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# Modified Ponchon-Savarit and McCabe-Thiele Methods for Distillation of Two-Phase Feeds

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Modified versions of the Ponchon-Savarit and McCabe-Thiele methods are presented which incorporate a correct analysis of the feed region in the case of partly vaporized feeds. The methods proposed are fully graphical, require no additional information or calculations, and do not complicate significantly the traditional procedures. It is shown that for optimal effect, the liquid and vapor parts are introduced to separate but contiguous stages. Numerical examples confirm that a small improvement in column performance is achieved, but the influence on the number of stages or reflux ratio required for a given separation is negligible, except in very special cases.

## Introduction

Despite the widespread availability of computer facilities and software for the design and calculation of distillation processes, traditional graphical methods such as the McCabe-Thiele and Ponchon-Savarit diagrams continue to be used, either as means to obtain quick preliminary estimates or as valuable teaching tools which allow the interrelationship of the several process variables to be easily grasped and understood.

Most, if not all, presentations of the subject, however, are based on the concept of a "feed stage", in effect assuming that a feed stream, whatever its physical condition, is always introduced in bulk to a single tray, where it mixes with liquid from the tray above and vapor from the tray below, before any phase separation can occur. This model obviously cannot be correct in the practically important case of a partially vaporized feed which, upon entering the column at some point between two trays, splits spontaneously into a vapor that flows to the tray immediately above, and a liquid that falls to the tray immediately below. In such cases, there are actually two feed stages, and the only way for the traditional model to be realized would be for the feed stream to be flash-separated, at the feed tray pressure outside the column, and for the resultant phases to be introduced separately, the liquid portion above and the vapor portion below the intended feed stage (Bennet and Myers, 1962).

Cavers (1965) identified this problem, and accurately pointed out that a correct treatment of partly-vapor feeds implies a discontinuity in the McCabe-Thiele method of stepping off stages. He failed, however, to propose a modified, fully graphical technique, suggesting only that the discontinuity be resolved by material balances around the feed section. It is the purpose of the present paper to show that all balance equations can be incorporated into both the McCabe-Thiele and the Ponchon-Savarit constructions to yield more general diagrams which, at the expense of negligible extra effort, will deal with two-phase feeds correctly.

## Modified Ponchon-Savarit Method

Consider the distillation of a binary mixture in a conventional column with a total condenser and an internal reboiler. The feed stream, introduced between stages "a" and "b", separates into mutually saturated liquid and vapor, as shown schematically in Figure 1. Trays are numbered starting at the column top, and are assumed to be ideal, including the reboiler. Additional assumptions in-

clude steady-state operation and negligible heat losses.

The following convention is adopted, for economy and compactness of notation. Any process stream, e.g., of mass flow rate  $P$ , composition of the more volatile component  $x_p$ , and enthalpy  $h_p$  is represented by a vector

$$\{P\} \equiv \{P, Px_p, Ph_p\}$$

so that the set of total mass, component mass, and energy balance equations for any control volume can be summarized in a single vector relation. The process stream is also represented by a point of the enthalpy-composition diagram

$$P \equiv (x_p, h_p)$$

Thus, the balance equation for a column section extending from any tray of the rectifying section up to the condenser is simply

$$\{V\}_{j+1} - \{L\}_j = \{\Delta_D\} \quad (j = 1, \dots, a) \quad (1)$$

where  $\{\Delta_D\}$  is the fictitious net upward flow

$$\{\Delta_D\} \equiv \{D, Dx_D, Dh_D + Q_D\} \quad (2)$$

which defines the rectifying-section difference point  $\Delta_D$  of coordinates  $(x_D, h_D + Q_D/D)$ .

Similarly, the balance equation

$$\{\bar{L}\}_{k-1} - \{\bar{V}\}_k = \{\Delta_W\} \quad (k = b, \dots, n) \quad (3)$$

is valid for any tray of the stripping section and defines the fictitious net downward flow

$$\{\Delta_W\} \equiv \{W, Wx_W, Wh_W - Q_W\} \quad (4)$$

to which corresponds the stripping-section difference point  $\Delta_W$  of coordinates  $(x_W, h_W - Q_W/W)$ . From the overall balance for the entire column

$$\{F\} = \{\Delta_W\} + \{\Delta_D\} \quad (5)$$

the feed point  $F$  is found to lie on the straight line which joins the two difference points,  $\Delta_D$  and  $\Delta_W$ .

The streams immediately above and below the feed are related by the equations

$$\{F\} = \{L_F\} + \{V_F\} \quad (6)$$

$$\{V\}_{a+1} = \{\bar{V}\}_b + \{V_F\} \quad (7)$$

$$\{\bar{L}\}_{b+1} = \{L\}_a + \{L_F\} \quad (8)$$

only two of which are linearly independent, in view of eq

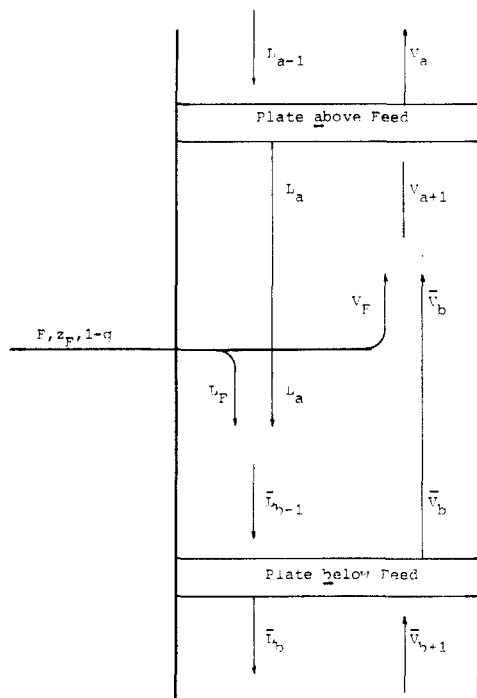


Figure 1. Two-phase feed introduction to a distillation column.

1, 3, and 5. The simplest way to connect the upper and lower column sections is to define a "changeover" net flow stream

$$\{\Delta_C\} \equiv \{\bar{V}\}_b - \{L\}_a \quad (9a)$$

Then from eq 1 and 7 there follows

$$\{\Delta_C\} = \{V\}_{a+1} - \{V_F\} - \{L\}_a = \{\Delta_D\} - \{V_F\} \quad (9b)$$

whereas from eq 5 and 6 and the above

$$\{\Delta_C\} = \{F\} - \{\Delta_W\} - \{V_F\} = \{L_F\} - \{\Delta_W\} \quad (9c)$$

Hence, the changeover difference point is the intersection point of two straight lines, one which passes through points  $\Delta_D$  and  $V_F$ , the other one passing through points  $L_F$  and  $\Delta_W$ .

Equations 2, 5, 6, and 9 determine the basic appearance of the modified Ponchon-Savarit diagram, as illustrated in Figure 2. With the main points in place, stages may be counted in the usual manner, starting at the top: (i) A point  $L_{j-1}$  is joined with one of the  $\Delta$  points by a straight line; this and the saturated vapor curve intersect at point  $V_j$ . (ii) The equilibrium tieline is drawn through  $V_j$ ; this defines  $L_j$  on the saturated liquid curve.

This sequence is repeated for  $j = 1, 2, \dots, n$ . The starting point is  $L_0$  at  $(x_D, h_D)$ , and the procedure is terminated when  $x_n \leq x_W$  for the first time. Upon reaching this condition, the total number of ideal stages is given by  $m - 1 < n \leq m$ .

Unless the stage numbers  $a$  and  $b$  are assigned a priori, in which case the introduction of the feed stream is probably nonoptimal, the difference point to use at each step (i) of the above sequence should be that one among  $\Delta_D$ ,  $\Delta_W$ , and  $\Delta_C$  which yields the lowest composition  $y_j$  (equivalently, the leftmost point  $V_j$ , or the largest slope of the line  $L_{j-1}V_j$ ) for the same  $L_{j-1}$ . Initially,  $L_{j-1}$  lies to the right of  $P$ , the point at which the line  $\Delta_D V_F \Delta_C$  intersects the saturated liquid curve. Difference point  $\Delta_D$  should be used (Figure 3a), giving  $V_j$  also to the right of  $V_F$ .

Eventually, a tieline  $V_j L_j$  will cross the line  $\Delta_D V_F \Delta_C$ , giving  $L_j$  to the left of  $P$ , but necessarily to the right of  $L_F$ .

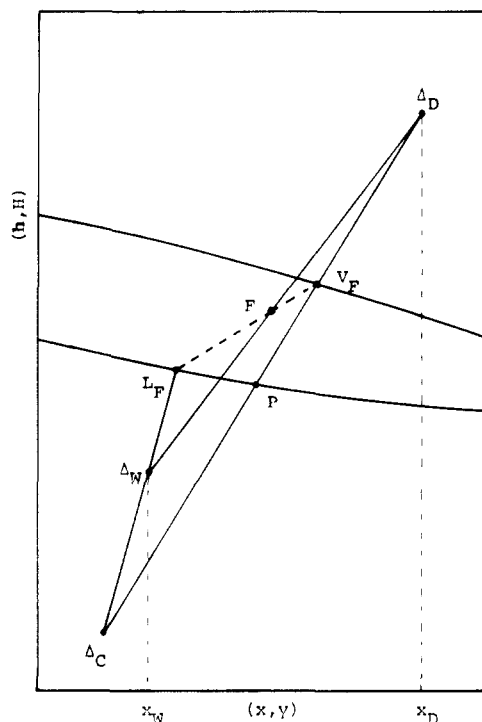


Figure 2. Basic Ponchon-Savarit diagram for two-phase feed.

Difference point  $\Delta_C$  should now be used (Figure 3b), which from eq 9b means that the vapor stream,  $V_F$ , has been introduced at its optimal position,  $a = j$ . Hence  $V_{j+1}$  is  $\bar{V}_b$  and falls to the left of  $V_F$ . The use of  $\Delta_C$ , not  $\Delta_W$ , at this point clearly indicates that the liquid portion of the feed should not be fed to the same tray that receives the vapor portion. Otherwise, the optimal changeover would be from  $\Delta_D$  to  $\Delta_W$  directly, skipping  $\Delta_C$ .

The equilibrium tieline through  $\bar{V}_b$  next fixes  $\bar{L}_b$  on the saturated liquid curve, necessarily to the left of  $L_F$ . Difference point  $\Delta_W$  should now be used (Figure 3c). This means that the liquid stream,  $L_F$ , has been introduced at its optimal position,  $b = a + 1$ ; i.e., the liquid portion of the feed is best input to the tray immediately below that which receives the vapor portion.

As shown in Figure 4, the straight line which joins  $L_a$  and  $\Delta_C$  directly gives  $\bar{V}_b$  at its intersection with the vapor locus. Points  $V_{a+1}$  and  $\bar{L}_{b-1}$  are not essential for continuation of the method. If wanted,  $V_{a+1}$  can be found at the intersection of the two lines,  $V_F \bar{V}_b$  and  $L_a \Delta_D$ , cf. eq 1 and 7, and  $\bar{L}_{b-1}$  can similarly be found at the intersection of  $L_F L_a$  and  $\bar{V}_b \Delta_W$ , cf. eq 3 and 8. It follows that  $V_{a+1}$  and  $\bar{L}_{b-1}$  will not be saturated phases unless the dew-point and bubble-point curves are perfectly straight lines. The rest of the procedure reverts to the traditional method,  $\Delta_W$  being used until the bottom composition  $x_W$  is attained.

### Modified McCabe-Thiele Method

Although the McCabe-Thiele diagram can always be generated trivially once the Ponchon-Savarit diagram is available, we specifically consider here the special case where the "usual simplifying assumptions" (Treybal, 1980) leading to constant molar overflow and vaporization are satisfied exactly. In such instances, material balance relations corresponding to the first two components of the vector eq 1 through 8 and the  $y$ - $x$  diagram are sufficient for a description of the distillation process, with no need to resort to enthalpy-related constructions.

In the rectifying section, the constant liquid and vapor flow rates are denoted simply by  $L$  and  $V$ . Equations 1

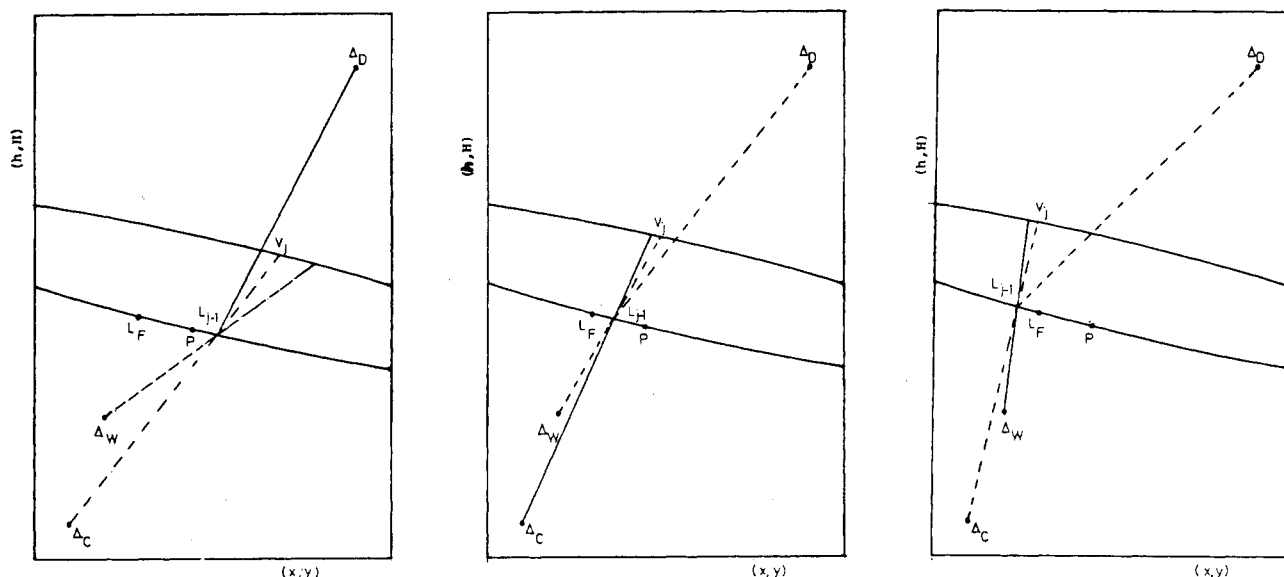


Figure 3. Optimal use of difference points in the Ponchon-Savarit diagram.

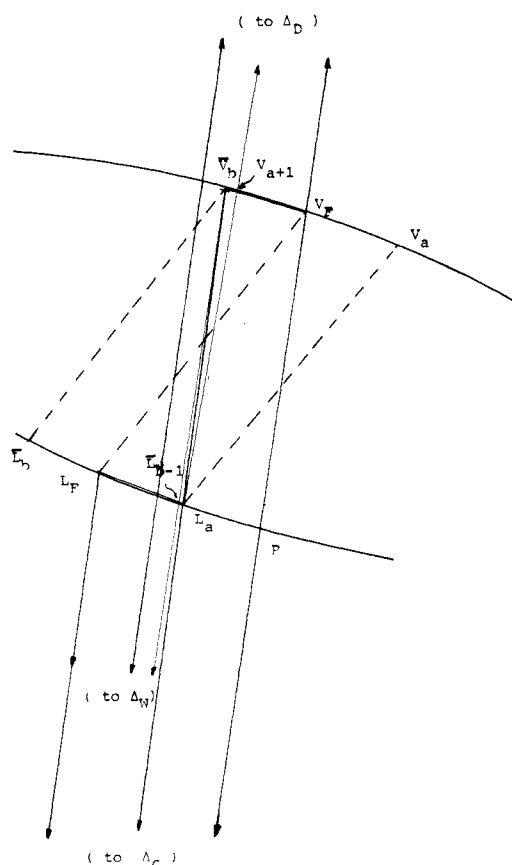


Figure 4. Detail of the feed region in the Ponchon-Savarit diagram.

and 2 then provide an equation for the operating line, or locus of counterflowing liquid and vapor compositions.

$$y_{j+1} = \frac{L}{L+D}x_j + \frac{Dx_D}{L+D} \quad (j = 1, \dots, a) \quad (10a)$$

On introducing the external reflux ratio,  $R = L/D$ , eq 10a becomes

$$y_{j+1} = \frac{R}{R+1}x_j + \frac{x_D}{R+1} \quad (j = 1, \dots, a) \quad (10b)$$

i.e., the rectifying line is a straight line of slope  $R/(R+1)$  that passes through point  $(x_D, x_D)$ .

Similarly, in the stripping section where flow rates are denoted by  $\bar{L}$  and  $\bar{V}$ , eq 3 and 4 yield the relation

$$\bar{y}_k = \frac{\bar{L}}{\bar{L} - \bar{W}}\bar{x}_{k+1} - \frac{Wx_W}{\bar{L} - \bar{W}} \quad (k = b, \dots, n) \quad (11a)$$

which, when combined with eq 5, 6, and 8 becomes

$$\bar{y}_k = \frac{RD + L_F}{(R+1)D - V_F}\bar{x}_{k+1} + \frac{Rx_D - Fz_F}{(R+1)D - V_F} \quad (k = b, \dots, n) \quad (11b)$$

Thus, the stripping line is a straight line that passes through point  $(x_W, x_W)$ , and whose slope can be calculated from known problem data. More usually, though, the liquid fraction of the feed,  $q = L_F/F$ , is defined as a measure of its thermal condition, and the feed  $q$  line is introduced from eq 6

$$y_F = -\frac{q}{1-q}x_F + \frac{z_F}{1-q} \quad (12)$$

This is the equation of a straight line which passes through point  $(z_F, z_F)$ , has a slope  $-q/(1-q)$ , and intersects the equilibrium curve at  $(x_F, y_F)$ . Since it may be shown that the three lines, eq 10, 11, and 12 meet in a single intersection point, the rectifying line and feed  $q$  line are traditionally drawn first, and their intersection point is located. The stripping line is then the straight line joining this point and  $(x_W, x_W)$ .

It should be clear, however, that the joint solution of eq 10 and 11 can never represent actual conjugate vapor and liquid compositions at any point along the column, since by definition both equations apply to separate column sections. The feed region between plates  $a$  and  $b$  must be represented by a third, "changeover" operating line, equivalent to the intermediate difference point  $\Delta_C$  of the revised Ponchon-Savarit method.

From eq 1, 2, and 7 total and light-component mass balances may be written for a column section extending from the condenser down to the vapor feed point, but excluding the liquid portion  $L_F$ . An equation for the changeover line is thus derived.

$$\bar{y}_b = \frac{RD}{(R+1)D - V_F}x_a + \frac{Dx_D - V_F y_F}{(R+1)D - V_F} \quad (13)$$

While this straight line can be constructed readily from problem data, it is easier and more instructive to consider



Table I. Comparison of Traditional and Modified McCabe-Thiele Calculations, Based on  $z_F = 0.50$  ( $x_F = 0.4142$ ,  $y_F = 0.5858$ ),  $q = 0.50$ ,  $R = 4.0$ ,  $\alpha = 2.0$ ,  $x_D = 0.95$ ,  $x_W = 0.05$ .

stage	traditional method		this work	
	y	x	y	x
1	0.9500	0.9048	0.9500	0.9048
2	0.9138	0.8413	0.9138	0.8413
3	0.8630	0.7591	0.8630	0.7591
4	0.7973	0.6629	0.7973	0.6629
5	0.7203	0.5629	0.7203	0.5629
6	0.6403	0.4709	0.6403	0.4709
7	0.5667	0.3954	0.5620	0.3908
8	0.4818	0.3173	0.4760	0.3123
9	0.3841	0.2377	0.3779	0.2330
10	0.2847	0.1659	0.2787	0.1619
11	0.1949	0.1080	0.1899	0.1049
12	0.1225	0.0652	0.1186	0.0631
13 <sup>a</sup>	0.0691	0.0358	0.0663	0.0343

<sup>a</sup> Internal reboiler.

Whenever the feed stream is an actual two-phase mixture at column conditions, however, the modified methods proposed here constitute the correct, fully graphical treatment of binary distillation. Inasmuch as they are based on a model of two feeds,  $V_F$  and  $L_F$  to separate stages, these methods are special cases of available procedures for multiple-feed columns. The distinguishing feature of the present analysis is that, being mutually saturated phases,  $V_F$  and  $L_F$  are shown to require introduction to adjacent stages for optimal effect.

Cavers (1965) stated that the separate introduction of the liquid and vapor portions should have but a small influence on the computed number of trays, and this is probably the main reason why the extremely simple modifications described have not been attempted before. That the difference is indeed minimal can be verified from a typical design calculation, e.g., the separation of an equimolar mixture into a distillate with  $x_D = 0.95$  and a bottom product with  $x_W = 0.05$ . The number of ideal stages was computed using the McCabe-Thiele method with  $q = 0.50$ ,  $R = 4.0$ , and an assumed constant relative volatility,  $\alpha = 2.0$ . For greater precision, analytical versions of both the traditional and the revised constructions were programmed into an HP-41C portable calculator. The computed compositions at each tray are reported in Table I, and the combined diagrams of the two methods are shown in Figure 7. The number of stages is the same in both cases,  $n = 13$ . Composition differences do not exceed 0.007 mole fraction, being understandably greater in the feed region and decreasing toward the reboiler as the driving forces become smaller.

The example given is a somewhat extreme one, in that Figure 7 shows point  $(\bar{y}_b, x_a)$  to lie roughly in the middle of the changeover line, where the distance to the rectifying and stripping lines is appreciable. Thus, in most cases one should expect even smaller differences in composition and number of stages between the textbook and the revised methods. In the above example, for instance, changing the reflux to  $R = 3.6838$  results in a vapor composition from stage 7 which exactly matches the vapor feed composition, thereby leading to complete coincidence between the two approaches, because point  $(\bar{y}_b, x_a)$  becomes identical with point T of Figure 5.

It should be noted that the modified procedures do not alter the definition of the minimum reflux ratio. Referring to the McCabe-Thiele diagram of Figure 5, as the reflux ratio is decreased, and a pinch is approached in the feed region, points S, T, and U slide respectively along the combined feed, vapor feed, and liquid feed  $q$  lines until

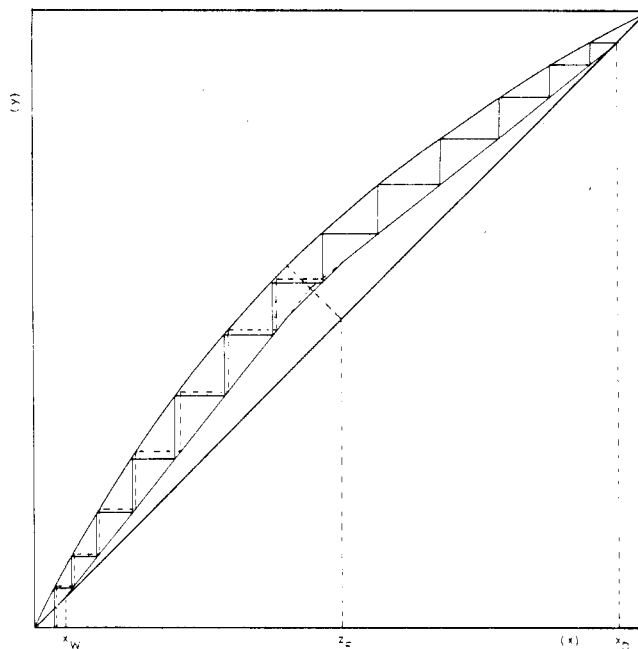


Figure 7. Comparison of traditional and modified McCabe-Thiele constructions, (design problem): (—) this work; (---) traditional method.

Table II. Comparison of Traditional and Modified McCabe-Thiele Calculations, Based on  $z_F = 0.50$  ( $x_F = 0.3090$ ,  $y_F = 0.6910$ ),  $q = 0.50$ ,  $n = 4$ ,  $\alpha = 4.0$ ,  $x_D = 0.95$ ,  $x_W = 0.05$ .

stage	traditional method, $R = 6.1763$		this work, $R = 4.9560$	
	y	x	y	x
1	0.9500	0.7917	0.9500	0.7917
2	0.8137	0.4663	0.8183	0.4738
3	0.5337	0.1863	0.5262	0.1817
4 <sup>a</sup>	0.2083	0.0500	0.2083	0.0500

<sup>a</sup> Internal reboiler.

they meet at the single point  $(x_F, y_F)$ , in agreement with the traditional version. At reflux ratios near the minimum, the composition difference between the changeover line and either operating line will be negligible, even if their slopes differ sharply, and both procedures will tend to give the same results.

For a noticeable contrast between the traditional and the proposed methods, one must turn to systems of high relative volatility, where each equilibrium stage (hence also the feed region) involves substantial changes in composition. This is most clearly seen from a typical rating calculation, in which the reflux ratio is computed for specified product compositions in a prescribed number of stages. The separation of an equimolar mixture into a distillate with  $x_D = 0.95$  and a bottom product with  $x_W = 0.05$  was again considered. The reflux ratio was computed by the McCabe-Thiele method with  $q = 0.50$ ,  $n = 4$ , and an assumed constant relative volatility,  $\alpha = 4.0$ . The previously mentioned HP-41C programs were used in an iterative process assuming a reflux ratio, counting 4 stages down from the known distillate composition and checking whether the resultant bottom product matched the specified purity. The calculated reflux ratios and tray compositions are reported in Table II, and the combined diagrams of the two methods are shown in Figure 8. The reflux ratio is nearly 20% smaller by the modified procedure,  $R = 4.9506$  as against 6.1763 in the usual construction, even if composition differences again do not exceed 0.008 mole fraction.

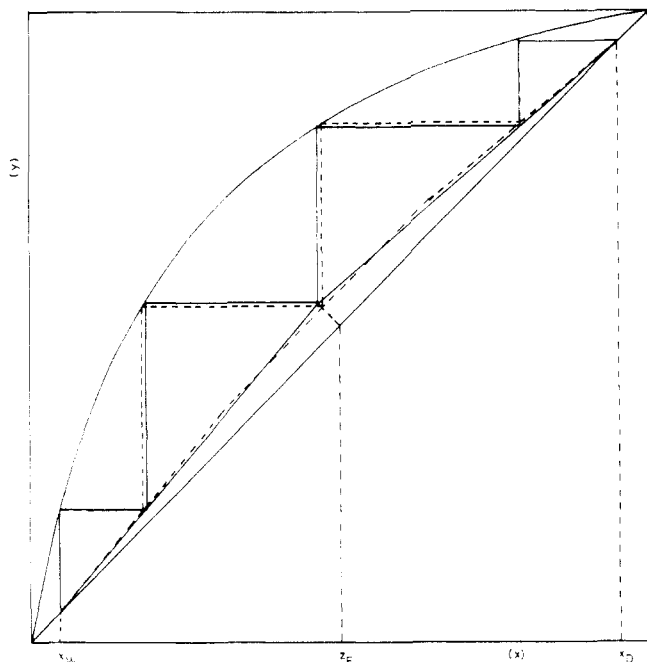


Figure 8. Comparison of traditional and modified McCabe-Thiele constructions, (rating problem): (---) this work; (—) traditional method.

The above illustration also constitutes an extreme case. Differences in reflux ratio will decrease rapidly as the relative volatility decreases and the number of stages increases. The same separation, for example, but with  $n = 10$  and  $\alpha = 2.0$  requires a reflux ratio only 2% smaller in the modified method,  $R = 3.0376$  compared with 3.1068 for the single-phase feed. This discrepancy is undoubtedly closer to what may be expected in most practical cases.

From the viewpoint of the present work, the traditional diagrams represent nonoptimal locations of the feed streams, with either early introduction of the liquid phase or delayed introduction of the vapor phase. The small differences encountered between the two methods are therefore probably typical of any analysis of the influence of feed position. Considering that the proposed methods correspond to a conceptually correct, and physically more realistic model, that they require no extra information and a veritable minimum of additional graphical work, and that they actually help to clarify the importance of feed location, it seems beneficial that they be included in treatments

and applications of the subject.

### Acknowledgment

The authors wish to thank their colleague, Mr. Julio Bastida, for helpful discussions during the preparation of this work.

### Nomenclature

Most symbols are also used as subscripts, to denote properties calculated for the appropriate stream or stage.

$a$  = stage immediately above feed point

$b$  = stage immediately below feed point

$D$  = distillate

$F$  = feed

$h$  = specific enthalpy of liquid

$H$  = specific enthalpy of vapor

$j$  = general stage, rectifying section

$k$  = general stage, stripping section

$L$  = liquid flow, rectifying section

$\bar{L}$  = liquid flow, stripping section

$L_F$  = liquid portion of feed

$m$  = final stage of graphical construction

$n$  = number of ideal stages in column

$P$  = general process stream

$q$  = fraction of liquid in feed

$Q_D$  = condenser heat load

$Q_W$  = reboiler heat input

$R$  = external reflux ratio

$S$  = intersection of stripping and rectifying lines

$T$  = intersection of changeover and rectifying lines

$U$  = intersection of changeover and stripping lines

$V$  = vapor flow, rectifying section

$\bar{V}$  = vapor flow, stripping section

$V_F$  = vapor portion of feed

$W$  = residue

$x$  = liquid composition, more volatile component

$y$  = vapor composition, more volatile component

$z$  = overall feed composition, more volatile component

$\Delta_C$  = net upward flow from stripping to rectifying section

$\Delta_D$  = net upward flow in rectifying section

$\Delta_W$  = net downward flow in stripping section

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