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Simultaneous Batching and Scheduling of Batch Plants That Operate in a Campaign-Mode, Considering Nonidentical Parallel Units and Sequence-Dependent Changeovers

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ABSTRACT: A mathematical modeling framework for the simultaneous batching and scheduling of multiproduct batch plants is proposed in this work. The scheduling decisions are formulated according to campaign-based operation mode. When a stable context can be assumed on the time horizon taken into account, this operation mode assures a more efficient production management. In addition, sequence-dependent changeover times and different unit sizes for parallel units in each stage are considered. Given the plant configuration and unit sizes, the total amount of each product to be produced and the product recipes, the proposed approach determines the number of batches that compose the production campaign and their sizes, the batches assignment to units, the sequencing of batches in each unit for each stage, and the timing of batches in each unit in order to minimize the campaign cycle time. A solution strategy is proposed to enhance the computational performance of the simultaneous optimization. The approach capabilities are shown through three numerical examples.

1. INTRODUCTION

The scheduling problem has received considerable attention from the research community and industry in the last decades due to the need to improve productivity and reduce costs in the chemical processing and manufacturing related industries. This problem represents a decision making process where different products have to be allocated to different shared and limited resources in an optimal manner. In the chemical industry there is a large variety of applications in both batch and continuous processes, like in pharmaceuticals, basic and specialty chemicals, and consumer products, etc.

Mathematical programming has become one of the most widely explored methods for process scheduling problems because of its rigorousness, flexibility, and extensive modeling capability. Also, the substantial advances of related modeling and solution techniques, as well as rapidly growing computational power have motivated the study and application of this kind of problems. Extensive details of the various aspects of the scheduling problem, modeling approach classification, and solution techniques are presented in the reviews of Floudas and Lin,[1,2] Méndez et al.,[3] Pan et al.,[4] Sundaramoorthy and Maravelias,[5] and Maravelias.[6]

According to Maravelias,[6] the scheduling problem in the context of batch processing involves the following decisions: (i) selection and sizing of batches to be carried out; (ii) assignment of batches to process units; (iii) sequencing of batches on units; and (iv) timing of batches. The former decision is commonly defined as "batching". Most scheduling models in the process systems engineering literature consider a special case of the problem, where the number and size of batches is fixed; that is, the batching problem is solved first and then obtained batches are used as inputs in the scheduling model. Also, there are some papers that solve both batching and scheduling problems through hierarchical approaches (Méndez et al.,[7] Schwindt and Trautmann,[8] Neumann et al.[9]); however, those approaches cannot guarantee an optimal solution. In contrast, Lim and Karimi[10] formulated a novel model for the simultaneous batching and scheduling of single-stage batch plants with parallel, nonidentical units, sequence-dependent changeover times, and multiple orders per product. They compared their approach with existing decomposition formulations and showed that the proposed model is more efficient both in model size (variables and constraints) and computational performance (nodes, iterations, and solution times). Prasad and Maravelias[11] presented the first approach for simultaneous batching and scheduling for multistage batch plants. The mixed-integer linear programming (MILP) model, which considers changeover times and nonidentical parallel units for each batch stage, is tested with several objective functions. Since then, some approaches, considering joint batching and scheduling for multistage batch plants, were presented.

Sundaramoorthy and Maravelias[12] presented a MILP model for simultaneous batching and scheduling of multistage batch plants with nonidentical parallel units and variable processing times. They also included sequence-dependent changeover costs, and proposed a method that allows improving the computational performance in large instances fixing certain sequencing variables and strengthening inequalities based on time-window information. In a later work, Sundaramoorthy and Maravelias[13] included storage constraints and provided general classification of storage policies in multistage processes for showing how their formulation can be adapted to address different types of problems. Sundaramoorthy et al.[14] presented a MILP model using discrete-time representation for the batching and scheduling of multistage batch plants with utility constraints.

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They considered limited storage, but changeover times/costs are not taken into account. Marchetti et al.[15] posed two MILP formulations for the simultaneous batching and scheduling of single-stage multiproduct batch facilities. Both models consider variable processing times and several customer orders per product with different delivery dates. The two proposed formulations deal with batch sequencing decisions in a different manner: one of them rigorously arranges individual batches assigned to the same unit, while the other sequences clusters of batches sharing the same product and due date and processed in the same equipment item. In the last case, grouping batches into clusters seeks to reduce the number of product changeovers. Thus, it was able to solve very large problems in an efficient manner. In a later work, Marchetti et al.[16] generalized the previous formulation to multistage batch plants with non-identical parallel units.

Usually, demand patterns combined with capacity availability are used to determine the production policy and the type of scheduling approach: short-term scheduling or cyclic scheduling. The former is more appropriate when products with irregular and/or small demand are produced to meet specific orders, that is, in a make-to-order production policy. On the other hand, the second type is used for the production of high-volume products with relatively constant demand, which lead to a more regular production mode, more appropriate for a make-to-stock production policy. It is worth highlighting that all the previous mentioned works correspond to the first type, that is, addressing short-term scheduling problems.

When product demands can be accurately forecasted during a relatively long time horizon due to a stable context (from a week until a month, for example), more efficient management and control of the production resources can be attained if the plant is operated in a periodic or cyclic mode, i.e. in a campaign-mode. In this case, the campaign consists of several batches of different products that are going to be manufactured, and the same pattern is repeated at a constant frequency over the time horizon. Fumero et al.[17] presented a MILP model for the optimal scheduling of batch plants operating under mixed product campaign (MPC) mode. Given a fixed number of batches of each product comprising the production campaign, the model determines the batches assignment, sequencing and timing in the campaign. They considered multistage batch plants with identical parallel units and no changeover times/costs were taken into account.

The MPCs-based operation is very useful, especially in production systems using the make-to-stock policy. This approach has several advantages, for example, more standardized production during certain periods of time, easier and profitable operations decisions, more efficient operation control and tracking, and adequate inventory levels without generating excessive costs and minimizing the possibility of stock-outs. From the computational point of view, the cyclic scheduling allows a reduction in the size of the overall scheduling problem, which is often intractable. On the other hand, one of the main differences between cyclic scheduling based on MPCs and short-term scheduling is the adopted objective function. While most of the approaches for short-term scheduling dealt with makespan minimization, tardiness, or earliness, the most appropriate performance measure for the scheduling problem using MPCs of cyclic repetition is the minimization of the campaign cycle time.[17] Taking in mind that in a planning context the campaign will be repeated over the time horizon, consecutive campaigns have to be overlapped in order to reduce idle times between them as much as possible.

The scheduling problem using MPCs, where the number and size of batches are known, was scarcely addressed in the literature. Besides the paper of Fumero et al.,[17] Birewar and Grossmann[18] developed slot-based formulations for the scheduling of multiproduct batch plants using production campaigns, considering different transfer policies (unlimited intermediate storage, UIS, and zero wait, ZW). They determined the optimal campaign cycle time, for simple plants including only one unit per processing stage, using a MILP formulation. Corsano et al.[19] presented a heuristic method for incorporating scheduling, using MPCs, in a batch plant design model. They formulated a detailed nonlinear programming (NLP) model including operational conditions and synthesis decisions. Mixed product campaigns are configured by the designer taking into account the ratios among the numbers of batches of the different products elaborated in the plant. Then, each suggested campaign is considered in the proposed formulation defining several alternatives. The best solution is determined after solving these cases and comparing them. In Fumero et al.[20] two MILP models for the simultaneous design and scheduling of a multistage batch plant are proposed. Both formulations consider MPCs: one in a simplified manner with the aim of reducing the solving time, while the other uses a rigorous formulation for the production campaign. The parallel units are considered identical and no changeover times are taken into account. The rest of the papers that mention the use of campaigns, do not refer to the determination of batches and its cyclic sequencing as is managed in this work. For example, some of them consider that a campaign is formed by several batches of the same product that follow each other (Kallrath,[21] Brandenburg and Tölle[22]). This assumption obviously simplifies the model formulation.

On the other hand, in network production environments, where recipes are rather complex, mixing and splitting operations are included, and material recycles are admitted, continuous-time formulations based on state-task-network (STN) and resource-task-network (RTN) are used to address the integrated problem. In this area, Wu and Ierapetritou[23] proposed a cyclic scheduling based on STN representation of the plant using a mixed-integer nonlinear programming (MINLP) formulation where sequence-dependent changeovers have not been incorporated in the formulation. Also, Castro and Grossmann[24] present continuous and discrete-time MILP formulations for the short-term scheduling of multistage multiproduct batch plants using the RTN representation, where sequence-dependent changeovers are neglected and the transfer policy adopted is UIS.

Although the multistage multiproduct topology is a special case of the network-type topology for which representations more generic and applicable to any type of scheduling problem exist, their computational efficiency for sequential batch processes cannot compete, so far, with those of methodologies explicitly exploiting the series structure. A current review on existing scheduling methodologies and solution techniques, presented by Harjunkoski et al.,[25] states that the vast majority of models developed to address problems in a batch process of sequential topology are based on some of the following representations: slot-based, unit-specific time event, and precedence-based.

In this work, the simultaneous batching and scheduling problem of multistage batch plants is addressed using mathematical programming. Nonidentical parallel units, ZW transfer policy, and sequence-dependent changeover times are

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considered. The scheduling is solved using the concept of MPC, in such a way that, given the plant configuration and unit sizes, the total amount of each product to be produced and the product recipes, the approach determines the number of batches that compose the production campaign and their sizes, the batches assignment to units, the sequencing of batches in each unit for each stage, and the initial and final times of the batches processed in each unit in order to minimize the campaign cycle time.

Taking into account the time requirement to solve the simultaneous optimization problem, a resolution strategy based in two MILP formulations is presented. With the aim of reducing the combinatorial complexity associated with the scheduling problem, a simplified model is first solved in order to obtain tight bounds on the campaign cycle time and the number of batches proposed for each product. Then, the simultaneous batching and scheduling approach is efficiently solved. From the computational point of view, the presented examples show that the proposed approach for addressing jointly batching and scheduling through MPCs considering sequence-dependent changeover times for multistage batch plant with nonidentical parallel units is a good option.

The paper is organized as follows. In the next section the description of the problem is presented. Then, a solution strategy is proposed in section 3. The MILP formulations used in the resolution process are described in section 4, and three illustrative examples to show the capabilities of the proposed approach are presented in section 5. Finally, conclusions of the work are drawn in the last section of this paper.

2. PROBLEM DEFINITION

The problem addressed in this article deals with a multiproduct batch plant where J denotes the set of processing stages that compose the plant and K is the set of all units in the plant. Stage $j \in J$ has a set K_j of nonidentical parallel batch units that operate out-of-phase, so $K = K_1 \cup K_2 \cup \cdots \cup K_{|J|}$. If the cardinality of K_j is N_j, then the elements of each set K_j are represented by $K_1 = \{1, 2, ..., N_1\}$, $K_2 = \{N_1+1, N_1+2, ..., N_1+N_2\}$, $K_3 = \{N_1+N_2+1, ..., N_1+N_2+N_3\}$, and so on, that is, using an ascending numerical order.

A set I of products must be manufactured in the plant following the same sequence of stages. The total amount required of each product in the campaign, Q_i ($i \in \mathrm{I}$), which allows maintaining adequate stocks levels taking into account the estimated demands, is a model parameter. Q_i can be fulfilled with one or more batches, then an index b is introduced to denote the bth batch required to meet production of the corresponding product.

In each stage, there are no restrictions about parallel unit sizes and, therefore, different unit sizes are admitted. Then, V_k is used to denote the size of unit k. The processing time of each batch of product i in unit k, t_{ik}, and the size factor SF_{ij} that denotes the required capacity of units in stage j to produce one mass unit of final product i, are problem data.

Considering the demand of product i, the nonidentical parallel unit sizes at each stage, the equipment utilization minimum rate for product i at each unit, denoted by α_{ik}, and the size factors of product i in each stage, the minimum and maximum numbers of batches required to fulfill the demand of product i can be calculated in order to ensure solution optimality. Thus, the minimum number of batches of product i at the campaign is calculated as follows:

$$NBC_i^{\mathrm{LOW}} = \left\lceil \frac{Q_i}{B_i^{\max}} \right\rceil \quad \forall\ i$$

where

$$B_i^{\max} = \min_{j \in J}\left\{ \max_{k \in K_j}\left\{ \frac{V_k}{SF_{ij}} \right\}\right\}$$

is the maximum feasible batch size for product i. Analogously, the maximum number of batches of product i at the campaign is given by

$$\mathrm{NBC}_i^{\mathrm{UP}} = \left\lceil \frac{Q_i}{B_i^{\min}} \right\rceil \quad \forall\ i$$

where

$$B_i^{\min} = \max_{j \in J}\left\{ \min_{k \in K_j}\left\{ \alpha_{ik} \frac{V_k}{\mathrm{SF}_{ij}} \right\}\right\}$$

is the minimum feasible batch size for product i.

Intermediate storage tanks are not allowed. Therefore, taking into account the configuration of the plant, there is no batch splitting or mixing, that is, each batch is treated as a discrete entity throughout the whole process. It is assumed that a batch cannot wait in a unit after finishing its processing. Therefore, the ZW transfer policy between stages is adopted, that is, after being processed in stage j, a batch b is immediately transferred to the next stage j+1. Besides, batch transfer times between units are assumed very small compared to process operation times and, consequently, they are included in the processing times.

Sequence-dependent changeover times, $c_{ii'k}$, are considered between consecutive batches processed in the same unit k, even of the same product. This transition time corresponds to the preparation or cleaning of the equipment to perform the following batch processing. It is necessary for various reasons: ensure product quality, maintain the equipment, safety reasons, etc.

As previously stated, the problem consists of solving simultaneously two decision levels often addressed sequentially. Through a holistic approach, the selection and sizing of batches of each product (*batching*), and the assignment of batches to units in each stage, the production sequence of assigned batches in each unit and initial and final processing times for batches that compose the campaign in each processing unit (*scheduling*) are jointly determined.

3. SOLUTION STRATEGY

Production scheduling for batch processes is a complex optimization problem, even more when the number of batches to be processed and their sizes are not problem data. Taking into account the combinatorial nature of the problem, most of the existing approaches in the area decouple the batching and scheduling decisions, which often leads to suboptimal solutions. Therefore, formulations and solution strategies for the simultaneous batching and scheduling of multiproduct batch processes are challenging and necessary.

With the aim of reducing the combinatorial complexity associated with the integrated problem, where the difficulty is mainly due to the scheduling decisions, a two-phase solution strategy is addressed in this work. First, a simplified MILP problem (SP) is solved, where the batching and scheduling decisions are made considering a preordering constraint in the assignment of batches in order to reduce the complexity related to the scheduling. Then, the campaign composition and the optimal value of the objective function of problem SP provide a better-estimation of the number of batches that must be proposed for the detailed model and a tighter upper bound on the objective function of this model. Thus, the detailed MILP formulation (DP) for the simultaneous batching and scheduling of the given plant is solved in a reasonable computational time. Model DP considers the exact scheduling of the batches selected for the optimal campaign composition, without preordering constraints. It is worth mentioning that the solution of model SP provides a tight upper bound on the objective function of problem DP, and, many times, it coincides with its global optimum.

For scheduling decisions of both formulations, an asynchronous slot-based continuous-time representation has been used. The slots correspond to time intervals of variable length in which batches will be assigned. In each slot l of a specific unit k at most one batch b of product i can be processed and, if no product is assigned to slot l, its length will be zero. In models based on this type of representation, the selection of the number of slots postulated for each unit is not a trivial decision since computational performance strongly depends on this parameter. Taking into account that in this work the parallel units at each stage have different capacities and processing times, the assumptions made and expressions proposed in previous works for the number of slots postulated for each unit at both models cannot be employed (Fumero et al., 2012). In this work, a strategy to define the set of postulated slots for model DP has been designed taking advantage of the optimal solution of model SP. The number of selected batches for each product from model SP allows tightly proposing the number of slots postulated for each unit in the detailed model. So, the model DP size is drastically reduced and it is solved in a reasonable CPU time.

4. MATHEMATICAL FORMULATIONS

In this section, continuous-time slot-based MILP formulations for both models previously mentioned are described.

The number of slots postulated for unit k of stage j, denoted by L_{kj}, is not a priori known because the set of batches to be processed for fulfilling the demands and the allocation of those to each unit are optimization variables. For the first model, this value can be approximated considering the estimation on the maximum number of batches of each product at the campaign, denoted by $\mathrm{NBC}_i^{\mathrm{UP}}$. Then, the number of slots postulated for all units of each stage is the same, and it is given by

$$L = \sum_{i \in I} \mathrm{NBC}_i^{\mathrm{UP}} \quad \forall\, j,\, k \in K_j$$

Although this value is an overestimation, a better approximation cannot be proposed taking into account that the parallel units are different and, on the other hand, the number and sizes of batches to be scheduled are optimization variables, unlike most of scheduling approaches presented in the literature in which they are considered as parameters. However, the lower bound on the number of batches of product i at the campaign, $\mathrm{NBC}_i^{\mathrm{LOW}}$ strongly reduces the number of combinations in the batching decisions and consequently improves the computational performance of the models.

Finally, taking into account the optimal solution of model SP, the option more conservative on the number of slots proposed for each unit of the batch plant for the first model can be relaxed and a better-estimation can be achieved for model DP. This is a significant result taking into account the impact of these values on the computational performance.

4.1. Simplified Model SP.

4.1.1. Batching Constraints. The number of batches of product i that must be manufactured at the campaign is a model variable. Taking into account the upper bound for this value, a set of generic batches associated with product i, IB_i, is proposed, where $|IB_i| = \mathrm{NBC}_i^{\mathrm{UP}}$. A binary variable z_{ib} is introduced, which takes value 1 if batch b of product i is selected to satisfy the demand requirements of that product and 0 otherwise. Then,

$$\mathrm{NBC}_i^{\mathrm{LOW}} \le \sum_{b \in IB_i} z_{ib} \le \mathrm{NBC}_i^{\mathrm{UP}} \quad \forall\, i \in I \tag{1}$$

Besides, without loss of generality and in order to reduce the number of alternative solutions, eq 2 guarantees that the selection of batches of a same product is made in ascending numerical order:

$$z_{ib+1} \le z_{ib} \quad \forall\, i \in I,\, b \in IB_i,\, b+1 \in IB_i \tag{2}$$

Let B_{ib} be the size of batch b of product i and Q_i the demand of product i that must be fulfilled, then

$$B_{ib} \le B_i^{\max} z_{ib} \quad \forall\, i \in I,\, b \in IB_i \tag{3}$$

$$Q_i = \sum_{b \in IB_i} B_{ib} \quad \forall\, i \in I \tag{4}$$

Taking into account that the size of unit k denoted by V_k and the size factor SF_{ij} are model parameters, if batch b of product i is processed in unit k of stage j the following inequalities limit the size B_{ib} of batch b between the minimum and maximum processing capacities of unit k:

$$\alpha_{ik}\frac{V_k}{\mathrm{SF}_{ij}} \le B_{ib} \le \frac{V_k}{\mathrm{SF}_{ij}} \quad \forall\, i \in I,\, b \in IB_i,\; k \in \{\text{units of stage } j \text{ used to process batch } b\} \tag{5}$$

where α_{ik} is the minimum filled rate required to process product i in unit k. Since the units selected to process the batches of each product are optimization variables and their sizes are different, eq 5 must be expressed through a variable that indicates this selection. Then, the allocation variables and constraints of the model are introduced.

For enhancing the resolution process by eliminating equivalent symmetric solutions, the batch b size of product i is forced to be greater than or equal to the batch b+1 size of the same product, that is

$$B_{ib+1} \le B_{ib} \quad \forall\, i \in I,\, b,\, b+1 \in IB_i \tag{6}$$

4.1.2. Assignment and Sequencing Constraints. Selected batches must be assigned, in each stage, to specific slots in the units. In this formulation, a preordering constraint is imposed in the assignment of batches, which allows simplifying the definition of the allocation variables, reducing the number of binary variables. This heuristic rule assures that each selected batch is assigned to the same slot on all stages. Then, the assignment of batches to slots on different units is defined through two sets of binary variables:

$$Z_{bl} = \begin{cases} 1 & \text{if batch } b \text{ is assigned to slot } l \\ 0 & \text{otherwise} \end{cases}$$

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$$X_{kl} = \begin{cases} 1 & \text{if slot } l \text{ of unit } k \text{ is employed for processing one batch} \\ 0 & \text{otherwise} \end{cases}$$

First, it is worth noting that if batch b of product i is not processed, variable Z_{bl} is zero for all slots:

$$Z_{bl} \leq z_{ib} \quad \forall\ i \in I,\ b \in IB_i,\ 1 \leq l \leq L \tag{7}$$

Variable Z_{bl} defines the relation batch-slot. Therefore, the proposed heuristic rule for the model SP, which states that each batch processed must be allocated to the same slot on different stages, is posed through the following condition:

$$\sum_{1 \leq l \leq L} Z_{bl} = z_{ib} \quad \forall\ i \in I,\ b \in IB_i \tag{8}$$

Moreover, for each stage of the plant, slot l is only used for processing at most one product batch on exactly one unit. Therefore, in the remaining units of that stage, this slot cannot be occupied. Then

$$\sum_{b \in \cup_i IB_i} Z_{bl} \leq 1 \quad 1 \leq l \leq L \tag{9}$$

$$\sum_{k \in K_j} X_{kl} \leq 1 \quad \forall\ j \in J,\ 1 \leq l \leq L \tag{10}$$

The proposed preordering constraint accelerates the model SP resolution, reducing the number of enumerated nodes. However, in some cases, suboptimal solutions can be obtained.

To reduce the number of alternative solutions and consequently improve the resolution process, in each stage the slots are used in ascending order. Without loss of generality, the following constraint is imposed:

$$\sum_{k \in K_j} X_{kl+1} \leq \sum_{k \in K_j} X_{kl} \quad \forall\ j \in J,\ 1 \leq l \leq L \tag{11}$$

The length of empty slots is zero, and, therefore, initial and final times are equal and coincide with the end time of the previous slot. Then, taking into account that the number of slots proposed in all units is overestimated, some of them will be empty.

Finally, the following equality ensures that in each stage, the number of batches selected to meet the demand of the corresponding product coincides with the total number of slots used at the units of each stage,

$$\sum_{k \in K_j} \sum_{1 \leq l \leq L_j} X_{kl} = \sum_{i \in I} \sum_{b \in IB_i} z_{ib} \quad \forall\ j \in J \tag{12}$$

The recently defined variables allow a correct expression of the inequalities posed in eq 5 as

$$\alpha_{ik} \frac{V_k}{\mathrm{SF}_{ij}} Z_{bl} X_{kl} \leq B_{ib} \qquad \forall\ i \in I,\ b \in IB_i,\ 1 \leq l \leq L,\ j \in J,\ k \in K_j \tag{13a}$$

$$B_{ib} \leq \frac{V_k}{\mathrm{SF}_{ij}} + \mathrm{BM}\left(1 - \sum_{1 \leq l \leq L_j} Z_{bl} X_{kl}\right) \qquad \forall\ i \in I,\ b \in IB_i,\ j \in J,\ k \in K_j \tag{13b}$$

where BM is a sufficiently large number that makes the constraint redundant when batch b is not assigned to any slot of unit k.

Constraints 13a and 13b are nonlinear because of the bilinear product $Z_{bl}\, X_{kl}$. Then, in order to eliminate this nonlinearity, a new non-negative variable Y_{bkl} is defined as

$$Y_{bkl} = \begin{cases} 1 & \text{if both } Z_{bl} \text{ and } X_{kl} \text{ are } 1 \\ 0 & \text{otherwise} \end{cases}$$

Thus, eq 13a and 13b can be respectively rewritten as

$$\alpha_{ik} \frac{V_k}{\mathrm{SF}_{ij}} Y_{bkl} \leq B_{ib} \quad \forall\ i \in I,\ b \in IB_i,\ j,\ k \in K_j,\ 1 \leq l \leq L \tag{14a}$$

$$B_{ib} \leq \frac{V_k}{\mathrm{SF}_{ij}} + \mathrm{BM}\left(1 - \sum_{1 \leq l \leq L} Y_{bkl}\right) \qquad \forall\ i \in I,\ b \in IB_i,\ j,\ k \in K_j,\ 1 \leq l \leq L \tag{14b}$$

Given that variables Z_{bl} and X_{kl} are binary, Y_{bkl} can be treated as a continuous variable in the interval [0, 1]. Thus, the number of binary variables is reduced. Taking into account that if batch b is not assigned to slot l, then none of the units of each stage can employ slot l to process batch b, and that if slot l of unit k at stage j is not utilized, then none of the products is processed in it, the following constraints must be posed:

$$\sum_{k \in K_j} Y_{bkl} = Z_{bl} \quad \forall\ i \in I,\ b \in IB_i,\ j \in J,\ 1 \leq l \leq L \tag{15}$$

$$\sum_{b \in \cup_i IB_i} Y_{bkl} = X_{kl} \quad \forall\ j \in J,\ k \in K_j,\ 1 \leq l \leq L \tag{16}$$

Reciprocally, Y_{bkl} must take a value of 1 only if both X_{kl} and Z_{bl} are 1. Therefore, the following linear inequality is imposed:

$$Y_{bkl} \geq X_{kl} + Z_{bl} - 1 \qquad \forall\ j \in J,\ k \in K_j,\ i \in I,\ b \in IB_i,\ 1 \leq l \leq L \tag{17}$$

In summary, if $Z_{bl} = 0$, then, taking into account constraint 15 and that variable Y_{bkl} is not negative, $Y_{bkl} = 0$ for all units. Similarly, if $X_{kl} = 0$, then, taking into account constraint 16, $Y_{bkl} = 0$ for all batches. In both cases, inequality 17 is redundant. On the contrary, if Z_{bl} and X_{kl} are simultaneously equal to 1, taking into account that the upper bound for continuous variable Y_{bkl} is equal to 1, constraint 17 ensures that $Y_{bkl} = 1$.

In this way, assignment variable Y_{bkl} does not need to be defined as binary, and hence the number of binary variables of the model remains unchanged.

4.1.3. Timing Constraints. Non-negative continuous variables, TI_{kl} and TF_{kl}, are used to represent the initial and final processing times, respectively, of the proposed slots in each unit k. When slot l is not the last slot used in unit k of stage j for processing one batch, knowing the processing times of products at each unit and the sequence-dependent changeover times between ordered pairs of products, the final processing time TF_{kl} of slot l in unit k is constrained by

$$\mathrm{TF}_{kl} = \mathrm{TI}_{kl} + \sum_{i \in I} \sum_{i' \in I} \sum_{b \in IB_i} \sum_{\substack{b' \in IB_{i'} \\ b \neq b'}} (t_{ik} + c_{ii'k}) Y_{bkl} Y_{b'kl'} \qquad \forall\ j \in J,\ k \in K_j,\ 1 \leq l < L,\ l < l' \leq L \tag{18}$$

where l' represents the first slot effectively used in unit k of stage j after slot l (Figure 1).

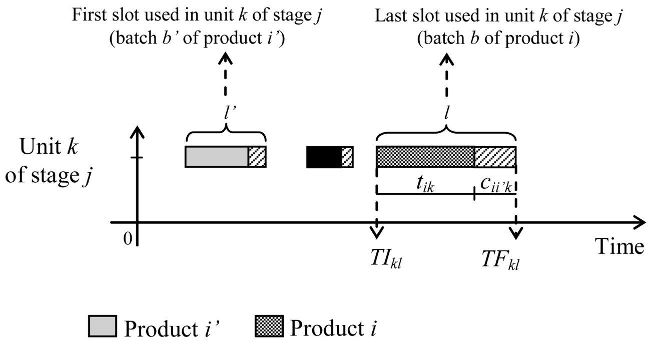

Figure 1. Illustration representing eq 18

When no batch is assigned to slot l at unit k (i.e., $Y_{bkl} = 0$ for all b), the initial and final times of this slot are equal, that is, $\mathrm{TI}_{kl} = \mathrm{TF}_{kl}$.

A new non-negative variable $YY_{blb'l'k}$ is defined to eliminate the bilinear term $Y_{bkl}\, Y_{b'kl'}$ in eq 18. This variable has to be linked to the assignment variables Y_{bkl} and $Y_{b'kl'}$ such that $YY_{blb'l'k}$ takes value 1 if both are 1, and 0 otherwise. To enforce its value, the following conditions are added:

$$YY_{blb'l'k} \geq Y_{bkl} + Y_{b'kl'} - 1 \quad \forall\, j \in J,\ 1 \leq k \leq K_j,\ i \in I,\ i' \in I,\ b \in IB_i,\ b' \in IB_{i'},\ 1 \leq l \leq l',\ 1 \leq l' \leq L \tag{19}$$

$$YY_{blb'l'k} \leq Y_{bkl} \quad \forall\, j \in J,\ 1 \leq k \leq K_j,\ i \in I,\ i' \in I,\ b \in IB_i,\ b' \in IB_{i'},\ 1 \leq l \leq l',\ 1 \leq l' \leq L \tag{20}$$

$$YY_{blb'l'k} \leq Y_{b'kl'} \quad \forall\, j \in J,\ 1 \leq k \leq K_j,\ i \in I,\ i' \in I,\ b \in IB_i,\ b' \in IB_{i'},\ 1 \leq l \leq l',\ 1 \leq l' \leq L \tag{21}$$

Taking into account the previous constraints, the new variable does not need to be declared as binary and can be treated as a continuous variable in the interval [0, 1].

Thus, eq 18 can be represented using the following Big-M representations:

$$\mathrm{TF}_{kl} - \mathrm{TI}_{kl} - \sum_{i\in I}\sum_{i'\in I}\sum_{b\in IB_i}\sum_{\substack{b'\in IB_{i'}\\ b\neq b'}} (t_{ik} + c_{ii'k})YY_{blb'l'k} \geq M_1\Big(X_{kl'} - 1 - \sum_{l<\tilde{l}<l'} X_{k\tilde{l}}\Big) \quad \forall\, j \in J,\ k \in K_j,\ 1 \leq l < L,\ l < l' \leq L \tag{22}$$

$$-\mathrm{TF}_{kl} + \mathrm{TI}_{kl} + \sum_{i\in I}\sum_{i'\in I}\sum_{b\in IB_i}\sum_{\substack{b'\in IB_{i'}\\ b\neq b'}} (t_{ik} + c_{ii'k})YY_{blb'l'k} \geq M_1\Big(X_{kl'} - 1 - \sum_{l<\tilde{l}<l'} X_{k\tilde{l}}\Big) \quad \forall\, j \in J,\ k \in K_j,\ 1 \leq l < L,\ l < l' \leq L \tag{23}$$

where M_1 is a sufficiently large number that makes the constraint redundant when slot l' is not used for processing any product batch.

On the other hand, when slot l is the last slot used at unit k of stage j for processing some batch of the campaign, the previous constraints are not valid. Now, taking into account that the campaign is cyclically repeated over a time horizon, the final processing time TF_{kl} of slot l in unit k is calculated considering the changeover time required for processing the batch assigned

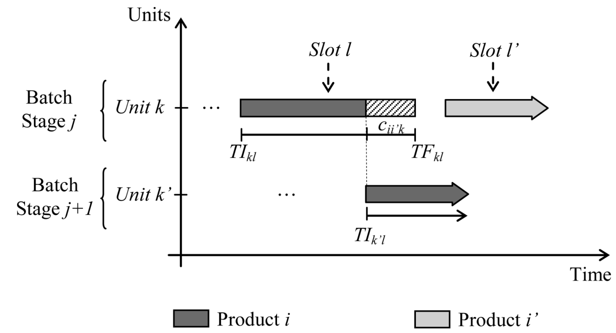

Figure 2. Illustration representing eqs 24a and 24b

to the first slot effectively used in unit k of stage j (Figure 2). Then, the following constraints must be satisfied:

$$\mathrm{TF}_{kl} - \mathrm{TI}_{kl} - \sum_{i\in I}\sum_{i'\in I}\sum_{b\in IB_i}\sum_{b'\in IB_{i'}} (t_{ik} + c_{ii'k})YY_{b'l'blk} \geq M_1\Big(X_{kl'} - 1 - \sum_{1\leq\hat{l}<l'} X_{k\hat{l}} - \sum_{l<\tilde{l}\leq L} X_{k\tilde{l}}\Big) \quad \forall\, j \in J,\ k \in K_j,\ 1 \leq l \leq L,\ 1 \leq l' \leq l \tag{24a}$$

$$-\mathrm{TF}_{kl} + \mathrm{TI}_{kl} + \sum_{i\in I}\sum_{i'\in I}\sum_{b\in IB_i}\sum_{b'\in IB_{i'}} (t_{ik} + c_{ii'k})YY_{b'l'blk} \geq M_1\Big(X_{kl'} - 1 - \sum_{1\leq\hat{l}<l'} X_{k\hat{l}} - \sum_{l<\tilde{l}\leq L} X_{k\tilde{l}}\Big) \quad \forall\, j \in J,\ k \in K_j,\ 1 \leq l \leq L,\ 1 \leq l' \leq l \tag{24b}$$

where the constraints are redundant when some slot following to slot l is used for processing one product batch or if slot l' is not the first slot used in unit k of stage j.

To avoid the overlapping between the processing times of different slots in a unit, the following constraint is added:

$$\mathrm{TF}_{kl} \leq \mathrm{TI}_{kl+1} \quad \forall\, j \in J,\ k \in K_j,\ 1 \leq l < L \tag{25}$$

Besides, if no batch is assigned to slot l+1 of unit k ($X_{kl+1} = 0$), then the initial time of this slot is enforced to be equal to the final time of slot l. Then, taking into account that eq 25 is satisfied for successive slots in a unit, this new condition is represented by

$$\mathrm{TF}_{kl} - \mathrm{TI}_{kl+1} \geq -M_2 X_{kl+1} \quad \forall\, j \in J,\ k \in K_j,\ 1 \leq l < L \tag{26}$$

where M_2 is a sufficiently large number that makes the constraint redundant when a batch is assigned to slot l+1.

As already mentioned, an operational feature of plants involved in this work is the ZW transfer policy. It is assumed that a batch, after finishing its processing at a stage, must be transferred immediately to the next stage. Therefore, when a batch processed in slot l utilizes unit k in stage j and k' in stage j +1, the following equation must be satisfied:

$$\mathrm{TF}_{kl} - \sum_{i \in I} \sum_{i' \in I} \sum_{b \in IB_i} \sum_{b' \in IB_{i'}} c_{ii'k} YY_{blb'l'k} = \mathrm{TI}_{k'l}$$
$$\forall\ j, j+1 \in J,\ k \in K_j,\ k' \in K_{j+1},$$
$$k/X_{kl} = 1,\ k'/X_{k'l} = 1,\ 1 \le l < L,\ l < l' \qquad (27)$$

where l' represents the first slot effectively used in unit k of stage j after slot l (Figure 3).

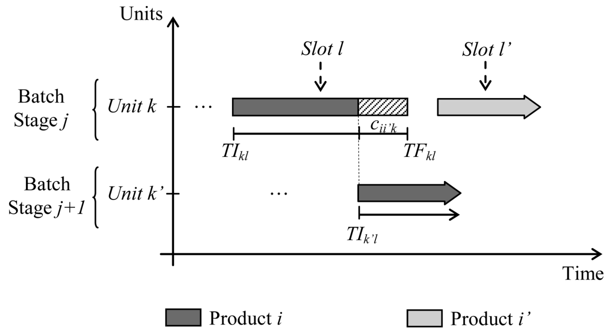

Figure 3. Representation of ZW policy between batch stages j and j+1.

Given that this constraint must be only satisfied when a batch is assigned to those units, then this condition can be expressed through constraints of Big-M type. Constraints 28a and 28b are active when some slot l' following to slot l is used for processing a product batch, while eq 29a and 29b when slot l is the last slot used at unit k of stage j.

$$\mathrm{TF}_{kl} - \sum_{i \in I} \sum_{i' \in I} \sum_{b \in IB_i} \sum_{b' \in IB_{i'}} c_{ii'k} YY_{blb'l'k} - \mathrm{TI}_{k'l}$$
$$\ge M_1\Big(X_{kl} + X_{k'l} + X_{kl'} - 3 - \sum_{l < \tilde{l} < l'} X_{k\tilde{l}}\Big)$$
$$\forall\ j, j+1 \in J,\ k \in K_j,\ k' \in K_{j+1},\ 1 \le l < L,\ l < l' \qquad (28a)$$

$$-\mathrm{TF}_{kl} + \sum_{i \in I} \sum_{i' \in I} \sum_{b \in IB_i} \sum_{b' \in IB_{i'}} c_{ii'k} YY_{blb'l'k} + \mathrm{TI}_{k'l}$$
$$\ge M_1\Big(X_{kl} + X_{k'l} + X_{kl'} - 3 - \sum_{l < \tilde{l} < l'} X_{k\tilde{l}}\Big)$$
$$\forall\ j, j+1 \in J,\ k \in K_j,\ k' \in K_{j+1},\ 1 \le l < L,\ l < l' \qquad (28b)$$

$$\mathrm{TF}_{kl} - \sum_{i \in I} \sum_{i' \in I} \sum_{b \in IB_i} \sum_{b' \in IB_{i'}} c_{ii'k} YY_{b'l'blk} - \mathrm{TI}_{k'l}$$
$$\ge M_1\Big(X_{kl} + X_{kl'} + X_{k'l} - 3 - \sum_{l < \tilde{l}} X_{k\tilde{l}} - \sum_{\tilde{l} < l'} X_{k\tilde{l}}\Big)$$
$$\forall\ j, j+1 \in J,\ k \in K_j,\ k' \in K_{j+1},\ 1 \le l \le L,\ l' \le l \qquad (29a)$$

$$-\mathrm{TF}_{kl} + \sum_{i \in I} \sum_{i' \in I} \sum_{b \in IB_i} \sum_{b' \in IB_{i'}} c_{ii'k} YY_{b'l'blk} + \mathrm{TI}_{k'l}$$
$$\ge M_1\Big(X_{kl} + X_{kl'} + X_{k'l} - 3 - \sum_{l < \tilde{l}} X_{k\tilde{l}} - \sum_{\tilde{l} < l'} X_{k\tilde{l}}\Big)$$
$$\forall\ j, j+1 \in J,\ k \in K_j,\ k' \in K_{j+1},\ 1 \le l \le L,\ l' \le l \qquad (29b)$$

In particular, if only a campaign is processed to fulfill the production targets, changeover times must not be considered to calculate TF_{kl} when l is the last slot used at unit k of stage j. Consequently, eqs 24a and 24b, and eqs 29a and 29b with regards to the ZW policy are simpler and the parameters $c_{ii'k}$ and the variables $Y_{blb'l'k}$ are not required.

4.1.4. Cycle Time Definition. The expression for the cycle time of the campaign, CT, is obtained using the initial and final times of the first slot used for processing batches and last slot proposed in unit k of stage j, respectively:

$$\mathrm{CT} = \max_{j \in J}\{\max_{k \in K}\{\mathrm{TF}_{kL} - \mathrm{TI}_{k\tilde{l}_k}\}\} \qquad (30)$$

where $\tilde{l}_k$ represents the first slot effectively used in unit k for processing one batch, that is, $\tilde{l}_k = \min\,\{1 \le l \le L/X_{kl} = 1\}$. This equation can be represented using a Big-M representation, as

$$\mathrm{CT} - \mathrm{TF}_{kL} + \mathrm{TI}_{kl} \ge M_1\Big(X_{kl} - 1 - \sum_{\substack{l' \\ 1 \le l' < l}} X_{kl'}\Big)$$
$$\forall\ j \in J,\ k \in K_j,\ 1 \le l \le L \qquad (31)$$

where the constraint is redundant for all the previous and subsequent slots, if any, to the first not empty one in unit k.

Additional constraints which establish tight bounds for timing variables TI_{kl}, TF_{kl}, CT are incorporated to the model in order to reduce the search space, and consequently achieve significant savings in computational time:

$$\mathrm{TI}_{kl} \le \mathrm{TFMax}_j - \min_i\{t_{ik}\} \quad \forall\ j \in J,\ k \in K_j,\ 1 \le l \le L \qquad (32)$$

$$\mathrm{TF}_{kl} \ge \min_i\{t_{ik}\} X_{kl} \quad \forall\ j \in J,\ k \in K_j,\ 1 \le l \le L \qquad (33)$$

$$\mathrm{CT} \ge \sum_{i \in I} \sum_{b \in IB_i} \sum_{l} t_{ik} Y_{bkl} \quad \forall\ j \in J,\ k \in K_j \qquad (34)$$

In this case, as the number of batches of each product in the campaign is a model decision, parameter TFMax_j is calculated by solving a simpler model where batching is not considered, each stage considers only the unit with longer total processing times and the number of batches to be scheduled is equal to the total set of proposed batches for all products. Then, TFMax_j corresponds to the final time of the last assigned slot to stage j.

4.1.5. Objective Function. The problem goal is to minimize the cycle time of the production campaign that fulfill the demands requirements, that is

$$\text{Minimize CT} \qquad (35)$$

where the proposed MILP formulation of model SP includes constraints 1−4, 6−12, 14a−17, 19−26, 28a, 29b, and 31−34.

The objective function value of this model represents a good upper bound for the cycle time of detailed problem DP, which is presented next.

4.2. Detailed Model DP. To complete the resolution methodology of the integrated batching and scheduling problem, the formulation of model DP is presented in this section. In this formulation, preordering constraints defined in SP, which affect the optimality of the scheduling decisions, are removed. On the contrary, DP result in a more complex model that considers an exact scheduling for the selected batches in the batching level as well as assumptions that keep the model generality. However, model SP provides significant information that allows solving the model DP in a reduced space in reasonable computation time.

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The optimal solution of the first model allows strongly reducing the number of slots proposed compared to model DP. On the other hand, the optimal objective function value obtained from simplified model SP provides a good upper bound for the campaign cycle time of the model DP.

Taking into account the preordering constraint of model SP is avoided, the variable Z_{bl} is not required. Now, batch b can be assigned to different slots in each stage. Equations involving this variable, as 7−9 and 15 are not valid in this new formulation and, therefore, this variable is not used. To assign batches to slots on each unit, variable Y_{bkl}, previously used in model SP, will be again employed in the same way. However, unlike the previous model, this variable is required to be binary. Although the binary variable Y_{bkl} is enough for formulating the detailed scheduling problem, the binary variable X_{kl}, which specifies the slots set utilized in unit k for processing batches, will be also used in order to reduce the search space and, therefore, to improve the computational performance.

Batching decisions are modeled through constraints 1−4, 14a, and 14b from model SP. However, due to that the preordering constraint is not considered for the detailed model, new assignment, sequencing and timing constraints are presented.

In this model, the handling of the slot concept is slightly different compared to model SP. Without loss of generality and in order to reduce the search space, it is assumed that slots of each unit are consecutively used in ascending numerical order. Hence, the optimal assignment of batches to slots obtained from model SP allows the tightening of the number of slots postulated for unit k of stage j, denoted by L_{kj}.

4.2.1. Assignment and Sequencing Constraints. Unlike model SP, it is assumed that slots of each unit are occupied in ascending order. Hence, the slots of zero length take place at the end of each unit. The following constraint establishes that for each unit k, slot l+1 is only used if slot l has been already allocated:

$$X_{kl} \geq X_{kl+1}, \quad \forall\, j \in J,\, k \in K_j,\, 1 \leq l \leq L_{kj} \tag{36}$$

Logical relations can be defined among binary variables z_{ib}, X_{kl} and Y_{bkl}. In fact, if slot l of unit k is not utilized, then none of the proposed batches is processed in it. Therefore, the following constraint is imposed:

$$\sum_{b \in \cup_{i \in I} IB_i} Y_{bkl} = X_{kl} \quad \forall\, j \in J,\, k \in K_j,\, 1 \leq l \leq L_{kj} \tag{37}$$

Moreover, eq 37 allows representing the opposite conditional of the logical relation recently mentioned, which indicates that if slot l of unit k is utilized, then only one of the proposed batches is processed in it.

On the other hand, if batch b of product i is selected (i.e., z_{ib} = 1), then this batch is processed, in each stage j, in only one slot of some of the available units at the stage. This condition is guaranteed by the following constraint:

$$\sum_{k \in K_j} \sum_{1 \leq l \leq L_{kj}} Y_{bkl} = z_{ib} \quad \forall\, j \in J,\, i \in I,\, b \in IB_i \tag{38}$$

Finally, considering constraints 37 and 38 the following constraint must be satisfied. Though it is redundant with the previous ones, it reduces the computing time:

$$\sum_{k \in K_j} \sum_{1 \leq l \leq L_{kj}} X_{kl} = \sum_{i \in I} \sum_{b \in IB_i} z_{ib} \quad \forall\, j \in J \tag{39}$$

4.2.2. Timing Constraints. Taking into account that the slots are used in ascending numerical order at each unit, the final processing time TF_{kl} of slot l in unit k is calculated by

$$\mathrm{TF}_{kl} = \mathrm{TI}_{kl} + \sum_{i \in I} \sum_{i' \in I} \sum_{b \in IB_i} \sum_{\substack{b' \in IB_{i'} \\ b \neq b'}} (t_{ik} + c_{ii'k}) Y_{bkl} Y_{b'kl+1}$$
$$\forall\, j \in J,\, k \in K_j,\, 1 \leq l < L_{kj} \tag{40}$$

This constraint is only valid if l is not the last slot used in unit k of stage j for processing one batch, that is, if $Y_{b'kl+1}$ take value 1 for some b' (Figure 4).

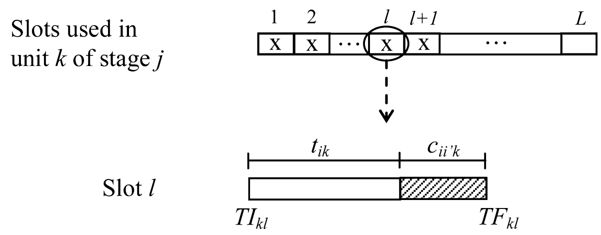

Figure 4. Illustration representing eq 40

On the other hand, when the sequence of slots used in unit k is 1, 2, ... l, that is, slot l is the last slot used at unit k of stage j to process some batch, taking into account that the campaign can be cyclical repeated over a time horizon, the final processing time TF_{kl} is calculated considering the changeover time required for processing the batch assigned to slot 1 in unit k of stage j. Then, the following constraints must be satisfied:

$$\mathrm{TF}_{kl} = \mathrm{TI}_{kl} + \sum_{i \in I} \sum_{i' \in I} \sum_{b \in IB_i} \sum_{\substack{b' \in IB_{i'} \\ b \neq b'}} (t_{ik} + c_{ii'k}) Y_{bkl} Y_{b'k1}$$
$$\forall\, j \in J,\, k \in K_j,\, 1 \leq l \leq L_{kj} \tag{41}$$

Nonnegative variable $YY_{blb'l'k}$ defined in model SP is used to eliminate the bilinear products in eqs 40 and 41. However, taking into account that in this formulation the slots of each unit are consecutively used in ascending order, it is only necessary to link the assignments variables Y_{bkl} and $Y_{b'kl+1}$, that is, those relative to consecutive slots on the same unit k as well as Y_{bkl} and $Y_{b'k1}$, for all slot l, in order to represent the previous constraints. Therefore, the number of variables $YY_{blb'l'k}$ required for this model is $|K||B|(|B|-1)(|L|-1).2 + |K||B|$, in contrast to model SP where $|K||B|(|B|-1)(|L|-1).|L|/2 + |K||B||L|$ variables are needed, which is strongly increased when $|L| \geq 4$.

Equations 40 and 41 can be represented using the following Big-M expressions, respectively:

$$\mathrm{TF}_{kl} - \mathrm{TI}_{kl} - \sum_{i \in I} \sum_{i' \in I} \sum_{b \in IB_i} \sum_{\substack{b' \in IB_{i'} \\ b \neq b'}} (t_{ik} + c_{ii'k}) YY_{blb'l+1k}$$
$$\geq M_3(X_{kl+1} - 1) \quad \forall\, j \in J,\, k \in K_j,\, 1 \leq l < L_{kj} \tag{42a}$$

$$-\mathrm{TF}_{kl} + \mathrm{TI}_{kl} + \sum_{i \in I} \sum_{i' \in I} \sum_{b \in IB_i} \sum_{\substack{b' \in IB_{i'} \\ b \neq b'}} (t_{ik} + c_{ii'k}) YY_{blb'l+1k}$$
$$\geq M_3(X_{kl+1} - 1) \quad \forall\, j \in J,\, k \in K_j,\, 1 \leq l < L_{kj} \tag{42b}$$

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$$\mathrm{TF}_{kl} - \mathrm{TI}_{kl} - \sum_{i\in I}\sum_{i'\in I}\sum_{b\in IB_i}\sum_{\substack{b'\in IB_{i'}\\ b\neq b'}} (t_{ik} + c_{ii'k})YY_{blb'1k} \geq -M_3X_{kl+1} \quad \forall\, j \in J,\, k \in K_j,\, 1 \leq l \leq L_{kj} \tag{43a}$$

$$-\mathrm{TF}_{kl} + \mathrm{TI}_{kl} + \sum_{i\in I}\sum_{i'\in I}\sum_{b\in IB_i}\sum_{\substack{b'\in IB_{i'}\\ b\neq b'}} (t_{ik} + c_{ii'k})YY_{blb'1k} \geq -M_3X_{kl+1} \quad \forall\, j \in J,\, k \in K_j,\, 1 \leq l \leq L_{kj} \tag{43b}$$

where M_3 is sufficiently large. Also, constraints 25 and 26 must be included in the formulation of this model.

To ensure ZW transfer policy, when a batch b is processed in slot l of unit k at stage j and in slot l' of unit k' at stage j+1, the following equation must be satisfied when slot l is not the last slot used at unit k of stage j:

$$\mathrm{TF}_{kl} - \sum_{i'\in I}\sum_{b'\in IB_{i'}} c_{ii'k}YY_{blb'l+1k} = \mathrm{TI}_{k'l'} \quad \forall\, j, j+1 \in J,\, k \in K_j,\, k' \in K_{j+1},\, 1 \leq l < L_{kj},\, 1 \leq l' \leq L_{k,j+1} i \in I,\, b \in IB_i,\, b/Y_{bkl} = 1,\, b/Y_{bk'l'} = 1 \tag{44}$$

As this constraint must be only satisfied when a batch is assigned to those units and slots, then this condition can be expressed through constraints of Big-M type. Constraints 45a and 45b are active when slot l+1 is used for processing a product batch, while eq 46a and 46b when slot l is the last slot used at unit k of stage j.

$$\mathrm{TF}_{kl} - \sum_{i'\in I}\sum_{b'\in IB_{i'}, b'\neq b} c_{ii'k}YY_{blb'l+1k} - \mathrm{TI}_{k'l'} \geq M_3(Y_{bkl} + Y_{bk'l'} + X_{kl+1} - 3) \quad \forall\, j, j+1 \in J,\, k \in K_j,\, k' \in K_{j+1},\, 1 \leq l < L_{kj},\, 1 \leq l' \leq L_{k'j+1},\, i \in I,\, b \in IB_i \tag{45a}$$

$$-\mathrm{TF}_{kl} + \sum_{i'\in I}\sum_{\substack{b'\in IB_{i'}\\ b'\neq b}} c_{ii'k}YY_{blb'l+1k} + \mathrm{TI}_{k'l'} \geq M_3(Y_{bkl} + Y_{bk'l'} + X_{kl+1} - 3) \quad \forall\, j, j+1 \in J,\, k \in K_j,\, k' \in K_{j+1},\, 1 \leq l < L_{kj},\, 1 \leq l' \leq L_{k'j+1},\, i \in I,\, b \in IB_i \tag{45b}$$

$$\mathrm{TF}_{kl} - \sum_{i'\in I}\sum_{b'\in IB_{i'}} c_{ii'}YY_{blb'1k} - \mathrm{TI}_{k'l'} \geq M_3(Y_{bkl} + Y_{bk'l'} - 2 - \sum_{\tilde{l}>l} X_{k\tilde{l}}) \quad \forall\, j, j+1 \in J,\, k \in K_j,\, k' \in K_{j+1},\, 1 \leq l \leq L_{kj},\, 1 \leq l' \leq L_{k'j+1},\, i \in I,\, b \in IB_i \tag{46a}$$

$$-\mathrm{TF}_{kl} + \sum_{i'\in I}\sum_{b'\in IB_{i'}} c_{ii'k}YY_{blb'1k} + \mathrm{TI}_{k'l'} \geq M_3(Y_{bkl} + Y_{bk'l'} - 2 - \sum_{\tilde{l}>l} X_{k\tilde{l}}) \quad \forall\, j, j+1 \in J,\, k \in K_j,\, k' \in K_{j+1},\, 1 \leq l \leq L_{kj},\, 1 \leq l' \leq L_{k'j+1},\, i \in I,\, b \in IB_i \tag{46b}$$

Analogously to model SP, if the campaign is not repeated over a time horizon, when slot l is the last slot used at unit k of stage j, the changeover times must not be considered to calculate the final time TF_{kl} of this slot. Constraints 43a, 43b, 46a, and 46b concerning to ZW policy are most simple and the parameters $c_{ii'k}$ and the variables $Y_{blb'l'k}$ are not required.

4.2.3. Cycle Time Definition. Taking into account that slots of each unit are used in ascending numerical order, the expression for the cycle time of the campaign, CT, is given by

$$\mathrm{CT} \geq \mathrm{TF}_{kL_{kj}} - \mathrm{TI}_{k1}, \quad \forall\, j \in J,\, k \in K_j \tag{47}$$

In addition, constraints 32, 33, and 34 of model SP, which proposed upper and lower bounds for the initial and final times, respectively, and a lower bound on the cycle time of the campaign, are also imposed.

The optimal value of the objective function of model SP, $\mathrm{CT}^{\mathrm{SP}}$, represents a tight upper bound for the cycle time of model DP presented in this section.

$$\mathrm{CT} \leq \mathrm{CT}^{\mathrm{SP}} \tag{48}$$

As will be shown in the Examples section, this bound strongly reduces the search space and allows the optimization of batching and scheduling decisions simultaneously in reasonable computing time.

4.2.4. Objective Function. The MILP formulation of model DP consists of minimizing the same objective function that models SP, subject to constraints 1−4, 6, 14a, 14b, 25, 26, 32−34, 36−39, 42a, 43b, 45a, 46b, 47, and 48.

Finally, this model allows a determination of the number and size of batches to satisfy the given production targets, the assignment of batches to units in each stage, the production sequence on each unit, and initial and final processing times for the batches that compose the campaign in each processing unit, in order to minimizing the production cycle time.

5. EXAMPLES

In this section, the capabilities of the proposed approach are highlighted through examples, which are of increasing size. For all examples, the batching and scheduling decisions of the given multiproduct plants, operating through mixed campaigns, have to be simultaneously solved to fulfill the required product demands. In all cases, coefficient α_{ik} representing the minimum filled rate required to process product i in unit k, is assumed to be 0.50 for all products and equipment items. That is, this parameter accounts for 50% of total available capacity of units. The last two examples emphasize the importance of the proposed resolution methodology from the computational point of view. All examples were implemented and solved in GAMS[26] version 24.1.3 with a 2.8 GHz Intel Core i7 processor. The CPLEX 12.5.1 solver was employed for solving the MILP problems, with a 0% optimality gap. The number of continuous and binary variables and constraints strongly depend on the total number of units, products and batches proposed to fulfill the required demands.

5.1. Example 1. In this example, the considered batch plant consists of three stages with nonidentical parallel units with known sizes that operate out-of-phase, as is illustrated in Figure 5.

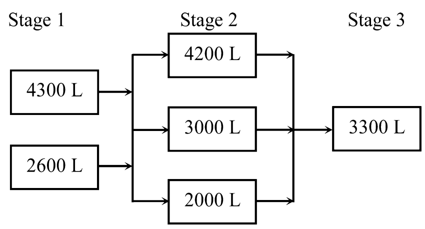

Figure 5. Plant structure for Example 1.

Available units at each stage are denoted by the sets: $K_1 = \{1, 2\}$, $K_2 = \{3, 4, 5\}$, and $K_3 = \{6\}$, respectively. Products A, B, and C have to be processed through all stages before being converted into final products. The required amounts in the campaign are $Q_A = 8000$ kg, $Q_B = 6000$ kg, and $Q_C = 3000$ kg. Data on processing times and size factors of each product are shown in Table 1, while the sequence-dependent changeover times are given in Table 2.

Considering the nonidentical parallel unit sizes at each stage, the size factors for each product in each stage and the equipment utilization minimum rate for all products, the minimum feasible batch sizes for products A, B, and C are

$$B_A^{min} = 0.5\max\{3714 \text{ kg}, 3333 \text{ kg}, 5076 \text{ kg}\} = 2538 \text{ kg}$$

$$B_B^{min} = 0.5\max\{4334 \text{ kg}, 2857 kg, 3882 \text{ kg}\} = 2167 \text{ kg}$$

$$B_C^{min} = 0.5\max\{3714 \text{ kg}, 3077 \text{ kg}, 4714 \text{ kg}\} = 2357 \text{ kg}$$

Then, the maximum number of batches of each product at the campaign is given by

$$\mathrm{NBC}_A^{UP} = \left\lceil \frac{8000}{2538} \right\rceil = 4, \quad \mathrm{NBC}_B^{UP} = \left\lceil \frac{6000}{2167} \right\rceil = 3, \quad \mathrm{NBC}_C^{UP} = \left\lceil \frac{3000}{2357} \right\rceil = 2$$

Thus, the sets of proposed batches are $\{b_1, b_2, b_3, b_4\}$, $\{b_5, b_6, b_7\}$, and $\{b_8, b_9\}$ for products A, B, and C, respectively, and consequently a total of nine batches must be postulated to guarantee the global optimality of the solution for this example.

Also, the maximum feasible batch sizes for all products allow a determination for the minimum number of batches of every product at the campaign. In this case, the maximum feasible batch sizes for all products are

$$B_A^{max} = \min\{6142 \text{ kg}, 7000 \text{ kg}, 5076 \text{ kg}\} = 5076 \text{ kg}$$

$$B_B^{max} = \min\{7167 \text{ kg}, 6000 \text{ kg}, 3882 \text{ kg}\} = 3882 \text{ kg}$$

$$B_C^{max} = \min\{6142 \text{ kg}, 6461 \text{ kg}, 4714 \text{ kg}\} = 4714 \text{ kg}$$

therefore, the required minimum number of batches for products A and B is two, while that for product C is one.

As was mentioned earlier in the paper, the optimization criteria used for both models that integrate the proposed solution strategy, is the minimization of the campaign cycle time, taking into account that the campaign is periodically repeated along a time horizon. The model SP comprises 75619 constraints, 22619 continuous variables, and 139 binary variables, and it was solved in 16.4 CPU seconds, while DP involves 6436 constraints, 1424 continuous variables, and 148 binary variables, and its solving time was equal to 3.6 s. The optimal objective function values are equal to 34.25 h for both models.

To assess the proposed strategy, model DP without considering the supplied information by model SP (upper bound for the cycle time introduced by eq 48 and tighter number of postulated slots) is solved. In this case, the same optimal solution is obtained in 35.1 CPU seconds, which increases the computational time compared with that obtained with the proposed solution strategy.

The optimal campaign for satisfying the required product amounts involves two batches of product A (b_1, b_2), two of B (b_5, b_6), and one of C (b_8); that is, the demands of all products are fulfilled with the minimum number of batches. The optimal production sequence obtained in each batch unit for the different stages, considering sequence-dependent changeover times, is illustrated in the Gantt chart of Figure 6. Taking into account that the optimal campaign is cyclically repeated over a time horizon, the changeover times between products processed in the last and first slot of each unit must be included in the optimization in order to achieve the accurate overlap of successive campaigns. For this example, as it can be seen from Figure 6, changeover times between pairs of campaigns are $c_{AB1} = 1$ h, $c_{AC2} = 1$ h for units of stage 1; $c_{BB3} = 0$ h, $c_{AC4} = 4$ h, $c_{AB5} = 2$ h for units of stage 2; and $c_{AC6} = 1$ h for stage 3.

The size for each batch that composes the campaign is equal to 5000 and 3000 kg for product A, 3833 and 2167 kg for product B, and 3000 kg for product C. The capacities used, in liters, in each unit of the different stages for processing the selected batches are summarized in Table 3. The batches that reach the minimum and maximum capacities are highlighted in boxes shaded in white and gray, respectively. Batch b_1 of product A is processed in units 1, 4, and 6, and its size is the maximum possible to be processed in unit 4 of stage 2. Then, batch b_2 fulfills the required amount of that product occupying approximately 81%, 90%, and 60% of the capacity of units 2, 5, and 6, respectively. On the other hand, two batches of product B are processed for meeting its demand. Batch b_5 size is larger than b_6 and additionally the size of b_6 is the required minimum. The latter batch is processed in units 2, 5, and 6 using 50%, 75.75%, and about 55% of their capacities, respectively.

Finally, from the computational point of view, this example shows that the simplified model SP may be used as a good heuristic for solving the simultaneous batching and scheduling of

Table 1. Processing Times and Size Factors of Products for Example 1

product	processing time: t_{ik} (h)						size factor: SF_{ij} (L/kg)		
	stage 1		stage 2			stage 3	stage 1	stage 2	stage 3
i	1	2	3	4	5	6	$k = 1, 2$	$k = 3, 4, 5$	$k = 6$
A	14	9	25	18	12	7	0.70	0.60	0.65
B	16	10	18	13	9	5	0.60	0.70	0.85
C	12	8	15	11	8	4	0.70	0.65	0.70

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Table 2. Changeover Times for Example 1

	Sequence-dependent changeover time: $c_{ii'k}$ (h)														
	stage 1			stage 2									stage 3		
	k = 1, 2			k = 3			k = 4			k = 5			k = 6		
i	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
A	1	1	1	0	6	6	0	4	4	0	2	2	0	2	1
B	0.25	2	1	6	0	6	4	0	4	2	0	2	2.15	0	2.25
C	2	0.5	0	6	6	0	1	1	0	2	2	0	0	1	0

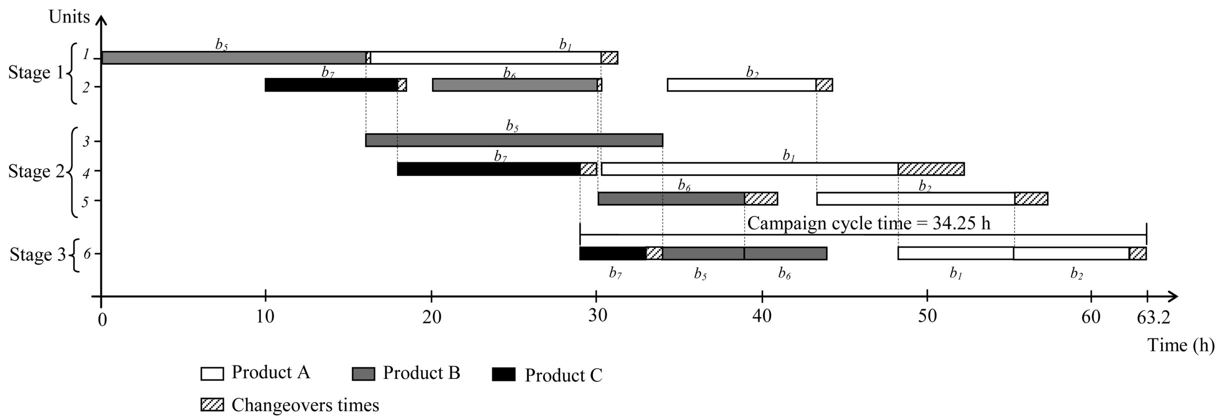

Figure 6. Optimal production schedule for the campaign of Example 1.

Table 3. Capacities Used in Each Unit of Each Stage for Example 1 (L)

		Stage 1		Stage 2			Stage 3
Product	Batch	k = 1	k = 2	k = 3	k = 4	k = 5	k = 6
A	b_1	3500			3000		3250
	b_2		2100			1800	1950
B	b_5	2300		2683			3258
	b_6		1300			1517	1842
C	b_8		2100		1950		2100

batch plants taking into account the high combinatorial complexity associated with this type of decisions.

5.2. Example 2. To illustrate the significant savings in CPU time that can be attained by using the proposed approach, a second case study is presented. In this example, the batch plant consists of four stages, with two nonidentical units operating out-of-phase on stages 1 and 3, and one unit on the remaining stages, as is illustrated in Figure 7.

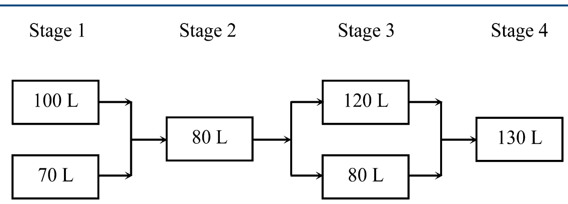

Figure 7. Plant structure for Example 2.

Products A, B, C, and D must be manufactured following the same sequence of stages. The required amounts of them in the campaign are Q_A = 110 kg, Q_B = 150 kg, Q_C = 85 kg, and Q_D = 80 kg. Data on processing times and size factors of each product are shown in Table 4, while the sequence-dependent changeover times are given in Table 5.

For this example, the minimum feasible batch sizes are 50, 72.22, 40.63, and 54.17 kg for products A, B, C, and D, respectively. Then, taking into account the required product amounts, the maximum number of batches for the campaign is three for products A, B, and C, and two for product D. Thus, a total of 11 batches partitioned into sets $\{b_1, b_2, b_3\}$, $\{b_4, b_5, b_6\}$, $\{b_7, b_8, b_9\}$ and $\{b_{10}, b_{11}\}$ must be postulated for products A, B, C, and D, respectively. On the other hand, considering that the maximum feasible batch sizes are 80 kg for product A, 100 kg for B, 60 kg for C, and 86.96 kg for product D, the minimum number of batches required for products A, B, and C is two, while that for product D is one.

Table 6 lists the objective function values, model sizes, and computational statistics for both formulations that integrate the proposed solution strategy, as well as for problem DP when the optimal solution achieved through model SP is not considered for its optimization.

As it can be observed from Table 6, the optimal objective values of models SP and DP are not equal. However, the number of selected batches for each product and the optimal campaign cycle time obtained from the first model, provide a better estimation of the number of slots that must be postulated for the detailed model and a tighter upper bound on the objective function of this model. Thus, model DP is solved in a reasonable

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Table 4. Processing Times and Size Factors of Products for Example 2

product	processing time: t_{ik} (h)						size factor: SF_{ij} (L/kg)			
	stage 1		stage 2	stage 3		stage 4	stage 1	stage 2	stage 3	stage 4
i	1	2	3	4	5	6	k = 1, 2	k = 3	k = 4, 5	k = 6
A	3	2	2	6	5	5	1.25	0.80	1.45	1.50
B	8	6	2	3	2	2	0.85	0.60	1.20	0.90
C	7	6	1	12	10	6	1.40	1.00	2.00	1.60
D	10	8	2	6	4	5	1.15	0.80	1.15	1.20

Table 5. Changeover Times for Example 2

	sequence-dependent changeover time: $c_{ii'k}$ (h)															
	stage 1				stage 2				stage 3				stage 4			
	k = 1, 2				k = 3				k = 4, 5				k = 6			
i	A	B	C	D	A	B	C	D	A	B	C	D	A	B	C	D
A	0	1	2	2	2	1.5	0	1	1	4	4	4	1	1.25	0	1
B	0	1	0.5	3	0	0	0	2	1.5	0	0	1.5	1	0.25	1.25	1
C	1	2	1	1	0	1.5	1.5	1	2	2	0	0	1	1	0.25	0
D	3	1.5	1	0.5	1	1	1	1	1	1	2	2	0.25	0	2	0.25

Table 6. Model Sizes and Computational Statistics for Example 2

model	objective function	constraints	variables: binary	variables: continuous	CPU time (s)
simplified (SP)	42	159467	191	48985	62.6
detailed (DP)	38	17363	289	3884	150.0
DP without considering SP	38	71425	741	14558	941.7

computing time. Also, this example allows showing the significant advantage of the proposed solution approach to obtain the simultaneous batching and scheduling of batch plants. In this sense, the last row of Table 6 shows that the CPU time is increased (more than 4 times) when it is compared with the required total time needed to solve proposed strategy.

For both models, 7 batches were selected out of a total of 11, at the optimal solution. Batches b_1 and b_2 satisfy the total required demand of product A with sizes of 55.17 and 54.83 kg, respectively. Batches b_4 and b_5 of product B with sizes equal to 77.78 and 72.22 kg, respectively, are processed for fulfilling the campaign demand of product B. Batches b_7 and b_8 are required to achieve the production of C with sizes equal to 44.38 and 40.63 kg, respectively. The assigned sizes to batches b_5 and b_8 satisfy the minimum filled capacity required for unit 6; that is, 50% of the total unit capacity is used for processing these batches. Finally, only one batch (b_{10}) is selected to accomplish the demand of product D. Information about the occupied capacity, in liters, in each unit of each stage for processing the selected batches is resumed in Tables 7 and 8 for both models, where the minimum and maximum capacities reached for processing the product batches are highlighted in boxes shaded in white and gray, respectively.

For each model, the optimal campaign scheduling with minimum cycle time is illustrated in Figure 8. The model SP considers a preordering constraint in the assignment of batches to slots, where each batch is assigned to exactly the same slot in all stages. Then, as it can be noted from Figure 8a, particularly on stages 2 and 4, the processing sequence of the batches assigned to the first seven slots is the same on those units, namely b_1–b_7–b_{10}–b_4–b_5–b_2–b_8. In contrast to model SP, model DP, which does not consider batch preordering, reduces the campaign cycle time in 4 h, varying the batch sequencing in the stages. This can be easily observed in Figure 8 b where the batch sequencing at stage 2 is b_7–b_1–b_{10}–b_4–b_5–b_8–b_2, while at stage 4 is b_1–b_7–b_{10}–b_4–b_5–b_2–b_8. As it can be observed from Figure 8 panels a and b, the batch sequencing on stage 4 is the same for both models (A–C–D–B–B–A–C). However, on stage 2, model DP allows the batches to be put such that the idle time among batches processed on stage 4 is reduced compared to model SP. It is worth mentioning that although the solution of model SP is suboptimal, it introduces an appropriate upper bound on the objective function and further reduces the number of postulated

Table 7. Capacities Used in Each Unit of Each Stage for Example 2 with SP (L)

Product	Batch	Stage 1: k = 1	Stage 1: k = 2	Stage 2: k = 3	Stage 3: k = 4	Stage 3: k = 5	Stage: k = 6
A	b_1	68.97		44.14		$\boxed{80}$	82.76
	b_2		68.53	43.86		79.50	82.24
B	b_4	66.11		46.67	93.33		70
	b_5		61.39	43.33	86.67		$\boxed{65}$
C	b_7		62.12	44.38	88.75		71
	b_8		56.87	40.63	81.25		$\boxed{65}$
D	b_{10}	92		64	92		96

Table 8. Capacities Used in Each Unit of Each Stage for Example 2 with DP (L)

Product	Batch	Stage 1: k = 1	Stage 1: k = 2	Stage 2: k = 3	Stage 3: k = 4	Stage 3: k = 5	Stage: k = 6
A	b_1		68.97	44.14		$\boxed{80}$	82.76
	b_2	68.53		43.86		79.50	82.24
B	b_4		66.11	46.67	93.33		70
	b_5	61.39		43.33	86.67		$\boxed{65}$
C	b_7	62.12		44.38	88.75		71
	b_8		56.87	40.63	81.25		$\boxed{65}$
D	b_{10}	92		64	92		96

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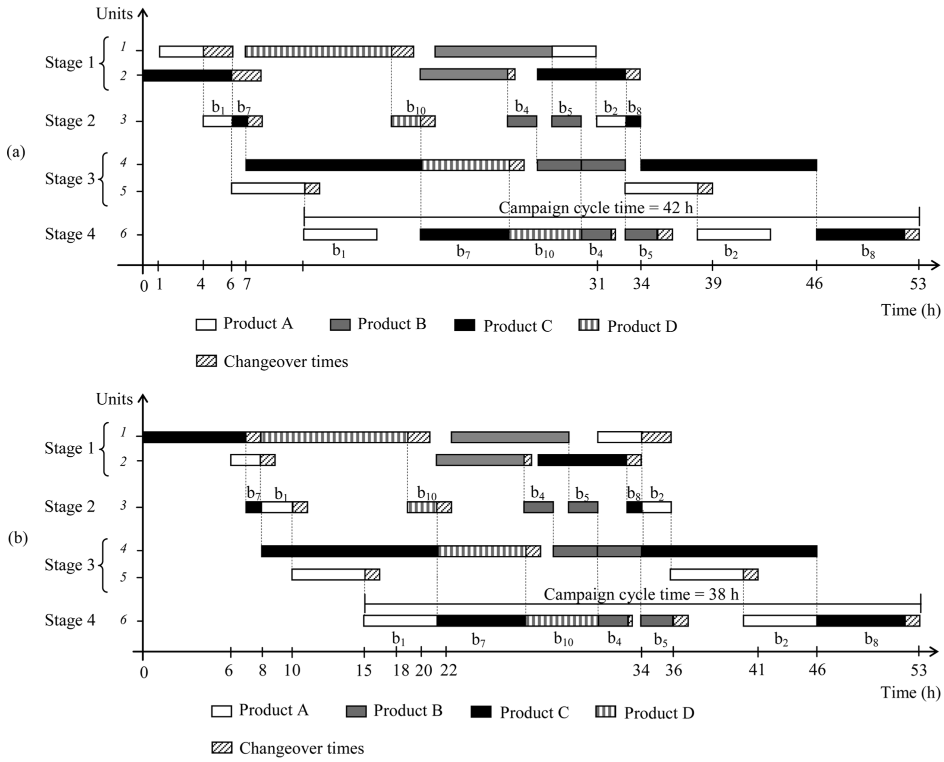

Figure 8. Gantt chart of the production campaign for example 2: (a) simplified model; (b) detailed model.

slots, which enable achieving a better performance in model DP, as it is shown in Table 6.

5.3. Example 3. In this case, a new large example is introduced. The plant consists of four batch stages with two nonidentical parallel units operating out-of-phase on stages 1 and 2, three nonidentical parallel units on stage 3, and one unit on stage 4. Units at each stage are denoted by the sets: $K_1 = \{1, 2\}$, $K_2 = \{3, 4\}$, $K_3 = \{5, 6, 7\}$, and $K_4 = \{8\}$, respectively, and units sizes are shown in Figure 9.

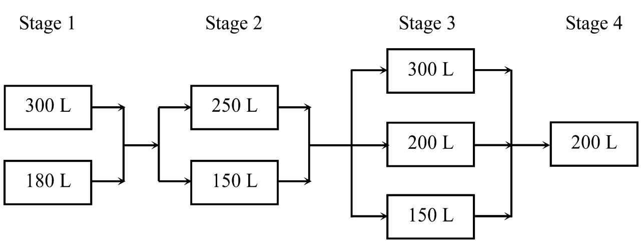

Figure 9. Plant structure for Example 3.

Products A, B, C, and D have to be manufactured following the same sequence of stages. The required amounts of them in the campaign are $Q_A = 560$ kg, $Q_B = 850$ kg, $Q_C = 250$ kg, and $Q_D = 300$ kg. Data on processing times and size factors of each product are shown in Table 9, while the sequence-dependent changeover times are given in Table 10.

The minimum feasible batch sizes are 200 kg for products A and C, 187.5 kg for product B, and 175 kg for D. Then, taking into account the required product amounts, the maximum number of batches for the campaign is three for product A, five for product B, and two for products C and D. Thus, a total of 12 batches partitioned into sets $\{b_1, b_2, b_3\}$, $\{b_4, b_5, b_6, b_7, b_8\}$, $\{b_9, b_{10}\}$ and $\{b_{11}, b_{12}\}$ must be postulated for products A, B, C and D, respectively. On the other hand, considering that the maximum feasible batch sizes are 357 kg for product A, 350 kg for B, 312.5 kg for C, and 500 kg for product D, the required minimum number of batches for product A is two, for product B is three, while for products C and D is one.

The optimal objective function values are equal to 39.75 h for models SP and DP. Table 11 lists model sizes and computational statistics for both formulations that integrate the proposed solution strategy, as well as for problem DP when the optimal solution achieved through model SP is not considered for its optimization. In this last case, the same optimal solution is obtained in 1589.1 CPU seconds, which increases more than 4

Table 9. Processing Times and Size Factors of Products for Example 3

	processing time: t_{ik} (h)								size factor: SF_{ij} (L/kg)			
	stage 1		stage 2		stage 3			stage 4	stage 1	stage 2	stage 3	stage 4
i	1	2	3	4	5	6	7	8	k = 1,2	k = 3,4	k = 5,6,7	k = 8
A	6	5	13	10	12	10	9	5	0.84	0.40	0.70	0.50
B	10	9	6	6	18	16	12	6	0.70	0.50	0.40	0.57
C	7	5	8	6	10	8	6	6	0.45	0.80	0.50	0.57
D	15	12	7	4	14	13	8	4	0.40	0.35	0.30	0.40

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Table 10. Changeover Times for Example 3

	sequence-dependent changeover time: $c_{ii'k}$ (h)															
	stage 1				stage 2				stage 3				stage 4			
	k = 1, 2				k = 3, 4				k = 5, 6, 7				k = 8			
i	A	B	C	D	A	B	C	D	A	B	C	D	A	B	C	D
A	0.5	2	2	3	1	1	1	2	1	2	2.5	3	0.5	0	0.25	1
B	1	1	2.5	1	0	0.5	2	1	2	0.5	1	0.25	0	0	2	0.5
C	0	2	0.5	2.5	3	1.5	0.5	2	2	1	1	2.5	2	1	0	2
D	2	2	2	1	3	0.5	2.5	0.5	1.75	2	0.5	0.75	0.25	2	1.5	0

Table 11. Model Sizes and Computational Statistics for Example 3

model	constraints	variables: binary	variables: continuous	CPU time (s)
simplified (SP)	296965	245	91466	267.1
detailed (DP)	14311	264	3036	79.6
DP without considering SP	141001	1123	24006	1589.1

times the computational time when it is compared with the proposed strategy.

The optimal campaign for satisfying the required product amounts involves two batches of product A (b_1, b_2), three of B (b_4, b_5, b_6), one of C (b_9), and one of D (b_{11}); that is, the demands of all products are fulfilled with the minimum number of batches. According to eq 2 the batches are selected in ascending numerical order. The optimal production sequence obtained in each batch unit for the different stages, considering sequence-dependent changeover times, is illustrated in the Gantt chart of Figure 10.

The size for each batch that composes the campaign is equal to 345.71 kg (b_1) and 214.29 kg (b_2) for product A, 300 kg (b_4), 292.86 kg (b_5), and 257.14 kg (b_6) for product B, 250 kg (b_9) for product C, and 300 kg (b_{11}) for product D. The capacities used, in liters, in each unit of the different stages for processing the selected batches are summarized in Table 12. The batches that reach the minimum and maximum capacities are highlighted in boxes shaded in white and gray, respectively. The size of batch b_2 of product A is the maximum possible following the route composed by the units 2, 5, 8, and its size satisfies the minimum filled capacity required for unit 5, that is, 50% of the total unit capacity is used for process batch b_2. Then, batch b_1 fulfills the required amount of that product occupying approximately 97%, 55%, 81%, and 86% of the capacity of units 1, 3, 5, and 8, respectively. On the other hand, three batches of product B are processed for meeting its demand. As it can be noted from Table 12, batches b_4 and b_5 size are the maximum possible to be processed in units 1, 4, 6, 8 and 2, 4, 6, 8, respectively, and batch b_5 fulfills the campaign demand of that product. Finally, only batches b_9 and b_{11} are selected to accomplish the demands of products C and D, respectively.

Finally, from the computational point of view, this example, as well as those previously presented, shows that the two-step solution strategy is a good option for the simultaneous optimization of batching and scheduling decisions of multistage, multiproduct batch plants, and it yields the optimal solution in less CPU time than solving the model DP at once. Also, it is worth mentioning that the proposed model formulations can be adapted in order to optimize the batching and scheduling of batch plants considering, for example, the makespan minimization as performance criteria.

6. CONCLUSIONS

In this work, the simultaneous batching and scheduling of multistage batch plants with nonidentical parallel units is faced. Scheduling is modeled according to campaign-based operation mode in such a way that the campaign cycle time minimization is an appropriate optimization criterion. Sequence-dependent changeover times are considered for each ordered pair of products in each unit of the different stages.

Taking into account the complexity of the simultaneous involved decisions, the optimization is tackled solving two MILP models. Both formulations handle binary and continuous variables for the batches selection and sizing, respectively, and

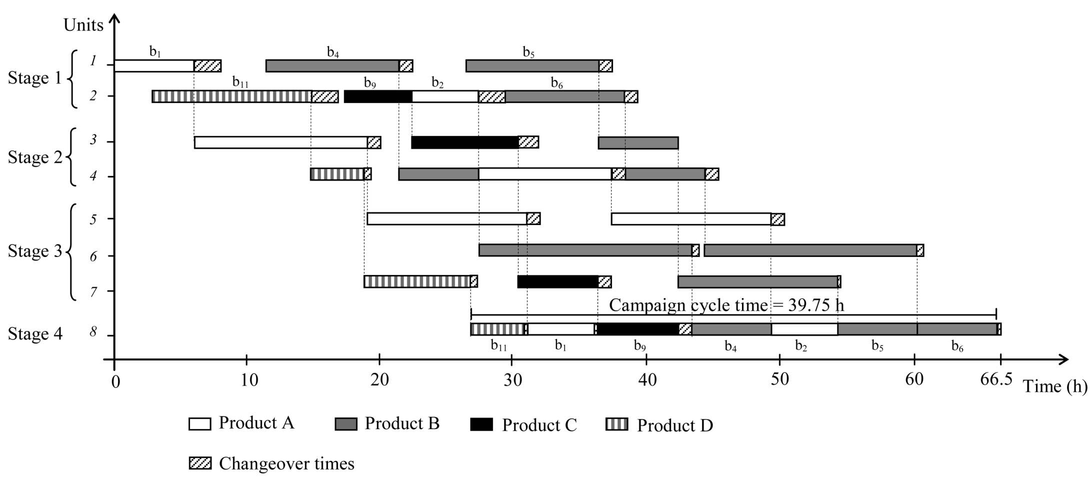

Figure 10. Gantt chart for the optimal production campaign of Example 3.

Table 12. Capacities Used in Each Unit of Each Stage for Example 3 (L)

		Stage 1		Stage 2		Stage 3			Stage 4
Product	Batch	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$	$k = 8$
A	b_1	290.40		138.29		242.00			172.86
	b_2		180.00		85.71	150.00			107.14
B	b_4	210.00			150.00		120.00		171.00
	b_5	205.00		146.43				117.14	166.93
	b_6		180.00		128.57		102.86		146.57
C	b_9		112.50	200.00				125.00	142.50
D	b_{11}		120.00		105.00			90.00	120.00

use a slot-based continuous-time representation for scheduling decisions. In particular, the first formulation, denoted by SP, considers preordering constraints for the assignment of batches to slots at the different stages. In contrast, the second model, called DP, considers the exact scheduling of the selected batches and the optimal objective value of model SP is used as a tight upper bound for the campaign cycle time. In addition, the number of selected batches for each product from the first model provides a better-estimation of the number of slots that must be postulated for the detailed model DP. In this sense, the computational performance is improved and the simultaneous optimization is efficiently solved.

In both mathematical models, various equations are reformulated in order to keep the problem linear and ensure the global optimality of the solution. Furthermore, the models could be adapted to optimize the batching and scheduling of batch plants using the makespan minimization as problem objective function.

Through the examples the capabilities of the proposed formulations and resolution methodology are shown. Furthermore, model SP represents a good heuristic for solving the integrated problem in complex contexts, taking into account that an appropriate approximated solution is achieved.

With the proposed formulation, an interesting problem has been solved. Many times, in made-to-stock contexts, the campaign-based operation mode is an appropriate alternative that allows taking advantage of the available resources with an ordered production management. The proposed models simultaneously solve batching and scheduling problems in reasonable computing time. Thus, this approach can be applied in real production systems that operate in campaign-mode taking into account the assumed suppositions as far as different unit sizes, changeovers, etc.

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Notes

The authors declare no competing financial interest.

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NOMENCLATURE

Indices

b = batch
i = product
j = stage
k = unit
l = slot

Sets

I = set of products
IB_i = set of batches proposed for product i
J = set of stages
K = set of units
K_j = nonidentical batch units operating in parallel out-of-phase in stage j

Parameters

α_{ik} = equipment utilization minimum rate for product i at unit k
B_i^{max} = maximum feasible batch size for product i
B_i^{min} = minimum feasible batch size for product i
BM = parameter used for constraint of Big-M type
$c_{ii'k}$ = sequence-dependent changeover time between products i and i' at unit k
L = number of slots postulated for all units in model SP
L_{kj} = number of slots postulated for unit k of stage j in model DP
M_n = parameter used for constraint of Big-M type, where $n = 1, ..., 7$
N_j = cardinality of K_j
NBC_i^{LOW} = minimum number of batches of product i in the campaign
NBC_i^{UP} = maximum number of batches of product i in the campaign
SF_{ij} = size factor for product i in stage j
t_{ik} = processing time of product i at unit k
V_k = size of unit k

Binary Variables

X_{kl} = indicates if slot l of unit k is employed
Y_{bkl} = indicates if batch b is assigned to slot l of unit k (defined for model DP)
Z_{bl} = indicates if batch b is assigned to slot l
z_{ib} = indicates if batch b of product i is selected

Continuous Variables

B_{ib} = size of batch b of product i
CT = cycle time of the campaign
TF_{kl} = final processing time of slot l in unit k
TI_{kl} = initial processing time of slot l in unit k

Y_{bkl} = represents the bilinear term $Z_{bl}\ X_{kl}$ (defined for model SP)

$YY_{blb'l'k}$ = represents the bilinear term $Y_{bkl}\ Y_{b'kl'}$

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