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# Refinery Production Scheduling Involving Operational Transitions of Mode Switching under Predictive Control System

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**ABSTRACT:** In the refinery industry, especially in large-scale modern refineries, production scheduling modeling is an essential tool in the operation and management. By scheduling systems, better economic performance can be achieved. In order to implement the scheduling schemes, the influence of control systems has to be considered in scheduling models. This paper presents a novel refinery scheduling model where several controllable and realizable operation modes for production units are established based on unitwide predictive control. Furthermore, operational transitions resulting from switching operation modes of production units are taken into account to reflect the dynamic nature of production. A novel mixed integer linear programming (MILP) discrete-time model is developed, and computational tests show the efficiency of the proposed model.

## 1. INTRODUCTION

In order to improve the overall management level and achieve better business objectives, many companies have built up real-time enterprise resource planning (ERP)–manufacturing execution system (MES)–process control system (PCS) three-layer systems. Scheduling is a main part of MES which links ERP and PCS. Well-organized schedules can achieve better economic performance. Short-term scheduling for refineries is one of the most challenging problems due to the complexity of the production process. Various scheduling models and approaches for refineries have been proposed in recent years. Zhang and Zhu<sup>1</sup> presented a decomposition approach to tackle large-scale overall refinery optimization problems, in which the overall plant model is decomposed into two levels. However, the model precision is hard to guarantee. Göthe-Lundgren et al.<sup>2</sup> described a production planning and scheduling problem in an oil refinery company, which involves the planning and utilization of a production process consisting of one distillation unit and two hydrotreatment units. The aim of the scheduling is to decide which operation mode to run in each processing unit at each time point. Pinto et al.<sup>3,4</sup> formulated the refinery scheduling problem as mixed integer programming (MIP) optimization models and relied on both continuous and discrete time representations, while different operation modes were not taken into consideration. Jia and Ierapetritou<sup>5,6</sup> developed a comprehensive mathematical programming model for crude oil short-term scheduling and the oil refinery operations scheduling. The overall oil refinery scheduling problem is decomposed into three domains including the crude oil unloading and blending, the production unit operations, and the product blending and delivery. Luo and Rong<sup>7</sup> presented a hierarchical approach with two decision levels for short-term scheduling problems in refineries. The optimization model at the upper level and the heuristics and rules adopted in a simulation system at the lower level are presented. However, they did not discuss how to realize the operation modes. Mouret et al.<sup>8</sup> introduced an integrated problem that contained refinery planning and crude oil operations scheduling. The refinery planning problem is regarded as a flow sheet optimization problem with multiple periods during which the refinery system is assumed to

operate in a steady state. Shah and Ierapetritou<sup>9</sup> present a comprehensive integrated optimization model based on continuous-time formulation for the scheduling problem of production units and end-product blending problem. The model incorporates quantity, quality, and logistics decisions related to real-life refinery operations. Besides these specific examples of the research, some excellent reviews have been published.<sup>10–13</sup> There is no doubt that all this research is valuable and meaningful for the development of refinery scheduling. Nonetheless, the realization of operation modes of units was not discussed. Also, their models do not capture the transient behavior between different operation modes.

The implementation of a scheduling result is crucial to a scheduling model. An optimal scheduling scheme makes sense only if it can be realized in real situations. For refinery scheduling models, it is crucial to implement the operation modes for production units. To achieve this goal, the advanced control of production units has to be taken into account in the modeling procedure. In our group's previous research, Lv et al.<sup>14</sup> proposed a new predictive control scheme by using the split ratio of distillate flow rate to that of bottoms as an essential controlled variable. Correspondingly, a new strategy of integrated control and online optimization is developed on a high-purity distillation process. With the strategy, the process achieves its steady state quickly. The proposed strategy has been successfully applied to a gas separation plant for more than three years. Gao et al.<sup>15</sup> proposed an integrated solution framework for refinery scheduling by combining with unitwide model predictive control. Based on the unitwide model predictive control, the finite optimal operating modes for units are realized. A case originated from a refinery in China is studied on the platform of Honeywell UniSim Operations. Multimode scheduling model is developed by the data from the simulation.

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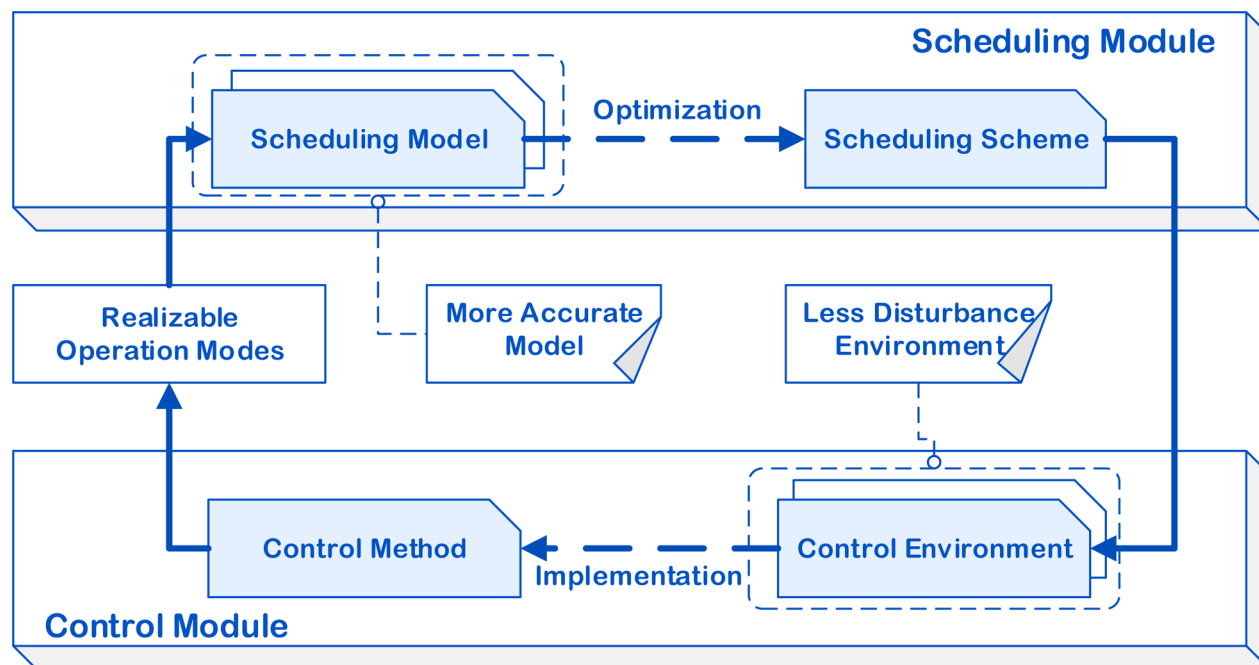


Figure 1. Interaction of scheduling and control.

The progress in unitwide predictive control provides several realizable and industrially meaningful operation modes for production units. Different operation modes result in the diversity of yields of products, key performance indicators, and operational costs. For production units with more than one operation mode, mode switching in the scheduling horizon is inevitable. In many papers, penalties are used to calculate the cost of mode switching in the objective function. However, because of the large inertia characteristic of the continuous process industry, the switching cannot be accomplished immediately. Therefore, operational transitions have to be considered to reflect the time factor in refinery scheduling problems. Mitra et al.<sup>16</sup> presented an optimal production planning for continuous power-intensive processes under time-sensitive electricity prices. They described the transitions between operating modes as transitional modes. The transitional modes help capture ramping behavior and obtain practical schedules.

In this paper, we propose a novel model for the whole refinery based on a predictive control system. The control system lays the basis for the refinery scheduling. On the other hand, a well-organized scheduling scheme can make operation modes better and the implementation of the control method is easier. In addition, an accurate and efficient modeling of transitions of mode switching is described. Compared with the models in previous work, we discuss the concept of operational transitions to allow for a more detailed modeling of transitional behavior that is due to process dynamics. Then the implementation of the obtained scheduling schemes of our model can be guaranteed. Meanwhile, the control method can be executed more easily because the running statuses of units are more stable owing to the optimized scheduling scheme.

This paper is organized as follows: first, the interaction of scheduling and control is introduced in section 2. In section 3, we describe the refinery scheduling problem. The objective and assumptions of the problem are discussed. In section 4, the time presentation is discussed. The whole refinery scheduling model is developed in section 5, and the obtained model is

reformulated in section 6. Some real scheduling cases are solved in section 7. Finally, conclusions are drawn and future developments are presented in section 8.

## 2. INTERACTION OF SCHEDULING AND CONTROL

There are two challenges for refinery scheduling problems. The first one is how to set up realizable and industrially meaningful operation modes. Some scheduling models involve many operation modes to describe the operation process. However, due to large inertia and uncertainty of production units, it is impossible to implement so many accurate modes with very slight differences. The second challenge is that the transition of mode switching resulting from the introduction of multimodes makes the scheduling model more complex to solve. It is necessary to find an efficient way to deal with multimodes and transitions.

To solve the above problems, an interaction of scheduling and control is proposed in this paper, and the structure is presented as follows.

In Figure 1, the control module provides several realizable operation modes to the scheduling model. The operation modes of production units determine the yield, operational cost, and key performance indicators of produced materials. The realizability of operation modes ensures the implementation of the scheduling scheme.

With the help of the unitwide predictive control, more realizable operation modes can be obtained. Therefore, a more practical scheduling model is available. On the other hand, the control method can also be implemented more easily because the running statuses of units are more stable after the adoption of an optimized scheduling scheme. In consequence, a benign circle is formed and the scheduling and control can integrate well as Figure 1 shows.

In our group's previous research, Lv et al.<sup>14,19</sup> proposed a strategy of integrated control and online optimization on high-purity distillation processes. For high-purity distillation processes, there are many difficulties such as long response

time, many unmeasurable disturbances, and the reliability and precision issues of product quality soft sensors. It is hard to achieve good direct product quality control using traditional proportional-integral-differential (PID) or multivariable predictive control techniques. In their research, a predictive control scheme is proposed by using the split ratio of the distillate flow rate to that of the bottoms as an essential controlled variable. Correspondingly, a strategy of integrated control and online optimization is developed on a high-purity distillation process. With the strategy, the process achieves its steady state quickly.

Simulation results of a propylene distillation column on the flow sheet simulation software UniSim Design, which was tuned by practical data, showed that the integrated scheme can improve the purities of propylene and decrease the fluctuation of the purities remarkably for feed composition disturbance. The root-mean-square error (RMSE) and lower limit of the propylene purity are shown in Table 1. The disturbance magnitude of the feed composition is 1.8 percentage points.

**Table 1. RMSE and Lower Limit of the Propylene Purity in the Simulation with the Proposed Strategy<sup>19</sup>**

disturbance type	step	sine
RMSE/%	0.0432	0.0454
lower limit/%	99.073	99.047

The required purity of propylene in the real world is 99.0%. When the soft limit of propylene purity is set to  $99.0\% + 3\sigma$ , where  $\sigma$  is approximated by the RMSE, we have the results shown in Table 2. Compared with those in the PID control

**Table 2. Comparisons between the Proposed Strategy and PID Control Strategy<sup>19</sup>**

	proposed strategy	PID control strategy
purity of propylene/%	99.199	99.691
yield of propylene	0.7486	0.7428
qualified rate/%	100	94.24

strategy, the yield and qualified rate of propylene of the proposed strategy were better and the purity of propylene is closer to the requirement.

On the basis of the implementation of unitwide model predictive control, Gao et al.<sup>15</sup> proposed finite numbers of optimal operating modes of the production units, which make it reasonable to represent refinery scheduling with a multimode model.

Mode switching of multimode production units leads to the operational transitions. In the continuous process industries, as a consequence of the large inertia characteristic, the mode switching cannot be achieved instantly. When an operation mode switches from A to B, a A–B transition is inevitable. Yield and key performance indicators of products have to change from the steady state value in mode A to that of mode B over time in the transition. The products manufactured during transitions are below standard and will affect the succeeding units and their yields. During transitions, the key performance indicators of products will fluctuate, and the accumulated indicators will lie between those in the preceding steady state and the succeeding one. The operation costs in transitions are higher compared with the steady states.

In realistic refineries, atmospheric distillation units (ATM) and vacuum distillation units (VDU) are usually upstream,

while hydrogen desulfurization units (HDS), etherification units (ETH), hydrorefining (HTU), catalytic reformers (RF), and some other units are downstream. On the basis of the production experience of refineries, the transitional time of upstream units is usually longer than that of the downstream ones. As the length of transitional time has a significant effect on the scheduling scheme, the transitions of mode switching have to be considered in the refinery scheduling model. The optimal schedule will ask for as few transitions as possible. The operation environments of processing units will be more smooth. It is helpful to the stability of the control system, and it is a promotion of the accuracy of operation modes in the scheduling model. As the integrated scheme runs on, more effective data will be provided for the scheduling model, and the scheduling model will provide more smooth environments. In consequence, the benign circle is formed and the scheduling module and the control module can integrate well.

In section 3, a specific example of refinery scheduling is introduced.

### 3. PROBLEM STATEMENT

The overall system of a typical refinery is decomposed into three parts as depicted in Figure 2. The first part is crude oil feeding. In this paper, this process is simplified as feeding from crude oil tanks. We assume that the amount of crude oil is sufficient. The second part is the production units including ATM, VDU, fluidized catalytic cracking unit (FCCU), HDS, ETH, RF, MTBE, HTU1, and HTU2. The third part involves the blending and delivery of product oil. There are one kind of crude oil and eight kinds of product oil, including five kinds of gasoline and three kinds of diesel. The product oil is stored in the oil tanks.

The objective is to minimize the total cost of production and penalties.

The variables to be determined are the following:

- sequencing and timing of operation modes of each processing unit
- processing amount of each processing unit at each time
- amount and type of component oil used in blending at each time
- amount and type of product oil being stored or delivered at each time

The given parameters available from external sources are the following:

- operation modes and corresponding transitions of each processing unit
- yield of output material of each processing unit at steady states and transitions
- operational cost of each processing unit at steady states and transitions
- time length of each transition
- key component concentration ranges of product oil
- amount of product oil and delivery time required for each order
- minimum and maximum flow rates
- capacity bounds of tanks
- minimum and maximum ratios of component oil used in blending
- price of crude oil
- price of material storages and penalty of order stockout
- scheduling time horizon

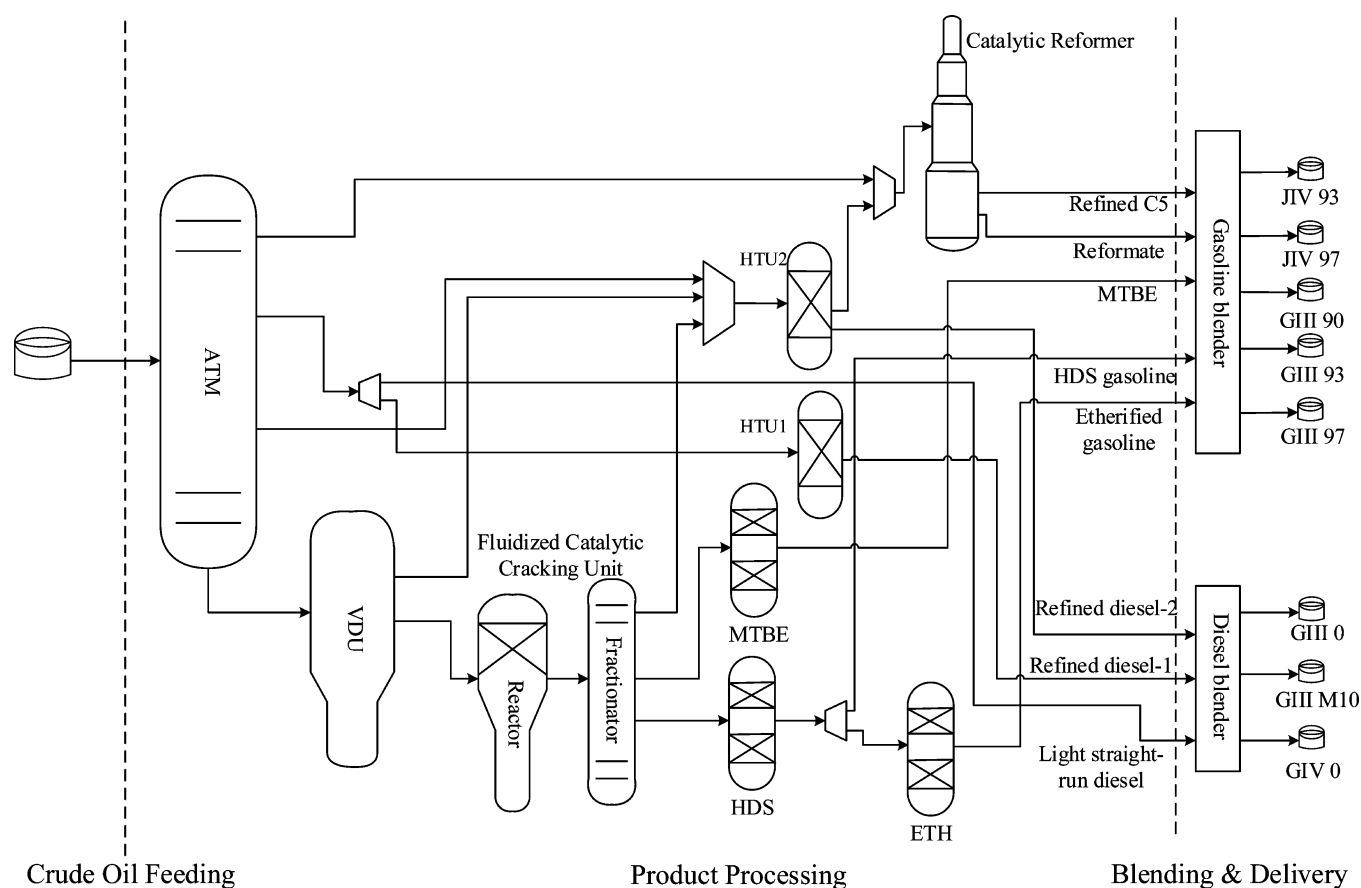


Figure 2. Simplified flow sheet of refinery system.

Different production units have distinct operation mode types. The details are as follows.

(a) **ATM and VDU.** For ATM and VDU, there are two operation modes of the fractionation: gasoline mode (G) and diesel mode (D). The gasoline mode means to get the gasoline fraction as much as possible, and the diesel mode means to get the diesel fraction as much as possible.

(b) **FCCU.** FCCU has two principal parts: reaction and fractionation. The operation modes of the two parts are gasoline mode and diesel mode, which have similarities with ATM and VDU. The gasoline mode means to get the gasoline fraction as much as possible, and the diesel mode means to get the diesel fraction as much as possible. Therefore, FCCU has four operation modes as a combination. The modes are named the gasoline–gasoline mode (GG), the gasoline–diesel mode (GD), the diesel–gasoline mode (DG), and the diesel–diesel mode (DD).

(c) **HDS and ETH.** For HDS, yields and key performance indicators of products are related to the kind of processing material from FCCU. If the operation mode of FCCU switches, the kind of output material from FCCU will change and the processing of HDS will change correspondingly. We define these different processes as different operation modes and name them the same as those of the FCCU.

The process of ETH is similar. Yields and key performance indicators of products are related to the kind of processing material from HDS. We define the operation modes of ETH in the same way as with HDS.

(d) **HTU1 and HTU2.** For HTU1 and HTU2, the processing has two modes: harsh mode (H) and mild mode (M). Compared with the mild mode, the output component oil

of the harsh mode has a lower sulfur concentration and a higher concentration of cetane number (CN). Correspondingly, the operational cost in the harsh mode is larger.

(e) **RF and MTBE.** RF and MTBE have only one operation mode.

The multimodes are shown in Table 3.

Table 3. Operation Modes for Production Units

	unit						
	ATM	VDU	FCCU	ETH	HDS	HTU1	HTU2
modes	G	G	GG	GG	GG	M	M
	D	D	GD	GD	GD	H	H
			DG	DG	DG		
			DD	DD	DD		

Because of the large inertia characteristic of the process, the mode switching of units cannot be achieved instantly. Therefore, if a processing unit changes its operation mode, it will lead to an operational transition between two steady states. During transitions, the key performance indicators of products will fluctuate, and the accumulated indicators will lie between those in the preceding steady state and the succeeding one. The operation costs in transitions are higher compared with the steady states. For transitions with the same preceding and succeeding steady states, the transition times are the same. However, for transitions with different preceding and succeeding steady states, the transition times may be different.

In this paper, we assume that during the transitions, the operational cost, and the yield change uniformly. To simplify



the scheduling mode, the operational cost and the yield are calculated averagely in the transitions.

From the definition above, the operational transitions of FCCU are shown in Table 4 as an example.

**Table 4. Operational Transitions of FCCU**

preceding	succeeding			
	mode GG	mode GD	mode DG	mode DD
mode GG		GG–GD	GG–DG	GG–DD
mode GD	GD–GG		GD–DG	GD–DD
mode DG	DG–GG	DG–GD		DG–DD
mode DD	DD–GG	DD–GD	DD–DG	

#### 4. TIME PRESENTATION

The discrete time representation is used in the scheduling model. Kondili et al.<sup>20</sup> proposed a general discrete time formulation based on the STN representation. The formulation is efficient for the sequence-dependent changeover which involves cleaning times. However, the sequence-dependent changeover did not involve the transition. Mitra et al.<sup>16</sup> presented an optimal production planning for continuous power-intensive processes under time-sensitive electricity prices. They described the transitions between operating modes as transitional modes. There is a cost coefficient for each transition and a transition cost term in the objective function. In our model, the transitions affect the calculation of product yield and operation cost. Therefore, more variables and constraints are needed to indicate whether the unit works in a transition. The description of the scheduling model is more complex.

We use  $y_{u,m,t}$  to represent whether unit  $u$  is in operation mode  $m$  during time interval  $t$ . Figure 3 is an illustration of the operation mode expression.

In Figure 3, unit  $u$  is in operation mode A at time intervals 7, 8, 9 and 10. Therefore

$$y_{u,A,t} = \begin{cases} 1, & t = 7, 8, 9, 10 \\ 0, & \text{otherwise} \end{cases}$$

In the same way

$$y_{u,B,t} = \begin{cases} 1, & t = 1, 2 \\ 0, & \text{otherwise} \end{cases}$$

$$y_{u,C,t} = \begin{cases} 1, & t = 3, 4, 5, 6 \\ 0, & \text{otherwise} \end{cases}$$

When we introduce the transitions in the scheduling model, the additional decision variables are needed and introduced below.

The decision variables  $x_{u,m,m',t}$  denote whether unit  $u$  is in the transition from operation mode  $m$  to  $m'$  during time interval  $t$ . If  $x_{u,m,m',t}$  is equal to 1, it means that unit  $u$  is in the transition from  $m$  to  $m'$  during time interval  $t$ .

Figure 4 shows an illustration of the operation mode expression involving the transitions. The operation modes at each time interval are the same as those in Figure 3. The length of each transition here is two time intervals. We use  $TT_{u,m,m'}$  to represent the length of transition from  $m$  to  $m'$  of unit  $u$ . In this example

$$TT_{u,m,m'} = \begin{cases} 0, & m = m' \\ 2, & m \neq m' \end{cases}$$

During time intervals 3 and 4, the operation mode is in the B–C transition.  $y_{u,C,3}$  and  $y_{u,C,4}$  are set to 1 in this situation. In the same way,  $y_{u,A,7}$  and  $y_{u,A,8}$  are set to 1. Therefore, the values of  $y_{u,m,t}$  are the same as the above in Figure 3.

There are two mode switchings in Figure 4. The operation mode switches from mode B to C at time 3 and from mode C to A at time 7. At time intervals 3 and 4, unit  $u$  are in the B–C transition, so

$$x_{u,B,C,t} = \begin{cases} 1, & t = 3, 4 \\ 0, & \text{otherwise} \end{cases}$$

In the same way

$$x_{u,C,A,t} = \begin{cases} 1, & t = 7, 8 \\ 0, & \text{otherwise} \end{cases}$$

#### 5. PROBLEM FORMULATION

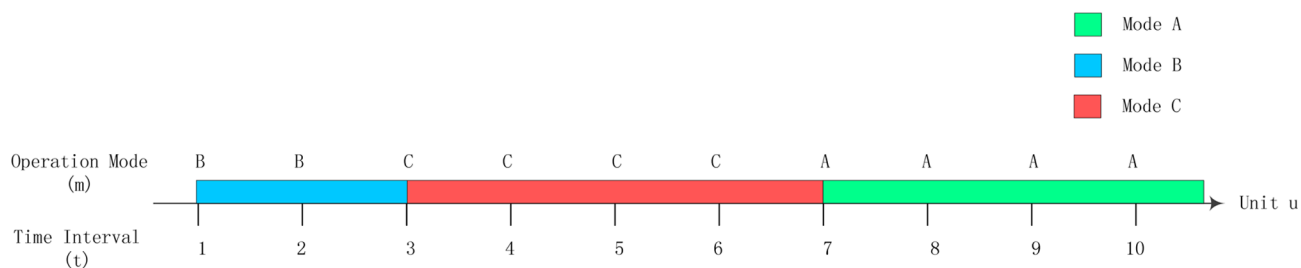
The model can be classified into two sets of constraints. The first set contains constraints for transitions between different operation modes. The second set describes the production constraints involving mass balance, capacity, blending, and delivery.

**5.1. Transition Constraints.** The scheduling model introduced in this paper involves the transitions of mode switching. To describe mode switching and transitions, some special variables and constraints should be introduced in the model. The variables have been discussed in section 4. Here we will present the constraints.

**5.1.1. Operation Mode Variable Constraints.** Only one operation mode can run at any time for any unit.

$$\sum_m y_{u,m,t} = 1, \quad \forall u \in U, t \in T \quad (1)$$

where  $y_{u,m,t}$  indicates whether unit  $u$  is in operation mode  $m$  during time interval  $t$ .



**Figure 3.** Illustration I of the operation mode expression.

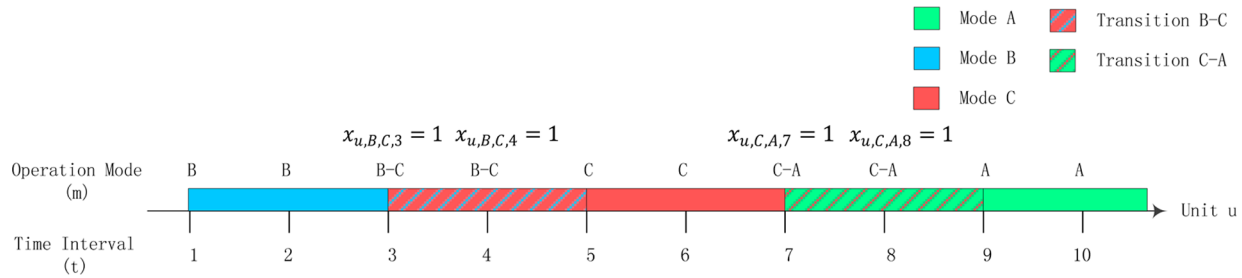


Figure 4. Illustration II of the operation mode expression.

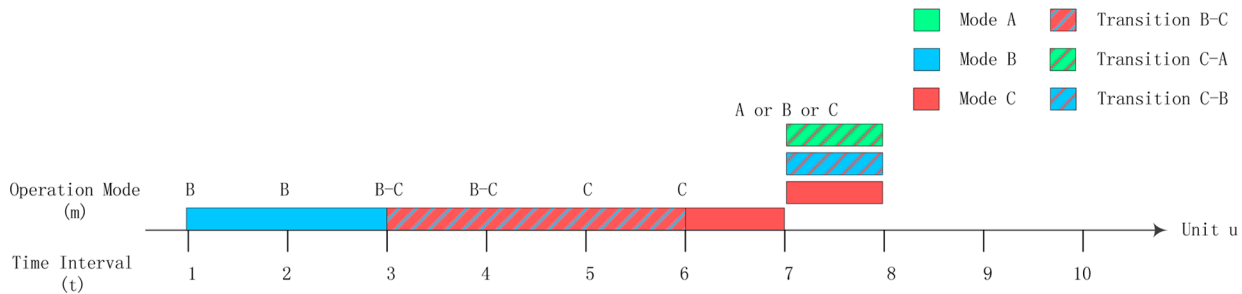


Figure 5. Illustration of minimum stay constraints.

**5.1.2. Transition Variable Constraints.** The relationship between  $x_{u,m,m',t}$  and  $y_{u,m,t}$  is

$$x_{u,m,m',t} \leq y_{u,m,t}, \quad \forall u \in U, t \geq 2, m \in M_u, m' \in M_u \quad (2)$$

$$x_{u,m,m',t} \leq y_{u,m,t-1}, \quad \forall u \in U, t > \text{TT}_{u,m,m'}, m \in M_u, m' \in M_u \quad (3)$$

$$x_{u,m,m',t} \leq y_{u,m,1}, \quad \forall u \in U, 2 \leq t \leq \text{TT}_{u,m,m'}, m \in M_u, m' \in M_u \quad (4)$$

where  $x_{u,m,m',t}$  indicates whether unit  $u$  is in the transition from operation mode  $m$  to  $m'$  during time interval  $t$ .

If  $x_{u,m,m',t}$  is equal to 1, it means that unit  $u$  is in the transition from mode  $m$  to mode  $m'$  during time interval  $t$ . Therefore,  $y_{u,m,t}$  must be equal to 1 and we have constraints 2.

If  $x_{u,m,m',t}$  is equal to 1, there must be a switch in the time period from  $t - \text{TT}_{u,m,m'} + 1$  to  $t$ . Therefore,  $y_{u,m,t-\text{TT}_{u,m,m'}}$  must be equal to 1 and we have constraints 3. If  $2 \leq t \leq \text{TT}_{u,m,m'}$ , constraints 3 turn to constraints 4.

The operation units start in steady states, so the value of  $x_{u,m,m',1}$  is 0.

$$x_{u,m,m',1} = 0, \quad \forall u \in U, m \in M_u, m' \in M_u \quad (5)$$

If  $m$  is equal to  $m'$ ,  $x_{u,m,m',t}$  is equal to 0.

$$x_{u,m,m,t} = 0, \quad \forall u \in U, m \in M_u, t \in T \quad (6)$$

The transition should be complete in the scheduling time horizon. This means that every unit is in a steady state at the last time interval.

$$\sum_m \sum_{m'} x_{u,m,m',T_{\max}} = 0, \quad \forall u \in U, m \in M_u, m' \in M_u \quad (7)$$

**5.1.3. Minimum Stay Constraints.** There cannot be new mode switching before a transition ends.

As the example shows in Figure 5, unit  $u$  is in mode B at times 1 and 2. The operation mode switches from mode B to mode C at

time 3, and the length of the transition is three intervals. Therefore, time intervals 3, 4, and 5 are all in the transition and no new switching can occur. At time interval 6, unit  $u$  is in the steady state of mode C. At time interval 7, unit  $u$  can switch to another operation mode or keep mode C. Therefore, we have the constraint 8:

$$\left[ \begin{array}{c} y_{u,m,t-1} \cdot y_{u,m',t} \\ \prod_{t'=t}^{t+\text{TT}_{u,m,m'}-1} x_{u,m,m',t'} = 1 \end{array} \right] \vee [\neg (y_{u,m,t-1} \cdot y_{u,m',t})] \quad (8)$$

where  $\text{TT}_{u,m,m'}$  indicates the length of the transition from operation mode  $m$  to  $m'$  of unit  $u$ .

If  $(y_{u,m,t-1} \cdot y_{u,m',t})$  is equal to 1, it means that unit  $u$  switches from mode  $m$  to mode  $m'$  at time  $t$ . Then  $x_{u,m,m',t'}$  should be equal to 1 from  $t$  to  $t + \text{TT}_{u,m,m'} - 1$ . If  $(y_{u,m,t-1} \cdot y_{u,m',t})$  is equal to 0, there are no constraints for  $x_{u,m,m',t'}$ .

We can reformulate  $\prod_{t'=t}^{t+\text{TT}_{u,m,m'}-1} x_{u,m,m',t'} = 1$  as  $\sum_{t'=t}^{t+\text{TT}_{u,m,m'}-1} x_{u,m,m',t'} = \text{TT}_{u,m,m'}$ . Then, constraints 8 can be reformulated into the following big  $M$  inequalities.

$$\begin{aligned} -M_1(1 - y_{u,m,t-1} \cdot y_{u,m',t}) &\leq \text{TT}_{u,m,m'} - \sum_{t'=t}^{t+\text{TT}_{u,m,m'}-1} x_{u,m,m',t'} \\ &\leq M_2(1 - y_{u,m,t-1} \cdot y_{u,m',t}) \end{aligned} \quad (9)$$

Since  $\sum_{t'=t}^{t+\text{TT}_{u,m,m'}-1} x_{u,m,m',t'}$  cannot be larger than  $\text{TT}_{u,m,m'}$ ,  $M_1$  in constraints 9 can be 0 and  $M_2$  can be  $\text{TT}_{u,m,m'}$ . Therefore, we have constraints 10.

$$\begin{aligned} \text{TT}_{u,m,m'} \cdot y_{u,m,t-1} \cdot y_{u,m',t} &\leq \sum_{t'=t}^{t+\text{TT}_{u,m,m'}-1} x_{u,m,m',t'} \\ \forall u \in U, m \in M_u, m' \in M_u, t \geq 2 \end{aligned} \quad (10)$$

$y_{u,m,t-1} \cdot y_{u,m',t}$  can be reformulated as  $y_{u,m,t-1} + y_{u,m',t} - 1$  to linearize constraints 10. Finally, we have constraints 11 to express the minimum stay constraints.

$$\text{TT}_{u,m',m} \cdot (y_{u,m,t-1} + y_{u,m',t} - 1) \leq \sum_{t'=t}^{t+\text{TT}_{u,m',m}-1} x_{u,m,m',t'} \quad \forall u \in U, m \in M_u, m' \in M_u, t \geq 2 \quad (11)$$

**5.2. Production Constraints.** The general constraints of refinery scheduling are introduced here. Due to the consideration of transitions, some constraints become more complex than usual (for example, the constraints of mass balance constraints of outflow ports for units).

**5.2.1. Mass Balance Constraints.** **5.2.1.1. Mass Balance Constraints for Outflow Ports of Units.** If the units have more than one operation mode

$$\begin{aligned} \text{QO}_{u,s,t} &= \sum_m \sum_{m'} x_{u,m,m',t} \text{QI}_{u,t} \text{tYield}_{u,s,m,m'} \\ &+ \sum_{m'} y_{u,m',t} (1 - \sum_m x_{u,m,m',t}) \text{QI}_{u,t} \text{Yield}_{u,s,m'} \\ \forall u \in U, t \in T, s \in S \end{aligned} \quad (12)$$

where  $\text{Yield}_{u,s,m'}$  indicates the yield of material leaving port  $s$  of unit  $u$  in the steady state of operation mode  $m'$  and  $\text{tYield}_{u,s,m,m'}$  indicates yield in the transition.

If the unit is in a steady state,  $x_{u,m,m',t}$  is 0; then the term  $\sum_m \sum_{m'} x_{u,m,m',t} \text{QI}_{u,t} \text{tYield}_{u,s,m,m'}$  is equal to 0. We have

$$\text{QO}_{u,s,t} = \sum_{m'} y_{u,m',t} (1 - \sum_m x_{u,m,m',t}) \text{QI}_{u,t} \text{Yield}_{u,s,m'}$$

If the unit is in a transition,  $(1 - \sum_m x_{u,m,m',t})$  is 0 and the term  $\sum_{m'} y_{u,m',t} (1 - \sum_m x_{u,m,m',t}) \text{QI}_{u,t} \text{Yield}_{u,s,m'}$  is equal to 0. We have

$$\text{QO}_{u,s,t} = \sum_m \sum_{m'} x_{u,m,m',t} \text{QI}_{u,t} \text{tYield}_{u,s,m,m'}$$

If the units have only one operation mode, i.e., no mode switching, constraints 12 turn to

$$\text{QO}_{u,s,t} = \text{QI}_{u,t} \text{Yield}_{u,s} \quad \forall u \in U, t \in T, s \in S$$

**5.2.1.2. Mass Balance Constraints for Intermediate Oil.** Intermediate oil contains the output flow of each production unit.

The constraints represent that the sum of output flow of intermediate oil “oi” from upstream units must be equal to the sum of its input flow to downstream units during time interval  $t$ .

$$\sum_u \text{QO}_{u,oi,t} = \sum_u \text{QI}_{u,oi,t'} \quad \forall oi \in \text{OI}, t \in T \quad (13)$$

**5.2.1.3. Mass Balance Constraints for Tanks.** The inventory of each storage tank at the end of time interval  $t$  is equal to the inventory at the end of time interval  $t - 1$  plus the amount of flows entering the tank during time interval  $t$  and minus the amount of flows leaving during time interval  $t$ .

When  $t = 1$

$$\begin{aligned} \text{INV}_{oc,1} &= \text{INV}_{oc,\text{ini}} + \sum_u \text{QI}_{u,oc,1} - \text{QO}_{oc,1} \\ \forall oc \in \text{OC}, u \in U \end{aligned} \quad (14)$$

$$\begin{aligned} \text{INV}_{o,1} &= \text{INV}_{o,\text{ini}} + \text{QI}_{o,1} - \sum_l D_{l,o,1} \\ \forall o \in O, l \in L \end{aligned} \quad (15)$$

When  $t \geq 2$

$$\begin{aligned} \text{INV}_{oc,t} &= \text{INV}_{oc,t-1} + \sum_u \text{QI}_{u,oc,t} - \text{QO}_{oc,t} \\ \forall oc \in \text{OC}, t \geq 2, u \in U \end{aligned} \quad (16)$$

$$\begin{aligned} \text{INV}_{o,t} &= \text{INV}_{o,t-1} + \text{QI}_{o,t} - \sum_l D_{l,o,t} \\ \forall o \in O, t \geq 2, l \in L \end{aligned} \quad (17)$$

The relationships between  $\text{QO}_{oc,t}$  and  $\text{QI}_{o,t}$  are

$$\sum_{oc} \text{Q}_{oc,o,t} = \text{QI}_{o,t}, \quad \forall o \in O, t \in T \quad (18)$$

$$\sum_o \text{Q}_{oc,o,t} = \text{QO}_{oc,t}, \quad \forall oc \in \text{OC}, t \in T \quad (19)$$

**5.2.2. Capacity Constraints.** **5.2.2.1. Capacity Constraints for Units.** The constraints specify that the charge size of the unit  $u$  during time interval  $t$  must satisfy the minimum and maximum capacity.

$$\text{QI}_u^{\min} \leq \text{QI}_{u,t} \leq \text{QI}_u^{\max}, \quad \forall u \in U, t \in T \quad (20)$$

**5.2.2.2. Capacity Constraints for Tanks.** The inventory must lie between its minimum and maximum bounds.

$$\text{INV}_{oc}^{\min} \leq \text{INV}_{oc,t} \leq \text{INV}_{oc}^{\max}, \quad \forall oc \in \text{OC}, t \in T \quad (21)$$

$$\text{INV}_o^{\min} \leq \text{INV}_{o,t} \leq \text{INV}_o^{\max}, \quad \forall o \in O, t \in T \quad (22)$$

**5.2.3. Blending Constraints.** **5.2.3.1. Component Oil Ratio Constraints in Blending.** The component oil used in blending has its minimum and maximum ratios

$$\begin{aligned} r_{oc,o}^{\min} \sum_{oc'} \text{Q}_{oc',o,t} &\leq \text{Q}_{oc,o,t} \leq r_{oc,o}^{\max} \sum_{oc'} \text{Q}_{oc',o,t} \\ \forall oc \in \text{OC}, o \in O, t \in T \end{aligned} \quad (23)$$

**5.2.3.2. Product Property Constraints.** The properties of the product oil must lie between its minimum and maximum bounds. When calculating the product property in the blending process, the research octane number (RON), the cetane number (CN), and the sulfur content of gasoline and diesel are used linear models. By introducing the condensation point factor, the condensation point of blending diesel can also be calculated linearly.<sup>17</sup> The properties of the flows entering the blenders are connected with the preceding production units.

$$\text{PRO}_{o,p}^{\min} \leq \text{PRO}_{o,p,t} \leq \text{PRO}_{o,p}^{\max},$$

$$\forall o \in O, p \in P, t \in T$$

where  $\text{PRO}_{o,p,t} = \sum_{oc} \text{PRO}_{oc,p} \text{Q}_{oc,o,t} / \sum_{oc} \text{Q}_{oc,o,t}$

The constraint can be expressed linearly in an alternative way by multiplying it by  $\sum_{oc} \text{Q}_{oc,o,t}$ :



$$\begin{aligned} \text{PRO}_{o,p}^{\min} \sum_{oc} Q_{oc,o,t} &\leq \sum_{oc} \text{PRO}_{oc,p} Q_{oc,o,t} \\ &\leq \text{PRO}_{o,p}^{\max} \sum_{oc} Q_{oc,o,t}, \quad \forall o \in O, p \in P, t \in T \end{aligned} \quad (24)$$

**5.2.4. Delivery Constraints: Product Oil Supply/Demand Constraints.** Each order has a start time and a due time for the delivery. Product oil cannot be delivered before the start time and after the due time. The stockout penalty is calculated at the end of the scheduling time.

$$D_{l,o,t} \geq 0, \quad \forall l \in L, o \in O, t \in T \quad (25)$$

$$\sum_{t=1}^{T_{l1}-1} D_{l,o,t} = 0, \quad \forall l \in L, o \in O \quad (26)$$

$$\sum_{t=T_{l2}+1}^{T_{\max}} D_{l,o,t} = 0, \quad \forall l \in L, o \in O \quad (27)$$

$$\sum_t D_{l,o,t} \leq R_{l,o}, \quad \forall l \in L, o \in O \quad (28)$$

$D_{l,o,t}$  is the product oil supply amount.  $R_{l,o}$  is the requirement of the product oil.  $T_{l1}$  is the beginning time interval for the delivery.  $T_{l2}$  is the due time interval for the delivery.

**5.3. Objective Function.** The objective function of the scheduling problem is to minimize the cost of production and material storage and penalties for stockout.

$$\begin{aligned} \min f = \min \sum_T & (\text{QI}_{\text{ATM},t} \text{OPC} \\ & + \sum_u \sum_m \sum_{m'} x_{u,m,m',t} \text{QI}_{u,t} \text{tOpCost}_{u,m,m'} \\ & + \sum_u \sum_{m'} y_{u,m',t} (1 - \sum_m x_{u,m,m',t}) \text{QI}_{u,t} \text{OpCost}_{u,m'}) \\ & + \sum_t \alpha (\sum_o \text{INV}_{o,t} + \sum_{oc} \text{INV}_{oc,t}) \\ & + \sum_l \sum_o \beta_l (R_{l,o} - \sum_t D_{l,o,t}) \end{aligned} \quad (29)$$

The first term in the objective function (29) is the cost of crude oil and the operational costs of units in transitions and steady states. The second term is the cost of material storages. The third term is the penalties of order stockout.

The mixed-integer nonlinear programming model is as follows:

(P0):

$$\min f$$

s.t. constraints 1–7 and 11–28.

## 6. LINEARIZATION

The MINLP model (P0) involves bilinear and trilinear terms. They are the products of two binary variables or the products of a binary variable and a continuous variable. You and Grossmann<sup>18</sup> presented a reformulation method for these kinds of bilinear terms by introducing additional variables. We reformulate similarly as follows.

According to P0, the constraints 12 and the objective function 29 involve the same bilinear and trilinear terms.

The bilinear terms are

$$x_{u,m,m',t} \text{QI}_{u,t}$$

$x_{u,m,m',t}$  is a binary variable and  $\text{QI}_{u,t}$  is a continuous variable.

The trilinear terms are

$$y_{u,m',t} (1 - \sum_m x_{u,m,m',t}) \text{QI}_{u,t}$$

By the definition of  $x_{u,m,m',t}$ , if  $x_{u,m,m',t}$  is equal to 1, it means that unit  $u$  is in the transition from mode  $m$  to mode  $m'$ . Because the operation mode in  $t-1$  is unique, the value of  $\sum_m x_{u,m,m',t}$  is no more than 1. Therefore,  $(1 - \sum_m x_{u,m,m',t})$  can be viewed as a binary variable.

$y_{u,m',t}$  is a binary variable and  $\text{QI}_{u,t}$  is a continuous variable.

We reformulate the two kinds of terms respectively as follows.

**6.1. Bilinear Terms.** The bilinear terms are the product of a binary variable  $x_{u,m,m',t}$  and a continuous variable  $\text{QI}_{u,t}$ . We introduce two auxiliary continuous variables  $\text{xQI}_{u,m,m',t}$  and  $\text{xQI1}_{u,m,m',t}$  and the following auxiliary constraints to realize the linearization.

$$\begin{aligned} \text{xQI}_{u,m,m',t} + \text{xQI1}_{u,m,m',t} &= \text{QI}_{u,t}, \\ \forall u \in U, m \in M_u, m' \in M_u, t \in T \end{aligned} \quad (30)$$

$$\begin{aligned} \text{xQI}_{u,m,m',t} &\leq x_{u,m,m',t} \text{QI}_{u,t}^{\max}, \\ \forall u \in U, m \in M_u, m' \in M_u, t \in T \end{aligned} \quad (31)$$

$$\begin{aligned} \text{xQI1}_{u,m,m',t} &\leq (1 - x_{u,m,m',t}) \text{QI}_{u,t}^{\max}, \\ \forall u \in U, m \in M_u, m' \in M_u, t \in T \end{aligned} \quad (32)$$

$$\text{xQI}_{u,m,m',t} \geq 0, \quad \forall u \in U, m \in M_u, m' \in M_u, t \in T \quad (33)$$

$$\begin{aligned} \text{xQI1}_{u,m,m',t} &\geq 0, \\ \forall u \in U, m \in M_u, m' \in M_u, t \in T \end{aligned} \quad (34)$$

The  $\text{QI}_{u,t}^{\max}$  terms in constraints 31 and 32 are the parameters that define the maximum value of  $\text{QI}_{u,t}$ .

The constraints 31–34 ensure that if  $x_{u,m,m',t}$  is 0,  $\text{xQI}_{u,m,m',t}$  should be 0; if  $x_{u,m,m',t}$  is 1,  $\text{xQI1}_{u,m,m',t}$  should be 0. Combining with the constraints 30, we can have  $\text{xQI}_{u,m,m',t}$  equivalent to the product of  $x_{u,m,m',t}$  and  $\text{QI}_{u,t}$ .

**6.2. Trilinear Terms.** The trilinear terms are the product of two binary variables,  $y_{u,m',t}$  and  $(1 - \sum_m x_{u,m,m',t})$ , and a continuous variable,  $\text{QI}_{u,t}$ .

To try to linearize the trilinear terms, first we use an auxiliary binary variable  $\text{xy}_{u,m',t}$  to express the product of  $y_{u,m',t}$  and  $(1 - \sum_m x_{u,m,m',t})$ .

The auxiliary constraints are as follows.

$$\text{xy}_{u,m',t} \leq y_{u,m',t}, \quad \forall u \in U, m' \in M_u, t \in T \quad (35)$$

$$\begin{aligned} \text{xy}_{u,m',t} &\leq 1 - \sum_m x_{u,m,m',t}, \\ \forall u \in U, m' \in M_u, t \in T \end{aligned} \quad (36)$$

$$xy_{u,m',t} \geq y_{u,m',t} + (1 - \sum_m x_{u,m,m',t}) - 1, \quad \forall u \in U, m' \in M_u, t \in T \quad (37)$$

$$xy_{u,m',t} \geq 0, \quad \forall u \in U, m' \in M_u, t \in T \quad (38)$$

The constraints 35, 36, and 38 ensure that if  $y_{u,m',t}$  or  $(1 - \sum_m x_{u,m,m',t})$  is 0,  $xy_{u,m',t}$  should be 0, while constraints 37 ensure that if  $y_{u,m',t}$  and  $(1 - \sum_m x_{u,m,m',t})$  are both equal to 1,  $xy_{u,m',t}$  should be 1.

Then another two auxiliary continuous variables,  $xyQI_{u,m',t}$  and  $xyQI_{u,m',t}$  are used to linearize the product of  $xy_{u,m',t}$  and  $QI_{u,t}$ . The auxiliary constraints are

$$xyQI_{u,m',t} + xyQI_{u,m',t} = QI_{u,t}, \quad \forall u \in U, m' \in M_u, t \in T \quad (39)$$

$$xyQI_{u,m',t} \leq xy_{u,m',t} QI_{u,t}^{\max}, \quad \forall u \in U, m' \in M_u, t \in T \quad (40)$$

$$xyQI_{u,m',t} \leq (1 - xy_{u,m',t}) QI_{u,t}^{\max}, \quad \forall u \in U, m' \in M_u, t \in T \quad (41)$$

$$xyQI_{u,m',t} \geq 0, \quad \forall u \in U, m' \in M_u, t \in T \quad (42)$$

$$xyQI_{u,m',t} \geq 0, \quad \forall u \in U, m' \in M_u, t \in T \quad (43)$$

The  $QI_{u,t}^{\max}$  terms in constraints 40 and 41 are the same as the terms in constraints 31 and 32.

The constraints 40–43 ensure that if  $xy_{u,m',t}$  is 0,  $xyQI_{u,m',t}$  should be 0; if  $xy_{u,m',t}$  is 1,  $xyQI_{u,m',t}$  should be 0. Combining with the constraints 39, we can have  $xyQI_{u,m',t}$  equivalent to the product of  $xy_{u,m',t}$  and  $QI_{u,t}$ .

Therefore, the calculations of processing flow in the mass balance constraints of outflow ports for units and the objective function can be rewritten as

$$QO_{u,s,t} = \sum_m \sum_{m'} xyQI_{u,m,m',t} tYield_{u,s,m,m'} + \sum_{m'} xyQI_{u,m',t} Yield_{u,s,m'} \quad (12')$$

and

$$\begin{aligned} \min f' = & \min \sum_T (QI_{ATM,t} OPC \\ & + \sum_u \sum_m \sum_{m'} xQI_{u,m,m',t} tOpCost_{u,m,m'} \\ & + \sum_u \sum_{m'} xyQI_{u,m',t} OpCost_{u,m,m'}) \\ & + \sum_t \alpha (\sum_o INV_{o,t} + \sum_{oc} INV_{oc,t}) \\ & + \sum_l \sum_o \beta_l (R_{l,o} - \sum_t D_{l,o,t}) \end{aligned} \quad (29')$$

Incorporating the above linearizations, we have the following reformulated MILP model (P1).

(P1):

$$\min f'$$

s.t. constraints 1–7, 11, 12', 13–28, and 30–43.

## 7. CASE STUDY

In this section, three cases are generated to test the proposed refinery scheduling model. The cases are based on the flow sheet shown in Figure 2. For all the three cases, the model proposed in this paper (denoted by “SMT”) is solved, and the results are analyzed. For case 1, a scheduling model without considering transitions (denoted by “SMnT”) is also solved to compare with SMT. The scheduling horizons are discretized uniformly with time intervals of 6 h. The cases are solved by CPLEX in GAMS 24.2.2 using a CPU Intel Xeon CPU E5-2609 v2 @2.5 GHz with RAM 32.0 GB.

Sizes of the three cases are shown in Table 5. From Table 5, we can see that case 1 and case 2 have the same number of time

Table 5. Sizes of Cases

case	no. of time intervals	no. of orders
1	8	2
2	8	3
3	10	2

intervals, while case 1 and case 3 have the same number of orders.

The key component concentration ranges of product oil are shown in Appendix B.

**7.1. Case 1.** The orders of product oil for case 1 are in Table 6. The statistics and solutions are shown in Table 7.

In Table 6, the difference between the demands of the two orders is not remarkable. The production schedule of units for case 1 is shown in Figure 6. In the Gantt chart, the duration of processing is represented as a rectangle. The rates of input flow and the operation mode are denoted above and below the rectangles, respectively. From Figure 6, we can see that the operation modes of production units keep the same one in the scheduling horizon. Only the amounts of input flow were changed to satisfy the different requirements of the two orders.

The Gantt chart for the operations of blenders in case 1 is shown in Figure 7. Rates of blending amounts are denoted in the rectangles.

The optimal schedule obtained by SMnT is shown in Figure 8. From Figure 8 we can see that the operation modes of FCCU, HDS, and ETH were changed four times. That is quite frequent. Besides, the mode switching of FCCU at times 18 and 36 h cannot be realized in actual operation because the operation just switched at times 12 and 30 h. The FCCU needs a transition of two time intervals (12 h) in mode switching (see Appendix C). Also, even if the mode switching can be realized, the processing yields in transitions are lower than those in steady states. Consequently, the product amounts of intermediate oil will be less than those in the schedule and may cause the order stockout. In a word, the schedule without considering the transitions will depart from the optimal, even a feasible one.

**7.2. Case 2.** The orders of product oil for case 2 are in Table 8. The statistics and solutions are shown in Table 9.

From Table 8, we can see that orders 1 and 3 involve more demand for gasoline and order 2 needs more diesel. As a result of adding an order, the numbers of constraints and variables are larger than those in case 1. Also, the consumed CPU time is quite longer.

The production schedule for case 2 is shown in Figure 9. The transitions are denoted as gray rectangles. From Figure 9, we can see that both the operation modes and the input flow

Table 6. Orders of Product Oil (Date and Quantity) for Case 1

order	$T_{11}$	$T_{12}$	gasoline/ton						diesel/ton	
			JIV93	JIV97	GIII90	GIII93	GIII97	GIII0	GIIM10	GIV0
1	1	6	100	400	200	100	550	1000	600	700
2	3	8	200	300	100	100	700	800	700	900

Table 7. Statistics and Solutions of SMT for Case 1

no. of constraints	7136
no. of variables	6460
no. of binary variables	584
objective value/¥	59,934,488.1
absolute gap	0
CPU time/s	863.9
no. of nodes	129 782

amounts are changed during the scheduling time horizon to satisfy the demand of orders. The operation modes of FCCU, HDS, and ETH are switched from mode DG to mode DD at time 12 h. Also, they switched from mode DD to mode DG at time 30 h. In the prior intervals of time horizon, more gasoline is produced for order 1. By the first mode switching, the units turn to produce more diesel to satisfy order 2. In the later intervals, more gasoline is produced for order 3.

The Gantt chart for the operations of blenders for case 2 is shown in Figure 10.

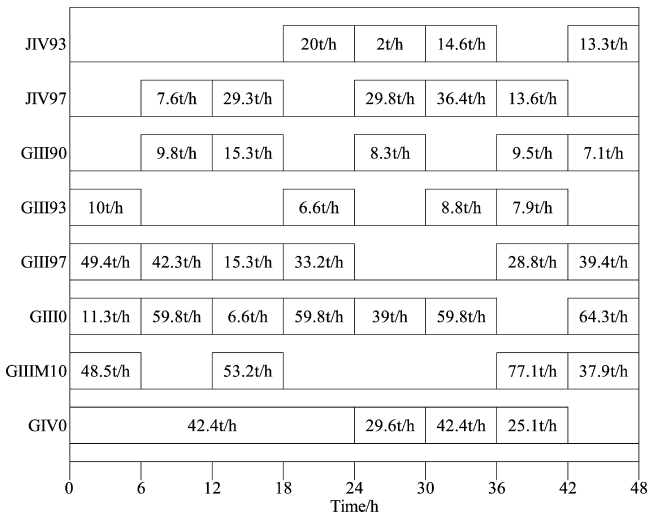


Figure 7. Gantt chart for the operations of blenders obtained by the model SMT for case 1.

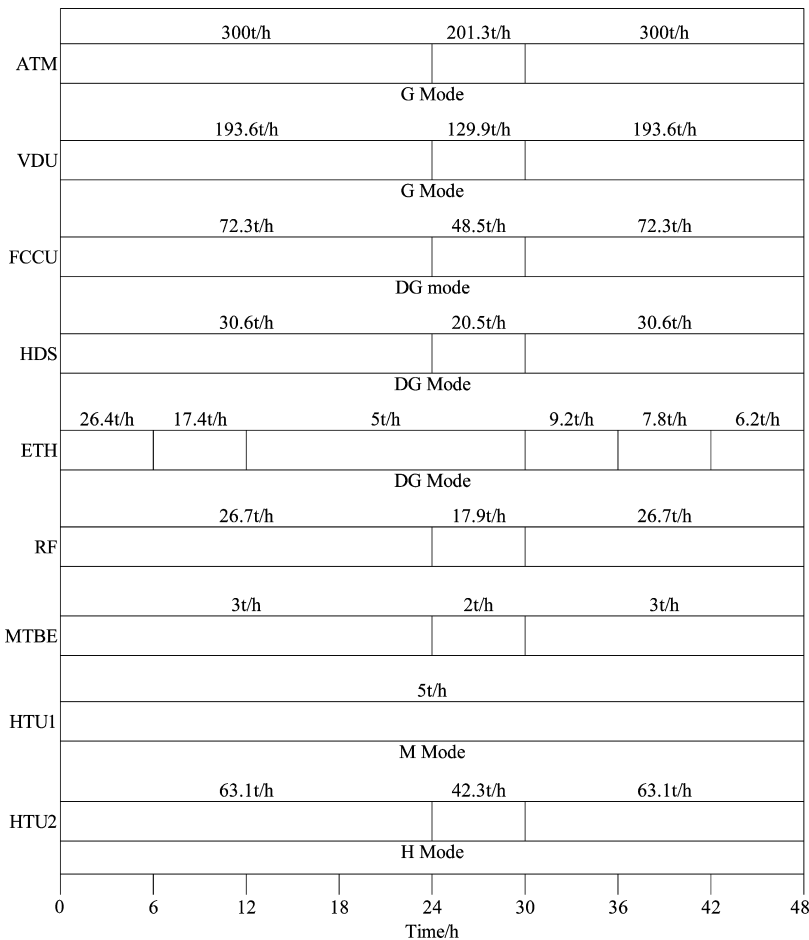


Figure 6. Production schedule of units obtained by the model SMT for case 1.

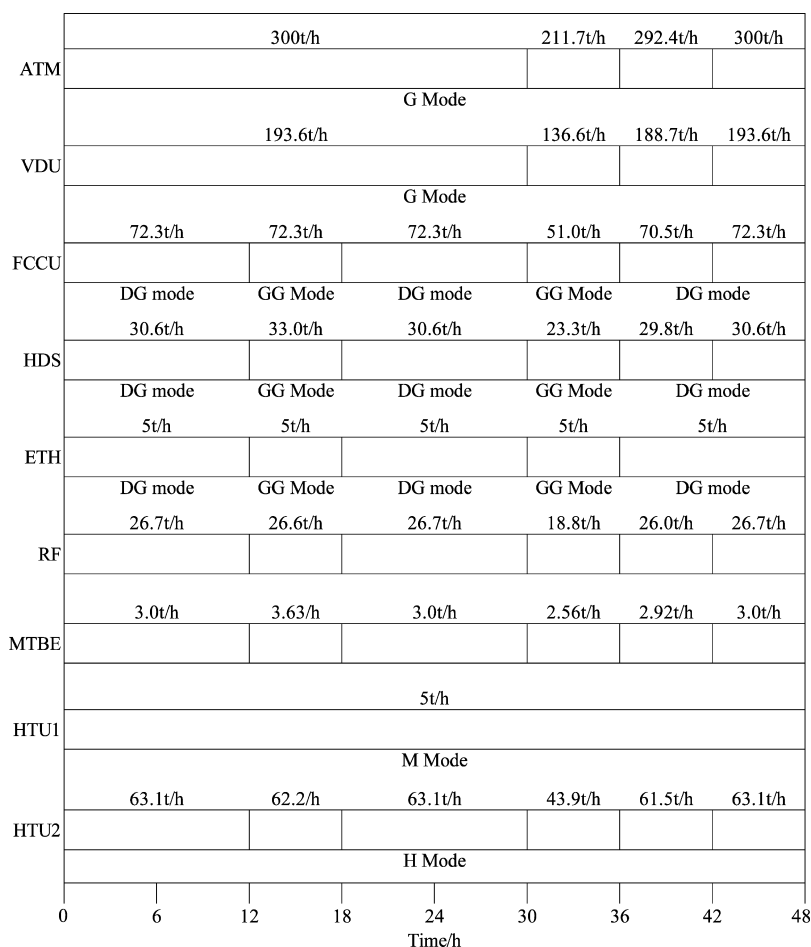


Figure 8. Production schedule of units obtained by the SMnT for case 1.

Table 8. Orders of Product Oil (Date and Quantity) for Case 2

order	$T_{l1}$	$T_{l2}$	gasoline/ton					diesel/ton		
			JIV93	JIV97	GIII90	GIII93	GIII97	GIII0	GIIIM10	GIV0
1	1	4	100	300	100	100	450	150	500	700
2	3	6	50	100	0	0	300	300	600	1000
3	5	8	150	300	0	100	500	150	500	800

Table 9. Statistics and solutions of SMT for Case 2

no. of constraints	7168
no. of variables	6532
no. of binary variables	584
objective value/¥	58,529,326.8
absolute gap	0
CPU time/s	4976.5
no. of nodes	4 769 804

**7.3. Case 3.** The orders of product oil for case 3 are in Table 10. The statistics and solutions are shown in Table 11.

From Table 10, we can see that the main components in order 1 are JIV97 and GIII97 gasoline while those in order 2 are GIII0, GIIIM10, and GIV0 diesel. In Table 11, with the number of time interval increases, the numbers of constraints and variables are larger than in the above two cases. The consumed CPU time is longer than those in case 1 and case 2.

The production schedule for case 3 is shown in Figure 11. From Figure 11, we can see that the operation modes of FCCU, HDS, and ETH were switched from mode GG to mode DG.

Owing to the wide differences of gasoline and diesel amounts in the two orders, the mode switchings are early and the end times of transitions are near the start time for delivery of order 2.

The Gantt chart for the operations of blenders for case 3 is shown in Figure 12.

From the three cases above, some conclusions are drawn about the transitions in the scheduling schemes.

1. In case 1, the demanded kinds and amounts of order 1 are similar to those of order 2, and the operation modes of units do not change during the time horizon. The reason is that the operational costs in transitional processes are higher and the yields are lower than those in steady processes. If the transitions were not taken into consideration in the model, the operation modes of FCCU, HDS, and ETH were changed four times and the obtained schedule would departure from the real optimal one.

2. In case 2, the demanded kinds and amounts of order 1 and order 3 are similar to each other, while the order 2 demand for more diesel and less gasoline compared with the other two orders. In the schedule, the operation modes of FCCU, HDS, and ETH changed twice. Similarly with the conclusion in case

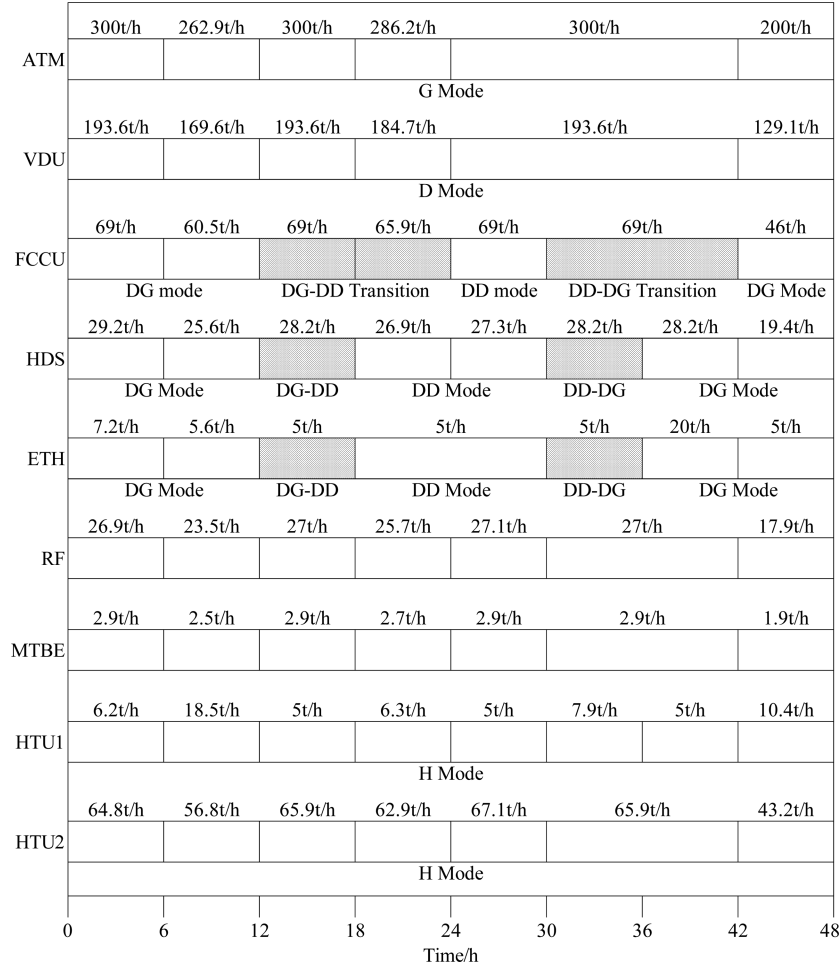


Figure 9. Production schedule of units obtained by the model SMT for case 2.

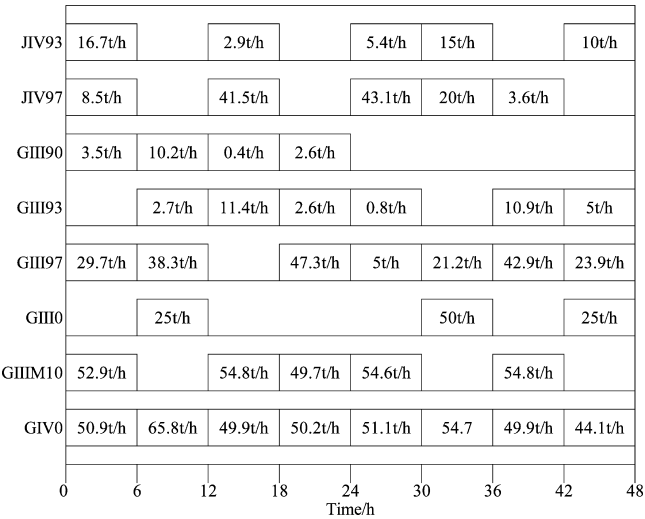


Figure 10. Gantt chart for the operations of blenders obtained by the model SMT for case 2.

Table 11. Statistics and Solutions of SMT for Case 3

no. of constraints	8940
no. of variables	8070
no. of binary variables	752
objective value/¥	69,816,226.5
absolute gap	0
CPU time/s	6830.0
no. of nodes	1 266 925

1, the operation modes of ATM, VDU, HTU1, and HTU2 did not change for the cost of transitions.

3. In case 3, the demands of order 1 and order 2 are significantly different. Order 1 required much more gasoline and less diesel than order 2. In the obtained schedule, the operation modes of FCCU and HDS switched early and the end times of transitions are close to the start time for delivery of order 2. While the yields of steady processes are higher than those of the transitional processes, earlier switches of operation modes can ensure the steady producing state for order 2 and

Table 10. Orders of Product Oil (Date and Quantity) for Case 3

order	$T_{l1}$	$T_{l2}$	gasoline/ton					diesel/ton		
			JIV93	JIV97	GIII90	GIII93	GIII97	GIII0	GIIIM10	GIV0
1	1	7	300	300	200	700	800	500	500	400
2	4	10	200	300	200	100	100	1000	1500	1500



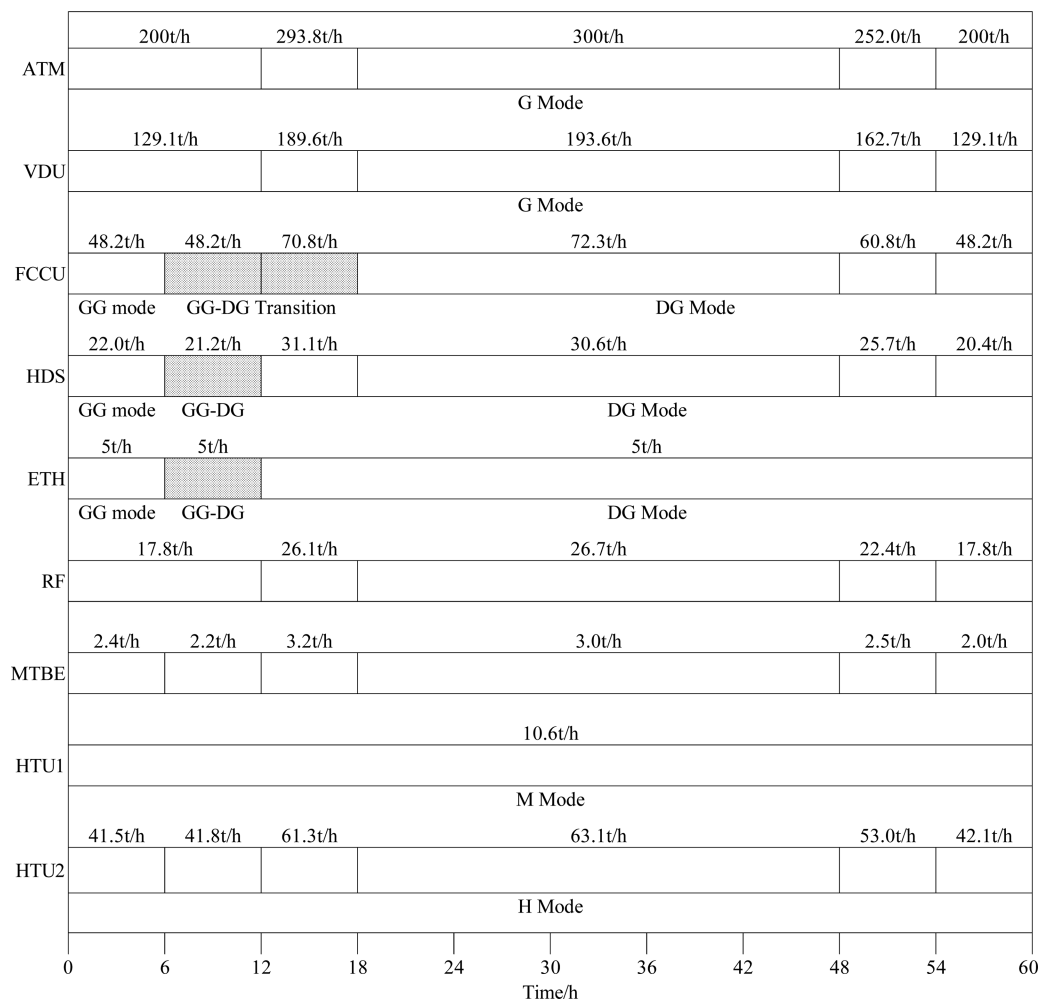


Figure 11. Production schedule of units obtained by the model SMT for case 3.

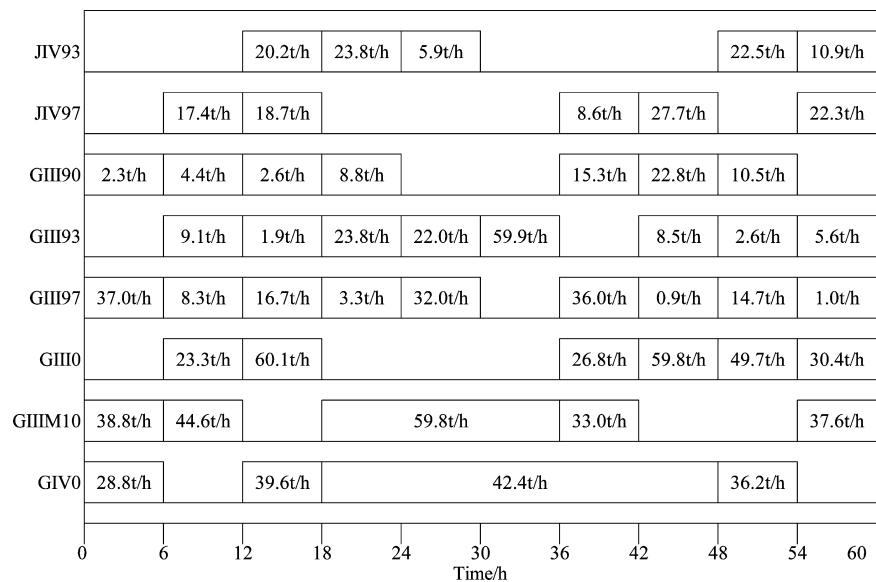


Figure 12. Gantt chart for the operations of blenders obtained by the model SMT for case 3.

produce more diesel to satisfy the demands. Also it has to be noted that the operation modes of ATM, VDU, etc. do not switch because of the cost of transitions.

Thus it can be seen that taking transitions into consideration influences the scheduling schemes in both cost and time factors.

Adding the description of transitions of mode switching in the scheduling model can reflect the dynamic nature of production units more accurately. The operation modes and amounts of production units were optimized according to the demand of orders.

Table 12. Yields and Operation Costs of ATM and VDU

mode	ATM				VDU		oper cost/(KgEo/t)
	SRD/%	LD/%	HD/%	AR/%	VHD/%	VR/%	
G	7.008	15.349	8.109	64.534	11.286	37.352	11
D	4.576	19.403	9.267	61.754	12.580	35.652	11.5

## 8. CONCLUSION

In this paper, a novel discrete-time scheduling model for refineries involving transitions of mode switching under the predictive control system is proposed to effectively handle the specific challenges in recent refinery scheduling problems. The focus of this paper is not only on the scheduling optimization itself, but also to obtain an efficient deterministic formulation that can handle transitional process behavior and realize the schedules. More realizable operation modes can be obtained with the help of the unitwide predictive control. Also, the implementation of an optimized scheduling scheme leads to less disturbance environment of control. If the transitions are not considered, the time and cost factors of mode switching will be ignored and lead to the infeasibility of scheduling schemes. Involving transitions of mode switching in the scheduling model can reflect the dynamic nature of production units more accurately. The effectiveness of the proposed scheduling model is validated by three cases of a typical refinery, and the effectiveness of transitions has been proved by a comparison. The operation modes and amounts of production units are adjusted according to the demand of orders. Because of the scheduling problems are large-scale and consume long CPU times, some decomposition algorithms are needed and will be the subject of future publications.

## ■ APPENDIX

### A. Yields and Processing Costs

The yields and operation costs of the processing units are listed in Tables 12–16. Operating costs are given in KgEo/t; 1 KgEo is equal to 10000 kcal.

Table 13. Yields and Operation Costs of FCCU

mode	FCCU			
	gasoline/%	diesel/%	rich gas/%	oper cost/(KgEo/t)
GG	45.664	22.216	5.02	58
GD	42.583	23.104	5.01	57
DG	42.261	23.418	4.15	56.5
DD	39.580	26.683	4.13	56

Table 14. Yields and Operation Costs of HDS and ETH

mode	HDS		ETH	
	gasoline/%	oper cost/(KgEo/t)	gasoline/%	oper cost/(KgEo/t)
GG	86.2	27.18	93.2	47.6
GD	79	28.98	90	49.56
DG	97	24.48	98.1	44.66
DD	88	26.73	94.1	47.11

Table 15. Yields and Operation Costs of HTU1 and HTU2

mode	HTU1		HTU2		
	diesel/%	oper cost/(KgEo/t)	diesel/%	naphtha/%	oper cost/(KgEo/t)
H	99.4	9	89	9	11
M	99.4	8	93	4	10

Table 16. Yields and Operation Costs of RF and MTBE

RF			MTBE	
gasoline/%	CS/%	oper cost/(KgEo/t)	MTBE/%	oper cost/(KgEo/t)
90	10	83	120	13.84

Table 17. Requirements for Property Values of Gasoline

gasoline	property	
	RON	sulfur content/%
JIV93	≥93	≤0.005
JIV97	≥97	≤0.006
GIII90	≥90	≤0.015
GIII93	≥93	≤0.015
GIII97	≥97	≤0.015

Table 18. Requirements for Property Values of Diesel

diesel	property		
	CN	sulfur content/%	condensation point factor
GIII0	≥49	≤0.035	1.6184
GIIIM10	≥49	≤0.035	1.1995
GIV0	≥51	≤0.01	1.6184

Table 19. Parameters of the Scheduling Model

parameter	value
$Q_{ATM}^{min}$	200 t/h
$Q_{ATM}^{max}$	300 t/h
$Q_{ETH}^{min}$	5 t/h
$Q_{HTU1}^{min}$	5 t/h
OPC	3882 ¥/t
$\alpha$	500
$\beta_1$	30 000
$r_{MTBE,o}^{max}$	0.1
$TT_{ATM,m,m'}, TT_{VDU,m,m'}$	3 time intervals
$TT_{FCCU,m,m'}$	2 time intervals
$TT_{HDS,m,m'}, TT_{ETH,m,m'}, TT_{HTU1,m,m'}, TT_{HTU2,m,m'}$	1 time intervals

### B. Concentration Ranges of Product Oil

Key component concentration ranges of the product oil are given in Tables 17 and 18.

### C. Parameters of Scheduling Model

The parameters of the scheduling model in section 7 are given in Table 19.

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### Notes

The authors declare no competing financial interest.

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## ■ NOTATION

### Sets/Indices

$M_u$  = set of operation modes for unit  $u$  with indices  $m$  and  $m'$   
 $T$  = set of time intervals being scheduled with index  $t$   
 $U$  = set of production units with index  $u$   
 $S$  = set of unit ports with index  $s$   
 $OI$  = set of intermediate oil with index  $oi$   
 $OC$  = set of component oil for blending with index  $oc$   
 $O$  = set of product oil with index  $o$   
 $L$  = set of orders with index  $l$   
 $P$  = set of properties with index  $p$

### Binary Variables

$x_{u,m,m',t}$  = 1 if unit  $u$  is in the transition from operation mode  $m$  to  $m'$  during time interval  $t$   
 $y_{u,m,t}$  = 1 if unit  $u$  is in operation mode  $m$  during time interval  $t$

### Continuous Variables

$INV_{oc,t}$  = inventory of component oil  $oc$  at the end of time interval  $t$   
 $INV_{o,t}$  = inventory of product oil  $o$  at the end of time interval  $t$   
 $PRO_{o,p,t}$  = value of property  $p$  for product oil  $o$  during time interval  $t$   
 $QI_{u,t}$  = input flow of unit  $u$  during time interval  $t$   
 $QO_{u,s,t}$  = output flow of port  $s$  of unit  $u$  during time interval  $t$   
 $QI_{u,oi,t}$  = input flow of intermediate oil  $oi$  of unit  $u$  during time interval  $t$   
 $QO_{u,oi,t}$  = output flow of intermediate oil  $oi$  of unit  $u$  during time interval  $t$   
 $QI_{u,oc,t}$  = input flow of component oil  $oc$  from unit  $u$  during time interval  $t$   
 $QO_{oc,t}$  = output flow of component oil  $oc$  during time interval  $t$   
 $D_{l,o,t}$  = delivery of product oil  $o$  for order  $l$  during time interval  $t$   
 $Q_{oc,o,t}$  = blending flow from component oil  $oc$  to product oil  $o$  during time interval  $t$   
 $QI_{o,t}$  = input flow of product oil  $o$  during time interval  $t$

### Auxiliary Binary Variable

$xy_{u,m',t}$  = product of  $y_{u,m',t}$  and  $(1 - \sum_m x_{u,m,m',t})$

### Auxiliary Continuous Variables

$xQI_{u,m,m',t}$  = product of  $x_{u,m,m',t}$  and  $QI_{u,t}$   
 $xQI1_{u,m,m',t}$  = auxiliary variable for linearization  
 $xyQI_{u,m',t}$  = the product of  $xy_{u,m',t}$  and  $QI_{u,t}$   
 $xyQI1_{u,m',t}$  = auxiliary variable for linearization

### Parameters

$\alpha$  = inventory cost of component oil and product oil per period  
 $\beta_l$  = penalty for stockout of order  $l$  per ton  
 $INV_{oc}^{\min}$  = minimum storage capacity of component oil  $oc$   
 $INV_{oc}^{\max}$  = maximum storage capacity of component oil  $oc$   
 $INV_o^{\min}$  = minimum storage capacity of product oil  $o$   
 $INV_o^{\max}$  = maximum storage capacity of product oil  $o$   
 $INV_{oc,ini}$  = initial storage of component oil  $oc$   
 $INV_{o,ini}$  = initial storage of product oil  $o$   
 $OPC$  = price of crude oil  $c$   
 $OpCost_{u,m}$  = operational cost of unit  $u$  in the steady state of operation mode  $m$   
 $tOpCost_{u,m,m'}$  = operational cost of unit  $u$  in the transition from operation mode  $m$  to  $m'$   
 $PRO_{o,p}^{\min}$  = minimum value of property  $p$  for product oil  $o$

$PRO_{o,p}^{\max}$  = maximum value of property  $p$  for product oil  $o$   
 $PRO_{oc,p}$  = value of property  $p$  for component oil  $oc$   
 $R_{l,o}$  = demand for product oil  $o$  of order  $l$   
 $TT_{u,m,m'}$  = time of the transition from operation mode  $m$  to  $m'$  of unit  $u$   
 $T_{l1}$  = start time interval for delivery of order  $l$   
 $T_{l2}$  = due time interval for delivery of order  $l$   
 $Yield_{u,s,m}$  = yield of material leaving port  $s$  of unit  $u$  in the steady state of operation mode  $m$   
 $tYield_{u,s,m,m'}$  = yield of material leaving port  $s$  of unit  $u$  in the transition from operation mode  $m$  to  $m'$   
 $Yield_{u,s}$  = yield of material leaving port  $s$  of unit  $u$   
 $QI_u^{\min}$  = minimum value of the input flow of unit  $u$   
 $QI_u^{\max}$  = maximum value of the input flow of unit  $u$   
 $r_{oc,o}^{\min}$  = minimum ratio of component oil  $oc$  in blending product oil  $o$   
 $r_{oc,o}^{\max}$  = maximum ratio of component oil  $oc$  in blending product oil  $o$

## ■ REFERENCES

- (1) Zhang, N.; Zhu, X. A Novel Modeling and Decomposition Strategy for Overall Refinery Optimization. *Comput. Chem. Eng.* **2000**, *24* (2–7), 1543–1548.
- (2) Göthe-Lundgren, M.; Lundgren, J. T.; Persson, J. A. An Optimization Model for Refinery Production Scheduling. *Int. J. Prod. Econ.* **2002**, *78* (3), 255–270.
- (3) Pinto, J. M.; Joly, M.; Moro, L. F. L. Planning and Scheduling Models for Refinery Operations. *Comput. Chem. Eng.* **2000**, *24* (9–10), 2259–2276.
- (4) Joly, M.; Moro, L.; Pinto, J. M. Planning and Scheduling for Petroleum Refineries using Mathematical Programming. *Braz. J. Chem. Eng.* **2002**, *19* (2), 207–228.
- (5) Jia, Z.; Ierapetritou, M.; Kelly, J. D. Refinery Short-term Scheduling using Continuous Time Formulation: Crude-oil Operations. *Ind. Eng. Chem. Res.* **2003**, *42* (13), 3085–3097.
- (6) Jia, Z.; Ierapetritou, M. Efficient Short-term Scheduling of Refinery Operations Based on a Continuous Time Formulation. *Comput. Chem. Eng.* **2004**, *28* (6–7), 1001–1019.
- (7) Luo, C.; Rong, G. Hierarchical Approach for Short-term Scheduling in Refineries. *Ind. Eng. Chem. Res.* **2007**, *46* (11), 3656–3668.
- (8) Mouret, S.; Grossmann, I. E.; Pectiaux, P. A New Lagrangian Decomposition Approach Applied to the Integration of Refinery Planning and Crude-oil Scheduling. *Comput. Chem. Eng.* **2011**, *35* (12), 2750–2766.
- (9) Shah, N. K.; Ierapetritou, M. Short-term Scheduling of a Large-scale Oil-refinery Operations: Incorporating Logistics Details. *AIChE J.* **2011**, *57* (6), 1570–1584.
- (10) Grossmann, I. Enterprise-wide Optimization: A New Frontier in Process Systems Engineering. *AIChE J.* **2005**, *51* (7), 1846–1857.
- (11) Grossmann, I. Advances in Mathematical Programming Models for Enterprise-wide Optimization. *Comput. Chem. Eng.* **2012**, *47*, 2–18.
- (12) Méndez, C. A.; Cerdá, J.; Grossmann, I. E.; Harjunkoski, I.; Fahl, M. State-of-the-art Review of Optimization Methods for Short-term Scheduling of Batch Processes. *Comput. Chem. Eng.* **2006**, *30*, 913–946.
- (13) Shah, N. K.; Li, Z.; Ierapetritou, M. G. Petroleum Refining Operations: Key Issues, Advances and Opportunities. *Ind. Eng. Chem. Res.* **2010**, *50*, 1161–1170.
- (14) Lv, W.; Zhu, Y.; Huang, D.; Jiang, Y.; Jin, Y. A New Strategy of Integrated Control and On-line Optimization on High-purity Distillation Process. *Chin. J. Chem. Eng.* **2010**, *18* (1), 66–79.
- (15) Gao, X.; Jiang, Y.; Yu, B.; Qi, L.; Chen, T.; Huang, D. A Step-wise Decision Making Approach to Oil Refinery Scheduling: Integrating Plant-wide Scheduling and Unit-wide Model Predictive Control. *Ind. Eng. Chem. Res.* Submitted for publication.

- (16) Mitra, S.; Grossmann, I. E.; Pinto, J. M.; Arora, N. Optimal Production Planning under Time-sensitive Electricity Prices for Continuous Power-intensive Processes. *Comput. Chem. Eng.* **2012**, *38*, 171–184.
- (17) Hou, X. L. *Chinese Refinery Technology*; China Petrochemical Press: Beijing, 1991; p 570.
- (18) You, F. Q.; Grossmann, I. E. Integrated Multi-Echelon Supply Chain Design with Inventories under Uncertainty: MINLP Models, Computational Strategies. *AIChE J.* **2010**, *56* (2), 419–440.
- (19) Lv, W. Study and Application on Integrated Control and Optimization Strategies for Distillation Process. Ph.D. Dissertation, Tsinghua University, 2010.
- (20) Kondili, E.; Pantelides, C.; Sargent, R. A General Algorithm for Short-term Scheduling of Batch-operations. 1. MILP Formulation. *Comput. Chem. Eng.* **1993**, *17* (2), 211–227.