

Comprehensive Benchmarking of a Density-Dependent Dispersion Correction

Stephan N. Steinmann and Clemence Corminboeuf*

Laboratory for Computational Molecular Design, Institut des Sciences et Ingénierie Chimiques, Ecole Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland



ABSTRACT: Standard density functional approximations cannot accurately describe interactions between nonoverlapping densities. A simple remedy consists in correcting for the missing interactions *a posteriori*, adding an attractive energy term summed over all atom pairs. The density-dependent energy correction, dDsC, presented herein, is constructed from dispersion coefficients computed on the basis of a generalized gradient approximation to Becke and Johnson's exchange-hole dipole moment formalism. dDsC also relies on an extended Tang and Toennies damping function accounting for charge-overlap effects. The comprehensive benchmarking on 341 diverse reaction energies divided into 18 illustrative test sets validates the robust performance and general accuracy of dDsC for describing various intra- and intermolecular interactions. With a total MAD of 1.3 kcal mol⁻¹, B97-dDsC slightly improves the results of M06-2X and B2PLYP-D3 (MAD = 1.4 kcal mol⁻¹ for both) at a lower computational cost. The density dependence of both the dispersion coefficients and the damping function makes the approach especially valuable for modeling redox reactions and charged species in general.

■ INTRODUCTION

Many chemical phenomena are dominated by weak interactions, as exemplified by the highly ordered structures of biomolecules (stacking of DNA, 1 protein folding 2) and supramolecular assemblies, 3 crystals arrangements of organic 4 and inorganic materials, or catalysis intermediates (see, e.g., ref 6). Because of the incomparable balance of accuracy and computational cost, Kohn-Sham density functional theory has emerged as the most widely applied methodology for investigating electronic structures and geometries of extended molecular systems. Despite this success, standard semilocal approximations do not properly describe attractive dispersion interactions that decay with R^{-6} at large intermolecular distances.⁸⁻¹¹ Even at the medium range, most semilocal density functionals fail to give an accurate description of weak interactions such as those dominating alkane isomerization energies and Pople's isodesmic bond separation equations (BSEs).12-17

Near the energy minimum, dispersion-corrected atom-centered potentials $(DCAPs)^{18-22}$ or carefully fitted density functionals²³⁻²⁸ $(M06-2X^{27})$ is certainly the most successful functional originating from this approach) give satisfactory results. Nevertheless, both approaches intrinsically lack the ability to recover the correct long-range $\sim R^{-6}$ attractive form. The simplest conceptual remedy, $^{29-33}$ first popularized by Grimme (motivated by HF-D)³⁴⁻³⁸ under the DFT-D acronym, 33,39,40 is to correct for the missing interaction a posteriori by adding an attractive energy term summed over all of the atom pairs in the system. The quest for the optimal parametrization is, however, still an active field of research. Recent DFT-D (e.g., D2³⁹ and D3⁴⁰) gives an accurate description for intermolecular interactions, but

the proper treatment of weak intramolecular interactions is trickier. 14,40,58-60 Over the past three years, our group has pioneered the design of corrections which give a balanced description of both inter- and intramolecular weak interactions. 43,50,57,61,62 Our most recent scheme combines dispersion coefficients (C) computed on the basis of an approximation to Becke and Johnson's 63-69 exchange-hole-dipole moment (XDM) formalism depending on the reduced density gradient $(s)^{70}$ and a genuine density dependent damping factor. 57 The resulting density dependent correction, called dDsC, promises substantial advantages over standard DFT computations for a broad range of applications. Following a careful validation of the dDsC scheme, we here introduce a few improvements to our original density dependent damping factor 57,70 and provide a comprehensive benchmarking of the density-dependent dispersion correction scheme. dDsC is tested on 18 diverse test sets featuring both intra- and intermolecular weak interaction energies together with a series of illustrative density functionals, i.e., BP86,^{71–73} BLYP,^{71,74} B3LYP,^{71,74–76} PBE,⁷⁷ B97⁷⁸ and the long-range corrected LC-ωPBELYP.^{74,79–81} Results for other schemes designed to better describe weak interactions are discussed as well: the local response dispersion (LRD) correction combined with LC-BOP, \$53,54 two fully nonlocal density functionals, VV10⁸² and vdW-DF-10, 83 the double hybrid functional B2PLYP-D3^{40,84} and M06-2X.²⁷ The benchmark is completed by a short assessment of the dDsC schemes on geometries.

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■ THEORY

The basic form of our correction is the Tang and Toennies (TT) damping function 85

$$E_{\text{disp}} = -\sum_{i=2}^{N_{\text{at}}} \sum_{j=1}^{i-1} \sum_{n=3}^{n=5} f_{2n}(bR_{ij}) \frac{C_{2n}^{ij}}{R_{ij}^{2n}}$$
(1)

where $N_{\rm at}$ is the number of atoms in the system and b is the TT-damping factor (*vide infra*). The correction is called dDsC if only the first term is retained in the multipole expansion (n = 3, corresponding to C_6), and dDsC10 otherwise (up to n = 5, i.e., up to C_{10}). $f_{2n}(bR_{ij})$ are the "universal damping functions" that are specific to each dispersion coefficient and that serve to attenuate the correction at short internuclear distances (to account for overlapping densities).

$$f_{2n}(x) = 1 - \exp(-x) \sum_{k=0}^{2n} \frac{x^k}{k!}$$
 (2)

This section describes the determination of the damping factor b in eq 1. The dispersion coefficients themselves are obtained as described previously 70 and rely on a classical Hirshfeld dominant partitioning of the electron density among the atomic centers.

Classical Hirshfeld weightings are defined as⁸⁶

$$w_i(r) = \frac{\rho_i^{\text{at}}(r)}{\sum_n \rho_n^{\text{at}}(r)}$$
(3)

where $\rho_i^{\rm at}$ is the sphericalized free (neutral) atomic density of atom i, weighted by the superposition of all $\rho_i^{\rm at}$ with all atoms n positioned as in the real molecule. The classical Hirshfeld dominant partitioning $w_i^{\rm D}$ is obtained by assigning each point exclusively to the atom which has the highest weight at that particular grid point. Such a partitioning is more appealing than the classical Hirshfeld populations, as it avoids overlapping atomic regions that conflict with the multipole expansion that is at the origin of the atom-pair-wise London dispersion correction 87

A key component of dDsC is the damping factor b. We showed previously^{50,57} that the performance of the TT-damping function is improved by the introduction of a second damping function, which prevents the corrections at regions of strong density overlap (i.e., covalent distances) that are better described by density functionals. Akin to our previous work, $b_{ij,asym}$, the asymptotic value of b, accounts for the short-range effect through a multiplicative function

$$b(x) = F(x) b_{ii, asym} (4)$$

x and F(x) are, respectively, the damping argument and function for $b_{ij,asym}$, the TT-damping factor associated with two separated atoms. $b_{ij,asym}$ is computed according to the combination rule ^{88,89}

$$b_{ij, \text{ asym}} = 2 \frac{b_{ii, \text{ asym}} b_{jj, \text{ asym}}}{b_{ii, \text{ asym}} + b_{jj, \text{ asym}}}$$
(5)

 $b_{ii,asym}$ is generally estimated from the square root of (atomic) ionization energies. However, the ionization energy does not correlate well with the size of an atom that is a determinant characteristic for the damping of a dispersion term. 31,39,49,95 We instead propose to compute $b_{ii,asym}$ on the basis of effective atomic polarizabilities. Note that polarizabilities as a measure of the "size" are extensively used in the closely related context of

Thole's interacting dipole moments. 96 After introduction of the parameter b_0 , which dictates the strength of the correction in the medium range, one obtains

$$b_{ii, \text{asym}} = b_0 \sqrt[3]{\frac{1}{\alpha_i}} = b_0 \sqrt[3]{\frac{1}{\alpha_{i, \text{free}}}} \sqrt[3]{\frac{V_{i, \text{free}}}{V_{i, \text{AIM}}}}$$
(6)

In the above definition, b_0 includes the conversion factor from Å³ to atomic units for α_i .

The effective atom in molecule polarizabilities are estimated from scaled free atomic polarizabilities ^{97,98}

$$\alpha_{i} = \frac{\langle r^{3} \rangle_{i}}{\langle r^{3} \rangle_{i, \text{free}}} \alpha_{i, \text{free}} = \frac{\int r^{3} w_{i}^{D}(r) \rho(r) d^{3}r}{\int r^{3} \rho_{i, \text{free}}(r) d^{3}r} \alpha_{i, \text{free}}$$
$$= \frac{V_{i, \text{AIM}}}{V_{i, \text{free}}} \alpha_{i, \text{free}}$$
(7)

A density cutoff of 0.002 au is applied to improve the consistency of atomic volumes between atoms at the surface and in the interior of a molecule. 70,99

The $b_{ii,asym}$ dependency on atomic polarizabilities (instead of atomic ionization energies) mostly benefits the treatment of highly polarizable atoms as shown later (e.g., neutral alkali-metal cluster like K_8 of the ALK6 test set). A similar relationship could also be an advantage in force fields specifically designed to predict crystal structures. In such force fields, atomic polarizabilities have already been introduced, but $b_{ii,asym}$ is usually determined from the molecular ionization energy with no dependency on the specific atom pair. $^{92-94}$ Along with the modified $b_{ii,asym}$, the secondary damping function is modified slightly and represented by a (steeper) exponential decay (see ref 100 for more discussion) rather than by the previously used arctan function

$$F(x) = \frac{2}{e^{a_0 x} + 1} \tag{8}$$

where the fitted parameter a_0 adjusts the short-range behavior of the correction.

The last element of the correction is the damping argument x

$$x = \left(2q_{ij} + \frac{abs((Z_i - N_i^{D})(Z_j - N_i^{D}))}{r_{ij}}\right) \frac{N_i^{D} + N_j^{D}}{N_i^{D}N_j^{D}}$$
(9)

where Z_i and N_i^D are the nuclear charge and Hirshfeld dominant population of atom i, respectively. $2q_{ij} = q_{ij} + q_{ji}$ is a covalent bond index 101 based on the overlap of classical Hirshfeld populations $w_i(r) \ q_{ij} = \int w_i(r) \ w_j(r) \ \rho(r) \ dr$, and the fractional term in the parentheses is a distance-dependent ionic bond index 102 taken as an absolute value. Classical Hirshfeld dominant charges in the damping function resolve the inconvenience of classical Hirshfeld charges that are generally too small. 57,103,104 The multiplicative factor, $(N_i^D + N_j^D)/(N_i^D N_j^D)$, serves to attenuate the damping of $b_{ii,asym}$ for heavier atoms (containing more electrons). Note that the damping function F(x) has the adequate form (i.e., F(0) = 1 and $F(\infty) = 0$), given that x is large when atoms are close to each other and goes to zero with increasing distance r_{ij} . In the present form, approximated dDsC gradients are available: All derivatives of the (density dependent) parameters (the damping parameter b and the dispersion coefficients) are set

to zero, or in other words, kept fixed at their values corresponding to the energy of the geometry for which the gradient is being computed. The approximation is expected to introduce only small errors, similar to those engendered by the use of a smaller basis set for geometry optimization, followed by energy refinement with a larger basis set. Exact gradients are computationally more expensive (although simpler than those derived for the original Becke—Roussel exchange hole in ref 105) given that the contributions to the Fock matrix are needed at each SCF cycle.

To summarize, the presented dDsC correction employs electronic structure information to determine dispersion coefficients and two fitted, functional dependent, damping parameters that are the strength of the TT-damping (b_0) and the steepness factor (a_0) .

■ DETERMINATION OF THE ADJUSTABLE PARAMETERS

In line with our former work, 43,50,57 the chosen fitting procedure ensures a successful treatment of both weak intra-(medium-range) and inter- (long-range) molecular interactions. The two parameters (a_0 and b_0) are fitted for each functional so as to minimize the mean absolute deviation (MAD) over a representative set of 48 reactions, assessing inter- and intramolecular interactions. The detailed list of reactions in the training set is given in the Supporting Information, but in summary, 3–6 entries are taken from the following test sets (*vide infra*): BSR36, RSE43, ISO34, NBPRC, WATER27, ACONF, CYCONF, SCONF, HEAVY28, and S22. The best fit parameters are given in the Supporting Information for dDsC and dDsC10.

■ TEST SETS

Eighteen test sets, corresponding to 341 reaction energies, were selected out of the 30 test sets from the GMTKN30 (database for general main group thermochemistry, kinetics, and noncovalent interactions) database 106,107 from where the geometries and reference values were taken. The sets are divided into three categories:

- i Intramolecular interactions (5 sets, 85 reactions): ISOL22 (isomerization energies of large organic molecules), 108 DARC (Diels—Alder reactions energies), 109 BSR36 (bond separation reactions of alkanes), 43,110 IDISP (intramolecular dispersion interactions), 14,106,111 and AL2X (dimerization energies of AlX₃ and AlHX₂ compounds, X = F, Cl, Br, and Me). 109
- ii Intermolecular interactions and conformational energies (7 sets, 108 reactions): S22 (binding energies of noncovalently bound dimers), 112-114 ADIM6 (interaction energies of *n*-alkane dimers), HEAVY28 (noncovalent interaction energies between heavy element hydrides), ACONF (relative energies of alkane conformers), SCONF (relative energies of sugar conformers), 116,117 PCONF (relative energies of PHE-GLY-GLY), 118 and CYCONF (relative energies of cysteine conformers).
- iii Mixed category of reaction energies (6 sets, 148 reactions): ALK6 (fragmentation and dissociation reactions of alkaline metal clusters and alkaline—cation benzene complexes), ⁴⁰ BHPERI (barrier heights of pericyclic reactions), ^{120–123} RSE43 (radical stabilization energies), ¹²⁴ NBPRC (oligomerizations and H₂ fragmentations of NH₃/BH₃ systems and H₂ activation reactions with PH₃/BH₃), ^{116,125} ISO34 (isomerization energies of small and medium-sized organic molecules), ^{125,126} and WATER27 (binding energies of water, H⁺(H₂O)_n and OH⁻(H₂O)_n clusters). ¹²⁷

Table 1. Mean Absolute Deviations for All Methods Tested, For All Test Sets (Overall), and the Three Individual Subcategories, i.e., Intramolecular Interactions (Intra), Intermolecular Interactions and Relative Conformational Energies (Inter+Conf), and the Mixed Test Sets (Mix)^a

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	Overall	Intra	Inter+Conf	Mix
HF	9.05	12.62	3.10	11.34
BLYP	6.85	14.38	2.53	5.67
revPBE	6.26	11.28	2.70	5.97
B3LYP	5.70	12.22	2.20	4.50
TPSSM	4.84	10.47	1.98	3.68
vdW-DF-10	4.80	11.13	0.61	4.00
BP86	4.54	9.07	2.14	3.68
B97	4.47	9.56	1.83	3.48
BHHLYP	4.40	9.10	1.77	3.63
HF-dDsC	3.74 (3.57)	5.82 (4.87)	1.25 (1.40)	4.37 (4.41)
LC-ωPBE	3.49	6.24	1.48	3.38
PBE	3.49	7.39	1.39	2.77
$LC-\omega PBELYP$	3.35	6.14	1.26	3.26
VV10	3.34	5.50	0.43	4.22
LC-BOP	3.32	5.36	1.45	3.52
PBE0	3.11	6.55	1.44	2.34
PW6B95	3.01	6.01	0.92	2.81
B3LYP-D3	2.96	6.82	0.28	2.70
LC-ωPBEB95	2.89	4.29	0.78	3.62
LC-BOP-LRD[10,0]	2.56	3.63	0.43	3.51
LC-BOP-LRD[10,6]	2.56	3.50	0.49	3.54
BLYP-dDsC	2.45 (2.65)	3.71 (4.26)	0.62 (0.63)	3.05 (3.21)
LC- ω PBEB95-dDsC	2.39 (2.39)	4.15 (4.11)	0.66 (0.67)	2.65 (2.66)
LC- ω PBE-dDsC	2.37 (2.37)	4.82 (4.87)	0.43 (0.41)	2.38 (2.37)
PBE-dDsC	2.19 (2.22)	1.94 (1.94)	0.52 (0.57)	3.56 (3.58)
$LC\text{-}\omegaPBELYP\text{-}dDsC$	2.14 (2.04)	2.35 (2.05)	0.71 (0.59)	3.06 (3.08)
revPBE-dDsC	2.12 (1.92)	1.83 (1.89)	0.70 (0.59)	3.32 (2.90)
BP86-dDsC	2.03 (2.01)	2.44 (2.47)	0.81 (0.72)	2.68 (2.69)
TPSSM-dDsC	1.96 (1.96)	2.54 (2.61)	0.65 (0.63)	2.59 (2.56)
B3LYP-dDsC	1.67 (1.86)	2.43 (2.85)	0.48 (0.58)	2.11 (2.23)
BHHLYP-dDsC	1.66 (1.73)	1.76 (1.81)	0.48 (0.53)	2.47 (2.55)
PBE0-dDsC	1.59 (1.66)	1.98 (2.04)	0.42 (0.52)	2.22 (2.28)
M06-2X	1.41	2.94	0.40	1.26
PW6B95-dDsC	1.39 (1.39)	1.70 (1.67)	0.62 (0.66)	1.78 (1.76)
B2PLYP-D3	1.37	3.41	0.16	1.08
B97-dDsC	1.30 (1.32)	1.78 (1.82)	0.48 (0.47)	1.62 (1.65)

^a Values in parentheses refer to the correction including coefficients up to C_{10} (dDsC10). All values are in kcal mol⁻¹. Results for B2PLYP-D3 and M06-2X are taken from refs 107 and 152.

Reaction energies and MADs for all methods tested are given in the Supporting Information, which also includes the corrected data with higher-order dispersion coefficients. Note that the effects of the higher-order terms strongly depend on the type of damping function. The TT-damping function applied herein "simulates" the missing higher-order dispersion terms by increasing the damping factor $b.^{128}$

■ COMPUTATIONAL METHODS

BLYP, 71,74 BP86, 71,72 PBE, 77 revPBE, 129 B3LYP, $^{71,74-76}$ and PBE0 77,130 computations were performed with a developmental

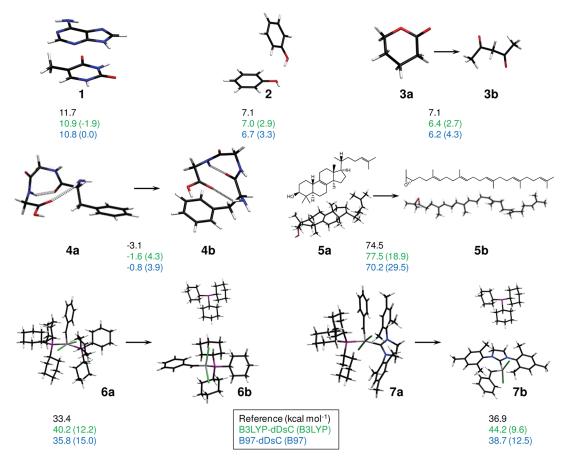


Figure 1. Set of illustrative examples of reactions poorly described by standard density functionals (e.g., B3LYP and B97) and corrected by dDsC. The reference values 108,113,126,149 are computed at the CCSD(T)/CBS level, except for 5, where SCS-MP3/CBS serves as the benchmark, and for 7, experimental values are used. 153 The DFT energies for 4–7 are computed with the def2-TZVP basis set.

version of ADF. 131,132 HF, BHHLYP, 133 Becke's hybrid B97⁷⁸ functional (that is to be distinguished from Grimme's GGA functional B97-D³⁹), PW6B95, 134 LC- ω PBE⁷⁹⁻⁸¹ (ω = 0.45), LC- ω PBELYP (ω = 0.45), LC- ω PBEB95¹³⁵ (ω = 0.45), VV10 (rPW86¹³⁶PBE⁷⁷+nonlocal correction), ⁸² and vdW-DF-10 (rPW86¹³⁶PW92¹³⁷+nonlocal correction)⁸³ were performed in a developmental version of Q-Chem, ¹³⁸ while LC-BOP, ^{71,139–141} LC-BOP-LRD, ^{53,54} and TPSSm¹⁴² and all geometry optimizations were run with a modified version of GAMESS. 143 A patch for GAMESS 2010 (version 1 Oct 2010) will be available on our Web site. Due to SCF convergence problems, computations in GAMESS use the cc-pVTZ basis ${\rm set}^{144-146}$ (augmented with diffuse functions, leading to aug-cc-pVTZ in order to minimize the BSSE for the WATER27 complexes and all but the benzene indole complexes of the S22 test set), except for potassium and the heavier elements for which the def2-QZVP(-g) basis set was used. All Q-Chem computations were done with the def2-QZVP(-g)¹⁴⁷ basis set except for the clusters involving OH⁻ from the WATER27 test set, for which the aug-cc-pVQZ basis set was used. In GAMESS and Q-Chem, the numerical integrations were performed on a fine 99/590 and 75/302 Euler-Maclaurin-Lebedev grid, respectively, with an integration threshold of 10⁻¹². In ADF, the QZ4P basis set was used for all systems except for the OH--containing WATER27 clusters, which were described by the ET-QZ3P-DIFFUSE basis set. All-electron computations in ADF for the HEAVY27 test set include the ZORA¹⁴⁸ relativistic corrections. The "dependency" and

"addDiffuseFit" keys were applied throughout and the integration accuracy set to 8. For the sake of clarity and brevity, only a selection of the tested functionals is included in the figures, but all of the statistics are collected in Table 1 and details given in the Supporting Information.

Geometries and reference values for the peptide conformational energies (4) and the cyclization reaction (5) are taken from ref 108 and refs 149 and 150, respectively. The Grubbs catalysts' (6 and 7) geometries and zero-point energy corrections are taken from ref 151.

The dDsC corrections are applied post-SCF, using atomic fragments computed on the fly with the same method and basis set as the molecular computation. All DFT-D3 40 and M06-2X 27 values are taken from the GMTKN30 Web page. 107

■ RESULTS AND DISCUSSION

The performance of dDsC is at first illustrated by Figure 1, which collects seven typical reactions for which a dispersion correction is essential. The first two reactions are taken from the S22 test set¹¹² and represent general π , π -stacking interactions (adenine—thymine base pair (1), which is unbound at the B3LYP level) and the phenol dimer (2) that features a combination of hydrogen-bond and other interactions often present in organic molecules. The isomerization reaction of δ -valerolactone (3a) into 2,4-penandione (3b)¹²⁶ is characteristic of typical organic isomerization reactions and is also in the training set.

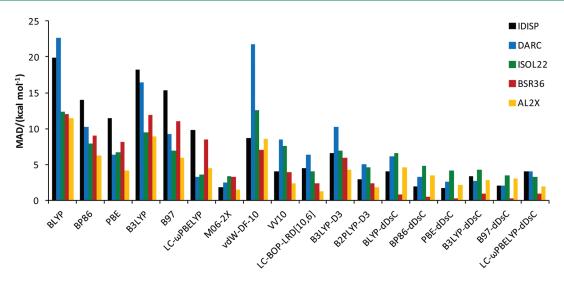


Figure 2. Mean absolute deviations for test sets dominated by intramolecular weak interactions.

The relative conformation energies of the two FGG tripeptides (4) is another example in which modeling of weak interactions is crucial to identifying the lower-lying conformer. The cascade reaction leading to the formation of the steroid framework 5a from the squalene precursor 5b is a striking case with an error of almost 50 kcal mol⁻¹ at the B3LYP level. Finally, the experimental senergy difference between the bond dissociation energies of PCy₃ from Grubbs' first (6a) and second generation (7a) catalysts senergy difference between the bond dissociation functional levels senergy difference between the bond dissociation that the standard density functional levels senergy difference between the bond dissociation for the standard density functional levels senergy difference between the bond dissociation that the senergy difference between the bond dissociation for the standard density functional levels senergy difference between the bond dissociation that the senergy difference between

Reaction energies associated with a considerable change in molecular size and shape are challenging cases for density functional approximations. As discussed previously, ⁶¹ the problem may be associated with over-repulsiveness in the short range, ^{109,157} but missing weak interactions in the medium and long-ranges are the largest contributors to the errors. ^{43,61,108,110,158} By including reactions accounting for weak intramolecular interactions into the training set, our aim is to (i) obtain additional information regarding the proper form of the damping that is empirical in nature and (ii) devise a robust correction that improves both reaction energies and weak intermolecular interactions that are generally the only focus of empirical dispersion corrections. ^{31,39,46,49,53,54}

dDsC reduces the MAD of the parent functional for intramolecular interactions (see Figure 2) by a factor of 3-6, depending on the functional. The dramatically low (<1.0 kcal mol⁻¹) MAD(BSR36) results from the highly systematic error in bond separation energies 15,43,61 along with the relatively large number (i.e., five) of such reactions included in the training set. The improvements for the intramolecular dispersion in hydrocarbons (IDISP) and the dimerizations of aluminum species (AL2X) as well as for the isomerizations of large organic molecules (ISOL22) highlight the high transferability of the density dependent scheme using the present parametrization. Long-range corrected functionals, such as LC- ω PBE, are among the bestuncorrected approximations (see Table 1 and the Supporting Information for more details). However, the remaining error is less systematic than that of standard functionals, and their combination with dDsC often leads to overcorrection.

LC-ωPBELYP-dDsC is the most accurate combination, but the variant does not present significant advantages over standard DFT-dDsC methods. The latter also clearly outperform the more sophisticated nonlocal van der Waals density functionals. The poorer performance of vdw-DF-10 as compared to VV10 is most likely related to the replacement of the local PW92 by the PBE correlation in VV10: The PBE correlation functional is known to capture intramolecular interactions involving weakly interacting densities that overlap reasonably well.⁶¹ The changes in bond types of the AL2X, DARC, and ISOL22 test sets might be more accurate with the PBE than the PW92 correlation functional as well. LC-BOP-LRD[10,6] further lowers the MAD to 3.5 kcal mol⁻¹ in this category. With a MAD of 2.9 and 3.4 kcal mol⁻¹ over the five "intramolecular" test sets, M06-2X and B2PLYP-D3, respectively, improve considerably over the standard density functionals (e.g., MAD(B3LYP) = 12.2 kcal mol⁻¹) but do not achieve the high accuracy of DFTdDsC, where most functionals are corrected to a MAD of only about 2 kcal mol⁻¹, with a minimum of 1.7 kcal mol⁻¹ for PW6B95-dDsC.

The improved energies for systems characterized by typical weak intermolecular interactions are collected in Figure 3. Most atom pairwise dispersion corrections and fully nonlocal van der Waals functionals are designed to improve the treatment of those interactions. Accordingly, the performance of methods such as B2PLYP-D3 is excellent, and VV10, vdW-DF-10, and LC-BOP-LRD[10,6] give relatively low errors as well. The remarkable performance of M06-2X is, on the other hand, illustrative of the success of extensive fitting. With an average MAD of 0.6 kcal mol⁻¹ (over 13 density functionals, excluding HF-dDsC), DFTdDsC also performs well for diverse types of weak intermolecular interactions and relative conformational energies (see Table 1 and Supporting Information). The small errors obtained for the S22 test set (assessing pure dispersion to H-bonding) along with those on the heavy atom hydrides confirm the general accuracy of the density dependent dispersion scheme. Alkane dimers (ADIM6) are, however, overcorrected by dDsC. Our careful analysis suggests that ADIM6 is an exception rather than the result of an overfitting toward intramolecular interactions dominating the training set. Subtle changes in nonbonded interactions such as those dictating the relative conformational energies

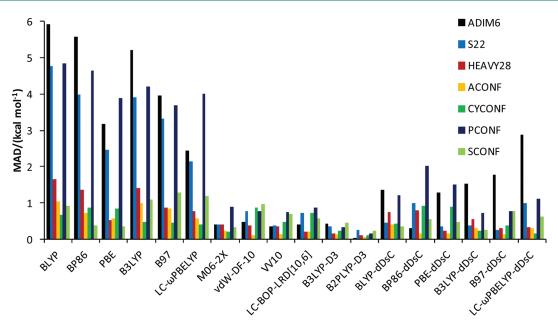


Figure 3. Mean absolute deviations for test sets featuring intermolecular weak interactions or relative conformational energies.

of alkanes (ACONF) are, for instance, well captured by dDsC, which shows that the strong correction needed for improving bond separation equations does not generally deteriorate longerrange interactions. To a much lesser extent, the D3 level also overcorrects alkane dimers, even though D3 is parametrized to perform well for these systems (see the detailed performance of D3 on the GMTKN Web site¹⁰⁷). The peculiarity of the ADMI6 test set is further illustrated by the contrasting trend in the performance of MP2/CBS (MAD = $0.27 \text{ kcal mol}^{-1}$) and SCS-MP2/CBS (MAD = $1.05 \text{ kcal mol}^{-1}$), which is opposite that of the S22 test set. 152 The modest performance of dDsC for the Phe-Gly-Gly-peptide conformations (PCONF) is, to a large extent, influenced by to the choice of "reference" conformer used in the relative energy computations. Standard functionals indeed identify the second lowest energy conformer instead of the correct conformation (at the CCSD(T) level) as the lowest energy one. The MADs are thus lowered by up to 50%, when considering the second lowest lying (0.14 kcal/mol higher according to the CCSD(T) reference values 118) as the "reference compound"!

Several additional interesting features of Figure 3 can be better understood by considering the characteristics of the parent functional. For instance, the accurate treatment of the relative conformational energies of cysteine (CYCONF) relies on a balanced description between strong (e.g., OH···N) intramolecular hydrogen bonds (that dominate some of the conformers) and weaker interactions (e.g., NH···S present in other conformers). The good description of OH···N and NH···O hydrogen bonds by PBE and BP86 versus their underestimation of weak interactions bias the relative conformation energies and result in the poorer performance of PBE(-dDsC) and BP86-(-dDsC) for CYCONF than for SCONF. The relative energies of sugar conformers, which are all dominated by strong hydrogen bonds, are indeed better described by these levels, 117 which do not benefit from the inclusion of a dispersion correction.

Figure 4 collects errors for the "mixed" category, regrouping six test sets, which are not all dominated by weak interactions but are nevertheless important for typical computational chemistry applications. The errors in radical stabilization energies (RSE43), isomerization energies of small molecules (ISO34), and the NBPRC test set, for instance, originate from subtle inaccuracies in, e.g., bond energies. The inaccurate treatment of barrier heights of pericyclic reactions (BHPERI) is generally attributed to the self-interaction error, ^{159–163} and to the delocalization error 109 (or the error in the repulsive wall 61) that is also at the origin of the poor assessment of the related Diels—Alder reaction energies (see DARC in Figure 3). For "repulsive" functionals such as BLYP or B3LYP, the dispersion correction stabilizes the transition state and leads to a clear improvement. The barrier heights are, however, overcorrected with more attractive approximation such as PBE. The unexpected poor performance of LC- ω PBELYP (LC- ω PBE and LC- ω PBEB95 perform better in this case, with a MAD of about 6.7 kcal mol⁻¹ vs 10.3 kcal mol⁻¹, but even BLYP (MAD= 5.8 kcal mol⁻¹) outperforms the long-range corrected functionals) results from a strong overestimation of the barrier heights in line with that of HF (23.2 kcal mol⁻¹ and 10.6 kcal mol⁻¹ with HF-dDsC). The high error for BHPERI along with the general difficulty of systematically improving the LC- ω PBE functional group by a dispersion correction (vide supra) reflects the need for a better-devised long-range correction parameter ω . A system dependence could be a strategy that would, however, cause size-extensivity problems important for reaction energies. At higher computational costs, the more balanced description of range-separated local hybrids 167 represents another alternative. Note that M06-2X, with a MAD of 2.8 kcal mol⁻¹, is also affected by the large amount of "exact" exchange (54%), while B97-dDsC (~19% "exact exchange") performs best for these barrier heights (MAD = 1.3 mol^{-1}).

ALK6 played an important role in cross-validating the proposed density dependent dispersion correction: the three benzene—alkaline cation (Li⁺, Na⁺, K⁺) complexes are dominated by electrostatic and inductive interactions and are thus well described by standard DFT levels. Such interactions are, however, problematic for classical dispersion correction schemes, which use dispersion coefficients and vdW radii corresponding (approximately) to the free (neutral) atoms, and not to the

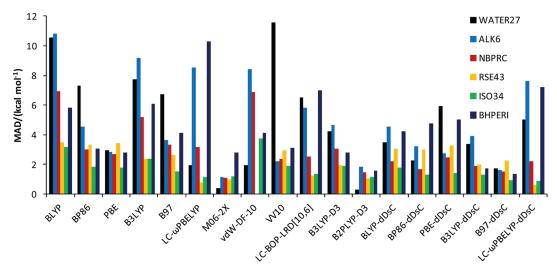


Figure 4. Mean absolute deviations over test sets assessing various reaction energies and barrier heights for pericyclic reactions. For vdW-DF-10, the RSE43 set could not be computed since it is not defined for open-shell systems.

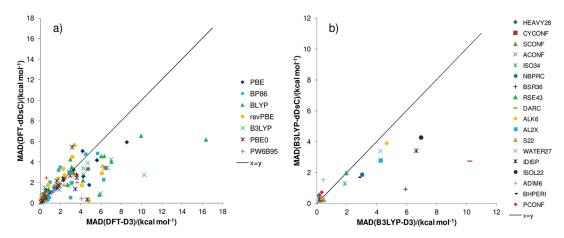


Figure 5. (a) Performance of DFT-dDsC versus DFT-D3 for seven functionals and the 18 selected test sets from the GMTKN30 database and (b) B3LYP-dDsC versus B3LYP-D3 with each test set as one point.

cations. The other three systems in the test set are the decomposition of Li₈, Na₈, and K₈, into their respective dimers. In our scheme, these clusters are characterized by relatively large dispersion coefficients and are almost as polarizable as free alkaline atoms. While most functionals underbind these clusters, our genuine damping factor, $b_{ij,asym}$, successfully avoids overcorrection due to its dependence on polarizability.

The overall description of test sets collected in the "mixed" category depends generally more strongly on the functional itself, than on the accuracy of the dispersion correction. For instance, the better performance of B2PLYP-D3 as compared to the dDsC corrected variants is due to B2PLYP, rather than to D3, as clearly illustrated by the comparison of B3LYP-D3 and B3LYP-dDsC (MADs of 2.7 and 2.1 kcal mol⁻¹, respectively). Similarly, even though the LRD scheme (independently from the use of multicenter contributions, i.e., LRD[10,0] or LRD-[10,6]) improves the overall performance on the 18 test sets (3.32 vs 2.56 kcal mol⁻¹), LC-BOP and LC-BOP-LRD[10,6], have almost the same MAD for these "mixed" test sets (3.52 and 3.54 kcal mol⁻¹, respectively). The relatively large error of PBE-dDsC originates from the overcorrected PBE energies for WATER27 and BHPERI. A similar overcorrection is at the origin of the relatively

poor performance of VV10 (total MAD of 4.2 kcal mol⁻¹). PBE-dDsC gives lower MAD than PBE-D3 for two reasons: (i) the ionic term in the damping function (eq 9) attenuates the correction for the strong and highly polarized hydrogen bonds of WATER27, and (ii) the polarizability-dependent damping factor prevents the energy overcorrection for the alkaline metal clusters (ALK6). Overall, B97-dDsC and PW6B95-dDsC (see Supporting Information) achieve MADs below 2.0 kcal mol⁻¹, which illustrate that dDsC leads to improvements for this most challenging mixed category, albeit less impressive than for inter- and especially intramolecular (weak) interactions.

Figure 5 provides a detailed comparison of the MADs obtained with dDsC and the geometry-dependent D3 correction for seven functionals (Figure 5a) and the individual test sets (Figure 5b). The D3 correction performs better than dDsC in cases for which the latter has a tendency to overcorrect (e.g., ADIM6 or BHPERI with PBE) or for which the former scheme uses quasi-exact dispersion coefficients (HEAVY28). As expected, D3 also performs well for its targeted interactions (weak interactions between neutral molecules and relative conformational energies are in the training set⁴⁰). On the other hand,

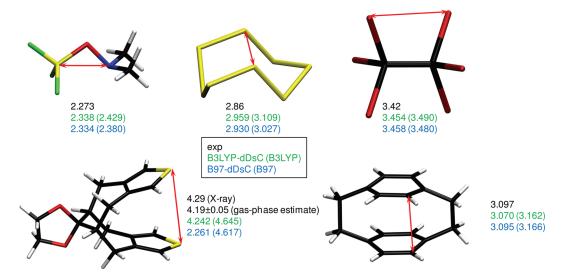


Figure 6. Geometrical structures of (CH₃)₂NOSiF₃, ¹⁷² S₈²⁺, ¹⁷¹ C₂Br₆ (first row), ¹⁷⁰ RESVAN, ^{174,175} and [2.2] paracyclophane ¹⁷³ (second row) with key nonbonded distances (in Ångstrom) indicated.

dDsC adjusts better to a given functional and provides a more robust performance, when considering both inter- and intramolecular interactions including challenging reaction energies (e.g., ISOL22, DARC, BSR36, IDISP, and AL2X).

The effect of dispersion corrections on thermochemistry has been thoroughly investigated. Geometries are usually less sensitive to the level of theory applied, but intramolecular nonbonded interactions are critical in certain cases. We thus compare the performance of two (un)corrected functionals, B3LYP and B97, for reproducing the geometry of five challenging molecules for which experimental structures are available: ${\rm C_2Br_6}, {\rm ^{170}S_8}^{2+171}$ (CH₃)₂NOSiF₃, ${\rm ^{172}}$ [2.2]paracyclophane, ${\rm ^{173}}$ and a bisthienofused molecule known under its CSD entry name RESVAN (see Figure 6). ${\rm ^{169,174-176}}$ B3LYP and B97 are overly repulsive for these intramolecular nonbonded contacts. The use of dDsC improves the geometries significantly, especially for the bisthieno-fused compound (RESVAN), mimicking stacked thiophene oligomers.

■ CONCLUSIONS

The final parametrization and refinement of the density dependent dispersion correction, dDsC, introducing a simple atomic partitioning, computationally efficient dispersion coefficients, and advanced damping functions, considerably improves the performance of standard density functionals for various reaction energies and weakly interacting systems. With a MAD of 1.3 kcal mol⁻¹ over the 18 test investigated sets, B97-dDsC performs slightly better than M06-2X and B2PLYP-D3 (MAD = 1.4 kcal mol⁻¹ for both) but at a lower computational cost. The performance of B97-dDsC is especially impressive for the five intramolecular test sets (MAD = 1.8 kcal mol⁻¹) for which M06-2X and B2PLYP-D3 are less satisfactory (MAD of 2.9 and 3.4 kcal mol⁻¹, respectively).

The correction is available for all elements of the periodic table. Due to its robust performance and general accuracy for various interactions, ranging from hydrocarbon reaction energies to heavy-atom hydride weak interaction energies, as well as for geometry optimization, we anticipate broad application of the dDsC scheme in diverse fields of computational

chemistry (e.g., organocatalysis, QM/MM hybrid schemes, prediction of crystal structures). The density dependence of both the dispersion coefficients and the damping function has been shown to be especially valuable for modeling oxygen reduction reactions by organic reducing agents, ¹⁷⁷ the splitting of water by metallocenes, ¹⁷⁸ as well as for the molecular receptors, ¹⁷⁹ which all involve charged species.

ASSOCIATED CONTENT

Supporting Information. The optimal parameters a_0 and b_0 for all functionals tested herein and all reaction energies and MADs are listed. This material is provided free of charge via the Internet at http://pubs.acs.org

■ AUTHOR INFORMATION

Corresponding Author

*E-mail: clemence.corminboeuf@epfl.ch.

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