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Fuzzy Control and Model Predictive Control Configurations for Effluent Violations Removal in Wastewater Treatment Plants

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ABSTRACT: In this paper the following new control objectives for biological wastewater treatment plants (WWTPs) have been established: to eliminate violations of total nitrogen in the effluent ($N_{tot,e}$) or ammonium and ammonia nitrogen concentration (NH) in the effluent (NH_e) and at the same time handle the customary requirements of improving effluent quality and reducing operational costs. The Benchmark Simulation Model No. 1 (BSM1) is used for evaluation, and the control is based on Model Predicitive Control (MPC) and fuzzy logic. To improve effluent quality and to reduce operational costs, a hierarchical control structure is implemented to regulate the dissolved oxygen (DO) on the three aerated tanks. The high level of this hierarchical structure is developed with a fuzzy controller that adapts the DO set points of the low level based on the NH concentration in the fifth tank (NH5). The low level is composed of three MPC controllers with feedforward control (MPC + FF). For avoiding violations of $N_{tot,e}$, a second fuzzy controller is used to manipulate the external carbon flow rate in the first tank (q_{EC1}) based on nitrate nitrogen in the fifth tank (NO5) plus NH5. For avoiding violations of NH_e, a third fuzzy controller is applied to manipulate the internal recirculation flow rate (Q_{rin}) based on NH5 and NH in the influent. Simulation results show the benefit of the proposed approach.

INTRODUCTION

Biological wastewater treatment plants (WWTPs) are considered complex nonlinear systems, and their control is very challenging, due to the complexity of the biological and biochemical processes that take place in the plant and the strong fluctuations of the influent flow rate. In addition, there are effluent requirements defined by the European Union (European Directive 91/271 Urban wastewater) with economic penalties.

In the literature there are several papers working on modeling of WWTPs.[1−4] In this work the evaluation and comparison of the different control strategies is based on Benchmark Simulation Model No. 1 (BSM1), developed by the International Association on Water Pollution Research and Control.[5−7] This benchmark defines a plant layout, influent loads, test procedures and evaluation criteria. It provides also a default control strategy that includes two Proportional-Integrative (PI) control loops: control of the dissolved oxygen concentration (DO) in the fifth tank (DO5) at a set point value of 2 g/m^3 by manipulating the oxygen transfer coefficient (K_La) in the fifth tank (K_La5), and control of the nitrate nitrogen concentration (NO) in the second anoxic reactor (NO2) at a set point value of 1 g/m^3 by manipulating the internal recycle flow rate (Q_{rin}). A complete review of results for PI control can be found in ref 8.

Many works can be found in the literature that propose different methods for controlling WWTPs. Some of them apply a direct control on the effluent variables, mainly ammonium and ammonia nitrogen (NH) and total nitrogen (N_{tot})[9−11]. The difficulty in this method is that the fixed values for the effluent variables are constraints and not set points to be tracked. Other studies deal with the basic control strategy (DO of the aerated tanks and NO of the last anoxic tank), but testing with different controllers such Model Predictive Controller (MPC) and fuzzy controller.[12−14] These methods provide an acceptable balance between quality and costs. Finally, other investigations propose a hierarchical control that regulates the DO set points, depending on some states of the plant, usually NH and NO concentration values in any tank or in the influent[15−20] or DO in other tanks[21].

The control objectives of previous works are usually based on achieving an improvement in the effluent quality and/or cost indices. However, it is of significant importance to avoid violations of pollution in the effluent, regarding the quality of the water from a legal point of view, and certainly in terms of cost, as these violations involve fines to be paid.

This work proposes a control strategy with the goal of eliminating violations of the effluent pollutants, while achieving an improvement of effluent quality and a reduction of operational costs compared to the default control of BSM1. The proposed approach is implemented by making use of fuzzy and MPC controllers. First, a hierarchical control structure is implemented. The low level is composed by three MPCs with feedforward compensation (MPC + FF) of the influent flow rate (Q_{in}), to control NO2, DO in the third tank (DO3), DO in the fourth tank (DO4), and DO5. The high level is built with a fuzzy controller that adjusts the DO set points according to NH in the fifth tank (NH5). A trade-off analysis is made, which determines a tuning region that simultaneously improves the results of effluent quality and operational cost compared to the default control of BSM1. Next, two fuzzy controllers are added in order to eliminate effluent violations. NH in the effluent (NH_e) and N_{tot} in the effluent ($N_{tot,e}$) are the pollutants that present more difficulties for being kept under the established limits. For reducing peaks of $N_{tot,e}$, external carbon flow rate in the first tank (q_{EC1}) is manipulated based on NO in the fifth tank (NO5)

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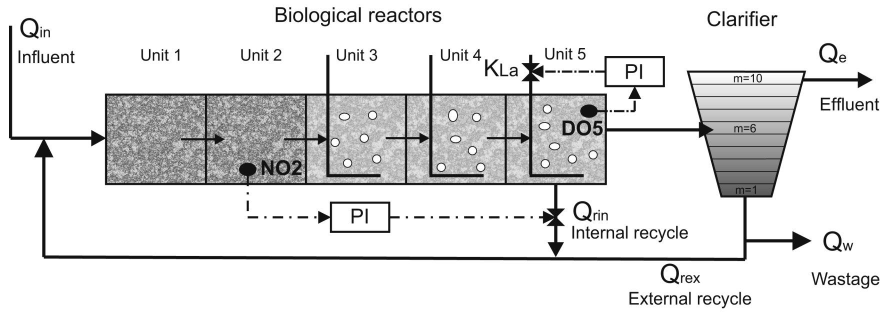

Figure 1. Benchmark Simulation Model No. 1.

plus NH5. For reducing peaks of NH_e, Q_{rin} is manipulated based on NH5, and the control of NO2 is removed.

Other works in the literature have presented proposals for avoiding effluent violations[9−11], with the quality indices as controlled variables. However, ref 9 does not provide costs results, and refs 10 and 11 present a cost increase. The novelty of this work is to simultaneously deal with the elimination of effluent violations, and the improvement of effluent quality and operational costs. Another meaningful novelty of this work is the regulation of Q_{rin} based in NH5 and NH in the influent (NH_{in}) in order to eliminate $N_{tot,e}$.

■ THE TESTING PLANT: BSM1

To make the evaluation and comparison of the different control strategies possible, BSM1[5−7] defines a plant layout, the influents loads of the plant, the procedures for carrying out the tests, and the criteria for evaluating the results. The more relevant aspects are described next:

Figure 1 shows the schematic representation of the wastewater treatment plant. It consists of five biological reactor tanks connected in series, followed by a secondary settler. The first two tanks have a volume of 1000 m^3 each and are anoxic and perfectly mixed. The other three tanks have a volume of 1333 m^3 each and are aerated. The settler has a total volume of 6000 m^3 and is modeled in ten layers, and the sixth layer from the bottom is the feed layer. Two recycle flows complete the system: the first from the last tank and the second from the underflow of the settler. The plant is designed for an average influent dry-weather flow rate of 18446 m^3/d and an average biodegradable chemical oxygen demand (COD) in the influent of 300 g/m^3. Its hydraulic retention time is 14.4 h, based on the average dry weather flow rate and the total tank and settler volume (12000 m^3). The default wastage flow rate (Q_w) is fixed to 385 m^3/d, which determines a biomass sludge age of about 9 days, based on the total amount of biomass present in the system. The nitrogen removal is achieved using a denitrification step performed in the anoxic tanks and a nitrification step carried out in the aerated tanks. The internal recycle is used to supply the denitrification step with NO.

The biological phenomena of the reactors are simulated by the Activated Sludge Model No. 1 (ASM1)[22] that considers eight different biological processes. The vertical transfers between layers in the settler are simulated by the double-exponential settling velocity model.[23] No biological reaction is considered in the settler. The two models are internationally accepted and include 13 state variables.

Despite the fact that BSM1 defines three different influent data, this paper only works with dry weather, that is the most common scenario, which contains 14 days of influent data with sampling intervals of 15 min.

A simulation protocol is established to ensure that results are obtained under the same conditions and can be compared. First, a 150 days period of stabilization has to be completed in a closed-loop using constant influent data to drive the system to a steady-state. Once the steady state is achieved, a simulation with dry weather is run and finally the desired influent data (dry, rain or storm) is tested. Only the results of the last 7 days are considered.

The performance assessment is made at two levels. The first level concerns the control. Basically, this serves as a proof that the proposed control strategy has been properly applied. It is assessed by the Integral of the Squared Error (ISE) criterion. The second level provides measures for the effect of the control strategy on plant performance. It includes the Effluent Quality Index (EQI) and Overall Cost Index (OCI).

The evaluation must include the percentage of time that the effluent limits are not met. The effluent concentrations of N_{tot}, Total Chemical Oxygen Demand (COD_t), NH, Total Suspended Solids (TSS) and Biochemical Oxygen Demand during 5 days (BOD_5) should obey the limits given in Table 1.

Table 1. Effluent Quality Limits

variable	value
N_{tot}	<18 g N·m^{-3}
COD_t	<100 g COD·m^{-3}
NH	<4 g N·m^{-3}
TSS	<30 g SS·m^{-3}
BOD_5	<10 g BOD·m^{-3}

N_{tot} is calculated as the sum of NO and Kjeldahl nitrogen (NKj), with this being the sum of organic nitrogen and NH.

Effluent Quality Index. EQI is defined to evaluate the quality of the effluent. It is related to the fines to be paid due to the discharge of pollution. EQI is averaged over a 7 days observation period, and it is calculated by weighting the different compounds of the effluent loads.

$$EQI = \frac{1}{1000T}\int_{t=7days}^{t=14days}(B_{TSS}\cdot TSS(t) + B_{COD}\cdot COD(t) + B_{NKj}\cdot NK_j(t) + B_{NO}\cdot NO(t) + B_{BOD_5}\cdot BOD_5(t))\cdot Q(t)\cdot dt \quad (1)$$

where B_i are weighting factors (Table 2) and T is the total time.

Overall Cost Index. OCI is defined as

$$OCI = AE + PE + 5\cdot SP + 3\cdot EC + ME \quad (2)$$

where AE is the aeration energy, PE is the pumping energy, SP is the sludge production to be disposed, EC is the external carbon source, and ME is the mixing energy.

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Table 2. B_i Values

factor	B_{TSS}	B_{COD}	B_{NKj}	B_{NO}	B_{BOD5}
Value (g pollution unit g^{-1})	2	1	30	10	2

AE is calculated according to the following relation:

$$AE = \frac{S_o^{sat}}{T\cdot 1.8\cdot 1000}\int_{t=7days}^{t=14days}\sum_{i=1}^{5} V_i\cdot K_La_i(t)\cdot dt \quad (3)$$

where V_i is the volume of the tank i.

PE is calculated as

$$PE = \frac{1}{T}\int_{7days}^{14days}(0.004\cdot Q_{in}(t) + 0.008\cdot Q_{rin}(t) + 0.05\cdot Q_w(t))\cdot dt \quad (4)$$

SP is calculated from the TSS in the flow wastage (TSS_w) and the solids accumulated in the system:

$$SP = \frac{1}{T}\cdot(TSS_a(14days) - TSS_a(7days) + TSS_s(14days) - TSS_s(7days) + \int_{t=7days}^{t=14days} TSS_w\cdot Q_w\cdot dt) \quad (5)$$

where TSS_a is the amount of solids in the reactors and TSS_s is the amount of solids in the settler.

EC refers to the carbon that could be added to improve denitrification.

$$EC = \frac{COD_{EC}}{T\cdot 1000}\int_{t=7days}^{t=14days}(\sum_{i=1}^{i=n} q_{EC,i})\cdot dt \quad (6)$$

where $q_{EC,i}$ is the flow rate of external carbon added to compartment i and COD_{EC} = 400 g COD·m^{-3} is the concentration of readily biodegradable substrate in the external carbon source.

ME is the energy employed to mix the anoxic tanks to avoid settling, and it is a function of the compartment volume:

$$ME = \frac{24}{T}\int_{t=7days}^{t=14days}\sum_{i=1}^{5}[0.005\cdot V_i \text{ if } K_La_i(t) < 20d^{-1} \text{ otherwise } 0]\cdot dt \quad (7)$$

■ CONTROL CONFIGURATION FOR THE PROPOSED OBJECTIVES

The original BSM1 definition includes the so-called default control strategy that is commonly used as a reference.[5−7] This strategy uses two PI control loops as shown in Figure 1. The first one involves the control of DO5 by manipulating K_La5. The set point for DO5 is 2 mg/L. The second control loop has to maintain NO2 at a set point of 1 mg/L by manipulating Q_{rin}.

The control techniques used in this work are based on MPC and fuzzy control. MPC controllers have been used in order to keep the NO2 and DO of the three aerobic reactors at the given set point. Fuzzy control has been applied, on one side, as high level controller in a hierarchical structure to vary the DO references tracked by the MPC controllers, and, on the other hand, to remove effluent violations by determining q_{EC1} and Q_{rin} values. The applied fuzzy controllers manipulate variables based on *if−then* rules, but without the goal of keeping the controlled variable at a set point given. In this case, the control objectives are the improvement of OCI and EQI, and the violations removal of $N_{tot,e}$ and NH_e.

Control Approaches. *Model Predictive Control.* The basis of MPC is the use of an optimization algorithm to solve the control problem and the use of a model of the plant to make predictions of the output variables.[24] At each control interval, Δt, for a prediction horizon, p, and a control horizon, m, ($m < p$), the MPC algorithm computes the sequences of control moves over the horizon m:

$$\Delta u(k),\ \Delta u(k+1),\ ...,\ \Delta u(k+m-1) \quad (8)$$

makes predictions of the outputs variables over a future horizon p (see Figure 2):

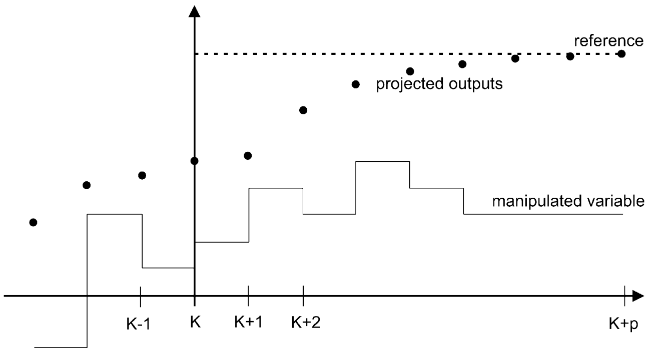

Figure 2. Model Predictive Control performance.

$$\hat{y}(k+1|k),\ \hat{y}(k+2|k),\ ...,\ \hat{y}(k+p|k) \quad (9)$$

and selects the sequence of control moves that minimizes a quadratic objective of the form

$$J = \sum_{l=1}^{p}\left\|\Gamma_y[y(k+l|k) - r(k+l)]\right\|^2 + \sum_{l=1}^{m}\left\|\Gamma_{\Delta u}[\Delta u(k+l-1)]\right\|^2 \quad (10)$$

where the output prediction $y(k+l|k)$ means a predicted controlled output for the future sampling instant $k + 1$, performed at the current instant k, and Γ_y and $\Gamma_{\Delta u}$ are the output weight and input rate weight, respectively, which penalize the residual between the future reference and the output variable prediction, and the control moves.

The MPC algorithm requires a state-space linear model to foresee how the plant outputs, $y(k)$, and reacts to the possible variations of the control variables, $u(k)$, and to compute the control moves at each Δt. WWTPs are nonlinear systems, but their operation can be approximated in the vicinity of a working point by a discrete-time state-space model as

$$x(k+1) = Ax(k) + Bu(k)$$
$$y(k) = Cx(k) + Du(k) \quad (11)$$

where $x(k)$ is the state vector, and A, B, C and D are the state-space matrices.

Fuzzy Control. Fuzzy logic is described as an interpretative system in which objects or elements are related with borders not clearly defined, granting them a relative membership degree and not strict, as is customary in traditional logic. The typical architecture of a fuzzy controller, shown in Figure 3, consists of a fuzzifier, a fuzzy rule base, an inference engine, and a defuzzifier[25,26].

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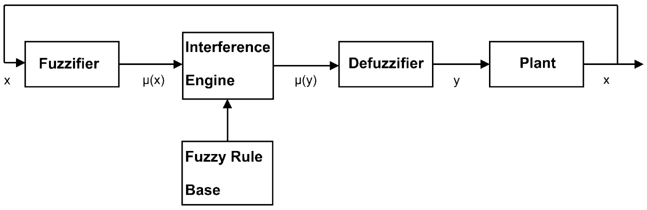

Figure 3. Architecture of a fuzzy controller.

Fuzzy control is defined as a control based on human expertise, determined by words instead of numbers and sentences instead of equations.[25,26] In fact, this does not mean at all that a knowledge of the process dynamics is not needed. Good knowledge of the dynamic behavior of the controlled plant is to be available to the designer. However, process variables are measured in numbers instead of words. For this reason, the fuzzifier adapts the input variables into suitable linguistic values by membership functions. There are different forms of membership functions, e.g. triangular, trapezoidal or Gaussian, and they are chosen according to the user's experience. Range of membership functions values are also set: minimum value of the input variable (*MinIn*), maximum value of the input variable (*MaxIn*), minimum value of the output variable (*MinOut*), maximum value of the output variable (*MaxOut*). The fuzzy rule base is a set of *if-then* rules that store the empirical knowledge of the experts about the operation of the process. A series of relationships that interprets common sense are also defined and can generate a desired action that is applied to the plant. First the fuzzy logic computes the grade of membership of each condition of a rule, and then it aggregates the partial results of each condition using a fuzzy set operator. The inference engine combines the results of the different rules to determine the actions to be carried out, and the defuzzifier converts the control actions of the inference engine into numerical variables, determining the final control action that is applied to the plant. There are two different methods to operate these modules: Mamdani[27] and Sugeno[28]. The Mamdani system aggregates the area determined by each rule, and the output is determined by the center of gravity of that area. In a Sugeno system the results of the *if-then* rules are already numbers determined by numerical functions of the input variables, and therefore, no deffuzifier is necessary. The output is determined weighting the results given by each rule with the values given by the *if* conditions.

For example, Figure 4 shows three triangular membership functions (*mf*1, *mf*2 and *mf*3) with *MinIn* = 0 and *MaxIn* = 5. Thus, an input of 1.5 can be transformed into fuzzy expressions as 0.25 of *mf*1 and simultaneously 0.5 of *mf*2. Figure 5 shows the three membership functions (*mf*4, *mf*5, *mf*6) of the Mamdani defuzzifier with *MinOut* = 0 and *MaxOut* = 5. The *if-then* rules implemented are as follows:

if (*Input* is *mf*1) **then** (*Output* is *mf*4)
if (*Input* is *mf*2) **then** (*Output* is *mf*5)
if (*Input* is *mf*3) **then** (*Output* is *mf*6)

The output is the result of the aggregation of two rules, one that gives 0.25 of *mf*4 and another that gives 0.5 of *mf*5.

EQI and OCI Improvement. To improve EQI and OCI, a hierarchical control is implemented (Figure 6). For the low level control, the two PI controllers of the default BSM1 control strategy are replaced by a MPC + FF configuration with DO5 and NO2 as controlled variables and K_La5 and Q_{rin} as manipulated variables. Two MPC + FF controllers are also added for controlling DO3 and DO4 by manipulating K_La in the third tank (K_La3) and in the fourth tank (K_La4), respectively. A fuzzy controller is proposed for the high level to regulate the DO set points of the low level based on NH5.

Low Level Control. MPC controllers are applied on the low level for the set points tracking of NO2 and DO of the three aerated tanks. Due to the presence of strong disturbances in WWTPs, MPC has difficulties in keeping the controlled variables at their reference level. To compensate the disturbances, a feedforward control is added, as in refs 9−12 and 20. MPC provides options for the feedforward compensation of the measured disturbances, in the same way as for the reference signals. Different variables have been considered for the feedforward action in those works, but in our case Q_{in} has been selected for its better results.

The variables of the state-space model (eq 11) for the three MPC controllers are described as follows: $u_1(k)$ is Q_{rin}, $u_2(k)$ is K_La5, $u_3(k)$ is Q_{in}, $y_1(k)$ is NO2, and $y_2(k)$ is DO5 in the controller of DO5 and NO2; $u_1(k)$ is K_La4, $u_2(k)$ is Q_{in}, and $y_1(k)$ is DO4 in the controller of DO4; and $u_1(k)$ is K_La3, $u_2(k)$ is Q_{in}, and $y_1(k)$ is DO3 in the controller of DO3.

The tuning parameters are Δt, m, p, $\Gamma_{\Delta u}$, Γ_y, and the overall estimator gain.

- Δt has a significant effect on the effectiveness of the controller. High Δt can give less controller performance,

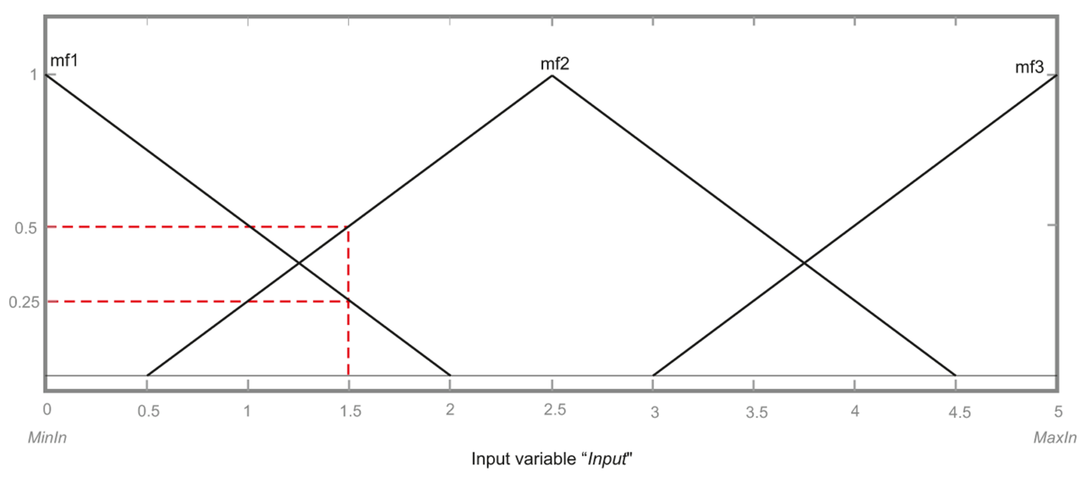

Figure 4. Example of membership functions of fuzzifier.

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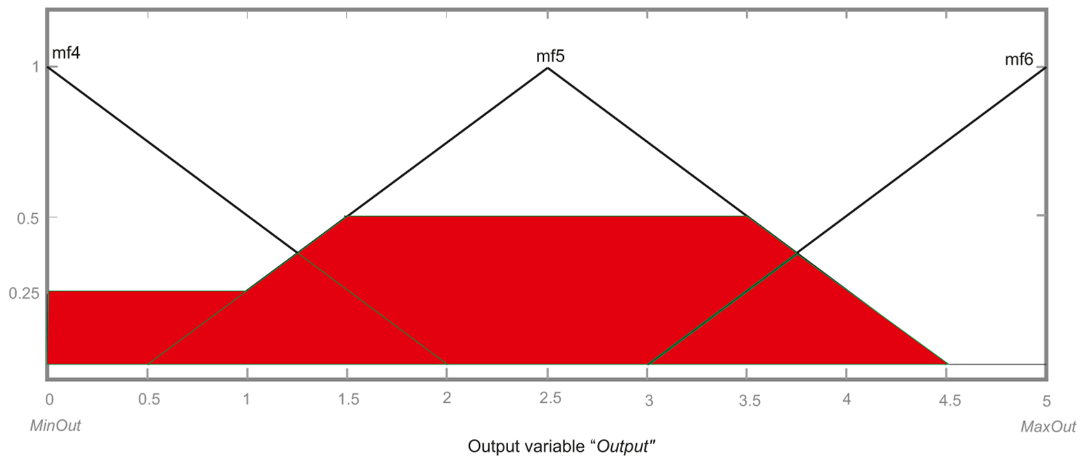

Figure 5. Example of membership functions of defuzzifier.

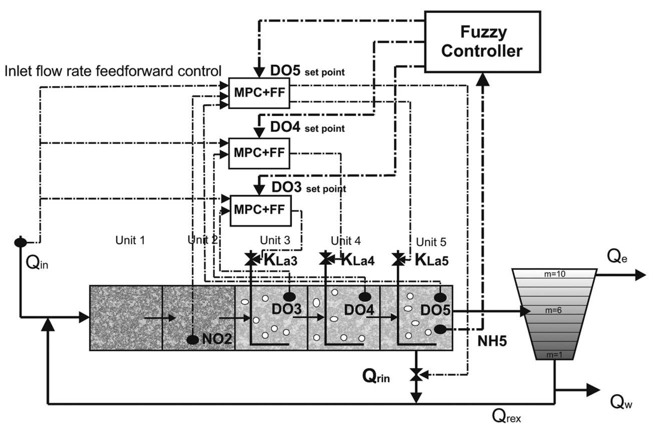

Figure 6. BSM1 with two-level hierarchical control for EQI and OCI improvement.

mainly when there are important input disturbances, and low Δt can produce changes too quickly in the actuators and also high energy consumption. Therefore, the chosen Δt has been the lowest one that allows achieving a successful tracking of the controlled variables, without abrupt changes in the actuators and without a significant aeration cost increase.

- Lower $\Gamma_{\Delta u}$ or higher Γ_y gives better performance of the controlled variable; otherwise, they could produce strong oscillations in the actuators that must be avoided.
- *m* and *p* should be adjusted in each case depending on the control system. However, values that are too high can increase the computational time in excess, and on the other hand, values that are too small may result in oscillatory responses or may not work at all.
- At each Δt the controller compares the measured values of the outputs with the expected values. The difference can be due to noise, to measurements errors, and to unmeasured disturbances. With the overall estimator gain parameter the percentage of this difference that is attributed to unmeasured disturbances is determined, and the calculation matrix is consequently adjusted. Higher overall estimator gains improve the results, but too high values can make the controller unfeasible.

High Level Control. The controller proposed for the high level is a fuzzy controller, that varies DO3, DO4 and DO5 set points based on NH5.

The values of NH and NO depend largely on their reaction rate, which is the result of several processes given by ASM1, which describes the biological phenomena that take place in the reactors. When NH increases, more DO is needed for nitrification. On the contrary, when NH decreases, less DO is required, producing less NO. NH5 is not forced to keep a fixed value since it has been observed that it is not possible by manipulating the DO set point due to the large disturbance[15−18,20].

Effluent Violations Removal. Two fuzzy controllers are applied for avoiding $N_{tot,e}$ or NH_e violations. These control strategies are implemented simultaneously with the hierarchical control structure, in order to achieve a reduction of EQI and OCI at the same time.

$N_{tot,e}$ Violation Removal. The variables with the highest influence in N_{tot} are NO and NH. Further efforts to reduce more NH increasing nitrification, result also in an increment of NO, and consequently $N_{tot,e}$ is not decreased. According to the biological processes of ASM1, an increase of substrate produces a growth of $X_{B,H}$ and therefore the denitrification process and the consequent reduction of NO are improved. Therefore, $N_{tot,e}$ is reduced with the dosage of EC in the first tank (EC1). However, dosing EC1 results in an increase of operational costs (eq 2), so it is important to dose EC1 only when a violation of $N_{tot,e}$ could take place. Consequently, the control strategy is based on the manipulation of q_{EC1} according to NH5 plus NO5. A fuzzy controller is proposed to regulate q_{EC1} as can be seen in Figure 7. The maximum q_{EC1} value was limited to $5m^3/d$. For this control objective, the tuning parameters of the high level fuzzy controller of the hierarchical structure are chosen with the aim to reduce as much as possible the percentage of time of NH_e violations and not to exceed the OCI value of the default PI controller when EC1 is added. A trade-off analysis is made for this tuning parameters selection.

NH_e Violations Removal. With the goal of removing NH_e violations, Q_{rin} is manipulated based on NH5 and NH_{in}. Therefore, the MPC of the low level that controls DO5 and NO2 by manipulating K_{La5} and Q_{rin} is replaced by a MPC with one input (DO5) and one output (K_{La5}) (see Figure 8).

To facilitate the understanding of the proposed solution some considerations about the propagation of the peaks in the reactor are provided: When a peak of pollution enters in the reactors, it is propagated through them with a delay determined by the retention time. So any change in the influent flow rate or in the Q_{rin} directly affects the propagation of the peaks of

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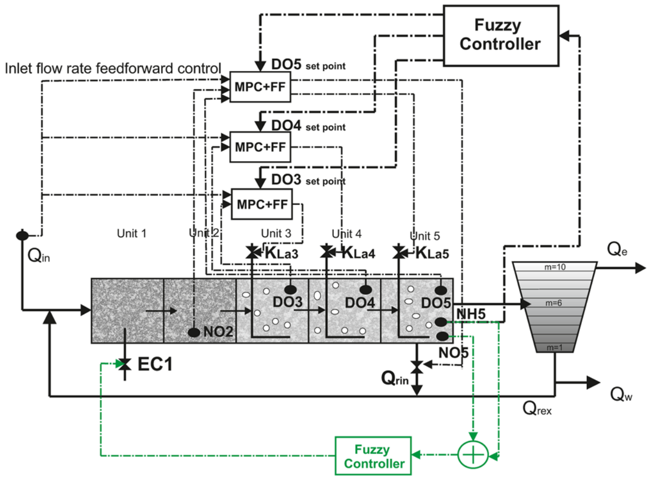

Figure 7. BSM1 with a control strategy for $N_{tot,e}$ removal.

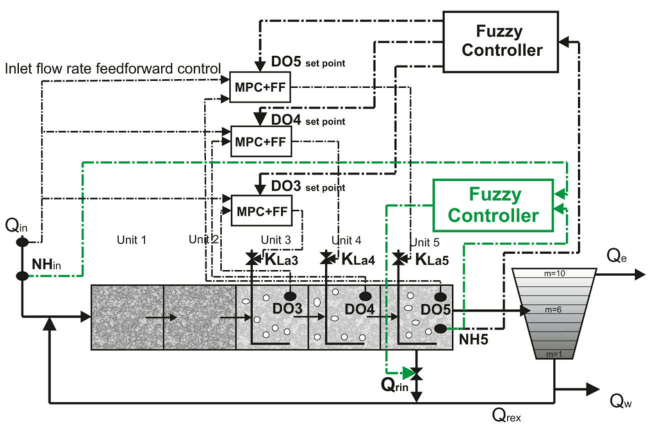

Figure 8. BSM1 with a control strategy for NH removal.

pollution inside the tanks. On the contrary, the peaks of flow rate are transmitted to all the plant immediately, because the system is always full and any variation in the influent causes an identical variation in the effluent and inside the system. Thus, according to the mass balance equation in the first reactor (eq 12), when NH_{in} increases, Q_{rin} is incremented to reduce the rise of NH in the first tank (NH1), and when the increase of NH arrives to the fifth tank, and Q_{rin} is reduced to increase the retention time and so to improve the de-nitrification process.

$$\frac{dNH1}{dt} = \frac{1}{V1}(Q_{rin}\cdot NH_{rin} + Q_{rex}\cdot NH_{rex} + Q_{in}\cdot NH_{in} + r_{NH1}\cdot V1 - Q1\cdot NH1) \tag{12}$$

$$Q1 = Q_{rin} + Q_{rex} + Q_{in}$$

where NH_{rin} is NH in the internal recirculation, NH_{rex} is NH in the external recirculation, r_{NH1} is r_{NH} in the first tank, and Q1 is the flow rate in the first tank.

A fuzzy controller is proposed for this control strategy. And the tuning parameters are different when there are peaks of NH_{in} or NH5 and the rest of the time. They are determined by a trade-off analysis of OCI and the percentage of operating time of $N_{tot,e}$ violation, reflecting only the results that avoid the NH_e violations.

■ SIMULATION RESULTS

In this section the control configurations proposed in the above section are tested and compared. Ideal sensors have been considered for the simulations. For the three fuzzy controllers applied, Mamdani[27] is the method selected to defuzzify. The design of the fuzzy controllers was based on the observation of the simulations results obtained by operating the plant with the default control of BSM1. MPC and fuzzy controllers are implemented using Matlab for the simulation and online control. Specifically, MPC controllers have been designed with a MPC tool, the identification of its prediction model with a System Identification Tool, and the fuzzy controllers with an FIS editor. To solve the quadratic objective of MPC in eq 10, the Quadratic Dynamic Matrix Control solver[29] with hard linear constraints in the inputs provided by Matlab MPC Tool has been used.

EQI and OCI Improvement. Here, the implementation of the hierarchical control with MPC + FF in the low level and the fuzzy controller in the high level stated in the previous sections is described.

The identification of the linear predictive models of the MPC controllers was performed using Matlab System Identification Tool. The data of the output variables (DO3, DO4, DO5 and NO2) are obtained by making changes to the input variables (K_La3, K_La4, K_La5 and Q_{rin}) with a maximum variation of 10% regarding its operating point, which is the value of K_La necessary to obtain 2 mg/L of DO and the value of Q_{rin} necessary to obtain 1 mg/L of NO2. Specifically, the working point is 264.09 day $^{-1}$, 209.23 day $^{-1}$, 131.65 day $^{-1}$ and 16486 m^3/day for K_La3, K_La4, K_La5 and Q_{rin} respectively. Different sources were tested to modify the input variables as random, sinusoidal or step, and finally the best fit was obtained with random source. These input variations are performed every 2.4 h, sufficient time to ensure the effect of these variations on the output signals. Furthermore, for the feedforward compensation, a step to Q_{in} of +10% is added over 18446 m^3/day, which is the average value during the stabilization period. Two methods were tested for determining the model with the obtained data, prediction error method (PEM)[30] and subspace state spacesystem identification (N4SID)[31]. Finally PEM were selected because it fits better with the real response of the plant. The order of the models was chosen from a trade-off between the best fit and the lowest order. Therefore, the following third order state-space models are obtained:

DO5 and NO2 control

$$A = \begin{bmatrix} 0.8748 & 0.04463 & 0.1314 \\ 0.04091 & 0.7331 & 0.1796 \\ 0.2617 & -0.1318 & 0.3007 \end{bmatrix}$$

$$B = \begin{bmatrix} 7.641 \times 10^{-6} & 0.004551 & -2.749 \times 10^{-5} \\ -2.631 \times 10^{-5} & 0.006562 & -4.551 \times 10^{-6} \\ -9.63 \times 10^{-6} & -0.02161 & 2.447 \times 10^{-5} \end{bmatrix}$$

$$C = \begin{bmatrix} 0.8812 & -0.5948 & 0.02114 \\ 1.187 & 0.9893 & -0.3754 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{13}$$

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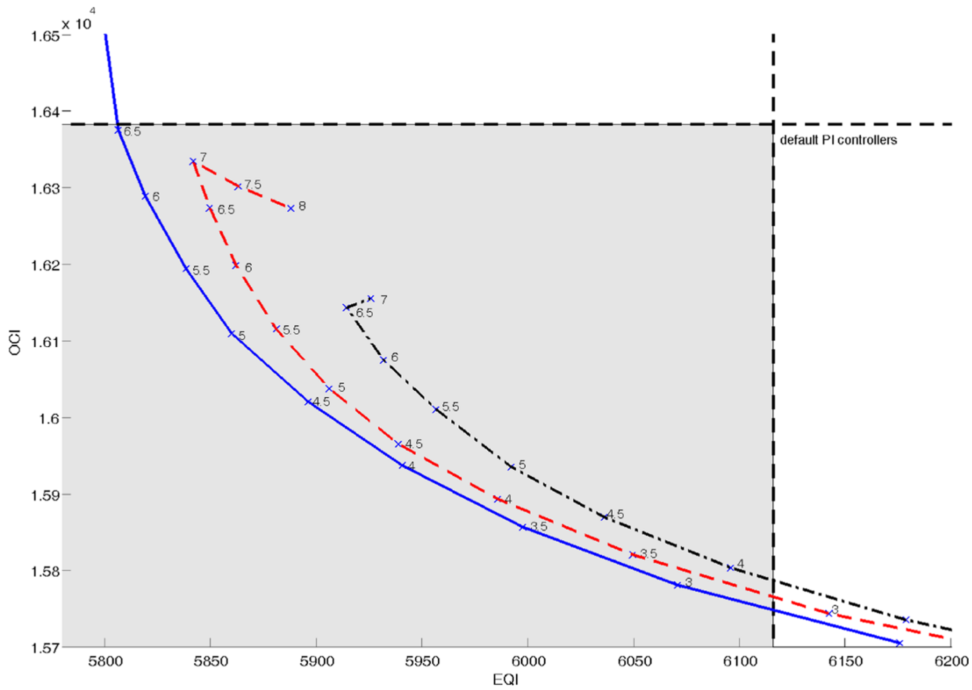

Figure 9. OCI and EQI trade-off with higher level fuzzy controller for a range of *MaxOut* from 2.5 to 8 with increments of 0.5 (points marked with crosses) and *MaxIn* = 3 (solid line), 5 (dashed line), and 7 (dash-dotted line).

DO3 control

$$A = \begin{bmatrix} 0.7859 & 0.4576 & -0.131 \\ 0.3334 & 0.2599 & 0.2718 \\ -0.003132 & 0.03235 & -1.003 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.009308 & -2.285 \times 10^{-5} \\ -0.01546 & 3.503 \times 10^{-6} \\ 0.003654 & -1.987 \times 10^{-5} \end{bmatrix}$$

$$C = \begin{bmatrix} 0.6376 & -0.4621 & 0.03698 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \end{bmatrix} \tag{14}$$

DO4 control

$$A = \begin{bmatrix} 0.8201 & 0.371 & -0.1016 \\ 0.3054 & 0.307 & 0.2544 \\ -0.003381 & 0.03144 & -0.9993 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.007712 & -4.65 \times 10^{-5} \\ -0.0148 & 8.164 \times 10^{-6} \\ 0.004523 & -2.526 \times 10^{-5} \end{bmatrix}$$

$$C = \begin{bmatrix} 0.947 & -0.496 & 0.02472 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \end{bmatrix} \tag{15}$$

The selected values to tune the MPC are $m = 5$, $p = 20$, $\Delta t = 0.00025$ days (21.6 s), $\Gamma_y = 1$ and $\Gamma_{\Delta u} = 0.01$ for DO3, DO4 and DO5 control and $\Gamma_y = 1$ and $\Gamma_{\Delta u} = 0.0001$ for NO2 control and overall estimator gain = 0.8. It should be noted that the values of m and p are not critical and they can be slightly changed with similar results.

Data acquisition for the model identification is based on simulations, as this work is a first step to be subsequently tested in a pilot plant and finally in a real plant. In order to predict the possible application in a real plant, the data acquisition for the identification is performed while the plant is kept at a certain desired operating point, whose values are considered suitable for the biological wastewater treatment of this plant, as the same way of K_La3 and K_La4. Therefore, what the identification needs is only the possibility of adding some incremental changes to those operating conditions. As mentioned before, the inputs used for identification purposes represent a maximum variation of 10%. Therefore, they will not disturb the actual plant operation. The generated outputs will reflect the effect of such input variables manipulation. Data for identification have been generated simulating 1 week. However, in the case of the real plant, the identification could be carried out in different periods and not necessarily in consecutive days. Plants operator knowledge can in addition be used to know the more appropriate days to perform the experiment.

For the high level fuzzy controller, three triangular membership functions for input and for output are used (low, medium and high). The rules implemented are as follows:

if (NH5 is *low*) **then** (DO is *low*)
if (NH5 is *medium*) **then** (DO is *medium*)
if (NH5 is *high*) **then** (DO is *high*)

MinIn and *MinOut* are 0.1 and 0.8, respectively. *MaxIn* and *MaxOut* have been determined with OCI and EQI trade-off

Table 3. EQI and OCI Results with Default PI Controllers and the Proposed Hierarchical Control

		Hierarchical Control		
	default PI controllers	lowest EQI	lowest OCI	% of improvement
EQI (kg pollutants/d)	6115.63	5804.38	6037.07	−5%
OCI	16381.93	16377.51	15743.27	−3.9%

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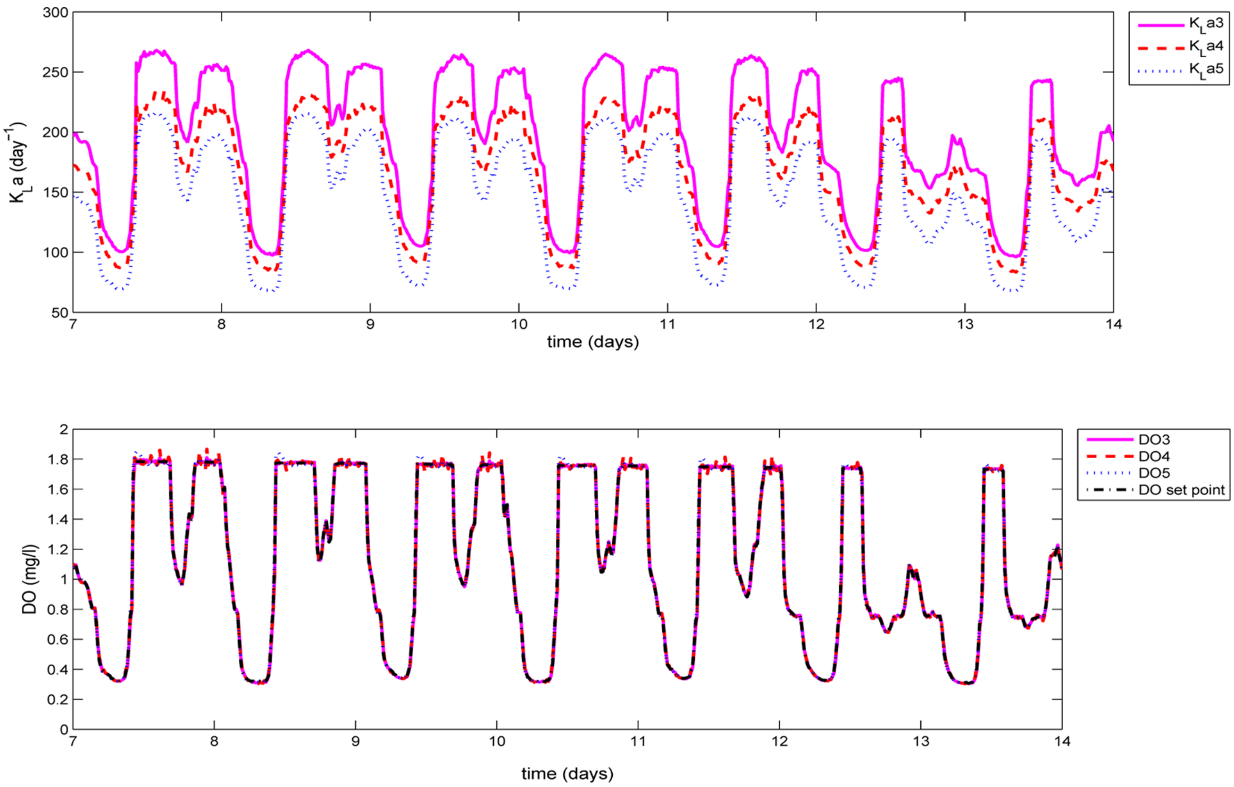

Figure 10. K_La and DO evolution of the three aerated tanks from day 7 to day 14 with hierarchical control structure.

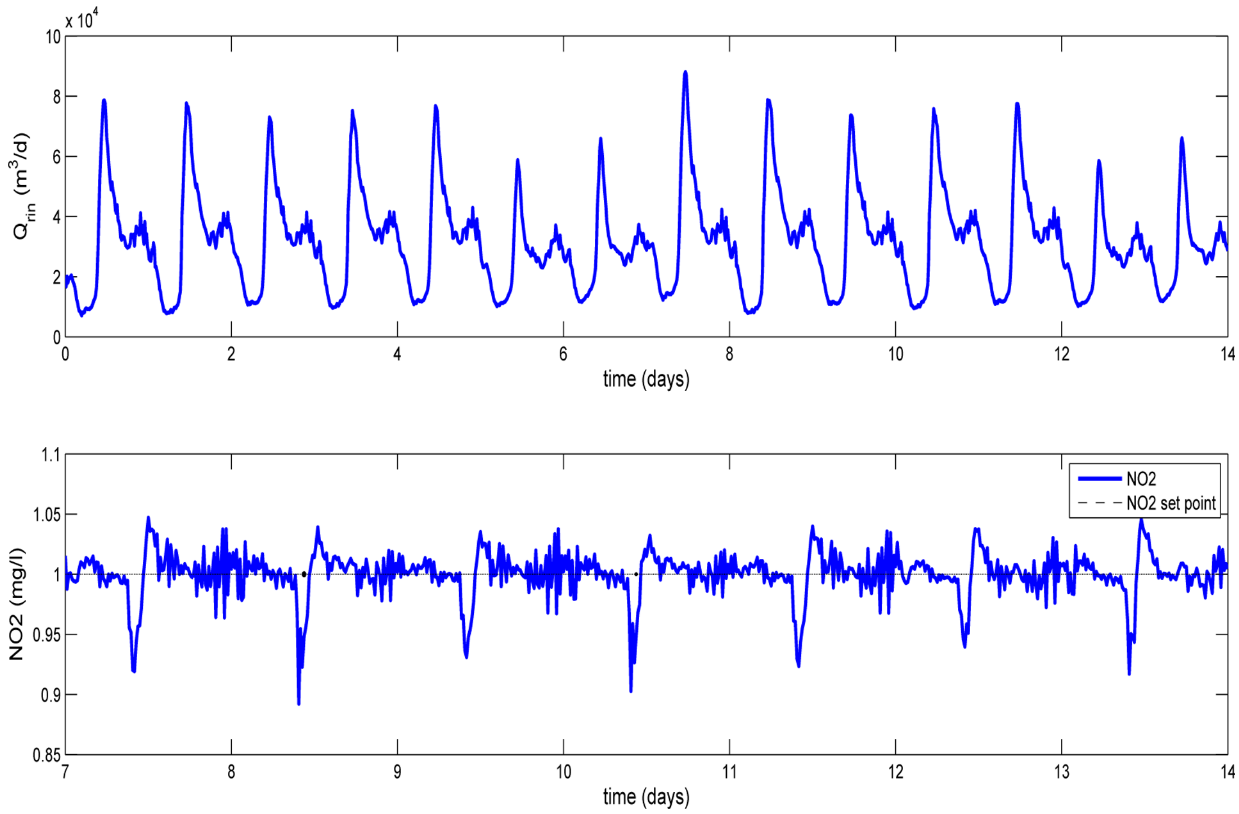

Figure 11. Q_{rin} and NO2 evolution from day 7 to day 14 with hierarchical control structure.

representations shown in Figure 9. Each one of the lines corresponds to the results obtained with different *MaxIn*, i.e. 3, 5, 7, and each one of the points marked with crosses is the result of a different *MaxOut* that varies from 2.5 to 8 with increments of 0.5. The results obtained with default PI controllers are also shown.

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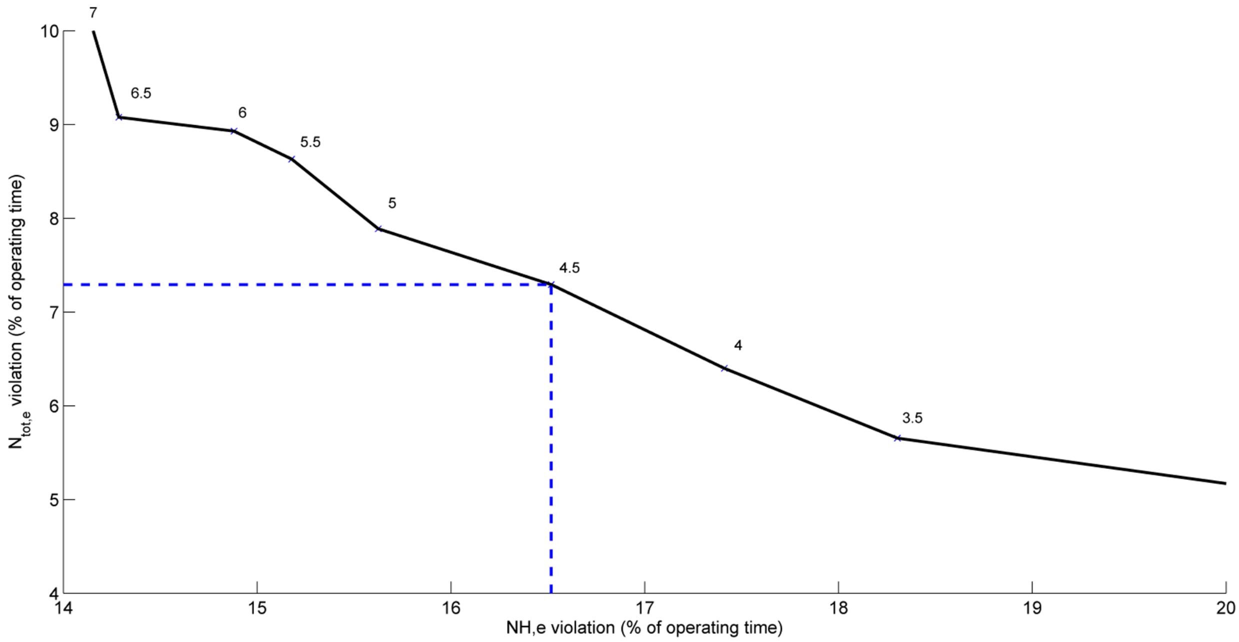

Figure 12. High level fuzzy controller: trade-off of the time percentage of NH_e and $N_{tot,e}$ violations for *MaxIn* = 3 and a range of *MaxOut* values from 3 to 7 with increments of 0.5 (points marked with crosses).

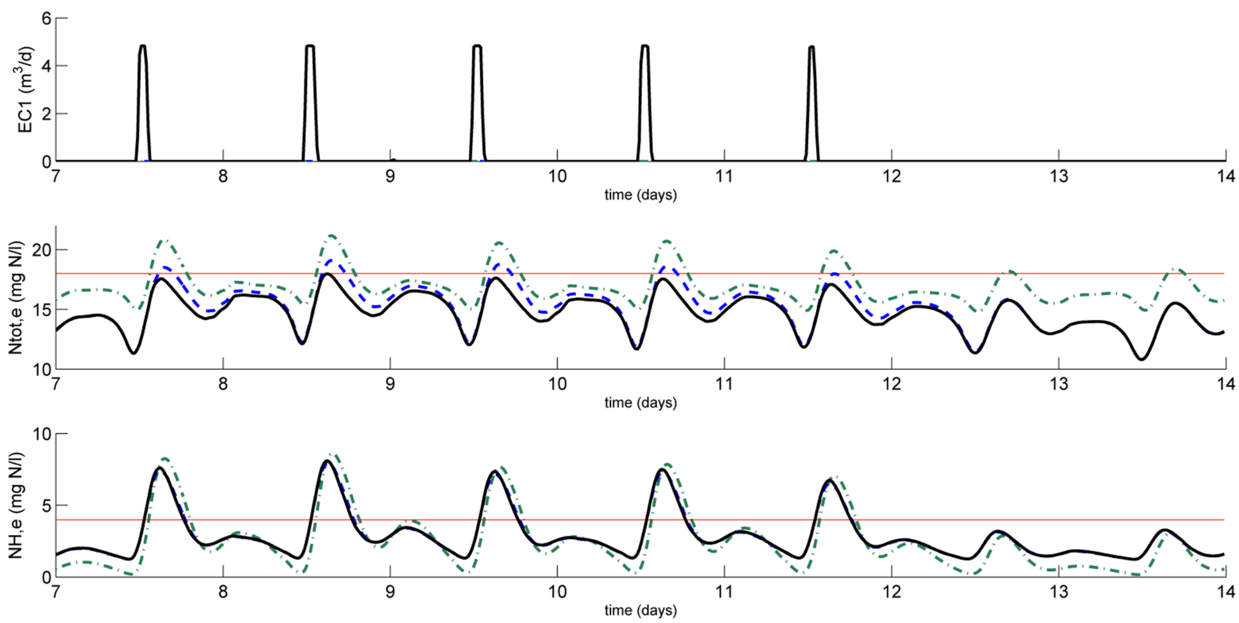

Figure 13. q_{EC1}, NH_e and $N_{tot,e}$ evolution from day 7 to day 14 with default PI controllers (dash-dotted line), with hierarchical control without adding EC1 (dashed line) and with hierarchical control adding EC1 (solid line).

The *MaxIn* and *MaxOut* values of the extreme cases of lowest EQI without increasing OCI and lowest OCI without worsening EQI in comparison with default PI controllers are *MaxIn* = 3 and *MaxOut* = 6.5 for the best EQI and *MaxIn* = 3 and *MaxOut* = 2.75 for the best OCI. Table 3 presents the results of best EQI without increasing OCI and best OCI without worsening EQI of high level fuzzy controller in comparison with default control strategy. The improvement of EQI is 5% and the reduction of OCI is 3.9%. Figures 10 and 11 show the evolution of the control and manipulated variables from day 7 to 14.

$N_{tot,e}$ Violations Removal. The objectives of this control strategy besides $N_{tot,e}$ violations removal are to improve EQI, not to increase OCI and to reduce the percentage of time of NH_e violations in comparison with the default control strategy. For this purpose, a trade-off analysis of the percentage of time over the limits of NH_e and $N_{tot,e}$ is made (see Figure 12). For this analysis, the hierarchical control strategy is included but not the addition of EC1. The tuning parameters of the high level fuzzy controller are selected at the point whose percentage of operating time of NH_e over the limits is the same as with the default control strategy (17.26%). These tuning parameters are *MaxIn* = 3 and *MaxOut* = 4.1, and the percentage of operating time of the $N_{tot,e}$ violation with these parameters is 6.39%.

In the OCI and EQI trade-off representation shown in Figure 9, in the points of the tuning parameters mentioned, a difference in OCI of 2.6% is observed regarding the default control strategy,

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which may be used for the EC1 dosage. The dashed line in Figure 13 shows the evolution of $N_{tot,e}$ from day 7 to 14 with the parameters selected for high level fuzzy controller and without adding EC1. The constant line is the $N_{tot,e}$ limit.

With these parameters selected for the high level, a fuzzy controller is added to manipulate q_{EC1}. For this controller, three triangular membership functions for input and for output are used (low, medium and high). The rules implemented are as follows:

if (NH5+NO5 is *low*) **then** (q_{EC1} is *low*)
if (NH5+NO5 is *medium*) **then** (q_{EC1} is *medium*)
if (NH5+NO5 is *high*) **then** (q_{EC1} is *high*)

The ranges of membership functions values are *MinIn* = 10, *MaxIn* = 17.5, *MinOut* = −8, *MaxOut* = 6.75. The solid lines of Figure 13 correspond to the evolution of q_{EC1}, $N_{tot,e}$ and NH_e from day 7 to 14. It is observed that $N_{tot,e}$ violations are removed.

Table 4 presents the results of EQI, OCI and the percentage of operating time out of the limits of NH and $N_{tot,e}$ obtained with hierarchical control adding EC1 and compared with the default control strategy of BSM1. It is shown that by adding EC1 and a hierarchical control of DO in the three aerated tanks, the violations of $N_{tot,e}$ can be avoided, also improving the results of EQI and OCI and the percentage of operating time of NH_e violations with respect to default PI controllers.

Table 4. Results with Default PI Controllers and with Control for $N_{tot,e}$ Violations Removal

	default PI controllers	control for $N_{tot,e}$ violations removal	% of improvement
EQI (kg pollutants/d)	6115.63	5862.03	−4.1%
OCI	16381.93	16336.36	−0.3%
$N_{tot,e}$ violations (% of operating time)	17.56	0	−100%
NH_e violations (% of operating time)	17.26	16.66	−3.4%

NH_e Violations Removal. As mentioned in the previous section, to perform the control for removing violations of NH_e, the MIMO MPC + FF, that controls DO5 and NO2 by manipulating K_La5 and $Q_{ri n}$, has been replaced by a SISO MPC + FF that controls DO5 by manipulating K_La5, because $Q_{ri n}$ is manipulated based on NH5 and NH_{in}.

The model identification of the new MPC + FF was performed with the same methodology as with the previous controller, but with one input and one output. However, in this case it is a second order state-space model:

$$A = \begin{bmatrix} 0.8349 & 0.2746 \\ 0.2512 & 0.2894 \end{bmatrix}$$
$$B = \begin{bmatrix} 0.008745 & -2.729 \times 10^{-5} \\ -0.02118 & 1.307 \times 10^{-5} \end{bmatrix}$$
$$C = [1.512 \quad -0.3525]$$
$$D = [0 \quad 0] \tag{16}$$

The *MaxIn* and *MaxOut* values of the high level fuzzy controller have been selected by a trade-off analysis of OCI and percentage of operating time of NH_e violation (see Figure 14), choosing the less percentage of NH_e violation in order to facilitate its later total elimination, but considering the increased costs that will be generated by the new control strategy. In this case the chosen parameters are *MaxIn* = 3 and *MaxOut* = 5.5. In the case of the fuzzy controller for the NH_e violations removal, two tunings are determined, one when there are peaks of NH_{in} or NH5, and the other the rest of the time. For both cases three triangular membership functions for input and for output are used (low, medium and high). The rules implemented are as follows:

if (NH5 is *low*) **then** ($Q_{ri n}$ is *high*)
if (NH5 is *medium*) **then** ($Q_{ri n}$ is *medium*)
if (NH5 is *high*) **then** ($Q_{ri n}$ is *low*)

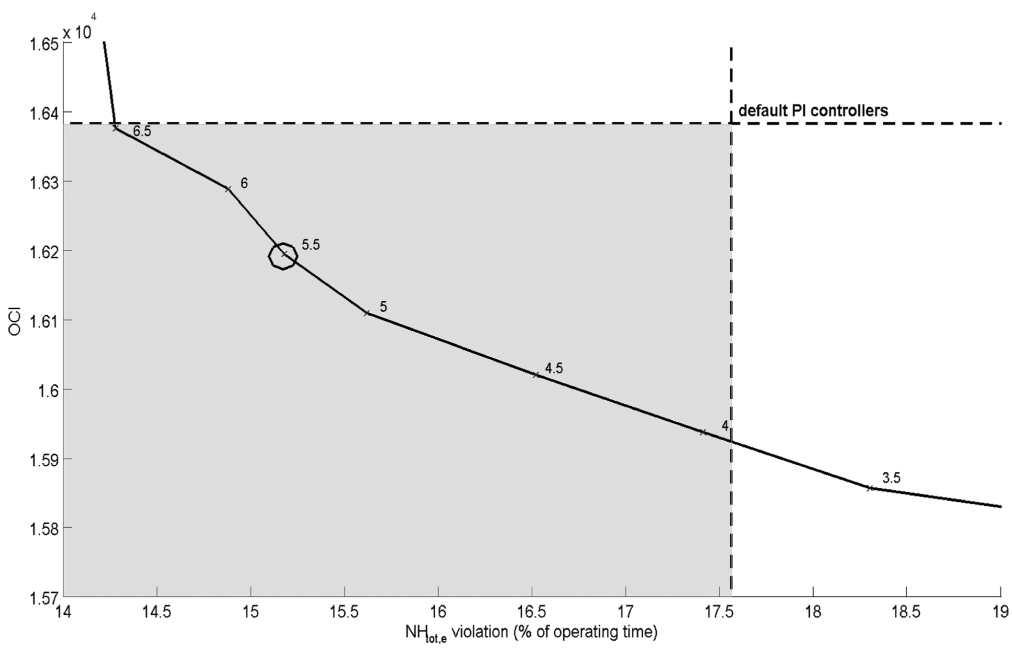

Figure 14. Trade-off representation of OCI and the percentage of operating time of NH_e violations for *MaxIn* = 3 and a range of *MaxOut* from 3 to 7 with increments of 0.5 (points marked with crosses).

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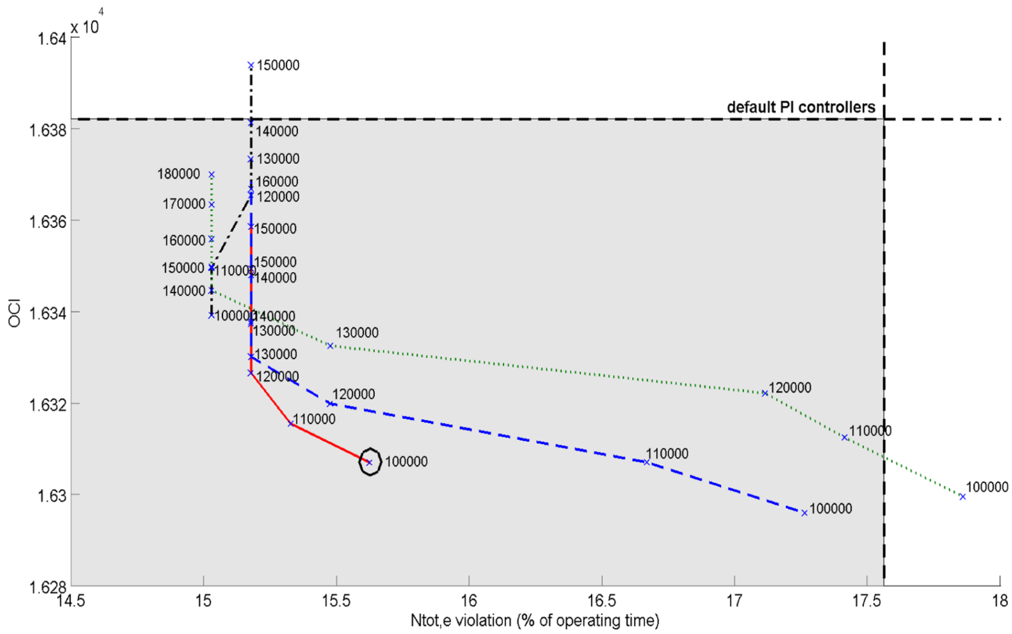

Figure 15. Trade-off representation of OCI and the percentage of operating time of $N_{tot,e}$ violations for a range of *MaxOut* from 90000 to 180000 with increments of 10000 (points marked with crosses) and *MaxIn* = 2 (dotted line), 2.2 (dashed line), 2.4 (solid line), 2.6 (dash-dotted line).

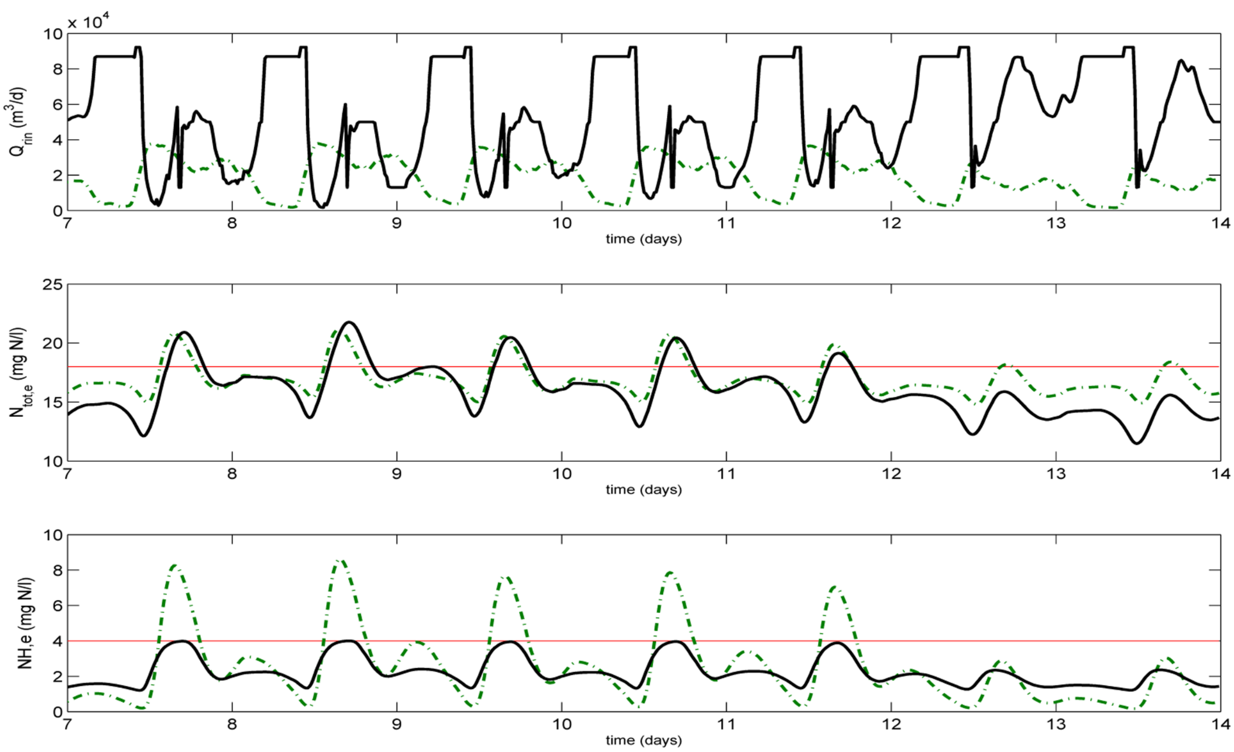

Figure 16. Q_{rin}, NH_e and $N_{tot,e}$ evolution from day 7 to day 14 with default PI controllers (dash-dotted line) and with the control for NH_e violations removal (solid line).

When there are peaks of NH_{in} or NH5, the tuning parameters are set looking for a great variation in Q_{rin} when NH_e is increasing. Therefore, *MinIn*, *MaxIn*, *MinOut* and *MaxOut* are 3.5, 4.1, -2×10^4, and 14×10^4, respectively. For the rest of the time, *MinOut* and *MaxOut* are set by a trade-off analysis of OCI and percentage of operating time of $N_{tot,e}$ violation, reflecting only the results that avoid the NH_e violations. An area is obtained where OCI and the operating time of the $N_{tot,e}$ violation are decreased compared to default PI controllers (see Figure 15). Each one of the lines corresponds to the results obtained with *MaxIn* = 2, 2.2, 2.4, and 2.6, and each one of the points marked with crosses is the result of a different *MaxOut* that varies from 90000 to 180000 with increments of 10000. The results obtained with default PI controllers alone are also shown. The parameters have been selected according to the Nash Solution[32]: *MaxIn* = 2.4 and *MaxOut* = 100000.

Q_{rin}, $N_{tot,e}$ and NH_e evolutions from day 7 to 14 are shown in Figure 16. The results with default PI controllers are also shown.

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Table 5. Results with Default PI Controllers and with Control for NH_e Violations Removal

	default PI controllers	control for NH_e violations removal	% of improvement
EQI (kg pollutants/d)	6115.63	5854.06	−4.26%
OCI	16381.93	16307.26	−0.45%
$N_{tot,e}$ violations (% of operating time)	17.56	15.62	−11.04%
NH_e violations (% of operating time)	17.26	0	−100%

It can be observed that, with this control strategy, NH_e peaks are reduced under the limits established.

Table 5 shows the results of EQI, OCI and percentage of time over the limits of NH_e and $N_{tot,e}$. It can be seen that it is possible to avoid NH_e violations with the regulation of Q_{rin} based on NH5 and NH_{in} applying a fuzzy controller with two different alternative settings of the tuning parameters and also with the hierarchical control of DO in the three aerated tanks. In addition, a reduction of 4.26% of EQI and 0.45% of OCI, with respect to the default control strategy of BSM1, is achieved.

■ CONCLUSION

In this paper, different control configurations based on MPC and fuzzy logic have been used in WWTPs to eliminate $N_{tot,e}$ violations or NH_e violations and at the same time improve the results of OCI and EQI in comparison with the default control strategy of BSM1.

The elimination of $N_{tot,e}$ violations is achieved by manipulating q_{EC1} based on NO5 plus NH5. The removal of NH_e violations is carried out by manipulating Q_{rin} based on NH5 and NH_{in}, which uses different tuning parameters depending on if there are peaks of pollution in the tanks or not.

In both cases, a two-level hierarchical control structure is simultaneously implemented to perform an EQI and OCI improvement. The low level of this structure is composed by three MPC + FF that manipulate K_La3, K_La4, K_La5 and Q_{rin} to control DO3, DO4, DO5 and NO2 in the first case and KLa3, KLa4 and KLa5 to control DO3, DO4 and DO5 in the second case. For the high level, a fuzzy controller is implemented to manipulate DO set points of the low level according to NH5.

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Notes

The authors declare no competing financial interest.

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