pnln101-500: bitflip_error_template

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Given an initial 1-qubit state:

$$|\psi\rangle = a|0\rangle + \sqrt{1-a^2}\,|1\rangle$$

we shell encode it in 3 -qubits as follows:

$$\left|\tilde{\psi}\right\rangle = |\psi\rangle \otimes |00\rangle = a|000\rangle + \sqrt{1-a^2}\,|100\rangle$$

$$\Downarrow CX_{0\to 1}$$

$$\left|\tilde{\psi}\right\rangle = a|000\rangle + \sqrt{1-a^2}\,|110\rangle$$

$$\Downarrow CX_{0\to 2}$$

$$\left|\tilde{\psi}\right\rangle = a|000\rangle + \sqrt{1-a^2}\,|111\rangle$$

$$\Downarrow BF$$

$$\left|\tilde{\psi}\right\rangle = \begin{cases} a|000\rangle + \sqrt{1-a^2}\,|111\rangle & 1-p_1-p_2-p_3 \\ a|100\rangle + \sqrt{1-a^2}\,|011\rangle & p_1 \\ a|010\rangle + \sqrt{1-a^2}\,|101\rangle & p_2 \\ a|001\rangle + \sqrt{1-a^2}\,|110\rangle & p_3 \end{cases}$$

Now we perform a sequence of unitary transformations as follows:

$$\Downarrow CX_{0\to 1}$$

$$\left|\tilde{\psi}\right\rangle = \begin{cases} a|000\rangle + \sqrt{1-a^2}\,|101\rangle & 1-p_1-p_2-p_3 \\ a|110\rangle + \sqrt{1-a^2}\,|011\rangle & p_1 \\ a|010\rangle + \sqrt{1-a^2}\,|111\rangle & p_2 \\ a|001\rangle + \sqrt{1-a^2}\,|100\rangle & p_3 \end{cases}$$

$$\Downarrow CX_{0\to 2}$$

$$\left|\tilde{\psi}\right\rangle = \begin{cases} a|000\rangle + \sqrt{1-a^2}\,|100\rangle & 1-p_1-p_2-p_3 \\ a|111\rangle + \sqrt{1-a^2}\,|011\rangle & p_1 \\ a|010\rangle + \sqrt{1-a^2}\,|110\rangle & p_2 \\ a|001\rangle + \sqrt{1-a^2}\,|101\rangle & p_3 \end{cases}$$

$$\Downarrow Toffoli_{(1,2)\to 0}$$

$$\left|\tilde{\psi}\right\rangle = \begin{cases} a\left|000\right\rangle + \sqrt{1-a^2}\left|100\right\rangle & 1-p_1-p_2-p_3 \\ a\left|011\right\rangle + \sqrt{1-a^2}\left|111\right\rangle & p_1 \\ a\left|010\right\rangle + \sqrt{1-a^2}\left|110\right\rangle & p_2 \\ a\left|001\right\rangle + \sqrt{1-a^2}\left|101\right\rangle & p_3 \end{cases} = \begin{cases} \left|\psi\right\rangle \otimes \left|00\right\rangle & 1-p_1-p_2-p_3 \\ \left|\psi\right\rangle \otimes \left|11\right\rangle & p_1 \\ \left|\psi\right\rangle \otimes \left|10\right\rangle & p_2 \\ \left|\psi\right\rangle \otimes \left|01\right\rangle & p_3 \end{cases}$$

Returning from the 'circuit' function the reduced density matrix on the two ancila qubits (1 & 2), turning to be:

$$p_0 = \mathrm{Tr}_0\left[\tilde{\rho}_{final}\right](0,0)$$

$$p_1 = \mathrm{Tr}_0\left[\tilde{\rho}_{final}\right](3,3)$$

$$p_2 = \mathrm{Tr}_0\left[\tilde{\rho}_{final}\right](2,2)$$

$$p_3 = \mathrm{Tr}_0\left[\tilde{\rho}_{final}\right](1,1)$$