Correlating Celsius and Fahrenheit Temperatures by the "Unit Calculus"

The Celsius and Fahrenheit temperature "scales" are defined by two fixed temperatures, that at the melting point of ice, 0 °C or 32 °F, and that at the boiling point of water, 100 °C or 212 °F, under normal atmospheric pressure (1 atm = 101 kPa).[1] If one chooses to describe these equalities by expressions such as (1) and (2):

$$0\ °C = 32\ °F \tag{1}$$

$$100\ °C = 212\ °F \tag{2}$$

it must be realized that these are *not* correctly formulated algebraic equations. In eqn. (1), if 0 had its customary meaning, the term (0 °C) would have to equal zero, and could not equal any finite quantity. Moreover, if eqn. (2) were algebraically valid, it would lead to the result (°C/°F) = 2.12, which is also incorrect.

In order to ascertain the correct relationship between a temperature determined on the Farenheit scale, t_F, and a temperature determined on the Celsius scale, t_C, one may first formulate an equation with two arbitrary constants:

$$t_F = at_C + b$$

The values of these constants can be determined by writing algebraically correct versions of eqns. (1) and (2), as follows:

$$32\ °F = a\ (0\ °C) + b$$

$$212\ °F = a\ (100\ °C) + b$$

and solving these equations simultaneously. This leads to:

$$b = 32\ °F$$

$$a = (9/5)(°F/°C)$$

Finally, then, the relation between t_F and t_C may be written as:

$$t_F = (9/5)(°F/°C)t_C + 32\ °F \tag{3}$$

and

$$t_C = (5/9)(°C/°F)(t_F - 32\ °F)$$

Substitution into these equations gives the correct results by the rules of the "unit calculus"; e.g. for t_C = 37 °C:

$$t_F = (9/5)(°F/°C)(37\ °C) + 32\ °F = 98.6\ °F$$

Equation (3) can be rearranged so that each term is a "pure" number, i.e. without units:

$$(t_F/°F) = (9/5)(t_C/°C) + 32 \tag{4}$$

Equation (4) resembles that which is conventionally given[2] for "converting" Celsius temperature into Fahrenheit:

$$°F = (9/5)°C + 32; \tag{5}$$

however, in eqn. (5) the meaning of the symbols °F and °C is ambiguous. The confusion results from failure to distinguish between the concepts of Celsius and Farenheit *temperatures*, which are defined in the opening paragraph, and temperature *units*. In consequence, eqn. (5) gives the correct *numerical* result, but not the appropriate units by the rules of the unit calculus.

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[1] At the present time, strictly speaking, all high-precision temperature measurements are referred to the thermodynamic temperature at the triple point of water, which is taken as 273.16 K *exactly*. The fixed points on the Celsius scale are still 0° and 100° to within 0.01 °C.

[2] By the "CRC Handbook of Chemistry and Physics," 60th Ed., CRC Press, Cleveland, OH, **1979,** p. F-134, and by numerous textbooks.

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