

observed for most of the bands in going from cyclohexane to methanol. This is possibly due to increased dipole-dipole or hydrogen-bonding interactions of the ground-state molecules with the solvent. Although there are some changes in extinction coefficients and oscillator strengths in the two solvents, no trends are apparent.

The spectra of the chromium-carbene complexes consist essentially of two intense bands with quite different oscillator strengths. The low-energy bands have oscillator strengths of  $\sim 0.1$ ; the high-energy bands,  $\sim 1.0$ . The spectra of the tungsten complexes are very similar to their chromium analogs except that the low-energy bands are split into two bands of comparable intensity. It is noteworthy that in both series the high-energy absorption bands at approximately  $41,000 \text{ cm}^{-1}$  are unaffected by changes in the carbene groups. At the same time the low-energy transitions are very sensitive to changes in the carbene ligand. When R goes from  $\text{CH}_3$  or  $n\text{-C}_4\text{H}_9$  to  $\text{C}_6\text{H}_5$ , a shift to lower energy of about  $2000 \text{ cm}^{-1}$  occurs.

In spite of the fact that a firm assignment of these transitions is not possible at this time, from the magnitude of the oscillator strengths they are all probably charge transfer in nature.

The spectra of  $\text{Cr}(\text{CO})_6$  and  $\text{W}(\text{CO})_6$  in solution are very similar to those reported here.<sup>37</sup> Although there are some discrepancies in the assignment of the electronic transitions in these cases, it is generally felt that the low-energy transition is primarily metal to ligand in character, whereas the high-energy transition is ligand to metal and/or metal to ligand.

A similar assignment of transitions in the carbene complexes is expected. However presently detailed MO calculations and spectral assignments in  $\text{M}(\text{CO})_5\text{L}$  compounds are not available.

**Acknowledgments.**—The authors are grateful to the donors of the Petroleum Research Fund, administered by the American Chemical Society, for financial support of this research.

(37) H. B. Gray and N. A. Beach, *J. Am. Chem. Soc.*, **85**, 2922 (1963).

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## Rates and Mechanism of Formation of Mono(acetylacetonato)copper(II) Ion in Water and Methanol<sup>1</sup>

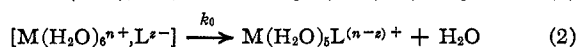
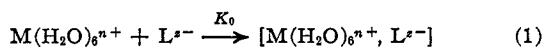
By RALPH G. PEARSON AND OREN P. ANDERSON

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Stopped-flow kinetic studies of the reaction between acetylacetone (2,4-pentanedione) and cupric ion to form the mono complex in water and in methanol show two separable reactions. Both reactions are much slower than the normal substitution kinetics of the  $\text{Cu}(\text{II})$  ion. The concentration dependence of the observed rate constants agrees with a mechanism involving direct, reversible reaction between the solvated  $\text{Cu}(\text{II})$  ion and both the keto and enol tautomers of acetylacetone, the enol tautomer reacting much faster. The second-order rate constants at  $25^\circ$  for the reaction between the enol form and the  $\text{Cu}(\text{II})$  ion are the same in water and in methanol ( $2 \times 10^4 \text{ M}^{-1} \text{ sec}^{-1}$ ), in agreement with a mechanism due to Kustin in which the rate-controlling process is the sterically difficult closure of the six-membered ring. For the reaction between the  $\text{Cu}(\text{II})$  ion and the keto form, the rate-limiting step is metal ion catalyzed proton transfer from the weakly bound keto tautomer. This is supported by the increase in rate constant of two orders of magnitude on changing from water ( $12 \text{ M}^{-1} \text{ sec}^{-1}$ ) to methanol ( $1300 \text{ M}^{-1} \text{ sec}^{-1}$ ) and the results of deuterium isotope substitution studies.

### Introduction

In the usual mechanism of complex formation for the six-coordinate transition metal ions, the first step is the reversible, diffusion-controlled formation of an ion pair between the solvated metal ion and the potential ligand.<sup>2</sup> If the metal ion then loses a water molecule, the complex may be formed. This scheme is outlined in eq 1 and 2. The loss of water from the metal ion in



the ion pair is thought to be the rate-controlling step in this process. In agreement with this interpretation, the rates of formation of transition metal complexes have normally been found to parallel the rates of water exchange for the metal ions. In the terminology of Langford and Gray, (2) is a dissociative interchange mechanism.

Most previous studies of the rates of formation of complexes of the cupric ion in aqueous solution agree on the extreme rapidity of these reactions. Measured values of formation constants for  $\text{Cu}(\text{II})$  complexes are in the range  $10^7$  to  $4 \times 10^9 \text{ M}^{-1} \text{ sec}^{-1}$ .<sup>2,3</sup> Exchange reactions of complexes of the cupric ion have also been found to be very fast.<sup>4</sup> This behavior is consistent

(1) Abstracted from the Ph.D. Thesis of O. P. Anderson, Northwestern University, 1969.

(2) M. Eigen and R. G. Wilkins in "Inorganic Reaction Mechanisms," Advances in Chemistry Series, No. 49, American Chemical Society, Washington, D. C., 1965, p 55.

(3) A. F. Pearlmuter and J. Stuehr, *J. Am. Chem. Soc.*, **90**, 858 (1968).

(4) R. G. Pearson and R. D. Lanier, *ibid.*, **86**, 765 (1964).

with the above mechanism, since the rate constant usually quoted for water exchange in the hexaaquocopper(II) ion is  $8 \times 10^9 \text{ sec}^{-1}$ .<sup>5</sup>

Recent studies, however, have indicated that the Cu(II) ion may sometimes react at a much slower rate than that expected from the above mechanism. Taft and Cook<sup>6</sup> found that the rate-determining step for reaction of the  $\beta$ -diketone thenoyltrifluoroacetone with the Cu(II) ion in aqueous solution was proton loss from the neutral ligand species to give the enolate anion, which then reacted very quickly with the Cu(II) ion. In a series of studies, Celiano, Cefola, and coworkers<sup>7</sup> succeeded in measuring the rate constants for reaction of the cupric ion with the enol forms of several  $\beta$ -diketones, among them acetylacetone (2,4-pentanedione = Hacac). These studies were carried out in alcohol-water mixed solvents at low temperatures ( $-27$  to  $0^\circ$ ), and under these conditions the reactions mentioned could be studied by employing flow techniques, with second-order rate constants of  $10^3$ – $10^4 \text{ M}^{-1} \text{ sec}^{-1}$ . These workers did not, however, advance any reason for the unusual slowness of these reactions, nor did they explore further than the initial stages of the reactions. A very recent study by Kustin and Pasternack<sup>8</sup> also indicated that the Cu(II) ion may react unusually slowly with the complicated ligand L-carnosine, and this was attributed to the difficulty of formation of a seven-membered chelate ring.

The earlier studies<sup>4</sup> indicated that the reaction between Cu(II) and acetylacetone should be observable at room temperature within the time range accessible to the stopped-flow technique in pure water as well as in alcoholic solvents. It was therefore decided to investigate the complete cupric ion-acetylacetone equilibrium system kinetically, employing both pure water and pure methanol as solvents to provide unambiguous solvent substitution and using deuterium isotope substitution where this might be suitable. It has been shown that relative rates in water and methanol are predictable, if the mechanism of reactions 1 and 2 is operating.<sup>9</sup>

### Experimental Section

Commercially available hexaaquocopper(II) perchlorate (G. F. Smith) was purified of insoluble material and acidic contaminants by recrystallization from aqueous solution. The purified salt was used to prepare stock aqueous cupric ion solutions, which were standardized by EDTA titration.<sup>10</sup> Acetylacetone (Eastman Kodak) was freshly distilled before each series of kinetic runs, and stock solutions were prepared by weighing a portion of the neat liquid in a volumetric flask. Aliquots of a stock sodium perchlorate solution and a standardized perchloric acid solution were used to adjust the ionic strength and the pH to the desired values in the reaction solutions, such that ionic strength and pH stayed essentially constant in the course of a kinetic run.

(5) R. E. Connick, "Symposium on Relaxation Techniques," Buffalo, N. Y., June 1965.

(6) R. W. Taft, Jr., and E. H. Cook, *J. Am. Chem. Soc.*, **81**, 46 (1959).

(7) (a) A. V. Celiano, M. Cefola, and P. S. Gentile, *J. Phys. Chem.*, **65**, 2194 (1961); (b) A. V. Celiano, M. Cefola, and P. S. Gentile, *ibid.*, **66**, 1132 (1962); (c) R. C. Barile, M. Cefola, P. S. Gentile, and A. V. Celiano, *ibid.*, **70**, 1358 (1966).

(8) R. G. Pasternack and K. Kustin, *J. Am. Chem. Soc.*, **90**, 2295 (1968).

(9) R. G. Pearson and P. Elgen, *Inorg. Chem.*, **6**, 1379 (1967).

(10) H. Flaschka, "EDTA Titrations. An Introduction to Theory and Practice," Pergamon Press, New York, N. Y., 1959, p 81.

Absolute methanol (99.5%) for use as solvent was further dried by refluxing over magnesium until reaction was complete, followed by distillation into vacuum-dried glassware which was protected from atmospheric water vapor. A stock anhydrous methanol solution of cupric perchlorate was prepared by a method analogous to literature methods for preparing anhydrous nickel<sup>11</sup> and cobalt<sup>12</sup> perchlorate. Dry methanolic solutions of anhydrous cupric chloride (Alfa Inorganic) and anhydrous silver perchlorate (G. F. Smith) were mixed, and the solid silver chloride was removed by filtration. The monohydrate of *p*-toluenesulfonic acid was used as a source of protons in methanol.

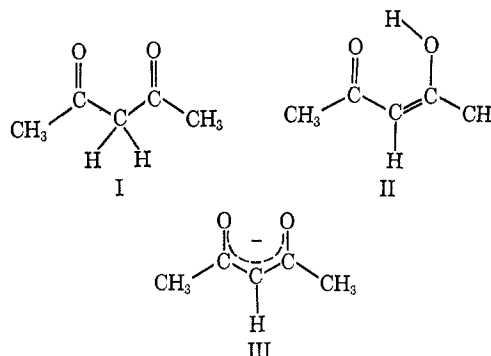
Deuterated acetylacetone was prepared by using deuterium oxide (Bio-Rad Laboratories, 99.84 mol %) to prepare a stock solution of acetylacetone. An nmr spectrum showed exchange of the acidic protons to be complete.

The reaction solutions of deuterated acetylacetone were prepared by adding aliquots of the solution of acetylacetone-*d*<sub>4</sub> in D<sub>2</sub>O to a volumetric flask which already contained H<sub>2</sub>O, acid, and inert electrolyte in the correct amounts. This reaction solution was then rapidly mixed and placed on the stopped-flow instrument, so that the first kinetic determination occurred within approximately 2 min of the beginning of deuterium-proton exchange on the ligand. This time is short enough that most of the keto tautomer of acetylacetone is still present in deuterated form.

The kinetic runs were performed on a commercially available stopped-flow instrument,<sup>13</sup> which has been described in the literature.<sup>14</sup> The reaction could be run in the "forward" direction by mixing a solution containing Cu<sup>2+</sup> and H<sup>+</sup> with one containing Hacac and H<sup>+</sup>, or in the "reverse" direction by mixing a solution containing the complex ion Cu(acac)<sup>+</sup> with one containing a large amount of hydrogen ion. In either case, the instrument could be set to follow either formation (forward) or hydrolysis (reverse) of the complex ion at 305 m $\mu$  or disappearance (forward) or appearance (reverse) of the enol form of acetylacetone at 260 m $\mu$ , where it is the principal absorbing species.<sup>15</sup> Usual reaction conditions involved large excesses of cupric ion, to ensure mono-complex formation only, and of hydrogen ion, to maintain constant pH throughout the approach to equilibrium. The use of buffers to control the pH of the solutions was unsuitable for the kinetic determinations, because of complications arising from coordination of the buffer anion with the metal ion and proton abstraction by the basic buffer component. The reactant reservoirs, drive syringes, and mixing chamber were maintained at a constant temperature of  $25.0 \pm 0.1^\circ$ .

### Mechanism

Acetylacetone (Hacac) exists in three forms in aqueous solution—the tautomeric keto (I) and enol (II) forms and the anionic form (III), which is the common product of dissociation of either protonated tautomer and may be referred to simply as "acac-." For



(11) G. Luz and S. Meiboom, *J. Chem. Phys.*, **40**, 2686 (1964).

(12) G. Luz and S. Meiboom, *ibid.*, **40**, 1058 (1964).

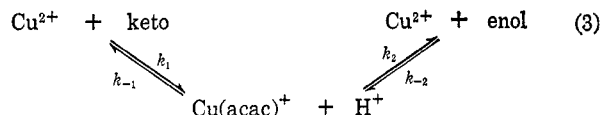
(13) Atom-Mech Machine Co., Patchogue, N. Y.

(14) G. Dulz and N. Sutin, *Inorg. Chem.*, **2**, 917 (1963).

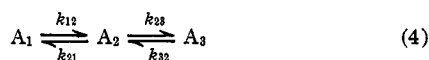
(15) R. G. Pearson and J. W. Moore, *ibid.*, **5**, 1523 (1966).

$\beta$ -diketones, such as acetylacetone, the enolization reaction is not acid catalyzed.<sup>16</sup> Thus, the mechanism of interconversion in acidic aqueous solution is simply dissociation of either protonated form to give the anion, followed by very fast recombination.

The most reasonable scheme to describe the reactions between the Cu(II) ion and acetylacetone in acidic aqueous solution is



Any trace of acac<sup>-</sup> present will react instantaneously. The rate of dissociation of Hacac is too slow to provide a significant alternate path (see below). Experimentally, the large excess of metal ion and hydrogen ion with respect to the ligand provides pseudo-first-order conditions for these species, and the mechanism takes the form shown



In (4), the following relationships hold

$$A_1 = \text{keto}; A_2 = \text{Cu}(\text{acac})^+; A_3 = \text{enol} \quad (5a)$$

$$k_{12} = k_1[\text{Cu}^{2+}]; k_{21} = k_{-1}[\text{H}^+];$$

$$k_{23} = k_2[\text{H}^+]; k_{32} = k_{-2}[\text{Cu}^{2+}] \quad (5b)$$

Methods for the solution of such a system have been described in the literature.<sup>17</sup> The method of secular equations was employed in this study.

If the symbol  $A_i$  is now allowed to represent the instantaneous concentration of species  $A_i$ ,  $A_i^e$  the equilibrium concentration, and  $A_i^0$  the initial concentration, the form of the final solution for any of the species in the reaction scheme is

$$A_i - A_i^e = C_{i2} \exp(-\lambda_2 t) + C_{i3} \exp(-\lambda_3 t) \quad (6)$$

If  $\lambda_2$  and  $\lambda_3$  are sufficiently well separated, the kinetic data will give the appearance of two reactions—one fast and one slow. If  $\lambda_2$  is much larger than  $\lambda_3$ , then the exponential term involving  $\lambda_2$  will be dominant (provide most of the observed change) at short times, and the term in  $\lambda_3$  will become dominant at long times. In this case, the usual kinetic plot of  $\ln(A_i - A_i^e)$  vs. time will yield an initial straight-line portion with slope of  $-\lambda_2$ , which will curve into a straight line with slope of  $-\lambda_3$  at longer times. The constants  $C_{i2}$  depend on the initial conditions and are listed in the Appendix. The observed rate constants  $\lambda_2$  and  $\lambda_3$  of this mechanism do not depend on initial conditions involving  $A_1$ ,  $A_2$ , or  $A_3$ , and have the same functional form whether the experiment is done in the "forward" direction (over-all formation of  $A_2$ ) or the "reverse" direction (over-all formation of  $A_1$  and  $A_3$ ).

The observed rate constants,  $\lambda_2$  and  $\lambda_3$ , are given by

$$\lambda_{2,3} = \frac{1}{2}(p \pm q) \quad (7)$$

where

$$p = k_{-1}[\text{H}^+] + k_2[\text{H}^+] + k_1[\text{Cu}^{2+}] + k_{-2}[\text{Cu}^{2+}] \quad (8)$$

$$q = \{p^2 - 4(k_1 k_2 [\text{Cu}^{2+}][\text{H}^+] + k_{-1} k_{-2} [\text{Cu}^{2+}][\text{H}^+] + k_1 k_{-2} [\text{Cu}^{2+}]^2)\}^{1/2}$$

$$= (p^2 - 4b)^{1/2} \quad (9)$$

From results on the concentration dependence of the observed rate constants, values for the second-order rate constants of interest,  $k_1$  and  $k_{-2}$ , must be extracted. This can be done by factoring the  $q$  term and expanding the factor  $(1 - (4b/p^2))^{1/2}$ . Retention of only the first two terms gives

$$q \approx p(1 - (2b/p^2)) \quad (10)$$

Substitution of this result in (7) will give the observed rate constants as

$$\lambda_2 = (k_{-1} + k_2)[\text{H}^+] + (k_1 + k_{-2})[\text{Cu}^{2+}] - \frac{(k_1 k_2 + k_{-1} k_{-2})[\text{H}^+][\text{Cu}^{2+}] + k_1 k_{-2} [\text{Cu}^{2+}]^2}{(k_{-1} + k_2)[\text{H}^+] + (k_1 + k_{-2})[\text{Cu}^{2+}]} \quad (11)$$

and

$$\lambda_3 = \frac{(k_1 k_2 + k_{-1} k_{-2})[\text{H}^+][\text{Cu}^{2+}] + k_1 k_{-2} [\text{Cu}^{2+}]^2}{(k_{-1} + k_2)[\text{H}^+] + (k_1 + k_{-2})[\text{Cu}^{2+}]} \quad (12)$$

The work of Taft and Cook<sup>6</sup> on the reaction of the cupric ion with the  $\beta$ -diketone thenoyltrifluoroacetone indicated the reaction with the enol form to be "instantaneous" in comparison with that of the keto form. If this holds true in the present work, then  $k_{-2}$  is much greater than  $k_1$ . This means also that  $k_2$  is much greater than  $k_{-1}$ , since  $K' (=k_{-2}/k_2)$  and  $K (=k_1/k_{-1})$  are similar in size,<sup>18,19</sup> only differing by the factor of  $\sim 5$  between the  $K_a$ 's for the two forms of acetylacetone.<sup>20</sup> If these assumptions are used to eliminate all terms but the largest in (11) and (12), the results, in terms of the rate constants of interest, simplify to

$$k_{-2} = \lambda_2 / \{[\text{Cu}^{2+}] + [\text{H}^+]/K'\} \quad (13)$$

$$k_1 = \lambda_3 \frac{[\text{H}^+] + K'[\text{Cu}^{2+}]}{(1 + (K'/K))[\text{H}^+][\text{Cu}^{2+}] + K'[\text{Cu}^{2+}]^2} \quad (14)$$

From these equations, the second-order rate constants for formation of the complex ion may be calculated from the observed rate constants and the experimental concentrations.

## Results

The results obtained fit the above mechanism quite satisfactorily. In all cases, two consecutive reactions were clearly involved. When concentrations of the reactants were high, only one continuous trace was directly observed. However, from the calculated value of the optical density of the mixed solution at zero time, together with the value of the observed rate constant, it was clear that this observed reaction was too slow to cause the total required change in optical density. When the concentrations of the reactants

(18)  $K' = K_1 K_{a,\text{enol}} = 1.0$ ;  $K = K_1 K_{a,\text{keto}} = 0.22$  in  $\text{H}_2\text{O}$  at  $25^\circ$ .  $K_1$  is the formation equilibrium constant for  $\text{Cu}(\text{acac})^+$ ;  $\log K_1 = 8.26$  at  $25^\circ$  in aqueous solution.<sup>19</sup>

(19) L. G. Sillén and A. E. Martell, "Stability Constants," Special Publication No. 17, The Chemical Society, London, 1964, p 443.

(20) M. Eigen and L. de Maeyer in "Technique of Organic Chemistry," Vol. VIII, A. Weissberger, Ed., 2nd ed, Interscience Publishers, New York, N. Y., 1963, Part II, p 1037.

(16) (a) R. P. Bell and O. M. Lidwell, *Proc. Roy. Soc. (London)*, **A176**, 88 (1940); (b) R. G. Pearson and J. M. Mills, *J. Am. Chem. Soc.*, **72**, 1892 (1950); (c) K. J. Pedersen, *J. Phys. Chem.*, **37**, 751 (1933).

(17) (a) F. A. Matsen and J. L. Franklin, *J. Am. Chem. Soc.*, **72**, 3337 (1950); (b) E. S. Lewis and M. D. Johnson, *ibid.*, **82**, 5406 (1960).

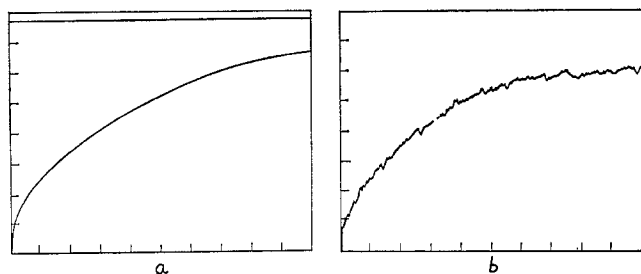


Figure 1.—Representations of typical oscilloscope tracings for the forward reaction: (a) the slow portion (corresponding to  $\lambda_3$ );  $x$  scale, 5 sec/division;  $y$  scale, 20 mV/division; (b) the fast portion (corresponding to  $\lambda_2$ );  $x$  scale, 10 msec/division;  $y$  scale, 10 mV/division.

were lowered, an initial fast reaction was brought within the upper limit of observation for the stopped-flow device. Representations of the two consecutive traces are shown in Figure 1, which represents what may be seen on the oscilloscope face simply by changing the beam sweep rate when the reaction is carried out in the "forward" direction and observed at a wavelength of  $305\text{ m}\mu$ , where only the complex,  $\text{Cu}(\text{acac})^+$ , absorbs. The wavelength chosen could equally well have been that of the absorption maximum of the enol tautomer ( $280\text{ m}\mu$ ), with the same kinetic results, though the traces would be in different directions. This behavior is entirely consistent with the mechanism discussed in the preceding section, which required observation of two apparently first-order reactions. Therefore, the observed rate constant derived from the faster portion of the trace is associated with  $\lambda_2$ , and that from the slower portion is associated with  $\lambda_3$ . From a collection of results for  $\lambda_2$  and  $\lambda_3$  as functions of the concentrations of metal ion and hydrogen ion,  $k_1$  and  $k_{-2}$  can be calculated.

Values for  $\lambda_3$  from the slow portions of the traces are found in Table I, together with the calculated values of  $k_1$ . The values of  $k_1$  are seen to be quite constant over a considerable concentration range, varying significantly only at the lowest concentrations of the cupric and hydrogen ions, where the requirement of pseudo-first-order conditions for these ions is not well satisfied. The two values of  $k_1$  marked by an asterisk in Table I show that the difference between the rate constants from the more concentrated and less concentrated solutions is largely due to a general ionic strength effect.

The results from the fast portion of the kinetic data,  $\lambda_2$  and  $k_{-2}$ , are given in Table II. The precision of these results is quite good, considering the rapidity of the reactions involved. It may be noticed here that the technique of observing the rate constant for the forward and reverse reaction directions has been employed to extend the concentration range to very high values of the hydrogen ion concentration, which would preclude observing any forward reaction, but, of course, provide a very large reverse reaction.

At  $25^\circ$  in water, the best value for  $k_1$  is  $12\text{ M}^{-1}\text{ sec}^{-1}$  at an ionic strength of  $0.1\text{ M}$  and  $15\text{ M}^{-1}\text{ sec}^{-1}$  at the higher ionic strength of  $1.0\text{ M}$ . The best value for

TABLE I  
OBSERVED ( $\lambda_3$ ) AND CALCULATED ( $k_1$ )  
RATE CONSTANTS FOR THE REACTION



IN  $\text{H}_2\text{O}$  AT  $25.0 \pm 0.1^\circ$

No.	$10^2[\text{Cu}^{2+}],$ $M$	$10^2[\text{H}^+],$ $M$	$\lambda_3,$ $\text{sec}^{-1}$	$k_1, M^{-1}$ $\text{sec}^{-1}$
1	2.199	2.332	1.3	18
2	4.398	2.332	1.8	16
3	6.597	2.332	2.1	15
4	8.796	2.332	2.5	15
5	11.00	2.332	2.8	14
6	2.199	4.664	1.3	14
7	4.398	4.664	2.8	15
8	6.597	4.664	3.2	14
9	8.796	4.664	3.7	16
10	11.00	4.664	4.5	16
11	2.685	11.66	2.4	18
12	5.370	11.66	3.9	17
13	7.955	11.66	4.9	15
14	10.62	11.66	5.6	15
15	13.28	11.66	6.6	15
16	5.370	23.32	3.8	14
17	7.955	23.32	5.1	14
18	10.62	23.32	6.6	14
19	13.28	23.32	7.9	15
20	1.094	0.494	0.45	17*
21	0.2188	0.0988	0.090	17
22	0.2188	0.1976	0.117	16
23	0.4376	0.0988	0.106	13
24	0.4376	0.1976	0.142	13
25	1.094	0.0988	0.154	10
26	1.094	0.1976	0.214	11
27	1.094	0.2964	0.232	11
28	1.094	0.4940	0.294	11*
29	1.094	0.6916	0.365	12
30	1.094	0.988	0.417	12
31	2.188	0.494	0.469	12
32	2.188	0.988	0.646	12
33	10.94	23.32	5.8	13
34	5.31	12.24	3.1	14
35	5.31	11.66	2.4	11

\* All reactions were run in the "forward" direction except no. 33–35, which were in the "reverse" direction. Wavelength of observation was  $305\text{ m}\mu$  in all cases. Numbers 1–20 were done with total  $[\text{Hacac}] = 5 \times 10^{-4}\text{ M}$  and ionic strength ( $\mu$ ) of  $1.0\text{ M}$ . Numbers 21–35 correspond to total  $[\text{Hacac}] = 10^{-4}\text{ M}$ . Numbers 21–32 have ionic strength of  $0.1\text{ M}$ , no. 33 and 34 have an ionic strength of  $1.0\text{ M}$ , and no. 35 has an ionic strength of  $0.2\text{ M}$ .

$k_{-2}$  is  $2 \times 10^4\text{ M}^{-1}\text{ sec}^{-1}$ . Both of these reactions are thus much slower than the normal substitutions discussed earlier.

Assuming the same mechanism holds, the same procedure may be followed in methanol as solvent. The lack of known equilibrium constants in methanol can be circumvented by employing the kinetic results to determine those equilibrium constants, specifically  $K$  and  $K'$ , which are needed in the calculation of the second-order rate constants  $k_1$  and  $k_{-2}$ . From eq 13 a plot of the observed rate constant for the fast portion of the reaction,  $\lambda_2$ , vs. the concentration of the cupric ion, with the concentration of hydrogen ion held constant, should be a straight line, with slope of  $k_{-2}$ . Furthermore, the intercept of this line will be  $k_{-2}[\text{H}^+]/K'$ , in which  $k_{-2}$  (the slope) and  $[\text{H}^+]$  (analytical data) are

TABLE II  
OBSERVED ( $\lambda_2$ ) AND CALCULATED ( $k_{-2}$ )  
RATE CONSTANTS FOR THE REACTION  
 $\text{Cu}^{2+} + \text{enol} \xrightleftharpoons[k_2]{k_{-2}} \text{Cu}(\text{acac})^+ + \text{H}^+$   
IN  $\text{H}_2\text{O}$  AT  $25.0 \pm 0.1^\circ\text{C}$

No.	$10^3[\text{Cu}^{2+}]$ , <i>M</i>	$10^3[\text{H}^+]$ , <i>M</i>	$\lambda_2$ , $\text{sec}^{-1}$	$10^{-4}k_{-2}$ , $\text{M}^{-1}\text{sec}^{-1}$
1	1.094	0.988	40	2.0
2	1.094	1.976	51	1.7
3	2.188	0.988	53	1.7
4	2.188	1.976	72	1.7
5	4.376	0.988	95	1.8
6	4.376	4.376	147	2.3
7	1.094	0.988	47	2.3
8	1.094	0.988	49	2.4
9	1.094	4.940	152	2.6
10	1.094	7.410	194	2.3

<sup>a</sup> All runs have total  $[\text{Hacac}] = 10^{-4} \text{ M}$  and an ionic strength of 0.1 *M*. Numbers 1-6 are in the forward direction; no. 7-10, reverse. Numbers 1-7 were observed at 305  $\mu$ ; no. 8-10, at 260  $\mu$ .

known, and  $K'$  may consequently be calculated. This value of  $K'$  may be combined with the known keto-enol distribution equilibrium constant in methanol<sup>7a</sup> to give a value for  $K$ . These values for the equilibrium constants for this system in methanol may then be used to calculate  $k_1$  in methanol from the data for the slow reaction.

The results from the kinetic experiments in methanol solution may be found in Table III. At the lower water concentration of  $1.09 \times 10^{-3} \text{ M}$ , a plot of  $\lambda_2$  vs. the concentration of the cupric ion shows the required straight-line behavior. The best (least-squares) straight line through the points has a slope of  $2.0 \times 10^4 \text{ M}^{-1} \text{ sec}^{-1}$ , which is thus the value of  $k_{-2}$  under these conditions. The intercept of  $6.2 \text{ sec}^{-1}$ , together with the hydrogen ion concentration of  $1.09 \times 10^{-3} \text{ M}$ , then gives  $K' = 3.5$ . The range of cupric ion concentration in these experiments is not great, due to the very rapid nature of these reactions. The observed rate constants reported are highly reliable, however, each being the average of at least ten separate determinations. At the higher water concentration of  $5.4 \times 10^{-3} \text{ M}$ ,  $k_{-2}$  is  $1.9 \times 10^4 \text{ M}^{-1} \text{ sec}^{-1}$ , and the value of the intercept,  $5.5 \text{ sec}^{-1}$ , gives  $K' = 3.8$ . The calculated values for  $k_1$  are also found in Table III. In methanol, the best value for  $k_{-2}$  is  $2 \times 10^4 \text{ M}^{-1} \text{ sec}^{-1}$ , and for  $k_1$  the best value is  $1.3 \times 10^3 \text{ M}^{-1} \text{ sec}^{-1}$ .

The data clearly show that at the low levels employed here the concentration of water is not an important variable affecting the kinetic behavior. Variation of ionic strength is also seen to have little effect on the reaction kinetics. The concentration of the hydrogen ion has a direct effect on the observed kinetic results for  $\lambda_2$ , in contrast to the behavior observed in the earlier work.<sup>7a</sup> The direct effect observed here is entirely consistent with the present mechanism.

The fast reaction observed in this work is definitely the same reaction that was followed by Celiano and coworkers<sup>7a</sup> and correctly attributed by them to the reaction between the  $\text{Cu}(\text{II})$  ion and the enol form of acetylacetone. Under their typical conditions of no

TABLE III  
KINETIC RESULTS IN METHANOL AT  $25.0 \pm 0.1^\circ\text{C}$

No.	$10^3[\text{H}^+]$ , <i>M</i>	$10^3[\text{Cu}^{2+}]$ , <i>M</i>	$\lambda_2$ , $\text{sec}^{-1}$	$\lambda_3$ , $\text{sec}^{-1}$	$k_1$ , <sup>b</sup> $\text{M}^{-1}\text{sec}^{-1}$
1	1.087	1.175	28	1.64	1330
2	1.087	1.410	36	2.21	1500
3	1.087	1.646	41	2.26	1260
4	1.087	1.881	43	2.64	1355
5	1.087	2.116	50	2.55	1170
6	1.087	2.351	53	2.76	1140
7	1.087	1.175	28	1.77	1440
8	1.087	1.646	37	2.02	1185
9	1.087	2.351	50	3.10	1285
10	1.087	2.351	50	3.10	1285
11	5.430	1.646	46	...	...
12	0.0	1.175	71	...	...

<sup>a</sup> All reactions are in the forward direction; total  $[\text{Hacac}] = 10^{-4} \text{ M}$ .  $[\text{H}_2\text{O}] = 1.09 \times 10^{-3} \text{ M}$  in no. 1-6,  $5.43 \times 10^{-3} \text{ M}$  in no. 7-11, 0.0 *M* in no. 12. Ionic strength of 0.1 *M* in no. 10, unadjusted in all others. <sup>b</sup> The value of  $k_{-2}$  is  $2.0 \times 10^4 \text{ M}^{-1} \text{ sec}^{-1}$  from a plot of  $\lambda_2$  vs.  $[\text{Cu}^{2+}]$  at constant  $[\text{H}^+]$ .

added hydrogen ion and calculating in the manner of these workers, the last entry in Table III indicates a value of  $k_{-2}$  of  $6 \times 10^4 \text{ M}^{-1} \text{ sec}^{-1}$ . Extrapolation of their low-temperature results in anhydrous methanol to  $25^\circ$  yields a value for  $k_{-2}$  of  $5.3 \times 10^4 \text{ M}^{-1} \text{ sec}^{-1}$ . The agreement between these values is regarded as quite satisfactory, but the absolute value of  $k_{-2}$  is calculated too large in this way.

More information on this system may be gained by consideration of experiments involving deuterated acetylacetone reacting with the copper(II) ion with normal water ( $\text{H}_2\text{O}$ ) as solvent. Deuteron loss from either tautomer of acetylacetone in these reaction solutions is essentially an irreversible process, since the percentage of  $\text{D}_2\text{O}$  in the solutions was always  $\leq 0.4\%$ . For enolization of the keto form of the 3-methyl analog of acetylacetone, the primary isotope effect in  $\text{H}_2\text{O}$  solvent is  $k_{\text{H}}/k_{\text{D}} = 3.5$ .<sup>21</sup> If the isotope effect is similar for acetylacetone, the rate constant for deuteron loss from the keto form of acetylacetone- $d_2$  would be approximately  $4 \times 10^{-3} \text{ sec}^{-1}$ , calculated from the measured value for proton loss from the keto form of acetylacetone.<sup>20,22</sup> Therefore, the half-life for exchange of the first deuteron of keto- $d_2$  for a proton is expected to be about 170 sec, which means that any effect of the isotopic substitution on the slow reaction should be observable. On the other hand, any reasonable isotope effect for the enol form gives a rate constant for deuteron loss (from OH group) approximately four orders of magnitude higher than the keto rate constant. This means that the enol form will have completely exchanged its deuterons for protons by the time an experiment is performed, and no effect of isotopic substitution on the fast portion of the reaction will be evident.

When the kinetic experiments were performed, the slow reaction (corresponding to  $\lambda_3$  and reaction with the keto form) showed a drastic perturbation when the deuterated keto form was initially present. The

(21) F. A. Long and D. Watson, *J. Chem. Soc.*, 2019 (1958).

(22) R. P. Bell, E. Gelles, and E. Moller, *Proc. Roy. Soc. (London)*, **A193**, 300 (1949).

reactions observed were several times slower than what was observed under the same concentration conditions when no deuterated species were present. A succession of kinetic experiments were performed using the same stock solution. At long times, as the amount of remaining deuterated acetylacetone became lower, less perturbation of the kinetics was observed, and finally the only behavior seen was that of the normal solution of protonated acetylacetone reacting with the copper(II) ion. It proved impossible, by any simple treatment, to obtain quantitative results from the reactions observed, as several reactions of similar rate are mixed together in these experiments. These qualitative results, however, establish that a kinetic isotope effect exists.

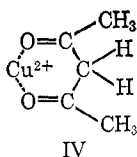
### Discussion

It is clear, from the results reported here, that the reactions of the tautomeric forms of acetylacetone with the copper(II) ion do not follow the normal pattern suggested earlier for mechanisms involving octahedral substitution. Both of these reactions occur with rate constants many orders of magnitude too low for the rate of water release from the cupric ion to be rate controlling.

The reaction between the cupric ion and the keto form of acetylacetone has been found here to be a direct reaction between these two species (see (3)). That the reaction can be described as such a direct process is unusual, as the work of Taft and Cook<sup>6</sup> on the kinetics of reaction between the  $\beta$ -diketone thenoyl-trifluoroacetone and the cupric ion leads to the expectation that ionization of the keto form to  $\text{acac}^-$  would be the rate-determining step. This is clearly not the case in the present work, however, as an inspection of the results for the slow reaction shows that it always proceeds much faster than the rate of ionization of the keto form of acetylacetone (about  $1.5 \times 10^{-2} \text{ sec}^{-1}$ ).<sup>22</sup>

Such a direct reaction for the keto tautomer with metal ions has received support independent of the present study. From kinetic results for mono-complex formation in the system  $\text{Fe(III)-keto}$  in aqueous solution, Ong and Prince<sup>23</sup> have postulated just such a direct reaction between the ferric ion and the keto tautomer, with a measured value for  $k_1$  in their system of  $2.6 \times 10^{-1} M^{-1} \text{ sec}^{-1}$ . This is rather similar to the result obtained for  $\text{Cu(II)-keto}$  in the present work of  $12 M^{-1} \text{ sec}^{-1}$ , especially after comparison of the water-exchange rate constant of  $3 \times 10^3 \text{ sec}^{-1}$  for  $\text{Fe(III)}$ <sup>5</sup> with that for  $\text{Cu(II)}$  of  $8 \times 10^9 \text{ sec}^{-1}$ .<sup>5</sup>

In the copper ion reaction, a logical precursor to the transition state of this direct reaction is a loosely held complex between the cupric ion and the keto tautomer,



which might be depicted as IV. Since the acidic proton is still present in this very weak complex, it would seem that this species might exist even in relatively concentrated acid solutions, where the usual mono complex involving the enolate anion cannot exist in any quantity. However, no spectral evidence for this species was found under these conditions.

Despite the lack of direct evidence for this intermediate in this system, other recent studies with acetylacetone and other metal ions have shown such a species to be reasonable. By employing solvents of weak coordinating power, Van Leeuwen<sup>24</sup> has been able to isolate  $\text{Ni(II)}$  complexes with the keto tautomer acting as a neutral ligand, just as in IV above. Similarly, Allred and Thompson<sup>25</sup> have demonstrated coordinative interaction between the keto form and tin tetrachloride. These observations make IV a reasonable intermediate for the slow reaction in this work.

From the intermediate IV, the reaction may be completed by loss of a proton from the 3-carbon atom to give the product, a complex of the enolate anion. The cupric ion, in the weak intermediate complex, will act as an electron sink to assist this deprotonation, as in the  $\text{Cu(II)}$ -catalyzed bromination of ethyl acetoacetate.<sup>26</sup> In direct analogy to this last study, the rate-determining step in the reaction between the cupric ion and the keto tautomer probably involves slow, metal ion catalyzed proton transfer from the carbon acid to the solvent acting as base, or to any other base present.

This conclusion is supported by the results of the deuterium isotope substitution experiment. As pointed out earlier, these experiments did not yield any quantitative results, due to the greatly increased complexity of the kinetic system which must be considered. This should not obscure the experimental fact that the isotope effect was present. If removal of the proton were unimportant (occurred far from the transition state), the observed kinetic behavior would have been the same whether a deuterium or a proton was lost. The fact that the deuterated system behaved so differently clearly shows that proton removal from the keto form is important in the transition state of the reaction with cupric ion.

The change in solvent from water to methanol should also give rise to effects interpretable on the basis of proton transfer as the rate-determining step. The rate constant,  $k_1$ , for reaction of the keto form with the cupric ion is  $12 M^{-1} \text{ sec}^{-1}$  in water and  $1300 M^{-1} \text{ sec}^{-1}$  in methanol. This increase in rate constant on going from water to methanol is exactly opposite to the behavior found to be the case<sup>9</sup> when the dissociative interchange mechanism is operating and clearly shows that this mechanism is not involved in the case under consideration. It has been shown<sup>27</sup> that methanol exchanges more slowly than water on  $\text{Cu(II)}$ .

The approximately 100-fold increase in the rate

(23) W. K. Ong and R. H. Prince, *J. Chem. Soc., A*, 458 (1966).

(24) P. W. N. M. Van Leeuwen, *Rec. Trav. Chim.*, **87**, 396 (1968).

(25) A. L. Allred and D. W. Thompson, *Inorg. Chem.*, **7**, 1196 (1968).

(26) K. J. Pedersen, *Acta Chem. Scand.*, **2**, 252, 385 (1948).

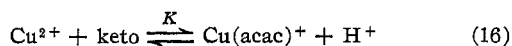
(27) F. W. Breivogel, *J. Chem. Phys.*, **51**, 445 (1969).

constant  $k_1$  is best explained by a combination of solvation effects and the basic strength of the solvent. It has been found in the present work that  $K'$ , the equilibrium constant for the reaction between the copper(II)



ion and the enol tautomer, increases by a factor of 3–4 on changing solvent from water to methanol. This increase is probably due to the electrostatic effect of the change in the bulk dielectric constant.

If  $K'$  is taken as 4 in methanol,  $K (=k_1/k_{-1})$  may be calculated in methanol from the keto–enol distribution equilibrium constant and is found to increase by a



factor of approximately 100 on going from water to methanol. On the basis of electrostatics alone,  $k_{-1}$  may be expected to decrease slightly from the water value, because of the increased difficulty of bringing positive charges together in the medium of lower dielectric constant. This decrease, however, will be small, and most of the increase in  $K$  must be caused by a large increase in  $k_1$ , which is indeed observed.

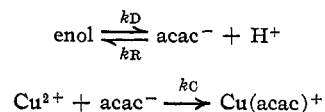
Thus, the reasons for the increase in the rate constant,  $k_1$ , are directly linked with the reasons for the large increase in  $K$ . There are at least two causes for this behavior. The more important is the large decrease in stability of the keto tautomer relative to the enol on changing from water to methanol as solvent. This is exemplified by the aqueous equilibrium keto:enol ratio of approximately 4,<sup>18,28</sup> compared with a ratio of approximately 0.25 in methanol<sup>7a</sup> (and in the neat liquid). Such a change is reasonable as the enol form (with its internal hydrogen-bonding arrangement) would be expected to feel little difference from solvent to solvent, while the keto form should be far more stable in the solvent of higher hydrogen-bonding capacity. Thus, a large part of the increase in  $k_1$  occurs because the keto tautomer is not as solvent stabilized in methanol as in water and will thus form an intermediate such as IV much more readily, perhaps by a factor of 16.

Solvent basicity is also expected to play a large role in the increase in  $k_1$ , since the deuterium isotope experiments have demonstrated the important role of proton removal in the transition state. If methanol is a stronger base than water, it is expected to be more effective at removing the proton. Unfortunately, the question of the relative basicities of water and methanol when considered as bulk solvents has not been resolved.<sup>29</sup> By analogy with the increase in basic strength on going from ammonia to methylamine, methanol is expected to be the stronger base. This then may provide an additional factor of 5–10 toward the increase in  $k_1$ , over and above the factor arising

from the hydrogen-bonding considerations just described.

Furthermore, if the reaction between the cupric ion and the enol form involved proton transfer from the oxygen as part of the rate-determining step, the change in solvent basic strength should also operate to change the rate constant  $k_{-2}$ . However changing solvents makes no difference in this rate constant. Also the rate of proton removal from oxygen in the enol form of acetylacetone is  $10^4$  times greater than the rate of removal from carbon in the keto form.<sup>20</sup> Thus it is unlikely that proton removal is energetically important in the ring closure to form complex. The role of proton removal cannot be completely assessed, however, since deuterium isotope effects cannot be studied at the oxygen atom.

The possibility might also be considered that the reaction, instead of occurring directly as written here, might occur by prior ionization of the free enol form, followed by reaction of the enolate anion with the metal ion species. This process was ruled out for the keto form on the basis of the relative rates of the complexation reaction and of the ionization from the carbon acid. The rate constant for ionization of the enol is, however, of approximately the same size ( $170 \text{ sec}^{-1}$  at  $25^\circ$ )<sup>20</sup> as the observed rate constants  $\lambda_2$  for the fast reaction in Table II. However, consideration of this possible process



quickly shows that this would lead to an incorrect prediction of the observed hydrogen ion concentration dependence. Further, because the protolytic recombination is so rapid ( $k_R = 3 \times 10^{10} \text{ M}^{-1} \text{ sec}^{-1}$  at  $25^\circ$ ),<sup>20</sup>  $k_C$  would have to be  $4 \times 10^4 \text{ M}^{-1} \text{ sec}^{-1}$  under typical reaction conditions in order that as much as 1% of the complex might be formed by this path. Thus, it appears unlikely that prior ionization of the enol form plays a kinetically important role in this reaction.

A possible explanation for the low rate of reaction of Cu(II) with the enol form of acetylacetone may be advanced by utilizing recent proposals of Kustin, *et al.*<sup>30</sup> Table IV provides a summary of the pertinent kinetic values found by these authors, together with the value for the enol reaction found in the present work. The results presented make a clear case for the fact that with metal ions more labile than Ni(II), seven-membered (L-carnosine) and six-membered (the  $\beta$ -amino acids and the enol form of Hacac) chelate rings are slow to form, compared with the formation of five-membered chelate rings. It may also be noticed in Table IV that the rate constants for six-membered chelate ring formation for the more labile ions, such as Mn(II), Co(II), and Cu(II), agree very well with each

(28) (a) F. C. Nachod, *Z. Physik. Chem.*, **A182**, 193 (1938); (b) G. Schwarzenbach and E. Felder, *Helv. Chim. Acta*, **27**, 1044 (1944).

(29) E. M. Arnett, *Progr. Phys. Org. Chem.*, **1**, 223 (1963); V. Gold and R. S. Satchell, *J. Chem. Soc.*, 1930 (1963); C. F. Wells, *Trans. Faraday Soc.*, **61**, 2194 (1965); N. C. Deno and J. O. Turner, *J. Org. Chem.*, **31**, 1969 (1966); R. E. Weston, Jr., S. Ehrenson, and K. Heinzinger, *J. Am. Chem. Soc.*, **89**, 481 (1967).

(30) (a) K. Kustin and R. F. Pasternack, *ibid.*, **90**, 2805 (1968); (b) K. Kustin, R. F. Pasternack, and E. M. Weinstock, *ibid.*, **88**, 4610 (1966); (c) A. Kowalak, K. Kustin, R. F. Pasternack, and S. Petrucci, *ibid.*, **89**, 3126 (1967).

TABLE IV  
 MONO-COMPLEX FORMATION RATE CONSTANTS IN WATER AT 25°

M	L	$k_t, M^{-1} \text{ sec}^{-1}$	Character <sup>a</sup>
Ni(II)	$\alpha$ -Aminobutyric acid	$1 \times 10^4$	Normal
Ni(II)	$\beta$ -Aminobutyric acid	$4 \times 10^3$	Normal
Co(II)	$\alpha$ -Aminobutyric acid	$2.5 \times 10^5$	Normal
Co(II)	$\beta$ -Aminobutyric acid	$2 \times 10^4$	Slow
Ni(II)	$\alpha$ -Alanine	$2 \times 10^4$	Normal
Ni(II)	$\beta$ -Alanine	$1 \times 10^4$	Normal
Co(II)	$\alpha$ -Alanine	$6 \times 10^5$	Normal
Co(II)	$\beta$ -Alanine	$7.5 \times 10^4$	Slow
Mn(II)	$\alpha$ -Alanine	Too fast to measure	Normal
Mn(II)	$\beta$ -Alanine	$5 \times 10^4$	Slow
Cu(II)	L-Carnosine (neutral form)	$3.5 \times 10^5$	Slow
Cu(II)	Acetylacetone (enol form)	$2 \times 10^4$	Slow
Ni(II)	Acetylacetone (enol form)	$2.7 \times 10^5$	Slow

<sup>a</sup> A "normal" character implies the rate constant to be consistent with the usual interchange mechanism. "Slow" character then corresponds to the cases which do not fit this mechanism, as discussed in the text. <sup>b</sup> From R. G. Pearson and J. W. Moore, *Inorg. Chem.*, **5**, 1523 (1966). Other data from ref 8 and 30 from this work.

other, suggesting that a value of  $k_t$  of approximately  $4 \times 10^4 M^{-1} \text{ sec}^{-1}$  might be considered a "characteristic" value for formation of rings of this size.<sup>31</sup>

This mechanism, which has been termed a "sterically controlled substitution" mechanism (SCS) by Kustin, *et al.*, postulates that steric requirements of the ligand (characteristic of the ring size) provide the main activation energy barrier for the reaction. From the evidence available, it seems that five-membered chelate rings form so easily that the rate of water release remains rate controlling for even the most labile transition metal ions. Seven-membered chelate ring formation is rate controlling for Cu(II) and from the numerical value of this result would also be predicted to be rate controlling with Cr(II) and Zn(II). Six-membered chelate ring formation has been demonstrated to be rate controlling for Co(II), Mn(II), and Cu(II) and would also be predicted to control the rate of reaction with Fe(II), Cr(II), and Zn(II) in aqueous solution. The slow reaction of Ni(II) with the enol form of acetylacetone ( $k_t = 2.7$ ) is also explainable by the SCS mechanism.<sup>30b</sup>

(31) However, a rate constant of  $2 \times 10^{-3} M^{-1} \text{ sec}^{-1}$  for the reaction of Cu(II) with  $\beta$ -alanine has recently been reported: W. B. Makinen, A. F. Pearlmutter, and J. E. Stuehr, *J. Am. Chem. Soc.*, **91**, 4083 (1969).

Furthermore, this mechanism predicts that the rate constant for the enol reaction should be much the same in both water and methanol, as was observed, since the rate-controlling process is only concerned with the steric problems of the ligand in completing the complex. This mechanism also predicts that the enolate anion, which must form the same six-membered ring, would also react at an abnormally slow rate. This prediction cannot be verified from the information available.

Thus, acetylacetone has been shown to exhibit two possible reasons for abnormally slow reaction with the Cu(II) ion. Reaction with the enol form demonstrates steric control of the reaction path, while reaction with the keto form exhibits the effect of slow, metal-catalyzed proton release. These effects lead to a departure from the accepted mechanism of six-coordinate metal ion substitution reactions by replacing water release from the metal ion as the rate-determining step.

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### Appendix

The coefficients  $C_{ij}$  which are obtained in the kinetic solution for the concentration of species  $A_i$  as a function of time (see eq 6) are as follows. All symbols are as defined in the text.

$$C_{12} = \frac{A_3^0 k_{21} k_{32}}{\lambda_2 (\lambda_2 - \lambda_3)} - \frac{A_1^0 k_{12} (k_{32} - \lambda_2) (k_{12} - \lambda_3)}{\lambda_2 (\lambda_2 - \lambda_3) (k_{12} - k_{32})}$$

$$C_{13} = \frac{A_1^0 k_{12} (k_{12} - \lambda_2) (k_{32} - \lambda_3)}{\lambda_3 (\lambda_2 - \lambda_3) (k_{12} - k_{32})} - \frac{A_3^0 k_{32} k_{21}}{\lambda_3 (\lambda_2 - \lambda_3)}$$

$$C_{22} = \frac{A_3^0 k_{32} (k_{12} - \lambda_2) + A_1^0 k_{12} (k_{32} - \lambda_2)}{\lambda_2 (\lambda_2 - \lambda_3)}$$

$$C_{23} = \frac{A_1^0 k_{12} (k_{32} - \lambda_3) + A_3^0 k_{32} (k_{12} - \lambda_3)}{\lambda_3 (\lambda_2 - \lambda_3)}$$

$$C_{32} = \frac{A_1^0 k_{23} k_{12}}{\lambda_2 (\lambda_2 - \lambda_3)} + \frac{A_3^0 k_{32} k_{23} (k_{12} - \lambda_2)}{\lambda_2 (k_{32} - \lambda_2) (\lambda_2 - \lambda_3)}$$

$$C_{33} = \frac{A_1^0 k_{12} k_{23}}{\lambda_3 (\lambda_2 - \lambda_3)} + \frac{A_3^0 k_{32} k_{23} (k_{12} - \lambda_3)}{\lambda_3 (k_{32} - \lambda_3) (\lambda_2 - \lambda_3)}$$