

## Characterization of the Chemical Behavior of the Low Excited States through a Local Chemical Potential

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**Abstract:** Exploiting the locality of the chemical potential of an excited state when it is evaluated using the ground-state density functional theory (DFT), a new descriptor for excited states has been proposed. This index is based on the assumption that the relaxation of the electronic density drives the chemical reactivity of excited states. The sign of the descriptor characterizes the electrophilic or nucleophilic behavior of the atomic regions. A relation between the new descriptor and the dual descriptor is derived and provides a posteriori justification of its use to rationalize the Woodward–Hoffmann rules for photochemical reactions within the conceptual DFT. Finally, the descriptor is successfully applied to some [2 + 2] photocycloadditions, like Paterno–Büchi reactions.

### 1. Introduction

During the past two decades, conceptual density functional theory (DFT)<sup>1–4</sup> has been a fruitful paradigm for the analysis of chemical processes. Through successive derivatives of the energy, with respect to either the number of electrons or the external potential, different reactivity and selectivity descriptors have been designed to account for the outcome of chemical reactions.<sup>5,6</sup> More importantly, descriptors previously proposed on an intuitive basis found a sharp definition within conceptual DFT. For instance, the electronegativity defined by Mulliken<sup>7</sup> has been identified as the opposite of the first derivative of the energy with respect to the number of electrons.<sup>8</sup> Another example is the proposal to measure the chemical hardness through the second derivative of the energy with respect to the number of electrons.<sup>9</sup> Besides, different older theories, such as the Frontier Molecular

Orbital (FMO) initiated by Fukui<sup>10–15</sup> and the Hard and Soft Acids and Bases (HSAB) proposed by Pearson,<sup>16–22</sup> have been unified within conceptual DFT.

However, some chemical processes are still out of the reach of conceptual DFT. Indeed, due to the lack of a suitably formulated excited-state DFT, all the chemical reactions that involve an excited molecule are difficult to rationalize. Even though some reactions have been investigated by transposing,<sup>23–25</sup> the traditional local descriptors for the ground state to the reactivity of the excited state, no formal theory has been designed to support their use. It must be noticed that a preliminary study of time-dependent DFT to design local descriptors has been proposed by Chattaraj and co-workers.<sup>26–28</sup> The purpose of this paper is then to provide some insight into both the reactivity and selectivity of low excited states using the locality of the excited state's chemical potential when evaluated using the energy density functional for ground states.

The paper is organized as follows. In Section 2, using the true density of an excited state as a trial function for the ground-state density, a local descriptor is proposed to characterize the electrophilicity or nucleophilicity of a region

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within an excited state. From a comparison between the proposed and dual descriptors, a justification for the use of the negative dual descriptor to rationalize the Woodward–Hoffmann rules appears. After summarizing the computational methods employed, the concepts developed in Section 2 are tested for prototypical excited-state processes, more specifically for the regioselectivity of [2 + 2] photocycloaddition and Paterno–Büchi reactions. The paper ends with a perspective on what has been achieved, and what the outstanding issues are for future work.

## 2. Chemical Potential of a Hot Excited State

The generation of an excited state can be seen as a two-step process. In the first step, a hot excited state (HES) is created by a vertical electronic transition in which the electronic density changes, while the external potential remains identical. Such processes are governed by the Franck–Condon principle.<sup>29,30</sup> The HES is actually both electronically and vibrationally excited. During the second step, the molecular geometry (external potential) relaxes, adapting itself to the new electronic density. This gives an optimized excited state (OES). In this section, an approximate formula for the chemical potential of the HES is presented. The analysis of this chemical potential formula provides a way to characterize the chemical reactivity of the different atomic sites within a HES.

Traditional DFT is grounded on the first and the second Hohenberg–Kohn theorems.<sup>31</sup> Unfortunately, the practical use of those theorems has only been established for the electronic ground state, and they are not believed to be valid for excited states. However, the situation is not as bad as it seems because different formulations of excited-state DFT have been published.<sup>32–35</sup> Indeed, since the electronic density of the ground state determines both the external potential and the number of electrons, it determines the Hamiltonian and, consequently, all of the eigenstates of the electronic system. One can even define a functional of the electronic density that gives the exact energy for all the stationary electronic excited states.<sup>36</sup> The most popular excited-state DFT is the formulation of Levy and Nagy.<sup>37</sup> In this formulation, the energy is computed as

$$E_k = \int \rho_k(\vec{r})v(\vec{r})d\vec{r} + F_k[\rho_k(\vec{r}), v(\vec{r})] \quad (1)$$

in which  $\rho_k(\vec{r})$  is the exact electronic density of the excited state.

The bifunctional  $F_k$  is defined as

$$F_k[\rho_k(\vec{r}), v(\vec{r})] = \min_{\psi_k \perp \rho_k} \langle \psi_k | T + V_{ee} | \psi_k \rangle$$

where it is understood that the  $k^{\text{th}}$  eigenfunction  $\psi_k$  is orthogonal to wave functions of the ground and lower excited states of  $v(\vec{r})$ . This assumption yields quite an important property: Unlike the Hohenberg–Kohn functional, the bifunctional  $F_k[\rho_k(\vec{r}), v(\vec{r})]$  is not universal and depends upon the external potential through the electronic density of the ground state. However, this property of the bifunctional  $F_k$  does not affect the definition of local descriptors. Even though  $F_k$  is not universal, the electronic density is still given by the first derivative of the energy of that excited state with

respect to the external potential at constant number of electron  $N$ :

$$\left( \frac{\delta E_k}{\delta v(\vec{r})} \right)_N = \rho_k(\vec{r}) + \left( \frac{\delta F_k[\rho_k(\vec{r}), v(\vec{r})]}{\delta v(\vec{r})} \right)_N \quad (2)$$

Indeed, using the Hellman–Feynman theorem, Ayers and Levy<sup>38</sup> have shown that the derivative of the bifunctional with respect to the external potential vanishes and, therefore,  $(\delta E_k / (\delta v(\vec{r})))_N = \rho_k(\vec{r})$ . Consequently, the Fukui functions and the dual descriptor can be generalized as second and third crossed derivatives of the energy. However, as the relaxation of the excited density toward the density of the ground state might be a phenomenon powerful enough to drive the reactivity of the excited state, the ground-state local descriptors could be not appropriate.

On the one hand, the variational principle applied to eq 1 under the constraint that  $\int \rho_k(\vec{r})d\vec{r} = N_0$  yields for all the stationary densities to

$$\left( \frac{\delta E_k}{\delta \rho(\vec{r})} \right)_N = \mu_k = v_k(\vec{r}) + \left( \frac{\delta F_k[\rho_k(\vec{r}), v(\vec{r})]}{\delta \rho(\vec{r})} \right)_N \quad (3)$$

in which  $\mu_k$  is a global property of the system that generalizes the chemical potential to the excited state  $k$ . If the bifunctional  $F_k$  for the excited states were known in a practical form, then the nonlocality of the quantity  $v_k(\vec{r}) + (\delta F_k[\rho_k(\vec{r}), v(\vec{r})] / \delta \rho(\vec{r}))_N$  would be an important criterion for finding stationary densities. Indeed, the nonlocality of the chemical potential is a consequence of the stationary character of the exact electronic density of the state  $k$ . In other words,  $\mu_k$  is global only for the true excited-state density.

On the other hand, given the electronic density of a chosen excited state, its use as a trial density in the ground-state DFT must, with rare exception,<sup>8,39</sup> lead to a nonconstant chemical potential:

$$\left( \frac{\delta E}{\delta \rho_k(\vec{r})} \right) = \lambda_k(\vec{r}) = v(\vec{r}) + \frac{\delta F_{\text{HK}}[\rho_k(\vec{r})]}{\delta \rho_k(\vec{r})} \quad (4)$$

The nonconstant chemical potential captures the tendency of the excited-state electronic density to relax toward the ground-state density. It is this trend we desire to exploit. In particular, we will use the equation:

$$dE_k = \int \lambda_k(\vec{r})\delta \rho_k(\vec{r})d\vec{r} \quad (5)$$

noting that  $dE_k$  does not refer to an excitation energy. Equation 5 is a lower bound to the true change in energy accompanying excitation.

This local chemical potential is related to the true global chemical potential of the ground state as

$$\lambda_k(\vec{r}) = \mu_0 + V(\vec{r}) \quad (6)$$

with  $V(\vec{r}) = (\delta F_{\text{HK}}[\rho_k(\vec{r})] / \delta \rho(\vec{r})) - (\delta F_{\text{HK}}[\rho_0(\vec{r})] / \delta \rho(\vec{r}))$ , where  $\delta F_{\text{HK}}[\rho_k(\vec{r})] / \delta \rho(\vec{r})$  is the electronic density potential for the system of interest and  $\delta F_{\text{HK}}[\rho_0(\vec{r})] / \delta \rho(\vec{r})$  is the electronic density potential for which  $\rho_0(\vec{r})$  is the  $N$ -electron ground-state density. The chemical meaning of the quantity  $V(\vec{r})$  is clearly related to electronic relaxation: regions associated with positive values of  $V(\vec{r})$  will decrease their electronic

densities and can be considered nucleophilic. Conversely, regions with negative values of  $V(\vec{r})$  will increase their electronic densities and can be considered electrophilic.

When a chemical reaction induces the relaxation of the electron density in an excited state, then the sign of  $V(\vec{r})$  provides a way to identify the nucleophilic and electrophilic sites within the excited-state molecule. Evaluating  $V(\vec{r})$  requires finding a relationship between the excited- and ground-state chemical potentials. The most obvious way to do this is to construct the excited-state density by distorting the ground-state density. Then, using the Taylor series for the universal functional:

$$\begin{aligned} \frac{\delta F_{\text{HK}}[\rho_k(\vec{r})]}{\delta \rho(\vec{r})} \Big|_{v(r)} &= \frac{\delta F_{\text{HK}}[\rho_0(\vec{r}) + \Delta_0^k \rho(\vec{r})]}{\delta \rho(\vec{r})} \Big|_{v(r)} = \\ &= \frac{\delta F_{\text{HK}}[\rho_0(\vec{r})]}{\delta \rho(\vec{r})} + \int \frac{\delta^2 F_{\text{HK}}[\rho_0(\vec{r})]}{\delta \rho(\vec{r}) \delta \rho(\vec{r}')} \Delta_0^k \rho(\vec{r}') d\vec{r}' + \\ &+ \frac{1}{2} \iint \frac{\delta^3 F_{\text{HK}}[\rho_0(\vec{r})]}{\delta \rho(\vec{r}') \delta \rho(\vec{r}) \delta \rho(\vec{r}'')} \Delta_0^k \rho(\vec{r}') d\vec{r}' \Delta_0^k \rho(\vec{r}'') d\vec{r}'' + \dots \end{aligned} \quad (7)$$

Here,  $\Delta_0^k \rho(\vec{r})$  represents the electronic density difference between the excited state labeled  $k$  and the ground state. The local chemical potential of the excited state is

$$\begin{aligned} \lambda_k(\vec{r}) &= v(\vec{r}) + \frac{\delta F_{\text{HK}}[\rho_0(\vec{r})]}{\delta \rho(\vec{r})} + \\ &+ \int \frac{\delta^2 F_{\text{HK}}[\rho_0(\vec{r})]}{\delta \rho(\vec{r}) \delta \rho(\vec{r}')} \Delta_0^k \rho(\vec{r}') d\vec{r}' + \\ &+ \frac{1}{2} \iint \frac{\delta^3 F_{\text{HK}}[\rho_0(\vec{r})]}{\delta \rho(\vec{r}') \delta \rho(\vec{r}) \delta \rho(\vec{r}'')} \Delta_0^k \rho(\vec{r}') d\vec{r}' \Delta_0^k \rho(\vec{r}'') d\vec{r}'' \end{aligned} \quad (8)$$

The first derivative occurring in eq 8 can be evaluated as

$$\frac{\delta F_{\text{HK}}[\rho_0(\vec{r})]}{\delta \rho(\vec{r})} = \mu_0 - v(\vec{r})$$

The second derivative occurring in eq 8 is the hardness kernel, which is commonly decomposed as

$$\frac{\delta^2 F_{\text{HK}}[\rho_0(\vec{r})]}{\delta \rho(\vec{r}) \delta \rho(\vec{r}')} = \eta_0(\vec{r}, \vec{r}') = \frac{1}{|\vec{r} - \vec{r}'|} + R(\vec{r}, \vec{r}') \quad (9)$$

The first term of the right-hand side of eq 9 is the Coulomb contribution; the second term gathers the kinetic, exchange, and correlation contributions.

The third derivative occurring in eq 8 is the (second-order) hyperhardness kernel:

$$\frac{\delta^3 F[\rho_0(\vec{r})]}{\delta \rho(\vec{r}) \delta \rho(\vec{r}') \delta \rho(\vec{r}'')} = \eta_0^{(2)}(\vec{r}, \vec{r}', \vec{r}'')$$

Substituting these results into eq 8 gives

$$\begin{aligned} \lambda_k(\vec{r}) &= \mu_0 + \int \frac{\Delta_0^k \rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d\vec{r}' + \int R(\vec{r}, \vec{r}') \Delta_0^k \rho(\vec{r}') d\vec{r}' + \\ &+ \frac{1}{2} \iint \eta_0^{(2)}(\vec{r}, \vec{r}', \vec{r}'') \Delta_0^k \rho(\vec{r}') d\vec{r}' \Delta_0^k \rho(\vec{r}'') d\vec{r}'' + \dots \end{aligned} \quad (11)$$

Note that if the electronic densities of the ground and excited states are similar, their chemical potentials are also similar. For this reason, one expects the deviation of  $\lambda(\vec{r})$  from  $\mu_0$  to be larger for higher-energy excited states. Referring back to the interpretation of  $V(\vec{r}) = \lambda(\vec{r}) - \mu_0$ , this suggests that the electrophilicity/nucleophilicity (ergo, the total reactivity) of excited states increases as the resemblance of the excited-state density to the ground-state density decreases.

While a first- or second-order truncation of the Taylor series might be sufficient for low-lying excited states, it is unlikely to be accurate for highly excited states. The ambition of the present paper is limited to the lower excited states, for which the following second-order truncation is *assumed* to be *qualitatively* accurate:

$$\lambda_k(\vec{r}) = \mu_0 + \int \frac{\Delta_0^k \rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d\vec{r}' + \int R(\vec{r}, \vec{r}') \Delta_0^k \rho(\vec{r}') d\vec{r}' \quad (12)$$

If one assumes that the approximation  $R(\vec{r}, \vec{r}') \approx 0$  suffices for qualitative purposes,<sup>40</sup> then eq 12 becomes

$$\lambda_k(\vec{r}) = \mu_0 + \int \frac{\Delta_0^k \rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d\vec{r}' \quad (13)$$

With:

$$V(\vec{r}) \approx \int \frac{\Delta_0^k \rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d\vec{r}'$$

In eq 13, the difference between the chemical potential of the excited and ground states is seen to be equal to the difference between the electrostatic potential of the ground and excited states.

Notice that eq 13 is consistent with the fact that when the molecule density shifts toward the ground state from the excited state, the energy decreases

$$\begin{aligned} dE_k^0 &\approx \int \lambda_k(\vec{r}) (-\Delta_0^k \rho(\vec{r})) d\vec{r} \\ &\approx - \iint \frac{\Delta_0^k \rho(\vec{r}) \Delta_0^k \rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d\vec{r} d\vec{r}' \end{aligned}$$

A favorable chemical process involving the excited-state will cause its energy to decrease. Therefore, for an “allowed” excited-state reaction:

$$\delta E_k \approx \int \lambda_k(r) \delta \rho_k(\vec{r}) d\vec{r} < 0 \quad (14)$$

Introducing the reduced expression for the local chemical potential one has

$$\delta E_k \approx \int \mu_0 \delta \rho_k(\vec{r}) d\vec{r} + \int V(\vec{r}) \delta \rho_k(\vec{r}) d\vec{r} \quad (15)$$

The first term of the right-hand side of eq 15 is a global reactivity term; it is zero unless there is electron transfer to/from the system. The second term provides information about regioselectivity. A site in which the excited-state electrostatic potential is lower than the ground-state electrostatic potential attracts electrons. A site in which the

excited-state electrostatic potential is higher than the ground-state electrostatic potential repels electrons. The sign of eq 15, thus, provides a physically consistent characterization of both the electrophilicity and the nucleophilicity of a chemical site.

Trying to rationalize the Grochala, Albrecht, and Hoffmann rule (GAH), Ayers and Parr<sup>41</sup> proposed an approximate relation between the densities of the first excited state  $\rho_{\text{es}}(\vec{r})$ , the ground state  $\rho_{\text{gs}}(\vec{r})$ , and the radical cation  $\rho_+(\vec{r})$  and anion  $\rho_-(\vec{r})$ :

$$\rho_{\text{gs}}(\vec{r}) + \rho_{\text{es}}(\vec{r}) - \rho_+(\vec{r}) - \rho_-(\vec{r}) \approx 0 \quad (16)$$

Within DFT, the relations between the Fukui functions and the densities of the cation and the anion are

$$\rho_+(\vec{r}) \approx \rho_{\text{gs}}(\vec{r}) + f^+(\vec{r}) \quad (17)$$

$$\rho_-(\vec{r}) \approx \rho_{\text{gs}}(\vec{r}) - f^-(\vec{r}) \quad (18)$$

Substitution of eqs 17 and 18 into eq 16 leads to

$$\rho_{\text{es}}(\vec{r}) \approx \rho_{\text{gs}}(\vec{r}) + f^+(\vec{r}) - f^-(\vec{r}) \quad (19)$$

Identifying the dual descriptor<sup>42,43</sup> as the difference between the electrophilic and nucleophilic Fukui functions leads to

$$\Delta_0^1 \rho(\vec{r}) \approx \Delta f(\vec{r}) \quad (20)$$

This result can be understood through a simple molecular orbital picture. Indeed, it corresponds to consider merely the first excited-state density as the one produced when one electron from the highest occupied molecular orbital (HOMO) is promoted to the lowest unoccupied molecular orbital (LUMO). Ayers and Parr further argue that eq 20 partially accommodates relaxation effects. Following the GAH rule, the first excited state combines the characteristics of both the cation and the anion. As expected,  $\Delta_0^1 \rho(\vec{r})$  and  $\Delta f(\vec{r})$  are quite similar (see Figure 1).

So for the specific case of the first excited state, one can define the dual potential as

$$V^{\Delta f}(\vec{r}) = \int \frac{\Delta f(\vec{r}')}{|\vec{r} - \vec{r}'|} d\vec{r}' \quad (21)$$

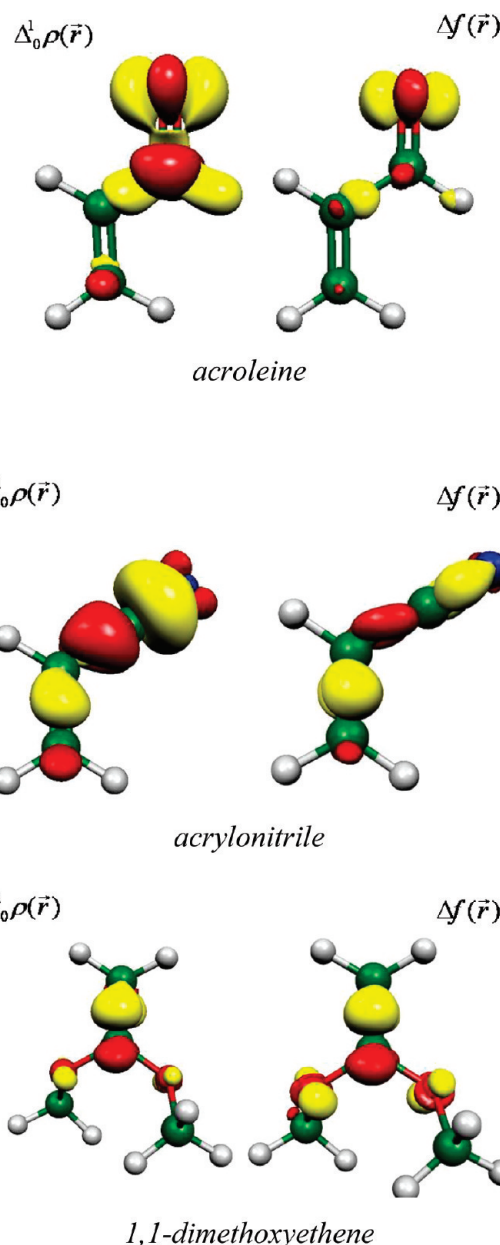
Then eq 15 reads

$$\delta E_1 \approx \mu_0 dN + \int V^{\Delta f}(\vec{r}) \delta \rho_1(\vec{r}) d\vec{r} \quad (22)$$

in which  $\delta \rho_1(\vec{r})$  stands for a variation of the first excited-state's electronic density. The second term of eq 22 can be seen as the contribution due to the variations of the electronic density weighted by the potential created by the dual descriptor. In this context, the best way to stabilize the excited state is to comply with the following criterion at each point  $\vec{r}$ :

$$\delta \rho_1(\vec{r}) V^{\Delta f}(\vec{r}) < 0 \quad (23)$$

This criterion enables us to sort out both the electrophilic and the nucleophilic atomic sites within an excited-state molecule. Indeed, electrophilic sites will have a positive

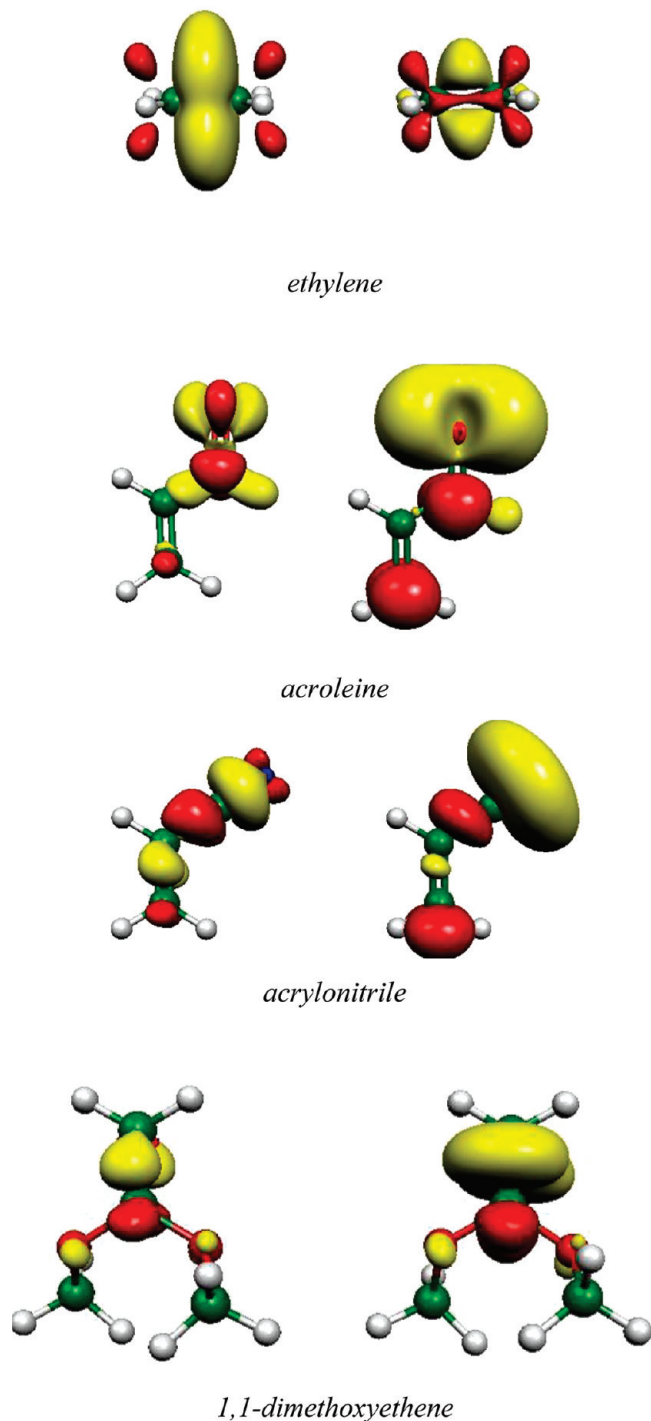


**Figure 1.** Comparison between  $\Delta_0^1 \rho(\vec{r})$  and  $\Delta f(\vec{r})$ .

variation of their electronic densities and, according to eq 23, the dual potential should be negative in those sites. Conversely, nucleophilic sites will experience a decreasing of their electronic densities and, according to the same criterion, should display positive values of the dual potential. The meanings of the signs of the dual potential for the first excited state are the opposite of the ones of the dual descriptor for the ground state.

Figure 2 displays both the dual potential and descriptor for some simple molecules. It can be seen that the general shapes of both functions are quite identical, except for the location of the nodes that separate the positive from negative regions. To explain those similarities, one can consider the dual potential at point  $\vec{r}$  as the weighted sum of the dual descriptor in all the other points  $\vec{r}'$ . The weighting coefficient has two important properties: (i) it is always positive, and (ii) the closer the point  $\vec{r}'$ , the higher its coefficient, and therefore, the more important its contribution to the dual potential. Taking into account those properties, it is no longer a surprise that both the dual potential





**Figure 2.** Comparison between  $\Delta_0\rho(\vec{r})$  and  $V^{\Delta_0\rho}(\vec{r}) = \int (\Delta_0\rho(\vec{r}')/|\vec{r} - \vec{r}'|)d\vec{r}'$ .

descriptor give the same description of the chemical behavior of the atomic sites within a molecule. A more elegant explanation of this similarity can be considered through the Poisson equation directly resulting from eq 21:  $\nabla V^{\Delta_0\rho}(\vec{r}) = -4\pi \Delta_0\rho(\vec{r})$ . As the dual potential and descriptor have approximately similar shapes, eq 22 is a posteriori theoretical justification of the use of the opposite of the dual descriptor as an index to characterize the behavior of reactive sites of the first excited state.<sup>44</sup>

The reactions studied in the application section always involve one molecule in either the first or second excited state and one molecule in the ground state. According to

**Table 1.** Favorable and Unfavorable Interactions between a Molecule in Both its Ground and Excited States

ground state $\Delta f(\vec{r})$		excited state $\Delta_0\rho(\vec{r})$		interactions
sign	character	sign	character	
+	electrophilic	+	nucleophilic	favorable
-	nucleophilic	-	electrophilic	favorable
$\pm$	el/nu	$\pm$	el/nu	unfavorable

the concepts developed in this section, the meaning of the sign of the electronic densities difference for the excited state is the opposite of the ones of the dual descriptor for the ground-state species. Therefore, the best interaction between molecules is obtained when regions with the same sign are aligned. Unfavorable interactions are occurring between regions with opposite signs of the density difference potential and dual descriptor. To express the previous consideration from a physical point of view the following criterion is proposed:

$$\delta E_1 \propto - \iint \frac{\Delta_0\rho(\vec{r}')\Delta f^{\text{gs}}(\vec{r})}{|\vec{r} - \vec{r}'|} d\vec{r} d\vec{r}' \quad (24)$$

From eq 24 it is easily seen that the energy goes downward when regions of the dual descriptor and electronic density difference of same sign are aligned. Those results are summarized in Table 1. Using this criterion, we will show that the regioselectivity of different photocycloadditions can be predicted.

### 3. Computational Details

All the studied molecules have been fully optimized at the B3LYP/6-31G\*\* level of theory using the Gaussian 03 software.<sup>45</sup> Both the excited and ground state electronic densities have been obtained through a CIS calculation with the same basis set. Then the 3-dimension cube files of the difference have been performed using the cubman facility program. The dual descriptor for the ground state has been calculated from the density of the radical cation and anion using the formula:

$$\Delta f(\vec{r}) = \rho_{N+1}(\vec{r}) + \rho_{N-1}(\vec{r}) - 2\rho_N(\vec{r})$$

For all the isodensity maps, the positive regions are colored in red, while the negative regions are colored in yellow.

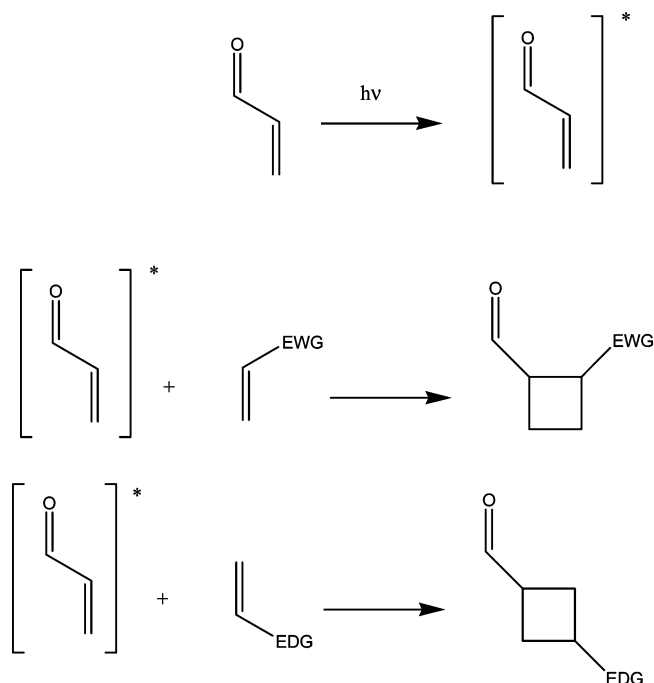
### 4. Application to Photocycloadditions

**4.1. Regioselectivity of [2 + 2] Photoreactions of Acroleine with Olefins.** The [2 + 2] cycloaddition between  $\alpha$ ,  $\beta$  unsaturated ketones and aldehydes with substituted olefins is one of the most versatile ways to produce cyclobutane derivatives. There are two main factors that control the regioselectivity of this reaction:

1) The steric interaction between the alkene and the  $\alpha$ ,  $\beta$  unsaturated ketone or aldehyde.

2) The electronic interaction between both reactants.

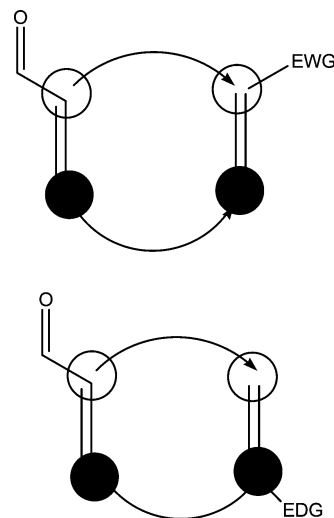
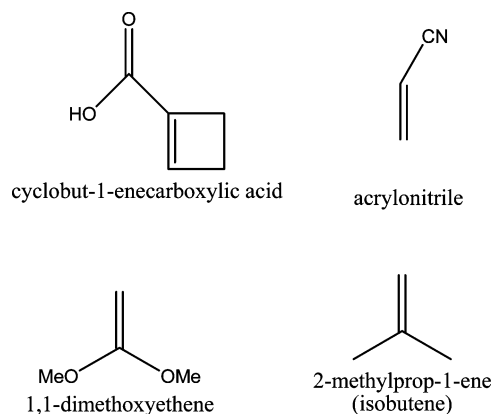
In the following study, only the reactions known to be controlled by electronic effects are discussed. According to either the electron-donating or electron-withdrawing character of the substituent on the alkene, the head-to-head or the head-

**Scheme 1.** Schematic Representation of the Regioselectivity of the [2 + 2] Photocycloaddition

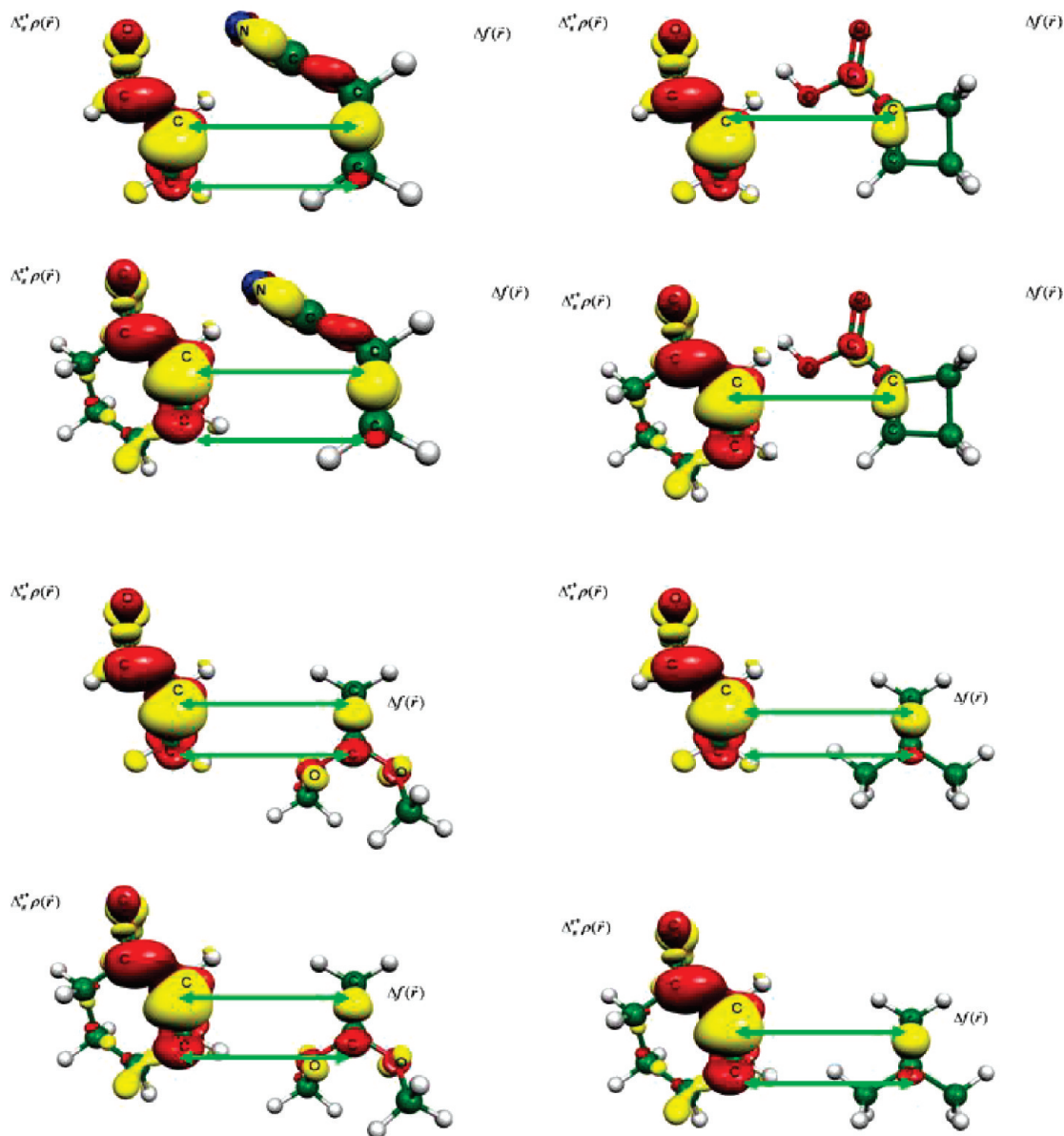
to-tail cyclobutane is formed.<sup>46</sup> As can be seen in Scheme 1, when the acroleine reacts with electron-rich alkenes the head-to-tail adduct is preferred. Conversely, the reaction between the acroleine with electron-poor alkenes leads preferentially to the head-to-head cyclobutane. Different conceptual DFT approaches of this reaction<sup>47</sup> have been performed through the softness-matching criterion and reaction force with rather good success.<sup>48</sup> The aim of this subpart is to show how the concept developed in Section 2 can be applied to predict the outcome of the reaction.

Even though specific calculations have been performed to exemplify the usefulness of the electronic density difference, the regioselectivity of those reactions can be elegantly rationalized without any calculation, only using a back-of-the-envelope analysis. Generally, the patterns of the dual descriptor for electron-poor alkenes are asymmetric, with the nucleophilic zones ( $\Delta f(\vec{r}) < 0$ ) centered on the substituted carbon, while the electrophilic character ( $\Delta f(\vec{r}) > 0$ ) is located mainly on the unsubstituted end. As acroleine can be seen as an electron-poor alkene, the position of the positive and negative zones should be located in the same areas for its electronic densities difference  $\Delta\rho(\vec{r})$ . Since acroleine reacts in its first excited state and following the rules gathered in Table 1, the best interaction between acroleine and electron-poor alkenes is achieved through a head-to-head interaction (see Figure 3). The inverse conclusions can be drawn up for electron-rich alkenes, since their dual descriptors are the opposite of electron-poor alkenes ones. These qualitative conclusions are confirmed by the calculated map of dual descriptors and electronic densities differences.

It is well established that the acroleine reactive state is the one corresponding to a  $\pi \rightarrow \pi^*$  transition. The corresponding electronic densities difference [ $\Delta\pi^*\rho(\vec{r})$ ] has been calculated for both acroleine and cyclohex-2-enone. It is important to notice that this electronic densities difference

**Figure 3.** Schematic interaction between the dual descriptor of acroleine and either electron-rich or -poor alkenes. Dark circles correspond to positive value of the dual descriptor, while white circles are attributed to negative values of the dual descriptor.**Figure 4.** Name and structures of the studied alkenes.

$[\Delta\pi^*\rho(\vec{r})]$  is not equivalent to the dual descriptor for the ground state, since the  $\pi$  orbital is not the HOMO. However, it appears that the positions of positive and negative values on the double bond are similar to the ones obtained for the dual potential. On the other hand, the ground-state dual descriptor calculations have been performed for different electron-rich and -poor alkenes: allene, acrylonitrile, isobutene, and 1,1-dimethoxyethene. The structures of all the compounds are drawn in Figure 4. The positions of positive and negative regions of the dual descriptor within the electron-rich and -poor alkenes follow the qualitative rule given previously. Namely, for electron-poor alkenes, the positive region is located on the unsubstituted end, while the negative area is centered on the substituted carbon. This is due to the asymmetry created on the HOMO and LUMO wave function by the electron-withdrawing group. Conversely, the effect of the electron-donating group (EDG) upon the shape of the dual descriptor is exactly the opposite. The negative regions are located on the unsubstituted carbon, while the positive regions cover mainly the substituted end of the double bond. Following the criterion defined in Section 2, the best interaction between the  $\alpha, \beta$  unsaturated carbonyl compound and the alkene leads to the head-to-tail adduct for the



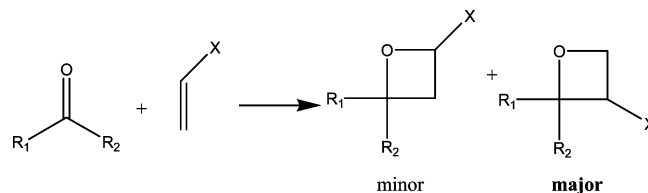
**Figure 5.** Favorable interactions between acroleine and cyclohex-2-enone with electron-poor (upper pictures) and -rich (lower pictures) alkenes. The isosurfaces are displayed for  $\Delta f(\vec{r}) = 0.01 \text{ au}$  and  $\Delta \pi^* \rho(\vec{r}) = 0.005 \text{ au}$ . The electron-poor alkenes taken are acrylonitrile and cyclobut-1-ene carboxylic acid. The electron-rich alkenes are 2-methylprop-1-ene and 1,1-dimethoxyethene.

electron-rich alkenes and head-to-head for electron-poor alkenes, as can be seen in Figure 5. This is in total agreement with experimental results.

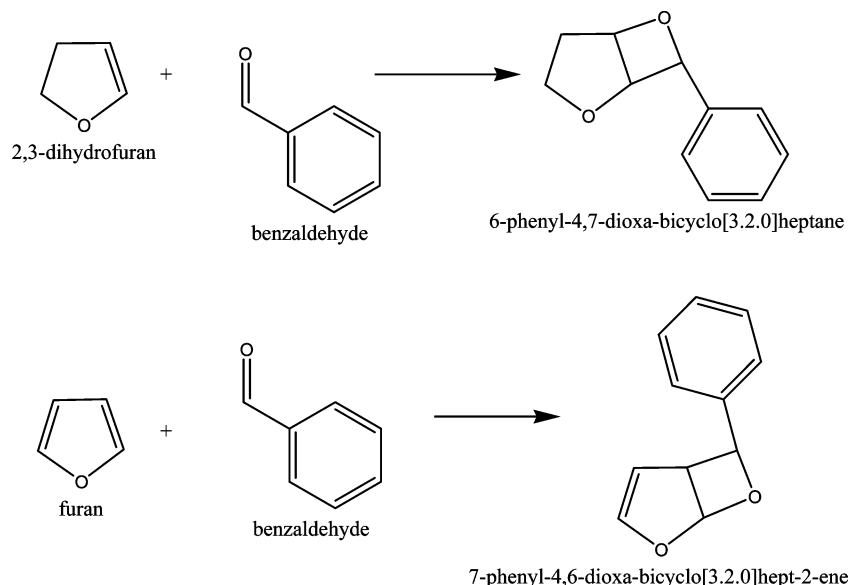
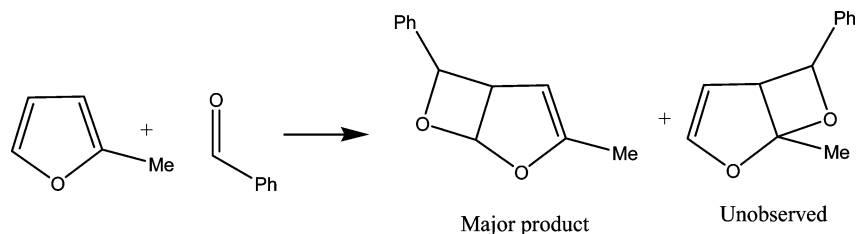
The next section is dedicated to the regio- and stereoselectivity of the Paterno–Büchi reaction.

**4.2. Application to the Regio- and the Stereoselectivity of the Paterno–Büchi Reaction.** The Paterno–Büchi (PB) is a versatile method to produce oxetane (see Scheme 2), i.e. four-membered oxygen heterocyclic rings with a good control over the regio- and stereoselectivity. This reaction involves a ketone or an aldehyde in its excited state and an olefin in its ground state. Numerous papers have been published on either the preparative or mechanistic aspect of the PB reaction. From all the results gathered, it is now generally admitted that the active excited state arises from the  $n \rightarrow \pi^*$  transition. In this specific case, and contrary to the  $[2 + 2]$  photocycloaddition, the electronic densities difference is roughly equal to the dual descriptor ( $\Delta \pi^* \rho(\vec{r})$ )

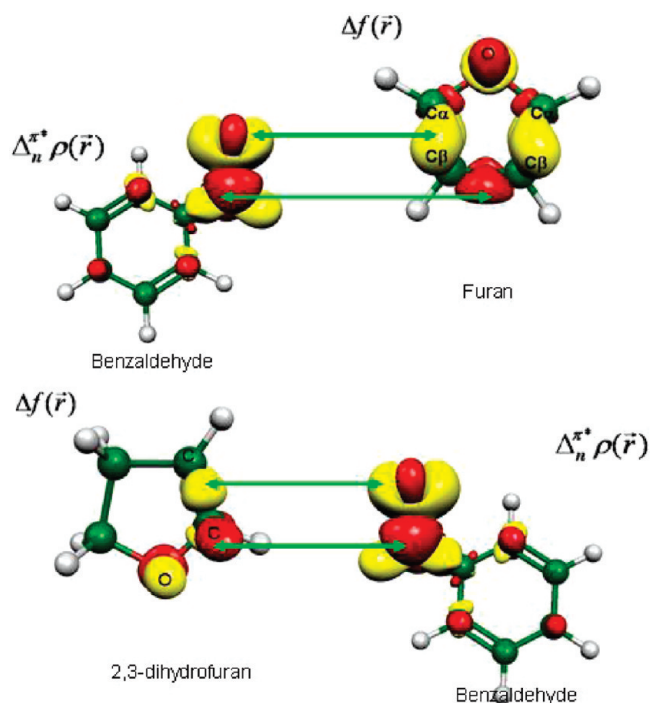
**Scheme 2.** Schematic Representation of the Paterno–Büchi Reaction



$\approx \Delta f(\vec{r})$ ). The PB reaction is particularly rapid when the ketone or aldehyde reacts with an electron-rich alkene and is highly regioselective. In this section, we will study the regioselectivity of the reaction between benzaldehyde on the one hand and furan, or 2,3-dihydrofuran, on the other hand. For those cases, it has been shown experimentally that only one regioisomer is produced<sup>49–51</sup> (see Scheme 3). The PB reaction can, therefore, be called regiospecific. Aiming to check the predictive capability of our method, the dual

**Scheme 3.** Schematic Representation of the Regioselectivity of the Paterno-Büchi Reaction**Scheme 4.** Schematic Representation of the Reaction between 2-Methylfuran and Benzaldehyde

descriptor for furan and its derivative and the density difference  $\Delta_n^{\pi^*} \rho(\vec{r})$  for benzaldehyde have been calculated. The results are displayed in Figure 6. As already said, the

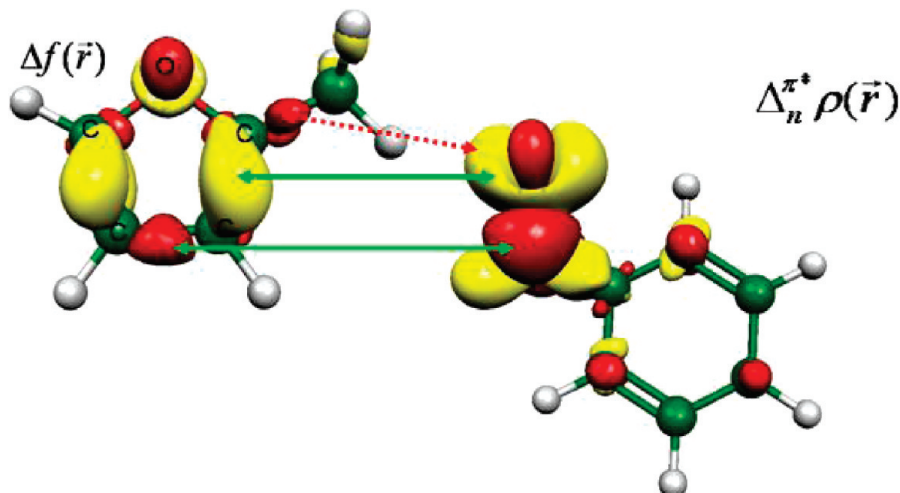
**Figure 6.** Favorable interaction between an excited benzaldehyde  $\Delta_n^{\pi^*} \rho(\vec{r}) = 0.01 \text{ au}$  with furan and 2,3-dihydrofuran  $\Delta f(\vec{r}) = 0.01 \text{ au}$ .

best interactions between both molecules are achieved when regions with same sign are aligned. One can easily infer from Figure 6 that, in both cases, the correct regioisomer is predicted. As guessed, the negative region of the  $\Delta_n^{\pi^*} \rho(\vec{r})$  of benzaldehyde is located on the oxygen, while the positive region is mainly on the carbonyl carbon. It looks very close to what was expected from the dual descriptor of benzaldehyde. For furan, the negative region is located between  $C\alpha$  and  $C\beta$ , namely between the double bond, while the positive region is in between the  $C\beta$ , i.e., located on the simple carbon-carbon bond. Therefore, the best interaction between furan and benzaldehyde is obtained when the oxygen of benzaldehyde reacts with the  $C\alpha$  of the furan and when the carbonyl carbon bonds with the  $C\beta$ . This must lead to the formation of 7-phenyl-4,6-dioxabicyclo[3.2.0]hept-2-ene.

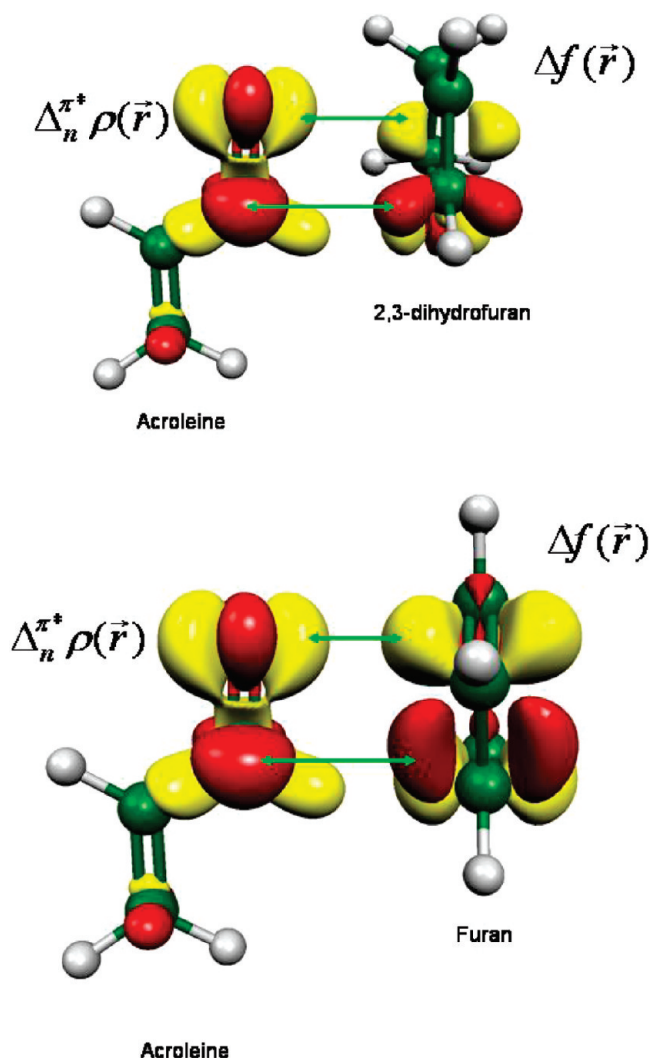
Since the position of the positive and negative region of the dual descriptor of 2,3-dihydrofuran are the opposite of the one obtained on furan, the best interaction is achieved when the oxygen of benzaldehyde reacts with  $C\beta$ , while the carbonyl carbon bonds with  $C\alpha$ . Again, the right regioisomer is predicted: 6-phenyl-4,7-dioxabicyclo[3.2.0]hept-2-ene.

When furan is methylated in position one, two regioisomers should be produced as shown in Scheme 4. From an analysis based on the frontier molecular orbital theory, D'auria et al.<sup>52</sup> concluded that the reaction is likely to take place in both positions  $C\alpha$ . However, this prediction is not in agreement with experimental results. Indeed, only the unsubstituted  $C\alpha$  carbon is attacked. This result can of course





**Figure 7.** Favorable and unfavorable interaction between an excited benzaldehyde  $\Delta\pi_n^*\rho(\vec{r}) = 0.01\text{au}$  and 2-methylfuran  $\Delta f(\vec{r}) = 0.01\text{au}$ .



**Figure 8.** Best approach geometry between an excited acrolein and either furan and 2,3-dihydrofuran.

be explained by the steric hindrance that occurs in the substituted carbon. Still an analysis based on the  $\Delta\pi_n^*\rho(\vec{r})$  can provide an additional explanation. As can be seen in Figure 7, an electrophilic zone [ $\Delta\pi_n^*\rho(\vec{r}) > 0$ , red] is located between the methyl group and the C $\alpha$ . As the benzaldehyde ap-

proaches the substituted C $\alpha$  carbon of methyl-furan, the favorable interaction between the oxygen of benzaldehyde with the C $\alpha$  carbon is counterbalanced by the unfavorable interaction between the electrophilic zone near the C $\alpha$  and the oxygen of the benzaldehyde. This unfavorable interaction is an additional effect that favors the experimentally observed isomer.

The geometry of approach can also be obtained from the interaction study between  $\Delta\pi_n^*\rho(\vec{r})$  benzaldehyde and the  $\Delta f(\vec{r})$  furan. Indeed, it is well-known that in PB reactions, the molecules approach each other with a face to edge geometry. As can be seen in Figure 8, this approach seems to be the one that maximizes the favorable interaction between both molecules.

## 5. Summary and Conclusions

In this paper, using the true excited-state density as a trial function for the ground-state DFT, a new local reactivity descriptor has been proposed. The main assumption is that the relaxation of the electronic density is a powerful enough phenomenon to drive the chemical behavior of an excited molecule. The locality of the chemical potential of an excited state is used to characterize the philicity of an atomic site within an excited molecule. It appears that for the first excited state the local chemical potential and the dual descriptor are related by a Poisson equation and look similar. This provides a posteriori justification of the use of the dual descriptor to rationalize the Woodward–Hoffmann rules for photochemical reactions. Finally, the application of the proposed concept of local chemical potential to [2 + 2] photocycloaddition and to Paterno–Büchi reaction predicts the correct regioselectivity. This paper can be considered as a first step toward a real conceptual DFT of excited states. To achieve this goal in full, however, it is necessary to derive a reliable time-independent functional of the energy that fully describes excited states. The local and global descriptors already defined for the ground state could then be generalized to excited states by differentiation of the excited-state functional.

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