

Estimation of Imbibition Capillary Pressure Curves from Spontaneous Imbibition Data

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A closed-form solution for calculating imbibition capillary pressure from experimental data has been derived on the basis of the assumption of a constant capillary diffusion coefficient combined with Corey functions for the relative permeability curves. The solution is almost similar to the capillary pressure correlation reported by Skjaeveland et al. (Skjaeveland, S. M.; Sigveland, L. M.; Kjosavik, A.; Hammervold Thomas, W. L.; Virnovsky, G. *SPE Reservoir Eval. Eng.* 2000, 3 (1), 60–67), but it allows for calculation of the empirical fit parameters from experimental data. Calculated imbibition capillary pressure curves show the correct qualitative behavior because of variations in the curvature of the relative permeability curves, the end-point values on the relative permeability curves, the absolute permeability, and the capillary diffusion coefficient. Capillary pressure threshold values calculated from the model are in accordance with experimental data, assuming reasonable input data. The model, however, needs to be tested more against experimental data to confirm its potential to calculate reliable imbibition capillary pressure curves and capillary pressure threshold values.

Introduction

The capillary pressure is of outermost importance regarding oil recovery from subterranean reservoirs. Modeling capillary pressure curves for drainage and imbibition is therefore a very important task in reservoir simulation when predicting oil recovery potential, improved oil recovery methods, transport properties for the fluid phases, etc. The subject in this paper is concerning the modeling of imbibition capillary pressure curves based on experimental data from spontaneous imbibition (SI) tests. It should be noticed that imbibition in this paper will refer to a process where water saturation in the porous medium is increasing regardless of wettability of the porous medium. Because drainage is the opposite of imbibition, it will in the same manner describe a process where water saturation is decreasing, again regardless of the wettability state of the actual porous medium.¹

Many capillary pressure correlations in the literature are based on the work by Brooks and Corey.² They observe that the following empirical relation held based on numerous experimental data:

$$S_e = \left[\frac{P_b}{P_C} \right]^\lambda \quad \text{for } P_C \geq P_b \quad (1)$$

where S_e is the effective wetting phase saturation, P_C is the capillary pressure (Pa), P_b is the threshold (entry) capillary pressure (Pa), and λ is the pore-size distribution index.

Skjaeveland et al.³ applied a modified form of eq 1 to correlate capillary pressure as a function of water saturation for mixed-wet reservoirs. Their expression reads for the

imbibition capillary pressure

$$P_c = \frac{c_{wi}}{\left(\frac{S_w - S_{wi}}{1 - S_{wi}} \right)^{a_{wi}}} + \frac{c_{oi}}{\left(\frac{S_o - S_{or}}{1 - S_{or}} \right)^{a_{oi}}} \quad (2)$$

where S_{wi} is the initial water saturation, S_{or} is the residual oil saturation, S_w is the water saturation, S_o is the oil saturation, and c_{wi} , c_{oi} , a_{wi} , and a_{oi} are the fit parameters.

Two branches modeled the imbibition capillary pressure curve, one for the positive and one for the negative part of the curve. The variables c_{wi} , c_{oi} , a_{wi} , and a_{oi} in the above equation are fit parameters, where the subscript i indicates that these parameters are valid for an imbibition-type process. Skjaeveland et al.³ used another set of parameters (c_{wd} , c_{od} , a_{wd} , and a_{od}) when modeling the drainage process. Most types of capillary pressure correlations are in general similar to the correlation given in eq 2, because they typically contain two adjustable parameters. One parameter is expressing the pore-size distribution, and the other parameter is expressing the entry or mean capillary pressure (a and c values in eq 2, respectively). The purpose of this work is to derive a closed form solution describing how the imbibition capillary pressure varies with the shape of the relative permeability curves, including end-point values, absolute permeability, and capillary diffusion coefficients. The development of the model is based on the continuity equation, Darcy's law, and Corey functions for the relative permeability curves together with certain assumptions, which will be outlined in the next section. This section will also provide a discussion of the relationships between the equations applied in this work in an attempt to clarify the matter presented herein. The empirical data applied as an empirical reference for the imbibition capillary pressure model devolved herein are reported previously by the author.⁴ Details related to the experimental procedures, core handling, etc. can be found there, and only a very brief description will be given here.

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(1) Morrow, N. R. *J. Pet. Technol.* **1990**, December, 1476–1484.
(2) Brooks, R. E.; Corey, A. T. *J. Irrig. Drain. Div., Am. Soc. Civ. Eng.* **1966**, June, 61–88.
(3) Skjaeveland, S. M.; Sigveland, L. M.; Kjosavik, A.; Hammervold Thomas, W. L.; Virnovsky, G. *SPE Reservoir Eval. Eng.* **2000**, 3 (1), 60–67.

(4) Standnes, D. C. *Energy Fuels* **2004**, 18, 271–282.

Table 1. Fluid Properties

liquid phase	density at 20 °C (g/mL)	viscosity at 20 °C (mPa s)	IFT using decane as the oil phase at 20 °C (mN/m)
distilled water	0.9982	1.0	46
<i>n</i> -decane	0.731	0.95	

Table 2. Core Data for the Spontaneous Imbibition Tests

test number	sample shape— boundary condition	diameter (<i>d</i>) and radius (<i>a</i>) (m)	height (m)	$V^b (\times 10^{-6}) (\text{m}^3)$	porosity %	PV ($\times 10^{-6}$) (m^3)
3	CY–2 FC	0.0199/0.00995	0.05	15.55	43.0	6.68
4	CY–2 FC	0.0339/0.01695	0.0496	44.77	42.6	19.06
5	CY–2 FC	0.06/0.03	0.032	90.48	42.4	38.36
6	CY–2 FC	0.1/0.05	0.0397	311.8	43.2	134.7

Experimental Section

Fluids. *n*-Decane from Riedel-deHäen with grade > 95% was used as the oil phase. Distilled water was used as the imbibing water phase. Fluid properties for the SI tests are summarized in Table 1.

Porous Medium. Outcrop chalk samples from Denmark were used as porous medium. The samples were taken from the same block. Porosities were all close to 42%, and absolute permeability was determined to be approximately 2 mD. Rock properties for all tests are shown in Table 2. It should be noticed that the test numbers are taken directly from the original work.⁴

***n*-Decane–Water IFT.** *n*-Decane–water IFT was determined to 46 mN/m at ambient temperature using a ring tensiometer.

Rock Sample Geometry. The geometry and boundary conditions are shown in Figure 1. All rock samples had a cylindrical shape, and they are referred to as CY, whereas the boundary conditions are referred to as 2 FC [two faces closed (top and bottom)].

Experimental Procedure. All SI tests were performed at ambient temperature (~20 °C). The tests were performed using Amott cells (Figure 2), except for the tests performed on the cylindrical cores with diameters of 6.00 and 10.00 cm. Tests in the Amott cells were performed by introducing 100% oil-saturated cores into the cell, adding distilled water, and monitoring oil production as a function of time. The oil production could be measured with an accuracy of ± 0.1 mL. In the case of the cylindrical cores having $d = 6.00$ and 10.00 cm, the oil-saturated rock samples were suspended from a balance and immersed in distilled water. A change in the weight of the core as a function of time was monitored. The accuracy of the balance was 0.01 g, corresponding to an uncertainty of approximately ± 0.02 mL of water imbibed. The mass of the rock sample was determined at the end of each run to check for material balance, and good agreement between the measured volume and the weight of the rock sample was obtained in all tests. This value was used to calibrate the intermediate recorded masses on the imbibition curve to correct for the effect of the downward moving (approximately 0.6 cm) of the oil–water contact during each run (the movement will induce decreasing buoyancy forces).

Theoretical Model

Consider a cylindrical core at initial water saturation (S_{wi}) with top and bottom faces closed for fluid exchange. If the porous medium is water-wet and the core is totally immersed in water at time $t = 0$, the normalized water saturation S_w at any point (x , y , and z) in the rock samples at $t \geq 0$ can be described by the following hyperbolic partial differential equation:⁵

$$\phi \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x} D(S_w) \frac{\partial S_w}{\partial x} + \frac{\partial}{\partial y} D(S_w) \frac{\partial S_w}{\partial y} + \frac{\partial}{\partial z} D(S_w) \frac{\partial S_w}{\partial z} \quad (3)$$

(5) Aziz, K.; Settari, A. *Petroleum reservoir simulation*, 1985.

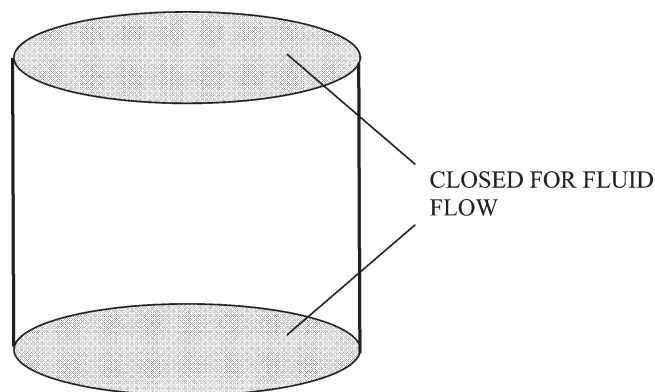


Figure 1. Rock sample shape and boundary condition in the spontaneous imbibition tests (cylindrical core sample with top and bottom faces closed for fluid flow).

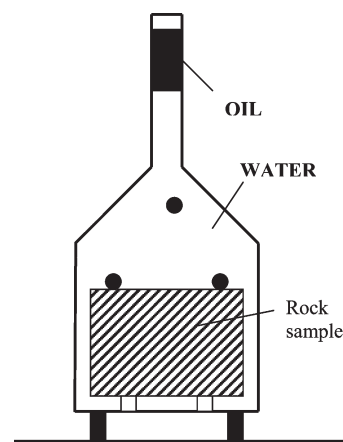


Figure 2. Amott cell with a rock sample immersed in the aqueous phase.

where

$$D(S_w) = - \frac{k k_{ro}}{\mu_o} \frac{1}{1 + \frac{k_{ro} \mu_w}{k_{rw} \mu_o}} \frac{dP_C}{dS_w} \quad (4)$$

where k is the absolute permeability (m^2), k_{ro} is the relative permeability to oil, k_{rw} is the relative permeability to water, μ_o is the oil viscosity (Pa s), μ_w is the water viscosity (Pa s), S_w is the water saturation, P_C is the capillary pressure (Pa), and D is the capillary diffusion coefficient (m^2/s).

The capillary diffusion coefficient (CDC) in eq 4 is reported in the literature to be a strong function of the water saturation.^{6,7} In this case, eq 3 is nonlinear and has to be

(6) Kashchiev, D.; Firoozabadi, A. *SPE Tech. Pap.* 75166, 2002.

(7) McWhorter, D. B.; Sunada, D. K. *Water Resour. Res.* **1990**, *26* (3), 399–413.

solved numerically, except for some few special cases.⁸ However, the approach used in this study is to assume that CDC is constant and independent of water saturation. The reason for doing so is that modeled results described by the author⁹ indicate that the CDC can be approximated reasonably well with a constant as a function of water saturation. Experimental and modeled oil production curves collapsed into one single curve over the whole saturation range when comparing calculated oil recovery versus time to oil recovery from SI tests into cylindrical chalk samples, with the diameter varying in the range from 1.99 to 10.00 cm. The modeled oil recovery curves used in the comparison were based on the assumption of a constant CDC. Before proceeding further, it is, however, very important to emphasize that the good match between experimental and modeled oil production curves mentioned above under no circumstances can be regarded as a confirmation of the correctness of applying a constant CDC. It merely serves as an indication or working hypothesis that the approximation under the prevailing circumstances may be acceptable. Therefore, if it is assumed that the CDC is constant independent of S_w , eq 4 can be used to calculate the imbibition capillary pressure by solving the following ordinary differential equation:

$$\frac{D\mu_o}{kk_{ro}} \left(1 + \frac{k_{ro}\mu_w}{k_{rw}\mu_o} \right) = -\frac{dP_C}{dS_w} \quad (5)$$

To find a solution to eq 5, relative permeability functions for the water and oil phases must be specified as a function of water saturation. These functions will be expressed here analytically using modified Corey functions.^{10,11} The main reasons for choosing this approach are that modified Corey functions are relatively simple mathematically and have a solid experimental basis. Thus, the relative permeability to water (k_{rw}) and oil (k_{ro}) as a function of water saturation can be expressed as

$$k_{rw} = k_{rwe} \left[\frac{S_w - S_{wi}}{1 - S_{or} - S_{wi}} \right]^{n_w} \quad (6)$$

$$k_{ro} = k_{roe} \left[\frac{1 - S_{or} - S_w}{1 - S_{or} - S_{wi}} \right]^{n_o} \quad (7)$$

where S_{wi} is the initial water saturation, S_w is the water saturation, k_{roe} is the end point of the oil relative permeability curve, k_{rwe} is the end point of the water relative permeability curve, n_w is the Corey exponent for the water relative permeability curve, and n_o is the Corey exponent for the oil relative permeability curve.

Inserting these expressions into eq 5 gives

$$\frac{dP_C}{dS_w} = -\frac{D}{k} \left[\frac{\Delta S^{n_o}}{\lambda_{oe}} \frac{1}{(1 - S_{or} - S_w)^{n_o}} + \frac{\Delta S^{n_w}}{\lambda_{we}} \frac{1}{(S_w - S_{wi})^{n_w}} \right] \quad (8)$$

where λ_{oe} is the mobility of the oil phase evaluated for k_{roe} , λ_{we} is the mobility of the water phase evaluated for k_{rwe} , and $\Delta S = 1 - S_{or} - S_{wi}$.

Performing the integration gives

$$P_C = \frac{D}{k} \left[\frac{\Delta S^{n_w}}{\lambda_{we}} \frac{1}{(n_w - 1)} \frac{1}{(S_w - S_{wi})^{n_w - 1}} - \frac{\Delta S^{n_o}}{\lambda_{oe}} \frac{1}{(n_o - 1)} \frac{1}{(1 - S_{or} - S_w)^{n_o - 1}} \right] \quad (9)$$

Both terms inside the bracket in eq 9 are positive (for $n_w > 1$ and $n_o > 1$, respectively), and the capillary pressure curve is the difference between these two terms. Their relative magnitude will therefore decide the magnitude and sign of the imbibition capillary pressure curve. It can be observed that the first term shows correct behavior when the water saturation is close to S_{wi} ; i.e., the capillary pressure goes to plus infinite for S_w approaching S_{wi} . It is expected that the same behavior should occur at the other extreme value of S_w when it approaches $1 - S_{or}$ (i.e., the capillary pressure should go to minus infinity on this end of the saturation scale). However, this is not the case in eq 9, but changing the saturation variables for this term ($S_w = 1 - S_o$) secures that this term also shows the right behavior when the oil saturation is approaching the critical value S_{or} . Equation 9 then transforms into

$$P_C = \frac{D}{k} \left[\frac{\Delta S^{n_w}}{\lambda_{we}} \frac{1}{(n_w - 1)} \frac{1}{(S_w - S_{wi})^{n_w - 1}} - \frac{\Delta S^{n_o}}{\lambda_{oe}} \frac{1}{(n_o - 1)} \frac{1}{(S_o - S_{or})^{n_o - 1}} \right] \quad (10)$$

Finally, writing eq 10 on the same form as Skjaeveland et al.³ gives the following relationship for the imbibition capillary pressure as a function of water saturation:

$$P_C = \frac{D}{k} \left[\frac{\Delta S}{\lambda_{we}} \frac{1}{(n_w - 1)} \frac{1}{\left(\frac{S_w - S_{wi}}{\Delta S} \right)^{n_w - 1}} - \frac{\Delta S}{\lambda_{oe}} \frac{1}{(n_o - 1)} \frac{1}{\left(\frac{S_o - S_{or}}{\Delta S} \right)^{n_o - 1}} \right] \quad (11)$$

The expression given in eq 11 is almost similar to the expression given by Skjaeveland et al.,³ except for the term ΔS (the difference is due to application of the modified Corey functions for relative permeability). They arrived at the expression in eq 2 by empirical generalization of the results found by Brooks and Corey² and putting reasonable constraints on the fitting parameters. The reason for the similarity between eqs 2 and 11 is however not coincidental and will be clear after looking into the background for the derivation of the Corey relative permeability functions as discussed below.

To understand the similarity of eqs 2 and 11, it is important to analyze and clarify the conditions and assumptions applied in deriving eq 11. The main point here is that the Corey functions for the relative permeability are derived using two equations: an equation based on an empirical observation in addition to the so-called Burdine's equation.¹² Brooks and Corey² observed that the following empirical relationship held for all of the core samples that they analyzed (logarithm to eq 1):

$$\ln S_e = -\lambda \ln P_C + \lambda \ln P_b \quad \text{for } P_C \geq P_b \quad (12)$$

(12) Burdine, N. T. *Trans. AIME* **1953**, *198*, 171–177.

(8) Knight, J. H.; Philip, J. R. *J. Eng. Math.* **1974**, *8* (3), 219–227.

(9) Standnes, D. C. *J. Pet. Sci. Eng.* **2006**, *50*, 151–160.

(10) Tveheyo, M. T.; Talukdar, M. S.; Torsæter, O.; Vafaeinezhad, Y. Presented at the 11th Oil, Gas, and Petrochemical Congress and Exhibition, Teheran, Iran, Oct 29–31, 2001.

(11) Yu, S. Y.; Akervoll, I.; Torsæter, O.; Stensen, J. A.; Kleppe, J.; Midtlyng, S. H. *SPE Tech. Pap.* 48842, 1998.

Table 3. Numerical Values of the Model Input Parameters for the Base Case

parameter	n_o	n_w	k_{rwe}	k_{roe}	μ_o (mPa s)
value	1.4	1.8	0.15	1.0	1.0
parameter	μ_w (mPa s)	S_{wi}	S_{or}	$k (\times 10^{-15}) (m^2)$	$D (\times 10^{-8}) (m^2/s)$
value	1.0	0	0.3	2	4

Taking the anti-log on both sides gives P_C expressed as a function of effective wetting saturation, which they applied in Burdine's equation reading

$$k_{rw} = \left[\frac{S - S_r}{1 - S_r} \right]^2 \frac{\int_0^S \frac{dS}{P_C^2}}{\int_0^1 \frac{dS}{P_C^2}} \quad (13)$$

where S_r is the residual saturation.

Brooks and Corey used eq 13 to derive their well-known relative permeability curves for wetting and nonwetting phases as a function of wetting phase saturation. Thus, assuming that relative permeability curves can be modeled as Corey functions is to say implicitly that the capillary pressure curve as a function of water saturation has a hyperbolic shape. Because Corey functions (or modified Corey functions) were applied for the relative permeability curves in this work during the derivation of eq 11, it follows from the discussion above that eq 11 had to show the same feature as eq 2, i.e., hyperbolic shape.

The improvements by performing the “recalculation” of eq 2 via eq 4 is that the result (eq 11) opens for the possibility to calculate imbibition capillary pressure curves and entry pressures based on SI data (plus some additional information about the relative permeability functions), because a direct comparison between eqs 2 and 11 gives the following expressions for c_{wi} , c_{oi} , a_{wi} , and a_{oi} :

$$c_{wi} = \frac{D\Delta S}{k\lambda_{we}} \frac{1}{(n_w - 1)} \quad \text{and} \quad a_{wi} = n_w - 1 \quad (14)$$

$$c_{oi} = \frac{D\Delta S}{k\lambda_{oe}} \frac{1}{(n_o - 1)} \quad \text{and} \quad a_{oi} = n_o - 1 \quad (15)$$

Results and Discussion

Parameter Input to the Imbibition Capillary Pressure Model. Equation 11 indicates that several parameters must be specified to calculate the imbibition capillary pressure curve as a function of water saturation. First, the CDC should be determined experimentally by performing SI tests using actual fluids and porous medium. It has previously been shown⁹ that the following equation describes the increase in water saturation because of SI if the rock sample has a cylindrical shape and the top and bottom faces are closed for fluid exchange (a constant CDC was also assumed when deriving the equation below):

$$S_{wAV}(t) = 1 - 4 \sum_{n=1}^{\infty} \frac{1}{Z_n^2} e^{-\frac{D_{AV}(Z_n/a)^2}{\phi} t} \quad (16)$$

where S_{wAV} is the average water saturation in the rock sample at time t , Z_n is the zeros of the Bessel function J_0 , where $n = 1, 2, 3, \dots$, a is the radius of the core (m), ϕ is the fractional porosity, D_{AV} is the average diffusion coefficient (m^2/s), and t is the imbibition time (s).

A good estimate for CDC can be found by fitting eq 16 to the experimental SI curve. It should be noticed that typical

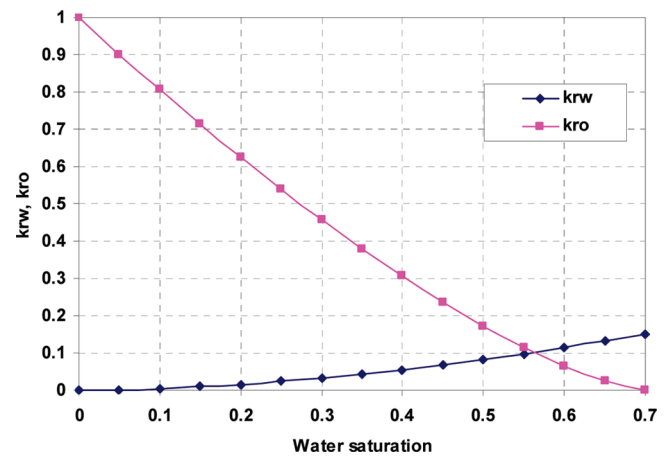


Figure 3. Corey-type relative permeability curves used in the base case (n_o , 1.4; n_w , 1.8).

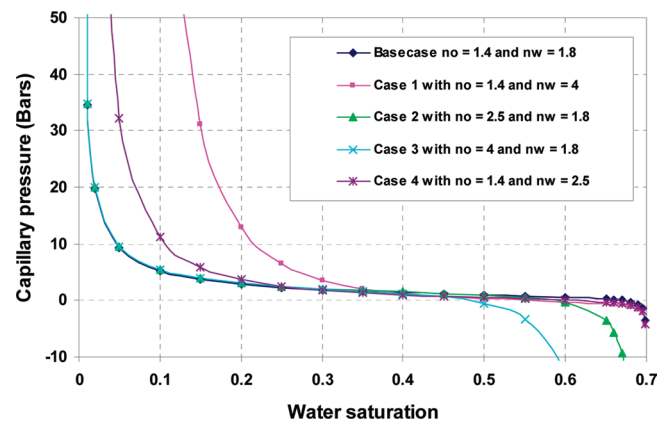


Figure 4. Variation of imbibition capillary pressure varying the curvature of the relative permeability curves.

values of the CDC for strongly water-wet media lie in the range from 1×10^{-8} to $1 \times 10^{-7} m^2/s$ (Table 4).¹³ On the basis of the data reported by Standnes,⁹ a value of $4 \times 10^{-8} m^2/s$ is used in this work as the base when looking at numerical examples.

Obeing the boundary conditions of closed top and bottom faces for the rock samples implicitly included in eq 16 can be obtained experimentally by closing these surfaces with epoxy or applying a core where the length/diameter ratio is large. The latter method is recommended because the core should be flooded with water at high rates after the SI process has ceased to determine S_{or} and ΔS in eq 11.

The next step is to measure or make an educated guess regarding the end points (k_{rwe} and k_{roe}) and the curvature of the relative permeability curves (n_w and n_o). All of this information together with the knowledge about the absolute permeability and fluid properties (viscosities) allows the model to determine the imbibition capillary pressure curve, the water saturation where the capillary pressure equals zero, and the threshold capillary pressure.

(13) Standnes, D. C. *Colloids Surf., A* **2004**, *251* (1–3), 93–101.

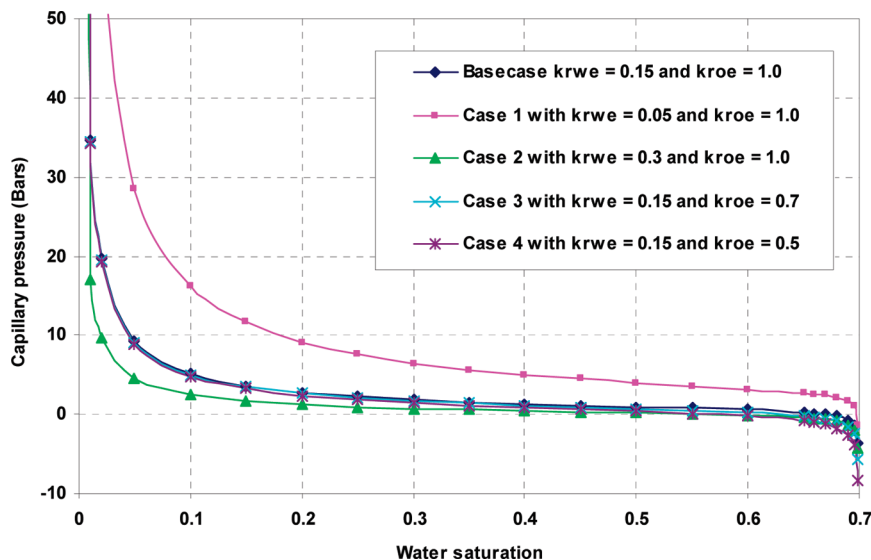


Figure 5. Variation of imbibition capillary pressure varying the end points of the relative permeability curves.

Unfortunately, to the knowledge of the author, there exists no comprehensive set of data in the literature that include all of the relevant parameters described above. Before such data are available, testing calculated imbibition capillary pressure curves from eq 11 can only be performed by assuming reasonable numerical values for some of the parameters and then testing if the resulting imbibition capillary pressure curve seems reasonable. In the next sections, the impact of changing Corey exponents, end-point relative permeability values, absolute permeability, and CDC on the imbibition capillary pressure curve model will be demonstrated. The last section contains the calculation of capillary threshold pressures because these values will give an indication of the validity of the model.

Variation of the Capillary Pressure Curve with Variation in Corey Exponents and End-Point Relative Permeability. The comparison of varying different parameters in the model will be compared to a base case. The chosen set of variables for the base case should be representative for the chalk samples used in the cited experimental data. It is assumed that the reservoir rock is strongly water-wet because it has never been in contact with crude oil. The numerical values given in Table 3 were chosen as the base case (relative permeability curves shown in Figure 3).

The calculated base case capillary pressure curve is shown in Figure 4 together with other curves where the Corey exponents have been varied. The base case capillary pressure curve shows high capillarity as expected from such low-permeability medium as chalk. The zero of the capillary pressure curve crosses the water saturation axis close to S_{or} as it should according to the wettability of the rock material. Increasing the Corey exponent for water does not affect the location of the zero capillary pressure point significantly. Increasing n_w shifts the capillary pressure to higher values at low water saturations, whereas the opposite effect is seen when varying n_o . For the latter case, there is no change in the capillary pressure for low water saturations but the zero point of the capillary pressure curve shifts to the left as n_o increases.

Increasing the end point on the relative permeability curve for water shifts the zero point of the capillary pressure curve slightly to the left (Figure 5). At the same time, the capillary pressure is lowered for lower water saturation. If the increase

in k_{rwe} is interpreted as a shift toward less water-wet conditions, the behavior of lower capillary pressure is in accordance with expectations because of the fact that the surface energy and, hence, the capillary pressure of the system are lowered by decreasing water wetness (see the discussion below). There is no significant change in the capillary pressure curve. The absolute value of the capillary pressure does not change nor does the zero capillary pressure point as the end-point relative permeability value to oil is reduced.

It is important to mention that imbibition capillary pressure curves and other results following from the model presented herein are only valid for the flow of oil and water in a medium where “one fluid phase wets the solid surface and the other does not” (directly cited from Brooks and Corey²). That means that the Corey formulation for the relative permeability curves is strictly speaking only valid for rock samples being water-wet or strongly water-wet. It is not exactly clear for which value of reduced water wetness the Corey formulation breaks down. The wettability can for example be quantified by the Amott–Harvey wettability index,¹ but it is reasonable to assume that only small perturbations in the wettability state from strongly water-wet conditions can be accepted when the Corey formulation is used to describe the flow of wetting and nonwetting phases in consolidated porous media.

Effect of k and CDC on the P_c Curve. It can be seen from eq 11 that there will be no shift in the point where the capillary pressure curve equals zero because of variations in the CDC and k . Variations in these two parameters will only increase or decrease the absolute value of the capillary pressure. An increase in CDC will increase the capillary pressure and vice versa for the absolute permeability k . Figures 6 and 7 show some examples on how variations in k and CDC affect the absolute value of the capillary pressure. The CDCs were varied from 1×10^{-9} to 2×10^{-7} m²/s, i.e., a factor of 200, resulting in capillary pressure curves showing very low and very high pressures, respectively. Whether such variations in CDC are realistic for different rock types having approximately the same wettability state is an open question. It is very interesting, however, to notice that Ma et al.¹⁴ reported

(14) Ma, S.; Morrow, N. R.; Zhang, X. *J. Pet. Sci. Eng.* **1997**, *18*, 165–178.

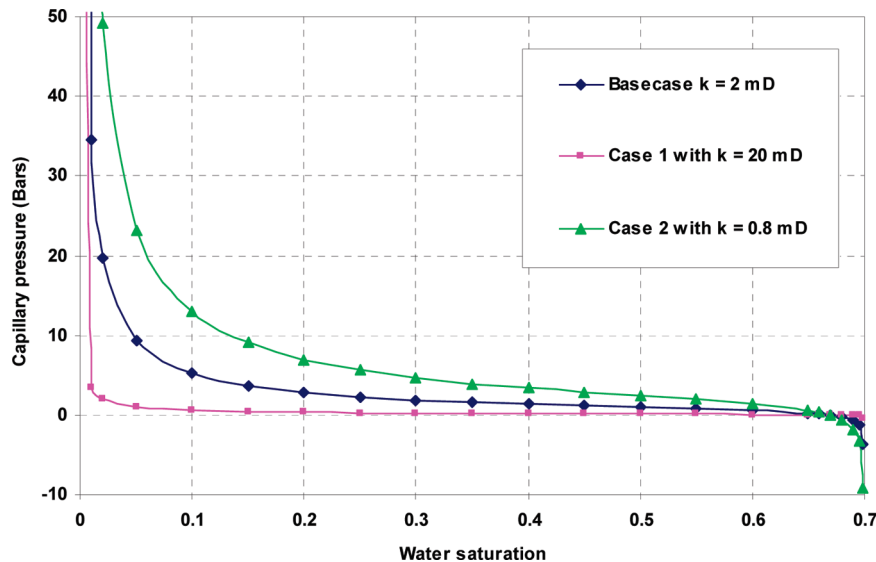


Figure 6. Variation of imbibition capillary pressure varying the absolute permeability k of the rock sample.

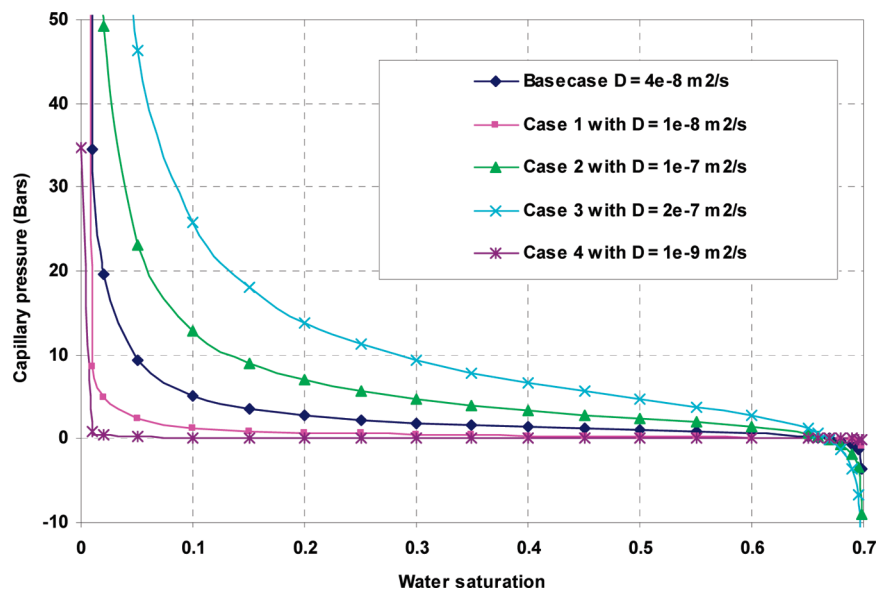


Figure 7. Variation of imbibition capillary pressure varying the capillary diffusion coefficient D .

that numerous experimental data from Mattax and Kyte,¹⁵ Hamon and Vidal,¹⁶ and Zhang et al.¹⁷ could all be reasonably approximated by the following expression:

$$\frac{R}{R_{\infty}} = 1 - e^{-0.05t_D} \quad (17)$$

where R is the oil recovery at dimensionless time t_D , R_{∞} is the maximum oil recovery at infinite time, and t_D is dimensionless time.

Equation 17 describes oil recovery as a function of recoverable oil as a function of dimensionless imbibition time. The numerical value of 0.05 in the exponent can be used to estimate the CDC for this type of rock. Using eq 6 in the paper by Standnes¹³ and typical parameters for Berea sandstone in the paper by Zhang et al.¹⁷ gives an approximate

value for the CDC for Berea sandstone equal to $7 \times 10^{-8} \text{ m}^2/\text{s}$. This value is remarkably close to the value obtained when analyzing experimental data using chalk. Furthermore, calculating CDC for some typical cases from the experimental data set of Mattax and Kyte¹⁵ and Hamon and Vidal¹⁶ all gave CDCs in the same range as the values obtained for chalk and Berea sandstone. Typical values of CDC reported in the literature also indicate the same trend as indicated in Table 4. The results presented here therefore supply more evidence to the hypothesis put forward by Ma et al.¹⁴ that SI rates into a wide range of porous media might be estimated reasonably well using the correlation given in eq 17, implicitly assuming that the porous medium is strongly water-wet.

Estimation of Threshold Capillary Pressure. Skjaeveland et al.³ modeled primary drainage using an expression similar to eq 11, with the last term inside the bracket equal to zero ($c_o = 0$). Thus, only the positive part of the capillary pressure curve was used to estimate the variation in

(15) Mattax, C. C.; Kyte, J. R. *SPE J.* **1962**, June, 177–184.
 (16) Hamon, G.; Vidal, J. V. *SPE Tech. Pap.* 15852, 1986.
 (17) Zhang, X.; Morrow, N. R.; Ma, S. *SPE Reservoir Eval. Eng.* **1996**, Nov, 280–285.

Table 4. Capillary Diffusion Coefficients from the Literature

reference	$D (\times 10^{-8}) (\text{m}^2/\text{s})$
18	maximum ≈ 0.7 based on the data from Kazemi et al. maximum ≈ 7 based on the data from Beckner et al.
19	Kleppe and Morse experiment, 50; Coats's example, 3.8 and 2; Kazemi's example, 4.8; Thomas' three-dimensional example, 3.8; SPE cross-sectional problem, 1.1

capillary pressure with a decreasing water saturation (drainage process). The capillary threshold pressure is calculated here because its value may give an indication of the validity of the present model. Hence, eq 14 gives (the subscript i has been removed to indicate that this process strictly speaking is a drainage process)

$$c_w = \frac{D\Delta S}{k\lambda_{we}} \frac{1}{(n_w - 1)} \quad (18)$$

Using the base case parameters to calculate the threshold capillary pressure (assuming $\Delta S = 0.7$; i.e., the irreducible water saturation after primary drainage starting at $S_w = 1$ is equal to 0.3) for the chalk material gives $c_w = 1.17$ bar, which is high but not in contradiction with experimental data by Tweheyo et al.¹⁰ They reported entry pressures of 1.01, 0.66, and 0.97 bar for three different chalk cores having comparable properties as the system used in this study.

Conclusion

The following conclusions can be drawn from this work: (1) A formula for imbibition capillary pressure curves has been derived from the expression for the capillary diffusion coefficient using Corey functions for the relative permeability under the assumption that the capillary diffusion coefficient is constant, independent of water saturation. (2) The present model gives the possibility to calculate the imbibition capillary pressure curve and the capillary threshold pressure based on experimental data. (3) The model cannot be fully tested because of the lack of experimental data. Capillary pressure curves calculated from the model assuming reasonable input parameters indicate that the model predicts the correct trends. Calculated capillary pressure threshold values are in accordance with experimental data.

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Nomenclature

a = radius of the core (m)
 a_{oi} = exponent related to pore-size distribution
 a_{wi} = exponent related to pore-size distribution
 c_{oi} = parameter related to the threshold or entry pressure for the negative part of the capillary pressure curve

c_w = parameter related to the threshold or entry pressure for the positive part of the capillary pressure curve
 c_{wi} = parameter related to the threshold or entry pressure for the negative part of the capillary pressure curve
CY = cylindrical
 D = capillary diffusion coefficient (m^2/s)
 D_{AV} = average capillary diffusion coefficient (m^2/s)
 d = diameter (m)
FC = faces closed
 H = height of the cylindrical rock sample (m)
IFT = interfacial tension (N/m)
 k = absolute permeability (m^2)
 k_{ro} = relative permeability to oil
 k_{rw} = relative permeability to water
 $k_{ro,e}$ = end point of the oil relative permeability curve
 k_{rwe} = end point of the water relative permeability curve
 n_w = Corey exponent for the water relative permeability curve
 n_o = Corey exponent for the oil relative permeability curve
 P_b = threshold capillary pressure (Pa)
 P_C = capillary pressure (Pa)
PV = pore volume (m^3)
 R = oil recovery at dimensionless time t_D (m^3)
 R_∞ = maximum oil recovery at infinite time (m^3)
 r = coordinate in the r direction
 S_e = effective wetting saturation
SI = spontaneous imbibition
 S_{or} = residual oil saturation
 S_w = water saturation
 S_{wi} = initial water saturation
 S_{wAV} = average water saturation in the rock sample at time t
 S_{wr} = residual wetting phase saturation
 S_r = residual saturation
 t = imbibition time (s)
 t_D = dimensionless time
 V_b = bulk volume of the rock sample (m^3)
 x = coordinate
 y = coordinate
 z = coordinate
 Z_n = zeros of the Bessel function J_0 , where $n = 1, 2, 3, \dots$
 ϕ = fractional porosity
 λ = pore-size distribution index
 λ_{oe} = mobility of the oil phase evaluated for $k_{ro,e}$ ($\text{Pa s})^{-1}$
 λ_{we} = mobility of the water phase evaluated for k_{rwe} ($\text{Pa s})^{-1}$
 μ_o = oil viscosity (Pa s)
 μ_w = water viscosity (Pa s)
 $\Delta S = 1 - S_{or} - S_{wi}$

(18) Chen, J.; Miller, M. A.; Sepehnoori, K. SPE Tech. Pap. 29141, 1995.

(19) Dutra, T. V.; Aziz, K. SPE Tech. Pap. 21248, 1991.