PROCESS DESIGN AND CONTROL

Improved Filter Design in Internal Model Control

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The widely published internal model control (IMC) proportional-integral-derivative (PID) tuning rules provide poor load disturbance suppression for processes in which the desired closed-loop dynamics is significantly faster than the open-loop dynamics. The IMC filter is modified to derive low-order controllers that provide effective disturbance suppression irrespective of the location at which the disturbances enter the closed-loop system.

1. Introduction

For decades engineers have worked to develop improved tuning rules for proportional-integral-derivative (PID) controllers (Chien and Fruehauf, 1990; Cohen and Coon, 1953; Hang et al., 1991; Smith and Corripio, 1985; Ziegler and Nichols, 1943). The well-known internal model control (IMC) PID tuning rules have the advantage that a clear tradeoff between closed-loop performance and robustness to model inaccuracies is achieved with a single tuning parameter (Rivera et al., 1986). The IMC-PID tuning rules are taught to most undergraduate chemical engineers (Ogunnaike and Ray, 1994; Seborg et al., 1989) and are widely applied in industry (Chien and Fruehauf, 1990).

However, several academic and industrial process control engineers (Åström and Hagglund, 1988; Åström et al., 1993; Bergh and MacGregor, 1987; Chien and Fruehauf, 1990; Ho et al., 1994; Ho and Zhou, 1995) have noted that the widely published IMC tuning rules (Ogunnaike and Ray, 1994; Rivera et al., 1986; Seborg et al., 1989), while providing adequate suppression of output disturbances, do a poor job suppressing load disturbances when the process dynamics are significantly slower than the desired closed-loop dynamics. Morari and co-workers proposed to address this problem by including an additional integrator in the output disturbance while performing the IMC design procedure (Scali et al., 1992). This method was found to provide adequate load disturbance suppression for many processes and has been applied to model predictive control (Morari et al., 1996). However, the resulting controllers do not have PID structure.

Here we develop a table of IMC tuning rules which provide adequate disturbance suppression irrespective of the location at which the disturbance enters the closed-loop system. The tuning rules provide the parameters for a PID controller in series with a filter and are derived by incorporating phase lead in the IMC filter. Such controllers are easily implemented using modern control hardware. The basic approach was first proposed and applied by Brosilow and Markale (Brosilow and Markale, 1992) to cascade controller design and provides performance similar to that obtained by the method of Morari and co-workers (Scali et al., 1992).

2. Controller Design

The classical control structure used for the feedback control of single-loop processes is shown in Figure 1. In the diagram, p refers to the transfer function of the process; d and l refer to the output and load disturbances, respectively; y refers to the controlled variable; n refers to measurement noise; r refers to the setpoint; and u refers to the manipulated variable specified by the controller k. The controlled variables are related to the setpoint, measurement noise, and unmeasured disturbances by

$$y = Sd + pSl + T(r - n) \tag{1}$$

where

$$S = \frac{1}{1 + pk}; \qquad T = 1 - S = \frac{pk}{1 + pk} \tag{2}$$

are the *sensitivity* and *complementary sensitivity* functions, respectively. In IMC, the process model $\tilde{p}$ is factored into an all-pass portion $\tilde{p}_A$ and a minimum phase portion $\tilde{p}_M$

$$\tilde{p} = \tilde{p}_A \tilde{p}_M \tag{3}$$

The all-pass portion $\tilde{p}_A$ includes all the open right-half-plane zeros and delays of $\tilde{p}$ and has the form

$$\tilde{p}_A = e^{-s\theta} \prod_i \frac{-s + z_i}{s + \bar{z}_i} \tag{4}$$

where $\theta > 0$ is the time delay, z_i is a right-half-plane zero ($\mathrm{Re}\{z_i\} > 0$) in the process model, and $\bar{z}_i$ is the complex conjugate of z_i.

In IMC, the complementary sensitivity is equal to (Garcia and Morari, 1982; Rivera et al., 1986)

$$T = \tilde{p}_A f \tag{5}$$

where the conventional IMC filter f is selected to have one of the following forms:

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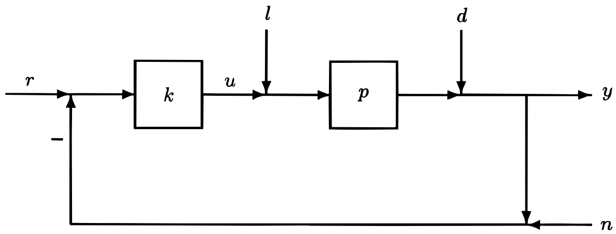

Figure 1. Classical control structure.

Type I: $$f(s) = \frac{1}{(\lambda s + 1)^n} \tag{6}$$

Type II: $$f(s) = \frac{n\lambda s + 1}{(\lambda s + 1)^n} \tag{7}$$

The type I filter ensures perfect setpoint tracking of steps, while the type II filter ensures perfect setpoint tracking of ramps. The filter order n is selected large enough to make $f/\tilde{p}_M$ proper, and the adjustable filter parameter λ provides the tradeoff between performance and robustness. From (5) we see that λ is the desired closed-loop time constant for setpoint tracking.

While the conventional filters provide good performance for setpoint tracking and output disturbance suppression, they provide poor performance for load disturbance suppression when the process contains a pole slower than the desired closed-loop speed of response. This can be seen from (1), where the slow process pole appears in pS irrespective of the selection of λ.

To fix this problem, we propose to use the alternative filter forms:

Type I: $$f(s) = \frac{\beta s + 1}{(\lambda s + 1)^n} \tag{8}$$

Type II: $$f(s) = \frac{\beta s^2 + n\lambda s + 1}{(\lambda s + 1)^n} \tag{9}$$

where β is chosen so that the slow pole of p is canceled by a zero in the sensitivity S.

The expression for T in (5) can be substituted into (2) and rearranged to arrive at the controller

$$k = \frac{T}{\tilde{p}(1 - T)} = \frac{\tilde{p}_A f(s)}{\tilde{p}_A \tilde{p}_M (1 - \tilde{p}_A f(s))} = \frac{f(s)}{\tilde{p}_M(1 - \tilde{p}_A f(s))} \tag{10}$$

Because most models for chemical processes are low order, IMC controllers based on these models are of low order and can be written in the form of a *proportional-integral-derivative* (PID) controller in series with a second-order filter

$$k = k_c\left(1 + \tau_D s + \frac{1}{\tau_I s}\right)\frac{1 + cs + ds^2}{1 + as + bs^2} \tag{11}$$

where k_c is the gain, τ_I is the integral time, τ_D is the derivative time constant, and a, b, c, and d are filter parameters. The second-order filter ensures that the nominal PID controller is proper (as must be required by any practical controller) and is easily implemented using modern control hardware. The form of the controller in (11) is an extension to the modified PID controller structure proposed by Rivera et al. (1986).

3. Example: Tuning Rules for First Order with Time Delay

Consider a first-order process with time delay:

$$p = \frac{k_p e^{-s\theta}}{\tau s + 1} \tag{12}$$

where k_p is the steady-state gain, τ is the time constant, θ is the time delay, and the desired closed-loop time constant λ is less than τ. The process model $\tilde{p}$ is factored into minimum-phase and all-pass portions

$$\tilde{p} = \frac{k_p}{\tau s + 1}\left(\frac{1 - \left(\frac{\theta}{2}\right)s}{1 + \left(\frac{\theta}{2}\right)s}\right); \qquad \tilde{p}_M = \frac{k_p}{\tau s + 1};$$
$$\tilde{p}_A = \frac{1 - \left(\frac{\theta}{2}\right)s}{1 + \left(\frac{\theta}{2}\right)s} \tag{13}$$

where the time delay has been modeled with a first-order Padé approximation. Using the *conventional* type I filter (6) with $n = 1$ and substituting into (10) gives

$$k = \frac{f}{\tilde{p}_M(1 - \tilde{p}_A f)} = \frac{\tau s + 1}{k_p}\,\frac{1}{1 + \lambda s}\,\frac{1}{1 - \dfrac{1 - \frac{\theta}{2}s}{\left(1 + \frac{\theta}{2}s\right)(\lambda s + 1)}} = \frac{1}{k_p}\,\frac{1 + \left(\tau + \frac{\theta}{2}\right)s + \left(\frac{\tau\theta}{2}\right)s^2}{(\lambda + \theta)s + \left(\frac{\lambda\theta}{2}\right)s^2} \tag{14}$$

which can be rearranged to be in the form of (11), with

$$k_c = \frac{2\tau + \theta}{2(\lambda + \theta)k_p}; \qquad \tau_I = \frac{\theta}{2} + \tau; \qquad \tau_D = \frac{\tau\theta}{2\tau + \theta};$$
$$a = \frac{\lambda\theta}{2(\lambda + \theta)}; \qquad b = c = d = 0 \tag{15}$$

Applying this controller (15) to the model (13) yields the nominal closed-loop relationship between the disturbances and the controlled output

$$\tilde{y} = \frac{s\left(\lambda + \theta + \frac{\lambda\theta}{2}s\right)}{\left(1 + \frac{\theta}{2}s\right)(\lambda s + 1)}d + \frac{k_p\left(1 - \frac{\theta}{2}s\right)}{(1 + \tau s)\left(1 + \frac{\theta}{2}s\right)}\,\frac{s\left(\lambda + \theta + \frac{\lambda\theta}{2}s\right)}{\left(1 + \frac{\theta}{2}s\right)(\lambda s + 1)}l \tag{16}$$

The dynamics between the load disturbance and the controlled output contains the slow process pole at $s = -1/\tau$. The effect of this slow process pole can be seen the long tail in Figure 2 for $\lambda = 20$ and $\lambda = 40$ (the controller k from (14) and the process transfer function p in (12) were substituted into (1) for the simulations). Note that selecting a higher order ($n > 1$) conventional filter (7) cannot remove the slow pole in (16) and hence will also result in IMC controllers with sluggish load disturbance suppression.

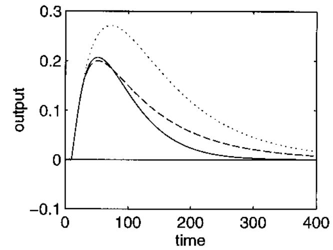

Figure 2. Output response to step load disturbance for first-order plus time delay process with $k_p = 1$, $\theta = 10$, and $\tau = 100$: conventional filter design ($\lambda = 20$, - - -; $\lambda = 40$, ···); alternative filter design ($\lambda = 40$, —). A solid horizontal line at $y = 0$ is included to make the comparison clearer.

The *alternative* type I filter (8) with $n = 2$ gives the nominal closed-loop relationship

$$\tilde{y} = \frac{\left(1 + \frac{\theta}{2}s\right)(\lambda s + 1)^2 - \left(1 - \frac{\theta}{2}s\right)(\beta s + 1)}{\left(1 + \frac{\theta}{2}s\right)(\lambda s + 1)^2} d + \frac{k_p\left(1 - \frac{\theta}{2}s\right)}{(\tau s + 1)\left(1 + \frac{\theta}{2}s\right)} \frac{\left(1 + \frac{\theta}{2}s\right)(\lambda s + 1)^2 - \left(1 - \frac{\theta}{2}s\right)(\beta s + 1)}{\left(1 + \frac{\theta}{2}s\right)(\lambda s + 1)^2} l \tag{17}$$

The extra degree of freedom β is selected to cancel the open-loop pole at $s = -1/\tau$ that causes the sluggish response to load disturbances:

$$\beta = \frac{\lambda^2\theta + 2\tau(\theta(\tau - \lambda) + \lambda(2\tau - \lambda))}{\tau(\theta + 2\tau)} \tag{18}$$

Note that the assumption that $\lambda < \tau$ implies that $\beta > 0$; thus, the filter will not introduce any undesirable RHP zeros into the closed-loop system.

Substituting β into (17) gives the nominal closed-loop relationship with the alternative filter

$$\tilde{y} = \frac{\left(1 + \frac{\theta}{2}s\right)(\lambda s + 1)^2 - \left(1 - \frac{\theta}{2}s\right)(\beta s + 1)}{\left(1 + \frac{\theta}{2}s\right)(\lambda s + 1)^2} d + \frac{k_p\left(1 - \frac{\theta}{2}s\right)}{\left(1 + \frac{\theta}{2}s\right)} \frac{\frac{\theta}{2\tau}\lambda^2 s + \frac{1}{\tau}\left(\lambda^2 + \theta\lambda + \frac{\theta\beta}{2} - \frac{\theta\lambda^2}{2\tau}\right)}{\left(1 + \frac{\theta}{2}s\right)(\lambda s + 1)^2} l \tag{19}$$

The controller with the alternative filter is computed from (10) and can be rearranged into the modified PID structure (11)

$$k_c = \frac{2\tau + \theta}{2(2\lambda + \theta - \beta)k_p}; \qquad \tau_{\mathrm{I}} = \tau + \frac{\theta}{2};$$
$$\tau_{\mathrm{D}} = \frac{\tau\theta}{2\tau + \theta}; \qquad a = \frac{2\lambda\theta + 2\lambda^2 + \beta\theta}{2(2\lambda + \theta - \beta)};$$
$$b = \frac{\lambda^2\theta}{2(2\lambda + \theta - \beta)}; \qquad c = \beta; \qquad d = 0 \tag{20}$$

The first-order Padé approximation of the time delay begins to deteriorate for frequencies greater than $1/\theta$, which implies that the desired closed-loop time constant λ should be selected greater than θ for the controller to be robust.

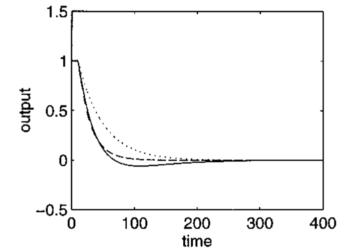

Figure 3. Output responses to step output disturbance for first-order plus time delay process with $k_p = 1$, $\theta = 10$, and $\tau = 100$: conventional filter design ($\lambda = 20$, - - -; $\lambda = 40$, ···); alternative filter design ($\lambda = 40$, —).

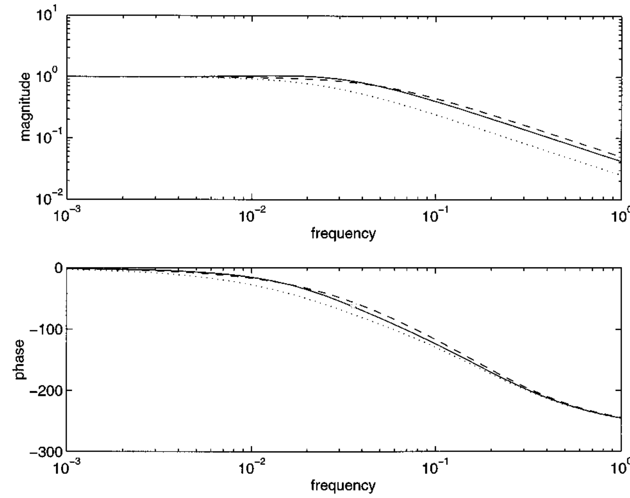

Figure 4. Bode magnitude and phase plots of T for process model with Padé approximation and $k_p = 1$, $\theta = 10$, and $\tau = 100$: conventional filter design ($\lambda = 20$, - - -; $\lambda = 40$, ···); alternative filter design ($\lambda = 40$, —).

The response to a load disturbance with the alternative filter does not contain the long tail that resulted when using the conventional filter (see Figure 2). The explanation for this is revealed in the output responses (16) and (19). The load disturbance transfer function in (16) contains the slow pole of the process, whereas β cancels this slow pole in (19). With the alternative filter, the settling time is approximately equal to $\theta + 5\lambda = 210$ s, which corresponds to the desired closed-loop time constant $\lambda = 40$. For the conventional filter, the response is not settled by 210 s, even with λ reduced to 20. With the conventional filter, the closed-loop speed of response to a load disturbance is a stronger function of the slow process pole than of the desired closed-loop time constant. The filter parameter must be very nearly zero for the long tail to disappear, which would lead to very poor robustness. Although the load disturbance responses are quite dissimilar for the different IMC filter designs, the output disturbance responses are all acceptable (see Figure 3).

Robustness vs Performance Tradeoff. The alternative IMC filter incorporates phase lead in the controller to offset the slow pole (phase lag) in the process. The time domain responses in Figures 2 and 3 show that the controller is robust to the plant/model mismatch caused by the Padé approximation.

The magnitude of T quantifies the robustness of single-loop systems. The Bode plots of T for different controller designs are shown in Figure 4. The magnitude of T for the conventional filter design with $\lambda = 20$ is very similar to that of the alternative filter design

Table 1. IMC Controllers for Processes with Open-Loop Dynamics Slower than Closed-Loop Dynamics ($\lambda < \tau$):[a]

$$k = k_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)\frac{1 + cs + ds^2}{1 + as + bs^2}$$

	model $\tilde{p}$	system type	filter f	k_c	τ_I	τ_D	a	b	c	d	β
A	$\frac{k_p}{\tau s + 1}$	I	$\frac{1 + \beta s}{(1 + \lambda s)^2}$	$\frac{\tau + \beta}{k_p(2\lambda - \beta)}$	$\tau + \beta$	$\frac{\tau\beta}{\tau + \beta}$	$\frac{\lambda^2}{2\lambda - \beta}$	0	0	0	$\frac{2\tau\lambda - \lambda^2}{\tau}$
B	$\frac{k_p(-s + z)}{(\tau_1 s + 1)(\tau_2 s + 1)}$	I	$\frac{1 + \beta s}{(1 + \lambda s)^2}$	$\frac{\tau_1 + \tau_2}{k_p(z(2\lambda - \beta) + 2)}$	$\tau_1 + \tau_2$	$\frac{\tau_1\tau_2}{\tau_1 + \tau_2}$	$\frac{z\lambda^2 + 2\lambda + \beta}{z(2\lambda - \beta) + 2}$	$\frac{\lambda^2}{z(2\lambda - \beta) + 2}$	0	0	$\frac{2\tau^2 z\lambda + 2\tau^2 - \tau z\lambda^2 - 2\tau\lambda + \lambda^2}{\tau^2 z + \tau}$
C	$\frac{k_p(-s + z)}{\tau s + 1}$	I	$\frac{1 + \beta s}{1 + \lambda s}$	$\frac{\tau + \beta}{k_p(z\lambda - z\beta + 2)}$	$\tau + \beta$	$\frac{\tau\beta}{\tau + \beta}$	$\frac{\lambda + \beta}{z(\lambda - \beta) + 2}$	0	0	0	$\frac{\tau(z\lambda + 2) - \lambda}{\tau z + 1}$
D	$\frac{k_p}{(\tau_1 s + 1)(\tau_2 s + 1)}$	I	$\frac{1 + \beta s}{(1 + \lambda s)^3}$	$\frac{\tau_1 + \tau_2}{k_p(3\lambda - \beta)}$	$\tau_1 + \tau_2$	$\frac{\tau_1\tau_2}{\tau_1 + \tau_2}$	$\frac{3\lambda^2}{3\lambda - \beta}$	$\frac{\lambda^3}{3\lambda - \beta}$	β	0	$\frac{3\tau^2\lambda - 3\tau\lambda^2 + \lambda^3}{\tau^2}$
E	$\frac{k_p}{s(\tau s + 1)}$	II	$\frac{1 + 4\lambda s + \beta s^2}{(1 + \lambda s)^4}$	$\frac{\tau}{k_p(6\lambda^2 - \beta)}$	τ	0	$\frac{4\lambda^3}{6\lambda^2 - \beta}$	$\frac{\lambda^4}{6\lambda^2 - \beta}$	4λ	β	$\frac{6\tau^2\lambda^2 - 4\tau\lambda^3 + \lambda^4}{\tau^2}$

[a] All process parameters are assumed to be positive and real (z, τ_2, τ_1, τ, $k_p > 0$). The τ in the β column of rows B and D represents the slowest pole in the process (either τ_1 or τ_2). All controller parameters are positive (K_c, τ_I, τ_D, a, b, c, $d > 0$) and provide the desired performance for $\lambda < \tau$.

with $\lambda = 40$, indicating that the two controllers provide very similar robustness. On the other hand, the conventional filter results in poor load disturbance suppression for any λ not nearly zero. In other words, the alternative filter provides a tradeoff between robustness and good performance, whereas the conventional filter provides a tradeoff between robustness and poor performance (in terms of load disturbance suppression).

4. Table of Improved Tuning Rules

The tuning parameters for common low-order process models have been calculated using the alternative filter forms (see Table 1). Straightforward manipulations of the algebraic expressions listed in Table 1 indicate that all the controller parameters are positive and well-posed (e.g., do not introduce right-half-plane zeros in the controller) when the desired closed-loop speed of response is faster than the open-loop speed of response (that is, $\lambda < \tau$). For $\lambda < \tau$, extensive simulations have confirmed that the controllers provide the desired performance for setpoint tracking and both load and output disturbance suppression. The conventional IMC tuning rules of Rivera et al. (1986) that are reported in undergraduate textbooks (Ogunnaike and Ray, 1994; Seborg et al., 1989) should be used for $\lambda > \tau$.

5. Extension to Higher Order Models

The version of IMC summarized in section 2 is that of (Garcia and Morari, 1982; Rivera et al., 1986), in which the IMC controller (commonly written as $\tilde{q}$) is set equal to the inverse of the minimum-phase portion of the process model (see references for details). This is equivalent to the direct synthesis method described in undergraduate textbooks (Ogunnaike and Ray, 1994; Seborg et al., 1989). A more general expression for the IMC controller (e.g., see eqs 4.1–7 of (Morari and Zafiriou, 1989) or eq 10.165 of (Braatz, 1995)) provides improved controllers for some high-order process models. For the process models in Table 1, the two IMC methods result in the same controller, so for brevity only the simpler version of IMC was described in section 2.

6. Conclusions

A table of IMC tuning rules was developed which provides good load disturbance suppression for processes in which the desired closed-loop speed of response is faster than the open-loop speed of response. For low-order process models, the controllers have the form of a PID controller in series with a filter. The resulting controllers recover the most desirable property of IMC, that a clear tradeoff between closed-loop performance and robustness to model inaccuracies is achieved with a single tuning parameter, the desired closed-loop time constant.

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