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# A Predictive Equation for the Tracer Diffusion of Various Solutes in Gases, Supercritical Fluids, and Liquids

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A predictive equation for the infinitely dilute diffusion coefficients of various solutes in gaseous and liquid phases is proposed. The expressions proposed previously for the tracer diffusion in binary hard sphere fluids are examined critically and a new expression, based on the molecular dynamics simulations of Alder et al. (1974), obtained. The obtained expression is extended first to a Lennard—Jones fluid and then, by introducing a coupling factor, to a real fluid. The molecular diameter and energy parameters used in the equation are calculated with generalized correlations and simple combining rules. The coupling factor is correlated with the acentric factor. The derived equation is tested against a large number of experimental data: 1443 points for 120 systems, including the diffusions of various solutes in gases, supercritical fluids, and liquids. Excellent predictions were obtained: the total absolute average deviation is only 8.42%. For most systems, the predicted errors are comparable to the experimental uncertainties.

#### Introduction

The infinitely dilute diffusion coefficient of a solute in a solvent is one of the most important transport properties. In some industrial processes, such as those encountered in supercritical fluid (SCF) extraction, the systems can be considered simply as infinitely dilute. For most concentrated liquid mixtures, the diffusion coefficients can be estimated on the basis of the tracer (infinitely dilute) diffusivities of each of the components, using the Darken (1948), the Vignes (1966), or any other of the equations reviewed recently by Pertler et al. (1996). For this reason, numerous equations have been suggested for this property.

For gaseous systems at low densities, the kinetic theory of Enskog provides reasonable estimations of the diffusion coefficient (Hirschfelder et al., 1964; Chapman and Cowling, 1970). For dense fluids, however, no acceptable theory is available even for the simple hard sphere (HS) fluid. The equations used for design are mostly empirical or semi-empirical.

For the infinitely dilute diffusion coefficient in a liquid solvent, the empirical Wilke—Chang (Wilke and Chang, 1955) equation and its modifications are usually employed. Semi-empirical expressions based on the Lennard—Jones (LJ) fluid have been more recently suggested (Salim and Trebble, 1995; Liu et al., 1997). The equation has the following form:

$$D_{12}^{\infty} = D_{12}^{\text{LJ}}(\rho_2, T; \sigma_{12}^{\text{LJ}}, \varepsilon_{12}^{\text{LJ}}/k)$$
 (1)

where  $D_{12}^{\infty}$  represents the infinitely dilute diffusion coefficient of solute 1 in solvent 2,  $\rho_2$  is the number density of the solvent molecules, T is the temperature (in K), and  $\sigma_{12}^{\rm LJ}$  and  $\varepsilon_{12}^{\rm LJ}/k$  are the binary molecular diameter and energy parameters, respectively, which can be evaluated from those of the pure substances using some combining rules. However, the predictions of eq 1 are not satisfactory. At least one of the molecular

parameters,  $\sigma_{12}^{LJ}$  or  $\varepsilon_{12}^{LJ}/k$ , had to be adjusted to fit the experimental data. For instance, the geometrical mean which was usually employed had to be replaced by

$$\varepsilon_{12}^{LJ}/k = (1 - I_{12})[(\varepsilon_{11}^{LJ}/k)(\varepsilon_{22}^{LJ}/k)]^{1/2}$$
 (2)

where  $I_{12}$  is an adjustable parameter which was regressed from diffusion data. However, the parameter  $I_{12}$  could not be related to the nature of the system (Salim and Trebble, 1995; Liu et al., 1997). The other models for liquid systems, such as the free volume models (Dymond, 1985), can be used only for correlation purposes, again because the fitted parameters cannot be related to the nature of the system (Liu et al., 1997).

Regarding the SCF systems, various methods have been adopted to predict the tracer diffusion coefficient (Liong et al., 1991). Some of them are based on a Stokes—Einstein type equation (Sun and Chen, 1986), and others contain two terms, one of the Stokes—Einstein form and another one which accounts for the singular behavior near the critical point (Liu and Ruckenstein, 1997). Many authors developed equations based on the rough hard sphere (RHS) theory (Chen, 1983; Erkey and Akgerman, 1989a,b; Erkey et al., 1990; Catchpole and King, 1994; Akgerman et al., 1996; Eaton and Akgerman, 1997). The basic equation, eq 3, which is similar to that proposed by Chandler (1975) for pure fluids, was suggested by Bertucci and Flygare (1975):

$$D_{12}^{\infty} \approx D_{12}^{\text{RHS}} = A_{\text{D}} D_{12}^{\text{HS}} (\rho_2, \sigma_{12})$$
 (3)

In eq 3,  $D_{12}^{\rm HS}(\rho_2,\sigma_{12})$  is the tracer diffusion coefficient in a hard sphere fluid, and  $A_{\rm D}$  is the translational—rotational coupling factor due to the nonsphericity of the molecules. The predictive capability of eq 3 is limited by the fact that  $A_{\rm D}$  is temperature-dependent (Dymond, 1985; Erkey and Akgerman, 1989a,b). To improve the correlation, the molecular diameter,  $\sigma_{12}$ , was also considered temperature-dependent (Akgerman et al., 1996; Eaton and Akgerman, 1997).

The goal of this paper is to extend our recent equation for the self-diffusion coefficient (Ruckenstein and Liu, 1997) to the tracer diffusion coefficient of solutes in various fluids. The paper is organized as follows. In the next section the expressions proposed previously for the tracer diffusion in binary hard sphere fluids will be

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examined and a new equation based on the molecular dynamics simulations of Alder et al. (1974) suggested. In the third section, the new equation for the HS fluid is transformed to one for a Lennard–Jones fluid and finally to one for a "real" fluid. The parameters needed for prediction are correlated with the critical constants and the acentric factor. The obtained equation is tested against a large number of experimental data: 1443 points for 120 diverse systems. The results are discussed in the fourth section and the conclusions are summarized in the last one.

#### **Expressions for Hard Sphere Fluids**

**Background.** As noted previously (Dymond, 1985; Ruckenstein and Liu, 1997), the expression for the HS fluid plays a key role in an equation for self-diffusivity. This observation is true for binary systems as well. The expression for a HS fluid can be written as a modification of the Enskog equation:

$$D_{12}^{\text{HS}} \equiv \left(\frac{D_{12}^{\text{HS}}}{D_{12}^{\text{E}}}\right) D_{12}^{\text{E}} \equiv F_{12} D_{12}^{\text{E}} \tag{4}$$

where  $D_{12}^{\rm E}$  is given by

$$D_{12}^{\rm E} = \frac{3}{8\rho_2 \sigma_{12}^2} \left(\frac{kT}{2\pi m_{12}}\right)^{1/2} \frac{1}{g(\sigma_{12})} \tag{5}$$

In eq 5,  $g(\sigma_{12})$  is the radial distribution function at contact for an infinitely dilute solution, and  $m_{12}$  (in kg) is related to the masses of the molecules of the pure substances,  $m_1$  and  $m_2$ , via the following equation:

$$m_{12} \equiv \frac{m_1 m_2}{m_1 + m_2} = \frac{0.001 M_1 M_2}{N_a (M_1 + M_2)}$$
 (6)

where  $M_i$  (i = 1,2) is the molecular weight in g/mol and  $N_a$  is the Avogadro number.

Generally, one can write the following expression:

$$F_{12} = F_{12}(\rho_2^*, \sigma_1/\sigma_2, m_1/m_2) \tag{7}$$

where the reduced density  $\rho_2^*$  is defined as

$$\rho_2^* \equiv \rho_2 \sigma_2^{\ 3} = N_{\rm a} \sigma_2^{\ 3} / V_2 = \sqrt{2} \, V_0 / V_2 \tag{8}$$

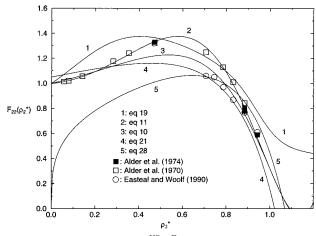
and  $V_2$  is the molar volume of the solvent and  $V_0$  the closed-packed molar volume of the solvent  $(V_0 = N_0 \sigma_0^3 / \sqrt{2})$ .

Obviously, for self-diffusion,  $F_{12}$  reduces to  $F_{22}$ :

$$\lim_{\substack{\sigma_1/\sigma_2=1\\ \mathrm{m}_1/\mathrm{m}_2=1}} F_{12} = \frac{D_{22}^{\mathrm{HS}}}{D_{22}^{\mathrm{E}}} \equiv F_{22}(\rho_2^*) \equiv F_{22} \tag{9}$$

Various expressions have been proposed for  $F_{22}(\rho_2^*)$  which are summarized elsewhere (Ruckenstein and Liu, 1997). Speedy (1987), using the molecular dynamics (MD) simulations of Easteal et al. (1983, 1984), suggested the following empirical expression:

$$F_{22}(\rho_2^*) = g(\sigma_{22})(1 - \rho_2^*/1.09)(1 + 0.4\rho_2^{*2} - 0.83\rho_2^{*4})$$
(10)



**Figure 1.** Plot of  $F_{22}(\rho_2^*)$  ( =  $D_{22}^{HS}/D_{22}^{E}$ ) against the reduced density  $\rho_2^*$ .

where  $g(\sigma_{22})$  is the radial distribution function at contact for the pure solvent. In our previous paper (Ruckenstein and Liu, 1997), another expression was obtained by using the MD simulations of Alder et al. (1970) and Erpenbeck and Wood (1991):

$$F_{22}(\rho_2^*) = 1 + 0.94605\rho_2^{*1.5} + 1.4022\rho_2^{*3} - 5.6898\rho_2^{*5} + 2.6626\rho_2^{*7} (11)$$

Equation 11 should be preferred to eq 10 because the two simulations on which the former is based are consistent with each other and inconsistent with the simulations of Easteal et al. (1983, 1984) on which the latter is based. In addition, as shown previously (Ruckenstein and Liu, 1997), the expression for the diffusion coefficient of a LJ fluid based on eq 11 provides more accurate results than that based on eq 10.

At low densities, the following requirement must be satisfied to recover the Enskog theory:

$$\lim_{\rho_2^* \to 0} F_{22}(\rho_2^*) = 1 \tag{12}$$

At high densities, the MD simulations of Woodcock (1981a,b) indicated that  $F_{22}(\rho_2^*)$  should approach zero for  $\rho_2^* = \rho_s^* \approx 1.1$ :

$$\lim_{\rho_2^{*} \to \rho_s^*} F_{22}(\rho_2^*) = 0 \tag{13}$$

As shown in Figure 1, both eqs 10 and 11 satisfy the above two constraints. However, in the middle-density range the two expressions provide different results.

A constraint similar to eq 12 should be also satisfied by  $F_{12}$ :

$$\lim_{\rho_{2}^{*} \to 0} F_{12}(\rho_{2}^{*}, \sigma_{1}/\sigma_{2}, m_{1}/m_{2}) = 1$$
 (14)

since the Enskog theory holds at low densities. In what follows, eqs 9.12-14 will be used to test the expressions proposed in the literature for  $F_{12}$ . A new expression for  $F_{12}$ , which reduces to eq 11 for pure substances and satisfies all the above constraints, is proposed.

Analysis of the Expressions for  $F_{12}$  Proposed in Literature. Sung and Stell (1984) derived the follow-

ing equation for the diffusion coefficient in infinitely dilute solutions for a hard sphere fluid:

$$F_{12} = \left(\frac{1 + \gamma_{\rm B} \frac{\sigma_2}{\sigma_1}}{1 + \gamma_{\rm B} \frac{\sigma_2}{\sigma_1}}\right)^2 \frac{g(\sigma_{12})}{1 + 4\eta_2 g(\sigma_{22})} + \frac{1 + \frac{\sigma_2}{\sigma_1}}{1 + \gamma_{\frac{\sigma_2}{\sigma_1}}} \frac{D_{12}^{\rm H}}{D_{12}^{\rm E}}$$
(15)

where

$$\eta_2 = \frac{1}{6}\pi \rho_2^* = \frac{0.7405 \, V_0}{V_2} \tag{16}$$

and  $\gamma$  and  $\gamma_B(\gamma_B = \lim_{\rho_2 \to 0} \gamma)$  are given by the following equation:

$$\frac{1}{(1-\gamma^{3}\eta_{2})} \left(1-\gamma^{3}\eta_{2}+\frac{2\gamma^{3}\eta_{2}}{1+\gamma\sigma_{2}/\sigma_{1}}\right) \left(1-\gamma^{3}\eta_{2}+\frac{\gamma^{3}\eta_{2}}{1+\gamma\sigma_{2}/\sigma_{1}}\right) = 1 + \frac{4\eta_{2}(1-0.5\eta_{2})}{(1-\eta_{2})^{3}} (17)$$

The first term in eq 15 represents the Enskog limit, while the second the hydrodynamic limit. The ratio  $D_{12}^H/D_{12}^E$  in the second term is given by (Sun and Chen, 1985a):

$$\frac{D_{12}^{H}}{D_{12}^{E}} = 2.881 \left(1 + \frac{\sigma_1}{\sigma_2}\right) \left(1 + \frac{m_2}{m_1}\right)^{-1/2} \frac{\eta_2 g(\sigma_{12})}{\mu_2^{HS}/\mu_0}$$
(18)

where  $\mu_2^{\rm HS}/\mu_0$  is the ratio of the viscosities of a HS solvent and a perfect gas.

For a pure solvent, eq 15 combined with 18 leads to

$$F_{22} = \left(\frac{1+\gamma_{\rm B}^0}{1+\gamma^0}\right)^2 \frac{g(\sigma_{22})}{1+4\eta_2 g(\sigma_{22})} + \frac{8.15\eta_2 g(\sigma_{22})}{(1+\gamma^0)\mu_2^{\rm HS}/\mu_0}$$
(19)

where the superscript "0" refers to the pure solvent and  $\gamma_{\rm B}^0=1.1909~(\gamma^0~{\rm should}$  be calculated from eq 17 for the pure solvent).

Equations 15 and 19 satisfy the zero density constraint, eqs 14 and 12, respectively. As shown in Figure 1, eq 19 provides quantitative agreement with the MD simulations of Alder et al. (1970) in the dilute and middle-density ranges. At high densities, it leads, however, to nonzero values, and hence it does not satisfy the constraint eq 13.

Table 1 and Figures 2 and 3 present the calculated values of  $F_{12}$  given by eq 15 and compare them with the MD simulations of Alder et al. (1974). One can see that the agreement is poor. For this reason, eq 15 will not be used further. While Sun and Chen (1985a) employed eq 15 for the SCF systems, the agreement with experimental data is most likely due to the two empirical parameters used to express the temperature-dependence of the molecular diameter, which strongly affects the diffusion coefficient.

In another paper, Sun and Chen (1985b) proposed an expression for  $F_{12}$  (eq 20) on the basis of the MD simulations of Herman and Alder (1972) and Alder et

Table 1. MD Data and Calculation Results for  $F_{12} = D_{19}^{HS}/D_{19}^{E}$ 

$\sigma_1/\sigma_2$	$m_1/m_2$	$ ho_2^*$	$\mathrm{MD}^a$	eq 32	eq 15	eq 20	ref for MD data
						-	
1.00	1.00	0.4714	1.33	1.33	1.40	1.16	b
1.00	0.10	0.4714	0.82	0.80	1.00	0.70	b
1.00	0.01	0.4714	0.39	0.39	0.81	0.33	b
1.00	1/5625	0.4714	0.19	0.20	0.73	0.08	b
0.75	1.00	0.4714	1.27	1.35	1.24	1.16	b
0.75	0.10	0.4714	0.91	0.87	0.93	0.70	b
0.75	0.01	0.4714	0.58	0.53	0.77	0.33	b
0.75	1/5625	0.4714	0.38	0.45	0.70	0.08	b
0.50	1.00	0.4714	1.22	1.32	1.09	1.23	b
$0.50 \\ 0.50$	0.10	0.4714	0.99	0.96	0.84	$0.75 \\ 0.35$	b b
0.50	0.01	0.4714	0.79	$0.74 \\ 0.74$	$0.72 \\ 0.67$		b b
0.30	1/5625 1.00	0.4714 $0.4714$	$0.75 \\ 1.13$	1.08	0.67	$0.09 \\ 1.61$	b b
0.25	0.10	0.4714	1.13	1.06	0.93	0.99	b b
0.25	0.10	0.4714	1.00	1.04	0.76	0.99	b b
0.25							b b
	1/5625	0.4714	1.03	1.03	0.64	0.12	b b
1.00	1.00	0.8839	0.78	0.81	0.95	0.71	b b
1.00	0.10	0.8839	0.40	0.47	$0.71 \\ 0.59$	0.33	b b
1.00 0.75	0.01 1.00	0.8839 0.8839	0.14	0.11 0.86	0.39	$0.11 \\ 0.79$	b b
0.75		0.8839	$0.83 \\ 0.47$	0.50	0.62	0.79	b b
	0.10						b b
0.75	0.01	0.8839	0.15	0.15	0.54	0.15	b b
0.50	1.00	0.8839	0.94	0.93	0.68	0.93	b b
0.50	0.10	0.8839	0.61	0.61	0.55	0.50	
$0.50 \\ 0.50$	0.01	0.8839	0.35	0.32	0.49	0.20	b b
0.30	1/5625 1.00	0.8839	0.13 1.08	$0.14 \\ 1.13$	$0.46 \\ 0.54$	$0.03 \\ 1.33$	b b
0.25		0.8839					b b
0.25	0.10 0.01	0.8839 0.8839	$0.93 \\ 0.75$	$0.98 \\ 0.80$	$0.46 \\ 0.41$	$0.75 \\ 0.32$	b b
0.25	1/5625	0.8839	0.73	0.69	0.41	0.32	b b
1.00	1.00	0.8639	0.59	0.57	0.40	0.00	b b
1.00	0.10	0.9428	0.35	0.37	0.73	0.48	b b
1.00	0.10	0.9428	0.33	0.37	0.55	0.10	b b
0.75	1.00	0.9428	0.12	0.64	0.68	0.61	$\stackrel{\scriptscriptstyle D}{b}$
0.75	0.10	0.9428	0.36	0.38	0.56	0.27	$\stackrel{\scriptscriptstyle D}{b}$
0.75	0.10	0.9428	0.30	0.15	0.51	0.27	$\stackrel{\scriptscriptstyle D}{b}$
0.50	1.00	0.9428	0.76	0.70	0.57	0.80	$\stackrel{D}{b}$
0.50	0.10	0.9428	0.49	0.48	0.49	0.40	$\stackrel{\scriptscriptstyle D}{b}$
0.50	0.01	0.9428	0.26	0.27	0.45	0.14	$\stackrel{\scriptscriptstyle D}{b}$
0.50	1/5625	0.9428	0.12	0.12	0.43	0.01	$\stackrel{\scriptscriptstyle D}{b}$
0.25	1.00	0.9428	0.93	0.91	0.46	1.23	$\stackrel{\scriptscriptstyle D}{b}$
0.25	0.10	0.9428	0.80	0.80	0.41	0.67	$\overset{\circ}{b}$
0.25	0.01	0.9428	0.71	0.67	0.38	0.27	$\stackrel{\scriptscriptstyle D}{b}$
0.25	1/5625	0.9428	0.56	0.55	0.37	0.05	$\overset{\scriptscriptstyle D}{b}$
1.00	4.00	0.4714	1.56	1.62	1.58	1.19	c
1.00	1.00	0.4714	1.34	1.33	1.40	1.16	c
1.00	0.25	0.4714	1.07	1.01	1.14	0.91	c
1.00	0.01	0.4714	0.40	0.39	0.81	0.33	c
1.00	1/5625	0.4714	0.24	0.20	0.73	0.08	c
1.00	4.00	0.8839	1.02	1.05	1.06	0.81	c
1.00	1.00	0.8839	0.80	0.81	0.95	0.71	c
1.00	0.25	0.8839	0.57	0.63	0.80	0.48	c
1.00	0.01	0.8839	0.12	0.11	0.59	0.11	c
	d AAD			4.97	75.62	34.51	

 $^a$  The extrapolated MD data for the infinite size system.  $^b$  Alder et al. (1974).  $^c$  Herman and Alder (1972).

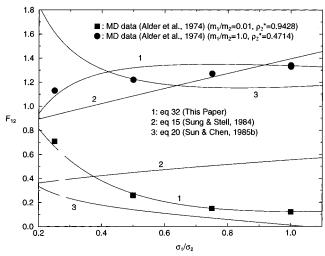
al. (1974) in the following ranges:  $0.5 < \sigma_1/\sigma_2 < 1.6$ , 0.5  $< m_1/m_2 < 4.0$ , and  $1.5 < V_2/V_0 < 3.0$ .

$$F_{12} = 1.0514 \left( \frac{\sigma_{12}^{2}}{\sigma_{1}\sigma_{2}} \right) \left( \frac{2}{1 + m_{2}/m_{1}} \right)^{1/2} g(\sigma_{12}) \left( \frac{m_{2}}{m_{1}} \right)^{0.167} \times \left[ 1 - 0.9779 \rho_{2}^{*} \left( \frac{m_{2}}{m_{1}} \right)^{0.0165} \left( \frac{\sigma_{1}}{\sigma_{2}} \right)^{0.129} \right] (20)$$

For a pure solvent, eq 20 reduces to

$$F_{22}(\rho_2^*) = 1.0514g(\sigma_{22})(1 - 0.9779\rho_2^*) \tag{21}$$

The predictions provided by eq 21 are also presented in



**Figure 2.** Plot of  $F_{12}$  (=  $D_{12}^{HS}/D_{12}^{E}$ ) against the molecular size ratio

Figure 1. While eq 21 satisfies approximately the constraints at both zero (with a slightly different numerical factor, 1.0514) and high densities, it provides too low values in the middle-density range. As shown by eqs 10 and 11, the ratio  $F_{22}/g(\sigma_{22})$  cannot be a linear function of density. Consequently, eqs 20 and 21 are not expected to provide reasonable values for  $F_{12}$  and  $F_{22}$ . Table 1 and Figures 2 and 3 present the calculated results.

Easteal and Woolf (1990) carried out MD simulations for tracer diffusion coefficients in HS systems and proposed the following empirical expression for the molecular-weight ratio dependence of  $F_{12}/F_{22}$ :

$$\ln\left(\frac{F_{12}}{F_{22}}\right) = a_0 + a_1 \ln\left(\frac{m_1}{m_2}\right) + a_2 \left[\ln\left(\frac{m_1}{m_2}\right)\right]^2$$
 (22)

where the coefficients,  $a_0$ ,  $a_1$ , and  $a_2$  are dependent on both the density  $\rho_2^*$  and the molecular size ratio  $\sigma_1/\sigma_2$ .

On the basis of the MD simulations of Easteal and Woolf (1990), Salim and Trebble (1995) proposed the following expressions for the coefficients in eq 22:

$$a_0 = -1.02564 \rho_2^{*4} \ln \left( \frac{\sigma_1}{\sigma_2} \right)$$
 (23)

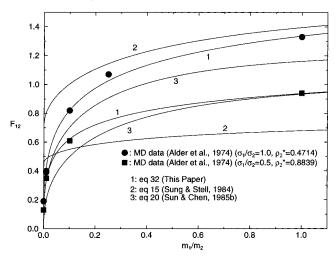
$$a_1 = -0.24107 + 1.27589 h_{12} - 1.35439 h_{12}^2 + \\ 0.62393 h_{12}^3 \ (24)$$

$$a_2 = -0.056 \tag{25}$$

where  $h_{12} = \rho_2^{*2} \ln(\sigma_1/\sigma_2)$ . For  $F_{22}$ , Salim and Trebble (1995) used the expression of Speedy (1987), eq 10, which, being based on the MD simulations of Easteal et al. (1983, 1984), is consistent with the MD simulations for mixtures (Easteal and Woolf, 1990). Nevertheless, one can easily see that because of eqs 24 and 25, eq 22 does not satisfy the constraint eq 14.

More recently, Eaton and Akgerman (1997) proposed the following expression, on the basis of the MD simulations of Easteal and Woolf (1990):

$$\frac{V_2}{V_0} \left( \frac{D_{12}^{\text{HS}}}{D_{12}^{\text{E}}} \right) = a \left[ \left( \frac{V_2}{V_0} \right)^{\alpha} - b_{12} \right]$$
 (26)



**Figure 3.** Plot of  $F_{12}$  ( =  $D_{12}^{HS}/D_{12}^{E}$ ) against the molecular weight ratio  $m_1/m_2$ 

where

$$\alpha = \frac{\sigma_2}{\sigma_1} - \frac{1}{3} \tag{27a}$$

$$b_{12} = \left(\frac{m_2}{m_1}\right)^{0.03587} \left[ 0.6001 + 0.8491 \left(\frac{\sigma_2}{\sigma_1}\right) - 0.244 \left(\frac{\sigma_2}{\sigma_1}\right)^2 \right]$$
(27b)

and a is a function of the molecular size ratio  $\sigma_1/\sigma_2$ . Using the MD simulations of Easteal and Woolf (1990), we obtained

$$a = 1.689(\sigma_1/\sigma_2)^{2.0674} \tag{27c}$$

For a pure solvent, eq 26 reduces to

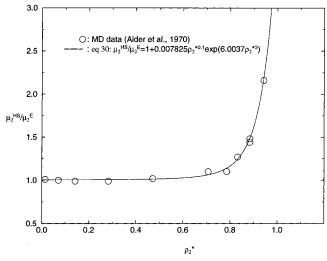
$$F_{22}(\rho_2^*) = 1.1943g(\sigma_{22})(1.26\rho_2^{*1/3} - 1.2052\rho_2^*)$$
 (28)

An obvious shortcoming of eq 28 is that it predicts  $F_{22}$ = 0 at zero density instead of 1. For this reason, eq 26 is valid only in the high-density range (see Figure 1).

A New Expression for  $F_{12}$ . Considering the limitations of the expressions summarized above, we propose an expression for  $F_{12}$  on the basis of the MD simulations of Herman and Alder (1972) and Alder et al. (1974). These simulations for binary HS mixtures are preferred to those of Easteal and Woolf (1990), because, as already noted, the recent MD simulations carried out by Erpenbeck and Wood (1991) for pure HS fluids are consistent with those of the former and inconsistent with those of the latter authors.

As shown by Alder et al. (1974), the dependence of  $F_{12}$  on  $\sigma_1/\sigma_2$  is complex. At low densities,  $F_{12}$  increases as  $\sigma_1/\sigma_2$  increases when  $m_1/m_2=1$ , while it decreases with increasing  $\sigma_1/\sigma_2$  when  $m_1/m_2 \leq 0.1$ ; at high densities,  $F_{12}$  always decreases as  $\sigma_1/\sigma_2$  increases. Instead of using a form which simply contains  $\rho_2^*$ ,  $\sigma_1/\sigma_2$ , and  $m_1/m_2$  as variables, as in the previous empirical correlations, eqs 20, 22, and 26, some terms involved in the theoretical expressions of Sung and Stell (1984) and Alder et al. (1974) will also be included.

The theoretical expression of Sung and Stell (1984) (eq 15) suggests that  $F_{12}$  should depend on the radial



**Figure 4.** Ratio of the viscosities for the hard sphere and Enskog fluids,  $\mu_2^{\rm HS}/\mu_2^{\rm E}$ , against the reduced density  $\rho_2^*$ .

distribution functions at contact and on the viscosity ratio,  $\mu_2^{\text{HS}}/\mu_0$ , given by

$$\frac{\mu_2^{\text{HS}}}{\mu_0} = \frac{\mu_2^{\text{HS}}}{\mu_2^{\text{E}}} [1/g(\sigma_{22}) + 3.2\eta_2 + 12.18\eta_2^2 g(\sigma_{22})]$$
 (29)

where  $\mu_2^{\rm HS}/\mu_2^{\rm E}$  is the ratio between the hard sphere and Enskog viscosities of the solvent. On the basis of the MD simulations of Alder et al. (1970) for an HS fluid, we obtained the expression

$$\frac{\mu_2^{\text{HS}}}{\mu_0^{\text{E}}} = 1 + 0.007825(\rho_2^*)^{0.1} \exp(6.0037\rho_2^{*3}) \quad (30)$$

which represents well the MD simulations (see Figure 4).

Alder et al. (1974) introduced a hydrodynamic vortex effect in  $F_{12}$  characterized by the ratios  $\sigma_1/\sigma_2$  and  $m_1/m_2$  and by the quanity  $\Delta$ :

$$\Delta = \frac{[\mu_2^{\rm HS}/\mu_0 + 1.2/g(\sigma_{22})]^{3/2}}{[\mu_2^{\rm HS}/\mu_0 + 1.2(m_2/2m_{12})^{1/2}(\sigma_{22}/\sigma_{12})^2g^{-1}(\sigma_{12})]^{3/2}} \tag{31}$$

Consequently,  $F_{12}$  is expected to depend not only on  $\sigma_1/\sigma_2$ ,  $m_1/m_2$ , and the solvent density but also on the radial distribution functions at contact, the viscosity ratio,  $\mu_2^{\rm HS}/\mu_0$ , and the quantity  $\Delta$ . After a long number of trials, the following expression was selected:

$$\frac{F_{12}}{F_{00}} = 1.0 + \sum_{i=1}^{10} b_i G_i \tag{32}$$

where the  $b_i$ 's are constants and  $G_i$ 's are functions listed in Table 2. The simulations of Alder et al. (1974) indicate logarithmic dependences on the ratios  $\sigma_1/\sigma_2$  and  $m_1/m_2$ . In addition, the density dependence should satisfy the constraints at low and high densities. For these reasons, most of the  $G_i$ 's contain logarithmic factors of one or both of the above ratios and some contain the density factor  $\beta_2^{*0.01}$ . The effects of the viscosity ratio, the radial distribution functions at contact, and the quantity  $\Delta$  are taken into account in

Table 2. Functions and Coefficients in Equation 32<sup>a</sup>

i	$b_i$	$G_i$
1	-0.293977	$\chi \ln(\sigma_1/\sigma_2)$
2	-0.106089	$\chi[\ln(\sigma_1/\sigma_2)]^2 \ln(m_1/m_2)$
3	0.111318	$\chi \ln(m_1/m_2)$
4	0.002865	$\ln(\mu_2^{\rm HS}/\mu_0)[\ln(m_1/m_2)]^2$
5	-0.0069724	$\chi \ln(m_1/m_2)(\sigma_2/\sigma_1)^{0.5}$
6	-0.0020652	$\chi \ln(\sigma_1/\sigma_2)(m_2/m_1)^{0.5}$
7	3.21576	$\{\ln[g(\sigma_{22})/g(\sigma_{12})]\}^2$
8	-0.334952	χ ln Δ
9	-0.26190	$\chi[\ln(\sigma_1/\sigma_2)]^2(m_2/m_1)^{1/6}$
10	0.143989	$\delta(\sigma_1/\sigma_2, m_1/m_2) \ln(\mu_2^{\rm HS}/\mu_2^{\rm E})$

$$^{a}\chi \equiv (\rho_{2}^{*})^{0.01}; \ \delta(\sigma_{1}/\sigma_{2}, m_{1}/m_{2}) = \begin{cases} 0, & (\sigma_{1}/\sigma_{2}) \text{ and } (m_{1}/m_{2}) = 1.0 \\ 1, & (\sigma_{1}/\sigma_{2}) \text{ or } (m_{1}/m_{2}) \neq 1.0 \end{cases}.$$

some of the  $G_i$ 's. Equation 11 was used for  $F_{22}$ . Equation 32 satisfies all the constraints noted in a previous section.

Throughout this paper, the Carnahan—Starling expression (Carnahan and Starling, 1969) for the radial distribution function was adopted for the pure solvent

$$g(\sigma_{22}) = \frac{1 - 0.5\eta_2}{\left(1 - \eta_2\right)^3} \tag{33}$$

and the Mansoori et al. expression (1971) for the radial distribution function of a binary mixture with a solute at infinite dilution

$$g(\sigma_{12}) = (1 - \eta_2)^{-3} \left( 1 - \eta_2 + \frac{2\eta_2}{1 + \sigma_2/\sigma_1} \right) \times \left( 1 - \eta_2 + \frac{\eta_2}{1 + \sigma_2/\sigma_1} \right)$$
(34)

Another expression for  $g(\sigma_{12})$ , proposed by Alder et al. (1974), provided the same results.

The results obtained on the basis of eq 32 are listed in Table 1 along with the MD data. The MD data provided by Alder et al. (1974), extrapolated to an infinite number of molecules, were employed. One can see from Table 1 that eq 32 can accurately represent the MD data over wide ranges of molecular size and weight ratios; eqs 15 and 20 provide unsatisfactory results. Only a few points for  $m_1/m_2 = 1/5625$  were disregarded since the values of  $F_{12}$  were too small (less than 0.02) and their simulation uncertainties comparable to those values (Alder et al., 1974).

Figure 2 illustrates the dependence of  $F_{12}$  on  $\sigma_1/\sigma_2$ . A comparison between the MD data (Alder et al., 1974) and those calculated using three different expressions is also included in the Figure. One can see that the theory of Sung and Stell (1984) (eq 15) and the empirical expression of Sun and Chen (1985b) (eq 20) provide wrong trends under some conditions.

Figure 3 presents the dependence of  $F_{12}$  on  $m_1/m_2$  and shows that eq 15 (Sung and Stell, 1984) overestimates  $F_{12}$  at high values of  $\sigma_1/\sigma_2$  and underestimates it at low values of  $\sigma_1/\sigma_2$ . Equation 20 (Sun and Chen, 1985b) always underestimates  $F_{12}$ .

Due to the range of the MD data used in obtaining our eq 32, this equation is recommended for calculating  $F_{12}$  in the following ranges: density,  $\rho_2^*$ , from 0.0 to 0.95; molecular size ratios,  $\sigma_1/\sigma_2$ , from 0.25 to 1.0; and molecular weight ratios,  $m_1/m_2$ , from 0.0002 to 4.0.

Table 3. MD Simulations and Summary of Expressions for  $F_{12}=D_{12}^{
m HS}/D_{12}^{
m E}$ 

MD Simulations							
author	$ ho^*$	$\sigma_1/\sigma_2$	$m_1/m_2$	NDP			
Herman and Alder (1972)	0.4714-0.8839	1.0	1/5625-4.0	10			
Alder et al. (1974)	0.4714 - 0.9428	$0.25{-}1.0$	1/5625 - 1.0	48			
Easteal and Woolf (1990)	0.7071 - 0.9428	0.5 - 1.0	0.100 - 5/3	360			

		Expressions	
author	expression	no. of eqs	MD data used
Sung and Stell (1984) Sun and Chen (1985b) Easteal and Woolf (1990) Salim and Trebble (1995) Eaton and Akgerman (1997) this paper	theoretical correlation correlation correlation correlation correlation correlation	eqs 15–18 eq 20 eq 22 eqs 22–25 eqs 26–27 eq 32	Herman and Alder (1972), Alder et al. (1974) Easteal and Woolf (1990) Easteal and Woolf (1990) Easteal and Woolf (1990) Herman and Alder (1972), Alder et al. (1974)

Table 4. Data for Pure Substances

substance	M	$T_{\rm C}$ (K)	P <sub>C</sub> (bar)	V <sub>C</sub> (cm <sup>3</sup> /mol)	ω	$\sigma_{\mathrm{IJ}}{}^{a}$ (Å)	$\varepsilon_{\mathrm{LJ}}/\mathit{k}^b$ (F
argon	39.948	150.80	48.70	74.90	0.001	3.32544	119.75
ktypton	83.800	209.40	55.00	91.20	0.005	3.63836	166.28
helium	3.017	3.31	1.14	72.90	-0.473	3.29557	2.63
hydrogen	2.016	33.00	12.90	64.30	-0.216	3.16052	26.21
xenon	131.30	289.70	58.40	118.40	0.008	3.96910	230.05
nitrogen	28.013	126.20	33.90	89.80	0.039	3.53276	100.21
carbon monoxide (CO)	28.010	132.90	35.00	93.20	0.067	3.57679	105.53
carbon dioxide (CO <sub>2</sub> )	44.010	304.19	72.85	94.07	0.239	3.58789	241.55
carbon disulfide	76.131	552.00	79.00	160.00	0.109	4.21440	438.34
methane	16.043	190.40	46.00	99.20	0.011	3.65196	151.20
ethylene (C <sub>2</sub> H <sub>4</sub> )	28.054	282.40	50.40	130.40	0.089	4.00050	224.25
ethane	30.070	305.40	48.80	148.30	0.099	4.17576	242.52
propane	44.094	369.80	42.50	203.00	0.153	4.63647	293.66
<i>n</i> -pentane	72.151	469.70	33.70	304.00	0.251	5.30453	372.98
<i>n</i> -hexane	86.178	507.50	30.10	370.00	0.299	5.66356	403.00
cyclohexane (cC <sub>6</sub> H <sub>12</sub> )	84.162	554.15	40.40	311.70	0.212	5.34894	440.05
<i>n</i> -heptane	100.205	540.30	27.40	432.00	0.349	5.96371	429.05
<i>n</i> -octane	114.232	568.80	24.90	492.00	0.398	6.22793	451.68
<i>n</i> -nonane	128.259	594.60	22.90	543.00	0.445	6.43609	472.17
<i>n</i> -decane	142.286	617.70	21.20	603.00	0.489	6.66491	490.51
<i>n</i> -undecane	156.313	638.80	19.70	660.00	0.535	6.86863	507.27
<i>n</i> -dodecane	170.340	658.20	18.20	713.00	0.575	7.04777	522.67
<i>n</i> -tetradecane	198.394	693.00	14.40	830.00	0.581	7.41392	<b>550.3</b> 1
<i>n</i> -hexadecane (C <sub>16</sub> H <sub>34</sub> )	226.448	722.00	14.10	944.00	0.742	7.73890	573.33
2,3-dimethylbutane (DMB)	86.178	500.00	31.30	358.00	0.247	5.60165	397.05
benzene	78.115	562.20	48.90	259.00	0.212	5.02869	446.44
toluene	92.141	591.80	41.00	316.00	0.263	5.37342	469.94
ethylbenzene	106.168	617.20	36.00	374.00	0.302	5.68389	490.11
<i>n</i> -propylbenzene	120.195	638.20	32.00	440.00	0.344	6.00030	506.79
isopropylbenzene	120.195	631.10	32.10	440.00	0.326	6.00030	501.15
<i>m</i> -xylene	106.168	617.10	35.40	376.00	0.325	5.83406	490.03
<i>p</i> -xylene	106.168	616.20	35.10	379.00	0.320	5.84953	489.32
o-xylene	106.168	630.30	37.30	369.00	0.310	5.65845	500.52
mesitylene	120.195	664.50	34.50	480.00	0.366	6.17688	527.67
hexachlorobenzene	284.780	847.70	32.28	557.50	0.568	6.49287	673.15
phenol	94.113	694.20	61.30	229.00	0.438	4.82652	551.20
benzoic acid	122.124	752.00	45.60	341.00	0.620	5.51155	597.10
2-naphthol	144.170	811.40	47.40	375.50	0.594	5.69148	644.33
naphthalene	128.174	748.40	40.50	413.00	0.303	5.87497	594.30
phenanthrene	178.200	873.00	33.30	554.00	0.517	6.47926	693.2
pyrene	202.260	907.48	30.73	619.50	0.612	6.72516	720.6
chrysene	228.290	990.83	27.41	701.50	0.641	7.00967	786.8
caffeine	194.200	855.60	41.50	488.00	0.906	6.21101	679.43
C4:0 ethyl ester	116.200	579.00	31.40	400.00	0.417	5.81267	459.78
C8:0 ethyl ester	172.300	655.70	21.18	621.50	0.670	6.73239	520.69
C10:0 ethyl ester	200.000	699.30	17.88	733.50	0.761	7.11468	555.3
C14:0 ethyl ester	256.430	788.40	13.23	957.50	0.897	7.77562	626.0
C16:0 ethyl ester	284.480	835.00	11.56	1069.5	0.926	8.06768	663.0
C18:0 ethyl ester	312.500	883.40	10.19	1181.5	0.934	8.34001	701.50
C22:0 ethyl ester	368.700	987.80	8.081	405.50	0.957	8.83688	784.4
C18:2 methyl ester	294.500	884.00	12.05	1065.5	0.902	8.05761	701.9
C20:5 ethyl ester	356.500	823.40	11.34	1137.5	0.824	8.23517	653.8
C22:6 methyl ester	342.500	852.40	10.35	1229.0	0.851	8.45031	676.8
C22:6 ethyl ester	356.500	867.10	11.02	1285.0	0.860	8.57676	688.5
chlorotrifluoromethane (CClF <sub>3</sub> )	104.459	302.00	38.70	180.40	0.198	4.45760	239.8
carbon tetrachloride (CCl <sub>4</sub> )	153.823	556.40	45.60	275.90	0.193	5.13577	441.83
sulfur hexafluoride (SF <sub>6</sub> )	146.054	318.70	37.60	198.80	0.286	4.60427	253.08
acetonitrile	41.053	545.50	48.30	173.00	0.327	4.32559	433.18
acetone	58.080	508.10	47.00	209.00	0.304	4.68171	403.48
ethyl acetate	88.107	523.20	38.30	286.00	0.362	5.19769	415.47

 $<sup>^</sup>a$  Calculated with eq 46.  $^b$  Calculated with eq 47.

**Table 5. Systems Employed and Calculation Results** 

				predi		correla		
system (solute + solvent)	$T_{ m r2}$	$ ho_{ m r2}$	NDP	$A_{ m D}^{hh}$	AAD	$A_D^{ii}$	AAD	ref
		Diffusion in						
argon + helium	74.9698-97.6284	0.0362 - 0.2740	16	0.9827	2.04	0.9631	1.07	a
nitrogen + helium	74.9698-97.6284	0.0362 - 0.2740	12	0.9856	1.39	0.9958	1.01	а
CO <sub>2</sub> + helium	74.9698-97.6284	0.0362 - 0.2297	16	0.9965	10.04	0.9055	0.80	a
methane + helium	74.9698-97.6284	0.0362-0.2740	16	$0.9835 \\ 0.9889$	9.32 5.94	1.0844 1.0203	1.25 5.51	a b
$\mathrm{CO_2} + \mathrm{ethylene}$ ethylene $+ \mathrm{CO_2}$	1.0559-1.2330	0.0514 - 1.5935	49 48	0.9889	5.94 7.45	1.0203	6.77	b b
ethylene + CO <sub>2</sub>	0.9803 - 1.1447	0.0365-1.6886		0.9000	7.43	1.0403	0.77	D
nonhtholono   othylono	1 0007 1 1966	Diffusion ir 0.8160-2.3935		0.0000	9.60	1 0099	9.70	a d
naphthalene + ethylene <i>n</i> -pentane + CO <sub>2</sub>	1.0097 - 1.1266 $0.9834 - 1.0130$	0.8160-2.3935 $1.5620-1.7482$	20 5	0.9833 $1.2020$	8.69 7.87	1.0023 1.1142	$8.79 \\ 3.92$	c,d
$n$ -pentane $+ CO_2$ $n$ -hexane $+ CO_2$	0.9834-1.0130	1.5620-1.7482	5 5	1.2020	3.79	1.1142	3.60	$e \\ e$
$n$ -heptane + $CO_2$	0.9834-1.0130	1.5620-1.7482	5	1.2614	3.50	1.2612	3.50	e
$n$ -octane + $CO_2$	0.9834-1.0130	1.5620-1.7482	5	1.2936	3.87	1.3011	3.86	e
$n$ -nonane + $CO_2$	0.9834-1.0130	1.5620-1.7482	5	1.3259	4.02	1.3255	4.02	e
$n$ -decane + $CO_2$	0.9834 - 1.0130	1.5620 - 1.7482	5	1.3574	4.06	1.3577	4.05	e
$n$ -undecane $+$ $ ilde{ ilde{ ilde{C}}} ilde{ ilde{O}}_2$	0.9834 - 1.0130	1.5620 - 1.7482	5	1.3918	4.78	1.4011	4.78	e
n-dodecane $+$ CO <sub>2</sub>	0.9834 - 1.0130	1.5620 - 1.7482	5	1.4228	6.74	1.4156	6.61	e
$n$ -tetradecane $+$ CO $_2$	0.9834 - 1.0130	1.5620 - 1.7482	5	1.4276	9.97	1.3319	8.12	e
$enzene + CO_2$	$0.9834 \! - \! 1.0952$	0.7481 - 2.0009	67	0.9784	8.97	0.9453	8.54	e, $f$ – $j$
$coluene + CO_2$	1.0063 - 1.0952	1.1564 - 2.0009	34	0.9736	5.42	1.0214	4.10	$g$ , $i$ , $\check{k}$
$ethylbenzene + CO_2$	1.0295 - 1.0952	1.2977 - 2.0009	15	0.9696	4.08	1.0103	1.99	g
$n$ -propylbenzene + $CO_2$	1.0295 - 1.0952	0.7652 - 2.0009	26	0.9650	8.93	0.9652	8.91	g
sopropylbenzene + CO <sub>2</sub>	1.0295 - 1.0952	1.2977 - 2.0009	15	0.9670	4.49	1.0119	2.13	$g_{\cdot}$
mesitylene + CO <sub>2</sub>	0.9966 - 1.0952	1.0901-1.8810	7	0.9625	9.58	1.0628	3.36	f,h
nexachlorobenzene $+ CO_2$	1.0125-1.0783	0.8550-1.9237	14	0.9350	8.98	0.9763	7.05	1
$O_2$	1.0125-1.0783	0.7032-1.7185	21	0.9535	8.63	0.8747	7.80	k
penzoic acid $+ CO_2$	0.9823-1.0477	1.1483-1.8359	7	0.9267	13.26	1.0675	2.67	m
2-naphthol + CO <sub>2</sub>	1.0132-1.0461	1.6284-1.9284	5	0.9309	34.10	0.6925	4.12	n
naphthalene + CO <sub>2</sub>	0.9151-1.0952	0.8567-2.2820	76	0.9695	11.96	0.9869	11.63	c,d,h,
ohenanthrene + CO <sub>2</sub>	0.9966-1.0952	0.8550 - 1.9237 $1.3893 - 1.8810$	21	$0.9426 \\ 0.9266$	14.04 16.87	1.0832 1.1115	$6.06 \\ 4.37$	h,1 h
$\mathrm{CO}_2$ $\mathrm{Chrysene} + \mathrm{CO}_2$	0.9966 - 1.0952 $0.9966 - 1.0952$	1.7100-1.8810	3 4	0.9232	13.55	1.1113	1.98	h
$caffeine + CO_2$	1.0125-1.0783	0.9127 - 1.7228	21	0.8724	6.21	0.8718	6.20	k
$1 + CO_2$	0.9834-1.0952	1.0508-1.8888	36	0.9694	7.57	0.9688	7.57	e, h, p,
ethyl acetate + CO <sub>2</sub>	1.0125-1.0783	0.4553 - 1.7228	15	0.9629	19.99	0.8146	19.13	k,11,p,
$C4:0$ ethyl ester $+ CO_2$	1.0125-1.0454	1.2825-1.8168	16	0.9562	4.99	0.9938	4.23	r
$C8:0$ ethyl ester $+ CO_2$	1.0125-1.0454	1.2825-1.8168	16	0.9183	6.37	0.9740	3.70	r
C10:0 ethyl ester $+$ CO <sub>2</sub>	1.0125 - 1.0454	1.2825 - 1.8168	16	0.9018	8.77	0.9851	4.70	r
$C14:0$ ethyl ester $+$ $CO_2$	1.0125 - 1.0454	1.2825 - 1.8168	16	0.8743	12.01	0.09901	5.03	r
C16:0 ethyl ester $+ CO_2$	1.0125 - 1.0454	1.2825 - 1.8168	17	0.8680	11.56	0.9785	4.48	r
$C18:0$ ethyl ester $+$ $CO_2$	1.0125 - 1.0454	1.2825 - 1.8168	17	0.8662	12.53	0.9872	4.86	r
$C22:0$ ethyl ester $+ CO_2$	1.0125 - 1.0454	1.2825 - 1.8168	17	0.8611	18.30	1.0498	5.29	r
C18:2 methyl ester $+$ CO <sub>2</sub>	1.0132 - 1.0789	1.5796 - 1.9857	19	0.8732	6.74	0.9360	1.44	S
$C20:5$ ethyl ester $+ CO_2$	1.0125 - 1.0454	1.2825 - 1.8168	17	0.8895	7.07	0.9490	4.97	$\boldsymbol{r}$
$C22:6$ methyl ester $+$ $CO_2$	1.0125 - 1.0454	1.2825 - 1.8168	17	0.8840	10.16	0.9813	4.43	r
$C22:6$ ethyl ester $+ CO_2$	1.0125-1.0454	1.2825-1.8168	17	0.8821	11.24	0.9909	4.64	r
p-xylene + CClF <sub>3</sub>	1.0535-1.0535	0.6908-1.7270	8	0.9720	28.11	0.7511	8.45	t
acetone + CClF <sub>3</sub>	1.0535-1.0535	0.6873-1.6510	10	0.9736	14.89	0.8456	3.82	t
$CCl_4 + SF_6$	1.0292-1.0292	0.6806-1.9056	6	0.9758	17.63	0.8267	4.89	t
penzene $+$ SF <sub>6</sub>	1.0292-1.0292	0.6806-1.9056	8	0.9740	4.83	0.9555	4.26	t
coluene + SF <sub>6</sub>	1.0292-1.0292	0.5445-1.9056	11 51	0.9688	3.78	0.9378	2.61	t
p-xylene + SF <sub>6</sub>	0.8885 - 1.0610	0.6806-2.2459	51	0.9623	6.44	0.9085	5.98	t
${ m nesitylene+SF_6} \ { m naphthalene+SF_6} \ { m naphthalene+SF_6}$	1.0292 - 1.0292	0.6806 - 1.9056 $1.5089 - 2.0278$	10 5	0.9567 0.9643	6.37 8.36	0.9048 0.9635	4.34 8.35	t n
naphthaiene + SF <sub>6</sub> penzoic acid + SF <sub>6</sub>	$0.9984 - 1.0298 \\ 1.0292 - 1.0612$	1.2485-1.9065	5 6	0.9643	8.36 19.89	0.9635	8.33 2.33	n n
penzene + DMB	1.0464-1.0964	1.4320-1.9084	11	0.9777	2.07	0.7636	2.33 1.25	u u
oluene + DMB	1.0464-1.0964	1.4320-1.9084	10	0.9728	2.61	0.9478	1.33	u
naphthalene + DMB	1.0464-1.0964	1.4320-1.9084	9	0.9686	4.28	0.9287	1.20	u
phenanthrene + DMB	1.0464-1.0964	1.4320-1.9084	11	0.9415	6.48	0.8840	1.29	u
urgan + cyclobovene	0 5380 <u></u> 0 7504	Diffusion in 1	⊥iquids 7	0 0044	16.65	U 8483	5 72	17
argon + cyclohexane krypton + cyclohexane	$0.5380 - 0.7504 \\ 0.5380 - 0.7504$	2.4147 - 2.8666 $2.4147 - 2.8666$	7	$0.9944 \\ 0.9942$	8.56	0.8483 1.0828	5.73 5.20	V
enon + cyclohexane	0.5380 - 0.7504 $0.5380 - 0.7504$	2.4147-2.8666	7	0.9942	5.17	0.9755	4.72	V = V
nethane + cyclohexane	0.5380 - 0.7504 $0.5380 - 0.7504$	2.4147-2.8666	7	0.9941 $0.9940$	12.76	1.1388	1.95	V V
nethane + <i>n</i> -octane	0.5345 - 0.7648	2.4707-2.9945	9	0.9820	11.59	1.1102	1.69	W
ethane + <i>n</i> -octane	0.5345 0.7648	2.4707 2.9945	6	0.9741	10.29	1.0848	2.63	W
propane + <i>n</i> -octane	0.5345 0.7648	2.4707 2.9945	6	0.9685	9.19	1.0655	2.71	W
$CO_2$ + toluene	0.4951 - 0.5542	2.8676-2.9797	10	0.9736	4.00	1.0035	3.08	X
nydrogen + n-dodecane	0.5654-0.8617	2.1714-2.8896	8	0.9858	4.32	1.0173	3.55	y
CO + n-dodecane	0.4621-0.8617	2.1714-3.0743	9	0.9581	6.21	1.0044	5.38	y
$CO_2 + n$ -dodecane	0.4621-0.8617	2.1714-3.0743	9	0.9339	13.06	1.0727	2.77	y
$\frac{1}{n}$ ydrogen + $n$ -hexadecane	0.4476 - 0.7814	2.3924-3.1789	10	0.9712	26.34	0.7635	6.74	y
CO + <i>n</i> -hexadecane	0.4476 - 0.7814	2.3924-3.1370	10	0.9348	7.03	0.8757	5.63	y
$CO_2 + n$ -hexadecane	0.4476 - 0.7814	2.3924 - 3.1456	10	0.9054	8.96	0.9914	4.66	$\boldsymbol{y}$

				prediction		correla	ation	
system (solute $+$ solvent)	$T_{ m r2}$	$ ho_{ m r2}$	NDP	$A_{ m D}^{hh}$	AAD	$A_{\mathrm{D}}^{ii}$	AAD	ref
		Diffusion in Li	guids					
benzene + cyclohexane	0.5381 - 0.9441	1.7777 - 2.8962	12	0.9807	7.25	0.9120	5.55	z,aa
toluene + cyclohexane	0.5381 - 0.9441	1.7777 - 2.8962	12	0.9762	5.33	0.9256	3.17	z,aa
<i>m</i> -xylene + cyclohexane	0.5471 - 0.6013	2.7999 - 2.8962	4	0.9700	3.15	0.8567	2.24	aa
<i>p</i> -xylene + cyclohexane	0.5381 - 0.9441	1.7777 - 2.8666	8	0.9706	2.84	0.9452	1.33	aa
mesitylene + cyclohexane	0.5381 - 0.9441	1.7777 - 2.8962	12	0.9656	11.25	0.8654	4.88	z,aa
naphthalene + cyclohexane	0.5381 - 0.9441	1.7777 - 2.8962	12	0.9723	8.88	0.8907	4.80	z,aa
phenanthrene + cyclohexane	0.5381 - 0.9441	1.7777 - 2.8666	8	0.9465	9.10	0.8656	4.03	$\boldsymbol{z}$
acetone + cyclohexane	0.5471 - 0.6013	2.7999 - 2.8962	4	0.9722	0.59	0.9665	0.08	aa
benzene + $n$ -hexane	0.5872 - 1.0703	1.3481 - 3.0561	24	0.9727	6.32	0.9712	6.31	aa,bb,cc
toluene $+ n$ -hexane	0.5974 - 1.0703	1.3481 - 2.8637	20	0.9673	4.72	0.9672	4.72	aa,bb
m-xylene + $n$ -hexane	0.5974 - 0.6566	2.7521 - 2.8637	5	0.9602	6.67	0.8994	2.79	aa
p-xylene + $n$ -hexane	0.6171 - 1.0703	1.3481 - 2.8637	17	0.9608	5.56	0.9613	5.55	aa,bb
mesitylene $+ n$ -hexane	0.5974 - 1.0703	1.3481 - 2.8637	20	0.9551	5.48	0.9182	4.04	aa,bb
naphthalene $+ n$ -hexane	0.5974 - 1.0703	1.3481 - 2.8637	20	0.9628	4.65	0.9637	4.65	aa,bb
phenanthrene $+ n$ -hexane	0.6566 - 1.0703	1.3481 - 2.6705	15	0.9336	4.71	0.9141	4.79	$bb^{'}$
acetone $+ n$ -hexane	0.5974 - 0.6566	2.7521 - 2.8637	5	0.9627	3.23	0.9854	2.83	aa
carbon disulfide $+ n$ -hexane	0.5876 - 0.5876	2.8122 - 3.4665	10	0.9820	15.14	0.8518	2.84	dd
acetonitrile $+ n$ -hexane	0.5876 - 0.5876	2.8122 - 3.4674	7	0.9600	11.37	0.8603	3.87	dd
n-octane + $n$ -heptane	0.5534 - 0.8828	2.2353 - 2.9290	4	0.9439	4.87	0.9916	2.54	ee
n-decane + $n$ -heptane	0.5534 - 0.8828	2.2353 - 2.9290	5	0.9301	2.68	0.9287	2.64	ee
n-dodecane + $n$ -heptane	0.5534 - 0.8828	2.2353 - 2.9437	5	0.9157	2.99	0.9122	2.98	ee
n-tetradecane + $n$ -heptane	0.5534 - 0.8828	2.2353-2.9437	5	0.9146	7.39	0.8506	3.39	ee
n-hexadecane $+ n$ -heptane	0.5534 - 0.8828	2.1638 - 2.9437	8	0.8838	8.45	0.8140	3.06	ee
n-pentane + $n$ -octane	0.5345 - 0.7648	2.4707 - 2.9945	6	0.9571	8.30	1.0425	2.94	W
n-heptane + $n$ -octane	0.5345 - 0.7648	2.4707 - 2.9945	6	0.9439	3.79	0.9438	3.79	W
n-decane + $n$ -octane	0.5345 - 0.7648	2.4707 - 2.9945	6	0.9221	3.59	0.9402	3.47	W
<i>n</i> -dodecane + <i>n</i> -octane	0.5345 - 0.7648	2.4707 - 2.9945	6	0.9069	2.24	0.8993	2.22	W
n-tetradecane + $n$ -octane	0.5345 - 0.7648	2.4707 - 2.9945	6	0.9057	5.33	0.8591	2.50	W
benzene $+ n$ -octane	0.5331 - 0.5858	2.8882 - 2.9974	4	0.9619	13.11	1.1059	2.93	ff
toluene $+ n$ -octane	0.5331 - 0.5858	2.8882 - 2.9974	4	0.9556	9.76	1.0575	3.03	ff
p-xylene + $n$ -octane	0.5331 - 0.5858	2.8882 - 2.9974	4	0.9480	13.42	1.0948	1.10	ff
o-xylene + n-octane	0.5331 - 0.5858	2.8882 - 2.9974	4	0.9494	7.17	1.0226	0.88	ff
ethylbenzene $+ n$ -octane	0.5331 - 0.5858	2.8882 - 2.9974	4	0.9505	3.19	0.9506	3.19	ff
mesitylene $+ n$ -octane	0.5331 - 0.5858	2.8882 - 2.9974	4	0.9415	1.75	0.9289	1.07	ff
n-octane + $n$ -dodecane	0.4619 - 0.8599	2.1896 - 3.1067	9	0.9069	1.94	0.8918	0.81	ee
n-decane + $n$ -dodecane	0.4619 - 0.8599	2.1896 - 3.1067	5	0.8893	5.06	0.8460	2.13	ee
n-tetradecane + $n$ -dodecane	0.4619 - 0.8599	2.1896 - 3.1067	5	0.8700	17.75	0.7368	4.76	ee
n-hexadecane + $n$ -dodecane	0.4919 - 0.8599	2.1896 - 3.1067	5	0.8324	18.02	0.7035	3.72	ee
acetone $+ n$ -dodecane	0.4607 - 0.5214	3.0389 - 3.1519	5	0.9234	6.44	0.8673	1.44	aa
benzene $+ n$ -dodecane	0.4607 - 0.5062	3.0682 - 3.1519	4	0.9381	10.15	0.8514	1.18	aa
toluene + <i>n</i> -dodecane	0.4607 - 0.5062	3.0682 - 3.1519	4	0.9301	6.52	0.8727	1.84	aa
m-xylene + $n$ -dodecane	0.4607 - 0.5062	3.0682 - 3.1519	4	0.9199	8.33	0.8486	1.99	aa
mesitylene + n-dodecane	0.4607 - 0.5062	3.0682 - 3.1519	4	0.9127	13.92	0.8007	2.36	aa
napthalene + n-dodecane	0.4607 - 0.5214	3.0389 - 3.1519	5	0.9236	9.53	0.8426	2.12	aa
n-octane + $n$ -hexadecane	0.4476 - 0.7814	2.3962 - 3.1416	10	0.8734	2.15	0.8579	1.71	gg
n-decane + $n$ -hexadecane	0.4476 - 0.7814	2.3962 - 3.1382	5	0.8531	4.07	0.8239	3.25	gg
<i>n</i> -dodecane + <i>n</i> -hexadecane	0.4476 - 0.7814	2.3962 - 3.1382	5	0.8324	4.30	0.7976	2.29	gg
n-tetradecane + $n$ -hexadecane	0.4476 - 0.7814	2.3962 - 3.1382	5	0.8310	7.76	0.7703	2.74	gg

<sup>a</sup> Hu and Kobayashi (1970). <sup>b</sup> Takahashi and Hongo (1982). <sup>c</sup> Imotev and Tsekhanskaya (1964). <sup>d</sup> Lamb et al. (1989). <sup>e</sup> Umezawa and Nagashima (1992). <sup>f</sup>Swaid and Schneider (1979). <sup>g</sup>Suarez et al. (1993). <sup>h</sup>Sassiat et al. (1987). <sup>i</sup>Levelt Sengers et al. (1993). <sup>j</sup>Bueno et al. (1993). <sup>k</sup>Lai and Tan (1995). <sup>l</sup>Akgerman et al. (1996). <sup>m</sup>Catchpole and King (1994). <sup>n</sup>Debenedetti and Reid (1986). <sup>o</sup>Lauer et al. (1983), <sup>p</sup> Dahmen et al. (1990), <sup>q</sup> Nishiumi et al. (1996), <sup>r</sup> Liong et al. (1992), <sup>s</sup> Funazukuri et al. (1991), <sup>t</sup> Kopner et al. (1987), <sup>u</sup> Sun and Chen (1985a). <sup>v</sup> Chen et al. (1981). <sup>w</sup> Erkey and Akgerman (1989a). <sup>x</sup> Littel et al. (1992). <sup>y</sup> Matthews et al. (1987b). <sup>z</sup> Sun and Chen (1985b). <sup>aa</sup> Funazukuri et al. (1994). <sup>bb</sup> Sun and Chen (1985c). <sup>cc</sup> Erkey and Akgerman (1989b). <sup>dd</sup> Dymond and Woolf (1982). <sup>ee</sup> Matthews and Akgerman (1987). "Fan et al. (1995). & Matthews et al. (1987a). hh Values calcaulated with eq 48. "Fitted values.

Equation 32 is not expected to provide reliable values for  $F_{12}$  at low (<0.25) and high (>1.0) molecular size

Table 3 summarizes the MD simulations and the expressions proposed by various authors for  $F_{12}$ .

#### **Equations for the Tracer Diffusion Coefficients** of Lennard-Jones and Real Fluids

As in a previous paper on the self-diffusion coefficient (Ruckenstein and Liu, 1997), we employ as the basic equation for the tracer diffusion coefficient of a solute in a real fluid the following rough Lennard–Jones (RLJ) expression:

$$D_{12}^{\infty} = A_{\rm D} D_{12}^{\rm LJ}(\rho_2^*, T^*; \sigma_{12}^{\rm LJ}, \varepsilon_{12}^{\rm LJ}/k)$$
 (35)

where  $A_D$  is the coupling factor treated as a constant, rather than a temperature-dependent function as in the original RHS theory. The tracer diffusion coefficient in the LJ fluid,  $D_{12}^{LJ}$ , is given by (Rice and Gray, 1965)

$$D_{12}^{\rm LJ} = \frac{kT}{\xi_{12}^{\rm R} + \xi_{12}^{\rm S}} \tag{36}$$

where  $\xi_{12}^R$  is the friction coefficient due to repulsive interactions and  $\xi_{12}^S$  the friction coefficient due to the soft attractive interaction. For a hard sphere fluid,  $\xi_{12}^{\rm S} = 0$ , and eq 36 reduces to eq 4.

The equation for a LJ fluid can be obtained by transforming the equation for a HS fluid in two steps: (1) introducing the attractive contribution,  $\xi_{12}^{S}$ , and (2)

replacing the molecular diameter with an effective hard sphere diameter (EHSD), which is dependent on temperature or on both temperature and density. In this way, the following equation for the self-diffusion in a LJ fluid was obtained (Ruckenstein and Liu, 1997):

$$D_{22}^{\text{LJ}} = \frac{kT}{8/_{3}\rho_{2}\sigma_{\text{e}22}^{2}(\pi m_{2}kT)^{1/2} \left[\frac{g(\sigma_{\text{e}22})}{F_{22}(\rho_{\text{e}2}^{*})} + \frac{0.4}{T_{2}^{1.5}}\right]}$$
(37)

where  $T_2^* = kT \varepsilon_{22}^{\text{LJ}}$ ,  $F_{22}$  is given by eq 11, the radial distribution function is given by eq 33, and the reduced density,  $\rho_{e2}^*$ , is defined as The reduced EHSD,  $\sigma_{e22}^*$ , is

$$\rho_{\rm e2}^* = \rho_2^* \sigma_{\rm e22}^* = \rho_2^* \frac{\sigma_{\rm e22}}{\sigma_{\rm e22}^{\rm LJ}} \tag{38}$$

given by the Boltzmann expression for the repulsive Weeks-Chandler-Anderson LJ fluid (Weeks et al., 1971; Ben-Amotz and Herschbach, 1990):

$$\sigma_{\rm e22}^* = 1.1532 \left[ 1 + \left( \frac{T_2^*}{0.527} \right)^{1/2} \right]^{-1/6}$$
 (39)

The parameters in eq 39 were determined by Ben-Amotz and Herschbach (1990) by fitting the Carnahan—Starling—van der Waals equation of state to the Monte Carlo simulations for the LJ fluid.

Equation 37 provides excellent predictions for the self-diffusion in a LJ fluid (Ruckenstein and Liu, 1997). By using a similar approach, the following equation can be obtained from eq 36:

$$D_{12}^{\text{LJ}} = \frac{kT}{\frac{8}{3\rho_2 \sigma_{\text{e}12}^2 (2\pi m_{12} kT)^{1/2} \left[ \frac{g(\sigma_{\text{e}12})}{F_{12}} + \frac{0.4}{T_{12}^{*1.5}} \right]}}$$
(40)

where  $m_{12}$  is given by eq 6,  $g(\sigma_{e12})$  by eq 34,  $F_{12}$  by eq 32,  $T_{12}^*$  by eq 41, and  $\sigma_{e12}$  by eq 42.

$$T_{12}^* = kT/\varepsilon_{12}^{\text{LJ}} \tag{41}$$

$$\sigma_{\rm e12} = \sigma_{12}^{\rm LJ} \sigma_{\rm e2}^* = 1.1532 \sigma_{12}^{\rm LJ} \left[ 1 + \left( \frac{T_{12}^*}{0.527} \right)^{1/2} \right]^{-1/6} \tag{42}$$

In  $g(\sigma_{\rm e12})$  and  $F_{12}$ , the molecular diameters,  $\sigma_1$  and  $\sigma_2$ , for a HS fluid have been replaced by the EHSDs,  $\sigma_{\rm e1} = \sigma_{11}^{\rm LJ}\sigma_{\rm e11}^*(T_{11}^*)$  and  $\sigma_{\rm e2} = \sigma_{22}^{\rm LJ}\sigma_{\rm e22}^*(T_{22}^*)$ . The combining rules used in this paper for  $\sigma_{12}^{\rm LJ}$  and  $\varepsilon_{12}^{\rm LJ}/k$  are

$$\sigma_{12}^{LJ} = \frac{\sigma_{11}^{LJ} + \sigma_{22}^{LJ}}{2} \tag{43}$$

and

$$\varepsilon_{12}^{\text{LJ}}/k = \sqrt{(\varepsilon_{11}^{\text{LJ}}/k)(\varepsilon_{22}^{\text{LJ}}/k)} \tag{44}$$

Consequently, the only parameter left to be determined from diffusion data is the coupling factor,  $A_{\rm D}$ , introduced in eq 35. As pointed out in the last section, the expression for  $F_{12}$ , eq 32, is valid in limited ranges of molecular size and mass ratios:  $\sigma_1/\sigma_2$  from 0.25 to 1.0, and  $m_1/m_2$  from 1/5625 to 4.0. For some of the real systems considered in this paper the molecular size and mass ratios are outside these ranges; for this reason, a factor, which was empirically determined by fitting the

experimental data, has been introduced to make the final equation valid in wide ranges of molecular size and weight ratios. As a result, instead of eq 35, the final equation has the following form:

$$D_{12}^{\infty} = A_{\rm D} \left(\frac{\sigma_1}{\sigma_2}\right)^{0.42} \left(\frac{m_1}{m_2}\right)^{0.06} D_{12}^{\rm LJ} = A_{\rm D} \left(\frac{V_{\rm C1}}{V_{\rm C2}}\right)^{0.14} \left(\frac{M_1}{M_2}\right)^{0.06} D_{12}^{\rm LJ}$$
(45)

where  $D_{12}^{\text{LJ}}$  is given by eq 40 and  $V_{\text{C}}$  is the critical volume. For obtaining a predictive equation, the following equations (Ruckenstein and Liu, 1997) for the molecular size and energy parameters are employed:

$$\sigma_{ii}^{\rm LJ} = 0.7889 \, V_{\rm Ci}^{1/3} \tag{46}$$

$$\varepsilon_{ii}^{\mathrm{LJ}}/k = T_{\mathrm{C}}/1.2593 \tag{47}$$

where i=1 or 2 and  $T_{\rm C}$  is the critical temperature. In eq 46, the critical volume is expressed in cm<sup>3</sup>/mol and  $\sigma_{ii}^{\rm LJ}$  in Å.

Finally, we found that the coupling factor,  $A_D$ , in eq 45 can be correlated with the acentric factor using the following simple expression:

$$A_{\rm D} = 1 + c_1 \omega_{12} + c_2 \omega_{12}^{2} \tag{48}$$

where  $c_1$  and  $c_2$  are universal constants and the binary acentric factor,  $\omega_{12}$ , is calculated from those of the pure substances using the following combining rule:

$$\omega_{12} = \frac{\omega_1 + \omega_2}{2} \tag{49}$$

The combination of eqs 45–49 leads to a three-parameter corresponding-state expression for the infinitely dilute diffusion coefficients of a solute in a solvent:

$$D_{\rm r12} = D_{\rm r12}(T_{\rm r12}, \, \rho_{\rm r2}, \, \omega_{12}) \tag{50}$$

where  $D_{\rm r12} \equiv (M_2/RT_{\rm C2})^{1/2}(D_{12}^{\circ\prime}/V_{\rm C2}^{-1/3})$  is a dimensionless quantity,  $T_{\rm r12} = T/T_{\rm C12}$  with  $T_{\rm C12} = \sqrt{T_{\rm C1}}T_{\rm C2}$ , and  $\rho_{\rm r2} = \rho_2/\rho_{\rm C2}$ . The input data needed for the prediction are the critical temperature, critical volume, and the acentric factor for both solute and solvent.

#### **Calculation Results and Discussion**

Equation 45, combined with eqs 6,32–34,40–49, represents the working equation obtained in this paper. For testing the equation a large number of experimental data from literature have been used, namely 1443 points for 120 systems, including the diffusion coefficients in gases (157 data points for 6 systems), in supercritical fluids (793 data points for 51 systems), and in liquids (493 data points for 63 systems). The solutes and solvents involved vary from simple gases to complex heavy compounds.

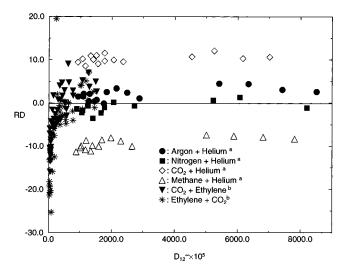
The critical constants and the acentric factor for pure compounds were obtained from Reid et al. (1987) when available there, or estimated by the methods suggested in the same reference (Reid et al., 1987) when not available there. The constants for the compounds considered in this paper are listed in Table 4.

For most systems, the values of the solvent densities can be found in the same references as those for the diffusion coefficient. For carbon dioxide and ethylene, when the density data were not reported in the same

**Table 6. Summary of the Calculation Results** 

			AAD				
system	$NS^a$	$NDP^b$	prediction	correlation			
diffusion in gases	6	157	6.42	4.18			
diffusion in SCFs	51	793	9.24	6.24			
diffusion in liquids	63	493	7.74	3.72			
total	120	1443	8.42	5.10			

<sup>a</sup> NS = number of systems. <sup>b</sup> NDP = number of data points.



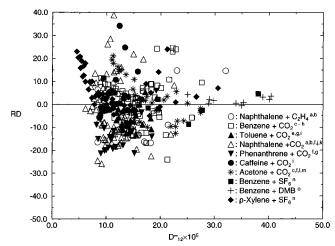
**Figure 5.** Relative deviations (RD =  $100(D_{12}^{\text{ccal}} - D_{12}^{\text{cexp}})/D_{12}^{\text{cexp}}$ ) for the diffusion coefficients of some solutes in gases. The  $D_{12}^{\text{cc}}$ 's (in cm²/s) are the experimental values. Data sources: (a) Hu and Kobayashi (1970); (b) Takahashi and Hongo (1982).

references as the diffusion coefficients, they were estimated using the equations of state of Pitzer and Schreiber (1988) and Jahangiri et al. (1986), respectively. Another equation for carbon dioxide is also available (Span and Wagner, 1996).

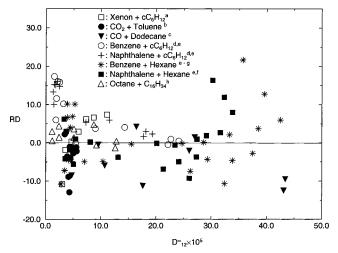
The systems, data sources, and the calculated results are listed in Table 5. For comparison purposes, both the predicted (using eq 48 to estimate  $A_{\rm D}$ ) and correlated (taking  $A_{\rm D}$  as an adjustable constant parameter for each pair) results are included in the table. For most systems, the values of the universal constants in eq 48 are  $c_1 = -0.0133$  and  $c_2 = -0.3662$ . For the data for alkanes (n- $C_5$ -n- $C_{14}$ ) + CO<sub>2</sub> taken from Umezawa and Nagashima (1992), different values had to be used:  $c_1 = 0.5$  and  $c_2 = 1.324$ . The same inconsistency was noted before for the same data when a different approach for diffusion coefficient in SCF systems (Liu and Ruckenstein, 1997) was employed. It is possible that the measured diffusion coefficients involved (Umezawa and Nagashima 1992) had a systematic error of 20%–30%.

Table 6 provides a summary of the deviations for each group of systems. Tables 5 and 6 show that, for all systems, the one-parameter equation, in which  $A_{\rm D}$  is determined from the diffusion data, provides an accurate correlation with deviations within the experimental uncertainties: the grand AAD for all the systems is only 5.1%. Higher deviations were observed for SCF systems, since the experimental uncertainties are higher for those cases (Liong et al., 1991). For most systems, the predictions based on eq 48 for  $A_{\rm D}$  are excellent with an AAD comparable to the experimental uncertainties. The grand AAD for all systems is only 8.42%.

As is well-known, the diffusion coefficient varies considerably from gaseous to liquid systems, typically from  $10^{-1}$  to  $10^{-5}$  cm<sup>2</sup>/s (see Figures 5–7). Nevertheless,



**Figure 6.** Relative deviations (RD =  $100(D_{12}^{ccal} - D_{12}^{cexp})/D_{12}^{cexp})$  for the diffusion coefficients of some solutes in supercritical fluids. The  $D_{12}^{\infty}$ 's (in cm²/s) are the experimental values. Data sources: (a) Imotev and Tsehanskaya (1964); (b) Lamb et al. (1989); (c) Umezawa and Nagashima (1992); (d) Swaid and Schneider (1979); (e) Suarez et al. (1993); (f) Sassiat et al. (1987); (g) Levelt Sengers et al. (1993); (h) Bueno et al. (1993); (i) Lai and Tan (1995); (j) Akgerman et al. (1996); (k) Lauer et al. (1983); (l) Dahmen et al. (1990); (m) Nishiumi et al. (1996); (n) Kopner et al. (1987); (o) Sun and Chen (1985a).



**Figure 7.** Relative deviations (RD =  $100(D_{12}^{\text{ocal}} - D_{12}^{\text{oexp}})/D_{12}^{\text{oexp}}$ ) for the diffusion coefficients of some solutes in liquids. The  $D_{12}^{\text{oc}}$ 's (in cm²/s) are the experimental values. Data sources: (a) Chen et al. (1981); (b) Littel et al. (1992); (c) Matthews et al. (1987b); (d) Sun and Chen (1985b); (e) Funazukuri et al. (1994); (f) Sun and Chen (1985c); (g) Erkey and Akgerman (1989b); (h) Matthews et al. (1987a).

eq 45 can represent the diffusion behavior over a wide range of densities,  $\rho_{\rm r2}$  from 0.036 to 3.15. Figures 5–7 show that the deviations for diffusion in gases, supercritical fluids, and liquids are small.

As noted in our previous paper (Ruckenstein and Liu, 1997), the self-diffusion coefficients are very sensitive to the molecular diameter,  $\sigma_{ii}^{\rm LJ}$  (i=1,2). For the binary systems considered here, it was found that the tracer diffusion coefficients are more sensitive to the molecular diameter of the solvent,  $\sigma_{22}^{\rm LJ}$ , than to that of the solute,  $\sigma_{11}^{\rm LJ}$ . Nevertheless, the sensitivity to  $\sigma_{22}^{\rm LJ}$  in the binary diffusion is lower than in the self-diffusion. Consequently, both the diameters of the solute and solvent can be estimated using eq 46. More accurate values for the critical volumes will be useful (Eaton and Akgerman, 1997), but as shown by Table 5, the sensitivity of

our equation to the value of the critical volume is not as high as that observed by Eaton and Akgerman (1997).

To avoid using the critical volume, some authors correlated the molecular diameter with the critical pressure and temperature (Hirschfelder et al., 1964; Tee et al., 1966). For this reason, we have also used the expression  $\sigma_{ii}^{\rm LJ} = 0.5058 (RT_{\rm C}/P_{\rm Ci})^{1/3}$  (Ruckenstein and Liu, 1997) in our prediction, but the deviations were higher.

Finally, as in the case of self-diffusion coefficients (Ruckenstein and Liu, 1997), the established equation leads to higher deviations (20%-30%) for hydrogenbonded solvents, such as alcohols and water.

#### **Conclusions**

A new expression was obtained for the tracer diffusion coefficient in hard sphere fluids on the basis of the molecular dynamics simulations of Herman and Alder (1972) and Alder et al. (1974). The expression is extended to a Lennard-Jones fluid by replacing the molecular sizes with the effective hard sphere diameters and by introducing the attractive contribution. Finally, the equation is extended to real fluids by introducing a coupling factor which accounts for the nonsphericity of the molecules. A large number of experimental data were employed to test the equation. It was shown that the obtained equation can be used with high accuracy to predict the tracer diffusion coefficients of various solutes in gases, supercritical fluids, and liquids.

The input data needed for prediction are the critical temperature, critical volume, and the acentric factor for pure compounds. Simple combining rules were used for the binary interaction parameters,  $\sigma_{12}^{\rm LJ}$  and  $\varepsilon_{12}^{\rm LJ}/k$ , and the only parameter which was determined from tracer diffusion data was the coupling factor  $A_D$ . This factor could be expressed in terms of the acentric factor  $\omega_{12}$ . For those systems for which reliable critical constants, especially the critical volume, are available, the prediction accuracy was comparable to the experimental one. The results obtained show that a three-parameter  $(T_{\rm C}, V_{\rm C}, \omega)$  expression is applicable to all systems with somewhat higher deviations for hydrogen-bonded solvents.

#### Nomenclature

CO = carbon monoxide

a =parameter given by eq 27c  $a_0$ ,  $a_1$ ,  $a_2$  = parameters in eqs 22-25  $A_{\rm D} = {\rm translational-rotational}$  coupling factor AAD = absolute average deviation, AAD  $(100/\text{NDP})\sum_{i=1}^{\text{NDP}}|(D_{12i}^{\text{ocal}}-D_{12i}^{\text{eexp}})/D_{12i}^{\text{eexp}}|$   $b_i = \text{constants in eq } 32 \text{ given in Table } 2$  $b_{12}$  = parameter given by eq 27b  $c_1$ ,  $c_2$  = constants in eq 48 (for most systems,  $c_1$  = -0.0133and  $c_2 = -0.3662$ ; for *n*-alkanes + CO<sub>2</sub>,  $c_1 = 0.5$  and  $c_2$ = 1.324) C4:0 = butyric acidC8:0 = caprylic acidC10:0 = capric acidC14:0 = myristic acidC16:0 = palmitic acidC18:0 = stearic acidC22:0 = behenic acidC18:2 = linoleic acidC20:5 = eicosapentaenoic acid C22:6 = docosahexaenoic acid  $C_2H_2$  = ethylene  $C_{16}H_{34} = n$ -hexadecane  $CCl_4 = carbon\ tetrachloride$  $CClF_3 = chlorotrifluoromethane$ 

 $CO_2$  = carbon dioxide  $D = diffusion coefficient, cm^2/s$ DMB = 2,3-dimethylbutane  $D_{12}^{\infty}$  = tracer diffusion coefficient of solute 1 in solvent 2, EHSD = effective hard sphere diameter  $F_{12}$ ,  $F_{22}$  = functions defined by eqs 4 and 9, respectively  $g(\sigma_{22}), g(\sigma_{12}) = \text{radial distribution function at contact for}$ pure solvent and infinitely dilute binary system, respectively  $G_i$  = functions defined in Table 2  $h_{12} =$ function in eq 24 HS = hard sphere $k = \text{Boltzmann constant}, 1.38048 \times 10^{-23}, \text{J/K}$ LJ = Lennard-Jones $I_{12}$  = binary interaction parameter in eq 2 m =mass of a molecule, kg M =molecular weight, g/mol MD = molecular dynamics  $N_{\rm a} = \text{Avogadro number, } 6.023 \times 10^{23}, \, \text{mol}^{-1}$ NDP = number of data points NS = number of systemsP = pressure, bar R = gas constantRLJ = rough Lennard-Jones RD = relative deviation, RD =  $100(D_{12}^{\text{ceal}} - D_{12}^{\text{cexp}})/D_{12}^{\text{cexp}}$ RHS = rough hard sphere  $SF_6 = sulfur hexafluoride$ T = temperature, K $T_{\rm r} = T/\hat{T}_{\rm C}$  $V = \text{molar volume, cm}^3/\text{mol}$  $V_0 = \text{closed-packed molar volume}, N_a \sigma_{22}^3 / \sqrt{2}, \text{cm}^3 / \text{mol}$ Greek Letters  $\alpha$  = parameter given by eq 27a

 $\eta_2 = \pi \rho_2 \sigma_{22}^3 / 6$  for HS fluid,  $\pi \rho_2 \sigma_{e22}^2 / 6$  for LJ and real fluids  $\Delta$  = function defined by eq 31  $\varepsilon^{\text{LJ}}/k$  = Lennard–Jones energy parameter, K  $\chi = \rho_2^{*0.01}$  $\gamma$ ,  $\gamma_{\rm B}$  = quantities introduced in eq 15  $\mu_2$  = viscosity of the HS solvent  $\mu_0$  = viscosity of gas at very low density  $\omega = acentric factor$  $\rho_2$  = number density of solvent,  $N_a/V_2$  $\rho_{\rm r} = \rho/\rho_c$  $\sigma^{LJ}$  = Lennard–Jones molecular diameter parameter, cm  $\xi$  = friction coefficient

### Subscripts

1 or 11 = solute2 or 22 = solvent12 = binary propertyC = critical constante = effective hard sphere diameter r = reduced quantity

#### Superscripts

\* = reduced property  $\infty = infinite dilution for solute$ 0 = solventE = Enskog theoryH = hydrodynamic contribution HS = hard sphere fluid LJ = Lennard-Jones fluid R = repulsive contribution

S = soft attractive contribution

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