

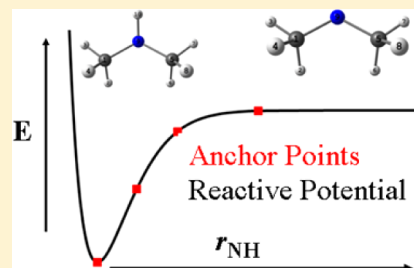
# Anchor Points Reactive Potential for Bond-Breaking Reactions

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**S** Supporting Information

**ABSTRACT:** We present a new method for fitting potential energy surfaces in molecular-mechanics-like internal coordinates based on data from electronic structure calculations. The method should be applicable to chemical reactions involving either bond dissociation or isomerization and is illustrated here for bond dissociation, in particular the breaking of an O–H bond in methanol and the breaking of an N–H bond in dimethylamine. As compared to previously available systematic methods for fitting global potential energy surfaces, it extends the maximum size of the system than can be treated by at least an order of magnitude.



## 1. INTRODUCTION

Potential energy surfaces (PESs) and their associated force fields play critical roles in chemical reactions and molecular interactions; the construction of a PES is the first step in computing the dynamics<sup>1–4</sup> or optimizing a geometry. The PES may be implicit, as in direct dynamics, or explicit, when an analytical PES is available. Molecular mechanics (MM), which denotes an analytical potential expressed in internal coordinates, is perhaps the simplest way to construct a PES, but conventional MM omits cross terms and is valid only near an equilibrium structure. Nevertheless MM finds a myriad of uses and is widely used in the study of protein folding, drug docking, and materials simulations. Many MM force fields, for instance, CHARMM,<sup>5–8</sup> AMBER,<sup>9–13</sup> GROMOS,<sup>14</sup> MM3,<sup>15–17</sup> OPLS,<sup>18–20</sup> UFF,<sup>21</sup> MMFF94,<sup>22–25</sup> GAFF,<sup>26</sup> and TraPPE,<sup>27,28</sup> have been developed for the study of biopolymers, other polymers, and small organic molecules. The key advantage of popular MM force fields is that they involve transferable parameters that need not be redetermined for each new molecule studied. Recently, MM-like force fields for specific molecules based on quantum mechanical electronic structure calculations of energies and Hessians at an equilibrium structure have been developed for more accurate simulations.<sup>29,30</sup> However, one of the limitations of even such molecule-specific internal-coordinate force fields is their inability to describe bond-breaking processes.

Recent development in potential energy surface fitting such as using global permutationally invariant polynomials<sup>31,32</sup> and the interpolated moving least-squares (IMSL) method,<sup>33–35</sup> can describe bond breaking processes and have been used successfully to generate global potential energy surfaces based on electronic structure calculations, but they require many such calculations. In particular, the number of electronic calculations required for PES fitting grows rapidly with the dimensionality of the system, and therefore, most fitted PESs are limited to systems with five or fewer atoms. For a system with  $N$  atoms, the number of internal degrees of freedom is  $F = 3N - 6$ , and

the number of data points grows as  $m^F$ , where  $m$  is the number of points needed to span a given degree of freedom. For a molecule with only ten atoms, using only four points in each dimension for the fit still requires  $4^{24} \approx 10^{14}$  data points.

Conventional MM can be extended to include bond breaking by substituting a Morse curve or similar one-dimensional potential energy curve for the usual bond stretching term, but this does not allow for the change in geometrical parameters and force constants in other modes as the breaking bond is extended, and so, it can be very inaccurate. Some work has been carried out to extend MM to bond breaking processes without assuming such separability of the bond breaking coordinate, but still allowing for the possibility of using general or specific-range parameters, as in the reactive empirical bond order method,<sup>36,37</sup> ReaXFF,<sup>38,39</sup> or the valence bond order (VBO) method,<sup>40</sup> but such methods have less accuracy for global or semiglobal PESs than conventional MM has near equilibrium structures. It would be desirable to combine analytical potentials in internal coordinates for degrees of freedom that involve small distortions from equilibrium structures with model potentials based on general functional forms to fit degrees of freedom involved in or closely coupled to bond breaking. Such an approach would be analogous to the combined quantum mechanics/molecular mechanics (QM/MM) method where MM is used for degrees of freedom of spectator atoms, and QM is used for degrees of freedom of active ones. Unlike QM/MM, which has been used widely for incorporating MM into electronic structure calculations of PESs of large systems such as enzymes and catalysis,<sup>41–47</sup> very few studies have been performed that incorporate MM-like force fields into the fitting of high-dimensional PESs.<sup>48</sup> In this article, we present a method called anchor points reactive potential (APRP) that combines fitting of quantum mechanical potential energy surfaces for selected degrees of freedom with molecule-specific MM-like

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force fields for other degrees of freedom to obtain a high-dimensional PES to treat bond-breaking processes.

## 2. METHODOLOGY

### 2.1. Anchor Points Reactive Potential (APRP) Method.

Conventional molecular mechanics force fields cannot describe chemical reactions in which bond breaking processes are involved. In our proposed anchor points reactive potential method, the contributions to a potential energy surface (PES) are divided according to their dependence on three predefined groups of internal coordinates:

1. Reaction coordinates:  $\mathbf{q}$
2. Secondary coordinates:  $\mathbf{s}$
3. Tertiary coordinates:  $\mathbf{Q}$

The potential energy is written as

$$V = V^{[1]}(\mathbf{q}) + V^{[2]}(\mathbf{s}|\mathbf{q}) + V^{[3]}(\mathbf{Q}|\mathbf{q}) \quad (1)$$

where  $f(x|y)$  denotes a function with a dependence on  $x$  and a parametric dependence on  $y$ . The three terms in eq 1 will be called the primary, secondary, and tertiary potentials; the primary and secondary terms can be completely general. In the present application, the reaction coordinate group consists of a single bond length  $r$ , and we will present the equations in the present subsection for this single-reaction-coordinate case to make the ideas more clear. To fit  $V^{[2]}(\mathbf{s}|r)$  and  $V^{[1]}(r)$ , we need to calculate many data points in the space of  $\mathbf{s}$  and  $r$  for a set of fixed values of the tertiary coordinates corresponding to a reference structure, and for the calculation of the torsional term in the secondary potential, we also calculate a grid of energies in a two-dimensional subspace with the other coordinates relaxed (see further details below).

The tertiary potential is described by analytic potentials in internal displacement coordinates  $\mathbf{Q}^{[a]}$  relative to a set of anchor structures  $a$  with the anchor structures spaced along the reaction coordinates at a set of anchor values  $\{r_a = r_i, i = 1, 2, \dots, N_A\}$ :

$$V^{[3]}(\mathbf{Q}|\mathbf{r}) = \sum_{a=1}^{N_A} V_a(\mathbf{Q}^{[a]}) T_a(r) \quad (2)$$

In eq 2,  $N_A$  is the number of anchor structures,  $V_a(\mathbf{Q}^{[a]})$  is a molecular mechanics-like potential with molecule-specific and anchor-point-specific parameters at anchor structure  $a$ , and  $T_a(r)$  is called a tent function. The tent function is defined by

$$T_1(r) \rightarrow \begin{cases} 1 & r < r_1 \\ \frac{(r - r_2)^4}{(r - r_2)^4 + (r - r_1)^4} & r_1 \leq r < r_2 \end{cases} \quad (3a)$$

$$T_a(r) \rightarrow \begin{cases} \frac{(r - r_{a-1})^4}{(r - r_a)^4 + (r - r_{a-1})^4} & r_{a-1} \leq r < r_a \\ \frac{(r - r_{a+1})^4}{(r - r_{a+1})^4 + (r - r_a)^4} & r_a \leq r < r_{a+1} \end{cases} \quad (3b)$$

for  $a = 2, \dots, N_A - 1$

$$T_{N_A}(r) \rightarrow \begin{cases} \frac{(r - r_{N_A-1})^4}{(r - r_{N_A})^4 + (r - r_{N_A-1})^4} & r_{N_A-1} \leq r < r_{N_A} \\ 1 & r_{N_A} \leq r \end{cases} \quad (3c)$$

Tertiary coordinates  $\mathbf{Q}$  are further divided into two subgroups: stiff coordinates  $\mathbf{Q}_{\text{stiff}}$  such as stretches, bends, and rigid torsions, and soft torsion coordinates  $\mathbf{Q}_{\text{ST}}$ , which involve wide-amplitude motion. The tertiary potential at anchor structure  $a$  is written as

$$V_a(\mathbf{Q}^{[a]}) = V_a^{\text{stiff}}(\mathbf{Q}_{\text{stiff}}^{[a]}) + V_a^{\text{ST}}(\mathbf{Q}_{\text{ST}}^{[a]}) \quad (4)$$

where  $V_a^{\text{stiff}}(\mathbf{Q}_{\text{stiff}}^{[a]})$  is the potential energy contribution from stiff coordinates, and  $V_a^{\text{ST}}(\mathbf{Q}_{\text{ST}}^{[a]})$  is the potential energy contribution from soft torsions.

Partial optimization is performed for each anchor structure  $a$ , with the only constraint being  $r = r_a$ , and the force constant matrix  $\mathbf{F}^{[a]}$  in internal displacement coordinates is calculated for the optimized structure. Element  $i$  of the stiff internal displacement coordinate vector  $\mathbf{Q}_{\text{stiff}}^{[a]}$  is defined as

$$Q_{\text{stiff},i}^{[a]} = S_i - S_{i,e}^{[a]} \quad (5)$$

where  $S_i$  is an internal coordinate, and  $S_{i,e}^{[a]}$  is the optimized value of  $S_i$  in the constrained optimized geometry of anchor point  $a$ .

Since  $V^{[2]}(\mathbf{s}|\mathbf{q})$  and  $V^{[1]}(\mathbf{q})$  are fitted to data corresponding to a rigid tertiary subspace corresponding to a reference structure, while the tertiary potential is expanded at partially optimized anchor structure  $a$ , a relaxation energy  $V^{[a]}$  needs to be considered; this is defined as the energy difference between partially optimized anchor structure  $a$  and the rigid geometry with the same  $r_a$ . Then,  $V_a^{\text{stiff}}(\mathbf{Q}_{\text{stiff}}^{[a]})$  is expanded near anchor structure  $a$  as

$$V_a^{\text{stiff}}(\mathbf{Q}_{\text{stiff}}^{[a]}) = V^{[a]} + \frac{1}{2} \mathbf{Q}_{\text{stiff}}^{[a]T} \mathbf{F}_{\text{stiff}}^{[a]} \mathbf{Q}_{\text{stiff}}^{[a]} \quad (6)$$

where  $\mathbf{F}_{\text{stiff}}^{[a]}$  is the partial force constant matrix in the stiff subspace. The other tertiary potential term is parametrized as

$$V_a^{\text{ST}}(\mathbf{Q}_{\text{ST}}^{[a]}) = \sum_{\tau=1}^{N_{\text{ST}}} V_{\tau}^{[a]} [1 - \cos n_{\tau}(\varphi_{\tau} - \varphi_{\tau,e}^{[a]})] \quad (7)$$

where  $N_{\text{ST}}$  is the number of soft torsions,  $n_{\tau}$  is the local periodicity of the torsion, and  $V_{\tau}^{[a]}$  and  $\varphi_{\tau,e}^{[a]}$  are respectively the torsion barrier and equilibrium torsion angle for torsion  $\tau$  at anchor structure  $a$ .

**2.2. Analytical Cartesian Gradient.** In order to perform efficient dynamic studies, we require analytic Cartesian gradients of the PES. Denoting a column vector containing  $3N$  Cartesian coordinates ( $\xi_{3i-2} = x_i$ ,  $\xi_{3i-1} = y_i$ ,  $\xi_{3i} = z_i$ ,  $i = 1, \dots, N$ , where  $N$  is the number of atoms) as  $\xi$ , eq 1 yields

$$\frac{dV}{d\xi} = \frac{dV^{[1]}(\mathbf{q})}{d\xi} + \frac{dV^{[2]}(\mathbf{s}|\mathbf{q})}{d\xi} + \frac{dV^{[3]}(\mathbf{Q}|\mathbf{q})}{d\xi} \quad (8)$$

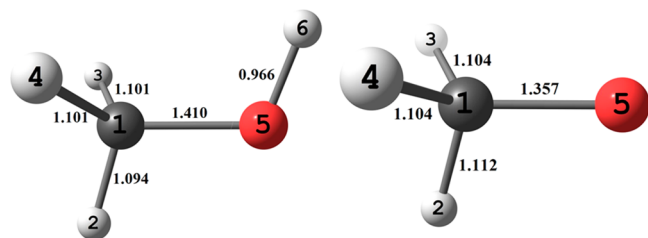
Each term in eq 8 is constructed with internal coordinates, so the chain rule must be applied to obtain the analytic Cartesian gradient. This yields

$$\frac{dV}{d\xi} = \left( \frac{\partial V^{[1]}(\mathbf{q})}{\partial \mathbf{q}} + \frac{\partial V^{[2]}(\mathbf{s}|\mathbf{q})}{\partial \mathbf{q}} + \frac{\partial V^{[3]}(\mathbf{Q}|\mathbf{q})}{\partial \mathbf{q}} \right) \frac{\partial \mathbf{q}}{\partial \xi} + \frac{\partial V^{[2]}(\mathbf{s}|\mathbf{q})}{\partial \mathbf{s}} \frac{\partial \mathbf{s}}{\partial \xi} + \frac{\partial V^{[3]}(\mathbf{Q}|\mathbf{q})}{\partial \mathbf{Q}} \frac{\partial \mathbf{Q}}{\partial \xi} \quad (9)$$

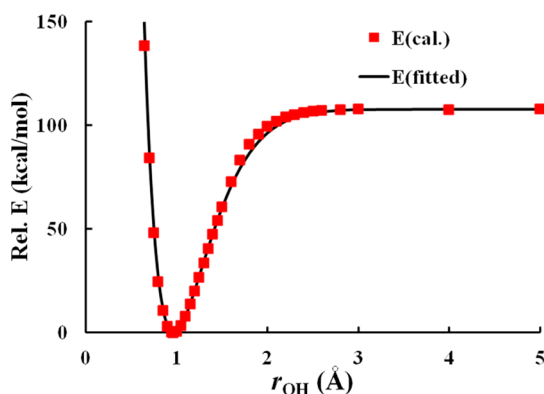
For the APRP PES of eq 1,  $(\partial V^{[1]}(\mathbf{q})/\partial \mathbf{q})$ ,  $(\partial V^{[2]}(\mathbf{s}|\mathbf{q})/\partial \mathbf{q})$ ,  $(\partial V^{[3]}(\mathbf{Q}|\mathbf{q})/\partial \mathbf{q})$ ,  $(\partial V^{[2]}(\mathbf{s}|\mathbf{q})/\partial \mathbf{s})$ , and  $(\partial V^{[3]}(\mathbf{Q}|\mathbf{q})/\partial \mathbf{Q})$  can be evaluated straightforwardly. The other derivatives,  $(\partial \mathbf{q}/\partial \xi)$ ,  $(\partial \mathbf{s}/\partial \xi)$ , and  $(\partial \mathbf{Q}/\partial \xi)$ , are Wilson's rectangular B matrices<sup>49</sup> for reaction, secondary, and tertiary coordinates, with dimensions of  $N_q \times 3N$ ,  $N_s \times 3N$ , and  $N_Q \times 3N$ , where  $N_q$ ,  $N_s$ , and  $N_Q$  are the number of reaction, secondary, and tertiary coordinates, and they can be evaluated analytically.<sup>50,51</sup>

### 3. RESULTS AND DISCUSSION

We provide two examples to illustrate the APRP method, namely the O–H dissociation in CH<sub>3</sub>OH and the N–H



**Figure 1.** Geometries of CH<sub>3</sub>OH (left) and CH<sub>3</sub>O (right) (both in *C<sub>s</sub>* symmetry).



**Figure 2.** Varshni potential to describe the O–H bond dissociation in CH<sub>3</sub>OH. All internal coordinates except the O–H bond distance are held constant along the path used for this figure.

dissociation in (H<sub>3</sub>C)<sub>2</sub>NH. The electronic structure calculations for geometry optimization and energy and force constant calculations were performed with the unrestricted Kohn–Sham formalism with *Gaussian 09*.<sup>52</sup> The M06 exchange–correlation functional<sup>53</sup> and the 6-31+G(d) basis set<sup>54</sup> were used. For integrations to compute the exchange–correlation energy, we used a pruned (99,590) grid [called the ultrafine grid] for all single-point energy calculations and a (150,974) grid [called the superfinegrid] for optimizations and force constant calculations.

**3.1. CH<sub>3</sub>OH → CH<sub>3</sub>O + H.** The reference structure for the parametrization of the primary and secondary potentials is taken to be the equilibrium geometry of CH<sub>3</sub>OH. The CH<sub>3</sub>OH molecule possesses *C<sub>s</sub>* symmetry. When the H atom is removed, the CH<sub>3</sub>O radical undergoes Jahn–Teller distortion

from *C<sub>3v</sub>* symmetry to *C<sub>s</sub>* symmetry.<sup>55</sup> The optimized geometries and atomic labeling are shown in Figure 1. To treat the O–H bond dissociation, the O–H distance ( $r_{\text{OH}} = r_{56}$ ) was chosen as the reaction coordinate; the bending and torsional coordinates that involve the separating H atom ( $\theta_{\text{HOC}} = \theta_{651}$  and  $\varphi_{\text{HOCH}} = \varphi_{6512}$ ), were chosen as secondary coordinates; and the other coordinates are tertiary ( $r_{\text{CH}} = r_{12}$ ,  $r_{13}$ ,  $r_{14}$ ,  $r_{\text{CO}} = r_{15}$ ,  $\theta_{\text{HCH}} = \theta_{213}$ ,  $\theta_{214}$ ,  $\theta_{314}$ , and  $\theta_{\text{HCO}} = \theta_{215}$ ,  $\theta_{315}$ , and  $\theta_{415}$ ) were chosen as tertiary coordinates.

The Varshni model potential, given by

$$V^{[1]}(r_{\text{OH}}) = D_1 \{ 1 - (r_1/r_{\text{OH}}) \exp[-\beta_1((r_1/r_{\text{OH}})^2 - 1)] \}^2 \quad (10)$$

was used to fit the O–H dissociation curve in CH<sub>3</sub>OH since it has been shown to provide better approximations than the widely used Morse model in diatomics<sup>56,57</sup> and in the C–H dissociation of CH<sub>4</sub>.<sup>58</sup> The optimized parameters are given in the Supporting Information, and the calculated and fitted O–H dissociation curves are shown in Figure 2. The H–O–C bending and H–O–C–H torsion in secondary coordinates both involve the dissociated H atom, so they were parametrized with flexible functional forms that depend parametrically on the O–H bond length.

A scan was performed for  $r = 0.65$ – $4.0$  Å and  $\theta = 60$ – $180^\circ$  with other internal coordinates of the equilibrium structure held rigid; the scan showed that at least a cubic term is needed to account for anharmonicity, so we used

$$V_{\text{bend}}^{[2]}(\theta|r) = k_2(r)[\cos \theta - \cos \theta_0(r)]^2 + k_3(r)[\cos \theta - \cos \theta_0(r)]^3 \quad (11)$$

Notice that eq 11 has physically correct behavior in the vicinity of  $\theta = 180^\circ$ , whereas a polynomial in  $\theta$  rather than  $\cos \theta$  would have unphysical behavior. The quadratic and cubic force constants,  $k_2(r)$  and  $k_3(r)$ , as well as the equilibrium bond angles  $\theta_0(r)$  depend parametrically on the O–H bond length. The parametric dependence of the equilibrium bond angle on the O–H bond length is fitted with a switching function:

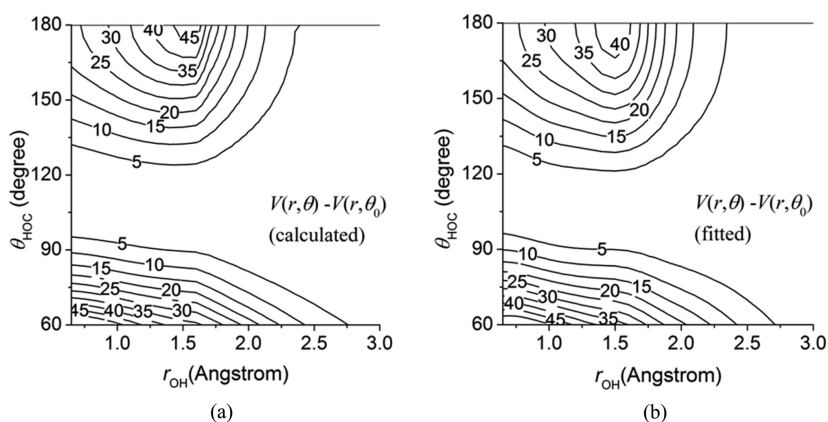
$$\cos \theta_0(r) = \cos \theta_1 + \frac{1 + \tan a(r - r_0)}{2} (\cos \theta_2 - \cos \theta_1) \quad (12)$$

where  $\cos \theta_1$ ,  $\cos \theta_2$ ,  $a$ , and  $r_0$  are adjustable parameters. The force constants were fitted with linear combinations of Gaussian functions:

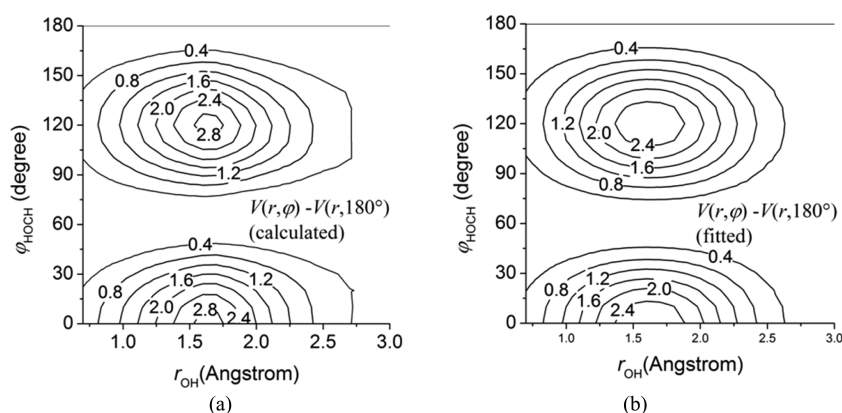
$$k_i(r) = \sum_{j=1}^{N_{G,i}} A_{i,j} \exp[-\alpha_{i,j}(r - r_{i,j})^2] \quad (13)$$

where  $N_{G,i}$  is the number of Gaussian functions used to fit  $k_i(r)$ . Contour plots of the calculated and fitted bending potentials are compared in Figure 3; they are in good agreement.

The H–O–C–H torsion potential was calculated for  $\varphi_{\text{HOCH}} = 0$ – $180^\circ$  with a step size of  $10^\circ$  for  $r = 0.65$ – $4.0$  Å. Contour plots of the torsion energy profiles along the O–H dissociation are shown in Figure 4a. The H–O–C–H torsion energy reaches its minimum for  $\varphi_{\text{HOCH}} = 180^\circ$ ,  $60^\circ$ , and by symmetry,  $-60^\circ$ . It is interesting that the torsion barrier increases first as the O–H bond length increases, reaching its maximum around  $1.7$  Å, and decreases as the O–H bond length increases further; this nonmonotonic trend is understandable since in the two limits of  $r_{\text{OH}}$  equal to 0 and to infinity, the barrier should be zero. A cosine function was used to fit the torsion energy



**Figure 3.** Contour plots of calculated and fitted H–O–C bending potential energies (in kcal/mol) along the O–H dissociation path (Note that  $\theta_0$  is a function of  $r_{\text{OH}}$ ). All internal coordinates except  $r_{\text{OH}}$  and  $\theta_{\text{HOC}}$  are held constant at their reference values for this figure.



**Figure 4.** Contour plots of calculated and fitted H–O–C–H torsional potential energies (in kcal/mol) along the O–H dissociation path. All internal coordinates except  $r_{\text{OH}}$  and  $\phi_{\text{HOCH}}$  are relaxed in the calculation of this secondary torsion.

$$V_{\text{tor}}^{[2]}(\phi|r) = V_3(r)[1 - \cos 3(\phi - \phi_0)] \quad (14)$$

The parametric dependence of torsion barrier  $V_3(r)$  on O–H bond length is fitted with linear combination of Gaussian functions:

$$V_3(r) = \sum_{i=1}^{N_G} A_i \exp[-\alpha_i(r - r_i)^2] \quad (15)$$

A contour plot of the fitted torsion energy is shown in Figure 4b. It reproduces the calculated result in Figure 4a very well. Unlike the other terms in the primary and secondary potentials, the parameters in the secondary torsion were explained with relaxed structures, that is, all internal coordinates except  $r_{\text{OH}}$  and  $\phi_{\text{HOCH}}$  are relaxed in the calculation.

Now we turn to  $V^{[3]}$ , which depends on tertiary coordinates and depends parametrically on  $r_{\text{OH}}$ . We optimized  $\text{CH}_3\text{OH}$  and  $\text{CH}_3\text{O}$  to obtain their equilibrium geometric parameters and force constant matrices (Hessians) needed for the molecular mechanics force fields in the tertiary subspace. In addition to  $\text{CH}_3\text{OH}$  and  $\text{CH}_3\text{O}$ , three more structures were partially optimized with fixed O–H bond lengths ( $r_{\text{OH}} = 1.5$ , 2.0, and 3.0 Å) to obtain the equilibrium geometric parameters and force constants matrices along the dissociation path. Five anchor structures were used to describe the distortion of tertiary coordinates along the O–H bond dissociation path. Four of these structures were the optimized structure with  $r_{\text{OH}} = 0.966$  Å and the partially optimized structures with  $r_{\text{OH}} = 1.5$ ,

2.0, and 3.0 Å; the fifth was an anchor structure at  $r_{\text{OH}} = 5.0$  Å, taken to have the energy and force constant matrix of  $\text{CH}_3\text{O}$ . Since there are no soft torsions in the tertiary coordinates for this system, eq 6 was used to describe distortions of tertiary coordinates in the vicinity of each anchor structure, and eq 2 was used to describe the tertiary potential all along the dissociation path.

As specified above, redundant internal coordinates are used as the tertiary coordinates for the construction of APRP PES. However, to test the performance of APRP, one needs to work with nonredundant coordinates that uniquely define a structure. A set of nonredundant internal coordinates are chosen as the following:

$$S_1\text{--}S_4: r_{12}, r_{13}, r_{14}, r_{15}$$

$$S_5: r_{56}$$

$$S_6: (\theta_{314} + \theta_{213} + \theta_{214} - \theta_{215} - \theta_{315} - \theta_{415})/\sqrt{6}$$

$$S_7: (2\theta_{314} - \theta_{213} - \theta_{214})/\sqrt{6}$$

$$S_8: (\theta_{213} - \theta_{214})/\sqrt{2}$$

$$S_9: (2\theta_{215} - \theta_{315} - \theta_{415})/\sqrt{6}$$

$$S_{10}: (\theta_{315} - \theta_{415})/\sqrt{2}$$

$$S_{11}: \theta_{651}$$



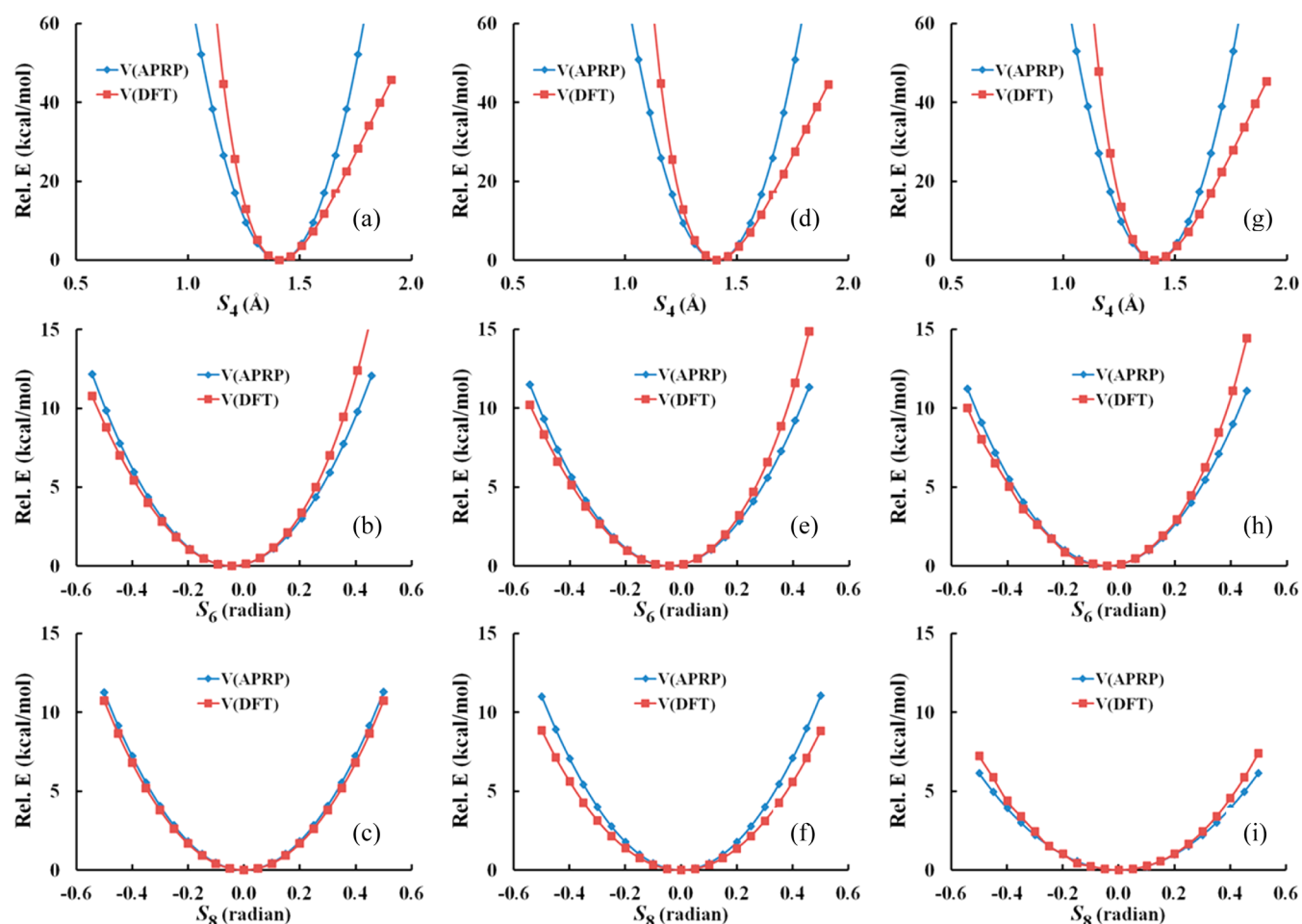


Figure 5. Slected slices along the degrees of freedom  $S_4$ ,  $S_6$ , and  $S_8$  (From left to right  $r_{\text{OH}} = r_e$ , 2.0, and 4.0 Å).

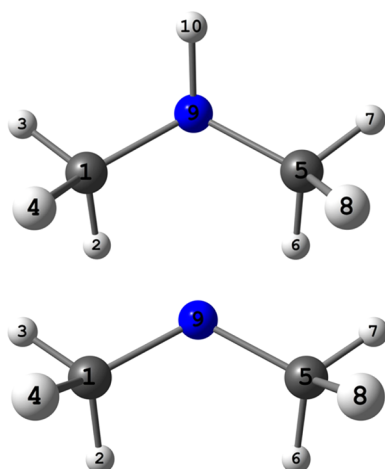


Figure 6. Geometries of  $\text{HN}(\text{CH}_3)_2$  (top,  $C_{2v}$ ) and  $\text{N}(\text{CH}_3)_2$  (bottom,  $C_{2v}$ ).

$$S_{12}: \varphi_{6512}$$

In this choice of nonredundant internal coordinates,  $S_5$  is the reaction coordinate equal to the O–H bond length,  $S_{11}$  and  $S_{12}$  are the H–O–C bending and H–O–C–H torsion included in the secondary coordinates.  $S_1$ – $S_4$  are bond lengths in tertiary coordinates, and  $S_6$ – $S_{10}$  are linear combinations of bond angles used to construct nonredundant internal coordinates as suggested by Pulay and co-workers.<sup>59</sup> Three slices of the

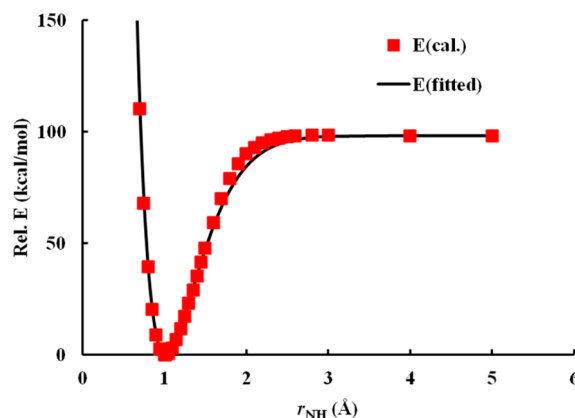


Figure 7. Varshni potential to describe the N–H bond dissociation in  $\text{HN}(\text{CH}_3)_2$ .

potential energy curves along the nonredundant internal coordinates,  $S_4$ ,  $S_6$ , and  $S_8$ , are shown in Figure 5 for three different O–H bond lengths ( $r_{\text{OH}} = r_e$ , 2.0, and 4.0 Å).

Figure 5a, d, and g show the comparison of DFT calculated potential energy curves and APRP potential energy curves along the C–O bond stretching. They agree with each other near the equilibrium bond distance, but the APRP curves are too repulsive for stretched bonds and not repulsive enough for compressed bonds, which is a typical result of bond stretch anharmonicity. Model potentials such as Morse potential or

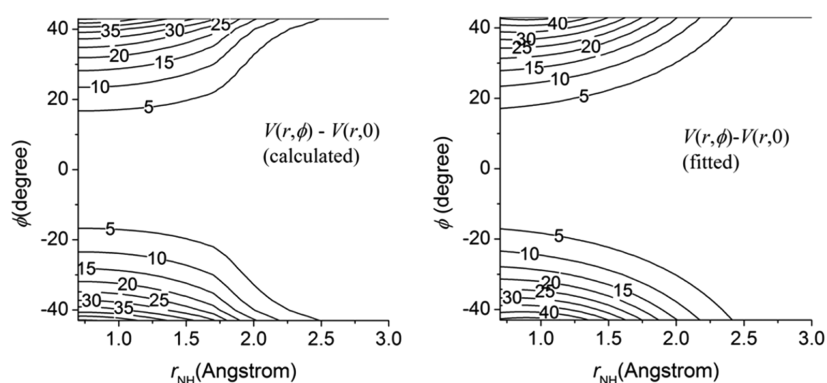


Figure 8. Contour plots of calculated and fitted H–N–C bending potential energies (in kcal/mol) along the N–H dissociation path.

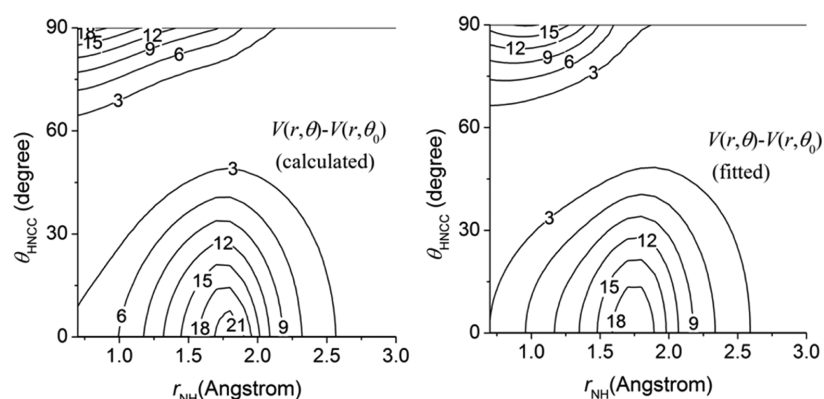


Figure 9. Contour plots of calculated and fitted H–N–C–C out-of-plane potential energies (in kcal/mol) along the N–H dissociation (Note that  $\theta_0$  is a function of  $r_{\text{NH}}$ ).

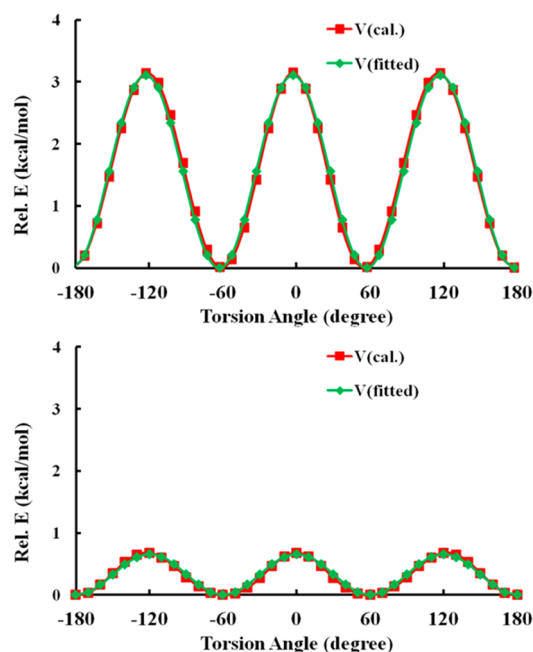


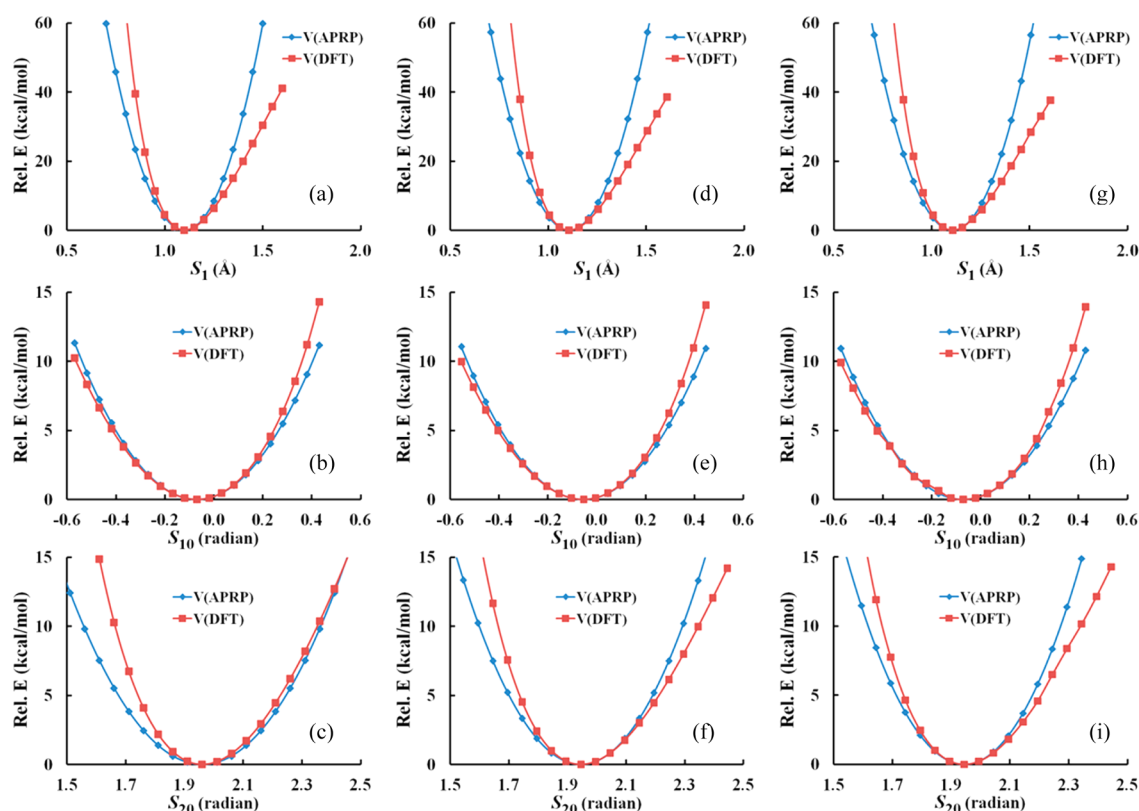
Figure 10. Calculated and fitted H–C–N–C torsion profiles of  $\text{HN}(\text{CH}_3)_2$  (top) and  $\text{N}(\text{CH}_3)_2$  (bottom).

Varshni potential can be used for bond stretching if more accurate descriptions are needed. However, in the photo-dissociation reaction under consideration, the displacements along these coordinates are expected to be small at the energies of interest, so the harmonic approximation should be adequate.

The comparison of DFT and APRP potential energy curves along  $S_6$  and  $S_8$  are shown in Figure 5b and c, e and f, and h and i, for  $r_{\text{OH}} = r_e$ , 2.0, and 4.0 Å, respectively. They agree well with each other in all cases. We note that no information (neither equilibrium geometric parameters nor Hessians) were input to the fit at  $r_{\text{OH}} = 4.0$  Å, but APRP nevertheless describes the potential energy curves quite well for this O–H distance.

**3.2.  $(\text{H}_3\text{C})_2\text{NH} \rightarrow (\text{H}_3\text{C})_2\text{N} + \text{H}$ .** The reference structure for the parametrization of the primary and secondary potentials is taken to be the equilibrium geometry of  $(\text{H}_3\text{C})_2\text{NH}$ . The  $(\text{H}_3\text{C})_2\text{NH}$  molecule has  $C_s$  symmetry. When the H atom is dissociated, the  $(\text{H}_3\text{C})_2\text{N}$  radical has  $C_{2v}$  symmetry. The optimized geometries and atomic labeling are shown in Figure 6. To treat the N–H bond dissociation, the N–H distance (which may be called either  $r_{\text{NH}}$  or  $r_{9-10}$ ) was chosen as the reaction coordinate; and the bends and out-of-plane bend that involve the dissociating H are chosen as secondary coordinates; in particular the secondary coordinates are two H–N–C bond angles  $\theta_{\text{HNC}}$  ( $\theta_{10-9-1}$  and  $\theta_{10-9-5}$ ) and the H–N–C–C out-of-plane bend  $\theta_{\text{HNCC}}$  ( $\theta_{10-9-1-5}$ ). These three coordinates cannot all change independent, so we used  $\theta_{\text{HNCC}}$  and a linear combination,  $\phi = (1/\sqrt{2})(\theta_{10-9-1} - \theta_{10-9-5})$ , of the other two coordinates as nonredundant coordinates for fitting  $V^{[2]}$ . The other 8 bond lengths, 13 bond angles, and 6 torsions, in which the dissociated H atom ( $\text{H}_{10}$ ) is not involved, were chosen as tertiary coordinates.

The Varshni model potential, eq 10, was used to fit the N–H dissociation curve in  $(\text{H}_3\text{C})_2\text{NH}$ . The calculated and fitted N–H dissociation curves are shown in Figure 7; they agree well.



**Figure 11.** Slices of the potential surface along the degrees of freedom  $S_1$ ,  $S_{10}$ , and  $S_{20}$ . (From left to right,  $r_{\text{NH}} = r_e$ , 2.0, and 4.0 Å.).

A scan was performed for  $r = 0.7$  Å to 4.0 Å and  $\phi = -43^\circ$  to  $43^\circ$  with other coordinates constrained to their values at the  $(\text{H}_3\text{C})_2\text{NH}$  equilibrium geometry. The calculated H–N–C bending potential energies along the N–H dissociation are shown in Figure 8a. The following functional form was chosen to fit the H–N–C bending potential energies

$$V_{\text{bend}}^{[2]}(\phi|r) = k_2(r)\phi^2 + k_4(r)\phi^4 \quad (16)$$

The cubic force constant,  $k_3(r)$ , is zero as a result of our definition of  $\phi$ . The quadratic and quartic force constants,  $k_2(r)$  and  $k_4(r)$ , depend parametrically on the N–H bond length, and they were fitted with linear combinations of Gaussian functions as shown in eq 13. A contour plot of the fitted H–N–C bending potential energies along the N–H dissociation is shown in Figure 8b, and it reproduces the main features of the calculated one in Figure 8a, where they are seen to agree well.

The H–N–C–C out-of-plane bend potentials are calculated for  $\theta_{\text{HNCC}} = 0-90^\circ$  and  $r = 0.7-4.0$  Å. Figure 9a shows 2D contour plots of the out-of-plane bending potential energy profiles along the N–H dissociation. Equation 11 was used to fit this potential, and a 2D contour plot of the fitted potential is shown in Figure 9b.

The tertiary potential for  $(\text{H}_3\text{C})_2\text{NH}$  was obtained by performing optimization and frequency analysis of  $(\text{H}_3\text{C})_2\text{NH}$  and  $(\text{H}_3\text{C})_2\text{N}$ , and partial optimization and frequency analysis of three other anchor structures with  $r_{\text{NH}} = 1.5$ , 2.0, and 3.0 Å. The bond stretches and bond angle bends were treated with eq 6 as in the case of  $\text{CH}_3\text{OH}$ . Since  $(\text{H}_3\text{C})_2\text{NH}$  has soft torsions, in particular HCNC torsions, in the tertiary subspace, eq 7 was used for this molecule although it was not needed for methanol. The HCNC torsion potentials of  $(\text{H}_3\text{C})_2\text{NH}$  at the five anchor structures were fitted with the following functional form

$$V_a^{\text{MM}}(\varphi) = V_i^{[a]}[1 - \cos 3(\varphi - \varphi_0)] \quad (17)$$

where  $a$  is the index of anchor structures. The calculated and fitted torsion potentials in  $(\text{H}_3\text{C})_2\text{NH}$  and  $(\text{H}_3\text{C})_2\text{N}$  are shown in Figure 10.

Again, to test the performance of APRP for tertiary coordinates, a set of nonredundant internal coordinates was chosen for  $(\text{H}_3\text{C})_2\text{NH}$ . Potential energy curves along three selected coordinates ( $S_1$ ,  $S_{10}$ , and  $S_{20}$ ) at  $r_{\text{NH}} = r_e$ , 2.0, and 4.0 Å were calculated with both DFT and APRP and are shown in Figure 11. The definition of nonredundant internal coordinates is described in detail in the Supporting Information and the potential dependence on the coordinates is shown for only three selected coordinates:

$$S_1: r_{1-2}$$

$$S_{10}: (\theta_{3-1-4} + \theta_{2-1-3} + \theta_{2-1-4} - \theta_{2-1-9} - \theta_{3-1-9} - \theta_{4-1-9})/\sqrt{6}$$

$$S_{20}: \theta_{1-9-5}$$

As in the case of  $\text{CH}_3\text{OH}$ , the APRP potential energy curves agree well with the DFT ones near the equilibrium bond lengths but fail to describe the anharmonicity for large bond stretches. The APRP potential energy curves along bond angle coordinates reproduce the DFT ones well, both for anchor structures and for nonanchor structures. These plots show that APRP is able to describe the potential well for small distortions along the bond dissociation coordinate by interpolating between anchor structures.

**3.3. Subroutines.** The two potential energy functions obtained in the present study are available (free) online in the POTLIB library.<sup>60,61</sup>

## 4. CONCLUDING REMARKS

By combining model potentials and internal coordinate force fields, we have developed a new scheme called anchor points reactive potential (APRP) to fit high-dimensional PESs. Analytic gradients of APRP PESs can be easily implemented for dynamic studies by using Wilson's B matrix. We illustrated the APRP method by applying it to construct the full dimensional PESs of CH<sub>3</sub>OH and (H<sub>3</sub>C)<sub>2</sub>NH to describe the X–H (X = O or N) bond breaking processes. It would be impractical to fit these potential energy surfaces with conventional PES fitting methods, but the present method is straightforwardly applicable and indeed would be applicable to much larger systems as well. The new method may be considered to be an extension of some previous methods<sup>29,30,62,63</sup> for fitting analytical potentials based in whole or in part on quantum mechanical Hessians, but those methods were developed for representing the potential in the vicinity of a single equilibrium structure, and the present method is designed for reactive potentials. It may also be compared to Shepard interpolation,<sup>64,65</sup> but it is more systematic in its design.

Although these examples involve only bond breaking, the method could be extended in a straightforward way to isomerization processes in which bonds are both broken and formed. Another generalization is that, although the APRP is applied here to Born–Oppenheimer processes occurring on the ground electronic state, it could also be applied to electronically nonadiabatic processes of large molecules, such as light-induced DNA damage or photodissociation. The method would be especially well suited to fitting diabatic PESs for photodissociation, such as those recently reported for phenol.<sup>66</sup>

## ■ ASSOCIATED CONTENT

### ● Supporting Information

Details of the functional forms used for fitting and the optimized fitting parameters, the definitions of internal coordinates, and the Hessians at anchor structures. This material is available free of charge via the Internet at <http://pubs.acs.org>.

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### Notes

The authors declare no competing financial interest.

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