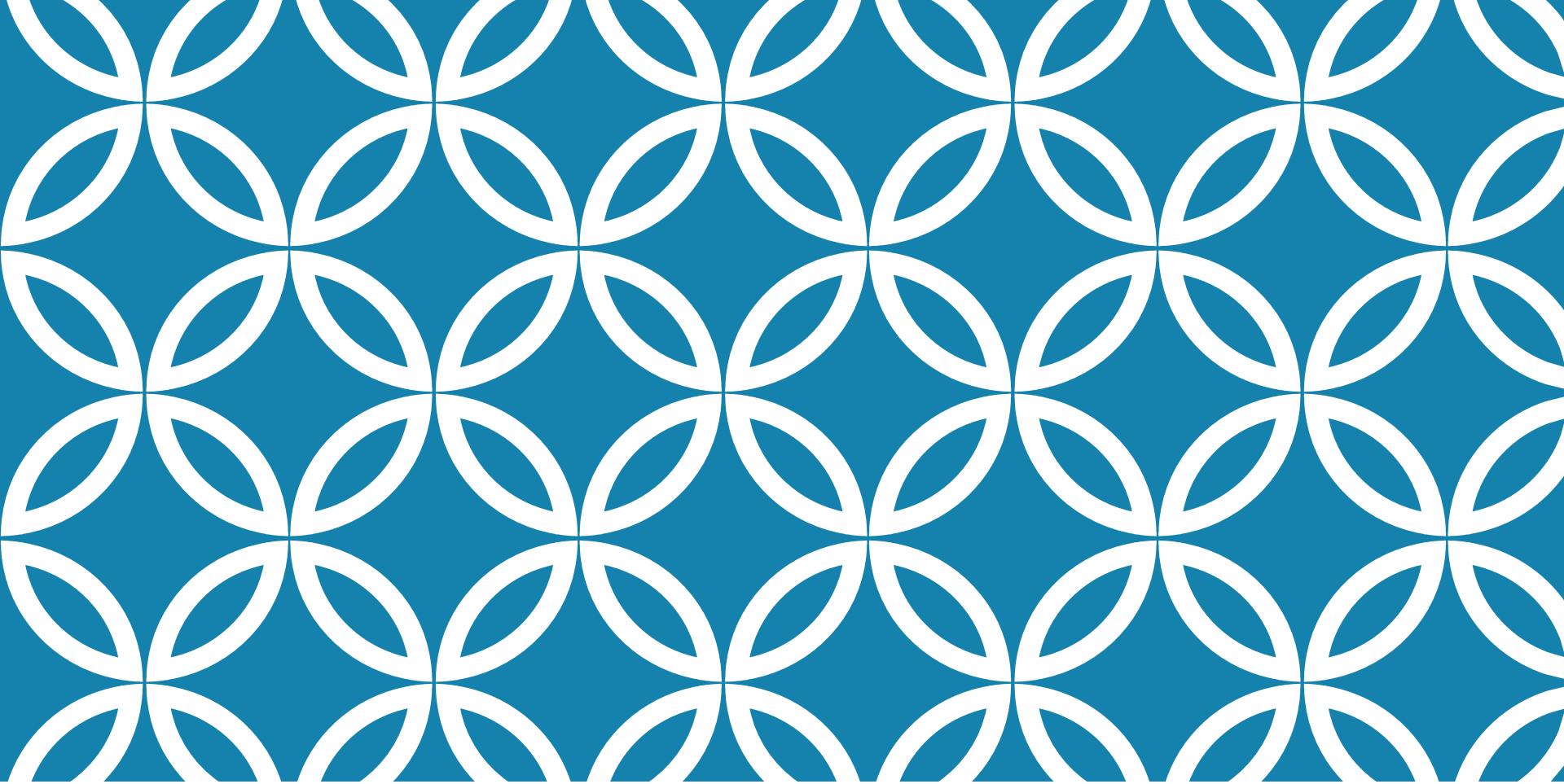


BASED NUMBER

AGENDA

Number bases

Range of possible numbers

Conversion between number bases

DEFINITIONS

The Base of a number system – how many different digits (incl. zero)

Base 2:	0, 1
Base 5:	0, 1, 2, 3, 4
Base 8:	0, 1, 2, 3, 4, 5, 6, 7
Base 10:	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Base 16:	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

COMMON NUMBER SYSTEMS

System	Base	Symbols	Used by humans?	Used in computers?
Decimal	10	0, 1, … 9	Yes	No
Binary	2	0, 1	No	Yes
Octal	8	0, 1, … 7	No	Yes
Hexa-decimal	16	0, 1, … 9, A, B, … F	No	Yes

POSITIONAL DECIMAL SYSTEM

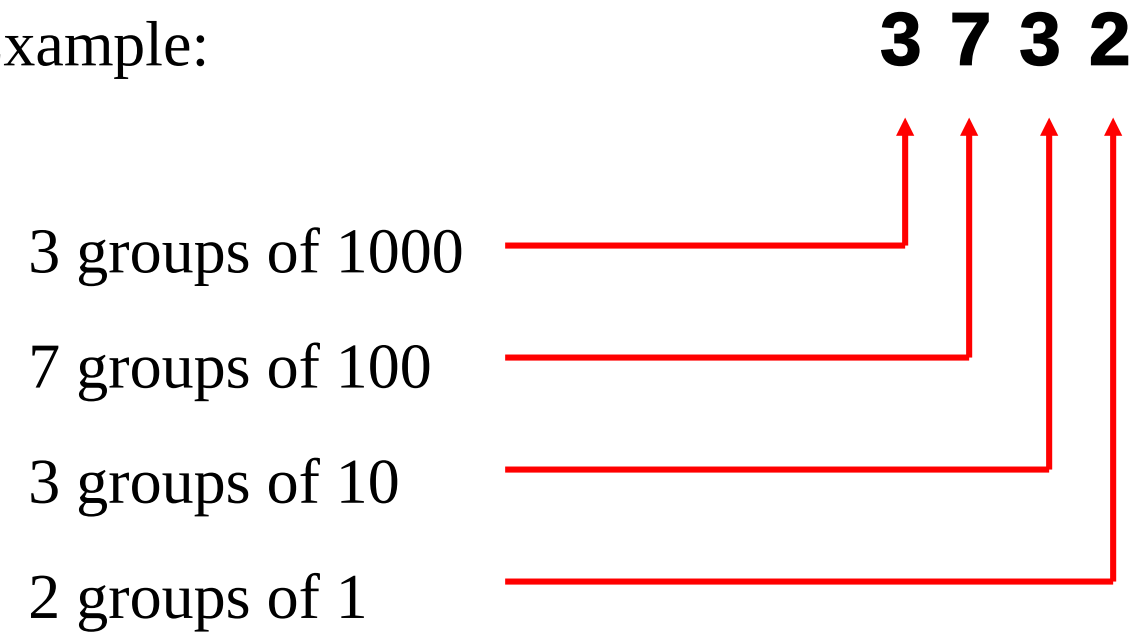
The number 125 means:

1 group of 100	($100 = 10^2$)
2 groups of 10	($10 = 10^1$)
5 groups of 1	($1 = 10^0$)

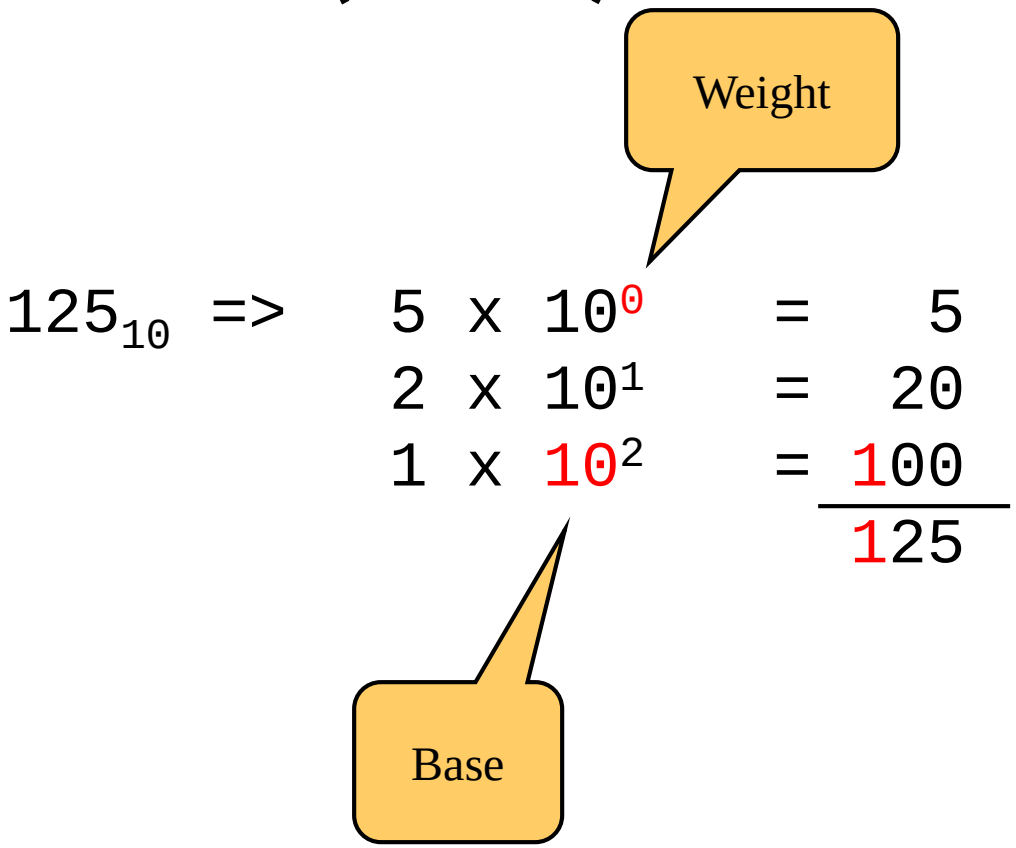
PLACE VALUES (1 OF 2)

In our usual positional number system, the meaning of a digit depends on where it is located in the number

Example:



PLACE VALUES (2 OF 2)



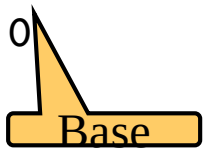
REPRESENTING IN BASES: 10, 2, 8, 16

$865_{10} = 8 \times 10^2 + 6 \times 10^1 + 5 \times 10^0 = 800 + 60 + 5$

$1011_2 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 8 + 2 + 1 = 11_{10}$

$25_8 = 2 \times 8^1 + 5 \times 8^0 = 16 + 5 = 21_{10}$

$A7_{16} = 10 \times 16^1 + 7 \times 16^0 = 160 + 7 = 167_{10}$



Note: The subscript naming the base is itself given in base ten (10), by convention.

Exercise: What is the base 10 number of 1234_{10}, 10101_2, 1234_8, $123F_{16}$?

ANSWER

What is the base 10 number of 1234_{10}, 10101_2, 1234_8, $123F_{16}$?

$1234_{10} = 1 \times 10^2 + 2 \times 10^1 + 2 \times 10^1 + 3 \times 10^0$

$= 1000 + 200 + 60 + 5$

$10101_2 = 1 \times 2^4 + 1 \times 2^2 + 1 \times 2^0 = 16 + 4 + 1 = 21_{10}$

$1234_8 = 1 \times 8^3 + 2 \times 8^2 + 3 \times 8^1 + 4 \times 8^0$

$= 512 + 128 + 24 + 4 = 668_{10}$

$123F_{16} = 1 \times 16^3 + 2 \times 16^2 + 3 \times 16^1 + 15 \times 16^0$

$= 4096 + 512 + 48 + 15 = 4671_{10}$

COUNTING IN BASES

Decimal	Binary	Octal	Hexa-decimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7

COUNTING IN BASES (CONT.)

Decimal	Binary	Octal	Hexa-decimal
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

COUNTING IN BASES (CONT.)

Decimal	Binary	Octal	Hexa-decimal
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14
21	10101	25	15
22	10110	26	16
23	10111	27	17

ESTIMATING MAGNITUDE: BINARY

$1101\ 0110_2 = 214_{10}$

$1101\ 0110_2 > 192_{10}$ (128 + 64 + additional bits to the right)

Place	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Value	128	64	32	16	8	4	2	1
Evaluate	1 x 128	1 x 64	0 x 32	1 x16	0 x 8	1 x 4	1 x 2	0 x 1
Sum for Base 10	128	64	0	16	0	4	2	0

RANGE OF POSSIBLE NUMBERS

$R = B^K$ where

- R = range
- B = base
- K = number of digits

Example #1: Base 10, 2 digits

- $R = 10^2 = 100$ different numbers (0…99)

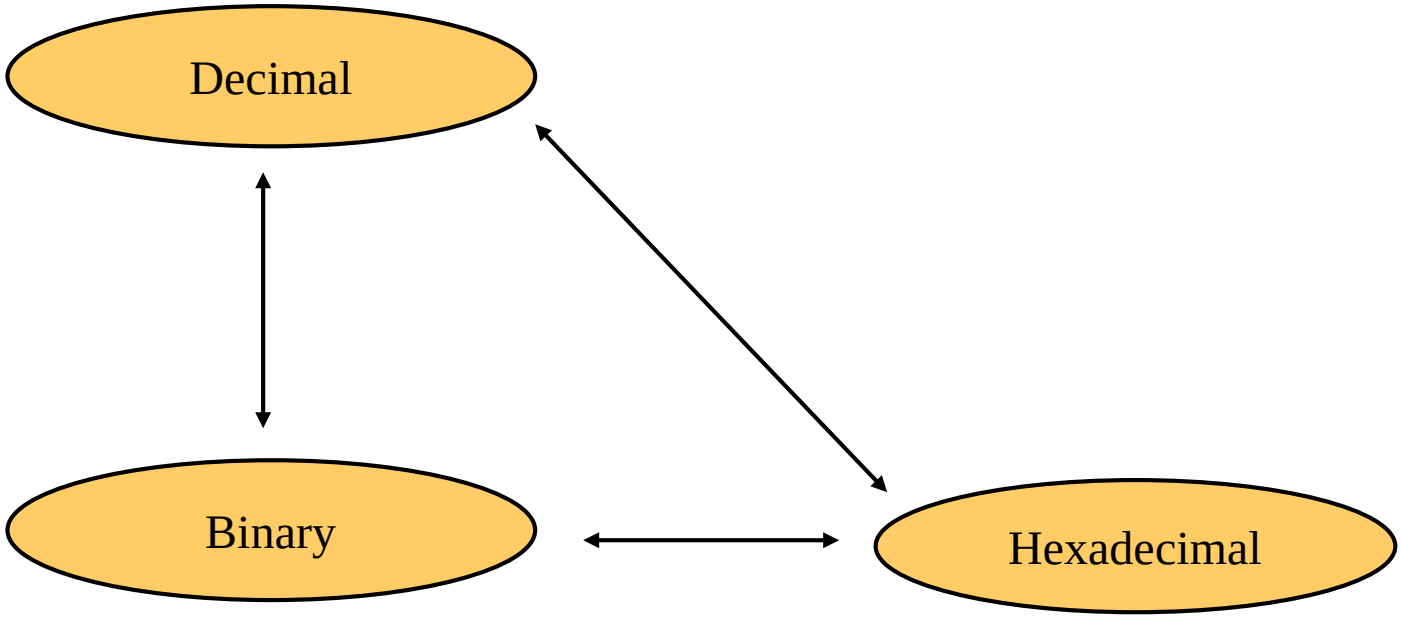
Example #2: Base 2, 16 digits

- $R = 2^{16} = 65{,}536$ or 64K
- 16-bit PC can store 65,536 different number values

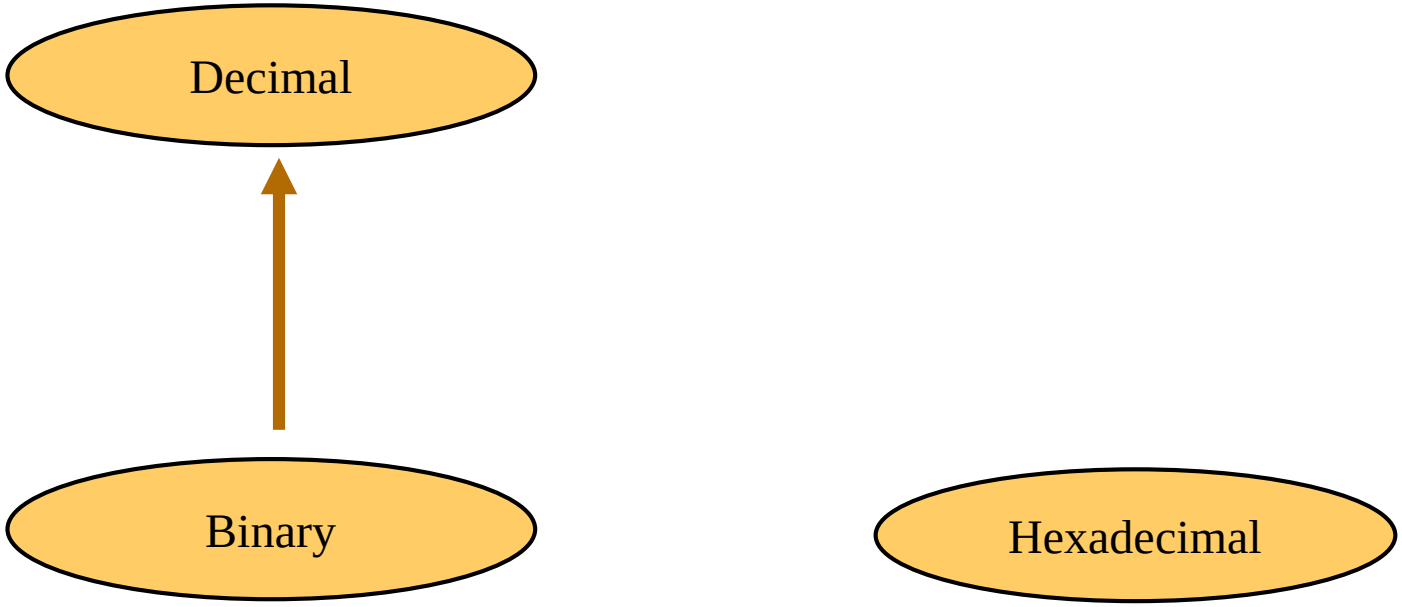
DECIMAL RANGE FOR BIT WIDTHS

Bits	Range
1	2 (0 and 1)
4	16 (0 to 15)
8	256
10	1,024 (1K)
16	65,536 (64K)
20	1,048,576 (1M)
32	4,294,967,296 (4G)
64	Approx. 1.6×10^{19}
128	Approx. 2.6×10^{38}

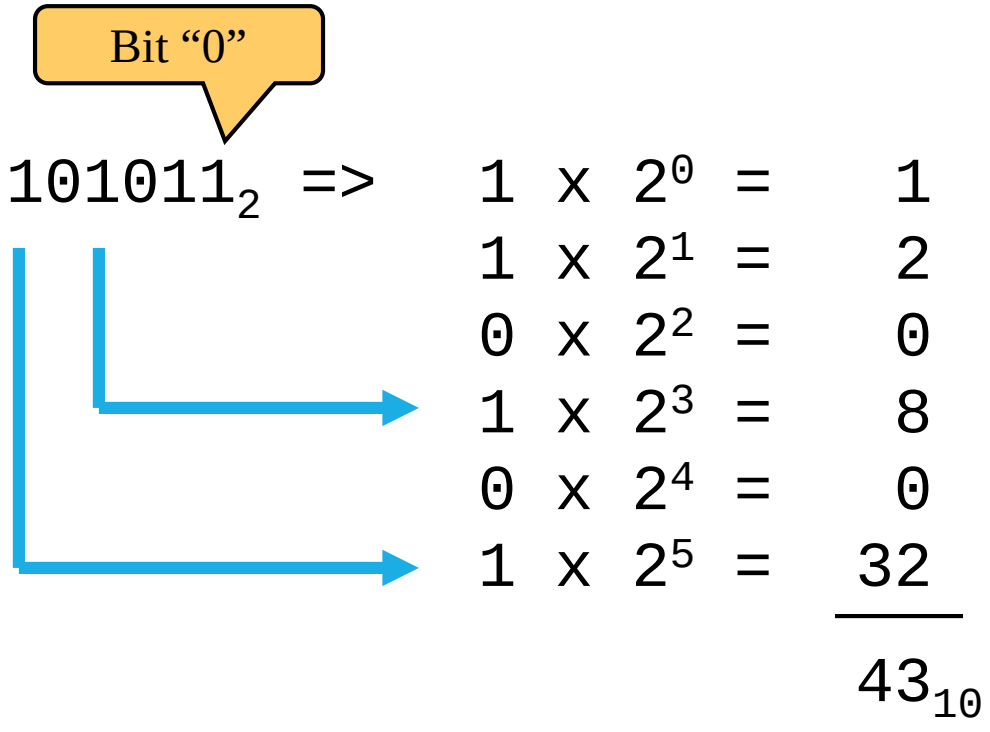
CONVERSION AMONG BASES



BINARY TO DECIMAL



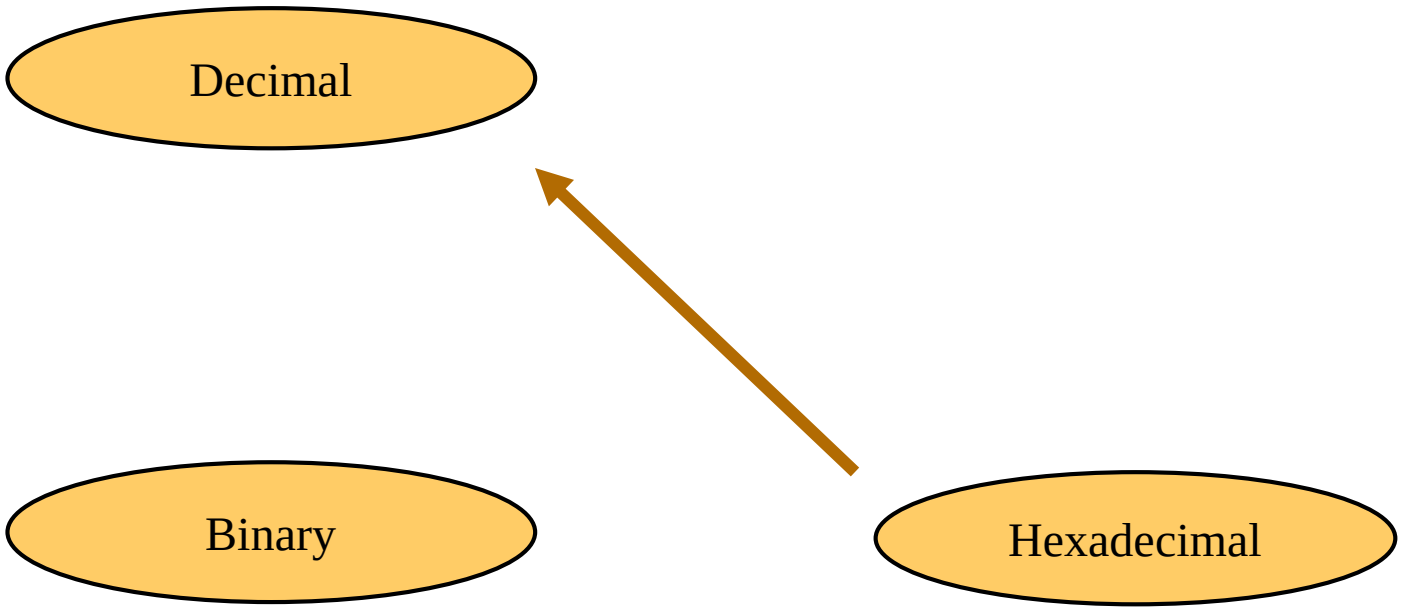
BINARY TO DECIMAL



Technique

- Multiply each bit by 2^n, where *n* is the "weight" of the bit
- The weight is the position of the bit, starting from 0 on the right
- Add the results

HEXADECIMAL TO DECIMAL



HEXADECIMAL TO DECIMAL

ABC_{16} =>

C x 16^0 = 12 x 1 = 12
B x 16^1 = 11 x 16 = 176
A x 16^2 = 10 x 256 = 2560

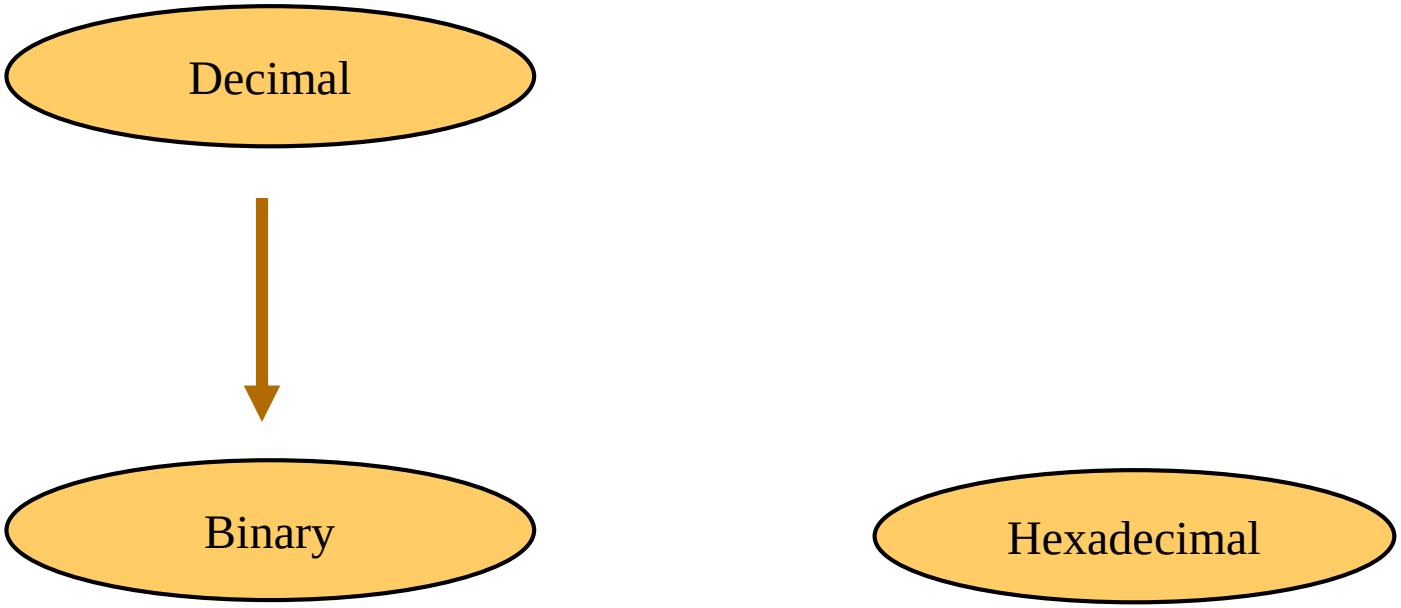
2748_{10}

Technique

- Multiply each bit by 16^n, where *n* is the "weight" of the bit
- The weight is the position of the bit, starting from 0 on the right
- Add the results

Note: $16^0 = 1$, $16^1 = 16$,
$16^2 = 256$ $16^3 = 4096$, *Etc.*

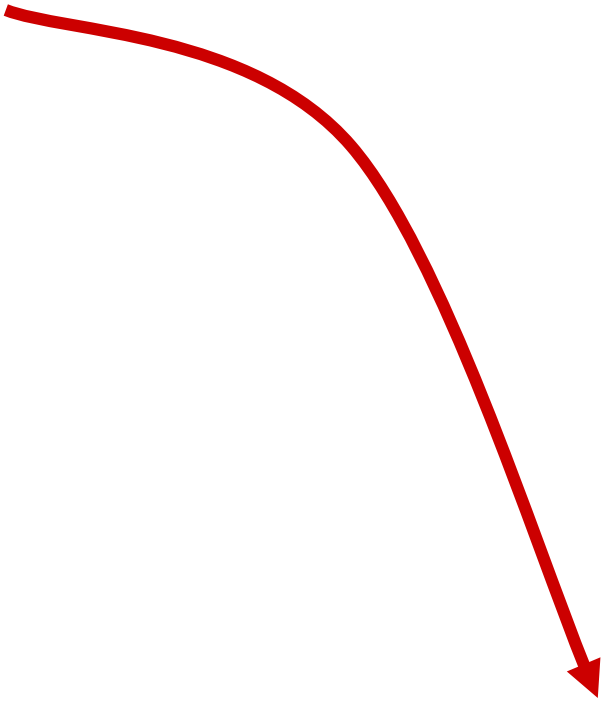
DECIMAL TO BINARY



DECIMAL TO BINARY

$125_{10} = ?_2$

2	125	
2	62	1
2	31	0
2	15	1
2	7	1
2	3	1
2	1	1
	0	1

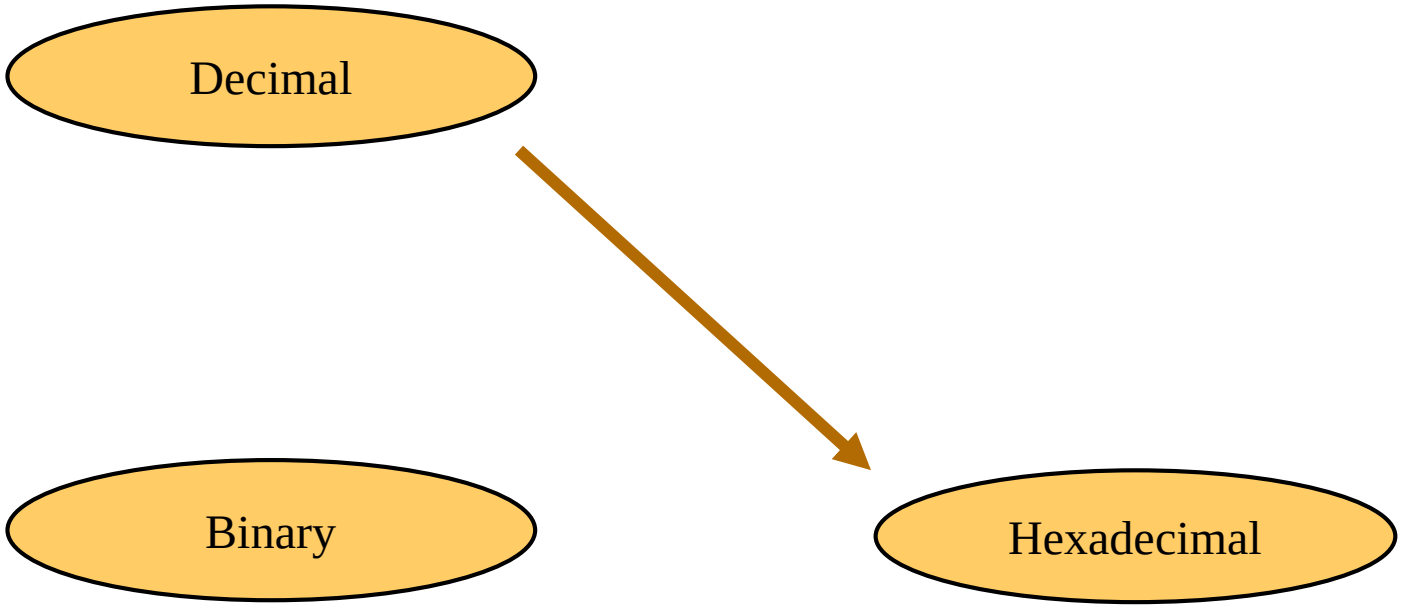


$125_{10} = 1111101_2$

Technique

- Divide by two, keep track of the remainder
- First remainder is bit 0 (LSB, least-significant bit)
- Second remainder is bit 1
- Etc.

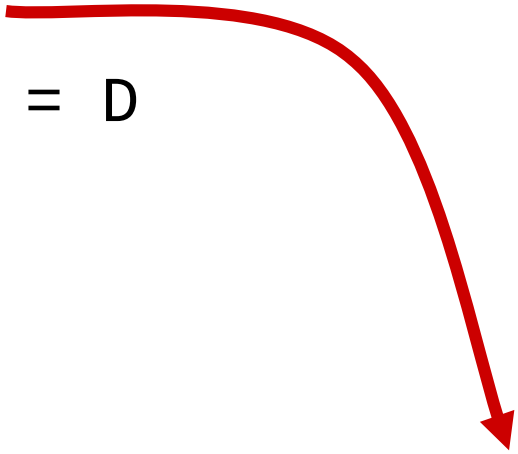
DECIMAL TO HEXADECIMAL



DECIMAL TO HEXADECIMAL

$1234_{10} = ?_{16}$

16	1234	
16	77	2
16	4	13 = D
	0	4

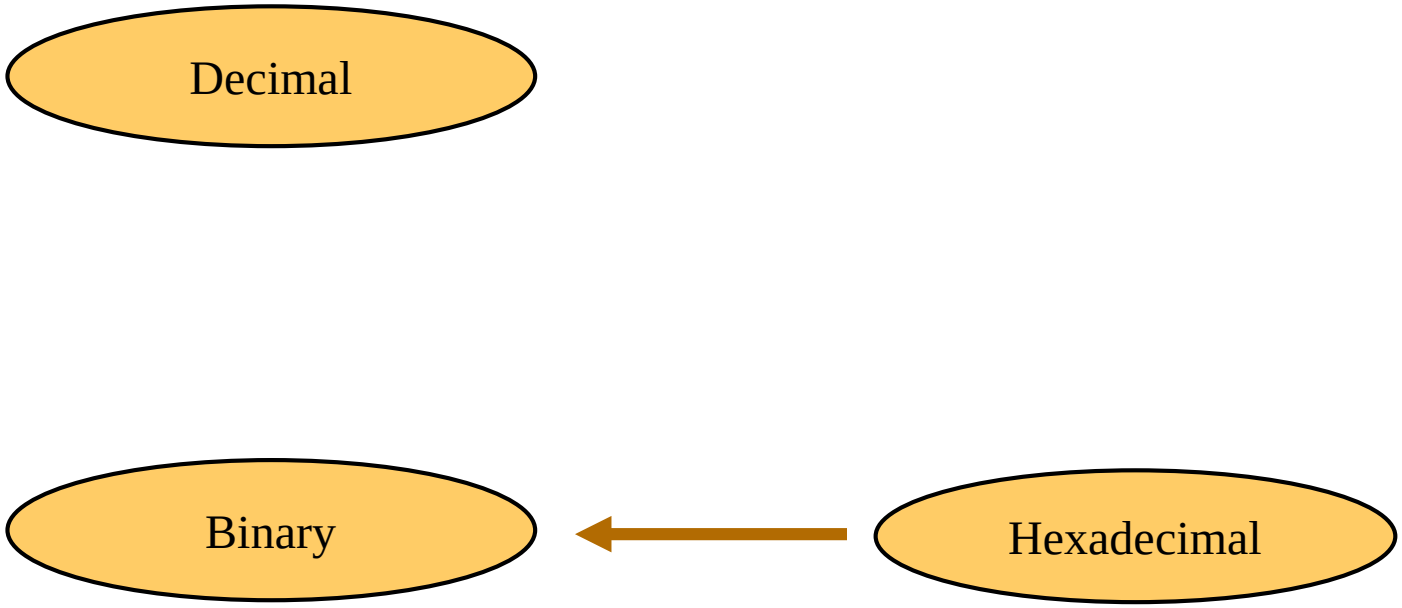


$1234_{10} = 4D2_{16}$

Technique

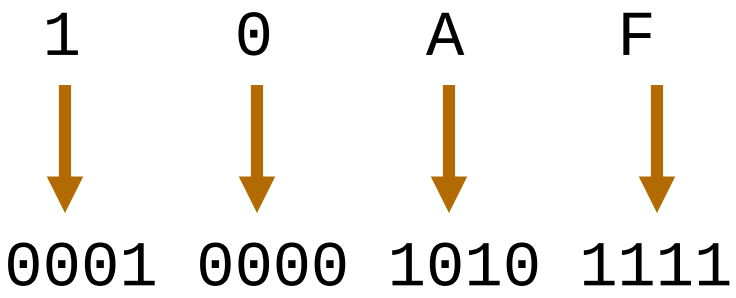
- Divide by 16
- Keep track of the remainder
- As in decimal to binary

HEXADECIMAL TO BINARY



HEXADECIMAL TO BINARY

$10AF_{16} = ?_2$

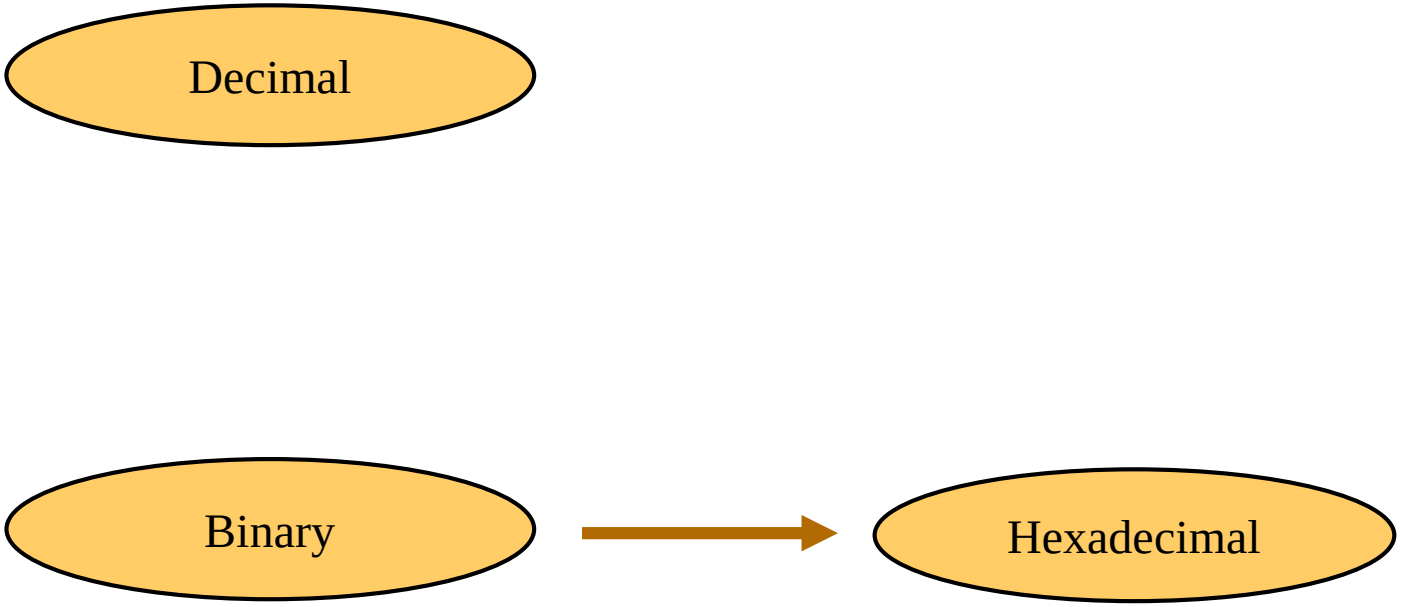


$10AF_{16} = 0001000010101111_2$

Technique

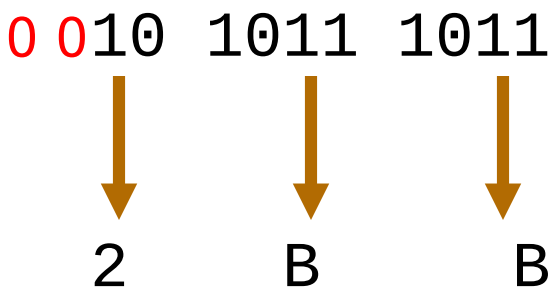
- Convert each hexadecimal digit to a 4-bit equivalent binary representation

BINARY TO HEXADECIMAL



BINARY TO HEXADECIMAL

$1010111011_2 = ?_{16}$



$1010111011_2 = 2BB_{16}$

Technique

- Group bits in fours, starting on right
- Convert to hexadecimal digits

EXERCISE – CONVERT …

Group as a 3 people, try to fill in the black

Write how to calculate each number in the paper

Random student will be picked to show the answer

Decimal	Binary	Hexa-decimal
33		
	1110101	
		1AF

COMMON POWERS

Base 10

Power	Preface	Symbol	Value
10^{-12}	pico	p	.000000000001
10^{-9}	nano	n	.000000001
10^{-6}	micro	μ	.000001
10^{-3}	milli	m	.001
10^{3}	kilo	k	1000
10^{6}	mega	M	1000000
10^{9}	giga	G	1000000000
10^{12}	tera	T	1000000000000

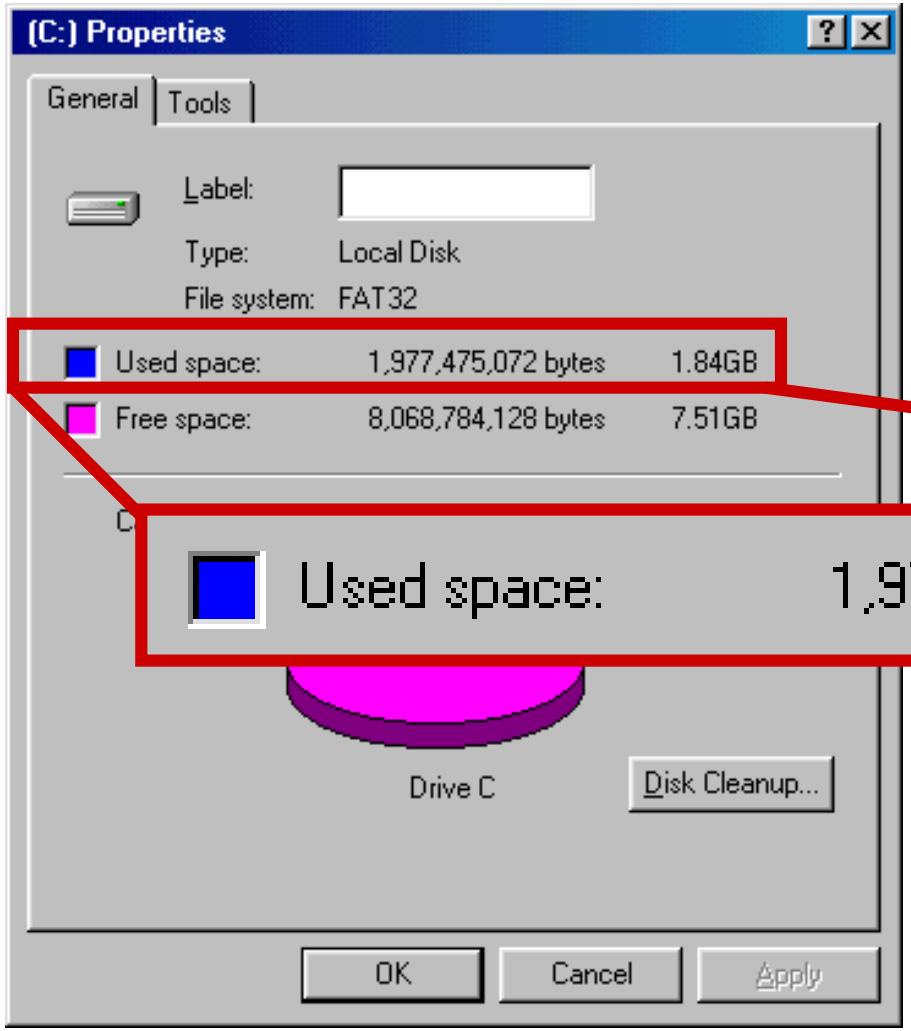
COMMON POWERS (2 OF 2)

Base 2

Power	Preface	Symbol	Value
2^{10}	kilo	k	1024
2^{20}	mega	M	1048576
2^{30}	Giga	G	1073741824

- What is the value of “k”, “M”, and “G”?
- In computing, particularly w.r.t. memory, the base-2 interpretation generally applies

EXAMPLE



1. Double click on My Computer
2. Right click on C:
3. Click on Properties

Used space: 1,977,475,072 bytes 1.84GB

$/ 2^{30} =$

Q&A