Modified Smith Predictor for Controlling Integrator/Time Delay Processes

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Astrom *et al.* have proposed a new Smith predictor to control the process with an integrator and a long time delay. Though the method provides significant performance improvement, it fails to give effective tuning rules even when the integral constant is equal to 1. In this paper the method is extended to the general integrator/time delay process. A clearer and more logical design procedure is formulated, and simple tuning rules are developed. It shows that there is a minimum-order compensator for the process. Illustrative examples show the superiority of the proposed scheme in terms of both closed-loop performance and robustness.

1. Introduction

In process control, there are two kinds of typical processes: one can be described by time constant plus time delay models, and the other, by integrator plus time delay models (Moore, 1985; Chien and Fruehauf, 1990). For a process consisting of a time constant and a time delay, many controllers have been developed. For example, when the time delay is small, a PID controller is commonly used; when the time delay is large, the Smith predictor (Smith, 1957) is an effective compensator. However, neither the PID controller nor the Smith predictor can be used directly for the integrator/time delay process. A PID controller tuned by Zigler−Nichols rules is found to be too oscillatory (Tyreus and Luyben, 1992), while the Smith predictor results in a steady-state error when there is a constant load disturbance. To solve the problem, Chien and Fruehauf (1990) developed an internal model control approach to selecting the tuning constants for the PID controller. Tyreus and Luyben (1992) pointed out that the approach could lead to poor control unless care was taken in selecting the parameters. They developed equations for the calculation of the optimum PID parameters. Unfortunately, we find that its response is so sluggish that it cannot be applied. Watanabe and Ito (1981) presented a modification of the Smith predictor. The system can reject a load disturbance provided that the time delay of the process is exactly known. Otherwise, there will be a small steady-state error. On the same lines, Astrom *et al.* (1994) proposed a new Smith predictor structure on the premise that the integral constant was normalized to 1. The controller provides superior performance. Its advantage is that the disturbance response is decoupled from the setpoint response and hence can be independently optimized. Nevertheless, the controller has four adjustable parameters. There are no effective rules to direct user to tune them.

In this paper, Astrom's Smith predictor is extended to the general integrator/time delay process. A clearer and more logical design procedure is formulated, and simple tuning rules are developed. It shows that there is a minimum-order compensator for the process. The new controller has only two adjustable parameters which allow the designer to shape the setpoint and disturbance response, respectively. Compared with that of Astrom's Smith predictor, they are simple and transparent in design and in the incorporation of performance and robustness. Furthermore, the new design procedure can be easily extended to a high-order process.
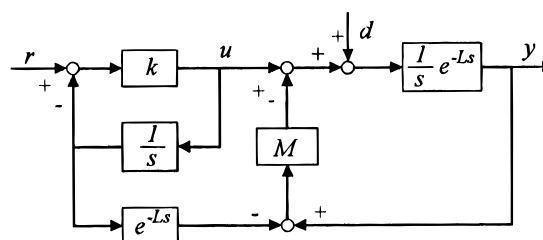

Figure 1. Structure of Astrom's Smith predictor.

The organization of the paper is as follows. A brief introduction to Astrom's method is undertaken first in section 2. Section 3 presents the main result of the paper where the extended structure is developed, and the tuning rules are significantly simplified. In section 4, simulation results are presented to illustrate the approach. Conclusions are given in section 5.

2. Astrom's Smith Predictor

The structure of Astrom's Smith predictor is shown in Figure 1, where k is the gain and $M(s)$ is the compensator. The plant transfer function is

$$G_m(s) = \frac{1}{s}e^{-Ls} \qquad (1)$$

The setpoint response is given by

$$H_r(s) = \frac{k}{s+k}e^{-Ls} \qquad (2)$$

Simply, it is a first-order system with time delay. The setpoint response of the system can be improved by choosing an appropriate value of the controller gain k. The disturbance response is given by

$$H_d(s) = \frac{\dfrac{1}{s}e^{-Ls}}{1 + M(s)\dfrac{1}{s}e^{-Ls}} \qquad (3)$$

Thus, the scheme has decoupled the disturbance response from the setpoint response. It is possible to improve the disturbance response by choosing a different compensator $M(s)$. Astrom *et al.* proposed the following transfer function with three adjustable parameters k_1, k_2, and k_3:
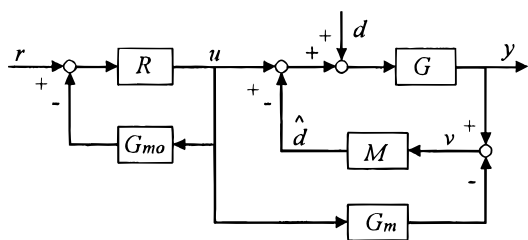
Figure 2. Structure of the modified Smith predictor.

$$M(s) =$$

$$\frac{k_4 + \dfrac{k_3}{s}}{1 + k_1 + \dfrac{k_2}{s} + \dfrac{k_3}{s^2} - \left(\dfrac{k_4}{s} + \dfrac{k_3}{s^2}\right) - \left(\dfrac{k_4}{s} + \dfrac{k_3}{s^2}\right)e^{-Ls}} \qquad (4)$$

where

$$k_4 = k_2 + k_3 L \qquad (5)$$

However, the scheme has two limitations. First, for controller tuning, simplicity, as well as optimality, is important. The three parameters cannot be easily translated into the desired performance and robustness characteristics which the control system designer has in mind. The presence of simple rules which relate model parameters and experimental data to controller parameters serves to simplify the task of the designer. Second, though a general integrator/time delay process can be normalized to (1), it may not be easy to see how to select the structure and parameters of $M(s)$.

3. Modified Smith Predictor

In this paper, we will extend Astrom's method to the general integrator/time delay process and develop simple tuning rules. The structure of a modified Smith predictor is shown in Figure 2, where $R(s)$ is the controller, $G(s)$ is the plant, $G_m(s)$ is the plant model, $G_{m0}(s)$ is the delay free part of $G_m(s)$, and $M(s)$ is the compensator. It can be seen that the proposed Smith predictor comprises an internal model structure. Here we assume that

$$G_m(s) = \frac{1}{Ts}e^{-Ls} \quad \text{and} \quad G_{m0}(s) = \frac{1}{Ts} \qquad (6)$$

where T is the integral constant. If the model is exact ($G_m(s) = G(s)$), the setpoint response is as follows:

$$H_r(s) = \frac{R(s)}{1 + G_{m0}(s)\,R(s)}G(s) = \frac{R(s)}{Ts + R(s)}e^{-Ls} \qquad (7)$$

For a first-order process a proportional controller can just provide good response. Let $R(s) = \lambda_1$, where $\lambda_1 > 0$ is a constant; then the bandwidth of the setpoint loop is determined by λ_1. Though any λ_1 will result in a stable setpoint response, significant plant uncertainty requires the designer to select a larger value of λ_1. Considering that $u = 0$ in case $r = 0$, thus λ_1 has little effect on the disturbance response even when uncertainty exists.

The disturbance response is given by the transfer function

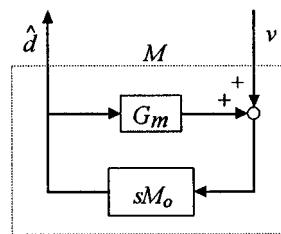$$H_d(s) = \frac{G(s)}{1 + M(s)\,G(s)} = \frac{e^{-Ls}}{Ts + M(s)\,e^{-Ls}} \qquad (8)$$

Figure 3. Compensator $M(s)$.

As we know, the principal advantage of the Smith predictor is that the time delay is eliminated from the characteristic equation of the closed-loop system by utilizing a plant model in the minor feedback loop. In conventional Smith predictor structure the characteristic equations of the setpoint response and disturbance response are the same, while in the new Smith predictor they are different. Thus, $M(s)$ is introduced and has to be selected such that it can eliminate the time delay from the characteristic equation of the disturbance response. The following form of $M(s)$ is proposed

$$M(s) = \frac{sM_0(s)}{1 - sM_0(s)\,G_m(s)} \qquad (9)$$

where $M_0(s)$ is a rational transfer function. In order to be physically realizable, $M_0(s)$ has to be strictly proper. Substitute (9) into (8) to get

$$H_d(s) = \left(1 - \frac{M_0(s)}{T}e^{-Ls}\right)\frac{1}{Ts}e^{-Ls} \qquad (10)$$

The time delay is eliminated from the characteristic equation of $H_d(s)$. Equation 10 implies that the right half-plane poles of disturbance response is just that of $M_0(s)$. Hence, $M_0(s)$ should be a stable, strictly proper and rational transfer function.

Why we select such a compensator can be explained intuitively by Figure 3. First, the effect of the disturbance d is separated from the system output y:

$$v = y - 0 = (d - \hat{d})G \qquad (11)$$

$$v_0 = v + \hat{d}G_m = dG \qquad (12)$$

then, $sM_0(s)$ provides an estimate of the disturbance:

$$\hat{d} = v_0 sM_0 = (dM_0)\frac{e^{-Ls}}{T} \qquad (13)$$

To get a zero steady-state error or a type I system, a constraint must be imposed on the transfer function $H_d(s)$.

$$\lim_{s \to 0} H_d(s) = \lim_{s \to 0}\left(1 - \frac{M_0(s)}{T}e^{-Ls}\right)\frac{1}{Ts}e^{-Ls} = 0 \quad (14)$$

which is equivalent to

$$\lim_{s \to 0}\left(1 - \frac{M_0(s)}{T}e^{-Ls}\right)\frac{1}{T}e^{-Ls} = 0 \qquad (15)$$

and

$$\lim_{s \to 0}\frac{d}{ds}\left(\left(1 - \frac{M_0(s)}{T}e^{-Ls}\right)\frac{1}{T}e^{-Ls}\right) = 0 \qquad (16)$$

With the Final Value Theorem and (15), we get

$$M_0(0) = T \qquad (17)$$

On the other hand, (16) gives

$$2LM_0(0) - LT - M_0(0) = 0 \qquad (18)$$

Thus, $M_0(s)$ should satisfy (17) and (18) simultaneously. Typically, the strictly proper stable function $M_0(s)$ is of the form

$$M_0(s) = \frac{\beta_m s^m + \dots + \beta_1 s + \beta_0}{(\lambda_2 s + 1)^n}, \qquad m < n, \quad \lambda_2 > 0 \qquad (19)$$

where both β_i ($i = 0, 1, \dots, m$) and λ_2 are constants. A little algebra shows that the minimum order of the transfer function $M_0(s)$ is 2. The simplest $M_0(s)$ is of the form

$$M_0(s) = \frac{\beta_1 s + T}{(\lambda_2 s + 1)^2} \qquad (20)$$

Substituting this into (18) yields

$$\beta_1 = LT + 2\lambda_2 T \qquad (21)$$

Therefore

$$M_0(s) = \frac{(LT + 2\lambda_2 T)s + T}{(\lambda_2 s + 1)^2} \qquad (22)$$

Then (10) becomes

$$H_d(s) = \frac{(\lambda_2 s + 1)^2 - ((L + 2\lambda_2)s + 1)e^{-Ls}}{(\lambda_2 s + 1)^2 Ts} e^{-Ls} \qquad (23)$$

By using a model of the process and a minimum-order compensator $M_0(s)$, time delay is removed from the characteristic equation of the disturbance response and a type I system is obtained.

Here, the parameter λ_2 is left to adjust the disturbance response. It is seen that $M_0(s)$ is in fact a low-pass transfer function, and its bandwidth is determined monotonously by λ_2. Then λ_2 has a monotonous relationship with the performance of disturbance response and the robustness of the closed-loop system. Equation 23 is in some sense very complex. It is difficult to give a rigorous theoretical treatment. When considering the plant uncertainty within the bandwidth of the controller, the recommendation of λ_2 is $0.5L - 1.5L$ which is a rule of thumb.

Sometimes, the process is described by a high-order model, i.e.

$$G_m(s) = \frac{1}{Ts(\tau s + 1)} e^{-Ls} \quad \text{and} \quad G_{m0}(s) = \frac{1}{Ts(\tau s + 1)} \qquad (24)$$

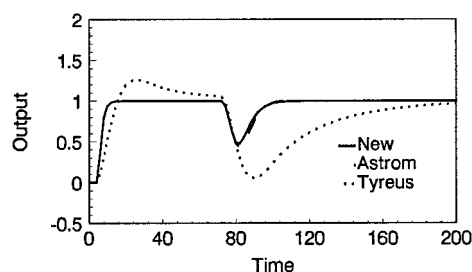a similar design procedure can be developed. Then $R(s)$

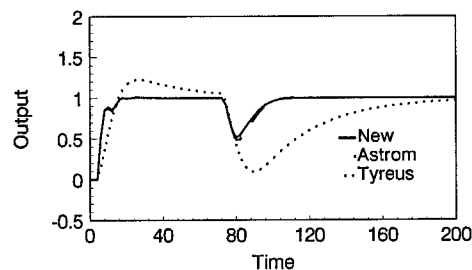**Figure 4.** Response of a nominal system ($T = 1$, $L = 5$).

Figure 5. Response of an uncertain system ($T = 1$, $L = 5$).

is a derivative or a derivative with lag, and a more complex $M(s)$ will result.

4. Simulations

In this section the performance of the modified Smith predictor is compared with the performances of Astrom's Smith predictor and Tyreus' PID controller. A unit step setpoint is introduced at time $t = 0$ values and a load disturbance $d(s) = -0.1/s$ is introduced at time $t = 70$ values.

Example 1. Consider an example of Astrom *et al.* The plant is described by the nominal transfer function

$$G_m(s) = \frac{1}{s} e^{-5s} \qquad (25)$$

i.e., $T = 1$ and $L = 5$. The parameters of Astrom's Smith predictor are $k = 0.6$, $k_1 = 10$, $k_2 = 4.0$, $k_3 = 0.5$, and $k_4 = 6.5$. The parameters of Tyreus' PID controller are $P = 0.1$ and $I = 43.8$. The parameters of the modified Smith predictor are given as $\lambda_1 = 0.6$ and $\lambda_2 = 4$. When the model is exact, the responses are given in the Figure 4. It shows that the results do reflect the expected asymptotic properties. Tyreus' PID controller has a very slow response. Since $\lambda_1 = k$, the setpoint response of the modified Smith predictor is the same as that of Astrom's Smith predictor. The disturbance response of the modified Smith predictor is better than that of Astrom's.

Now assume that there is a 10% error when estimating the time delay of the system. The corresponding process time delay is 4.5 and the estimated time delay is 5. The effect of the error on the system responses is shown in Figure 5. The new controller is the best.

Though the new Smith predictor does not provide much better performance than Astrom's, the tuning rules of the former are significantly simplified. As discussed above, the parameters λ_1 and λ_2 determine the bandwidth of the setpoint loop and disturbance loop; hence, they can be used to optimize setpoint response and disturbance response, respectively. When uncertainty exists, they have a direct relationship with the system robustness. The system response to different λ_1 is shown in Figure 6, and that to different λ_2 is shown in Figure 7. Notice that λ_2 plays a central role, while
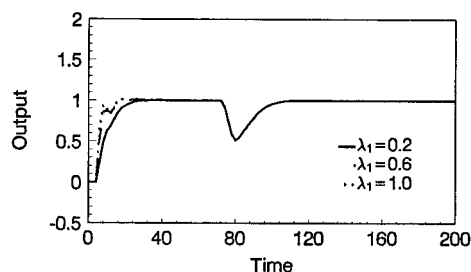
Figure 6. Response for different λ_1.
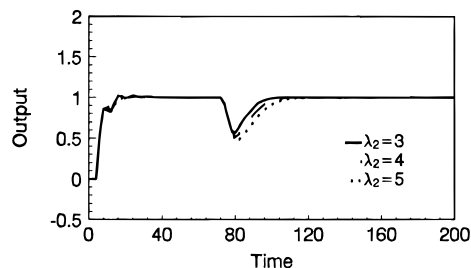

Figure 7. Response for different λ_2.

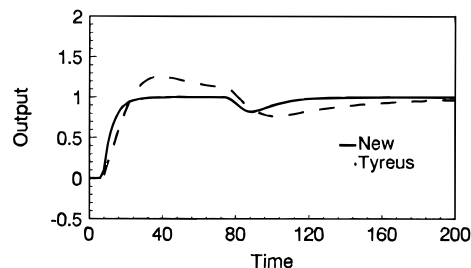**Figure 8.** Response of a nominal system ($T = 5$, $L = 7.4$).

λ_1 has a relationship only with the setpoint response. The larger λ_1 or the smaller λ_2 corresponds to a high speed of response and poor robustness, while the smaller λ_1 or the larger λ_2 corresponds to a low speed of response and good robustness.

Example 2. Consider the plant studied by Tyreus and Luyben

$$G_{\mathrm{m}}(s) = \frac{1}{5s}e^{-7.4s} \qquad (26)$$

i.e., $T = 5$ and $L = 7.4$. The PID controller settings given by Tyreus and Luyben are $P = 0.33$ and $I = 64.7$. For Astrom's Smith predictor, it is hard to see how to select the structure and parameters. However, for the modified Smith predictor, a good choice is that $\lambda_1 = 1$ and $\lambda_2 = 8$. The nominal performances are shown in Figure 8. Suppose that there are 20% errors when estimating the model. The process integrator constant is 4, and the time delay is 5.92. The estimated values
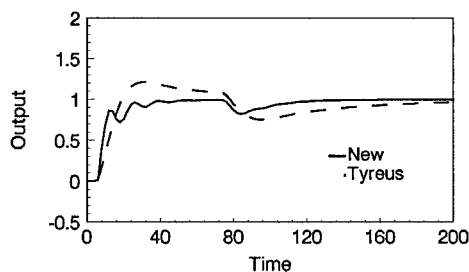

Figure 9. Response of an uncertain system ($T = 5$, $L = 7.4$).

are 5 and 7.4, respectively. The responses when modeling error exists are shown in Figure 9. The response of the new controller is much better than that of Tyreus' PID controller.

5. Conclusions

A general form of Astrom's Smith predictor is successfully developed in this paper. Design methodology for the new Smith predictor is given and a minimum-order compensator is obtained. The important improvement of the proposed modified Smith predictor is that it provides an easier possibility of tuning the parameters, and the setpoint response and the disturbance response can be optimized by only one parameter, respectively. The two parameters have a direct relationship with the system bandwidths or equivalently the performance and robustness of the closed-loop system. Simulations show that the new controller is superior to the previous algorithms.

Literature Cited

Astrom, K. J.; Hang, C. C.; Lim, B. C. A New Smith Predictor for Controlling a Process with an Integrator and Long Dead Time. *IEEE Trans. Autom. Control* **1993**, *39*, 343.

Chien, I. L.; Fruehauf, P. S. Consider IMC Tuning to Improve Performance. *Chem. Eng. Prog.* **1990**, *10*, 33.

Moore, R. L. *The Dynamic Analysis of Automatic Process Control*; ISA: Research Triangle Park, NC, 1985.

Smith, O. J. M. Close Control of Loops with Dead Time. *Chem. Eng. Prog.* **1957**, *53*, 217.

Tyreus, B. D.; Luyben, W. L. Tuning PI Controllers for Integrator/Dead Time Processes. *Ind. Eng. Chem. Res.* **1992**, *31*, 2625.

Watanabe, K.; Ito, M. A Process Model Control for Linear Systems with Delay. *IEEE Trans. Autom. Control* **1981**, *26*, 1261.