HERCULES 2021 - TUTORIAL

Modelling SR beamlines using OASYS

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Outline

- Introduction to OASYS
- XOPPY toolbox: Source emission and characteristics of optical elements. Power transport
- ShadowOUI: Ray tracing SR beamlines
  - Mirror aberrations
  - Crystal systems
  - Lenses
- WOFRY and SRW: coherent beam transport:
  - Simple examples with WOFRY
  - Introduction to Partial coherence

Outcome

Calculate main characteristics of synchrotron sources (dipoles and IDs)

Simulating beamline optics by ray-tracing to obtain main parameters of beam size, energy resolution and flux

Understand basic principles of x-ray optics: Reflective (aberrations, slope errors), Diffractive (dispersion) and Refractive (chromatic aberrations)

A few concepts about coherence

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The OASYS Project

https://oasys-kit.github.io/

Luca Rebuffi   Manuel Sanchez del Rio

- ✓ OASYS = OrAnge SYnchrotron Suite
- ✓ A common platform to build synchrotron-oriented User Interfaces ***that communicate***
- ✓ The upper layer of the application presented to the user
- ✓ Open Source & Python technology

Luca Rebuffi, Manuel Sanchez del Rio (2017)
OASYS (OrAnge SYnchrotron Suite) : an open-source graphical environment for x-ray virtual experiments
Proc.SPIE 10388: 10388-10388. http://dx.doi.org/10.1117/12.2274263

Synchrotron Virtual Experiments

Beamline Experiment Chain

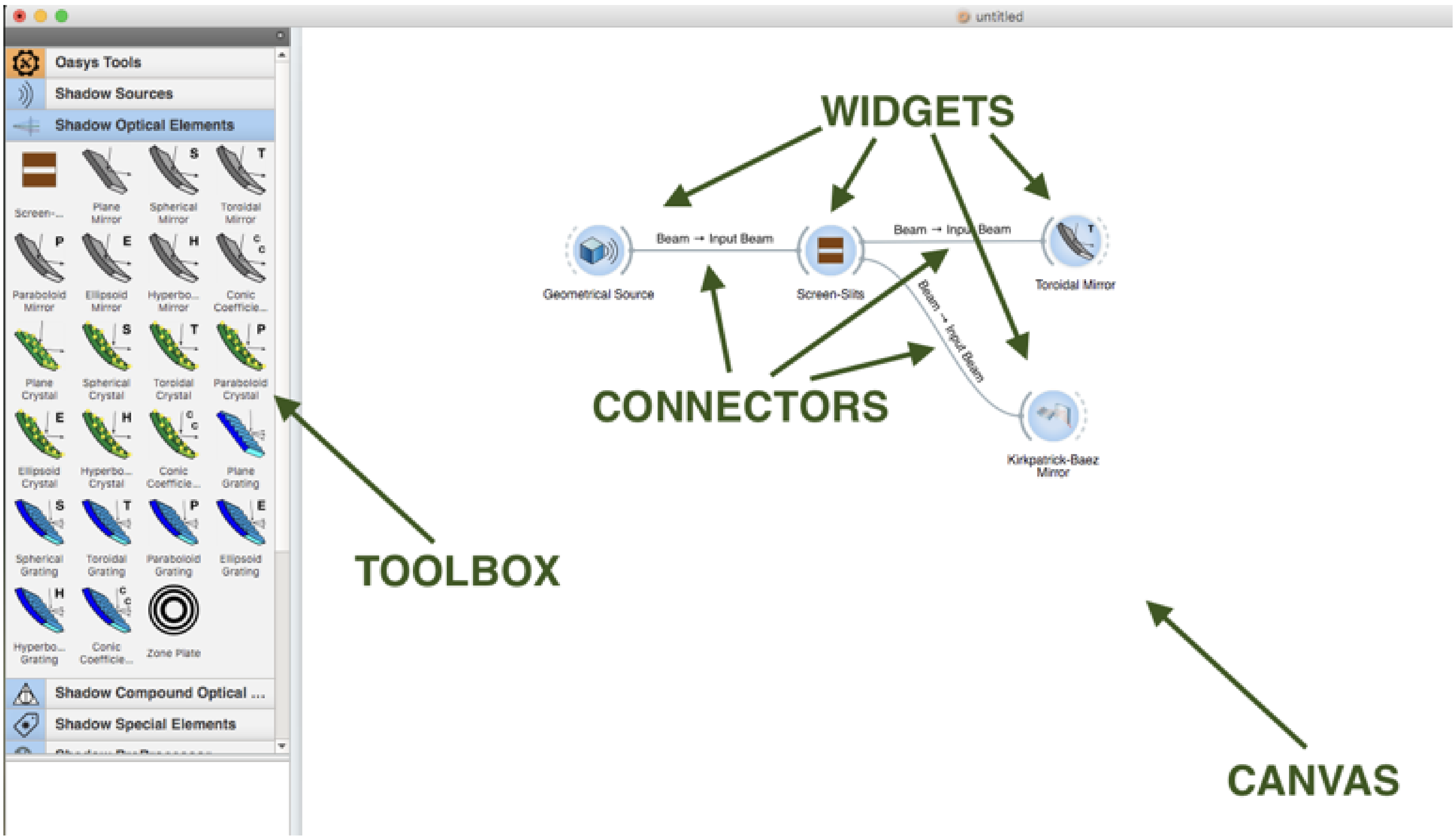
Storage Ring (e^- optics) → Radiation devices (e^- -> γ) → Beamline (γ optics) → Sample (γ –matter interactions)

Upper layer based on ORANGE (U Ljubljana)

Suite of Add-ons

XOPPY | ShadowOui | WOFRY | SRW | X-ray server | WISE | COMSYL

The OASYS language



Multiple tools in the same environment

L. Rebuffi & M. Sanchez del Rio, Proc. SPIE 10388, 103880S (2017)

X. Shi et al., J. Synchrotron Rad. 21, 669 (2014)

L. Rebuffi & M. Sanchez del Rio, J. Synchrotron Rad. 23, 1357 (2016)

M. Sanchez del Rio et al., J. Synchrotron Rad. 23, 665 (2016)

References

Official Web Page

https://www.aps.anl.gov/Science/Scientific-Software/OASYS
or https://oasys-kit.github.io/

OASYS Publications

M. Sanchez del Rio, L. Rebuffi, J. Demšar, N.Canestrari and O. Chubar, *A proposal for an open source graphical environment for simulating X-ray optics*, Proc. SPIE 9209, 92090X (2014)

X. Shi, R. Reininger, M. Sanchez del Rio, L. Assoufid, *A hybrid method for X-ray optics simulation: combining geometric ray-tracing and wavefront propagation*, J. Synchrotron Rad. 21, 669 (2014)

X. Shi, R. Reininger, M. Sanchez del Rio, J. Qian, L. Assoufid, *X-ray optics simulation and beamline design using a hybrid method: diffraction-limited focusing mirrors*, Proc. SPIE, 9209, 920909 (2014)

M. Sanchez del Rio, D. Bianchi, D. Cocco, M. Glass, M. Idir, J. Metz, L. Raimondi, L. Rebuffi, R. Reininger, X. Shi, F. Siewert, S. Spielmann-Jaeggi, P. Takacs, M. Tomasset, T. Tonnessen, A. Vivo and V. Yashchuk, *DABAM: an open-source database of X-ray mirrors metrology*, J. Synchrotron Rad. 23 (2016).

L. Rebuffi, M. Sanchez del Rio, *ShadowOui: A new visual environment for X-ray optics and synchrotron beamline simulations*, J. Synchrotron Rad. 23 (2016)

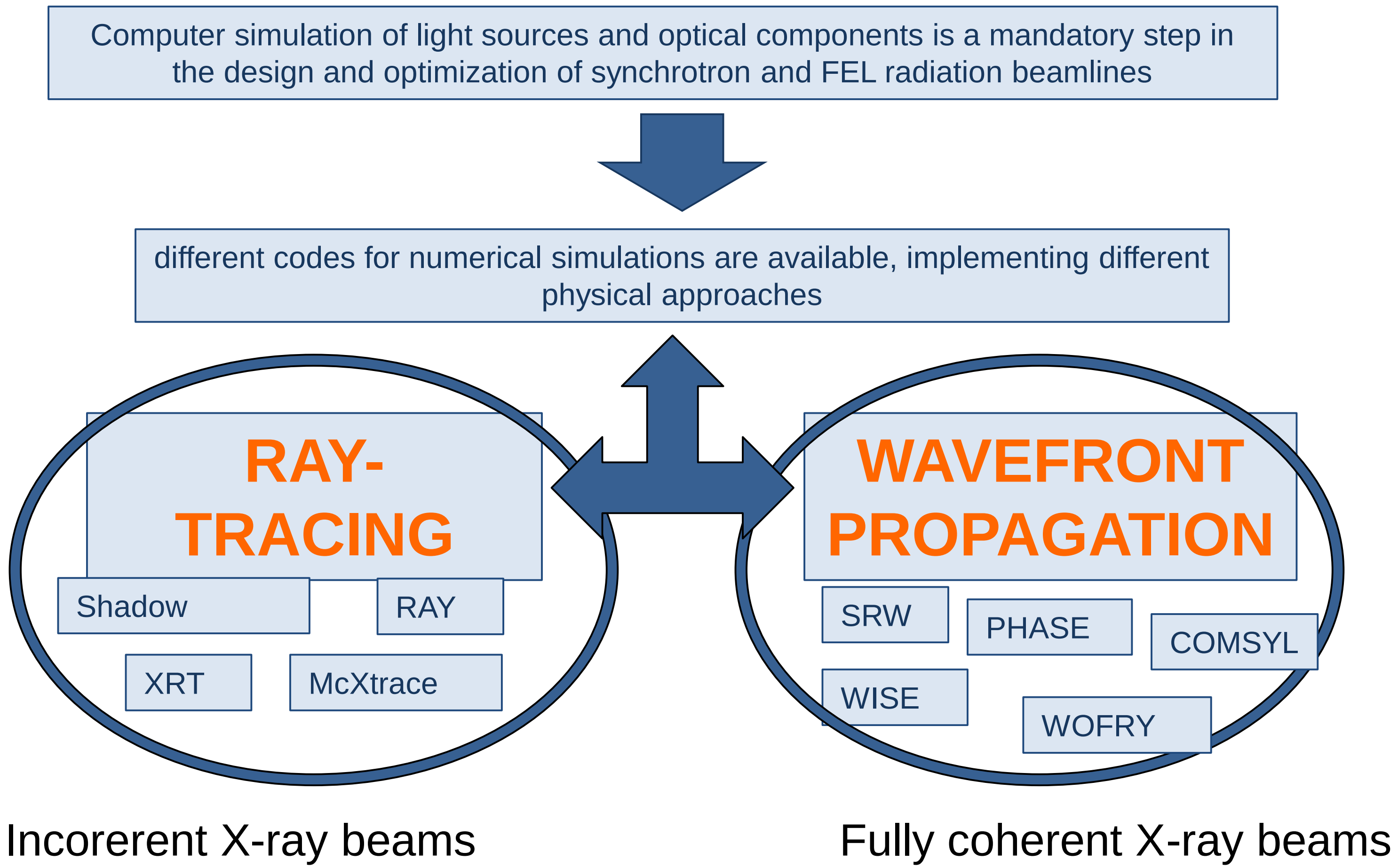
L. Rebuffi, M. Sanchez del Rio, *Interoperability and complementarity of simulation tools for beamline design in the OASYS environment*, Proc. SPIE 10388, 1038808 (2017).

L. Rebuffi, M. Sanchez del Rio, *OASYS (OrAnge SYnchrotron Suite): an open-source graphical environment for x-ray virtual xperiments*, Proc. SPIE 10388, 103880S (2017).

Three sessions

- XOPPY (tools) hercules_xoppy.ows
- ShadowOUI (ray tracing) hercules_shadowoui.ows
- WOFRY (wave optics) hercules_wofry.ows

X-ray optics for synchrotron and XFEL beamlines

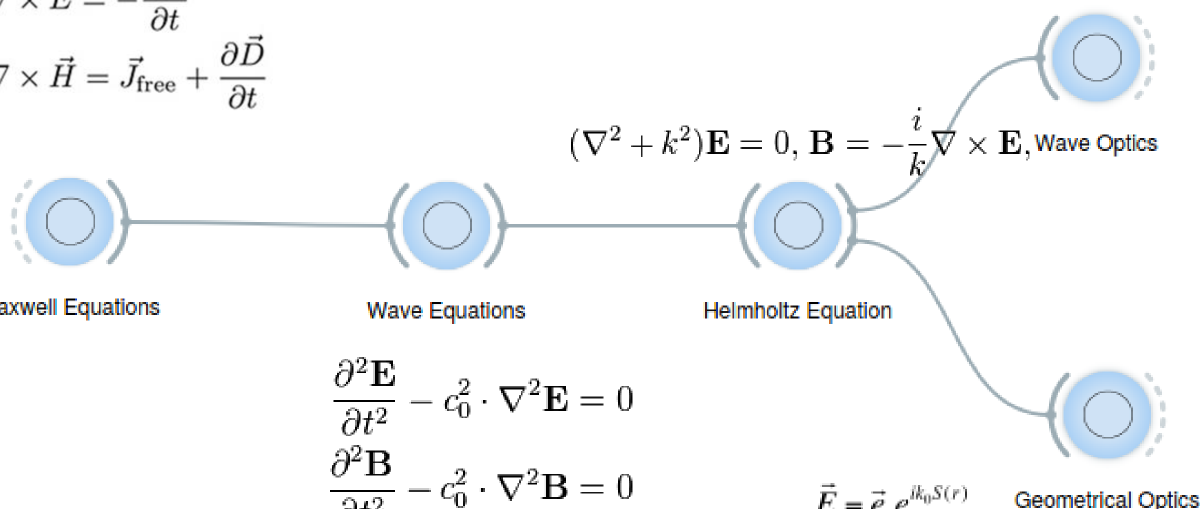


Propagation of one wavefront. One wavefront is coherent by nature.

$$\nabla \cdot \vec{D} = \rho_{\text{free}}$$
$$\nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\nabla \times \vec{H} = \vec{J}_{\text{free}} + \frac{\partial \vec{D}}{\partial t}$$

$$\int \phi_n(\mathbf{r}_1', \omega) G(\mathbf{r}_1, \mathbf{r}_1', \omega)\, \mathrm{d}^2 r_1'$$

$$(\nabla^2 + k^2)\mathbf{E} = 0,\ \mathbf{B} = -\frac{i}{k}\nabla \times \mathbf{E},$$



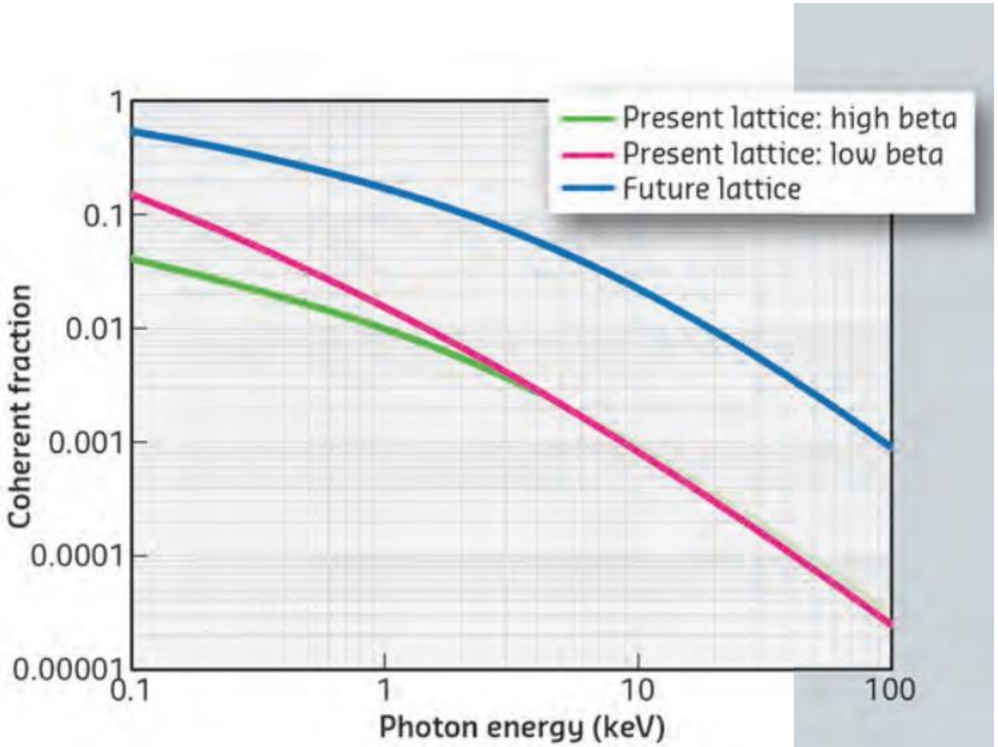
$$\frac{\partial^2 \mathbf{E}}{\partial t^2} - c_0^2 \cdot \nabla^2 \mathbf{E} = 0$$
$$\frac{\partial^2 \mathbf{B}}{\partial t^2} - c_0^2 \cdot \nabla^2 \mathbf{B} = 0$$

$$\vec{E} = \vec{e}\, e^{ik_0 S(r)}$$
$$\vec{H} = \vec{h}\, e^{ik_0 S(r)}$$
$$(\nabla S)^2 = n^2$$
$$\nabla S = n\,\vec{s}$$
$$\frac{d}{ds}\left(n \frac{d\vec{r}}{ds}\right) = \nabla n$$
$$\nabla n = 0 \Rightarrow \frac{d\vec{r}}{ds} = 0 \Rightarrow \vec{r} = s\vec{a} + \vec{b}$$

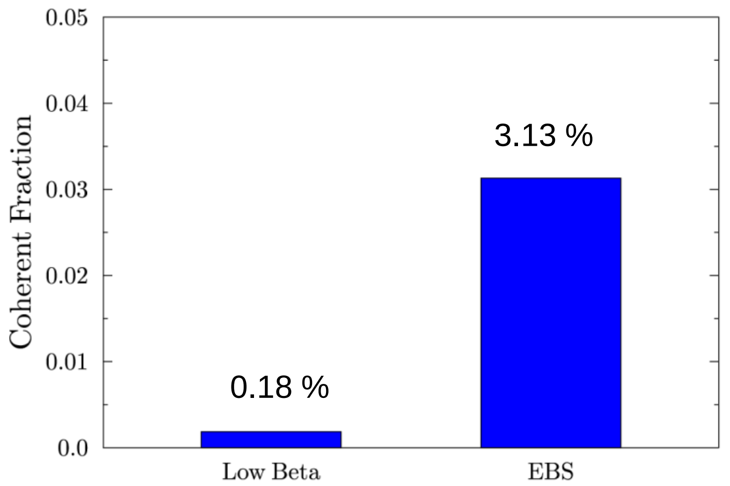
A wavefront is approximated by a ray. Many rays are summed up to form a beam, that is incoherent.

AT HIGH ENERGIES, WE ARE FAR FROM DIFFRACTION-LIMIT (=FULLY COHERENCE)

THEREFORE, ANY BEAMLINE SIMULATION MUST START WITH RAY TRACING (INCOHERENT BEAMS)

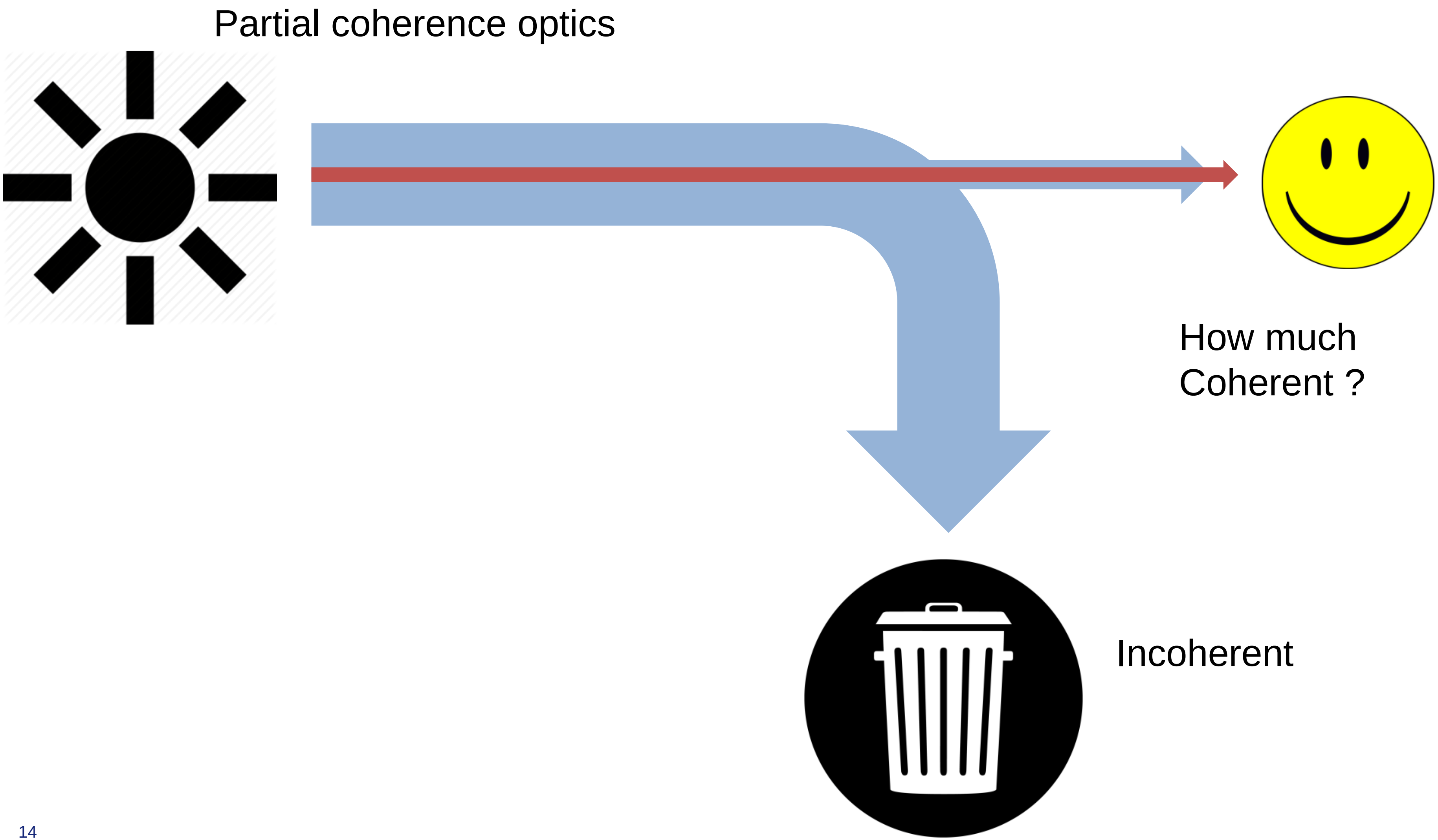


Source: ESRF Orange book



COMSYL calculation
U17 2m @ 17 keV (K=0.4842) L=2m

Typical coherence beamline



Hierarchical approach to study beamline optics

- Hand calculations (sizes divergences and distances define magnification, optical element reflectivities)
- Ray tracing (starting from point sources, monochromatic beams, ideal optics, etc. then integrate finite sizes, polychromaticity, real optics with surface errors)
- If coherence is important, apply wave optics
  - Hybrid method: corrections to ray tracing
  - 1D prototyping (fast)
  - 2D full simulation
  - Partial coherence (multielectron, coherent mode decomposition)

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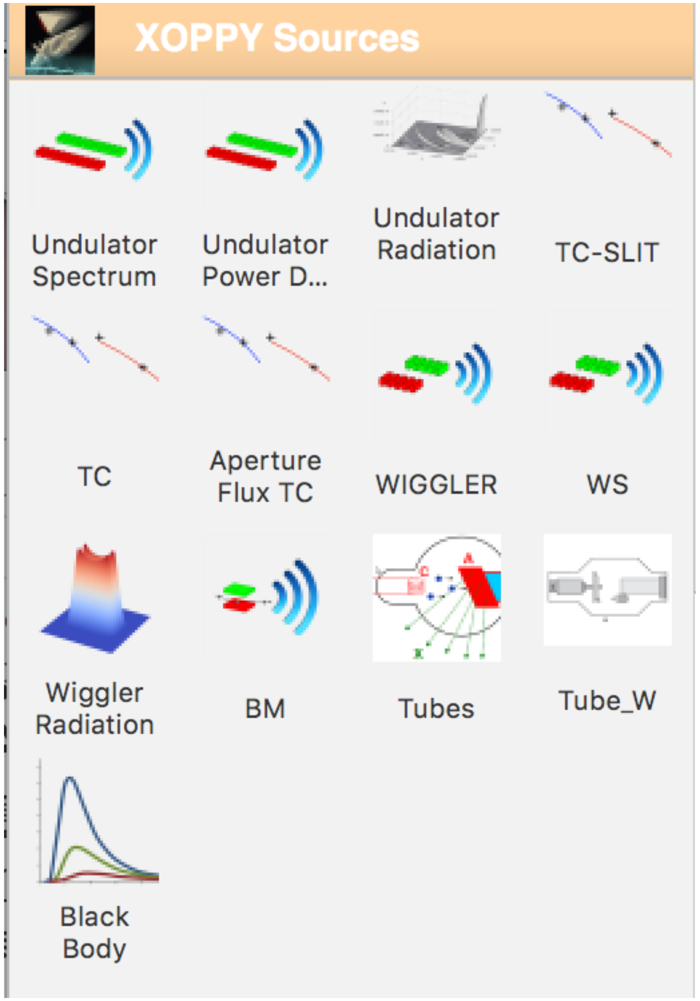
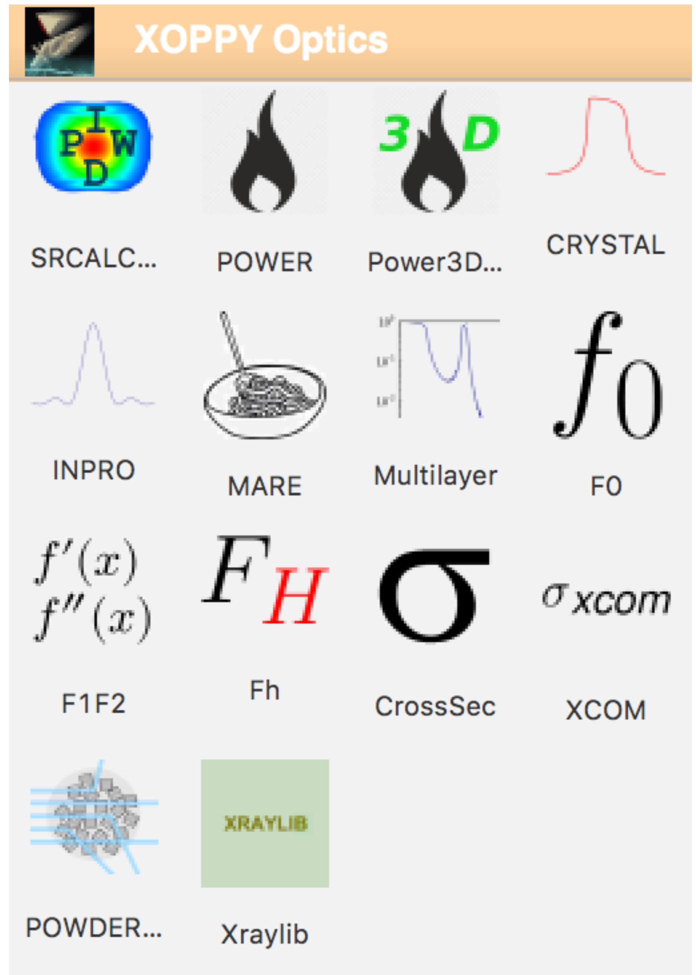
Source emission (XOPPY)

- Undulator spectrum (flux or power versus E)
- Undulator power density (power versus X,Y)
- Undulator Spectrum (flux/power vs X,Y,E)
- Undulator tuning curves
- Wiggler spectrum Spectrum (flux/power vs X,Y,E)
- BM Spectrum (flux/power vs E, angle)
- Others



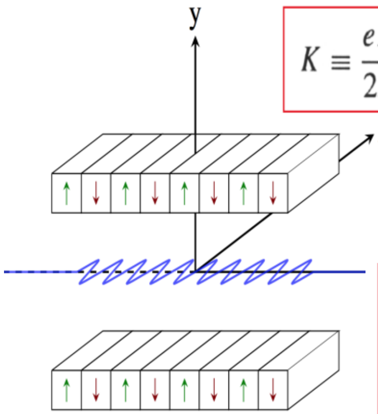
Optical elements (XOPPY)

- Attenuators
- Mirrors
- Multilayers
- Crystals



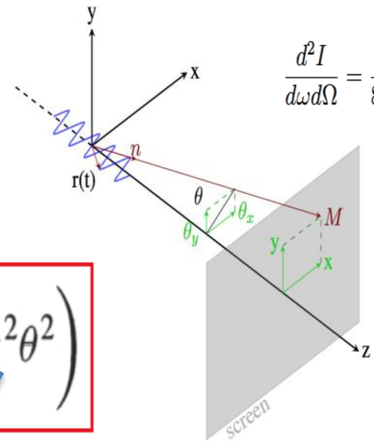
Undulator Memorandum

Undulator emission, after classical electrodynamics (e.g., Jackson, etc)



$$\boxed{K \equiv \frac{eB_0\lambda_u}{2\pi mc} = 0.9337 B_0(\mathrm{T})\lambda_u(\mathrm{cm})}$$

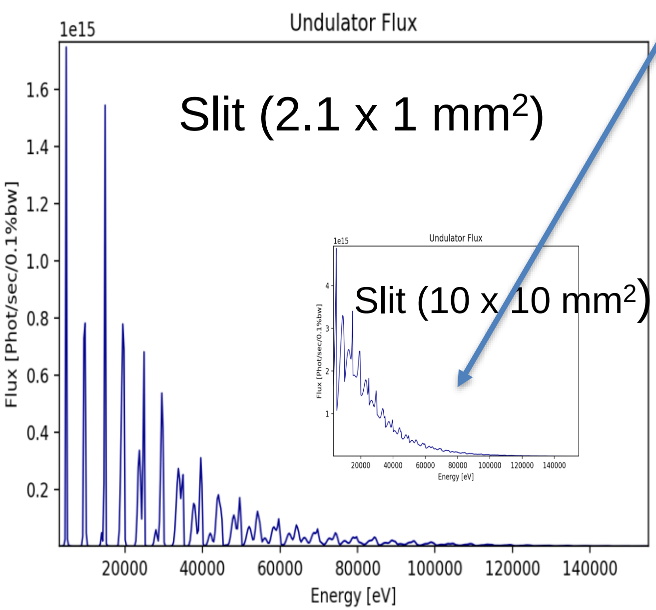
$$\boxed{\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)}$$

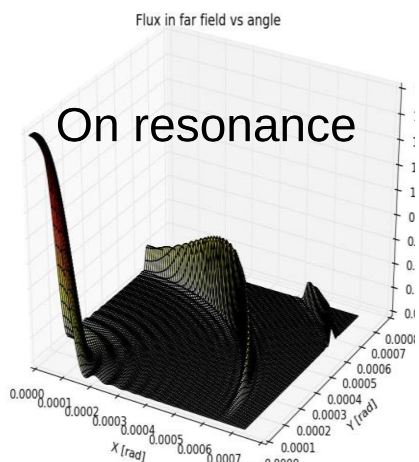


$$\frac{d^2I}{d\omega d\Omega} = \frac{eI}{8\pi^2 c\epsilon_0 h}10^{-9}\left|\int_{-\infty}^{\infty}\left[\frac{n\times\left[(n-\beta)\times\dot{\beta}\right]}{(1-\beta\cdot n)^2} + \frac{c}{\gamma^2 R}\frac{(n-\beta)}{(1-\beta\cdot n)^2}\right]e^{i\omega(t'+R(t')/c)}dt'\right|^2$$

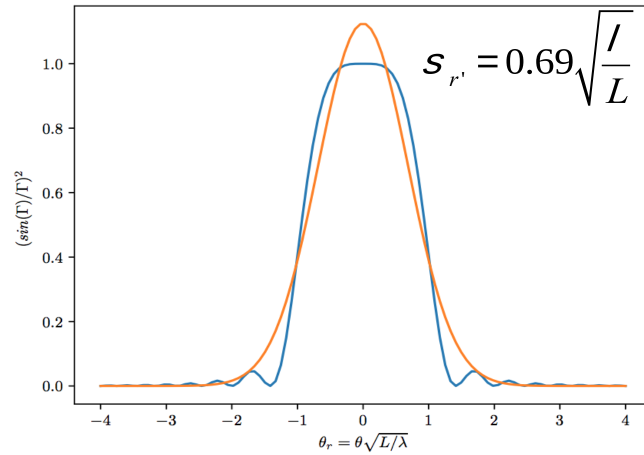
$$E = \gamma m_e c^2$$

$m_e c^2$ = 0.51 MeV





On resonance approximation



$$s_{r'} = 0.69\sqrt{\frac{l}{L}} \sim \sqrt{\frac{l}{2L}}$$

Onuki & Elleaume Undulators, Wigglers and their applications, CRC press, 2002

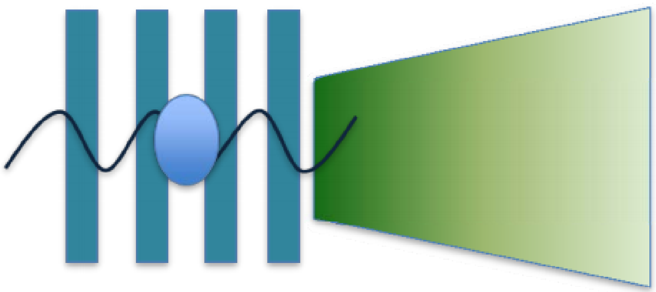
Photon beam size and divergence is determined by a combination of electron beam and single electron emission

$$\Sigma_x^2 = \sigma_{x,elec}^2 + \sigma_{x,photon}^2$$

$$\Sigma_{x'}^2 = \sigma_{x',elec}^2 + \sigma_{x',photon}^2$$

$$\Sigma_z^2 = \sigma_{z,elec}^2 + \sigma_{z,photon}^2$$

$$\Sigma_{z'}^2 = \sigma_{z',elec}^2 + \sigma_{z',photon}^2$$



Courtesy: Boaz Nash

These are at source. A distance D away, beam size become: $\Sigma_{x,0}^2 + \Sigma_{x',0}^2 D^2$

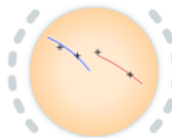
(FOR UNDULATORS, THESE FORMULAS ARE VALID AT THE WAIST, AT THE UNDULATOR RESONANCE, AND SUPOSSING GAUSSIAN EMISSION OF PHOTONS)

ShadowOui performs "numeric convolution" by Monte Carlo sampling of the electron beam [Gaussian] and photon emission [non Gaussian]

Undulators: Power and spectral calculations

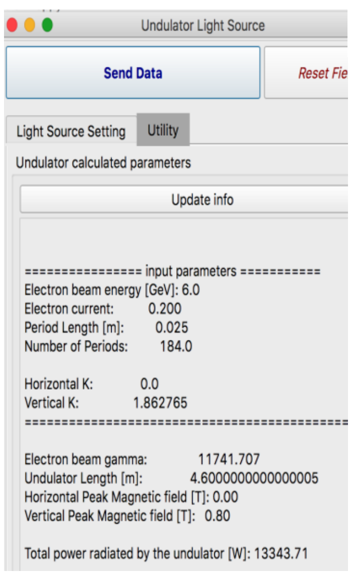
$$Power[kW] \approx 0.633 \cdot B^2\,[T] \cdot E^2[GeV] \cdot I[A] \cdot L[m]$$

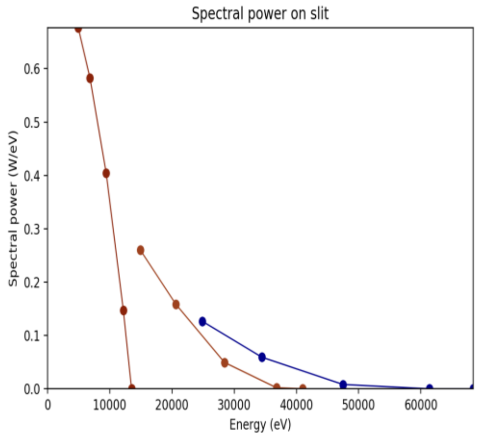
$$\frac{Power}{Solid\ Angle}[W/mrad^2] \approx 10.84 \cdot B\,[T] \cdot E^4[GeV] \cdot I[A] \cdot L[m] \cdot N \cdot G(K)$$



TC-SLIT

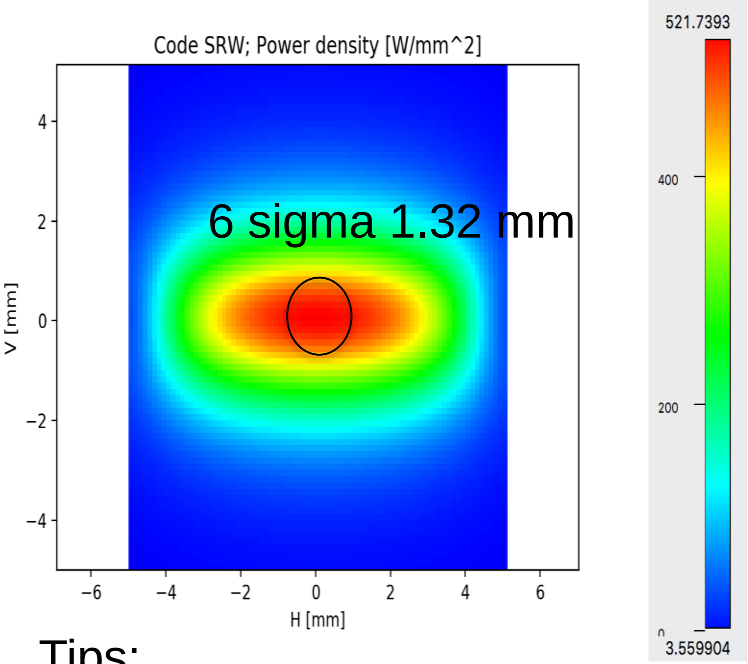
- Maximum power is obtained at highest K (or B) corresponding to the minimum photon energy
- The spectral range covered by undulator is shown by the Tuning Curves.



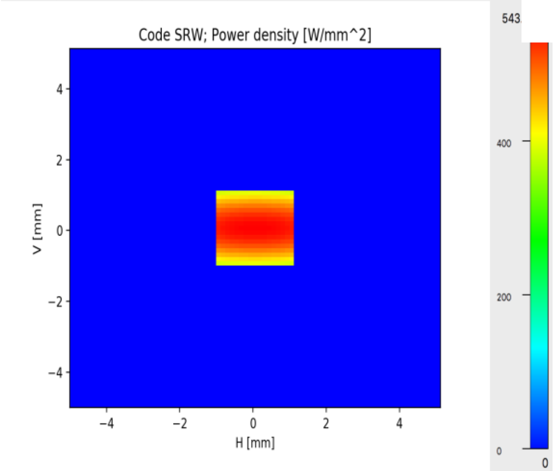


Power density (Power vs X and Y)

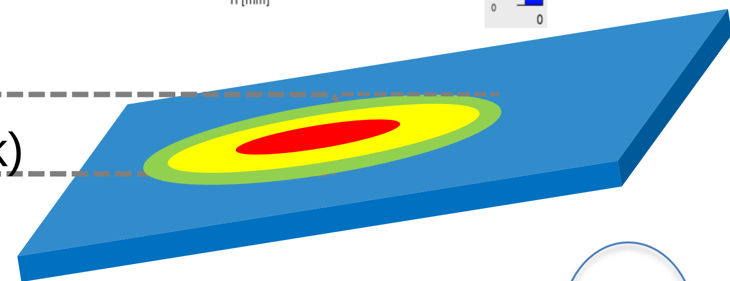
Undulator Power Density

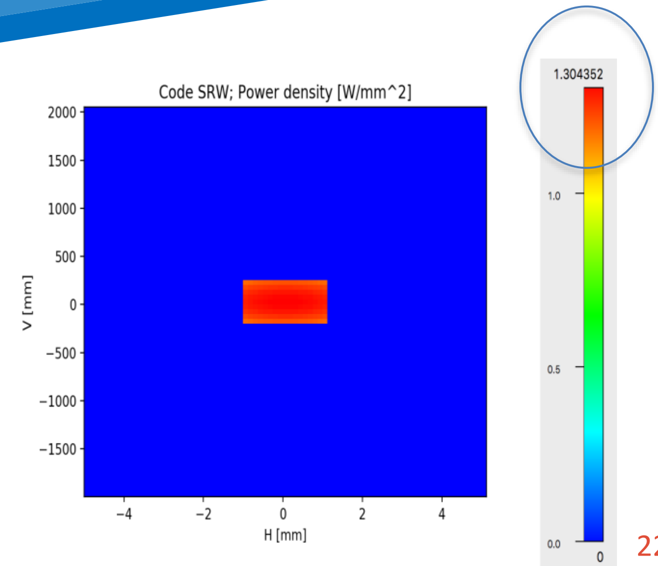


Mask
2x2 mm @
25.4,=m
Or
2.1x2.1 @
28m



Mirror:
400mm
2.1mm (mask)





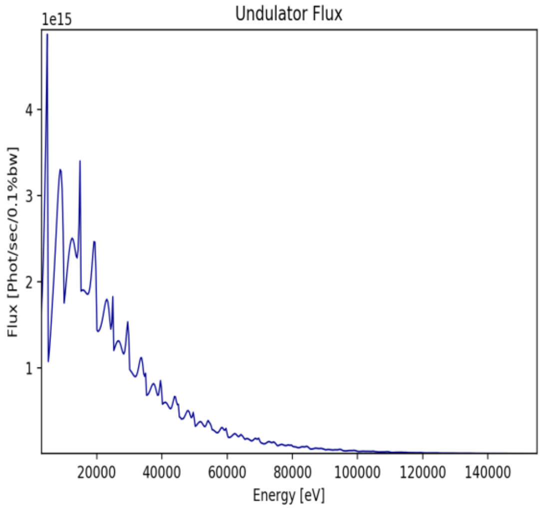
Tips:

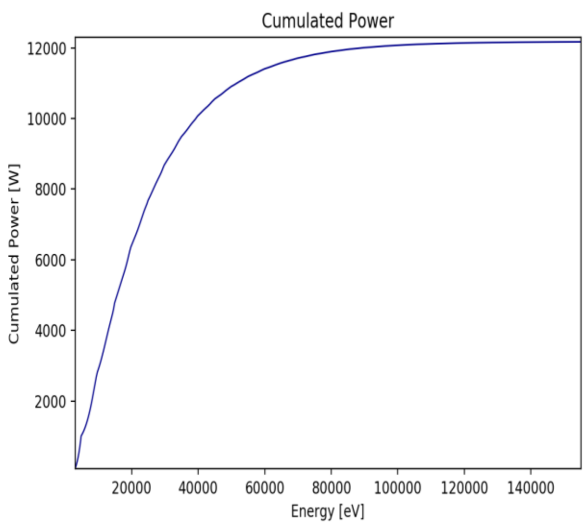
- Calculate on aperture large enough to accept all the radiation
- Select “modify slit” to calculate power on smaller slit and to calculate power density on mirror surface

Flux and Spectral Power

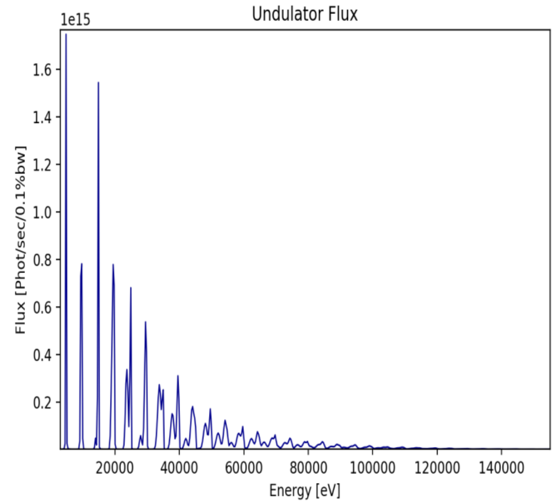
Undulator Spectrum

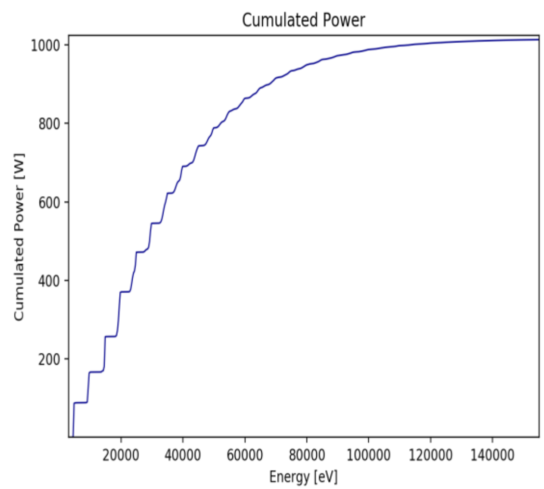
Open Slit (10 x 10 mm^2)





Slit (2.1 x 1 mm^2)

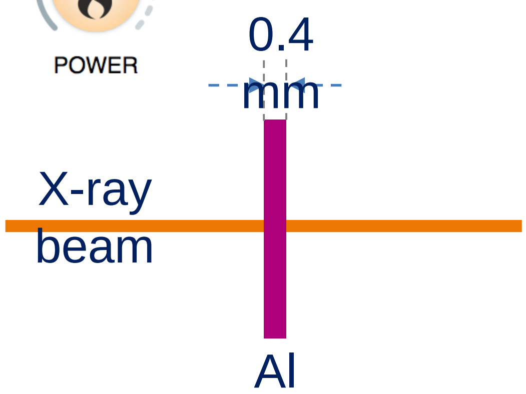


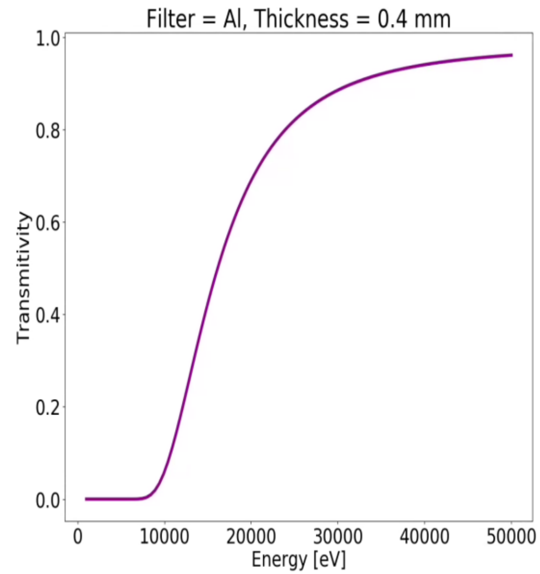


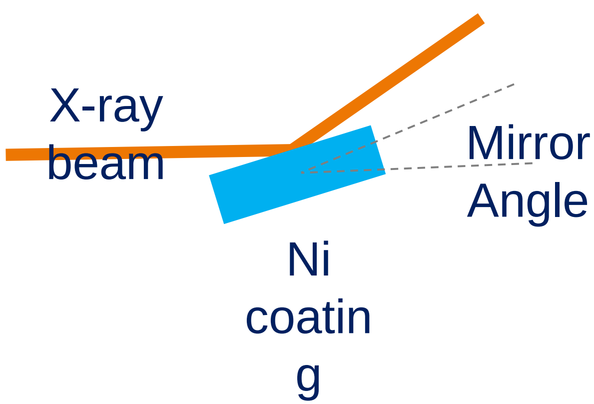
Spectral response of mirrors and attenuators (filters)

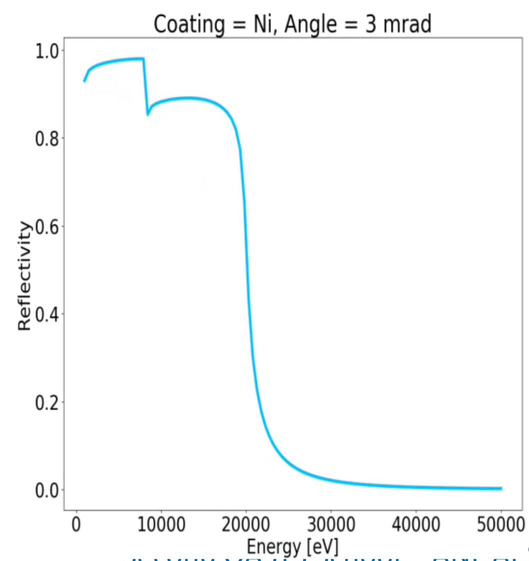
XOPPY

Interaction of radiation with matter

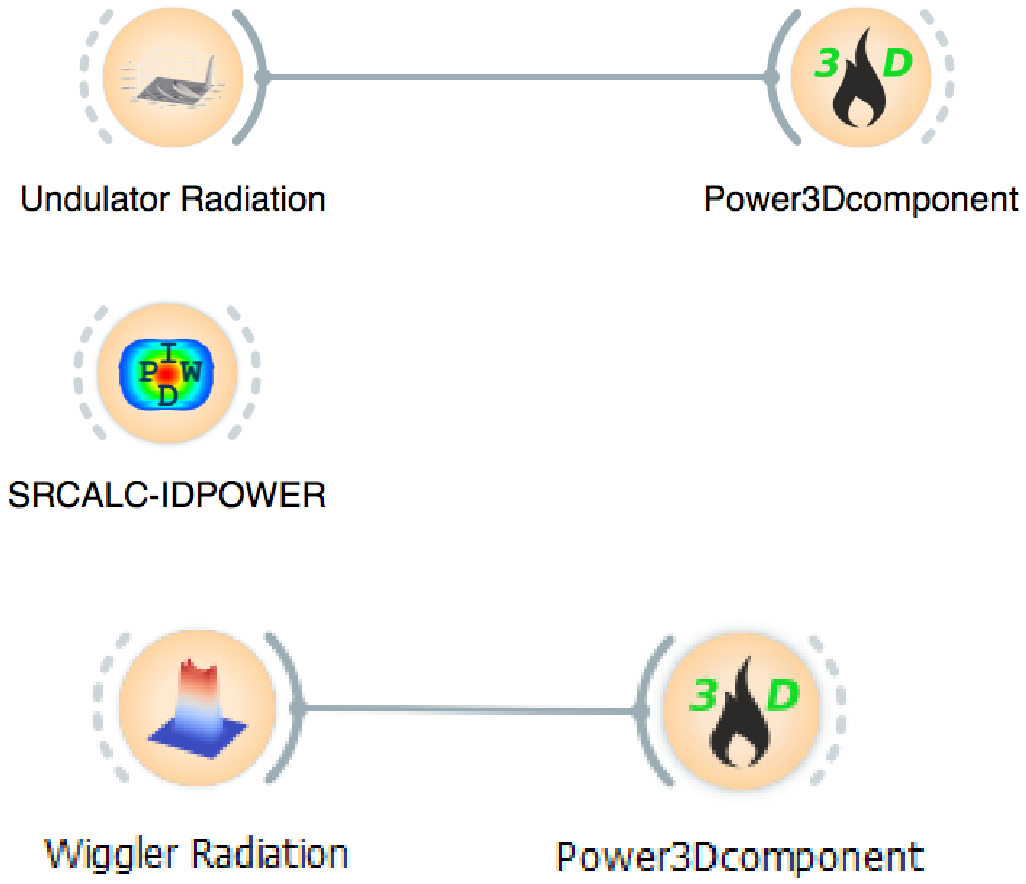








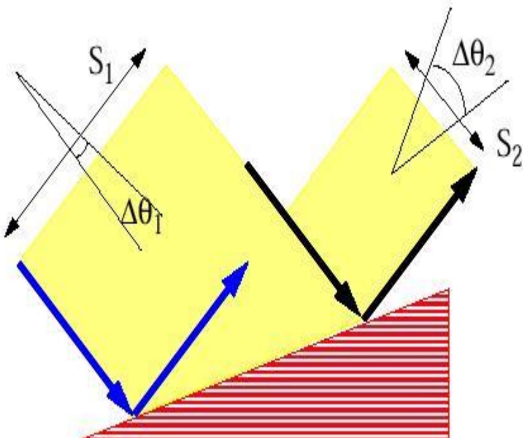
Combined (X,Y,E) effect of optical elements



Crystals: Zachariasen treatment for plane crystals

BRAGG or reflection

LAUE or transmission



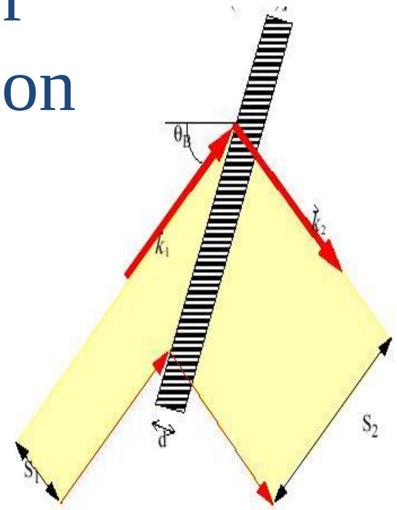
$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{-z \pm (qP^2 + z^2)^{1/2}}{P\Psi_{\overline{H}}}$$

$$c_1 = \exp(-i\varphi_1 T), \quad c_2 = \exp(-i\varphi_2 T),$$

$$\varphi_1 = -\frac{2\pi k^0 \delta_0'}{\;} \qquad \varphi_2 = -\frac{2\pi k^0 \delta_0''}{\;},$$

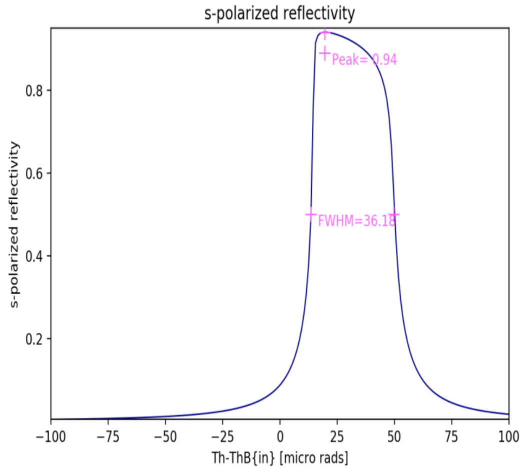
$$\begin{pmatrix} \delta_0' \\ \delta_0'' \end{pmatrix} = \frac{1}{2}\left[\Psi_0 - z \pm (qP^2 + z^2)^{1/2}\right]$$

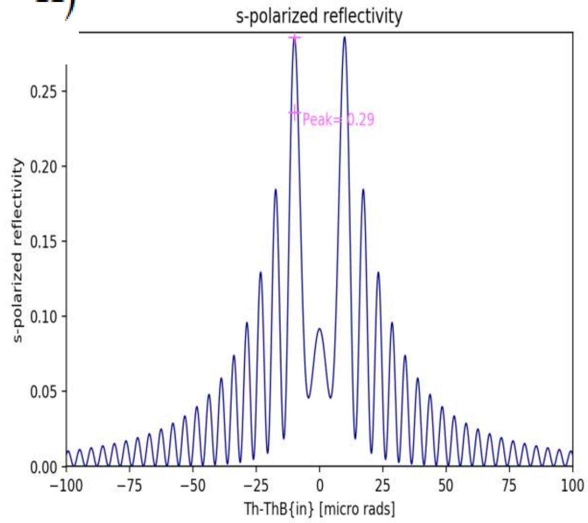
$$\alpha_Z = \frac{1}{|\mathbf{k}^0|^2}\left(|\mathbf{H}|^2 + 2\mathbf{k}^0 \cdot \mathbf{H}\right)$$



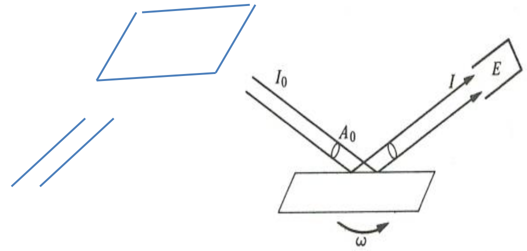
$$r^{\text{Laue}}(\alpha_Z) \equiv \frac{1}{|b|}\frac{I^H}{I^0} = \frac{1}{|b|}\left|\frac{x_1 x_2 (c_1 - c_2)}{x_2 - x_1}\right|^2$$

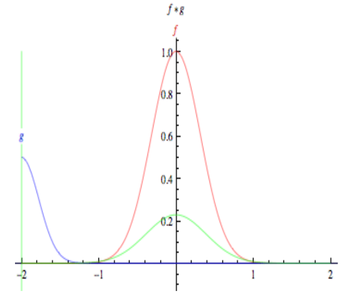
$$r^{\text{Bragg}}(\alpha_Z) \equiv \frac{1}{|b|}\frac{I^H}{I^0} = \frac{1}{|b|}\left|\frac{x_1 x_2 (c_1 - c_2)}{c_2 x_2 - c_1 x_1}\right|^2$$



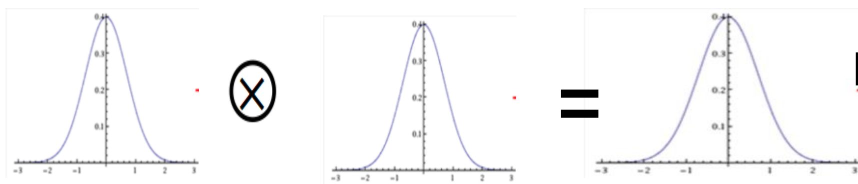


Crystals: Rocking curves

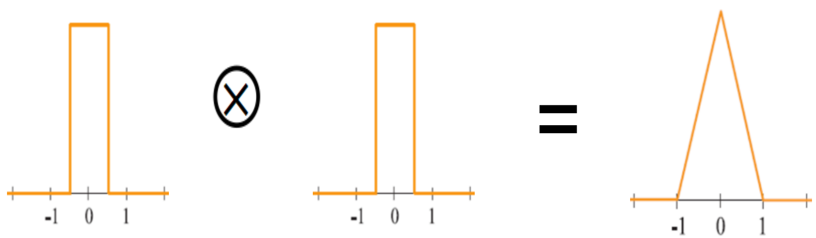




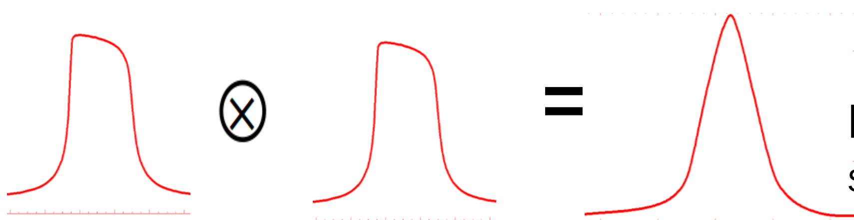
The rocking curve is the **CONVOLUTION** of the two diffraction profiles.



$FWHM_R$ = 1.4 $FWHM_1$



$FWHM_R$ = 1 $FWHM_1$

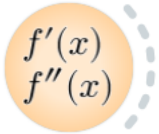


$FWHM_R$ ~ 1.5 $FWHM_1$

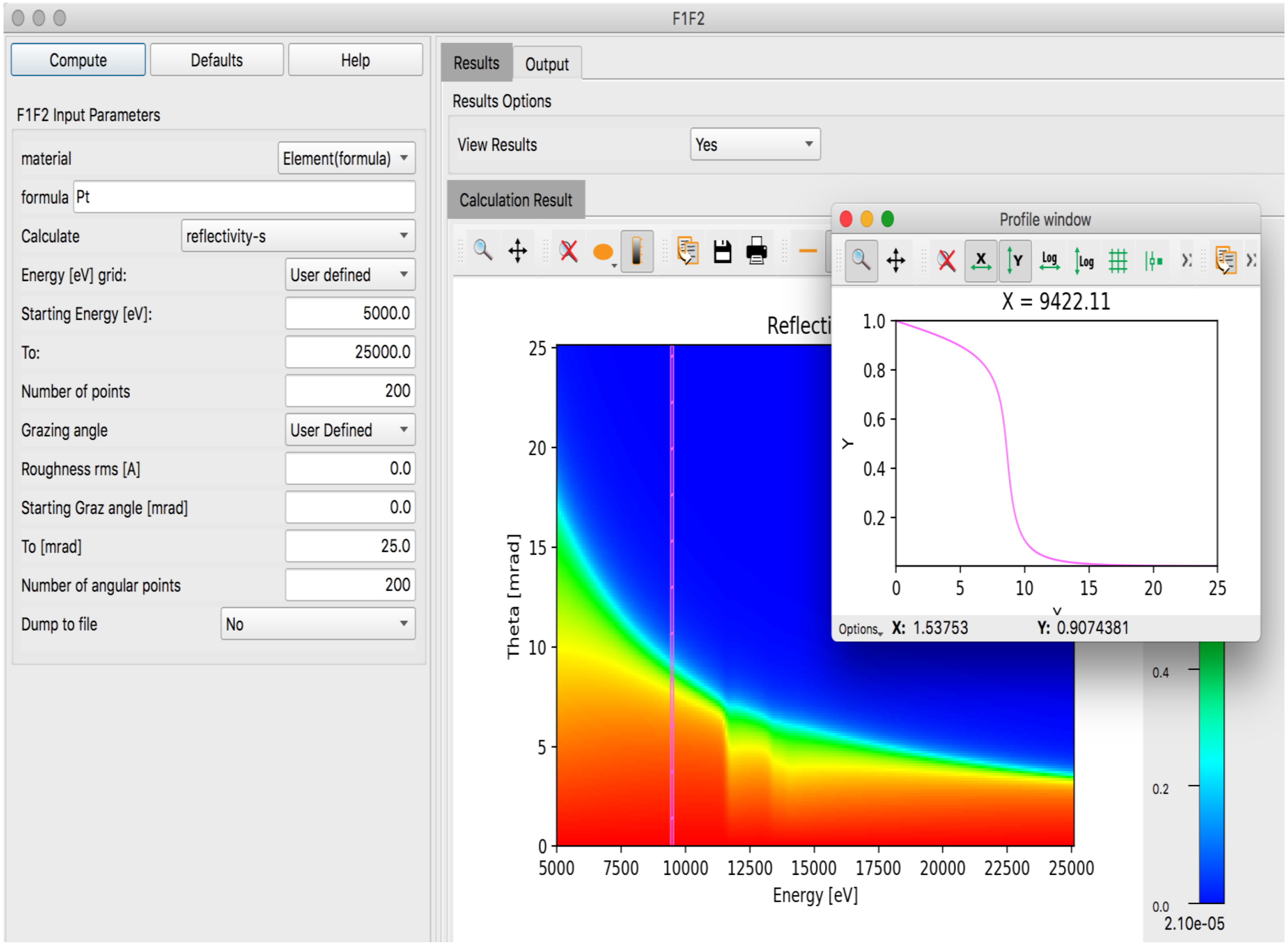
Si111 8 keV

For a full discussion about rocking curves see: Masiello *et al.* J. Appl. Crystall. 47, 1304-1314 (2014)

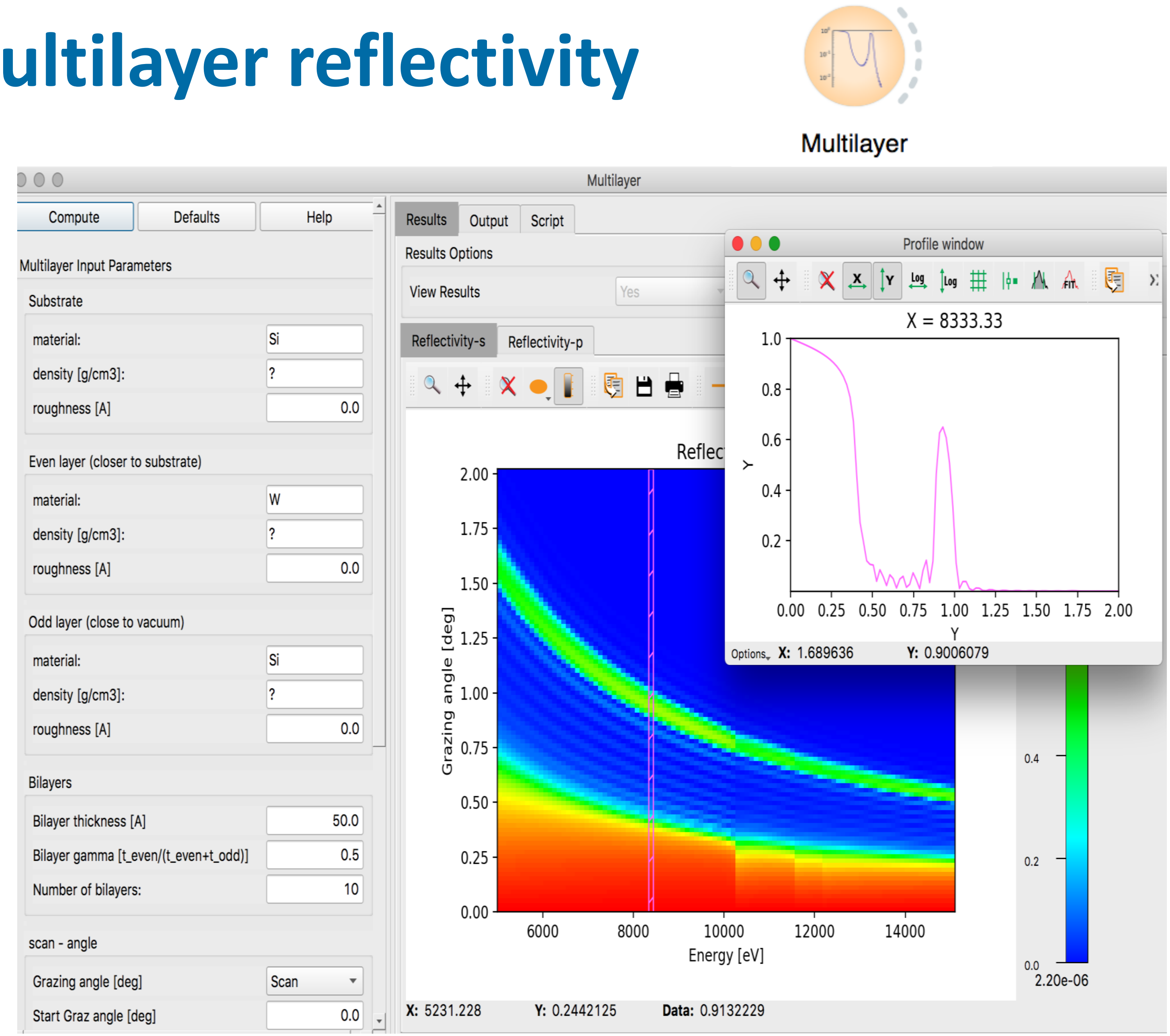
Mirror reflectivity F1F2



F1F2



Multilayer reflectivity



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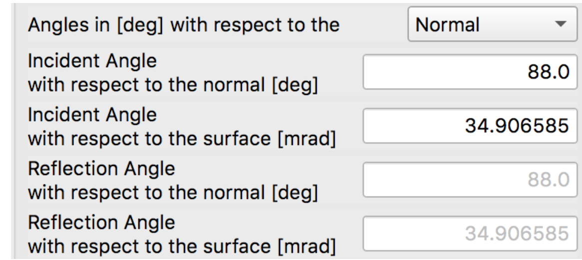
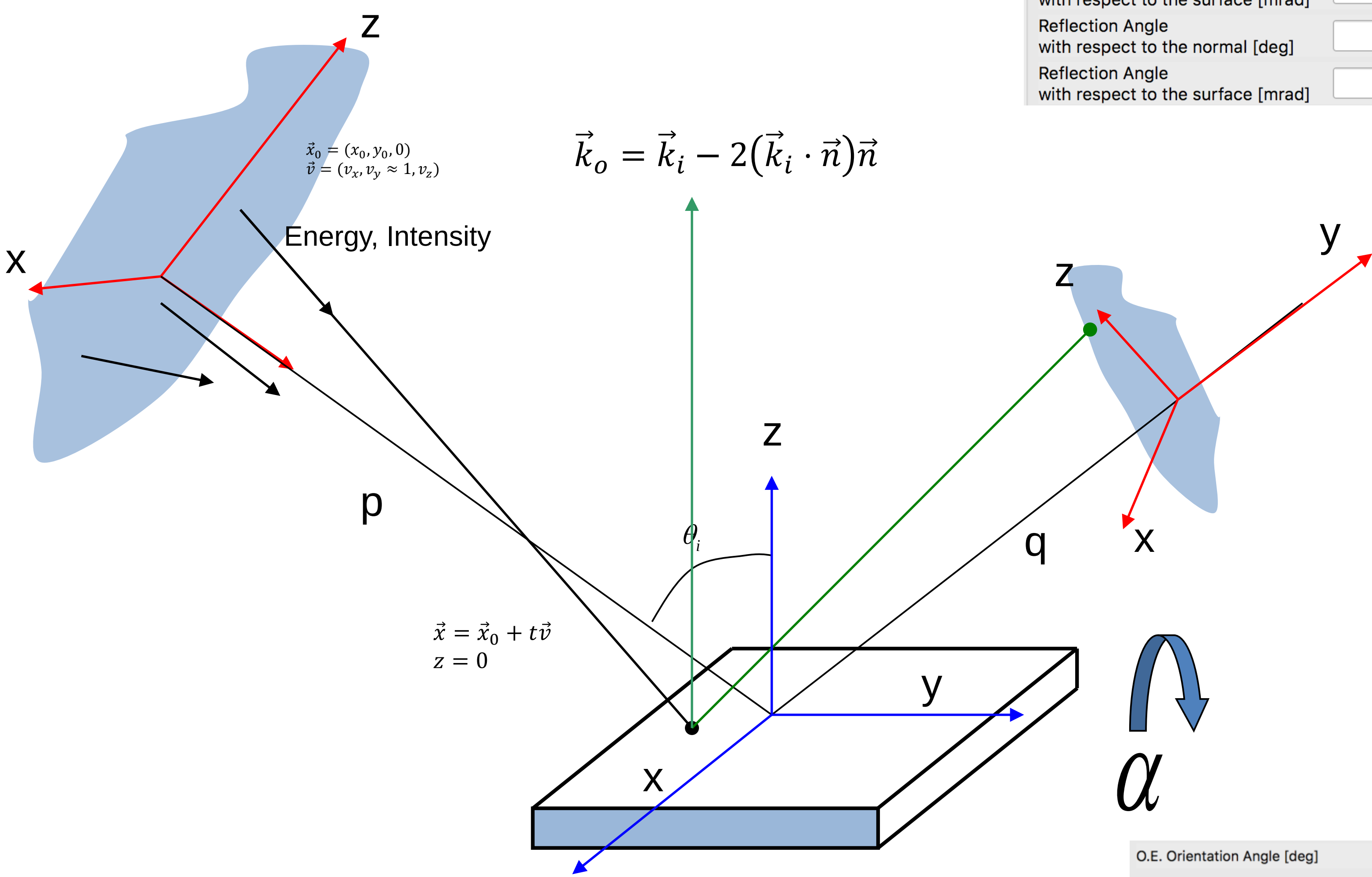
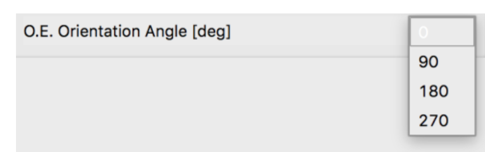
  - Introduction to Partial coherence

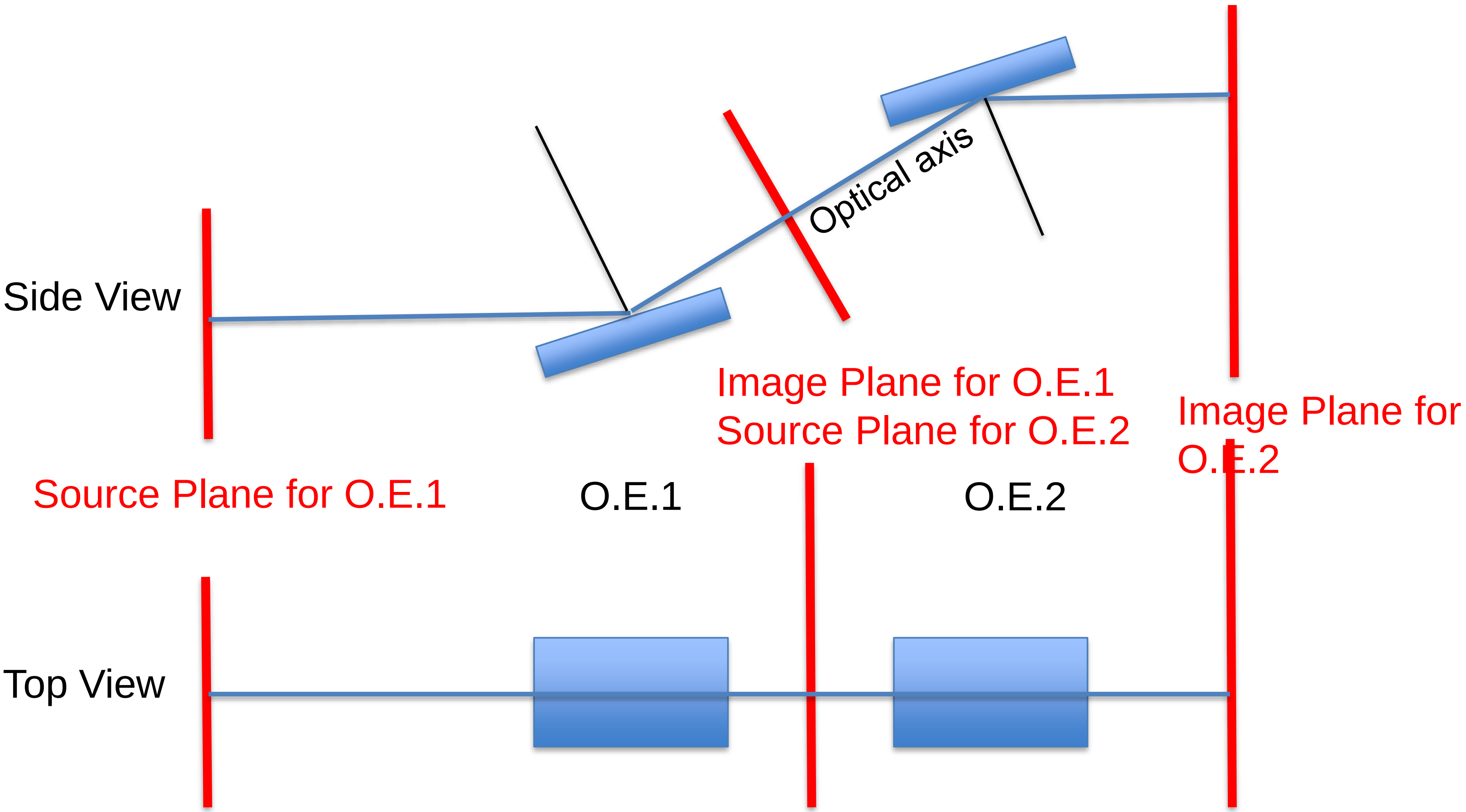
Trace (the beamline)

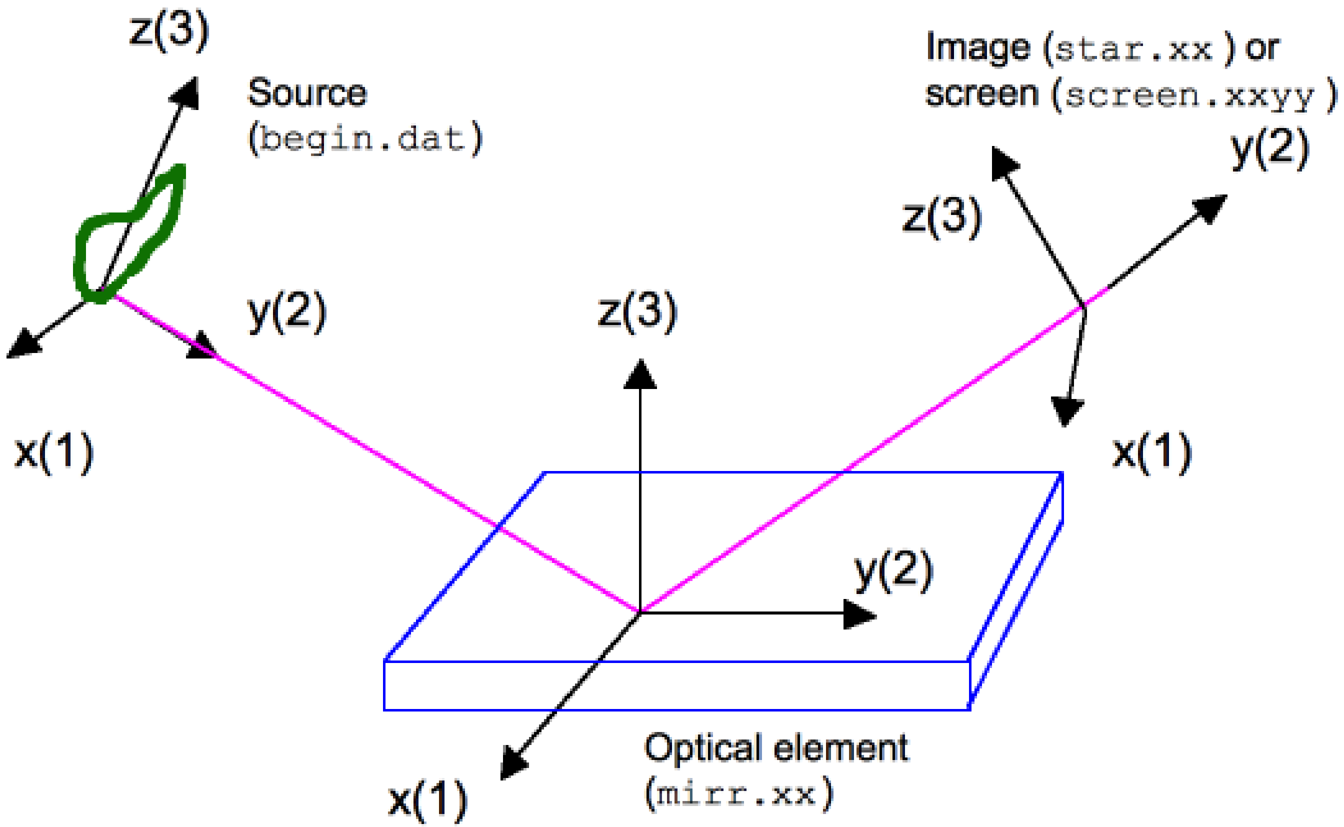






Continuation planes





Note that (VERY IMPORTANT!):

- The y(2) coordinate is along the beam direction
- The position (Source Plane Distance), orientation (O.E. Orientation Angle) of any o.e. is always referred to the previous one
- Source Plane and Image Plane for each optical element are the “Continuation Planes”
- The frame is rotated if one o.e. is rotated

SHADOW ray's variables (*columns*)

Stored:

1: X
2: Y
3: Z
4: X' *
5: Y'
6: Z'
7: Eσ X
8: Eσ Y
9: Eσ Z
10: Ray Flag
11: Energy **
12: Ray Index
13: Optical Path
14: Phase σ
15: Phase π
16: Eπ X
17: Eπ Y
18: Eπ Z

Computed:

19: Wavelength
20: R = sqrt($X^2 + Y^2 + Z^2$)
21: Theta (angle from Y axis)
22: Magnitude = |Eσ| + |Eπ|
23: Total Intensity = $|E\sigma|^2 + |E\pi|^2$
24: Σ Intensity = $|E\sigma|^2$
25: Π Intensity = $|E\pi|^2$
26: |K|
27: K X
28: K Y
29: K Z
30: S0-stokes = $|E\pi|^2 + |E\sigma|^2$
31: S1-stokes = $|E\pi|^2 - |E\sigma|^2$
32: S2-stokes = 2|Eσ||Eπ|cos(Phase σ-Phase π)
33: S3-stokes = 2|Eσ||Eπ|sin(Phase σ-Phase π)
34: Power = Intensity * Energy

* X',Y',Z' is the direction vector (unitary), for small angles (always in SR) Y'<~1, and X' and Z' can be considered "divergences"

** Column 11 is energy in eV. Internally SHADOW stores the wavenumber $2\pi/\lambda$ in cm^{-1}

Optical elements

For each optics element SHADOW includes:

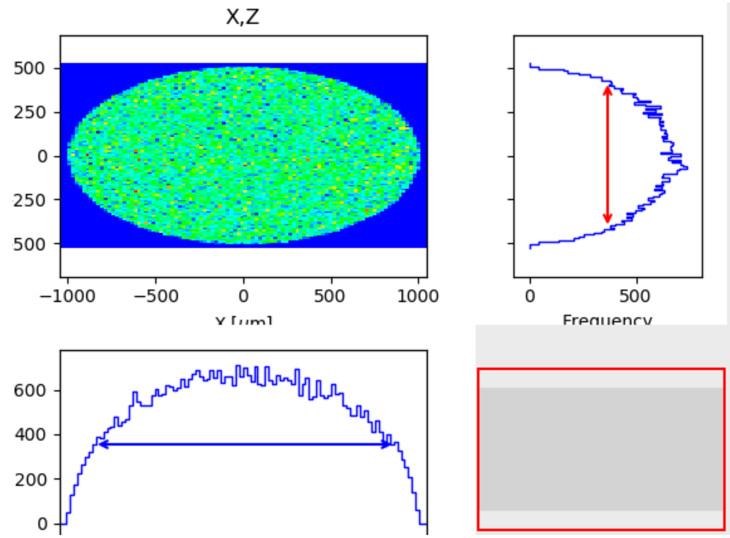
- Geometrical model: how the direction of the rays are changed (reflected, refracted or diffracted)
- Physical model: how the ray intensity (in fact electric fields) decreases because of the interaction

•Structures along the surface =>playing with the direction

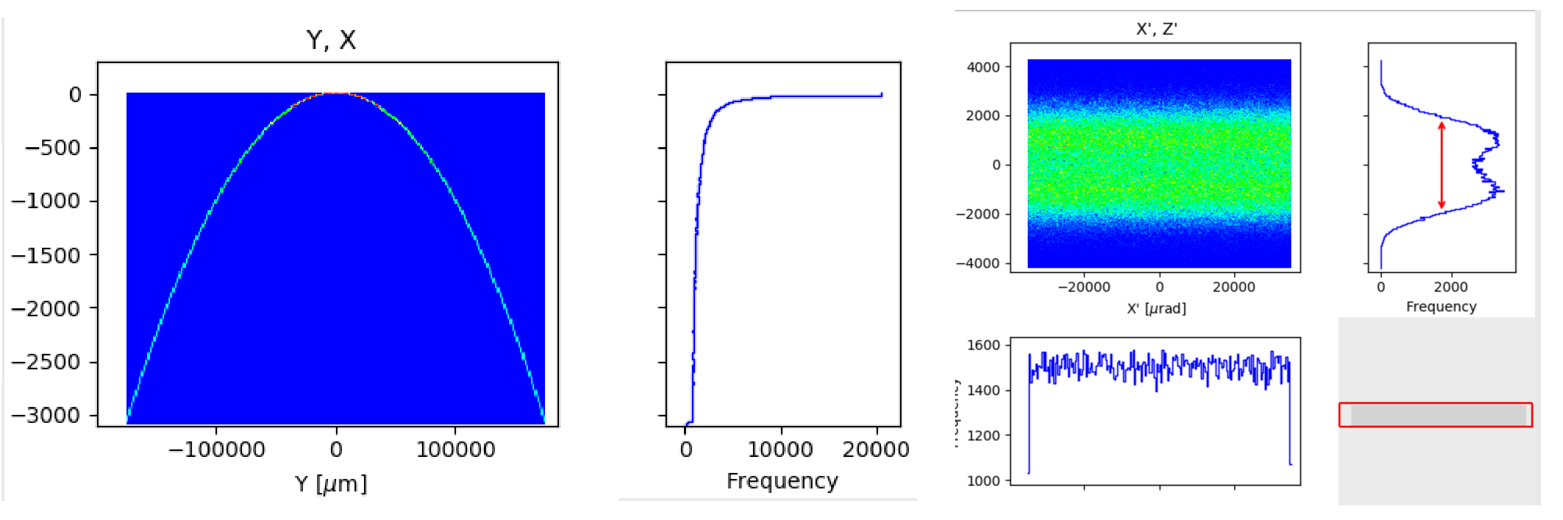
•Structures in depth => playing with the reflectivity

ShadowOUI Sources I

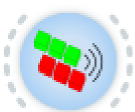
Geometrical Source



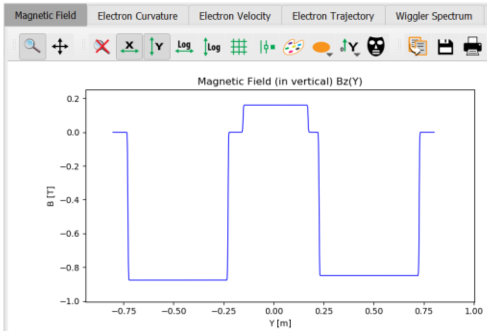
Bending Magnet

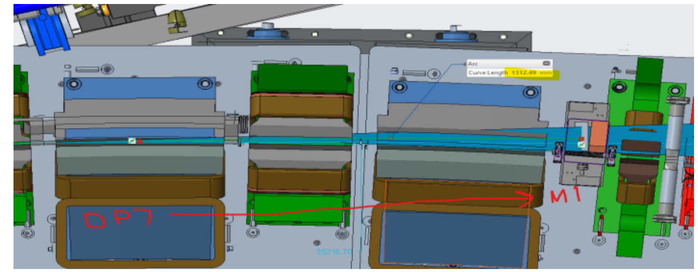


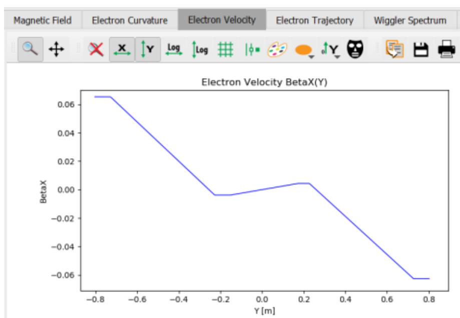
Sources II (Wiggler and also mutiple BMs and Short IDs)

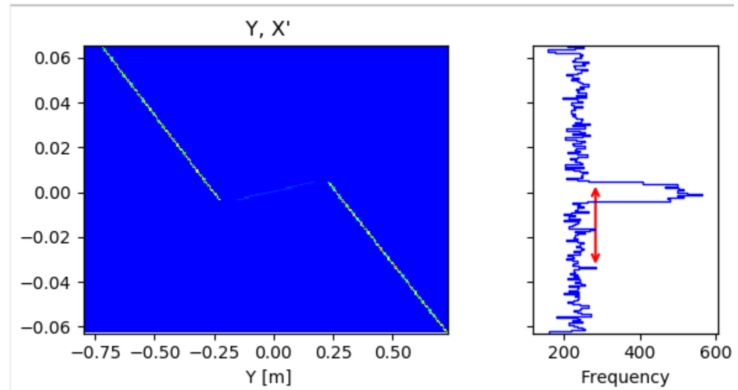


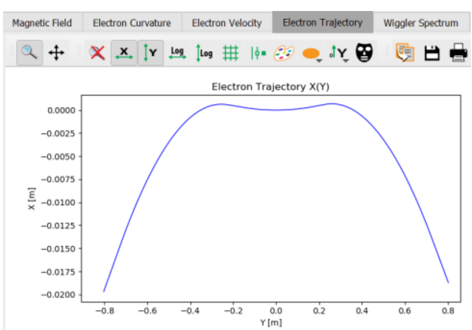
Wiggler

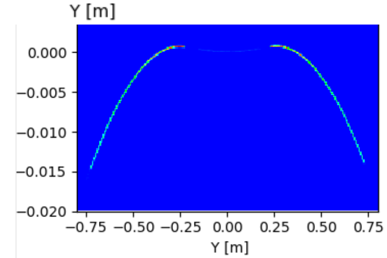


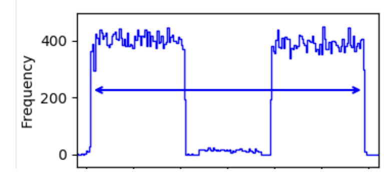












Sources III (Undulator)

Fake Geometrical Source
(Gaussian spatial + Gaussian divergency)

Undulator Gaussian

Single e⁻

Divergence:

$$\sigma_{r'} = 0.69\sqrt{\frac{\lambda}{L}} \approx \sqrt{\frac{\lambda}{2L}}$$

Size:

$$\sigma_{r} = \frac{2.704}{4\pi}\sqrt{\lambda L} \approx \sqrt{\frac{\lambda L}{2\pi^2}}$$

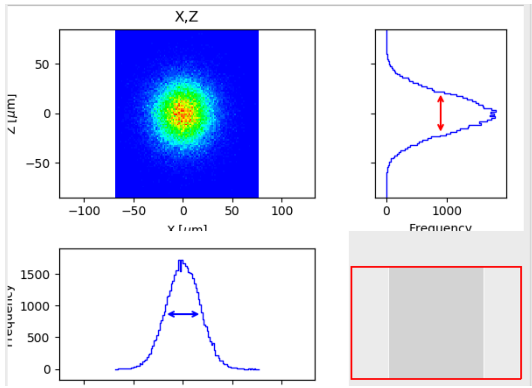
Photon source:

$$\Sigma_x^2 = \sigma_{x,elec}^2 + \sigma_{x,photon}^2$$

$$\Sigma_{x'}^2 = \sigma_{x',elec}^2 + \sigma_{x',photon}^2$$

$$\Sigma_z^2 = \sigma_{z,elec}^2 + \sigma_{z,photon}^2$$

$$\Sigma_{z'}^2 = \sigma_{z',elec}^2 + \sigma_{z',photon}^2$$



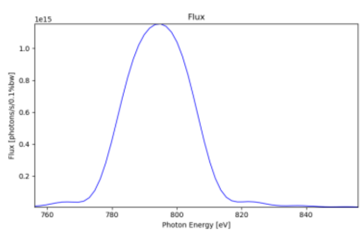
Plots | Output

System Output

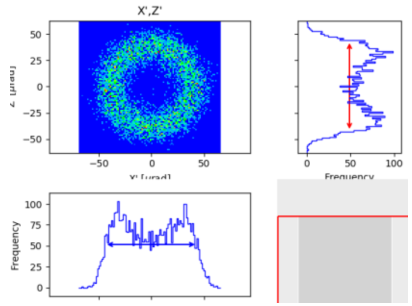
Photon single electron emission at wavelength 15.382655 A:
sigma_u: 12.2503 um
sigma_uprime: 18.8919 urad
Electron sizes:
sigma_x: 12.1 um
sigma_z: 14.7 um
sigma_x': 5.7 urad
sigma_z': 4.7 urad
Photon source sizes (convolution):
Sigma_x: 17.2186 um
Sigma_z: 19.1353 um
Sigma_x': 19.7331 urad
Sigma_z': 19.4678 urad

Full Undulator

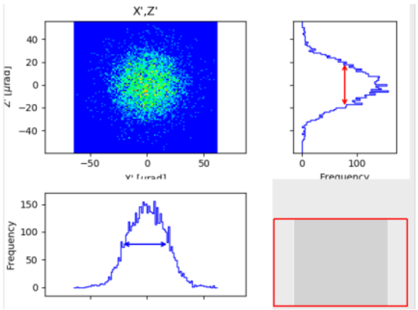
Full Undulator



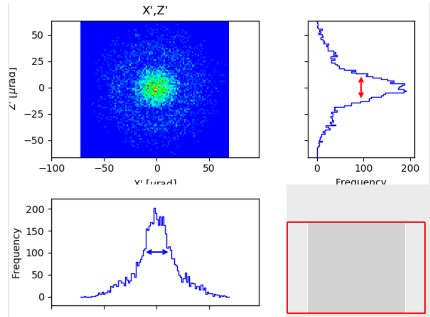
789.938eV



806.060eV

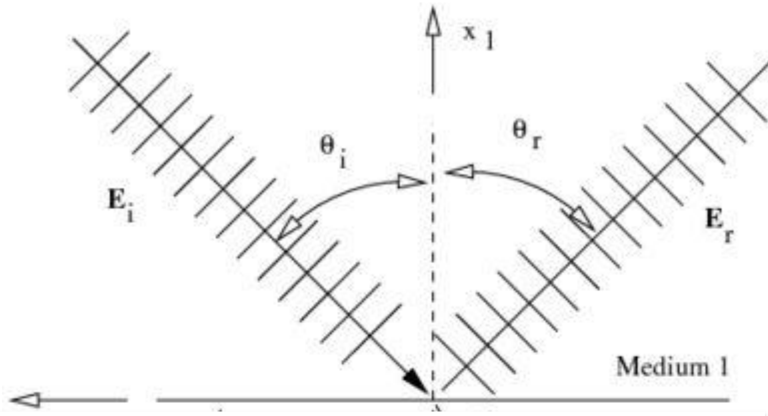


814.121eV



Optical Elements (mirrors)

Geometrical model

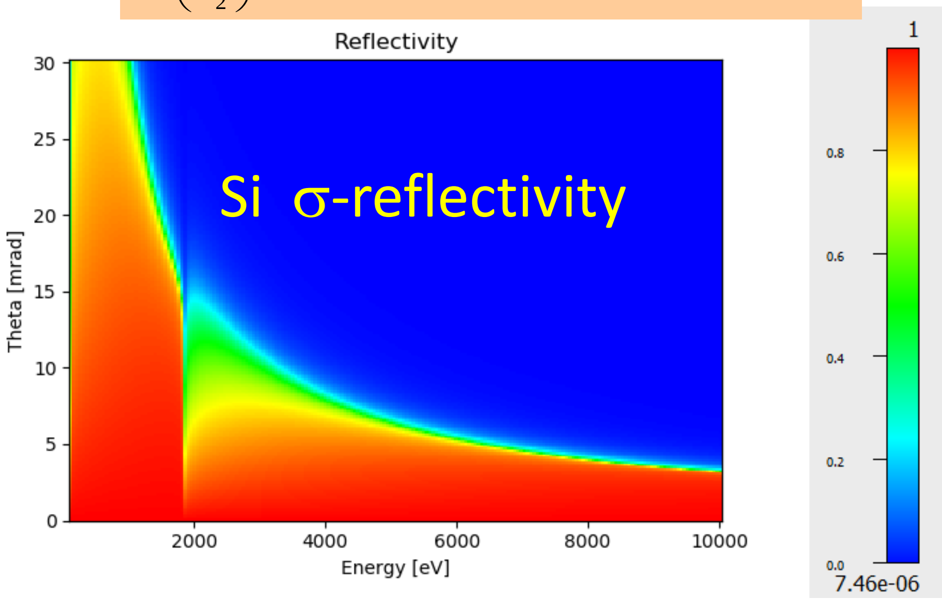


$$\hat{\mathbf{d}}_s = 2\left(\hat{\mathbf{d}}_n \cdot \hat{\mathbf{d}}_i\right)\hat{\mathbf{d}}_n - \hat{\mathbf{d}}_i,$$

Physical model

Fresnel equations give the reflectivity as a function of angle and photon energy. As a consequence, one gets the critical angle:

$$1 = \left(\frac{n_1}{n_2}\right)^2 \cos^2\theta_c \quad \Leftrightarrow \quad \sin\theta_c = \sqrt{2\delta - \delta^2} \approx \sqrt{2\delta}$$



Optical Elements (gratings)

Geometrical model

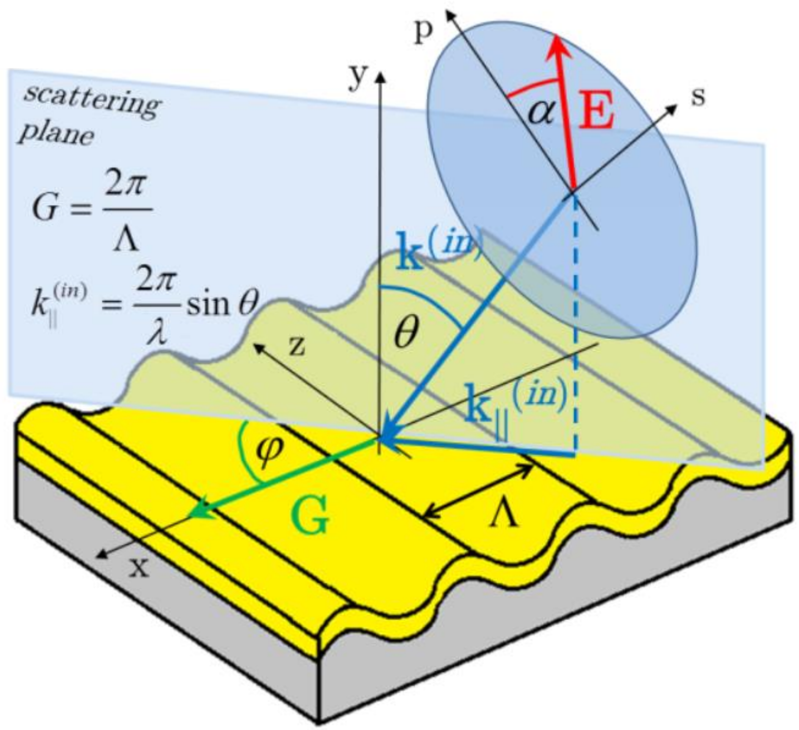


Figure from doi.org/10.5772/51044

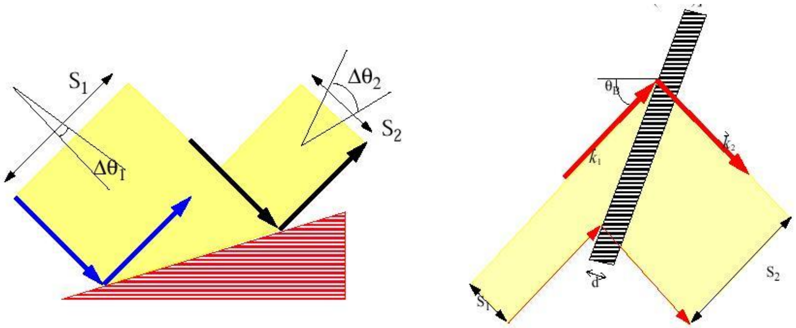
Physical model

No model (yet) in SHADOW for grating efficiency

SHADOW uses Fresnel equations like for mirrors

Optical Elements (crystals)

Geometrical model

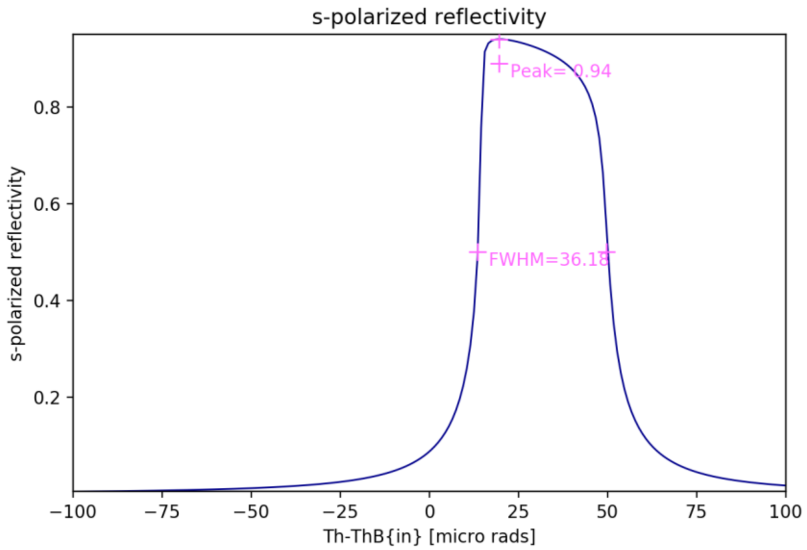


$$|\vec{k}_1| = |\vec{k}_2|$$

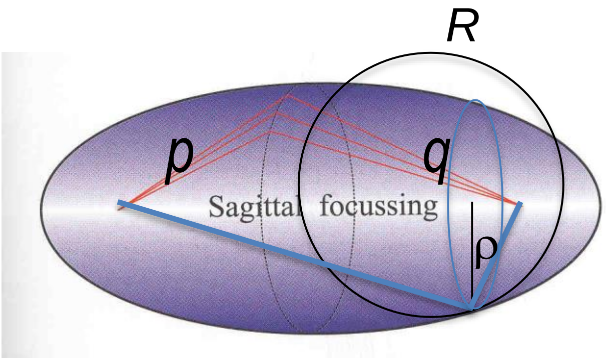
$$\vec{k}_{2,||} = \vec{k}_{1,||} + \vec{G}_{||}$$

Physical model

$$r^{\text{Bragg}}(\alpha_Z) \equiv \frac{1}{|b|}\frac{I^H}{I^0} = \frac{1}{|b|}\left|\frac{x_1 x_2 (c_1 - c_2)}{c_2 x_2 - c_1 x_1}\right|^2$$



Mirrors and aberrations



- Ellipsoid: Point to point focusing
- Paraboloid: Collimating
- Focalization in two planes
  - Tangential or Meridional (ellipse or parabola)
  - Sagittal (circle)
- Demagnification: M=p/q
- Easier manufacturing:
  - 2D: Ellipsoid => Toroid
  - Only one plane: cylinder Ellipsoid (ellipse)=> cylinder (circle)
  - Sagittal radius: non-linear (ellipsoid) => constant (cylinder) or linear (cone),
- All mirrors produce aberrations

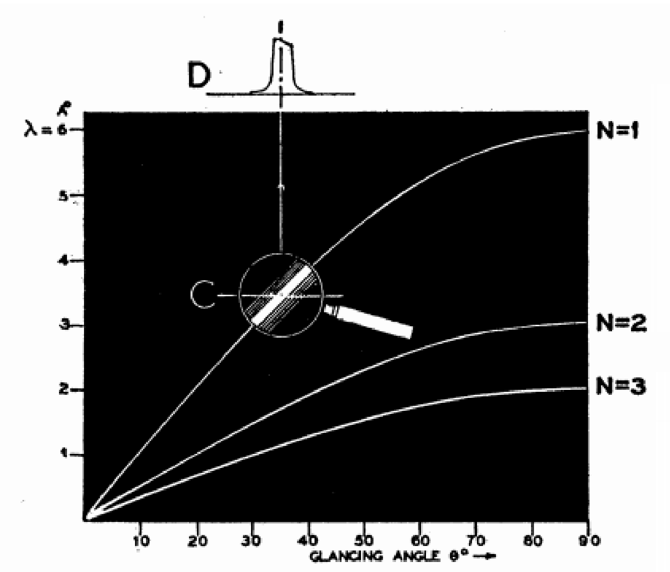
$$\frac{1}{p}+\frac{1}{q}=\frac{2}{R\sin\theta}$$

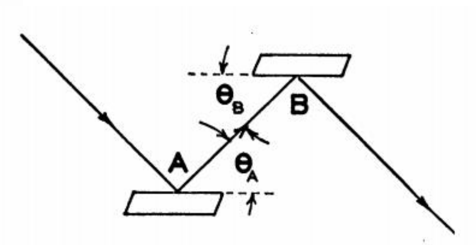
$$\frac{1}{p}+\frac{1}{q}=\frac{2\sin\theta}{\rho}$$

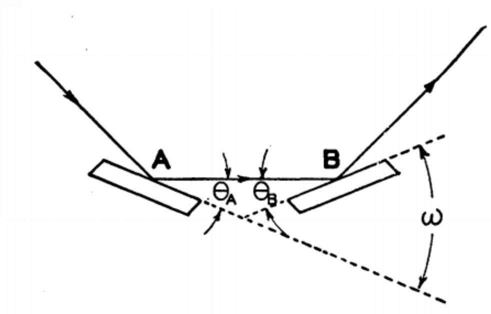
Crystal monochromators

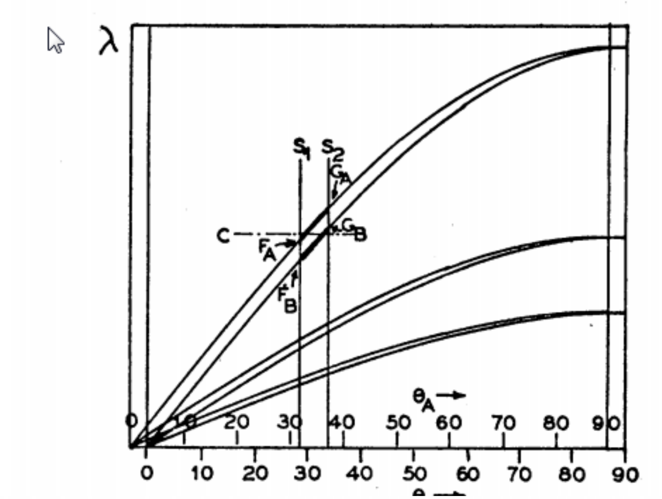
ex17_crystalmono.ows

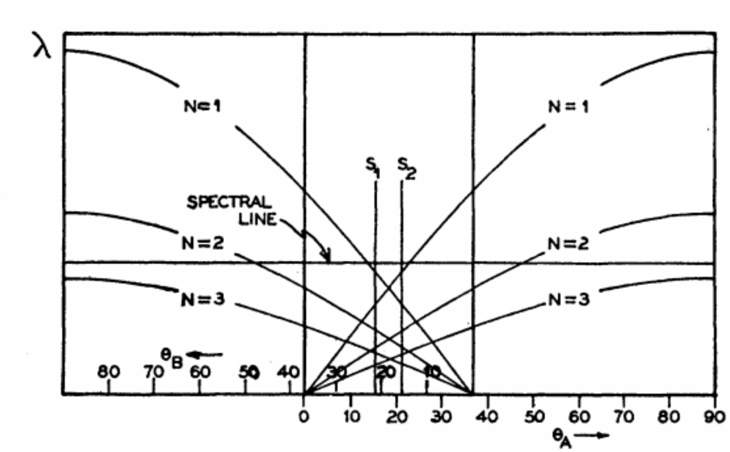
Theory of the Use of More Than Two Successive X-Ray Crystal Reflections to Obtain Increased Resolving Power
J W. M. DuMond Phys. Rev. 52, 872 – (1937)
http://dx.doi.org/10.1103/PhysRev.52.872











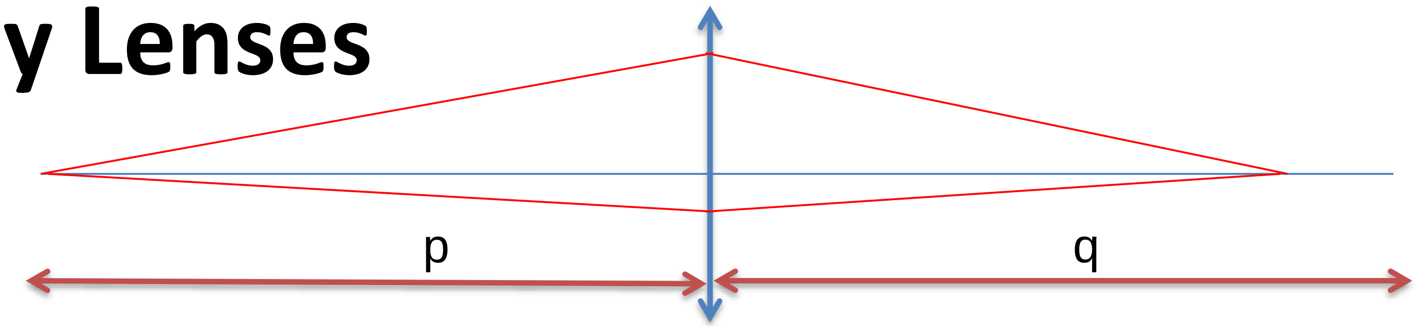
Crystal curvature and spectral resolution

$$\frac{\Delta\lambda}{\lambda_0} = \frac{\Delta E}{E_0} = (\Delta_{src} + \omega_D)\cot\theta_0$$

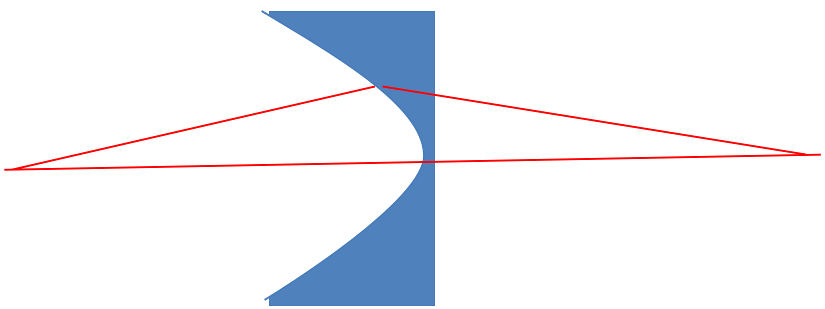
$$\frac{\Delta\lambda}{\lambda_0} = \frac{\Delta E}{E_0} \approx \sqrt{\omega_D^2 + (\Delta_{geom} + \Delta_{ss})^2}\cot\theta_0 = \sqrt{\omega_D^2 + \left[\boxed{\left(\frac{p}{R\sin\theta_1} - 1\right)\Delta_{src}} + \frac{s_1}{p}\right]^2}\cot\theta_0$$

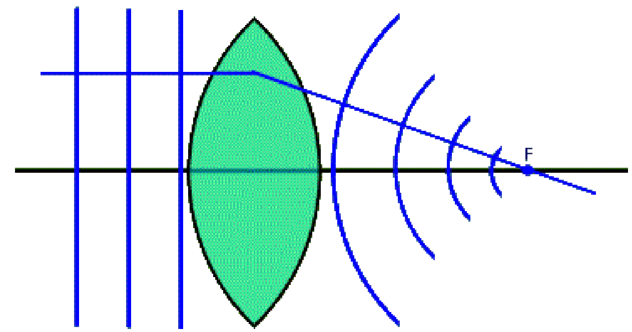
Zero for Rowland mounting:
p = R sin(θ) that for Bragg
symmetric reflection means
1:1 magnification

X-ray Lenses



$$\frac{1}{F} = \frac{1}{p} + \frac{1}{q}$$





$$\sin\theta_1 = (1-\delta)\sin\theta_2$$

$$\frac{1}{F} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right) \approx \frac{\delta}{R_1}$$

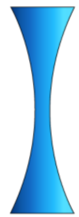
$$E = E_0 e^{i\frac{kr^2}{2f}}$$

CRL (COMPOUND REFRACTIVE LENSES) = replicate N lenses

Single interface

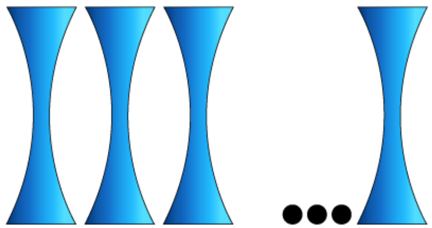
$$F = \frac{R}{\delta}$$

Lens



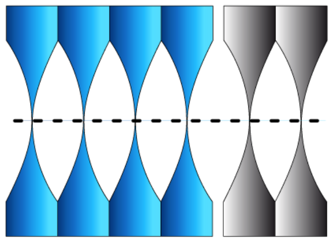
$$F = \frac{R}{2\delta}$$

Compound Refractive Lens (CRL)



$$F = \frac{R}{2N\delta}$$

Transfocator



$$\frac{1}{F} = \frac{1}{F_1} + \frac{1}{F_2} + \dots$$

1996 Experimental demonstration of CRL

- A. Snigirev *et al* Nature 384 (1996) 49

2011 Transfocator

Outline

- Introduction to OASYS
- XOPPY toolbox: Source emission and characteristics of optical elements. Power transport
- ShadowOUI: Ray tracing SR beamlines
  - Mirror aberrations
  - Crystal systems
  - Lenses
- WOFRY and SRW: coherent beam transport:
  - Simple examples with WOFRY
  - Introduction to Partial coherence

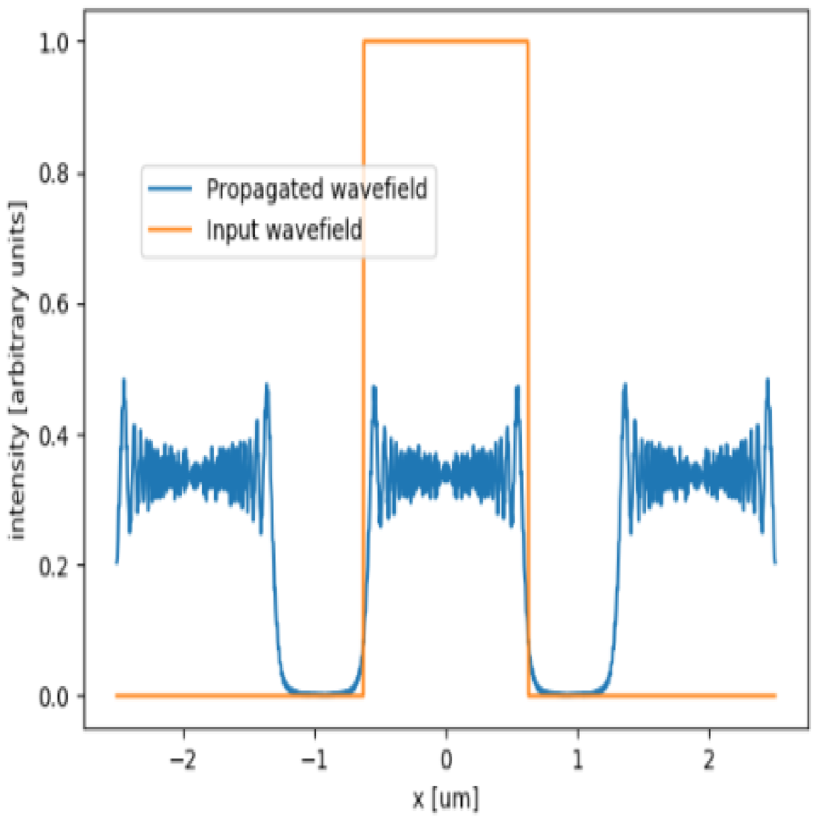
Wave optics

- Based on sampling a wavefront (monochromatoc and coherent), and propagate it (in free space and trough optical elements)
- 2D wavefront sampling is costly ~N^2
- Propagation is expensive N^4 or N^2 if FFT
- Optical elements modelled as thin objects
- Parameter setting is complex

Wavefront and Propagator discretization

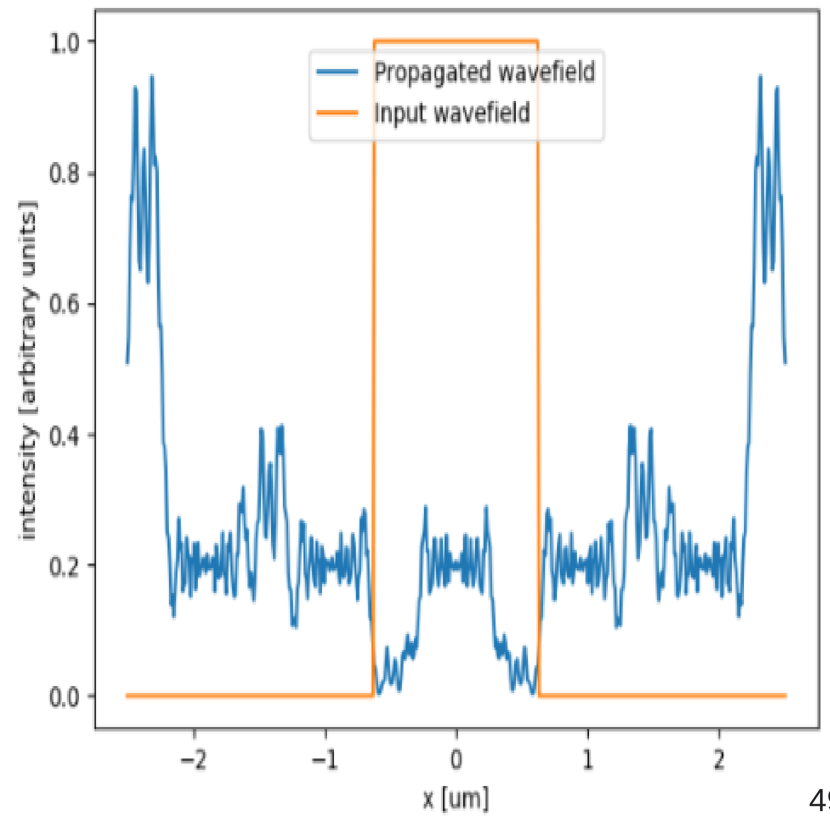
Effect of the discretization: Replicas

Replicas can be avoided by increasing the sampling or by isolating the main feature at the image plane, if the replica are well separated.



Effect of under-sampling: Aliasing

Under-sampling produces an overlapping of the replicas, thus completely distorting the expected result. This effect is called *aliasing.*



SRW (Oleg Chubar, BNL)

https://github.com/ochubar/SRW

Chubar, O. and Ellaume, P.: *Accurate and efficient computation of synchrotron radiation in the near field region.* Proceedings of the 6th European Particle Accelerator Conference - EPAC-98, pages 1177-1179

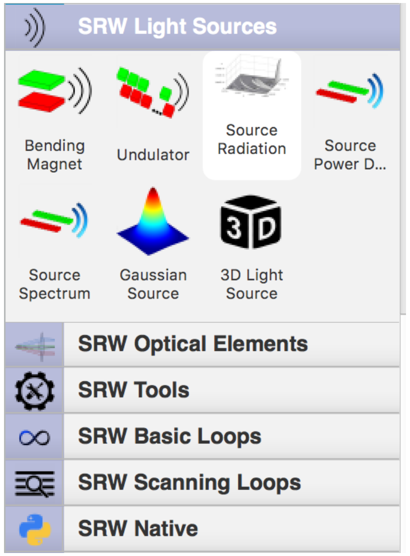
Chubar, O.: *Wavefront calculations*. Proc. SPIE, 4143:48-59 (2001)

Chubar, O., Berman, L., Chu, Y. S., Fluerasu, A., Hubert, S., Idir, M., Kaznatcheev, K., Shapiro, D. Shen, Q. and Baltser, J.: *Development of partially-coherent wavefront propagation simulation methods for 3rd and 4th generation synchrotron radiation sources.* Proc. SPIE, 8141:814107 (2011)

Chubar, O.: *Recent updates in the "Synchrotron Radiation Workshop" code, on-going developments, simulation activities, and plans for the future.* Proc. SPIE, 9209:920907 (2014)

Chubar, O., Rakitin, M., Chen-Wiegart, Y.K., Chu, Y.S., Fluerasu, A., Hidas, D. and Wiegart, L.: *Main functions, recent updates, and applications of Synchrotron Radiation Workshop code.* Proc. SPIE, 10388:1038805 (2017)

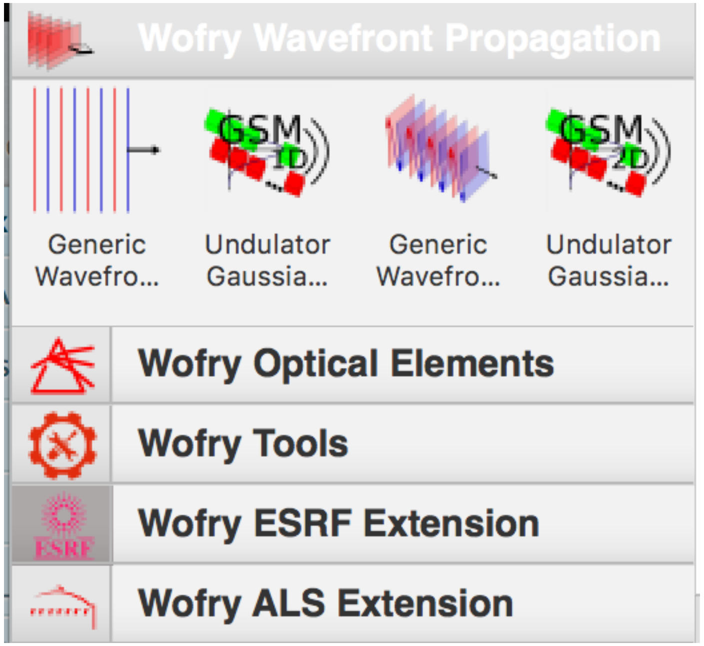
Chubar, O. and Celestre, R.: *Memory and CPU efficient computation of the Fresnel free-space propagator in Fourier optics simulation.* Opt. Express 27, 28750-28759 (2019)



OASYS interface by Luca Rebuffi

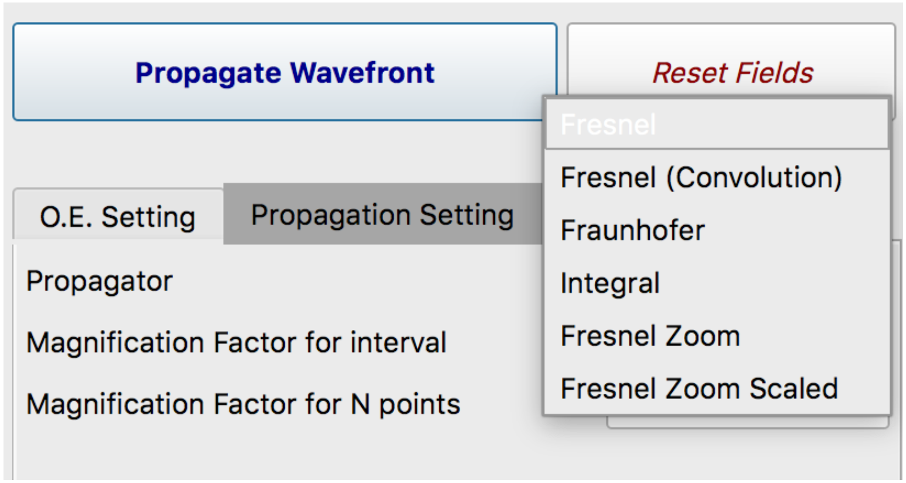
WOFRY (OASYS team)

- Simpler use
- Basic wavefronts (plane waves, spherical waves)
- 1D and 2D modes (less expensive)
- Approximated undulator sources
- Effective propagators
- Not as complete as SRW but growing fast
- Good for prototyping beamlines

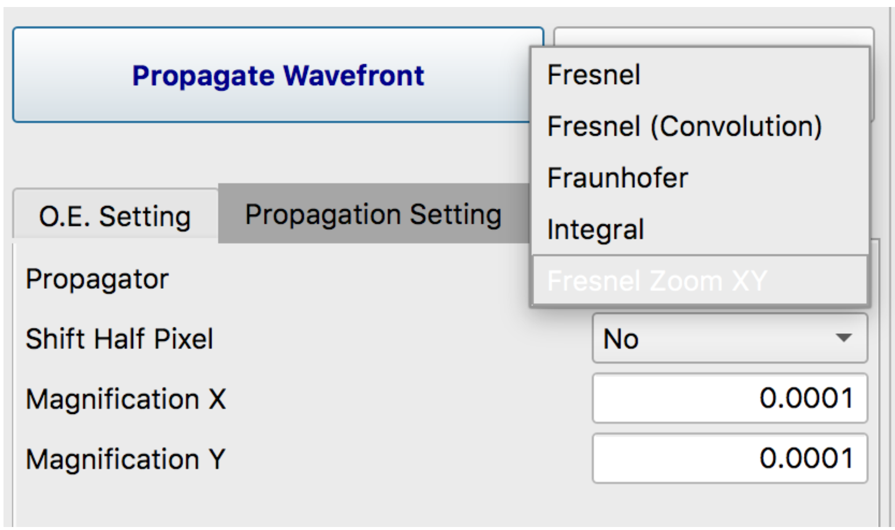


Propagators

1D



2D



Fresnel

Fresnel (convolution)

Integral

Fresnel Zoom

Fresnel Zoom Scaled

Zoom Propagator

$$\boxed{\begin{aligned} U(x_2, y_2) =& \frac{e^{ik\Delta z}}{\sqrt{m_x m_y}} e^{i\frac{k}{2\Delta z}[\frac{m_x - 1}{m_x}x_2^2 + \frac{m_y - 1}{m_y}y_2^2]} \\ & \mathcal{F}^{-1}\left[\mathcal{F}\left[U(x_1, y_1)e^{i\frac{k}{2\Delta z}[(1-m_x)x_1^2 + (1-m_y)y_1^2]}\right] \times e^{-i\pi\lambda\Delta z(\frac{f_x^2}{m_x} + \frac{f_y^2}{m_y})}\right] \end{aligned}}$$

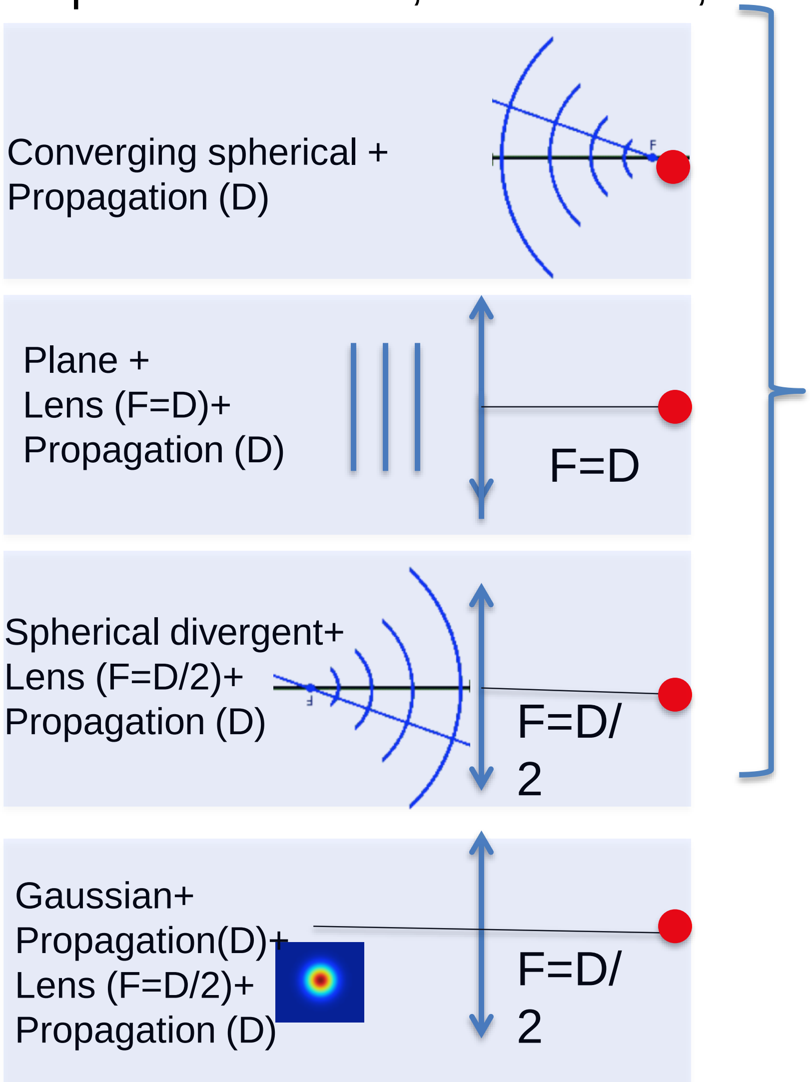
Jason D. Schmidt. *Numerical Simulation of Optical Wave Propagation*. SPIE Press, Bellingham, WA, USA, 2010.

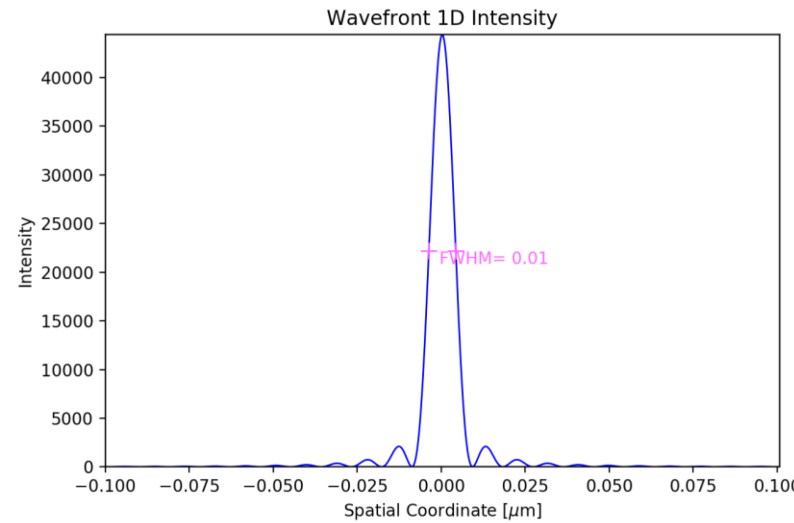
G. Pirro, Master Thesis (2017)

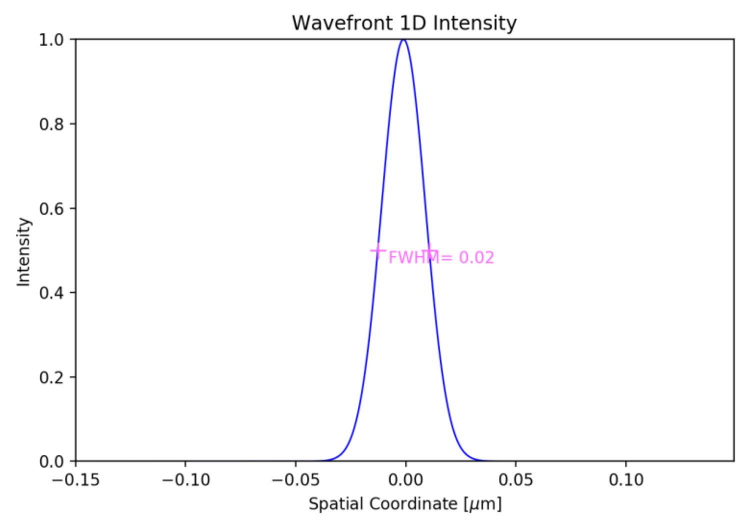
https://github.com/oasys-kit/documents/blob/master/zoom_propagator_pirro_thesis.pdf

wofry_examples.ows simple propagation cases

Aperture=0.4 mm, E=17225 eV, D=5cm

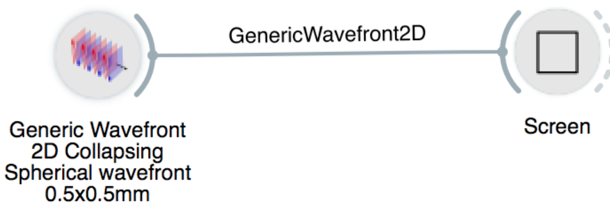


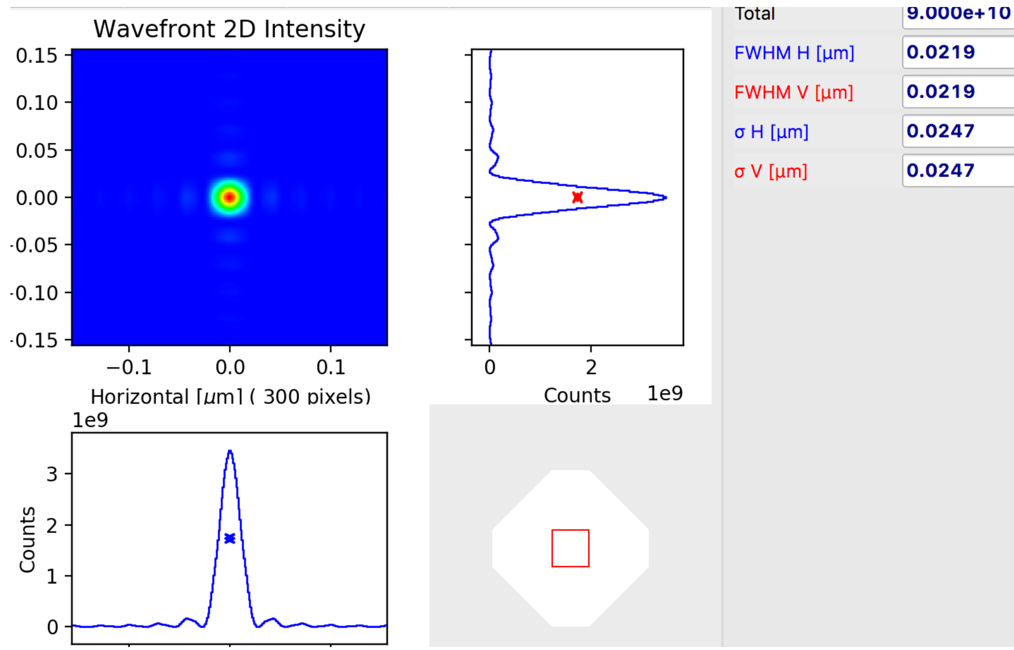




54

2D Oversimplified Beamline





Theoretical consideration

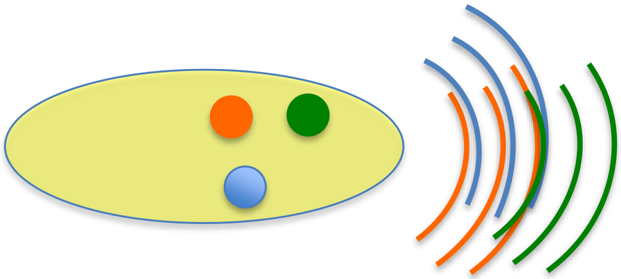
The diffraction by a DxD square aperture of a collapsing spherical wavefront has an intensity distribution proportional to $\mathrm{sinc}^2(kDx/2f)\ \mathrm{sinc}^2(kDy/2f)$, where $k=2\pi/\lambda$, x(y) is the horizontal (vertical) coordinate, and f is the distance aperture-focus (D/f is the divergence). Considering that the FWHM of $\mathrm{sinc}^2(x)$ is approximately 2.78, one obtains a FWHM=0.885 * wavelength / divergence=18nm

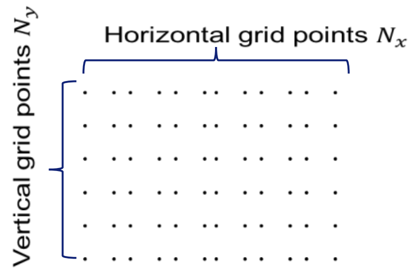
Partial coherence

Partial coherence is due to the electron beam emittance

Multielectron Calculation (SRW)

$$W(x_1, y_1, x_2, y_2, z, \omega) = \langle E^*(x_1, y_1, z, \omega) E(x_2, y_2, z, \omega) \rangle$$

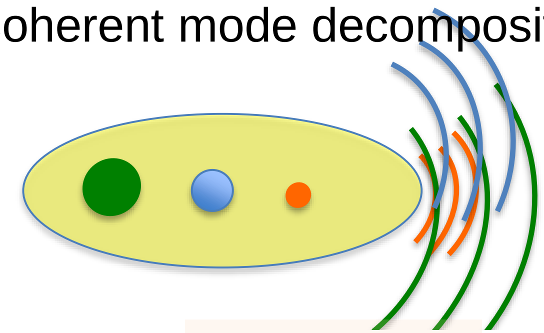




$N_x, N_y \in [100{,}1000]$

W ~ 10^8 – 10^{12} (Gb-Tb)
Propagate: 4D integrals

Coherent mode decomposition



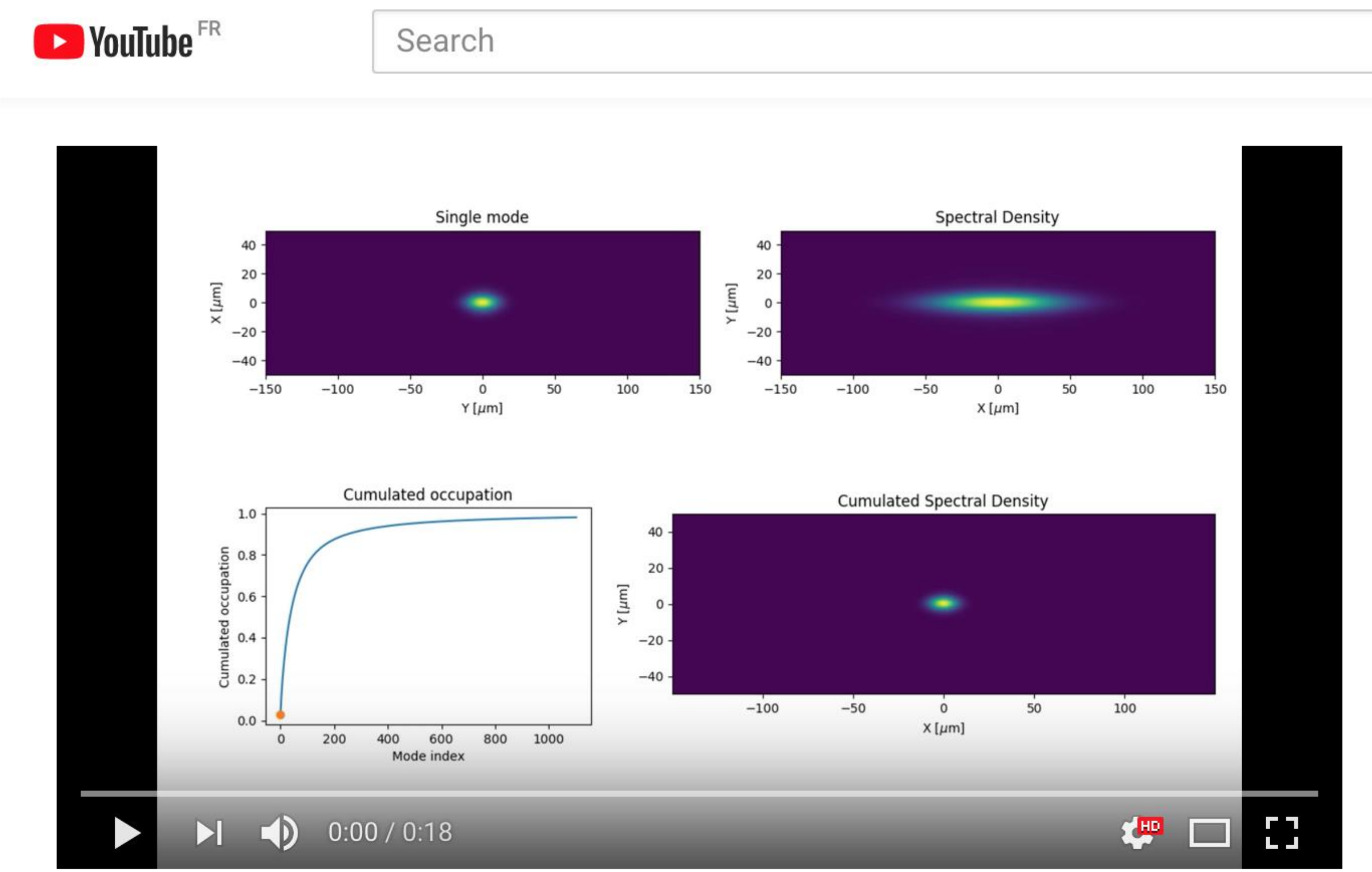
http://dx.doi.org/10.1209/0295-5075/119/34004
Mark Glass, Manuel Sanchez del Rio (2017) Coherent modes of X-ray beams emitted by undulators in new storage rings EPL (Europhysics Letters) 119

4D function

2D functions

$$W(x_1, y_1, x_2, y_2, \omega) = \sum_m \lambda_m(\omega) \phi_m^*(x_1, y_1, \omega) \phi_m(x_2, y_2, \omega)$$

Store m x N x N Propagate: 2D integrals



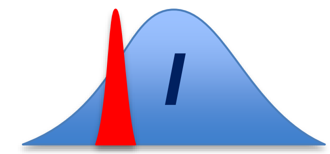
Coherent modes of synchrotron radiation for EBS

https://youtu.be/h24RrJZaQ80

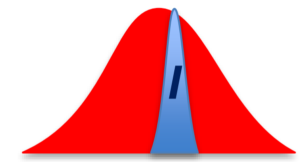
Example approximating the Undulator by the Gaussian Shell-model

SIMPLE CASE: 1D Gaussian SHELL-MODEL

$$W(x_1, x_2, \omega) = A^2 e^{-(x_1^2+x_2^2)/(4\sigma_I^2)} e^{-(x_2-x_1)^2/(2\sigma_\mu^2)}$$



Both intensity (Spectral Density) and correlation (Spectral Degree of Coherence) are Gaussians



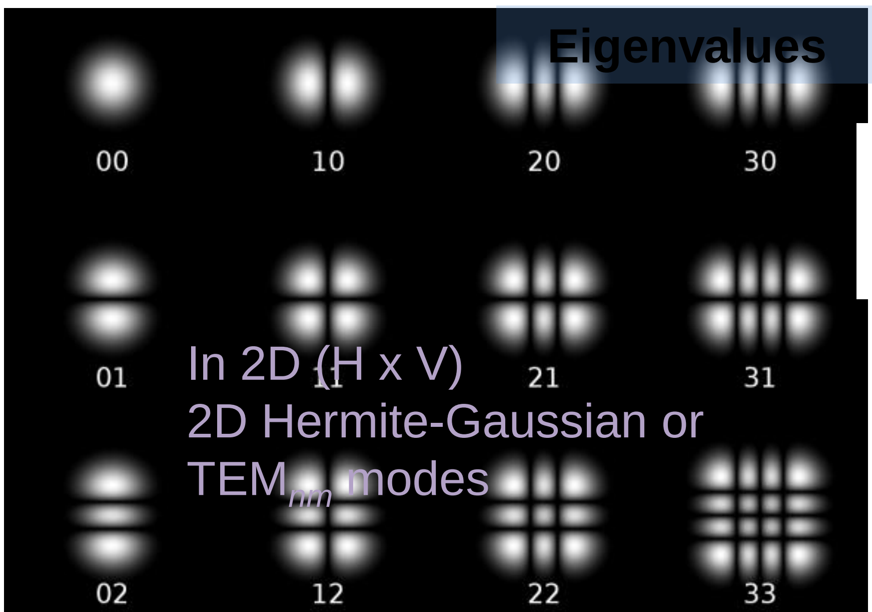
$\sigma_I >> \sigma_\mu$ the source is mostly incoherent (quasi homogeneous)

$\sigma_I << \sigma_\mu$ is mostly coherent

$$W(x_1, x_2, \omega) = \sum_n \boxed{\lambda_n(\omega)} \boxed{\phi_n^*(x_1, \omega)} \boxed{\phi_n(x_2, \omega)},$$

$$\beta = \frac{\sigma_\mu}{\sigma_I}$$

Eigenvalues



In 2D (H x V)
2D Hermite-Gaussian or TEM_{nm} modes

Eigenfunctions (Hermite-Gaussian modes)

$$\phi_n(x) = \left(\frac{2c}{\pi}\right)^{1/4} \frac{1}{\sqrt{2^n n!}} H_n(x\sqrt{2c}) e^{-cx^2}$$

Magic property: Propagation invariance

In the first mode (Gaussian) : $\boxed{s_x s_q = \frac{l}{4p}}$

The spectrum of coherent modes

Eigenvalues

$$\lambda_n = A\left(\frac{\pi}{a+b+c}\right)^{1/2}\left(\frac{b}{a+b+c}\right)^n$$

$$a(\omega) = \frac{1}{4{\sigma_I}^2(\omega)}, \qquad b(\omega) = \frac{1}{2{\sigma_\mu}^2(\omega)} \qquad c = (a^2 + 2ab)^{1/2}$$

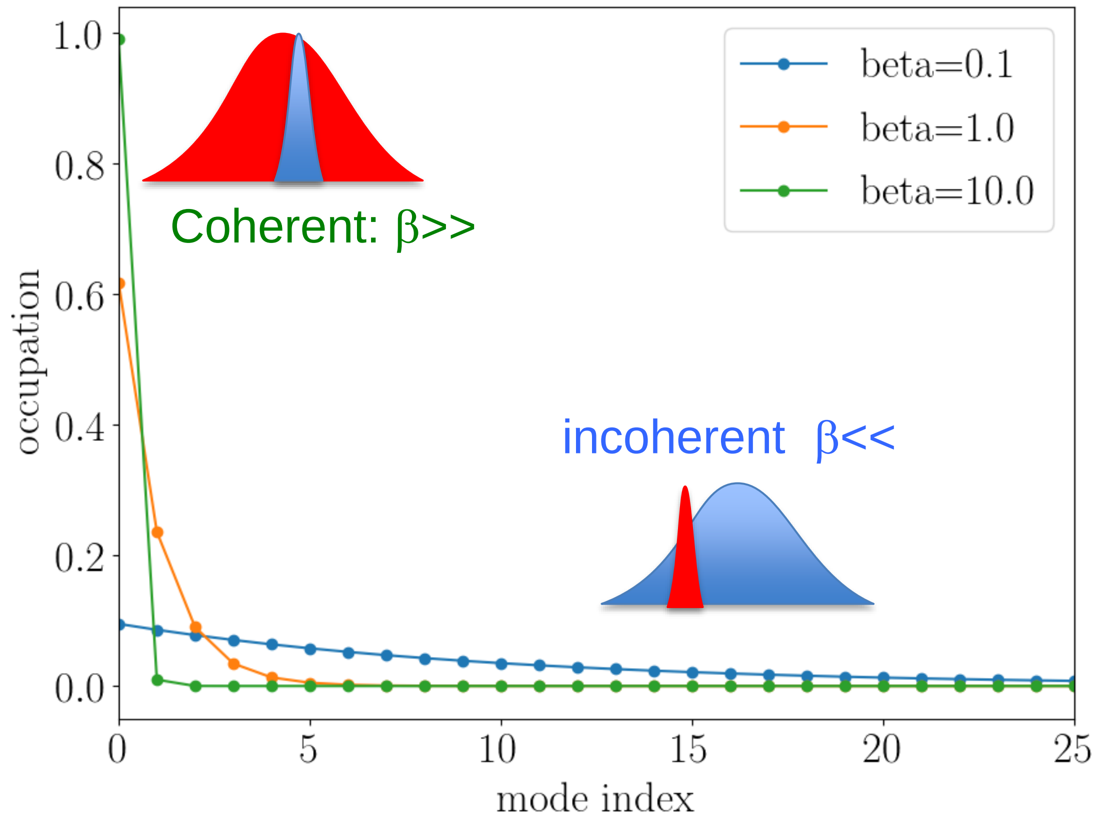
$$\beta = \frac{\sigma_\mu}{\sigma_I}$$

Mode occupation:

$$\eta_i(\omega) = \frac{\lambda_i(\omega)}{\sum_{n=0}^{\infty} \lambda_n(\omega)}$$

Coherent fraction:

$$CF = \frac{\lambda_0}{\sum \lambda_n}$$



In general we cannot apply Gaussian Shell-model to synchrotron