Optimization of Operations in Supply Chain Systems Using Hybrid Systems Approach and Model Predictive Control

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PROCESS DESIGN AND CONTROL

Optimization of Operations in Supply Chain Systems Using Hybrid Systems Approach and Model Predictive Control

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This paper addresses the optimal operation of multiproduct supply chain systems, using Model Predictive Control (MPC). The supply chain considered in this paper is a hybrid system governed by continuous/discrete dynamics and logic rules. For optimization purposes, it is modeled within the framework of the Mixed Logical Dynamical (MLD) system and the overall profit is optimized through MPC. Dynamic responses of the different nodes of the supply chain are analyzed when the supply chain is subjected to unknown but measurable changes in customer demand. The performances of a centralized decision-making scheme and two types of decentralized decision making schemes are compared.

1. Introduction

A typical supply chain system consists of customers, retailers, distributors, warehouses, production facilities, and suppliers that can be represented as nodes in a highly integrated dynamic network, as shown in Figure 1. Optimization of the performance of supply chain systems includes addressing issues such as forecasting the demand for the product(s), determining production plans and schedules, setting inventory levels at different nodes, determining material transfer amounts among different nodes, and optimizing the overall system operation to achieve the desired profit margins and/or customer satisfaction.

The optimization of supply chain systems involves a variety of decisions at three different levels, with respect to their impact and time scale: the strategic level, the tactical level, and the operational level.[1] The strategic level decisions have long-term impact, spanning two to five years such as the development of a new product and investment in a new facility. The tactical level decisions have medium-term impact, spanning six months to two years, such as capacity increases and the determination of inventory management policies. The operational level decisions have short-term impact, spanning days to several months, such as the shipment quantities among the nodes in the supply-chain network and production schedules. This paper addresses operational level decisions in a supply chain system.

Supply chain systems are traditionally optimized in a decentralized fashion by treating the performance of each node separately, ignoring the dynamic interaction among them. Such an approach results in poor performance of the supply chain system and in infeasibility to meet customer demand. The success of a supply chain network is dependent on its ability to integrate and coordinate the network of business relationships among its participants.[1] Supply chain networks are integrated dynamic systems, and advanced control approaches can be very effective in optimizing the performance of the supply chains, subject to time-varying demand conditions. The optimization models for the operation of supply chain networks should consider the physical constraints that may be present while maintaining a certain degree of robustness against uncertainty in demand. The dynamic nature of supply chain systems was discussed and the need to develop discrete-event simulation models to depict the supply chain dynamics was elaborated by Towill.[2] The applicability of continuous-time differential equations to modeling the dynamic nature of supply chain systems was presented by Riddalls et al.[3]

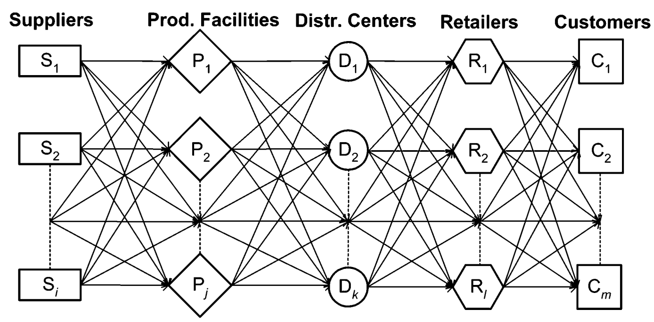

Figure 1. Schematic of a typical supply chain network.

Traditional methods for the analysis of supply chain systems consider individual nodes of the supply chain network separately from each other, because of modeling difficulties and bottlenecks in computational tools. Backx et al.[4] included the manufacturing plant as an integral part of the supply chain. Perea-Lopez et al.[5] addressed decentralized control of a multiple-product supply chain system using classical control laws. They analyzed performance in terms of total cost, customer satisfaction, and demand amplification under dynamical changes in demand. Their work was later extended by proposing a Model Predictive Control (MPC) strategy for optimization of a centralized supply chain using mixed-integer programming models.[6] It was concluded that the centralized control scheme gives more satisfactory results, compared with decentralized control scheme, in terms of overall efficiency of the supply chain. It is also reported that a decentralized MPC implementation for inventory management, under simultaneous demand forecast inaccuracies and plant-model mismatch, stabilizes supply chains and allows nodes to buffer the inventory appropriately before large changes in

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demand happen. MPC methodology was also used to ensure desired customer service level while minimizing average inventory for a single product in the consumer-goods supply chain.[7] An MPC approach was shown to be very effective in the microprocessor manufacturing industry, where a partially decentralized implementation has been discussed.[8] Seferlis and Giannelos[9] presented a centralized control strategy for a two-layered multiechelon network that uses dedicated feedback controllers to maintain inventory levels at predetermined set points at all nodes of the supply chain. In the work of Ryu et al.,[10] the decisions in the operation of supply chain networks have been separated into two categories: production planning in plants and inventory decisions in the distribution network. The resulting problem is solved as a parametric optimization problem. In the work of Bemporad et al.,[11] a general purpose hybrid system model has been given and the applicability of this model for supply chain systems has been discussed. Symbolic techniques and event-based formulations of hybrid system models are shown to result in improved numerical computation schemes.

In this paper, the operation of multiproduct supply chain networks is modeled using the Mixed Logic Dynamical (MLD) systems approach.[12] The discrete dynamic model and switching between different operating conditions represent the nature of multiproduct, multiechelon supply chain systems. The objective of the supply chain system is considered to be maximization of the total value generated. The resulting system of differential, algebraic, and logical equations is used for optimization within the MPC framework under centralized, decentralized, and semi-decentralized information systems.

The problem description, with its underlying dynamic behavior, and the detailed description of the supply chain network nodes, and the formulation of the demand and objective function are given in the next section. The differences between different MPC configurations, which include centralized, decentralized, and semi-centralized approaches, then are discussed in detail. Next, different MPC configurations are illustrated on a multiproduct supply chain. The last section of the paper summarizes the major conclusions.

2. Problem Description

In a typical supply chain network, shown in Figure 1, each product has production system, retailer, distribution center, and warehouse nodes. The production facility is considered to contain a single production line that is capable of producing different products. The link between these different nodes is established by material transfer from an upstream node to a downstream node. The inventory balance equation at node j for product m is given by the following discrete dynamic model, which is obtained after discretizing the continuous inventory balance:

$$I_{jm}(k) = I_{jm}(k-1) + \sum_i Y_{ijm}(k) - \sum_l Y_{jlm}(k) \tag{1}$$

where $Y_{ijm}(k)$ is the material transfer of product m from the upper node i to node j in period k and $Y_{jlm}(k)$ is the material transfer of product m from node j to its lower node l in period k.

Similarly, the order accumulation at node j for product m in period k is defined as $S_{jm}(k)$ and is given as

$$S_{jm}(k) = S_{jm}(k-1) + \sum_l O_{jlm}(k) - \sum_l Y_{jlm}(k) \qquad \forall\, j,m,k \tag{2}$$

where $O_{ilm}(k)$ is the order placed to node j for product m from its lower node l in period k, and $Y_{jlm}(k)$ is the amount of product m transferred from node j to a downstream node l in period k.

In response to an order placed by a downstream node and available inventory levels, the quantities of material that will be transferred to the downstream nodes (i.e., shipping rates) are calculated from

$$Y_{jlm}(k) = a_{jm}I_{jm}(k) + b_{jm}S_{jlm}(k) + c_{jm} \qquad \forall\, j,l,m,k \tag{3}$$

This empirical expression reflects the real behavior observed by companies.[5] The parameters a_{jm}, b_{jm} and c_{jm} are predetermined from the data given in Perea-Lopez et al.[5]

Because a node can transfer material in larger or smaller quantities than that ordered from a downstream node, the slack or excess amount of material transfer is modeled by

$$M^{+}_{jlm}(k) - M^{-}_{jlm}(k) = Y_{jlm}(k) - O_{jlm}(k) \qquad \forall\, j,l,m,k \tag{4}$$

The slack ($M^{-}_{jlm}(k)$) or excess ($M^{+}_{jlm}(k)$) quantities are then used in the calculation of the cost of customer satisfaction for each node of the supply chain network.

Finally, there also exist constraints on the inventory levels, order quantities, and material transfer rates, as given by

$$I^{L}_{jm} \le I_{jm}(k) \le I^{U}_{jm} \qquad \forall\, j,m,k \tag{5}$$

$$O^{L}_{jlm} \le O_{jlm}(k) \le O^{U}_{jlm} \qquad \forall\, j,l,m,k \tag{6}$$

$$Y^{L}_{jlm} \le Y_{jlm}(k) \le Y^{U}_{jlm} \qquad \forall\, j,l,m,k \tag{7}$$

2.1. Production Subsystem Model. The production facility is capable of producing different products, based on the optimal schedule generated by the optimizer. Kallrath[13] and Shah[14] have provided extensive reviews of production systems in process industries. In this section, we provide a production system model for multiproduct supply chains. The plant can only manufacture one type of product in a specific production time, and each product may have a different production time. The plant is designed to produce a quantity less than maximum production capacity for each product, r_m, after the production order for that product is received. The binary variable $P_m(k)$ is the production variable for product m, which has a value of 1 if the plant produces product j in period k and 0 otherwise. To model both the discrete process dynamics and switching between different operating conditions, first propositional logic[15,16] is used to develop the logic rules. Next, the equations and the inequalities that comprise the system model are arranged in MLD form.[12]

The discrete-time formulation of the MLD system allows development of numerically tractable schemes for solving complex problems such as stability,[17] state estimation,[18] and control.[12] In particular, MLD models were proven successful for recasting hybrid dynamic optimization problems, such as this particular problem, into mixed-integer linear and quadratic programming models. The first step in the derivation of the MLD form of a hybrid system is to associate with a logical clause, S, which is a binary variable,[19] such as the $P_m(k)$ production variable in this case. Then elementary statements S_1, ..., S_q are combined in a compound statement via the Boolean operators AND($\wedge$), OR($\vee$), or NOT($\neg$), as given in eq 8 below:

$$\begin{bmatrix} P_m(k) \\ YP_m(k) \le r_m \\ PC_m(k) = (f_m + v_m YP_m(k)) \end{bmatrix} \vee \begin{bmatrix} \neg\, P_m(k) \\ YP_m(k) = 0 \\ PC_m(k) = 0 \end{bmatrix} \tag{8}$$

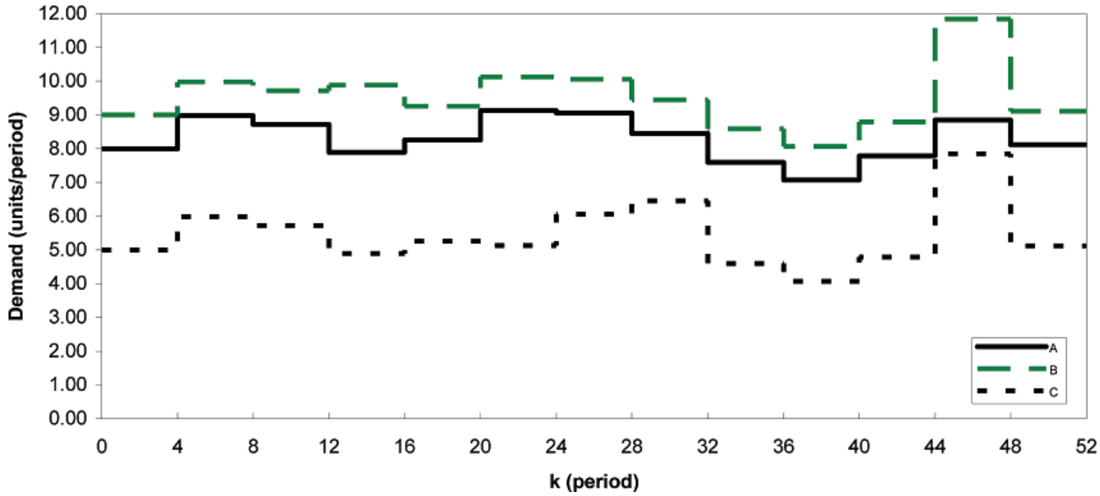

Figure 2. Demand profile for the three-product supply chain system.

where the operator ¬ negates the variable $P_m(k)$ (i.e., if $P_m(k)$ is True, ¬ $P_m(k)$ is False, and vice versa) and the operator ∨ selects one of the conditions. The statement in eq 8 models the condition that either $P_m(k)$ is True or $P_m(k)$ is False. When $P_m(k)$ is True, production occurs with amount $YP_m(k)$ for product m in period k (where r_{m} is the upper bound on the production capacity), and the production cost $PC_m(k)$ is the sum of fixed (f_{m}) and variable costs (υ_{m}). When $P_m(k)$ is False, the production amount and production costs are zero for product m at period k.

The required production time for each product (l_m) can be different, and, after the production of a particular product starts at period k, it is not possible to produce any other products during periods (k, ..., $k + l_m$). The production schedule in the production node is subject to the following logical conditions:

$$P_m(k) \Rightarrow \neg \left(\bigwedge_{m'} \left(\bigwedge_{k'=k,...,k+l_m} P_{m'}(k') \right) \right) \qquad \forall\ m,k \tag{9}$$

The propositional logic expression in eq 9 indicates that whenever the production of product m starts at time period k, the production of any product m', including the product m, cannot start in the time interval [k, ..., $k + l_m$]. The convex hull formulation of the disjunction in eq 8 and algebraic expression of the logical conditions in eq 9 can be derived as follows:[15,16]

$$PC_m(k) = (f_m P_m(k) + \upsilon_m YP_m(k)) \qquad \forall\ m,k \tag{10}$$

$$YP_m(k) \le r_m P_m(k) \qquad \forall\ m \tag{11}$$

$$\sum_m P_m(k) \le 1 \qquad \forall\ k \tag{12}$$

$$\sum_{k'=k}^{k'=k+l_m} P_m(k') + \sum_{m' \neq m} \sum_{k'=k}^{k'=k+l_m} P_{m'}(k') \le 1 \qquad \forall\ k \tag{13}$$

Equation 10 gives the cost of producing product m at time period k including fixed costs f_m and variable costs υ_m. The amount of product m produced at time period k is given by eq 11. Equation 12 models the conditions that, at most, one of the products can be produced, and eq 13 models that, while the plant is producing a product, the production of any product cannot start. When a single production line is not sufficient to satisfy the demand for the products, it may be necessary to have more capacity by adding more lines to the production system. In such a case, the production system model is extended to include multiple production lines by replicating eqs 10−13 for each line.

We define the binary state vector $p(k) \in (0,1)$:

$$p(k) = \begin{bmatrix} P_1(k) \\ P_2(k) \\ \cdot \\ \cdot \\ P_m(k) \end{bmatrix} \tag{14}$$

The linear inequalities 12 and 13 then can be expressed by

$$E_1 p(k) + E_2 p(k+1) + ... + E_q p(k+q) \le e \tag{15}$$

where the matrixes E_i and vector e are dependent on the problem at hand.

2.2. Demand Model. The customer demand has an important role in the supply chain operation, because it introduces uncertainties. We assume that, at current instances, demands are known exactly; i.e., they are measured, but the future demands are uncertain. To incorporate this uncertainty, the demand for each product is modeled in discrete time as random steps:

$$d_i(k+1) = d_i(k) + w(k) \tag{16}$$

where $w(k)$ is a zero-mean random shock $\delta(k)$ that affects the demand value every four time intervals:

$$w(4k) = \delta(4k) \text{ and } w(k) = 0 \text{ otherwise} \tag{17}$$

Figure 2 shows the demand profile that has been used in our optimizations. In the MPC optimization, the demand forecasts are obtained by setting them equal to the current demand value that is available exactly. Such a prediction is optimal because demand is measured exactly and future values of $w(k)$ are random. Thus, the forecasts at measurement time k are computed from

$$\hat{d}_i(k+j|k) = d_{i,m}(k) = d_i(k) \qquad (\text{for } j = 1, 2, ..., P) \tag{18}$$

where $d_{i,m}$ is the measured value of the demand for product i and P is the optimization horizon.

2.3. Mixed Logical Dynamic (MLD) Model. The discrete dynamical equations described above and the inequalities over the continuous and binary variables can be cast into the Mixed Logical Dynamic (MLD) form:

$$x(k+1) = Ax(k) + B_1u(k) + B_2p(k) \tag{19a}$$

$$y(k) = Cx(k) + D_1u(k) + D_2p(k) \tag{19b}$$

$$E_1p(k) + E_2p(k+1) + ... + E_qp(k+q) \leq e \tag{19c}$$

with constraints

$$h^{\mathrm{L}} \leq Hx(k) \leq h^{\mathrm{U}} \tag{20a}$$

$$q^{\mathrm{L}} \leq Qu(k) \leq q^{\mathrm{U}} \tag{20b}$$

$$p(k) \in (0,1) \tag{20c}$$

The state variables ($x(k)$) include the accumulated inventories $I_{jm}(k)$, orders $S_{jm}(k)$, and demand $d_i(k)$. The decision (control) variables are $u(k)$ and $p(k)$: $u(k)$ includes the orders placed ($O_{ijm}(k)$) between the nodes and $p(k)$ includes the discrete decision variables that represent the production schedule in the plant. The optimal control problem given in eq 20 that contains discrete and continuous decision variables is classified as a hybrid system.[12]

2.4. Centralized Optimization Model. For the optimization of the supply chain problem, MPC framework is used to predict the future behavior of the system within a given time horizon and determine the optimal values of decision variables.

Selection of the performance criteria for supply chain optimization problem is very important. The total value generated in the supply chain is one of the most meaningful performance measures, as discussed in the work of Chopra and Meindl.[20] Therefore, the objective in the centralized MPC formulation is the maximization of overall profit, i.e., the difference between the revenue and all costs associated with the activities in the operation of the supply chain network:

$$J = R_{\mathrm{sales}} - C_{\mathrm{inventory}} - C_{\mathrm{transfer}} - C_{\mathrm{purchase}} - C_{\mathrm{production}} - C_{\mathrm{customer\ satisfaction}} \tag{21}$$

We let t to be the current time instant and H be the length of the optimization horizon, respectively. The revenue that will be generated from the future sales then is given by

$$R_{\mathrm{sales}}(t) = \sum_{k=1}^{H-1}\sum_{i=1}^{m} pr_i y_{\mathrm{R}i}(t+k) \tag{22}$$

where pr_i is the unit price of product i, and $y_{\mathrm{R}i}$ is the amount of product i sold by the retailer.

An important cost item is the inventory holding:

$$C_{\mathrm{inventory}}(t) = \sum_{k=1}^{H-1}\sum_{i=1}^{m}\sum_{j=1}^{n} ci_{ji}I_{ji}(t+k) \tag{23}$$

where n is the number of nodes where inventory is held and ci_{ji} is the unit inventory cost of product i at node j.

The cost of transferring material between the nodes and from the retailer node to the customer (C_{transfer}) is given as

$$C_{\mathrm{transfer}}(t) = \sum_{k=1}^{H-1}\sum_{i=1}^{m}\sum_{j=1}^{n} ct_{ji}y_{ji}(t+k) \tag{24}$$

where ct_{ji} is the unit transfer cost of the product i incurred at the node, and y_{ji} is the total amount of product i transferred to all the nodes from node j.

The cost of purchasing (C_{purchase}) includes the unit purchasing cost (cr_m), which is the cost of purchasing all of the raw materials and intermediate products needed to produce one unit of product m:

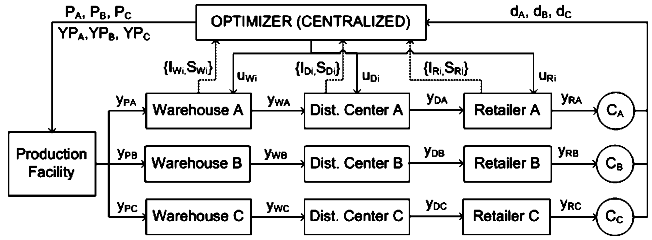

Figure 3. Centralized operation of the three-product supply chain system.

$$C_{\mathrm{purchase}}(t) = \sum_{k=1}^{H-1}\sum_{i=1}^{m} cr_i YP_i(t+k) \tag{25}$$

The production cost, $C_{\mathrm{production}}$, consists of the fixed cost (fc_i), which is incurred each time product i is produced, regardless of the production quantity, and variable cost (vc_i), which is dependent on the amount of production:

$$C_{\mathrm{production}}(t) = \sum_{k=1}^{H-1}\sum_{i=1}^{m} fc_iP_i(t+k) + vc_iYP_i(t+k) \tag{26}$$

A customer satisfaction term ($C_{\mathrm{customer\ satisfaction}}$) is included to penalize for selling less or more than the actual demand d_i to the customer:

$$C_{\mathrm{customer\ satisfaction}} = \sum_{k=1}^{H-1}\sum_{i=1}^{m} cs_i|y_{ri}(t+k) - d_i(t+k)| \tag{27}$$

The customer satisfaction cost is determined by multiplying the absolute value of the difference between the amount of product i sold by the retailer, $y_{\mathrm{R}i}$, and the demand d_i with the unit cost of customer dissatisfaction for product i (cs_i).

At time t, the centralized MPC maximizes the overall profit $J(t)$, subject to the aforementioned cost model described by eq 21, the system dynamics (given in MLD form), and constraints. It implements the first computed control moves (or decision variables) $u(k)$ and $p(k)$; and inventories, orders, and material transfer rates are time-updated. This process is repeated at the next sampling time when new customer demand is measured and demand predictions are updated.

For a three-product supply chain, centralized operation is depicted in Figure 3. The central optimizer (in this case MPC) receives all of the demand information from customers, monitors inventory and order levels at each node, and sets the production schedule of the plant, inventory, and material transfer rates at each node, as shown in Figure 3. The production facility receives the production schedules in the form that t_m represents the period that the production of product m starts with quantity $YP_m(k)$. Because there is no inventory holding capacity in production facility, the products are immediately transferred to the warehouses. The warehouses receive orders, O_{Wm}, and transfer the distribution center the optimized quantities, y_{Wm} for product m. Similarly, the distribution centers receive orders, O_{Dm}, and transfer the retailer the optimized quantities, y_{Dm} for product m. The retailer supplies the customers with the optimized material transfer amounts (y_{Rm}), in response to the demand for product m (d_m).

2.5. Decentralized Supply Chain Optimization Models. Many contemporary supply chain systems consist of independently acting nodes, each seeking to optimize its own local performance, using the information available at each node in a

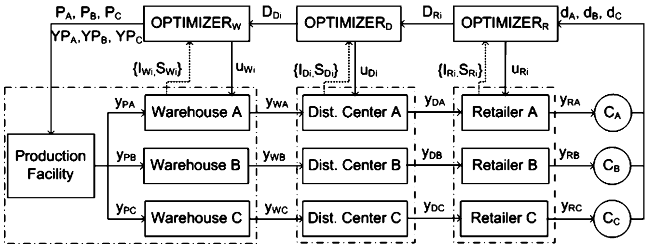

Figure 4. Decentralized operation of the three-product supply chain system.

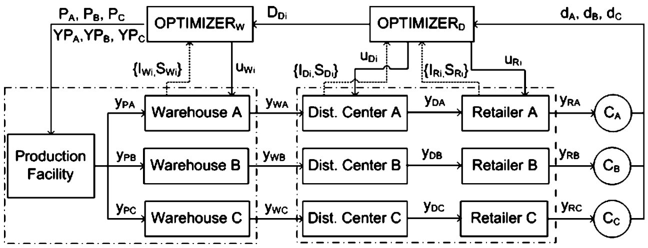

Figure 5. Semi-decentralized operation of the three-product supply chain system.

decentralized fashion. The decentralized scheme includes three decision units: the retailer, the distribution center, and production facility/warehouse as shown in Figure 4. The decision unit for the retailer observes the demand for the products and maximizes the profit for the retailer only. In this process, the retailer determines its optimal demand values (D_{Ri}) for each period in the MPC horizon but places its order for the current period to the distribution center. The decision unit for the distribution center receives the order quantities from the decision unit for the retailers and performs its own optimization. The decision unit for the plant and warehouse decides on the optimal production schedule, material transfer rates, and inventory levels. Each downstream node in the network assumes that the upstream nodes will transfer as much material as demanded during the optimization horizon. However, the actual amounts transferred to retailer and distribution center nodes are determined by the local optimization of the distribution center and plant/warehouse nodes, respectively. This discrepancy is the consequence of decentralized decision making, which results in sub-optimality of the end result.

The semi-decentralized configuration is an intermediate form of integration between the fully decentralized and fully centralized operations previously discussed. The supply chain is decomposed to two major decision units: the first decision unit optimizes the performance of distribution centers and retailers, and the second decision unit optimizes the performance of the production/warehouse facility, as shown in Figure 5. The first decision unit observes the demand for products from the customers and maximizes the profit for the given horizon length. The material-transfer requests are optimized, and these requests constitute the demand placed to the upstream decision unit for the production facility and warehouse. The second decision unit responds to the demand by computing the optimal production schedule.

The objective of the decision units in the decentralized and semi-decentralized configurations is to optimize their local objectives only. Therefore, the previous objective function given for the centralized case must be modified for the individual decision units to be consistent with local optimizations. For example, in the decentralized case, the retailer maximizes its profit using the objective function given in eq 21. In this equation, the sales revenue, R_{sales}, is generated by selling material to customers. The total cost of purchasing, C_{purchase}, is the cost of purchasing the material from the immediate upstream distribution center, rather than the cost of purchasing raw materials and intermediate products, which would be the case in the case of centralized optimization. When the distribution center performs its local optimization using eq 21, the revenue is generated by selling material to the retailer. The cost of purchase for the distribution center corresponds to the total cost of material that is transferred from the upstream subsystem production/warehouse facility.

Table 1. Sales Prices, Costs, and Production Amounts for the Three Products

parameter	Value product A	product B	product C
sales price	50	50	50
transportation cost	1	1	1
inventory holding cost	2	2	2
customer satisfaction cost	50	50	50
variable production cost	24	12	84
fixed production cost	18	6	42
max. batch production quantity	100	100	50

Table 2. Initial Inventory and Material Transfer Levels

node	initial inventory
Product A	
retailer	150
distribution center	150
warehouse	150
Product B	
retailer	150
distribution center	150
warehouse	150
Product C	
retailer	150
distribution center	150
warehouse	150

In the decentralized and semi-decentralized cases, the sales price is considered to decrease by 10% at each successive node of the supply chain network opposite to the material flow. For example, if the sale price of a product is 50 monetary units per unit material sold to the customer at the retailer node, the distribution center charges 45 monetary units per unit material transferred to the retailer. It is important to note that these intermediate prices can be optimally computed as well, but this defeats the purpose of decentralization, because such an optimization would require coordination among the subsystems and result in another centralized scheme. Therefore, in the sequel, we compare decentralized and semi-decentralized optimizations with this intermediate price policy with centralized optimization, where such pricing is not needed to solve the problem.

3. Three-Product Supply Chain Network

The Hybrid Systems approach for the optimization of supply chain systems described in the previous section is illustrated on a supply chain network with three products. The supply chain system consists of a manufacturing plant that can only produce one of the products at a time: a warehouse that can store these three products, a distribution center, or retailers for the products and customer with uncertain demand. The problem data are given in Tables 1 and 2. The system is coded in OPL Studio with CPLEX as a mixed-integer linear programming (MILP) solver.[21] The optimization is performed for a total horizon of 52 time periods, and for each MPC run, the horizon length of 20 time periods is considered. The system is optimized for the horizon length and only the decisions for the first time period

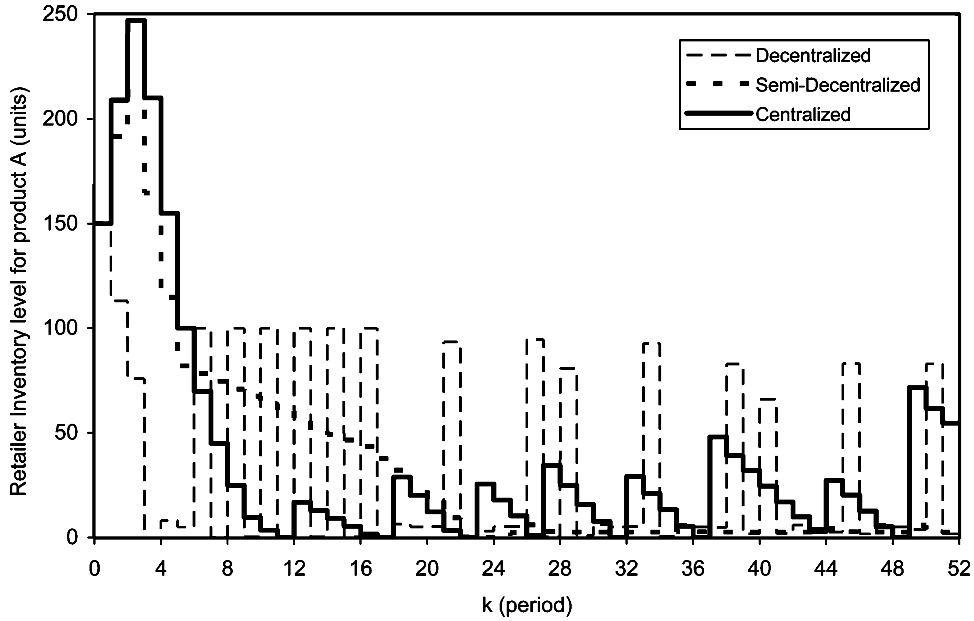

Figure 6. Inventory profile of product A at the retailer.

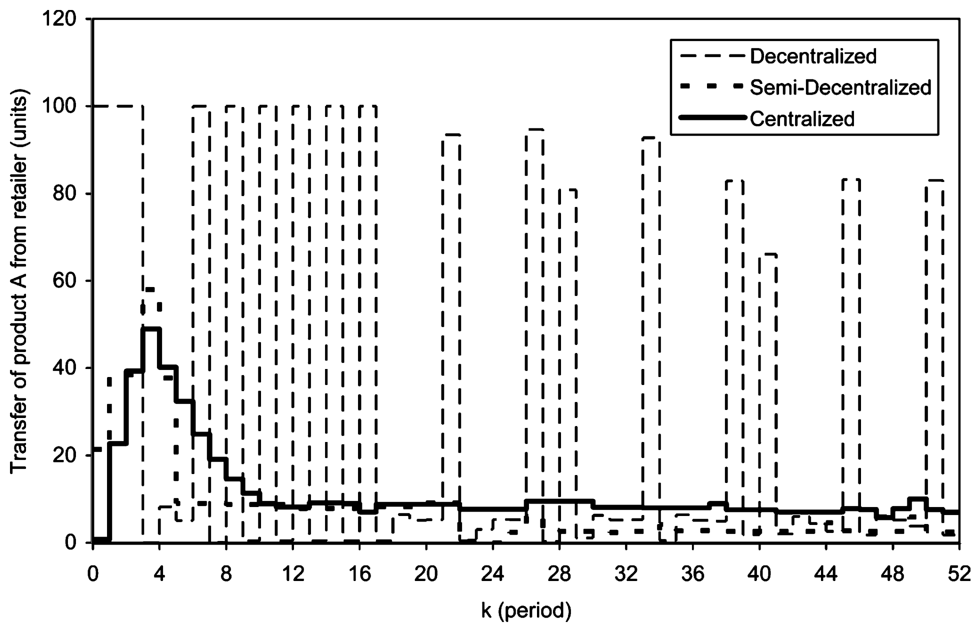

Figure 7. Material transfer profile for product A from the retailer.

are implemented. At the end of each period, inventory levels, order states, and demands are updated and the system is re-optimized for another optimization horizon.

3.1. Results for Model Predictive Control (MPC) of Centralized and Decentralized Supply Chain System Configurations. The MPC approach, under three different configurations, is applied to the three-product supply chain network. An important measure of efficiency in the operation of supply chain networks is the inventory levels. The inventory profile for product A at the retailer node is given in Figure 6. Similar trends hold for products B and C.[22] Both semi-decentralized and centralized configurations initially increase the inventories at the retailer, compared to the decentralized configuration. The initial inventory buildup at the retailer level is greater for the centralized configuration than for the semi-decentralized system. The reason for this difference is the way inventories are minimized by different decision units. The centralized configuration, which has a single decision unit for all of the nodes in the supply chain network, transfers all initial inventories to the retailer as quickly as possible. The semi-decentralized configuration, which has two separate decision units, transfers only part of the inventory to the retailer. In the case of decentralized control, inventory is depleted immediately, because of local optimization of the retailer.

After the initial inventory buildup, inventory is managed much better by centralized and semi-decentralized configurations than by decentralized configuration. This result is expected, because the decentralized configuration optimizes the material transfer out of the retailer by just knowing the amount of material in the retailer, whereas the semi-decentralized configuration optimizes the material transfer level out of the retailer by knowing the amount of material in the distribution center and retailer, thus optimizing the inventory level by utilizing more information. The inventory profile of product A at the retailer has peaks at periods 12, 18, 23, 27, 32, 37, 44, and 49, as shown in Figure 6. The main reason for these peaks is the production of product A in the production facility. The production of product A occurs precisely eight times, which is equal to the number of peaks,

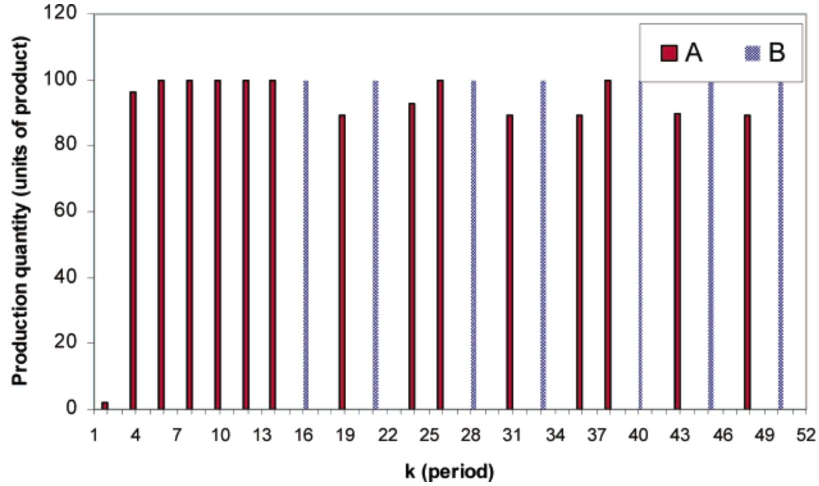

Figure 8. Production schedule and quantities under the decentralized control configuration.

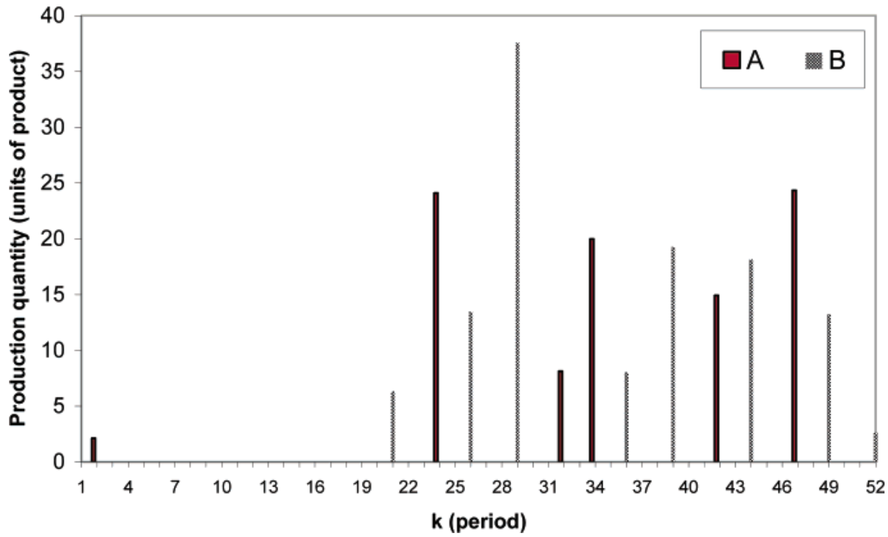

Figure 9. Production schedule and quantities under the semi-decentralized control configuration.

and the inventory levels at the retailer sharply increase exactly after two periods from production. Two periods are needed to transfer the product from the warehouse to the retailer. This observation leads to the conclusion that it is better to keep the inventory of products at the retailer, thus eliminating the customer satisfaction cost that is incurred because of the uncertain behavior of the demand and the time delay in transferring material from an upstream node. As a consequence, the centralized configuration tries to accumulate inventories at the retailer. The semi-decentralized configuration, on the other hand, which has two separate decision units, has a smoother inventory profile, which indicates that there is a frequent transfer of material between the warehouse and the distribution center/retailer.

Material transfer of product A from the retailer to the customer is shown in Figure 7. The decentralized configuration accumulates inventory to the full capacity frequently, as shown in Figure 6. To eliminate this inventory, it transfers the material to the customer in the same fashion, as shown in Figure 7. Initially, semi-decentralized and centralized schemes deliver less material out of the retailer, which also contributes to their initial inventory buildup, as observed in Figure 6.

The production schedules under different configurations exhibit significant differences, as shown in Figures 8−10. There is frequent production of product A at the maximum capacity under the decentralized configuration. This is due to the fact that inventories are depleted frequently to reduce costs; however, production is required to satisfy demand. Whenever there are insufficient inventories, a production is scheduled. This behavior is different under semi-decentralized and centralized configurations. The centralized configuration schedules production after inventories are depleted at the retailer. Therefore, a balance between the production cost and inventory holding cost is established that maximizes the overall profit of the supply chain, rather than individual decision units. The semi-decentralized configuration also shows similar behavior. From the fixed and variable production costs, it is a clear fact that the system must try to produce as much as possible but in small amounts, because the variable production cost is more than the fixed production cost. Product C is never produced in decentralized and semi-decentralized systems, because it is not profitable (because of its production costs; see Table 1).

Except for the beginning, the order levels of the semi-decentralized system from the plant and warehouse are always lower than those of the decentralized system, as shown in Figure 11. First, in the decentralized configuration, the distribution center must forecast the demand of the retailer. Whenever the retailer makes a material transfer request, the distribution center assumes that the demand level is going to be maintained in the next optimization horizon, which may exaggerate the needs of the retailer, as perceived by the distribution center, hence forcing the distribution center to place high order levels to the plant

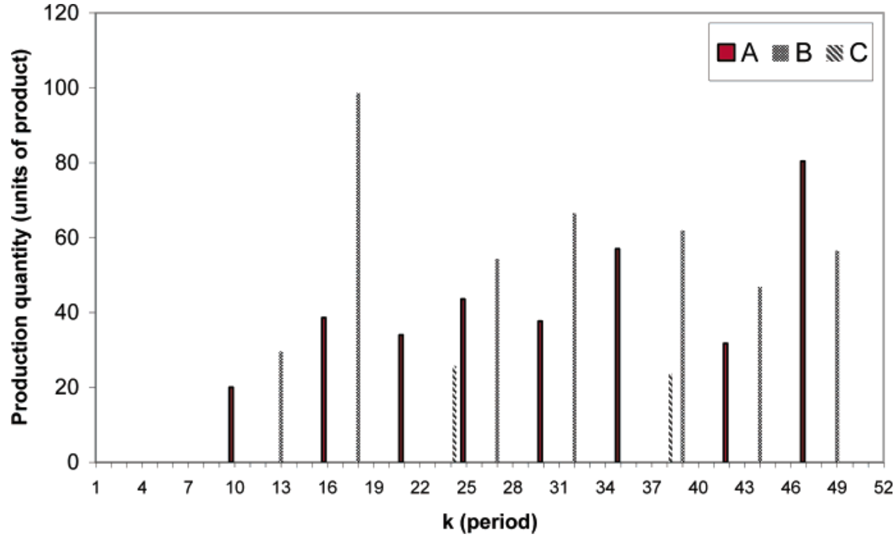

Figure 10. Production schedule and quantities under the centralized control configuration.

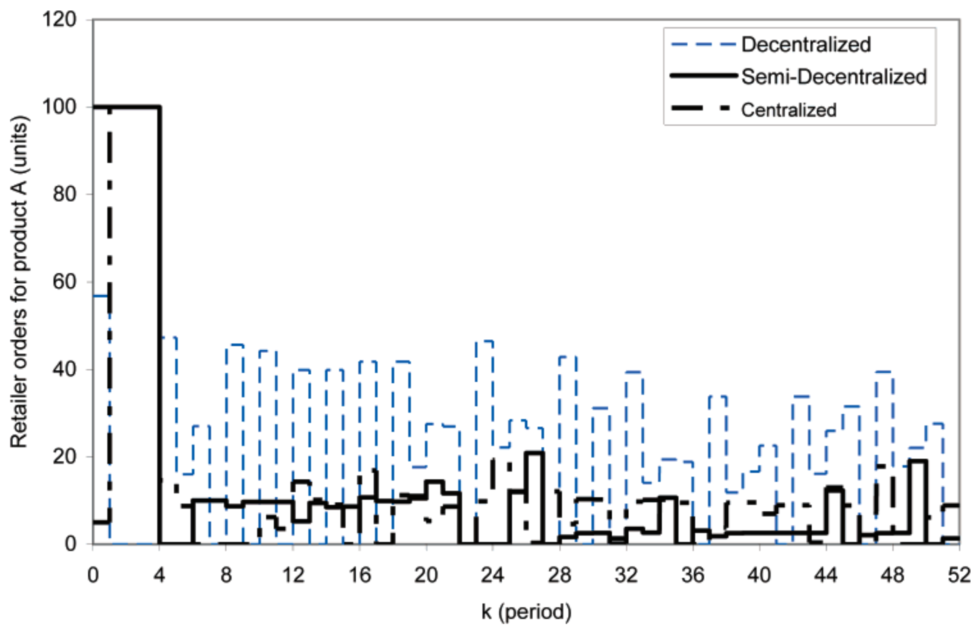

Figure 11. Profile of orders placed by the retailer for product A.

and warehouse. In the semi-decentralized configuration, the optimizer knows the exact needs of the retailer and distribution center, because they are in the same complex, hence avoiding the exaggeration of the needs. The second point is the fact that, in the semi-decentralized configuration, the retailer and distribution center complex must pay the entire inventory holding costs and material transfer costs throughout the travel of the material. On the other hand, in the decentralized configuration, the distribution center does not consider the inventory holding cost and the material transfer cost of the retailer and only optimizes its own profit. As a result, the distribution center can place high order levels. The observed phenomenon—the exaggeration of demand levels and, therefore, material transfer levels—is called the demand amplification problem or the bullwhip effect.[23] The measure of bullwhip effect is given in eq 28.

$$bwr_{jm}(k) = \left| \frac{\sum_i O_{ijm}(k)}{\sum_l O_{jlm}(k)} - 1 \right| \tag{28}$$

The measure of the bullwhip effect generated by node j for product m, $bwr_{jm}(k)$, is a function of the ratio of orders that are placed from node j to all upstream nodes i for product m ($\Sigma_i\ O_{ijm}(k)$) to the cumulative orders from all downstream nodes l to node j for product m ($\Sigma_l\ O_{jlm}(k)$). For the retailer, the denominator corresponds to the demand in period k, because the orders are placed by the customers to the retailer. The profile of the bullwhip effect that is created by the retailer for product A is shown in Figure 12. Ideally, the order-to-demand ratio should be 1 for a perfect supply chain, where all of the information is available to the decision maker, demand is certain, and the production and transfer of materials is instantaneous. The more this ratio deviates from 1, the less effective the supply chain system is, in terms of customer satisfaction, coordination, and finances. As shown in Table 3, the decentralized configuration has the worst performance, compared to others at the retailer for all products.

The best average performance for all products is obtained under the centralized configuration.

In all of the system variables, it is easily observable that the decentralized system has more variability than the semi-

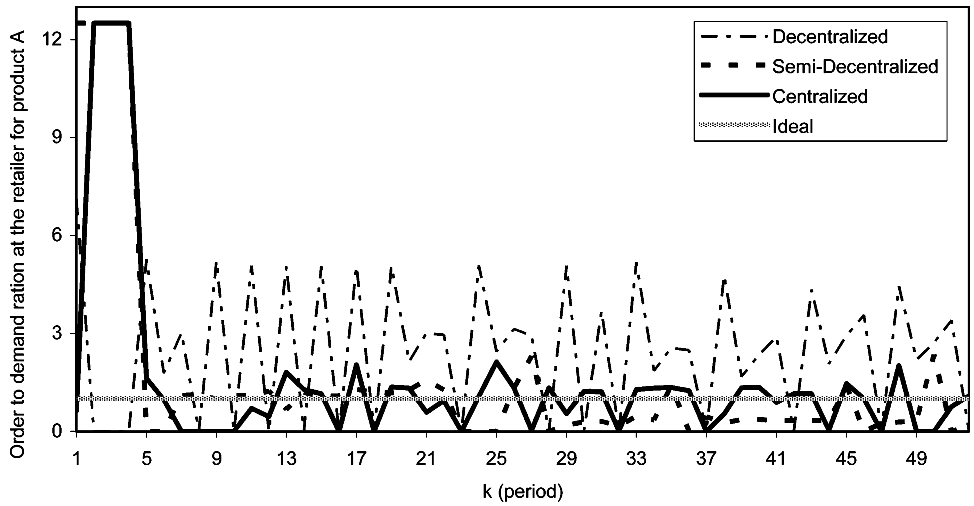

Figure 12. Ratio of demand and orders placed by the retailer for product A.

Table 3. Comparison of the Bullwhip Effect under Different Configurations

	Average Measure of the Bullwhip Effect		
product	decentralized	semi-decentralized	centralized
A	2.10	1.43	1.17
B	3.06	1.54	1.22
C	2.55	2.62	2.13

decentralized system, in the sense that the variables take higher and lower values and the frequency of change is more intense. It must be stated that the bullwhip effect is more severe in the decentralized configuration. The reason for the repetitive occurrence of this phenomenon is the “inability of the system to learn”. That is, initially the decentralized configuration observes that the inventory levels are high and tries to transfer this material to the downstream nodes, including the end customers. Because of the bullwhip effect, the plant and warehouse configuration produces a large batch of products, which creates a large amount of inventory in the warehouse. Then, in the next time step, seeing the large amount of inventory in the warehouse and the demand from the distribution center, most of the materials are transferred to the distribution center, similar to that which occurs with the initial inventory. When the material arrives at the distribution center, the distribution center observes the large amount of material in the inventory and the demand from the retailer, it imitates what happened with the initial inventory levels, gets rid of the large inventory, and assumes that the plant and warehouse will deliver as much as wanted whenever desired. The same procedure occurs for the retailer, which leads to the large variability in the decentralized configuration. Although the same repetitive occurrence is also present in the centralized and semi-decentralized configurations, because the bullwhip effect is not as severe as in the decentralized configuration, the same amount of variability is not observed, despite the fact that both the variability and the repetitive occurrence is present.

The customer satisfaction levels can be determined by comparing the absolute value of the difference between material transfers at the retailer and the customer demand. Figure 13 shows the profile of customer satisfaction, which is represented as the absolute difference between the materials transferred and the demand for product A. The decentralized configuration has the worst customer satisfaction performance, whereas the centralized configuration has the best customer satisfaction performance. Similar behavior is observed for all products, as shown in Table 4.

The overall performance measure of the supply chain system is the value that is generated. Among the three MPC configurations, the centralized system finishes the total optimization horizon with profit, as shown in Table 5. The decentralized and semi-decentralized configurations do not incur profit, with the former exhibiting the worst performance.

The statistics on the problem size and computer processing unit (CPU) times is given in Table 6.

The effect of MPC horizon length on the optimal supply chain operation has been extensively studied by Mestan.[22] Here, we summarize only the major trends. As the horizon length decreases, MPC takes short-sighted decisions and allocates its resources, i.e., inventories in earlier periods, as shown in Figure 14. A smaller horizon length depletes inventory faster, initially, which leads to lower inventory holding cost and high sales revenues. At the same time, the production frequency and production amounts increase and more material than actually needed is delivered to the customer. This results in a high customer dissatisfaction penalty, which decreases the total profit. With a longer horizon, the material transfers are closer to the actual demand and customer satisfaction and total profit are higher. Yet the CPU times increase with horizon length, causing a trade-off between optimal results and allowable solution times. The results are reported here for a horizon length of 20, which is chosen after such a trade-off analysis.

3.2. Measures for Performance Improvement of Decentralized Supply Chain Configurations. The centralized supply chain configurations give better performance, compared to the decentralized configurations, with respect to the bullwhip effect, customer satisfaction, and profit, as shown in Tables 3, 4, and 5, respectively. It is important to explore ways to improve the performance of the decentralized configurations, because of difficulties in establishing a centralized decision unit for the supply chain system. We consider the following measures to improve the decentralized performance: (i) move suppression and (ii) information sharing.

Move suppression is a widely used, robustifying action in classical MPC.[24] Within a given horizon P, the control actions in M out of P periods (where $M < P$) are kept free while the remaining control actions in periods $M + 1$ to P are fixed to the levels in period M. Therefore, a less-aggressive control is computed and applied on the system.

Information sharing has an important role in the decisions related to operation of supply chain systems. After calculating its optimal future orders, a downstream node can decide to share

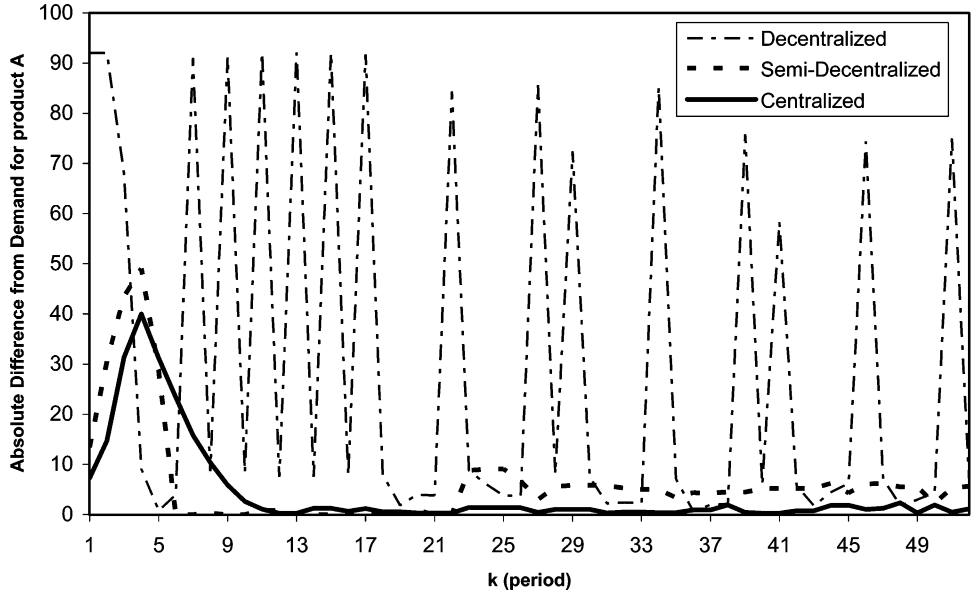

Figure 13. Customer satisfaction level for product A.

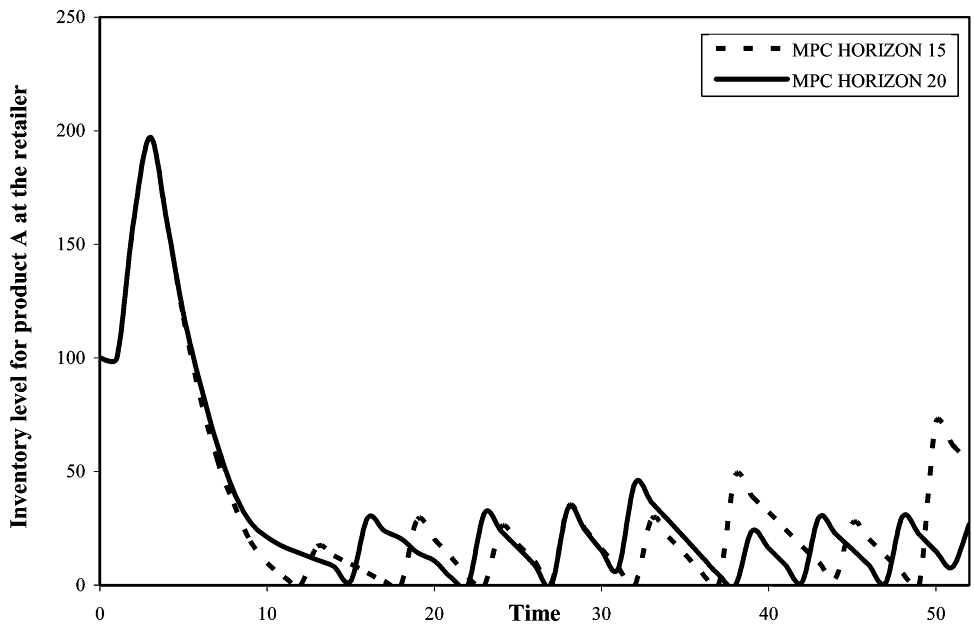

Figure 14. Effect of model predictive control (MPC) horizon length on inventory levels at the retailer for product A.

Table 4. Comparison of Customer Satisfaction Levels under Different Configurations

	Average Measure of Customer Satisfaction		
product	decentralized	semi-decentralized	centralized
A	30.53	6.39	4.24
B	18.90	10.32	4.18
C	8.81	7.54	6.67

Table 5. Total Profit of the Supply Chain Network

configuration	monetary units
centralized	1687
semi-decentralized	−35279
decentralized	−984300

its future orders up to *s* periods into the future at any period *k*. Although the orders of the downstream node can be revised between periods $k + 1$ to *s* in future, this information may provide more-accurate inputs for improved performance. The value for *s* was taken to be 1 in the results presented in the previous section. This means that each downstream unit places

Table 6. Characteristics of the Optimization Model

item	value
number of continuous variables	19969
number of binary variables	208
number of constraints	38821
CPU time[a]	1874 s

[a] Determined with a personal computer with 1.5 GHz Intel Pentium 4 processor and 512 MB of RAM.

its optimal order for the current period only to the upstream unit. Here, we increase *s* and allow the units to share more information and analyze the outcome. The effects of these measures are summarized in Table 7.

Move suppression (i.e., smaller *M* values in Table 7) reduces the bullwhip effect and increases customer satisfaction, but reduces the total profit. The main reason for a decrease in the total profit is due to the availability of fewer decision variables for the optimization of the objective function: the total profit. Information sharing, on the other hand, can simultaneously improve all three performance criteria for the decentralized

Table 7. Effect of Move Suppression and Information Sharing[a]

configuration/ action	average measure of bullwhip effect at the retailer	average measure of customer satisfaction	total profit
centralized	4.52	15.09	1687
decentralized ($s = 1$)	7.71	58.24	−984300
move supression			
M =15	5.02	43.00	−998522
$M = 5$	4.96	42.36	−1003598
$M = 3$	4.83	42.49	−1045831
information sharing			
$s = 2$	5.83	43.00	−656293
$s = 5$	4.19	24.36	−75,652
$s = 10$	4.17	24.25	−98,193

[a] Values are for all three products combined.

configuration. When $s = 2$, the improvement is minor, because the amount of information is small. However, when $s = 5$, there is noticeable improvement in all three performance criteria. The total profit increases from −984 300 to −75 652 and the bullwhip effect is reduced from 7.71 to 4.19. Similarly, customer satisfaction improves as well (compare 58.24 and 24.36). When $s = 10$, the additional improvements in the bullwhip effect and customer satisfaction are minor. The total profit gets worse when information that is likely to change in the distant future is shared among the nodes during optimization.

4. Conclusions

In this paper, supply chain systems that include both discrete-events and continuous-time dynamics are modeled using a hybrid systems approach. The production system model is developed in a disjunctive programming framework that integrates both discrete decisions and continuous variables. The methodology is illustrated on a three-product supply chain system that contains retailer, distribution center, and warehouse nodes, along with a production facility. The value generated in the supply chain system is optimized under centralized, fully decentralized, and semi-decentralized model predictive control (MPC) configurations.

Optimal production schedules, inventories at different nodes of the supply chain network, and material transfer rates are illustrated and compared for different MPC schemes. Centralized MPC configuration results in better inventory management and production scheduling. Results also are compared in terms of total profit and customer satisfaction. In the fully decentralized and semi-decentralized models, a lack of coordination is the key reason for obtaining inferior results, compared to the centralized model. This lack of coordination causes the bullwhip effect, especially in the fully decentralized model, which damages the optimal performance of the supply chain.

The effects of several measures, including move suppression and information sharing, on the performance of decentralized supply chain configurations are investigated. It is shown that, although move suppression results in an improvement of the bullwhip effect and customer satisfaction, the total profit for the system decreases, because of the decreased degrees of freedom. On the other hand, information sharing is an effective way to improve the decentralized configurations, because it can improve all of the performance criteria, if the degree of sharing is properly chosen.

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