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## End-Tethered Chains in Polymeric Matrices

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### I. Introduction

At high enough coverage polymer chains tethered by one end to a flat solid surface stretch away from the surface forming a polymer "brush". These brushes—which are of both practical and theoretical interest—have been the subject of intense study since the pioneering work of Alexander and de Gennes,<sup>1,2</sup> and several review articles on the subject are now available.<sup>3–5</sup> Most of the existing studies focus on brushes exposed to a low molecular weight solvent and less is known of the behavior of surface-tethered polymers in polymeric matrices.<sup>6–11</sup> The case where the grafted chains are in contact with a melt of shorter, chemically identical chains was studied theoretically by de Gennes some years ago<sup>2</sup> (see also the work of Leibler<sup>12</sup>) and more recently by Raphaël, Pincus, and Fredrickson<sup>13</sup> (hereafter referred to as RPF). In addition to a Flory-type approach based on a virial expansion for the entropy of mixing, RPF developed a blob analysis of the various existing regimes and introduced the concept of "cylindrical blobs". In this Note we would like to propose a slightly different scaling picture based entirely on spherical subunits. At sufficiently high grafting densities, the subunit diameter  $\Lambda$  is no longer proportional to the average distance between two grafting sites  $D$ . In particular, in the regime where the mobile chains are not present in the brush,  $\Lambda$  varies as  $\Lambda \approx a(D/a)^2$ , where  $a$  is the monomer size. Throughout this Note we assume a steplike concentration profile and impose that all the free ends be at the same distance from the surface. In all our formulas the exact prefactors remain undetermined.

### II. Flory-Type Argument

Consider a brush made of chains, with degree of polymerization  $N$ , terminally grafted onto a flat surface and exposed to a polymeric solvent made of chemically identical chains, with degree of polymerization  $P < N$ . The number of terminally grafted chains per unit area is  $\sigma a^{-2}$ , where  $a$  is the monomer size. The average distance between two grafting sites is given by  $D =$

$a\sigma^{-1/2}$ . At sufficiently low  $\sigma$ , the grafted chains do not overlap (the so-called mushroom regime<sup>2</sup>) and the layer thickness  $L$  is given by  $L \approx aN^{1/2}$  for  $P > N^{1/2}$  and by  $L \approx aN^{3/5}P^{-1/5}$  for  $P < N^{1/2}$  (see regions 1 and 2 in Figure 1). As  $\sigma$  increases, the different chains begin to overlap for  $D \approx L$ . This defines an overlap concentration:  $\sigma_{ov} \approx N^{-1}$  for  $P > N^{1/2}$  and  $\sigma_{ov} \approx N^{-6/5}P^{2/5}$  for  $P < N^{1/2}$ . In a Flory-type approach, the free energy per chain is then given by<sup>2</sup>

$$\frac{F}{kT} \approx \frac{L^2}{a^2 N} + \frac{a^3}{P} \frac{N^2}{LD^2} \quad (1)$$

The first term in eq 1 represents the elastic contribution. The second term corresponds to the effect of the (screened) two-body interactions. From eq 1 one can easily construct the  $(P, \sigma)$  diagram represented in Figure 1.<sup>2,11,13</sup> In region 3 (i.e., in regions 3a and 3b), two-body interactions are relevant and the brush thickness is obtained by minimizing eq 1:  $L \approx aNP^{-1/3}\sigma^{1/3}$ . In region 4, repulsive interactions are not sufficient to swell the brush and the conformation of a grafted chain remains Gaussian:  $L \approx aN^{1/2}$ . At higher  $\sigma$ , we reach region 5 where the  $P$  chains are almost completely expelled from the brush: the brush is "dry". In this region the volume fraction of the grafted polymer is of order unity and  $L \approx aN\sigma$ . The boundaries between regions 3 and 4 and between regions 3 and 5 are given by  $\sigma \approx PN^{-3/2}$  and  $\sigma \approx P^{-1/2}$ , respectively.

### III. Scaling Analysis

We now try to build a scaling analysis for the moderate-coverage regime represented by region 3. Let us assume that the fundamental distance of the problem is  $D$ , the average distance between two grafting sites. A grafted chain may then be subdivided into spherical blobs of size  $D$ , each containing  $g_D$  monomers. Within one blob, the chain behaves like a free chain and can therefore be pictured as a self-avoiding walk of subunits called *melt blobs*.<sup>13–14</sup> This leads to the relation  $D \approx ag_D^{3/5}P^{-1/5}$ . Different blobs repel each other and the brush is essentially a closely packed system of blobs. The volume fraction  $\phi$  of monomers belonging to grafted chains is given by  $\phi \approx g_D(a/D)^3 \approx \sigma^{2/3}P^{1/3}$ . Since, on the other hand,  $\phi \approx Na\sigma/L$ , we obtain for the equilibrium brush thickness

$$L \approx aNP^{-1/3}\sigma^{1/3} \quad (2)$$

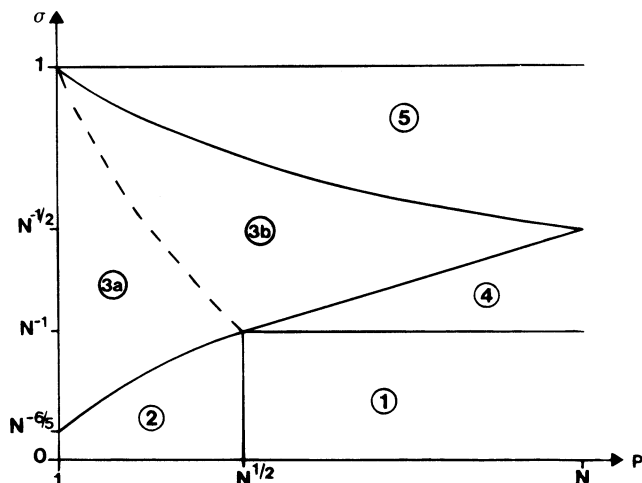
Note that eq 2 can be rewritten as  $L \approx (N/g_D)D$ . This last result indicates that the chain can be viewed as a string of blobs almost fully stretched along the normal to the wall. The  $kT$  per blob ansatz leads to a free energy per chain  $F/kT \approx NP^{-1/3}\sigma^{5/6}$ .

When the grafting density increases, the blob size  $D$  progressively decreases up to a point where it is of the order of the melt blob size  $l_c \approx aP$ . The crossover occurs for  $D \approx aP$ , that is for a grafting density  $\sigma^* \approx P^{-2}$  (this grafting density corresponds to the broken line of Figure

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**Figure 1.** Schematic  $(P, \sigma)$  diagram for a brush exposed to a chemically identical high molecular weight solvent. The thickness of the layer in the different regions is given as follows: region 1,  $L \approx aN^{1/2}$ ; region 2,  $L \approx aN^{3/5}P^{-1/5}$ ; region 3,  $L \approx aNP^{-1/3}\sigma^{1/3}$ ; region 4,  $L \approx aN^{1/2}$ ; region 5,  $L \approx aN\sigma$ . According to the scaling analysis, region 3 is subdivided into regions 3a and 3b (see text).

1). Therefore, the blob picture developed above is only valid in region 3a of the  $(P, \sigma)$  diagram.

What happens in region 3b? By analogy with the blob picture developed for region 3a, one might consider describing the chain as a linear string of subunits of size  $D$ . Since  $D$  is now smaller than the melt blob size  $l_c$ , the chain behavior at a scale smaller than  $D$  is ideal ( $g_D \approx (D/a)^2$  monomers per subunit). This picture leads to a brush thickness

$$L \approx (N/g_D)D \approx aN\sigma^{1/2} \quad (3)$$

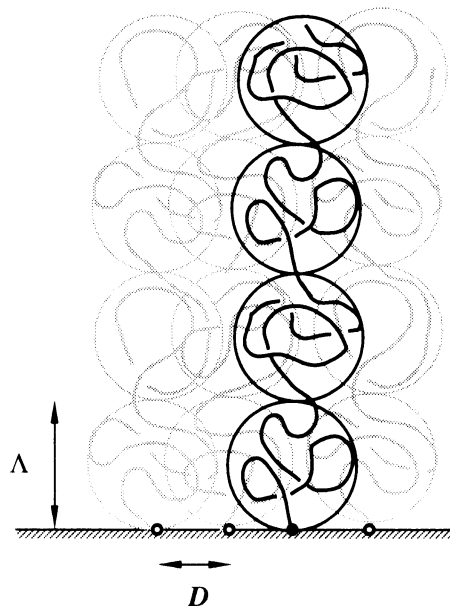
This result is however in disagreement with the Flory prediction  $L \approx aNP^{-1/3}\sigma^{1/3}$ . In order to solve that problem, RPF introduced the concept of "cylindrical blobs".<sup>15</sup> We here propose another approach where one has to give up the idea that the subunit size is proportional to the average distance between two grafting sites  $D$ . Our picture is as follows (see Figure 2): (i) Each chain can be viewed as a string of nonoverlapping spherical subunits, of size  $\Lambda$  larger than  $D$ , almost fully stretched along the normal to the wall. (ii) At a scale smaller than  $\Lambda$ , the chain behaves like an ideal chain of  $g_\Lambda \approx (\Lambda/a)^2$  monomers. It is important to notice that in a plane parallel to the wall, subunits of different chains do overlap and therefore the brush as a whole cannot be described as a closely packed system of subunits (see Figure 2). In order to calculate the subunit size  $\Lambda$ , we have to consider the perturbation parameter  $\zeta \approx a^3P^{-1}g_\Lambda C$ , where  $C$  is the average concentration of monomers belonging to grafted chains. Within a given subunit the total number of monomers (belonging to grafted chains) is given by  $G_\Lambda \approx g_\Lambda(\Lambda/D)^2$ , leading to  $C \approx G_\Lambda/\Lambda^3 \approx \Lambda/(aD)^2$ . At a scale larger than  $\Lambda$ , the chain ceases to be ideal. The subunit size  $\Lambda$  is therefore determined by the condition  $\zeta \approx 1$ . Thus

$$\Lambda \approx aP^{1/3}\sigma^{-1/3} \quad (4)$$

and

$$g_\Lambda \approx (\Lambda/a)^2 \approx P^{2/3}\sigma^{-2/3} \quad (5)$$

Since the different subunits of a grafted chain repel each



**Figure 2.** Schematic representation of a brush in region 3b. Each chain can be pictured as a string of nonoverlapping spherical subunits of size  $\Lambda > D$ , almost fully stretched along the normal to the wall. In a plane parallel to the wall, subunits of different chains overlap.

other, the brush thickness is given by

$$L \approx (N/g_\Lambda)\Lambda \approx aNP^{-1/3}\sigma^{1/3} \quad (6)$$

in agreement with the Flory result. The  $kT$  per subunit ansatz leads to a free energy per chain

$$\frac{F}{kT} \approx \frac{N}{g_\Lambda} \approx NP^{-2/3}\sigma^{2/3} \quad (7)$$

Note that eqs 6 and 7 are in accordance with the predictions of RFP. The present approach is more satisfactory from the physical point of view, however.<sup>16</sup>

We now consider what happens at larger grafting densities  $\sigma > P^{-1/2}$  (region 5). In this regime the  $P$  chains are almost completely expelled from the brush and the thickness  $L$  is related to the grafting density  $\sigma$  by the requirement  $\phi \approx Na\sigma/L \approx 1$ . This leads to  $L \approx aN\sigma$ . The free energy per chain consists merely of an elastic term

$$\frac{F}{kT} \approx \frac{L^2}{a^2N} \approx N\sigma^2 \quad (8)$$

It is of some interest to build up a scaling picture for this high-coverage regime. As in region 3b, each chain can be viewed as a string of subunits of size  $\Lambda$  larger than  $D$  almost fully stretched along the normal to the wall; at a scale smaller than  $\Lambda$ , the chain behaves like an ideal chain (the corresponding number of monomers is given by  $g_\Lambda \approx (\Lambda/a)^2$ ). Within a given subunit, there is on average a number  $(\Lambda/D)^2$  of other chains. The subunit size  $\Lambda$  can be determined as follows. For  $\sigma \approx P^{-1/2}$ ,  $\Lambda$  is given by  $aP^{1/2}$  (see eq 4). Assuming the scaling form  $\Lambda \approx aP^{1/2}(\sigma/P^{-1/2})^m$  and imposing the condition that  $\Lambda$  is independent of  $P$ , we find  $m = -1$ . This leads to

$$\Lambda \approx a(D/a)^2 \quad (9)$$

Note that the  $kT$  per subunit ansatz leads to a free

energy per chain  $F/kT \cong N/g_{\Lambda} \cong N(a/D)^4$ , in agreement with eq 8. The result (9) might be of some relevance in the study of the structures formed by diblock copolymers.<sup>17</sup>

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