Obtención Verosimilitud

$$x_1, x_2, \dots x_n \sim N(0, \sigma^2) \rightarrow f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-x^2}{2\sigma^2}}$$

$$L(\sigma^2; x_1, x_2, \dots, x_n) = \prod_{i=1}^{150} f(x_i; \sigma^2) = \prod_{i=1}^{150} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-x_i^2}{2\sigma^2}}$$

$$= \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^{150} e^{-\sum_{i=1}^{150} x_i / 2\sigma^2}$$

$$\log\left(L(\sigma^2; x_1, x_2, \dots x_{150})\right) = 150 \cdot \log\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) + \frac{-\sum_{i=1}^{150} x_i^2}{2\sigma^2}$$

Obtención de parámetros gamma Inversa

Si $X \sim$ Gamma Inversa $\Rightarrow f(x) = \dfrac{\beta^{\alpha}}{\Gamma(\alpha)} x^{-\alpha-1} e^{-\frac{\beta}{x}}$

$$E(x) = \frac{\beta}{\alpha - 1} \quad y \quad \sigma^2 = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)}$$

$$\Rightarrow \beta = E(x)(\alpha-1) \qquad \sigma^2 = \frac{E(x)^2(\alpha-1)^2}{(\alpha-1)^2(\alpha-2)} = \frac{E(x)^2}{\alpha-2} \therefore \alpha = \frac{E(x)^2}{\sigma^2} + 2$$

Nota: en R $\beta$=rate
$\alpha$=shape.

Obtencion de Posterior

$$x_1, x_2, x_3, \ldots, x_{150} \sim N(0, \sigma^2) \Rightarrow f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$$

$$P(x_1, x_2, \ldots x_{150} | \sigma^2) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^{150} e^{-\sum_{i=1}^{150} x_i^2 / 2\sigma^2} = \frac{(\sigma^2)^{-\frac{150}{2}}}{(\sqrt{2\pi})^{150}} e^{-\sum_{i=1}^{150} x_i^2 / 2\sigma^2}$$

Por otro lado:

$$P(\sigma^2) = \frac{\beta^\alpha}{\Gamma(\alpha)} (\sigma^2)^{-\alpha-1} e^{-\frac{\beta}{\sigma^2}}$$

Calculamos la posterior

$$P(\sigma^2 | x_1, x_2, \ldots x_{150}) \propto \frac{(\sigma^2)^{-75}}{(\sqrt{2\pi})^{150}} e^{-\sum_{i=1}^{150} x_i^2 / 2\sigma^2} \frac{\beta^\alpha}{\Gamma(\alpha)} (\sigma^2)^{-\alpha-1} e^{-\beta/\sigma^2}$$

$$\propto (\sigma^2)^{-\alpha-76} e^{-\frac{\sum_{i=1}^{150} x_i^2 + 2\beta}{2\sigma^2}}$$

$$\propto (\sigma^2)^{-(\alpha+75)-1} e^{-\frac{\frac{\sum_{i=1}^{150} x_i^2 + 2\beta}{2}}{\sigma^2}}$$

$$\propto \text{gammainv}\left(\alpha' = \alpha + 75, \beta' = \frac{\sum_{i=1}^{150} x_i^2 + 2\beta}{2}\right)$$

Posterior para 2.3

$$X_1, X_2, \dots, X_{150} \sim N(0, \sigma^2) \Rightarrow f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2}$$

$$\tau = \log(\sigma) \Rightarrow \sigma = e^{\tau}$$

$$\therefore f(x) = \frac{1}{e^{\tau}\sqrt{2\pi}} e^{-x^2/2e^{2\tau}} = \frac{1}{\sqrt{2\pi}} e^{-x^2/2e^{2\tau} - \tau}$$

$$L(\tau | x_1, x_2, \dots, x_{150}) = \left(\frac{1}{\sqrt{2\pi}}\right)^{150} e^{-\frac{\Sigma x_i^2}{2e^{2\tau}} - 150\tau}$$

$$\log(L(\tau | x_1, x_2, \dots, x_{150}) = 150 \log\left(\frac{1}{\sqrt{2\pi}}\right) - \frac{\sum_{i=1}^{150} x_i^2}{2e^{2\tau}} - 150\tau$$

Tomando la misma ~~posterior~~ inicial (gamma inversa)

$$P(\sigma^{-2}) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} (\sigma^{-2})^{-\alpha-1} e^{-\beta/\sigma^2} = \frac{\beta^{\alpha}}{\Gamma(\alpha)} (e^{2\tau})^{-\alpha-1} e^{-\beta/e^{2\tau}} = P(\tau)$$

$$P(\tau | x_1, x_2, \dots, x_{150}) \propto \left(\frac{1}{\sqrt{2\pi}}\right)^{150} e^{-150\tau} e^{-\frac{\Sigma x_i^2}{2e^{2\tau}}} \frac{\beta^{\alpha}}{\Gamma(\alpha)} (e^{2\tau})^{-\alpha-1} e^{-\beta/e^{2\tau}}$$

$$\propto (e^{2\tau})^{-75} (e^{2\tau})^{-\alpha-1} e^{-\frac{\Sigma x_i^2/2 + \beta}{e^{2\tau}}} = (e^{2\tau})^{-(\alpha+75)-1} e^{-\frac{\frac{\Sigma x_i^2}{2} + \beta}{e^{2\tau}}}$$

si $V = e^{2\tau}$ $\quad P(V) \sim \text{gamma inv}\left(\alpha + 75, \frac{\Sigma x_i^2}{2} + \beta\right)$