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PROCESS DESIGN AND CONTROL

Multivariable Inferential Feed-Forward Control

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Two multivariable inferential feed-forward control strategies are proposed in this paper. In the first strategy, the effects of disturbances on the primary process variables are inferred from uncontrolled secondary process variables that are measured on-line. In the second approach, the effects of disturbances on the primary process variables are inferred from the manipulated variables for those controlled secondary process variables that have fast dynamics. The proposed strategies are particularly useful in situations where some disturbances cannot be easily and quickly measured. Robustness analysis of the inferential feed-forward controllers and the selection of appropriate secondary measurements are discussed. Structured singular value analysis is used in assessing the robustness of the inferential feed-forward control systems. The performance characteristics of the two inferential feed-forward control systems are demonstrated by application to a simulated methanol−water separation column. In the first system, the effects of disturbances in feed composition (and feed rate) are inferred from tray temperatures, whereas in the second system, the disturbance effects are inferred from inventory manipulations. Nonlinear dynamic simulation results demonstrate the superior performance of these strategies. Robustness analysis shows that using multiple tray temperatures can improve the robustness of the inferential feed-forward controller, and this conclusion is confirmed by simulation.

1. Introduction

The primary function of a process control system is to maintain the controlled process variables at their desired values in the presence of disturbances. Process plants can have large time constants and long time delays. Substantial measurement delays in some process variables such as composition often exist. When long time delays or large time constants between the disturbances and the controlled variables exist, the effects of disturbances might not be satisfactorily rejected through feedback control alone. A strategy widely used in process control is feed-forward control,[1−3] in which disturbances are measured and anticipatory control actions are taken before the controlled variables are actually affected by disturbances. In many situations, however, disturbances in some variables, such as composition, cannot be easily measured. In such cases, it would not be feasible to implement feed-forward control.

In many process plants, there are usually some secondary process variables that are measured on-line and that might or might not be controlled. The correlation between disturbances and some uncontrolled secondary process variables make it possible to infer disturbance effects from these variables. If the secondary process variables are controlled, then the changes in their manipulated variables can be used to infer the effects of disturbances. On the basis of these inferred disturbances, feed-forward control can be implemented indirectly. For example, Yu and co-workers[4−6] proposed an indirect feed-forward control strategy in which the effects of disturbances were inferred from changes in uncontrolled secondary process variables. McAvoy et al.[7] proposed a nonlinear inferential cascade control to consider the nonlinear aspects in many industrial processes.

This paper presents two methods for implementing inferential feed-forward control. In the first method, the effects of disturbances are inferred from the measurements of certain uncontrolled secondary process variables that can be measured easily. In the second method, the effects of disturbances are inferred from the manipulated variables associated with fast secondary controlled variables. Although the first approach shares some common ideas with the indirect feed-forward control strategy of Yu and co-workers,[4−6] some important aspects, such as robustness analysis, that were not addressed by them are discussed here.

The inferential feed-forward control strategies proposed here differ from the inferential control widely reported in the literature.[8−11] In inferential control, the variables to be controlled cannot be measured quickly enough or, in some cases, cannot be measured at all. These variables are estimated using some readily available process variables such as temperatures, pressures, and flows, and then, the estimated values are used in feedback control loops. In the inferential feed-forward control schemes proposed here, some disturbances cannot be measured fast enough for feed-forward control

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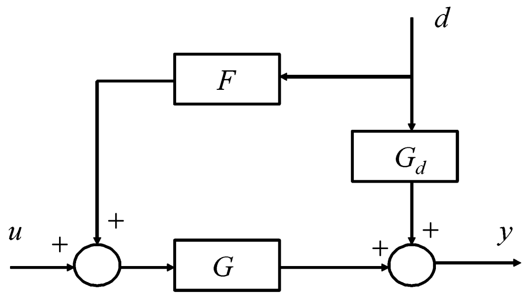

Figure 1. Feed-forward control system.

and their effects on the controlled variables are estimated to calculate feed-forward control actions.

The paper is organized as follows. Section 2 presents two approaches for implementing inferential feed-forward control. The robustness of the first approach is also analyzed. A procedure for secondary measurement selection taking into account the robustness issues is also presented. Section 3 describes the applications of these approaches to a comprehensive nonlinear simulator of a methanol−water distillation column. In the first approach, tray temperatures are used to infer the effects of disturbances. In the second approach, the effects of disturbances are inferred from inventory manipulators. Both approaches improve the disturbance rejection capability of the distillation composition control system. The final section contains some concluding remarks.

2. Inferential Feed-Forward Control

2.1. Feed-Forward Control. A feed-forward control system is shown in Figure 1, where disturbances are measured and compensating control actions are taken through the feed-forward controller $F(s)$. Deviations in the controlled variables due to disturbance can be calculated as

$$\Delta y(s) = G(s)\, F(s)\, \Delta d(s) + G_d(s)\, \Delta d(s) \qquad (1)$$

where y is a vector of controlled variables; $G(s)$ is the model transfer function between a vector of manipulated variables, u, and y; and $G_d(s)$ is the model transfer function between a vector of disturbances d and y. For the sake of simplicity, the dependence on s, i.e., (s), is omitted in the rest of the paper.

To make Δy equal zero, a feed-forward controller of the following form is usually designed, provided that G^{-1} is stable, i.e., G has no zeros in the right half plane and is proper or semiproper

$$F = -G^{-1}G_d \qquad (2)$$

In many practical applications, F is chosen to be of static form. In this case, the controller F is obtained as

$$F = -G^{-1}(0)\, G_d(0) \qquad (3)$$

The feed-forward controller of eq 3 can ensure that Δy is equal to zero at the steady state provided that the process gain, $G(0)$, and disturbance gain, $G_d(0)$, are without errors. In this study, a static inferential feed-forward controller is considered.

2.2. Inferential Feed-Forward Control Using Measurements of Uncontrolled Secondary Process Variables. 2.2.1. Structure. The framework of this control strategy is shown in Figure 2, where y represents the primary controlled variables and y_s the secondary process variables, d is a vector of unmeasured disturbances, and F is the inferential feed-forward controller. The feed-forward control actions can be calculated as

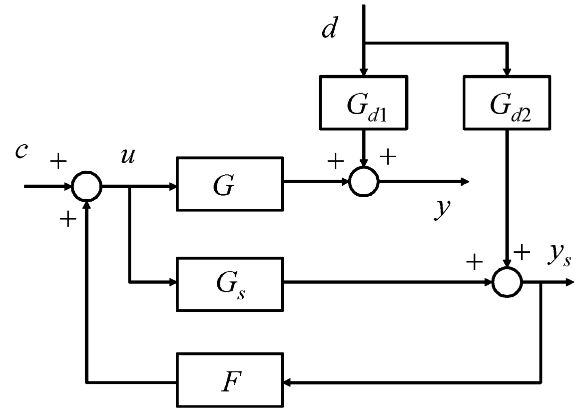

Figure 2. Inferential feed-forward control using uncontrolled secondary variables.

$$\Delta u = F\Delta y_s = F(G_{d2}\Delta d + G_s\Delta u) \qquad (4)$$

$$\Delta u = (I - FG_s)^{-1}FG_{d2}\Delta d \qquad (5)$$

The deviations in the primary process variables are given by

$$\begin{aligned}\Delta y &= G\Delta u + G_{d1}\Delta d \\ &= [G(I - FG_s)^{-1}FG_{d2} + G_{d1}]\Delta d \qquad (6)\end{aligned}$$

To make $\Delta y(0)$ equal zero, it is required that

$$G(0)[I - FG_s(0)]^{-1}FG_{d2}(0) + G_{d1}(0) = 0 \qquad (7)$$

Premultiplying both sides of eq 7 by $G(0)^{-1}$ and rearranging gives

$$[I - FG_s(0)]^{-1}FG_{d2}(0) = -G(0)^{-1}\, G_{d1}(0) \qquad (8)$$

Denoting $[I - FG_s(0)]^{-1}F$ as A, eq 8 then becomes

$$AG_{d2}(0) = -G(0)^{-1}G_{d1}(0) \qquad (9)$$

Depending on the rank of $G_{d2}(0)$, there can be a unique solution, many solutions, or no solution at all for A. Here, we chose A as

$$A = -G(0)^{-1}\, G_{d1}(0)\, G_{d2}(0)^{+} \qquad (10)$$

where X^{+} denotes the pseudo-inverse of X.[12]

If $G_{d2}(0)$ is square and nonsingular, then its pseudo-inverse is the standard inverse and the A determined by eq 10 is the unique solution to eq 9. If $G_{d2}(0)$ has more independent columns than independent rows, then the A obtained from eq 10 solves eq 9 in the least-squares sense. If $G_{d2}(0)$ has more independent rows than columns, then the A calculated from eq 10 is the solution of eq 9 having the smallest norm.[12] A small-norm matrix $A = [I - FG_s(0)]^{-1}F$ will provide increased robustness, as will be shown later.

Once A is obtained, F can be determined by solving the equation

$$[I - FG_s(0)]^{-1}F = A \tag{11}$$

Premultiplying both sides of eq 11 by $[I - FG_s(0)]$ and rearranging gives

$$F[I + G_s(0)A] = A \tag{12}$$

The inferential feed-forward controller F can then be obtained as

$$F = A[I + G_s(0)A]^{-1} = -G(0)^{-1}\, G_{d1}(0)\, G_{d2}(0)^{+}[I - G_s(0)\, G(0)^{-1} G_{d1}(0)\, G_{d2}(0)^{+}]^{-1} \tag{13}$$

It can be seen that the feed-forward controller designed according to eq 13 will eliminate the (steady-state) effects of disturbances on the primary process variables. If the models are perfect, then the (steady-state) effects of disturbances on the primary process variables will be either completely rejected when $G_{d2}(0)$ has more independent rows than independent columns or maximally rejected when $G_{d2}(0)$ has more independent columns than independent rows. However, model−plant mismatches exist in most process plants, and therefore, an inferential feed-forward controller alone cannot completely eliminate control offsets. Rather, it should be used in conjunction with a feedback controller.

Remark. For the single-variable case, the inferential feed-forward controller given by eq 13 can be shown to be equivalent to the inferential control described in the literature.[1] However, the inferential control law for a general multivariable case is not presented in the literature.[1]

2.2.2. Robustness. Studies of the effects of model uncertainties in process models and disturbance models are particularly important and it is necessary to consider the impact of model uncertainties on the inferential feed-forward controller. Model−plant mismatches always exist in practice, especially when a nonlinear process is approximated by a linear model around a particular operating point. Model uncertainties are considered here as norm-bounded multiplicative uncertainties of the form

$$\bar{G} = (I + W\Delta)G, \quad \bar{\sigma}(\Delta) \le 1 \tag{14}$$

$$\bar{G}_s = (I + W_s\Delta_s)G_s, \quad \bar{\sigma}(\Delta_s) \le 1 \tag{15}$$

$$\bar{G}_{d1} = (I + W_1\Delta_1)G_{d1}, \quad \bar{\sigma}(\Delta_1) \le 1 \tag{16}$$

$$\bar{G}_{d2} = (I + W_2\Delta_2)G_{d2}, \quad \bar{\sigma}(\Delta_2) \le 1 \tag{17}$$

where $\bar{G}$, $\bar{G}_s$, $\bar{G}_{d1}$, and $\bar{G}_{d2}$ are the true models; G, G_s, G_{d1}, and G_{d2} are the corresponding nominal models; Δ, Δ_s, Δ_1, and Δ_2 are the corresponding model uncertainties; and W, W_s, W_1, and W_2 are the corresponding uncertainty weights representing the degrees of model uncertainties. It should be stressed here that the most appropriate form of model uncertainty representation depends on the particular circumstance of the considered process.

The deviations in the primary process variables can be calculated from eq 5 as

$$\begin{aligned}\Delta y = {} & \{G[I - (I - FG_s)^{-1}FW_s\Delta_sG_s]^{-1}(I - FG_s)^{-1}FG_{d2} + G_{d1}\}\Delta d \\ & + \{G[I - (I - FG_s)^{-1}FW_s\Delta_sG_s]^{-1}(I - FG_s)^{-1}FW_2\Delta_2G_{d2} + W_1\Delta_1G_{d1}\}\Delta d \\ & + W\Delta G[I - (I - FG_s)^{-1}FW_s\Delta_sG_s]^{-1}(I - FG_s)^{-1}FG_{d2}\Delta d \\ & + W\Delta G[I - (I - FG_s)^{-1}FW_s\Delta_sG_s]^{-1}(I - FG_s)^{-1}FW_2\Delta_2G_{d2}\Delta d\end{aligned} \tag{18}$$

To make the effects of model uncertainties small, $(I - FG_s)^{-1}F$ should be small. At a steady state, eqs 10 and 11 indicate that

$$[I - FG_s(0)]^{-1}F = -G(0)^{-1}\, G_{d1}(0)\, G_{d2}(0)^{+} \tag{19}$$

Thus, to minimize the effects of model uncertainties, it is necessary that $\bar{\sigma}[G(0)^{-1}\, G_{d1}(0)\, G_{d2}{}^{+}(0)]$ be small. To ensure controller robustness over a wide frequency range, it is necessary that $\sup_\omega \bar{\sigma}[(I - FG_s)^{-1}F]$ be small.

The above analysis is for open-loop robustness. Feed-forward control is typically used in conjunction with feedback control. For closed-loop robustness analysis, structured singular value analysis (also known as μ-analysis)[13,14] can be used. The closed-loop control system with the above-mentioned model uncertainties is represented in Figure 3, where C represents a feedback controller. Figure 3 can be rearranged into the form shown in Figure 4 for structured singular value analysis. The connection matrix N in Figure 4 is obtained as

$$N = \begin{bmatrix} N_{11} & N_{12} & N_{13} & N_{14} & N_{15} \\ N_{21} & N_{22} & N_{23} & N_{24} & N_{25} \\ 0 & 0 & 0 & 0 & W_1G_{d1} \\ 0 & 0 & 0 & 0 & W_2G_{d2} \\ N_{51} & N_{52} & N_{53} & N_{54} & N_{55} \end{bmatrix} \tag{20}$$

with

$$N_{11} = N_{13} = -W_oS_4GS_3C$$
$$N_{12} = N_{14} = W_oGS_2S_1F$$
$$N_{15} = -W_oS_4GS_3CG_{d1}W_d + W_oGS_2S_1FG_{d2}W_d$$
$$N_{21} = N_{23} = W_sG_sS_4S_3C$$
$$N_{22} = N_{24} = W_s(I - G_sS_1F)G_sS_1F$$
$$N_{25} = W_sG_sS_4S_3CG_{d1}W_d + W_s(I - G_sS_1F)G_sS_1FG_{d2}W_d$$
$$N_{51} = N_{53} = -W_pS_4$$
$$N_{52} = N_{54} = -W_pGS_2S_1F$$
$$N_{55} = -W_pS_4G_{d1}W_d - W_pGS_2S_1FG_{d2}W_d$$
$$S_1 = (I + CG)^{-1}$$
$$S_2 = (I - S_1FG_s)^{-1}$$

$$S_3 = (I - FG_s)^{-1}$$

$$S_4 = (I + GS_3C)^{-1}$$

where W_d is disturbance weight defined so that the weighted disturbances are less than 1 at all frequencies and the resulting normalized control errors are also less that 1 at all frequencies.

Applying the robust stability and robust performance conditions[14] to N, the control system will be robustly stable if and only if

$$\sup_{\omega} \mu\left(\begin{bmatrix} N_{11} & N_{12} & N_{13} & N_{14} \\ N_{21} & N_{22} & N_{23} & N_{24} \end{bmatrix}\right) \le 1 \tag{21}$$

where the structured singular value is calculated with respect to the perturbation diag$\{\Delta, \Delta_s, \Delta_1, \Delta_2\}$. The control system will achieve robust performance if and only if

$$\sup_{\omega} \mu(N) \le 1 \tag{22}$$

where the structured singular value is calculated with respect to the augmented perturbation diag$\{\Delta, \Delta_s, \Delta_1, \Delta_2, \Delta_p\}$. The smaller the value of $\sup_\omega \bar{\sigma}(N)$, the better the control performance that can be achieved.

2.2.3. Feedback Controller Tuning. The addition of an inferential feed-forward controller changes the overall process model, and the effects of the inferential feed-forward controller need to be taken into account when the feedback controller is tuned. With the inferential feed-forward controller in place, deviations in y_s are given by

$$\Delta y_s = G_s(\Delta u + F\Delta y_s) + G_{d2}\Delta d \tag{23}$$

Rearranging eq 23 gives

$$\Delta y_s = (I - G_s F)^{-1} G_s \Delta u + (I - G_s F)^{-1} G_{d2}\Delta d \tag{24}$$

Deviations in the primary process variables are calculated as

$$\begin{aligned} \Delta y &= G(\Delta u + F\Delta y_s) + G_{d1}\Delta d \\ &= G[I + (I - FG_s)^{-1} FG_s]\Delta u + \\ &\qquad [G(I - FG_s)^{-1} FG_{d2} + G_{d1}]\Delta d \end{aligned} \tag{25}$$

The process model is thus changed to

$$G_N = G[I + (I - FG_s)^{-1} FG_s] \tag{26}$$

and the feedback controller should be tuned for the new model, G_N, for example, using the tuning method proposed by Luyben[15] involving the largest logarithmic modulus.

From eqs 13 and 26, the steady-state G_N is obtained as

$$G_N(0) = G(0)[I - G(0)^{-1}\, G_{d1}(0)\, G_{d2}(0)^{+} G_s(0)] \tag{27}$$

The controller can also be tuned so as to optimize the achievable robust control performance. This is accomplished by solving the following optimization problem

$$\min_{C} \sup_{\omega} \mu(N) \tag{28}$$

$$\text{s.t. robust stability eq 21}$$

where C represents the controller parameters and the controller can be a diagonal PI controller, a multivariable IMC (internal model control) controller, or any other type of controller.

2.2.4. Secondary Measurement Selection. The inferential feed-forward controller is usually designed for specific disturbances. Suppose that d_n is a vector of other disturbances that were not considered in the design of the inferential feed-forward controller. The disturbance gains from d_n to the primary process variables are G_{d1n}, and those to the secondary process variables are G_{d2n}. The deviations in the primary process variables caused by d_n can be calculated from eq 5 as

$$\begin{aligned} \Delta y &= [G(I - FG_s)^{-1} FG_{d2n} + G_{d1n}]\Delta d_n \\ &= [G_{d1n} - G_{d1}\, G_{d2}^{+}\, G_{d2n}]\Delta d_n \end{aligned} \tag{29}$$

It is then desirable that $G_{dN} = G_{d1n} - G_{d1}\, G_{d2}^{+}\, G_{d2n}$ be small.

A number of secondary process variables that can be measured might also be available. Secondary measurements can be selected so as to minimize the effects of model uncertainties and unconsidered disturbances. Shen and Yu[5] proposed a procedure in which secondary measurements are selected so as to reduce the effects of unconsidered disturbances. However, the robustness

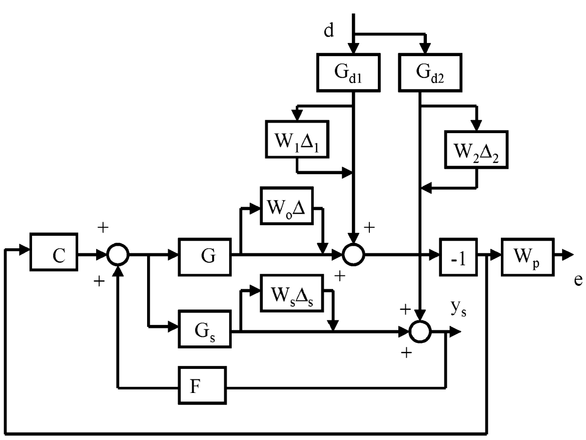

Figure 3. Closed-loop control with model uncertainties.

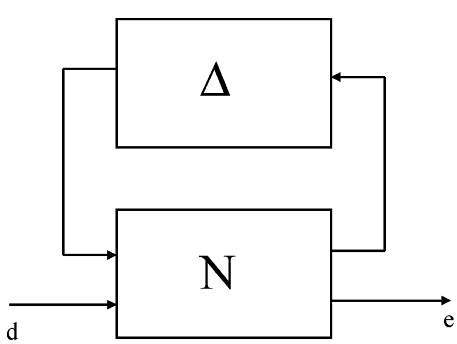

Figure 4. Framework for structured singular value analysis.

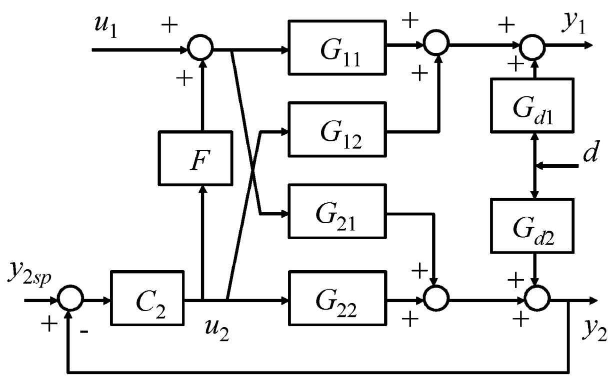

Figure 5. Inferential feed-forward control using manipulated variables of secondary controlled variables.

is also an important factor to be considered in selecting secondary measurements.

Equation 26 indicates that the selection of secondary measurements will also affect the overall process model. If the primary controllers used are diagonal controllers, then it would be desirable that the off-diagonal elements in the process model be relatively small so that control loop interactions are small. Because the objective of employing a feed-forward controller is to counteract a disturbance as soon as possible, the secondary measurements should have a rapid dynamic response to the considered disturbances so that the presence of the disturbance can be sensed quickly. The following are therefore some factors that ought to be considered when selecting secondary measurements: (1) The selected secondary measurements should have a rapid response to the considered disturbances. (2) The robustness measures $\sup_\omega \bar{\sigma}[(I - FG_s)^{-1}F]$ and $\sup_\omega \bar{\sigma}(N)$ should be small. (3) Elements of $G_{dN} = G_{d1n} - G_{d1}G_{d2}^{+}G_{d2n}$ should be small. (4) Off-diagonal elements of $G_N = G(I - G^{-1}G_{d1}G_{d2}^{+})$ should be relatively small if diagonal controllers are used.

2.3. Inferential Feed-Forward Control Using the Manipulated Variables for Secondary Controlled Variables. The effects of disturbances can also be inferred from the manipulated variables for some fast secondary controlled variables. The framework for this strategy is shown in Figure 5, where y_1 is a vector of controlled primary process variables; y_2 is a vector of controlled secondary process variables; d is a vector of disturbances; u_1 and u_2 are the manipulated variables for y_1 and y_2, respectively; and F is the inferential feed-forward controller.

Suppose that there are no set-point changes in y_2; then

$$\Delta u_2 = -C_2\Delta y_2 = -C_2[G_{22}\Delta u_2 + G_{21}(\Delta u_1 + F\Delta u_2) + G_{d2}\Delta d] \quad (30)$$

It follows from eq 30 that

$$\Delta u_2 = -(I + C_2G_{22} + C_2G_{21}F)^{-1}C_2(G_{21}\Delta u_1 + G_{d2}\Delta d) \quad (31)$$

Suppose that y_2 has fast dynamics and is tightly controlled. Then, $(I + C_2G_{22} + C_2G_{21}F)$ can be approximated by $(C_2G_{22} + C_2G_{21}F)$. Equation 31 then becomes

$$\Delta u_2 = -(G_{22} + G_{21}F)^{-1}C_2^{-1}C_2(G_{21}\Delta u_1 + G_{d2}\Delta d) = -(G_{22} + G_{21}F)^{-1}(G_{21}\Delta u_1 + G_{d2}\Delta d) \quad (32)$$

Deviations in the primary process variables can then be calculated as

$$\Delta y_1 = G_{11}(\Delta u_1 + F\Delta u_2) + G_{12}\Delta u_2 + G_{d1}\Delta d = [G_{11} - (F + G_{12})(G_{22} + G_{21}F)^{-1}G_{21}]\Delta u_1 + [G_{d1} - (G_{11}F + G_{12})(G_{22} + G_{21}F)^{-1}G_{d2}]\Delta d \quad (33)$$

To eliminate the (steady-state) effects of disturbances on the primary process variables, eq 33 indicates that the following relationship should be satisfied

$$G_{d1}(0) - [G_{11}(0)F + G_{12}(0)][G_{22}(0) + G_{21}(0)F]^{-1}G_{d2}(0) = 0 \quad (34)$$

The desired feed-forward controller F can then be calculated from eq 34. Denoting $[G_{11}(0)F + G_{12}(0)][G_{22}(0) + G_{21}(0)F]^{-1}$ as B and rearranging eq 34, we have

$$BG_{d2}(0) = G_{d1}(0) \quad (35)$$

Equation 35 might have many solutions, or only one solution, or no solutions at all. Using the concept of the pseudo-inverse, a solution can be obtained as

$$B = G_{d1}(0)[G_{d2}(0)]^{+} \quad (36)$$

If eq 35 has a unique solution, then it is given by eq 36. If eq 35 has no solution, then the B obtained from eq 36 solves eq 35 in the least-squares sense. If multiple solutions exist for eq 35, then the solution provided by eq 36 is the one having the smallest size, i.e., the smallest norm. Once B is obtained, F can be solved from the following equation

$$[G_{11}(0)F + G_{12}(0)][G_{22}(0) + G_{21}(0)F]^{-1} = B \quad (37)$$

Post-multiplying the two sides of eq 37 by $[G_{22}(0) + G_{21}(0)F]$ and rearranging, we have

$$[BG_{21}(0) - G_{11}(0)]F = G_{12}(0) - BG_{22}(0) \quad (38)$$

Supposing now that the inverse of $[BG_{21}(0) - G_{11}(0)]$ exists, then F can be obtained as

$$F = [BG_{21}(0) - G_{11}(0)]^{-1}[G_{12}(0) - BG_{22}(0)] \quad (39)$$

The inferential feed-forward controller F determined by eq 39 then compensates for the disturbance effects on the primary process variables.

Remark. This inferential feed-forward control strategy is not required in cases where an overall multivariable controller is developed, that is, all of the controlled variables are controlled by a single multivariable controller. However, many industrial plants are not controlled by a single multivariable controller for several reasons, including that a high dimensional multivariable controller (e.g., a 10 × 10 multivariable controller) is usually difficult to maintain by process operators.

Table 1. Nominal Distillation Column Operating Data

parameter	value
no. of theoretical stages	10
feed tray	5
feed composition (z)	50% methanol
feed flow rate (F)	18.23 g/s
top composition (y)	95% methanol
bottom composition (x)	5% methanol
top product flow rate (D)	9.13 g/s
bottom product flow rate (B)	9.1 g/s
reflux flow rate (L)	10.0 g/s
steam flow rate (V)	13.8 g/s

Thus, this inferential feed-forward control strategy is still of some practical value. It should also be emphasized that this inferential feed-forward controller could add complexity and raise the possibility of instability from poorly tuned secondary controllers. The possibility of instability can be assessed using structured singular value analysis. Judging from the robustness analysis result, the user can decide whether to adopt this inferential feed-forward controller or not.

3. Application to a Binary Distillation Column

3.1. Distillation Column. The distillation column studied in this paper is a comprehensive nonlinear simulation of a methanol–water separation column. A nonlinear tray-by-tray dynamic model has been developed using mass and energy balances. This simulation has been validated against pilot-plant tests and is very well-known for its use in control system performance studies.[16,17] The following assumptions are imposed: negligible vapor hold-up, perfect mixing in each stage, and constant liquid hold-up. The steady-state conditions for this column are listed in Table 1.

The process model and the disturbance model for the LV configuration are obtained through the application of a series of step response tests. In the LV configuration, the top composition, y_D, is controlled by the reflux flow rate, L, and the bottom composition, x_B, is controlled by the steam flow rate to the reboiler, V. The condenser level is controlled by the top product flow rate, and the reboiler level is controlled by the bottom product flow rate. Product compositions are measured, and it is assumed that there is a 5-min time delay in the composition analyzers. Disturbances in this column are feed rate disturbances and feed composition disturbances. The sampling interval is 1 min. For the purpose of control system synthesis, the process is approximated by the following linear model

$$\begin{pmatrix}\Delta y_D\\ \Delta x_B\end{pmatrix} = G(s)\begin{pmatrix}\Delta L\\ \Delta V\end{pmatrix} + G_{d1}(s)\begin{pmatrix}\Delta F\\ \Delta z\end{pmatrix} \tag{40}$$

To identify the transfer function models $G(s)$ and $G_{d1}(s)$, step changes in reflux flow, steam flow to the reboiler, feed flow rate, and feed composition were imposed on the column, and the resulting process data were recorded. The process and disturbance models were approximated by first-order lag plus delay models, in line with a number of previous studies using this column. Because the distillation column exhibits some degrees of nonlinearity, a series of positive and negative step changes was applied. From these plant test data, linear discrete time models were identified using least-squares regression and converted into continuous-time transfer function models.

Table 2. Tray Temperature Transfer Function Models

tray	L	V	F	z
1	$\frac{-2.89}{16.69s+1}$	$\frac{6.40}{16.81s+1}$	$\frac{-4.46}{27.78s+1}$	$\frac{-1.02}{21.93s+1}$
2	$\frac{-1.86}{19.53s+1}$	$\frac{6.0}{38.31s+1}$	$\frac{-3.92}{52.91s+1}$	$\frac{-0.75}{22.03s+1}$
3	$\frac{-0.87}{17.16s+1}$	$\frac{3.23}{44.25s+1}$	$\frac{-1.92}{68.03s+1}$	$\frac{-0.42}{10.78s+1}$
4	$\frac{-0.46}{9.54s+1}$	$\frac{1.22}{39.37s+1}$	$\frac{-0.71}{65.36s+1}$	$\frac{-0.28}{8.86s+1}$
5	$\frac{-0.81}{9.61s+1}$	$\frac{1.53}{29.41s+1}$	$\frac{-0.75}{86.96s+1}$	$\frac{-0.24}{17.48s+1}$
6	$\frac{-1.04}{6.54s+1}$	$\frac{1.39}{17.54s+1}$	$\frac{-0.57}{82.64s+1}$	$\frac{-0.18}{13.77s+1}$
7	$\frac{-0.98}{7.89s+1}$	$\frac{1.21}{20.04s+1}$	$\frac{-0.40}{94.34s+1}$	$\frac{-0.12}{15.90s+1}$
8	$\frac{-0.64}{4.14s+1}$	$\frac{0.73}{10.58s+1}$	$\frac{-0.20}{106.38s+1}$	$\frac{-0.06}{12.95s+1}$

The identified process model is

$$G(s) = \begin{bmatrix}\dfrac{1.09e^{-5s}}{5.51s+1} & \dfrac{-1.30e^{-5s}}{13.72s+1}\\ \dfrac{2.27e^{-5s}}{17.15s+1} & \dfrac{-7.18e^{-5s}}{29.50s+1}\end{bmatrix} \tag{41}$$

and the identified disturbance model is

$$G_{d1}(s) = \begin{bmatrix}\dfrac{0.34e^{-5s}}{89.29s+1} & \dfrac{0.11e^{-5s}}{15.43s+1}\\ \dfrac{2.64e^{-5s}}{16.67s+1} & \dfrac{0.70e^{-5s}}{26.25s+1}\end{bmatrix} \tag{42}$$

Dynamic models for tray temperatures were also identified and are given in Table 2.

3.2. Inferential Feed-Forward Control Using Tray Temperatures. In the first approach, tray temperatures, which are not controlled, are used to infer the effects of feed composition disturbances. Concentrations are usually difficult to measure rapidly on-line. Concentration analyzers are usually expensive and have long time delays. It is therefore difficult, and also not effective, to implement direct feed-forward control for feed composition disturbances.

This particular column has eight trays, and therefore, up to eight tray temperature measurements can be used. The selection of tray temperature measurements is based on the discussion in the previous section. From Table 2, it can be seen that the following trays have fast dynamic responses to feed composition disturbances: 3, 4, 6, 7, and 8. Therefore, one or several of these trays should be selected. Table 3 compares the robustness measures, the disturbance gains to the feed rate disturbance (not considered), and the relative gains of several possible tray selection schemes. If a single tray is to be selected, then it can be seen from Tables 2 and 3 that tray 4 is the most appropriate choice because it has fast dynamics for the feed composition disturbance, good relative gain, small disturbance gain to the feed rate, and good robustness measures. Trays 6–8 are not appropriate because of their poor relative gains. Table 3 indicates that more appropriate configurations

Table 3. Comparison of Different Tray Temperature Selection Schemes

tray(s)	$\sup_{\omega} \bar{\sigma}[(I - FG_s)^{-1}F]$	$G_{dN}(0)$	$\lambda_{11}[G_N(0)]$
3	1.1208	−0.1618 −0.5857	1.3288
4	0.6759	0.0608 0.8558	1.3297
6	1.7546	−0.0032 0.4413	0.5088
7	5.1679	−0.0144 0.3687	0.0723
8	36.1269	−0.0091 0.4029	−0.0301
3, 4	0.6380	−0.0926 −0.1376	1.3146
1−8	0.2275	−0.1502 −0.5106	1.1168

can also be obtained by selecting more than one tray with improved robustness measures and relative gains. Three inferential feed-forward controllers were developed and compared. The first one, F_4, is based on tray 4; the second one, F_6, is based on tray 6; and the third one, F_{1-8}, is based on all eight trays.

From eq 13, these inferential feed-forward controllers were obtained as

$$F_4 = -\begin{bmatrix} 0.1705 \\ 0.6526 \end{bmatrix} \tag{43}$$

$$F_6 = -\begin{bmatrix} 0.4431 \\ 1.6960 \end{bmatrix} \tag{44}$$

$$F_{1-8} = -\begin{bmatrix} 0.0418 & 0.0308 & 0.0171 & 0.0115 & 0.0097 & 0.0075 & 0.0050 & 0.0026 \\ 0.1601 & 0.1181 & 0.0654 & 0.0439 & 0.0371 & 0.0285 & 0.0192 & 0.0099 \end{bmatrix} \tag{45}$$

These inferential feed-forward controllers were separately implemented in conjunction with IMC controllers. For the purpose of comparison, a composition feedback control system using an IMC controller without inferential feed-forward control was also developed. The IMC controllers are of the following form

$$C = G^{-1}F(I - F)^{-1} \tag{46}$$

In the above equation, F is an IMC filter of the form

$$F = \mathrm{diag}[1/(\epsilon_1 s + 1),\ 1/(\epsilon_2 s + 1)] \tag{47}$$

where ϵ_1 and ϵ_2 are tuning parameters. In the implementation of the controller, the reflux rate is constrained within the interval [0, 18], and the steam rate is constrained within the interval [0, 20].

All of these IMC controllers were tuned to optimize the achievable robust control performance as shown in eq 28. In the robustness analysis using the structured singular values, the model uncertainty weights are selected as follows

$$W_o = W_s = W_1 = W_2 = 0.2\frac{1 + 4.5s}{1 + 0.8s} \tag{48}$$

The selected uncertainty weight indicates that there will be up to 20% uncertainty at steady state and that the degree of uncertainty will be larger at high frequencies. It should be noted that, although the four model

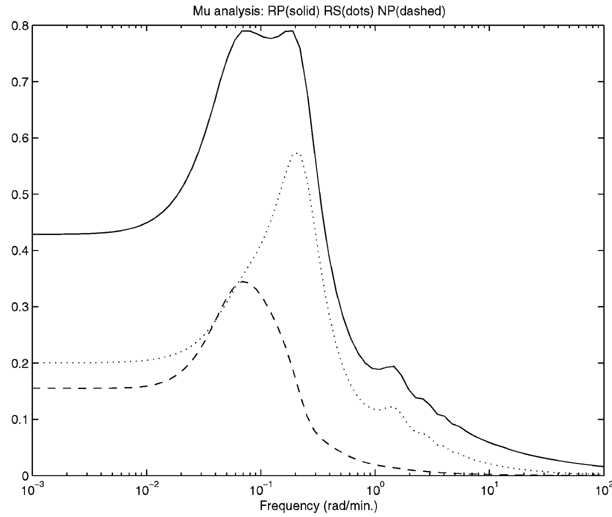

Figure 6. μ-analysis results: feedback control only.

Table 4. Controller Parameters

control scheme	ϵ_1	ϵ_2	$\sup_{\omega} \mu(N)$
feedback control only	7.25	6.99	0.79
with F_4	16.31	8.20	0.71
with F_6	29.75	5.94	1.18
with F_{1-8}	15.48	4.75	0.74
with F_{DB}	13.00	20.00	0.68

uncertainty weights were selected to be the same in this study, they can be, and usually are, different.

The performance weight is selected as

$$W_p = 0.5\frac{1 + 15s}{15s} \tag{49}$$

which indicates that integral action is required at low frequency and an amplification of disturbances at high frequency by a factor of at most 2 is allowed. This performance weight is selected in accordance with the discussions in the literature.[18] The disturbance weights for feed composition and feed rate disturbances are selected as

$$W_d = \mathrm{diag}(0.046\alpha,\ 0.25\beta) \tag{50}$$

where α, $\beta \in [0, 1]$ are used to indicate which disturbance is more important or occurs more frequently. For example, if the feed composition disturbance is more important or occurs more frequently than the feed rate disturbance, α will take a larger value than β. Both parameters are set to 1 in this study.

The tuned controller parameters, together with the achievable robust control performance, are reported in Table 4. The structured singular value analysis results are shown in Figures 6−9, where the solid, dotted, and dashed lines represent, respectively, the robust control performance, the robust stability, and the nominal control performance. From Table 4 and Figures 6−9, it can be seen that all inferential feed-forward controllers, except F_6, can achieve better robust control performance than can be obtained without using inferential feed-forward control. The structured singular value analysis results indicate that developing inferential feed-forward control using tray 6 temperature measurements would not be appropriate and might not help in improving control performance. Among the three inferential feed-forward controllers based on tray temperature measure-

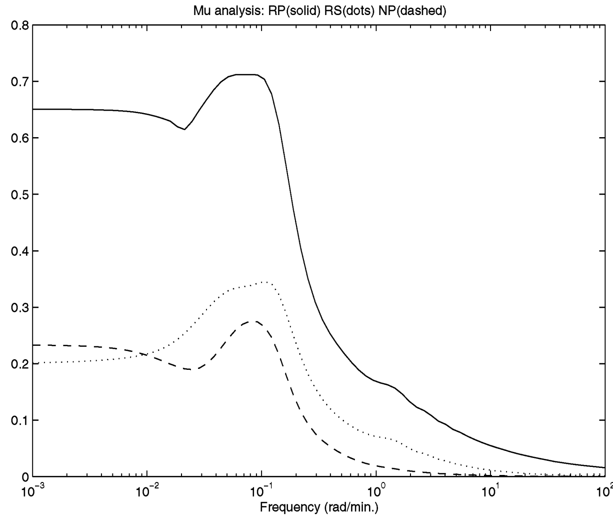

Figure 7. μ-analysis results with F_4.

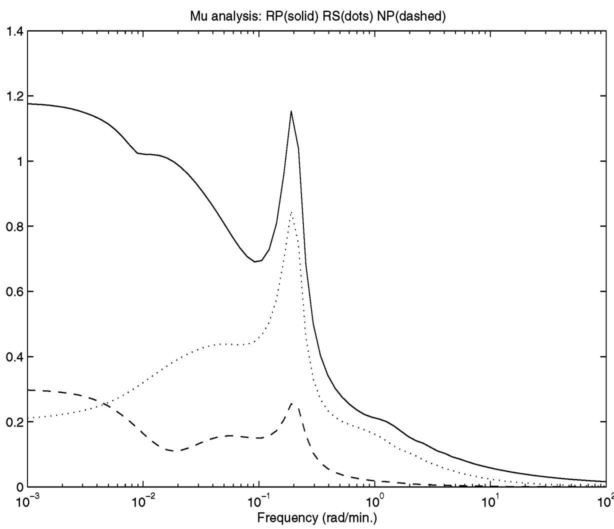

Figure 8. μ-analysis results with F_6.

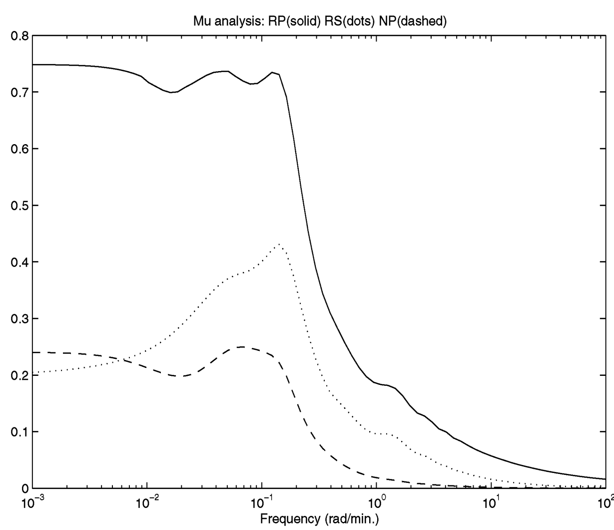

Figure 9. μ-analysis results with F_{1-8}.

ments, the one based on tray 4 gives the best achievable robust control performance. These results are verified in the following simulations.

To test the performance of the inferential feed-forward controllers, the disturbances shown in Figure 10 were applied to the simulation. These disturbances represent

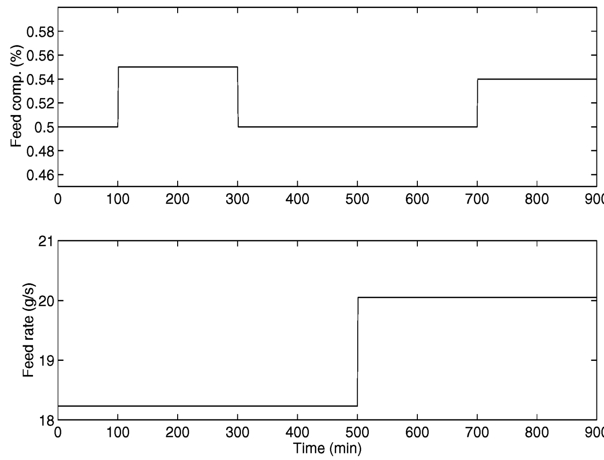

Figure 10. Disturbance sequences.

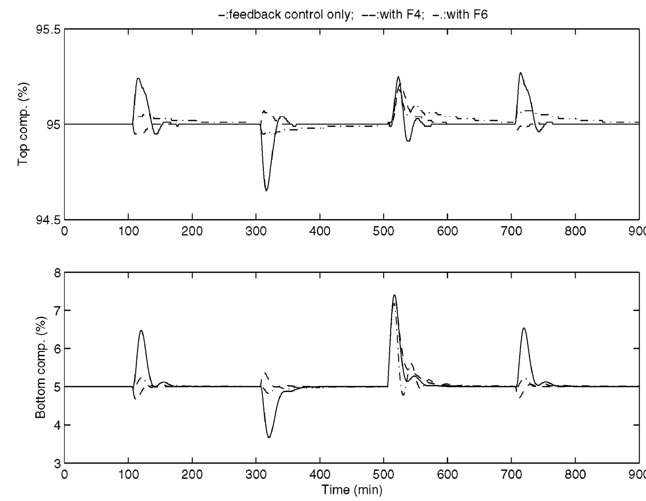

Figure 11. Control performance at the operating point (95%, 5%) for feedback control only with F_4 and with F_6.

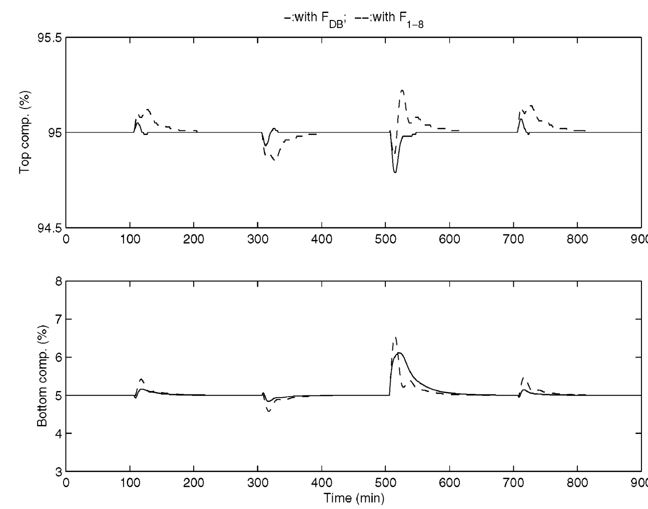

Figure 12. Control performance at the operating point (95%, 5%) for feedback control with F_{1-8} and with F_{DB}.

a 10% increase in feed composition at 100 min, a 10% decrease in feed composition at 300 min, a 10% increase in feed rate at 500 min, and finally a 10% increase in feed composition at 700 min. Figures 11 and 12 show the performance of these control systems. The sums of squared controller errors of these control systems are given in Table 5. It can be seen from Figures 11 and 12 and Table 5 that all inferential feed-forward controllers

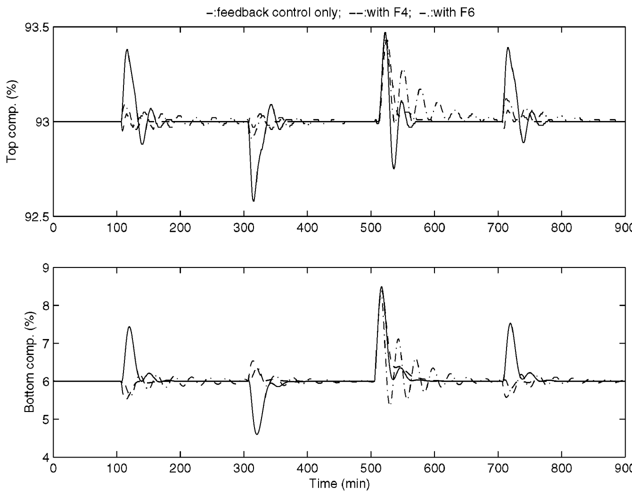

Figure 13. Control performance at the operating point (93%, 6%) for feedback control only with F_4 and with F_6.

Table 5. Sums of Squared Control Errors

control scheme	operating point (95%, 5%) top	operating point (95%, 5%) bottom	operating point (93%, 6%) top	operating point (93%, 6%) bottom
feedback control only	3.72	145.85	7.69	145.60
with F_4	0.76	73.54	2.31	79.75
with F_6	1.17	49.50	3.71	70.62
with F_{1-8}	1.95	30.78	6.86	36.44
with F_{DB}	0.45	33.07	0.94	33.13

can improve the control system performance in the presence of disturbances, especially F_4 and F_{1-8}. The performance of the control system with F_6 is considerably worse than that of the control system with F_4. This simulation result confirms the robustness analysis. Table 3 shows that both F_4 and F_{1-8} can also reduce the disturbance gains from the feed rate disturbance to the top and bottom product compositions. As is confirmed by the simulation results shown in Figures 11 and 12, both F_4 and F_{1-8} can also improve the control performance under feed rate disturbances.

To further test the robustness of the inferential controllers, the operating conditions were changed to the top composition at 93% and the bottom composition at 6%. The same disturbances shown in Figure 10 were then applied. Figures 13 and 14 show the responses of these control systems. The sums of squared control errors for these operating conditions are given in Table 5. Figures 13 and 14 and Table 5 show that, under these new operating conditions, the control system with F_6 deteriorates quite significantly in performance and, thus, is not robust to process operating condition variations. The control systems with F_4 and F_{1-8} can still give good control performance. These simulation results confirm the robustness analysis presented earlier.

To test the performance of the inferential feed-forward controllers under measurement noises, normally distributed random noises with a mean of 0 and a standard deviation of 0.1 were added to the eight tray temperature measurements, and normally distributed random noises with a mean of 0 and a standard deviation of 0.01 were added to the product composition measurement. The sums of squared control errors of different control schemes under measurement noises are reported in Table 6. Figure 15 shows the performance of the two control systems with F_4 and F_6 under the disturbances shown in Figure 10 at the operating point (93%, 6%). It

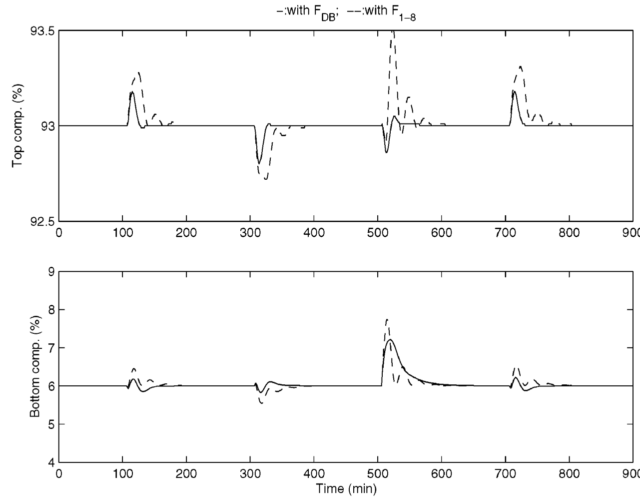

Figure 14. Control performance at the operating point (93%, 6%) for feedback control with F_{1-8} and with F_{DB}.

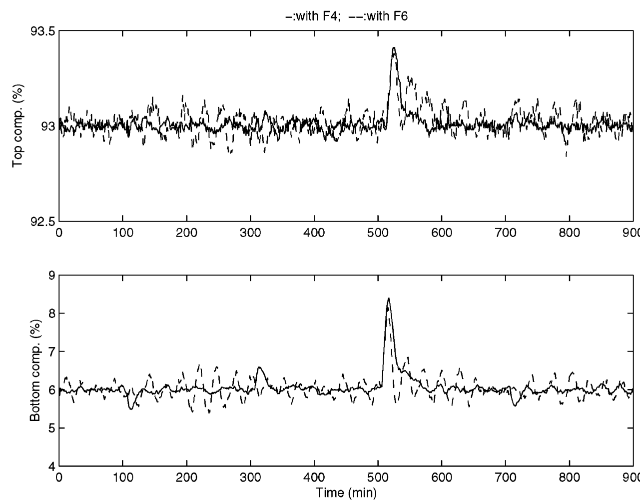

Figure 15. Control performance under tray temperature measurement noise.

Table 6. Sums of Squared Control Errors for the Cases with Measurement Noise

control scheme	operating point (95%, 5%) top	operating point (95%, 5%) bottom	operating point (93%, 6%) top	operating point (93%, 6%) bottom
feedback control only	3.87	146.26	7.78	145.92
with F_4	0.98	69.44	2.52	81.75
with F_6	2.28	66.16	5.47	88.61
with F_{1-8}	2.04	30.48	6.92	36.36
with F_{DB}	0.56	33.10	1.04	33.32

can be seen that F_4 is much less sensitive to temperature measurement noises than F_6. This is due to the fact that the control system with F_4 is much more robust than that with F_6.

Set-point tracking performance is shown in Figures 16 and 17, where the set points for the top and bottom compositions change to 93 and 6%, respectively, at 101 min. The sums of squared control errors of the different control schemes are given in Table 7. From Figures 16 and 17 and Table 7, it can be seen that, apart from F_6, the other inferential feed-forward controllers can also improve the set-point tracking performance. It should be noted here that the objective of inferential feed-forward control is disturbance rejection.

3.3. Inferential Feed-Forward Control Using Inventory Manipulators. Disturbances will also affect

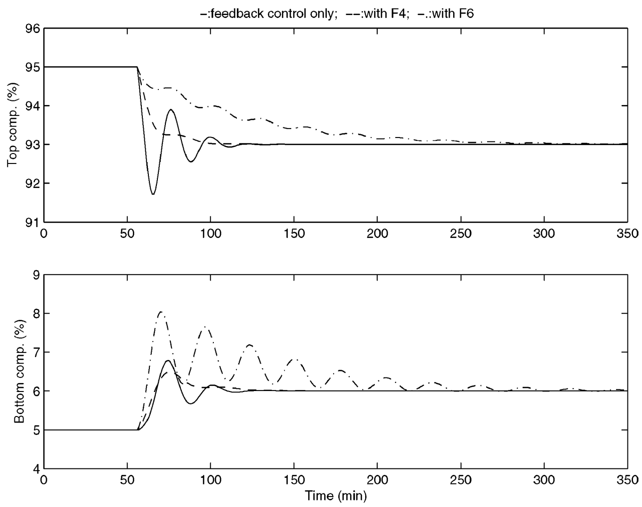

Figure 16. Set-point tracking performance for feedback control only with F_4 and with F_6.

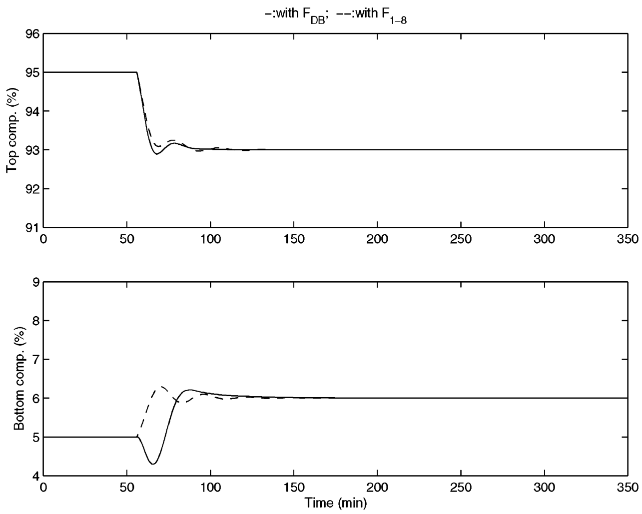

Figure 17. Set-point tracking performance for feedback control with F_{1-8} and with F_{DB}.

Table 7. Sums of Squared Control Errors for Set-Point Tracking

control scheme	top	bottom
feedback control only	23.50	12.16
with F_4	19.33	7.93
with F_6	110.81	102.12
with F_{1-8}	15.58	4.32
with F_{DB}	13.54	37.43

inventories in the reboiler and condenser. The effects of disturbances can, therefore, be inferred from changes in inventory manipulators. In this approach, we use inventory manipulator variations to infer the effects of feed rate and feed composition disturbances. We consider the distillation column as a 5 × 5 system described by the following model

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{pmatrix} G_{d1} \\ G_{d2} \end{pmatrix} d \tag{51}$$

where $y_1 = (y_D\ x_B)^T$ represents product compositions; $y_2 = (M_D\ M_B\ M_P)^T$ represents the holdups in the condenser and reboiler and the column pressure, respectively; $u_1 = (L\ V)^T$; $u_2 = (D\ B\ V_T)^T$; $d = (F\ z)^T$; G_{11} is given by eq 41; $G_{12} = 0$ because inventory and pressure manipulators cannot affect product compositions; and G_{d1} is given by eq 42. If G_{21}, G_{22}, and G_{d2} are available, then the inferential feed-forward controller can be obtained using eq 39.

Here, we present an alternative approach that does not require knowledge of G_{21}, G_{22}, and G_{d2}. In a distillation column, the following relationship between composition manipulators, inventory manipulators, and disturbances can be shown to hold

$$\begin{pmatrix} \Delta D \\ \Delta B \end{pmatrix} = M\begin{pmatrix} \Delta L \\ \Delta V \end{pmatrix} + N\begin{pmatrix} \Delta F \\ \Delta z \end{pmatrix} \tag{52}$$

In eq 52, $M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$ and $N = \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix}$ can be calculated from steady-state column operating data as[19]

$$m_{11} = -m_{21} = -\frac{\bar{D}g_{11}(0) + \bar{B}g_{21}(0)}{\bar{y}_D - \bar{x}_B} \tag{53}$$

$$m_{12} = -m_{22} = -\frac{\bar{D}g_{12}(0) + \bar{B}g_{22}(0)}{\bar{y}_D - \bar{x}_B} \tag{54}$$

$$n_{11} = 1 - n_{21} = -\frac{\bar{z} - \bar{x}_B - \bar{D}g_{d11}(0) - \bar{B}g_{d21}(0)}{\bar{y}_D - \bar{x}_B} \tag{55}$$

$$n_{12} = -n_{22} = \frac{\bar{F} - \bar{D}g_{d12}(0) - \bar{B}g_{d22}(0)}{\bar{y}_D - \bar{x}_B} \tag{56}$$

where $g_{11}-g_{22}$ are given by eq 41 and $g_{d11}-g_{d22}$ are given by eq 42, and the overlined variables represent the nominal steady-state values. It is, however, easier to obtain M and N than to obtain G_{21}, G_{22}, and G_{d2}. For this particular column, M and N are calculated as[19]

$$M = \begin{pmatrix} -0.3401 & 0.8579 \\ 0.3401 & -0.8579 \end{pmatrix} \tag{57}$$

$$N = \begin{pmatrix} 0.1986 & 0.1205 \\ 0.8014 & -0.1205 \end{pmatrix} \tag{58}$$

The inferential feed-forward control strategy can now be considered as a new control configuration having the following composition manipulators

$$\begin{pmatrix} \Delta u_{c1} \\ \Delta u_{c2} \end{pmatrix} = \begin{pmatrix} \Delta L \\ \Delta V \end{pmatrix} + F_{DB}\begin{pmatrix} \Delta D \\ \Delta B \end{pmatrix} \tag{59}$$

Substituting eq 52 into eq 59 gives

$$\begin{pmatrix} \Delta u_{c1} \\ \Delta u_{c2} \end{pmatrix} = (I + F_{DB}M)\begin{pmatrix} \Delta L \\ \Delta V \end{pmatrix} + F_{DB}N\begin{pmatrix} \Delta F \\ \Delta z \end{pmatrix} \tag{60}$$

It follows from eq 60 that

$$\begin{pmatrix} \Delta L \\ \Delta V \end{pmatrix} = (I + F_{DB}M)^{-1}\begin{pmatrix} \Delta u_{c1} \\ \Delta u_{c2} \end{pmatrix} - (I + F_{DB}M)^{-1}F_{DB}N\begin{pmatrix} \Delta F \\ \Delta z \end{pmatrix} \tag{61}$$

Substituting eq 61 into eq 40, we have

$$\begin{pmatrix}\Delta y_D \\ \Delta x_B\end{pmatrix} = G\begin{pmatrix}\Delta L \\ \Delta V\end{pmatrix} + G_{d1}\begin{pmatrix}\Delta F \\ \Delta z\end{pmatrix} = G(I + F_{DB}M)^{-1}\begin{pmatrix}\Delta u_{c1} \\ \Delta u_{c2}\end{pmatrix} + [G_{d1} - G(I + F_{DB}M)^{-1}F_{DB}N]\begin{pmatrix}\Delta F \\ \Delta z\end{pmatrix} \quad (62)$$

where G and G_{d1} are given by eqs 41 and 42, respectively. To make the steady-state disturbance gains zero, the inferential feed-forward controller can be calculated as

$$F_{DB} = G(0)^{-1}G_{d1}(0)[N - MG(0)^{-1}G_{d1}(0)]^{-1} \quad (63)$$

For the column studied here, this becomes

$$F_{DB} = -\begin{pmatrix}0.2713 & 0.1352 \\ 0.6950 & 0.1689\end{pmatrix} \quad (64)$$

This inferential feed-forward controller is also used in conjunction with an IMC controller tuned by optimizing the robust control performance. The controller parameters are given in Table 4. This inferential feed-forward controller was tested under the sequence of disturbances shown in Figure 10 at the two operating conditions (95%, 5%) and (93%, 6%). The control performance results are shown in Figures 12 and 14, and the corresponding sums of squared control errors are given in Table 5. The sums of squared control errors for the cases with measurement noise are given in Table 6. Figure 17 shows the set-point tracking performance with the sum of squared control errors given in Table 7. From Figures 12 and 14 and Tables 5 and 6, it can be seen that F_{DB} can also offer very good control performance in disturbance rejection, particularly for feed rate disturbances. This is because F_{DB} is designed for rejecting both feed composition and feed rate disturbances. Compared to the inferential feed-forward controllers using tray temperatures, the control system with F_{DB} exhibits better control performance in disturbance rejection. This confirms the robustness analysis results shown in Table 4. Tables 5 and 6 show that the control performance with F_{DB} is insensitive to measurement noise. This is because tray temperature measurements are not used in F_{DB}, and thus, the control performance is affected only by composition measurement noise. Figure 17 and Table 7 show that the control system with F_{DB} has better set-point tracking performance for the top product composition than the control system without inferential feed-forward control, but the same is not true for the bottom product. It should be emphasized here that the inferential feed-forward control is designed for the rejection of unmeasured disturbances.

4. Conclusions

Two inferential feed-forward control strategies are proposed. One uses uncontrolled secondary process variables, whereas the other uses the manipulated variables for the controlled secondary process variables with fast dynamics. These strategies are useful when disturbances cannot easily be measured and, hence, direct feed-forward control cannot be applied. The effects of disturbances on the primary process variables are inferred from certain easily available measurements of uncontrolled secondary process variables or from the manipulated variables for certain controlled secondary process variables with fast dynamics. The main advantage of such inferential feed-forward control strategies is that measurements of disturbances are not needed. A robustness analysis is presented, and it is shown that robustness is an important factor to be considered when selecting secondary process variables. Secondary process variable selection and feedback controller tuning can be performed by optimizing the achievable robust control performance represented by the structured singular value of the overall control system.

The proposed strategies have been applied to a simulated methanol−water separation column. Nonlinear dynamic simulations demonstrate that the proposed strategies can significantly improve the disturbance rejection capabilities of the distillation composition control system. Robustness analysis shows that using multiple tray temperature measurements can significantly improve the robustness of the control scheme, which is confirmed by simulations.

The present work considers only static inferential feed-forward controllers aimed at removing the steady-state effects of disturbances on the controlled variables. Extension to dynamic inferential feed-forward control is currently under study and will be reported in the future.

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