	I2	OI20	OI2
43	0	I2	OI20	OI3

Figure 12 shows the mSTN representation and the relevant information concerning eStates, iStates, oStates, eTasks, tTasks (π) and associated connections c, as well as sets T_s^{in}, T_s^{out}, Π_s^{in}, and Π_s^{out}.

Production requirement bounds $Q_{s\text{STNProd}}^{\min}$, $Q_{s\text{STNProd}}^{\max}$, $D_{s\text{STN},t}^{\min}$, $D_{s\text{STN},t}^{\max}$, $R_{s\text{STN},t}^{\min}$, and $R_{s\text{STN},t}^{\max}$ are all assumed to be null, with the exception of product STN states P1 and P2, which have the following minimum and maximum production requirements: $Q_{P1}^{\min} = 20$, $Q_{P1}^{\max} = 300$, $Q_{P2}^{\min} = 10$, and $Q_{P2}^{\max} = 200$. All utilization factors $\phi_{i,g}^{\max}$ are equal to 1, and $\phi_{i,g}^{\min}$ equals 0. All size factors $\phi_{s,g}$ and $\phi_{\pi,c}$ are equal to unity.

The final plant is characterized by the choice of processing units H, R2, and St along with vessels VV1, VV2, VV3, VV5, V8, and V9 (see Figure 13). These units and related capacities (V_g), along with the connections installed and respective capacities (BT_c), are depicted in Figure 13. This corresponds to a capital cost of 40 956.395 cu.

The operational schedule is illustrated in Figure 14. In operational terms, unit H performs task TT1; unit R2 is a multitask unit performing tasks TT2, TT3, and TT4; and finally, unit St performs task TT5. In terms of storage, the choice of VV5 indicates the need to store intermediate material I3, whereas the remaining storage vessels are allocated to raw materials (VV1, VV2, VV3) and final products (V8 and V9).

The optimal plant produces 32.3 tonne of P1, stored in vessel V8, and 10 tonne of P2, stored in vessel V9, thus meeting the required production requirements.

The optimal layout obtained is depicted in Figure 15.

In terms of computer statistics the final results are shown in Table 7 (with a 0.001% margin of optimality). The problem is solved quite efficiently.

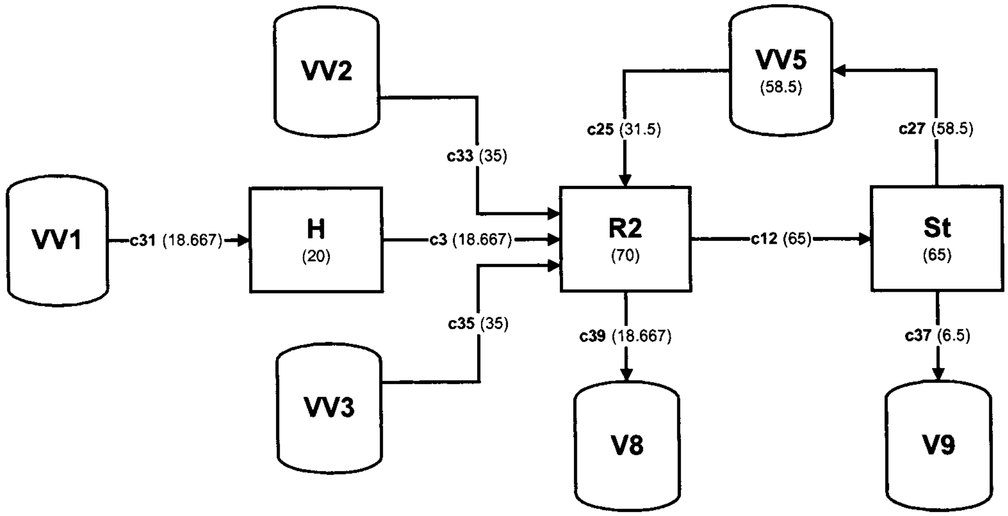

Figure 13. Optimal plant topology.

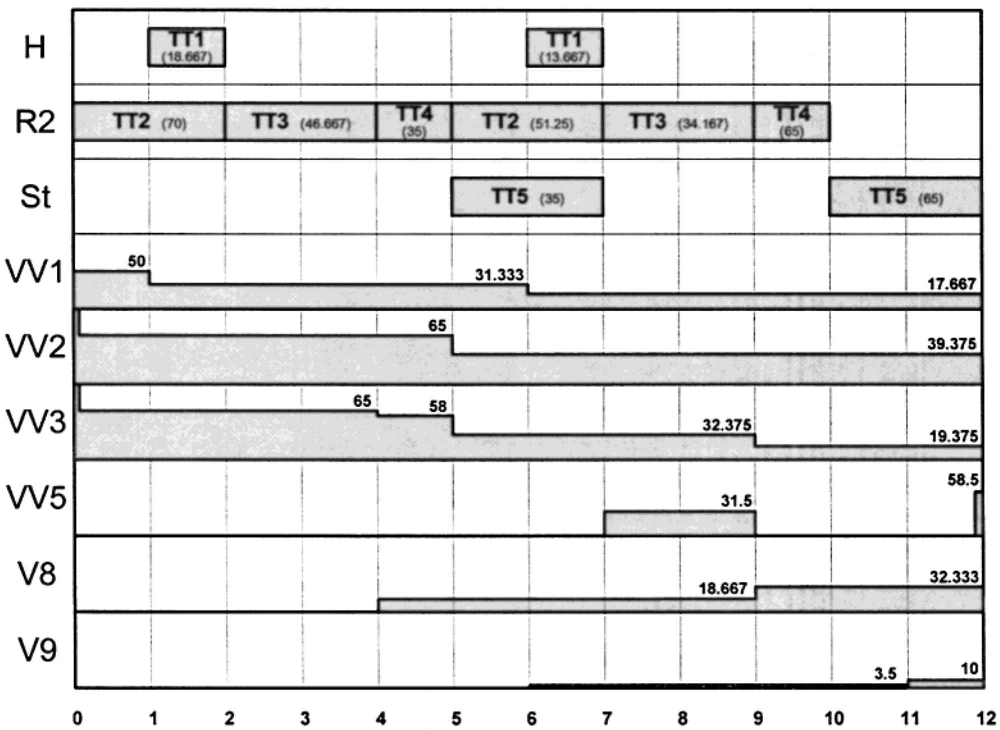

Figure 14. Optimal plant scheduling.

Table 7. Problem Statistics

OF	CPU s	nodes	iterations	NIV	NV	NC
57 400.59	47.51	2462	39 871	356	1 599	2 315

[a] OF = objective function, NIV/NV = number of integer/total variables, NC = number of constraints.

6. Conclusions and Future Work

The simultaneous approach to the solution of the design and layout problems of industrial facilities was applied to a number of illustrative examples that address aspects to be found in real-case multipurpose batch processing facilities. This approach was based on the detailed design model developed by Barbosa-Póvoa and Macchietto.[1]

The simultaneous design and layout model considered the layout problem over a two-dimensional continuous area, as presented in part I of this work,[2] along with design constraints such as the processing units existence, capacity, and batch size, as well as storage, connectivity, mass balance, and production requirements.

Again, the problem results in an MILP that provides the optimal plant layout, design, and schedule based on a specified economic goal: minimization of costs. Several examples were solved, and good results were obtained.

In conclusion, it can be stated that the integration of the variable layout formulation with a specific design case, multipurpose batch facilities, was achieved with success.

A matter for future work is the generalization of the proposed model to include other layout characteristics such as three-dimensional designs, along with the more generic multipurpose batch processing characteristics as presented in Barbosa-Póvoa.[4] The authors have

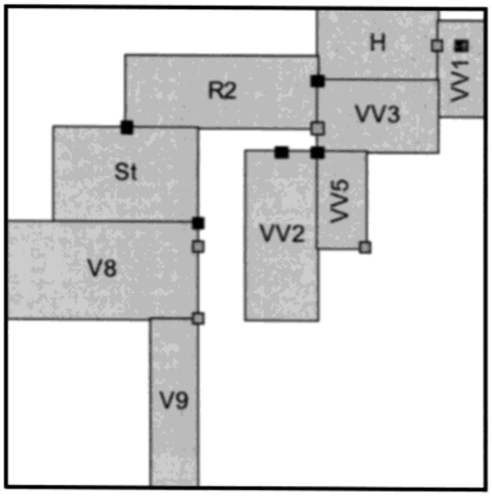

Figure 15. Optimal plant layout.

already investigated the three-dimensional extension with promising results.

Nomenclature

Global Indices

t, t' = time
i = processing and storage task (eTask)
π = transfer task (tTask)
s = mSTN state (including iStates $\gg$ oStates $\gg$ eStates)
s^{STN} = STN state
s^{STNProd} = product STN state

Sets

PU = {g: set of processing equipment units}
DSV = {g: set of dedicated storage vessels}
I_g = {i: set of eTasks i which can be performed in equipment unit g}
e^{state} = {s: set of eStates}
i^{state} = {s: set of iStates}
o^{state} = {s: set of oStates}
s^{STN} = {s: set of STN states}
S_s^{STN} = {s: set of mSTN eStates generated from STN state $s \in s^{\mathrm{STN}}$}
S_s^{STNProd} = {s: set of mSTN eStates generated from STN product state $s \in s^{\mathrm{STNProd}}$}
K_i = {g: set of processing equipment units g suitable for eTask i}
K_s = {g: set of dedicated storage vessels g suitable for storing eState s}
I_c = {π: set of tTasks π that can be performed in connection c}
Π_g^{sink} = {π: set of tTasks π for which equipment unit g is a sink}
T_s^{in} = {i: set of eTasks i receiving material from state s}
T_s^{out} = {i: set of eTasks i producing material to state s}
Π_s^{in} = {π: set of tTasks π transferring material from state s}
Π_s^{out} = {π: set of tTasks π transferring material to state s}

Parameters

CC_g^0 = fixed capital cost of equipment unit g
CC_g^1 = size-dependent capital cost of equipment unit g
CC_c^0 = fixed capital cost of connection c
CC_c^2 = distance-dependent capital cost of connection c
$p_{i,s}^{\mathrm{lag}}$ = time lag of the input state s entering eTask i relative to the start of the eTask i
$p_{i,s}^{\mathrm{proc}}$ = processing time for eTask i to produce the output state s relative to the start of eTask i
p_i = duration of eTask i: $\max\{p_{i,s}^{\mathrm{proc}}\}$
ρ_{is} = proportion of material from input state s entering eTask i
$\bar{\rho}_{is}$ = proportion of material to output state s leaving eTask i
$\phi_{i,g}^{\max}$ = maximum utilization factor of eTask i in equipment unit g [this parameter can be considered as a conversion factor between the maximum batch size units and the capacity units of the processing vessel (e.g., kg/L) or just a maximum fraction of the capacity that can be used for eTask i]
$\phi_{i,g}{}^{\min}$ = minimum utilization factor of eTask i in equipment unit g (minimum capacity fraction that can be used)
$\phi_{s,g}$ = size factor of eState s [this parameter can be considered as a conversion factor between the maximum amount stored and the capacity units of the storage vessel g dedicated to eState s (i.e., kg/L) or the just fraction used]
$\phi_{\pi,c}$ = size factor of tTask π in connection c [this parameter can be considered as a conversion factor between the maximum amount transferred and the capacity units of the connection (i.e., kg/L) or just the fraction used]
$V_g^{\min}$ = minimum capacity of equipment unit g
$V_g^{\max}$ = maximum capacity of equipment unit g
$\mathrm{BT}_c^{\min}$ = minimum capacity of connection c
$\mathrm{BT}_c^{\max}$ = maximum capacity of connection c
S_s^{ini} = initial amount of material in eState s at the beginning of production (before $t = 0$)
$S_{s^{\mathrm{STNProd}}}^{\mathrm{ini}}$ = initial amount of material in product STN state $s \in s^{\mathrm{STNProd}}$ at the beginning of the production (before $t = 0$)
$Q_{s^{\mathrm{STNProd}}}^{\min}$ = minimum production requirement of product STN state $s \in s^{\mathrm{STNProd}}$ at the end of the horizon
$Q_{s^{\mathrm{STNProd}}}^{\max}$ = maximum production requirement of product STN state $s \in s^{\mathrm{STNProd}}$ at the end of the horizon
$D_{s^{\mathrm{STN}},t}^{\min}$ = minimum amount of intermediate material delivered to the outside from STN state $s \in s^{\mathrm{STN}}$ at the beginning of period t
$D_{s^{\mathrm{STN}},t}^{\max}$ = maximum amount of intermediate material delivered to the outside from STN state $s \in s^{\mathrm{STN}}$ at the beginning of period t
$R_{s^{\mathrm{STN}},t}^{\min}$ = minimum amount of intermediate material received from the outside into STN state $s \in s^{\mathrm{STN}}$ at the beginning of period t
$R_{s^{\mathrm{STN}},t}^{\max}$ = maximum amount of intermediate material received from the outside into STN state $s \in s^{\mathrm{STN}}$ at the beginning of period t

Variables

Continuous Variables (All Defined as Positive)

$B_{i,g,t}$ = amount of material that starts undergoing eTask i in unit g at the beginning of period t ($t = 0, ..., H - p_i$)
$S_{s,t}$ = amount of material in eState s at the beginning of period t ($t = 0, ..., H$)
$\mathrm{BT}_{\pi,t}$ = amount of material transferred by transfer task π at the beginning of period t ($t = 0, ..., H$)
$D_{s,t}$ = amount of material delivered from state s to the outside at the beginning of period t ($t = 0, ..., H$)
$R_{s,t}$ = amount of material received from the outside into state s at the beginning of period t ($t = 0, ..., H$)
V_g = capacity of equipment unit g
BT_c = capacity of connection c

Binary Variables

$W_{i,g,t}$ = allocation variable of eTask i in unit g at t, which equals 1 if unit g starts processing eTask i at the beginning of period t and 0 otherwise ($t = 0, ..., H - p_i$)

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Literature Cited

(1) Barbosa-Póvoa, A. P. F. D.; Macchietto, S. Detailed design of multipurpose batch plants. *Comput. Chem. Eng.* **1994**, *18*, 1013−1042.

(2) Barbosa-Póvoa, A. P.; Mateus, R.; Novais, A. Q. Optimal design and layout of industrial facilities. Simultaneous approach. *Ind. Eng. Chem. Res.* **2002**, *41*, 3601−3609.

(3) Papageorgaki, S.; Reklaitis, G. V. Optimal design of multipurpose batch plants. 1. Problem formulation. *Ind. Eng. Chem. Res.* **1991**, *20* (10), 4852−486.

(4) Barbosa-Póvoa, A. P. F. D. Detailed Design and Retrofit of Multipurpose Batch Plants. Ph.D. Thesis, Imperial College, University of London, London, U.K., 1994.

(5) Lin, X.; Floudas, C. A. Design synthesis and scheduling of multipurpose batch Plants via an effective continuous-time formulation. In *Computer-Aided Chemical Engineering 8*; Pierucci, S., Ed.; Elesevier: New York, 2001.

(6) Georgiadis, M. C.; Rotstein, G. E.; Macchietto, S. Optimal layout design in multipurpose batch plants. *Ind. Eng. Chem. Res.* **1997**, *11*, 4852−4863.

(7) Papageorgiou, L.; Rotstein, G. E. Continuous domain mathematical models for optimal process plant layout. *Ind. Eng. Chem. Res.* **1998**, *37*, 3631−3639.

(8) Realff, M. J.; Shah, N.; Pantelides, C. C. Simultaneous design, layout and scheduling of pipeless batch plants. *Comput. Chem. Eng.* **1996**, *20*, 6−7, 869−883.

(9) Barbosa-Póvoa, A. P.; Mateus, R.; Novais, A. Q. Simultaneous design and layout of batch processing facilities. In *Foundations of Computer-Aided Process Design*; Malone, M., Trainhaim, J., Carnham, B., Eds.; AIChE Symposium Series; American Institute of Chemical Engineers (AIChE): New York, 1999; Vol. 323, p 96.

(10) Kondili, E.; Pantelides, C. C.; Sargent, R. W. H. A General algorithm for scheduling batch operations. In *Proceedings of the 3rd International Symposium on Process Systems Engineering*; 1988; pp 253−274.

(11) Shah, N. Efficient scheduling, planning and design of multipurpose batch plants. Ph.D. Thesis, Imperial College, University of London, London, U.K., 1992.

(12) Brooke, A.; Kendrick, D.; Meeraus, A. *GAMS: A User's Guide*; The Scientific Press: San Francisco, CA, 1988.

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