953103 Programming Logical Thinking

Based Number

Agenda

- Number bases
- Range of possible numbers
- Conversion between number bases

Definitions

- The Base of a number system - how many different digits (incl. zero) are used in the system.

Base 2:	0, 1
Base 5:	0, 1, 2, 3, 4
Base 8:	0, 1, 2, 3, 4, 5, 6, 7
Base 10:	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Base 16:	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Common Number Systems

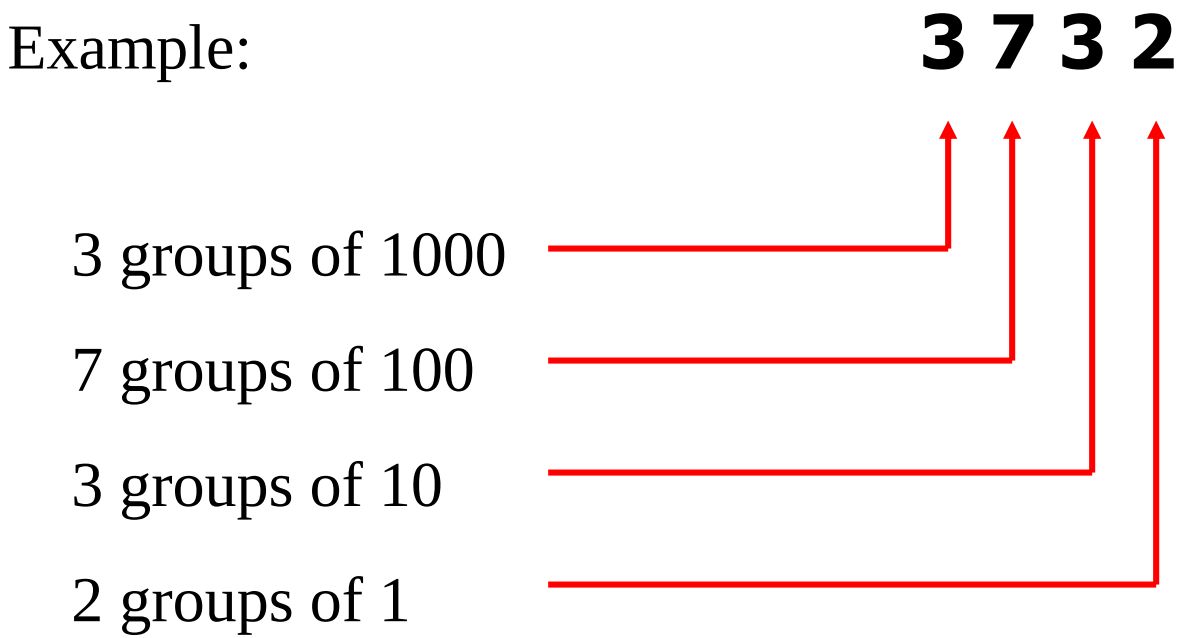
System	Base	Symbols	Used by humans?	Used in computers?
Decimal	10	0, 1, … 9	Yes	No
Binary	2	0, 1	No	Yes
Octal	8	0, 1, … 7	No	Yes
Hexa-decimal	16	0, 1, … 9, A, B, … F	No	Yes

Positional decimal system

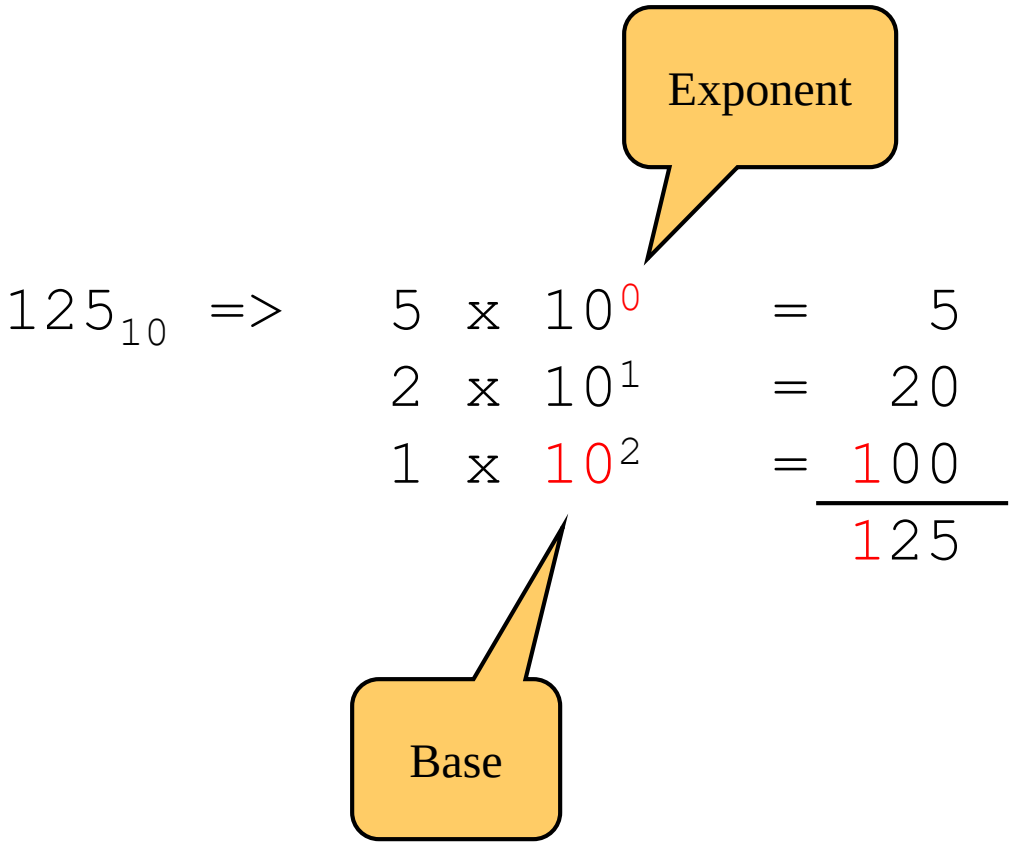
- The number 125 means:
  - 1 group of 100 ($100 = 10^2$)
  - 2 groups of 10 ($10 = 10^1$)
  - 5 groups of 1 ($1 = 10^0$)

Place values (1 of 2)

- In our usual positional number system, the meaning of a digit depends on where it is located in the number



Place values (2 of 2)



Representing in bases: 10, 2, 8, 16

- $865_{10} = 8 \times 10^2 + 6 \times 10^1 + 5 \times 10^0 = 800 + 60 + 5$
- $1011_2 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 8 + 2 + 1 = 11_{10}$
- $25_8 = 2 \times 8^1 + 5 \times 8^0 = 16 + 5 = 21_{10}$
- $A7_{16} = 10 \times 16^1 + 7 \times 16^0 = 160 + 7 = 167_{10}$

Base

Note: The subscript naming the base is itself given in base ten (10), by convention.

Counting in bases (1 of 3)

Decimal	Binary	Octal	Hexa-decimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7

Counting in bases (2 of 3)

Decimal	Binary	Octal	Hexa-decimal
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Counting in bases (3 of 3)

Decimal	Binary	Octal	Hexa-decimal
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14
21	10101	25	15
22	10110	26	16
23	10111	27	17

Estimating magnitude: Binary

$1101\ 0110_2 = 214_{10}$

$1101\ 0110_2 > 192_{10}$ (128 + 64 + additional bits to the right)

Place	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Value	128	64	32	16	8	4	2	1
Evaluate	1 x 128	1 x 64	0 x 32	1 x16	0 x 8	1 x 4	1 x 2	0 x 1
Sum for Base 10	128	64	0	16	0	4	2	0

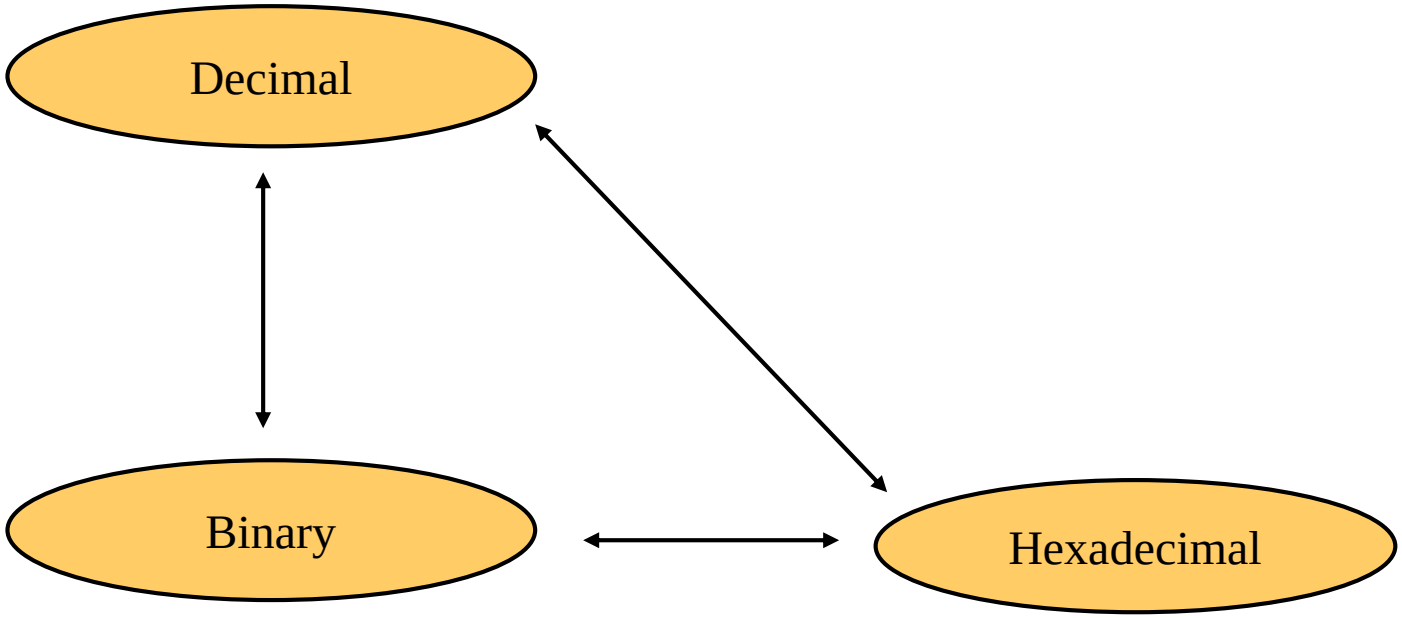
Range of possible numbers

- $R = B^K$ where
  - R = range
  - B = base
  - K = number of digits
- Example #1: Base 10, 2 digits
  - $R = 10^2 = 100$ different numbers (0...99)
- Example #2: Base 2, 16 digits
  - $R = 2^{16} = 65{,}536$ or 64K
  - 16-bit PC can store 65,536 different number values

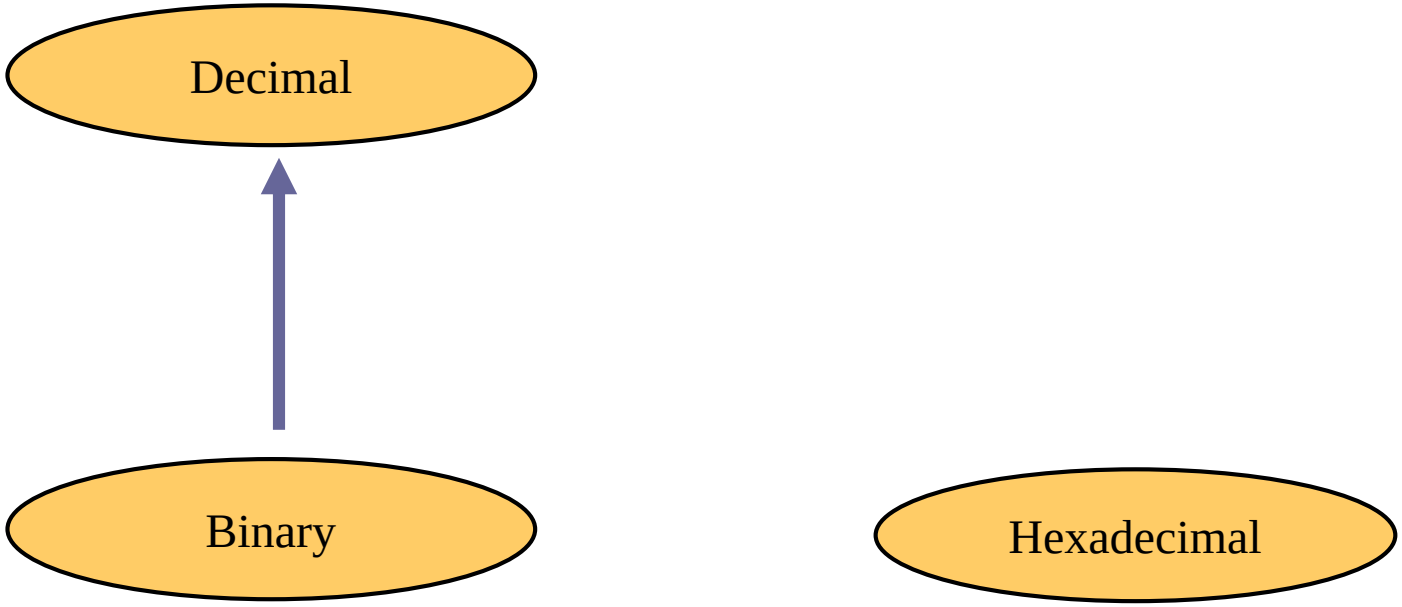
Decimal Range for Bit Widths

Bits	Digits	Range
1	0+	2 (0 and 1)
4	1+	16 (0 to 15)
8	2+	256
10	3	1,024 (1K)
16	4+	65,536 (64K)
20	6	1,048,576 (1M)
32	9+	4,294,967,296 (4G)
64	19+	Approx. 1.6×10^{19}
128	38+	Approx. 2.6×10^{38}

Conversion Among Bases



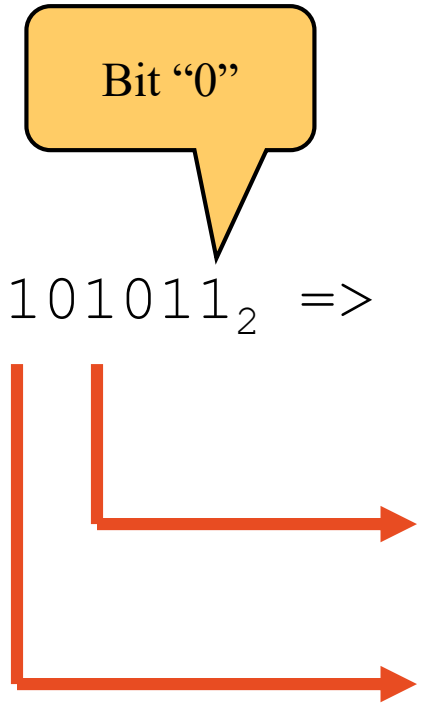
Binary to Decimal (1 of 3)



Binary to Decimal (2 of 3)

- Technique
  - Multiply each bit by 2^n, where *n* is the "exponent" of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

Binary to Decimal (3 of 3)



101011_2 =>

$1 \times 2^0 = 1$

$1 \times 2^1 = 2$

$0 \times 2^2 = 0$

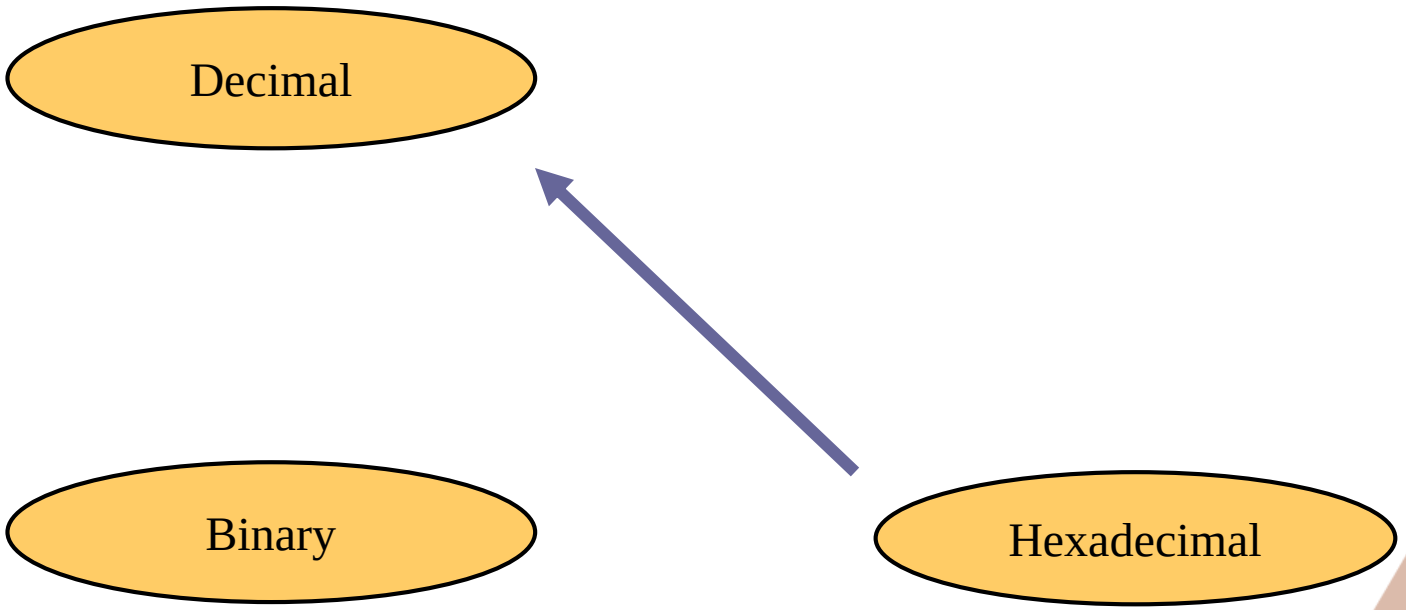
$1 \times 2^3 = 8$

$0 \times 2^4 = 0$

$1 \times 2^5 = 32$

43_{10}

Hexadecimal to Decimal (1 of 3)



Hexadecimal to Decimal (2 of 3)

- Technique
  - Multiply each bit by 16^n, where *n* is the "exponent" of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

Note: $16^0 = 1$, $16^1 = 16$,
$16^2 = 256$ $16^3 = 4096$, *Etc.*

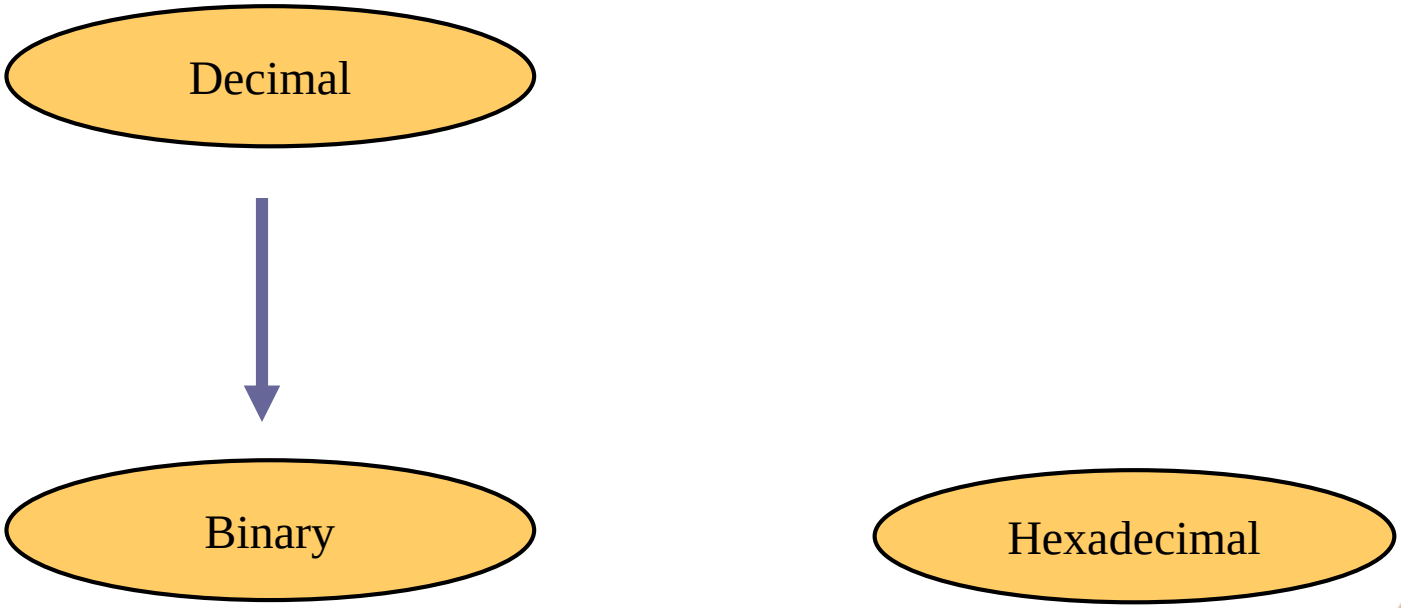
Hexadecimal to Decimal (3 of 3)

ABC_{16} =>

C	x	16^0	=	12	x	1 = 12
B	x	16^1	=	11	x	16 = 176
A	x	16^2	=	10	x	256 = 2560

2748_{10}

Decimal to Binary (1 of 3)



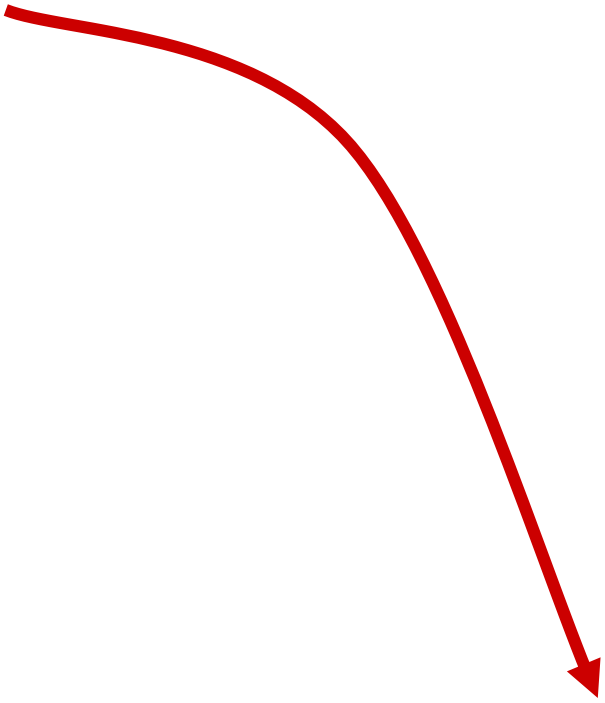
Decimal to Binary (2 of 3)

- Technique
  - Divide by two, keep track of the remainder
  - First remainder is bit 0 (LSB, least-significant bit)
  - Second remainder is bit 1
  - Etc.

Decimal to Binary (3 of 3)

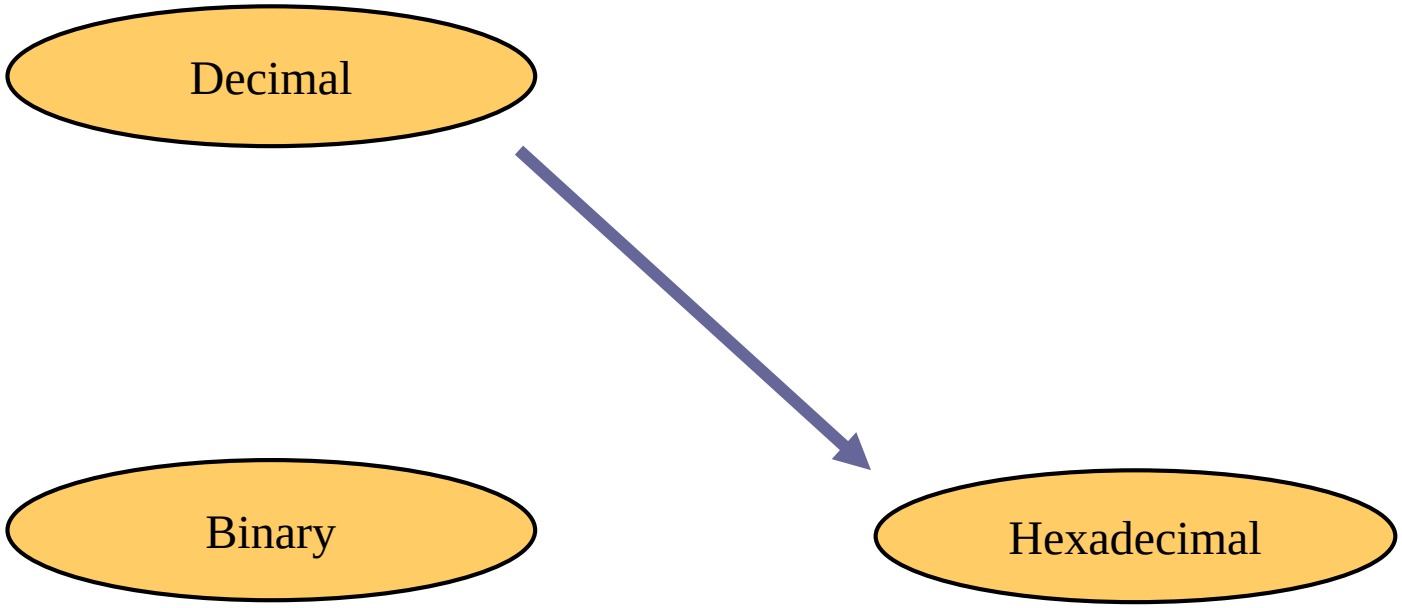
$125_{10} = ?_2$

2	125	
2	62	1
2	31	0
2	15	1
2	7	1
2	3	1
2	1	1
	0	1



$125_{10} = 1111101_2$

Decimal to Hexadecimal (1 of 3)



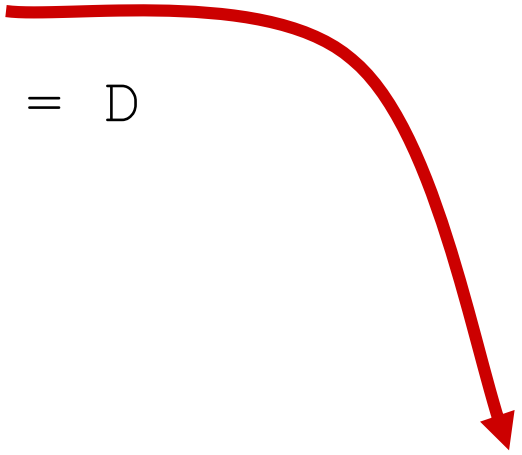
Decimal to Hexadecimal (2 of 3)

- Technique
  - Divide by 16
  - Keep track of the remainder
  - As in decimal to binary

Decimal to Hexadecimal (3 of 3)

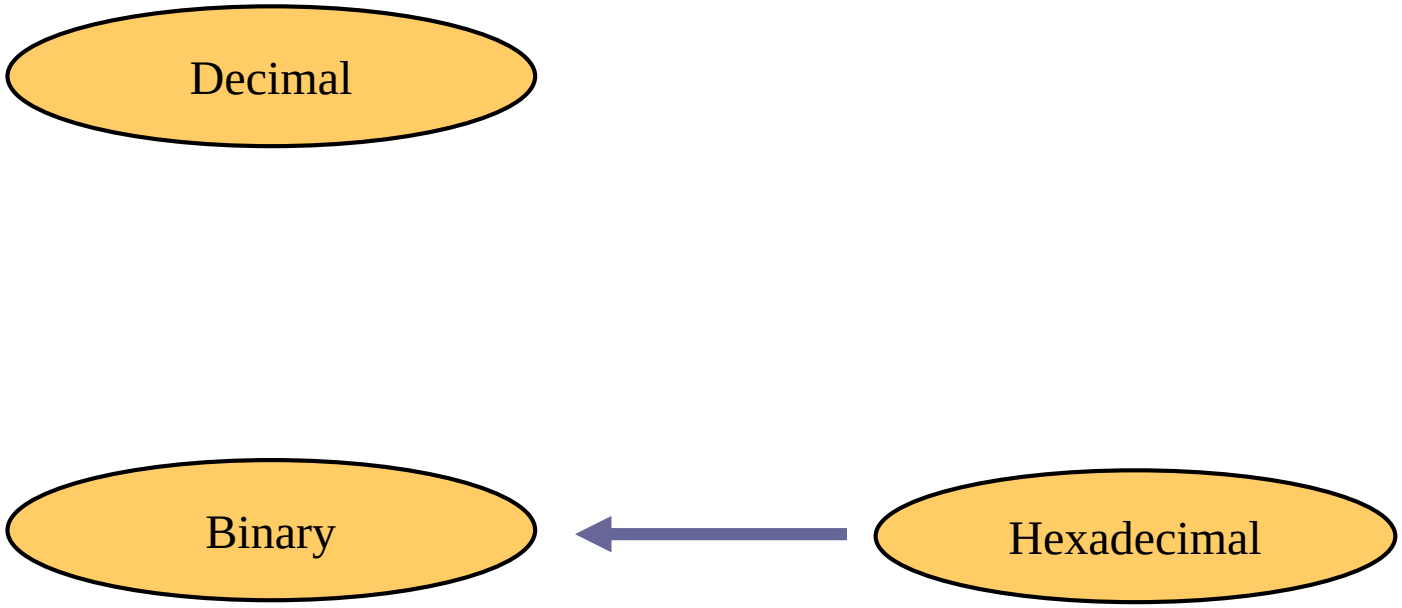
$1234_{10} = ?_{16}$

16	1234	
16	77	2
16	4	13 = D
	0	4



$1234_{10} = 4D2_{16}$

Hexadecimal to Binary (1 of 3)

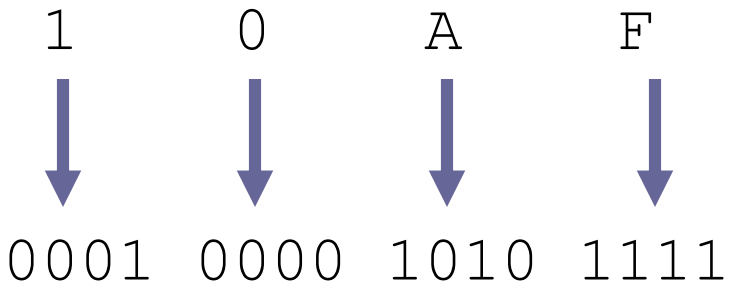


Hexadecimal to Binary (2 of 3)

- Technique
  - Convert each hexadecimal digit to a 4-bit equivalent binary representation

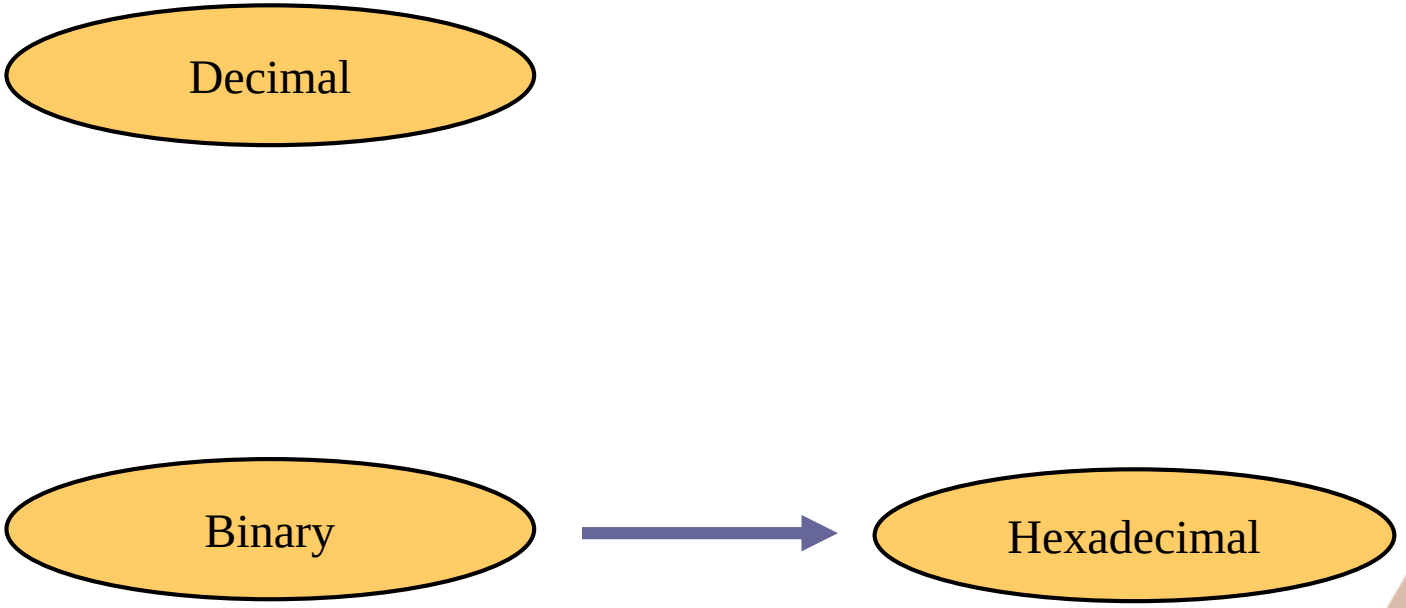
Hexadecimal to Binary (3 of 3)

$10AF_{16} = ?_2$



$10AF_{16} = 0001000010101111_2$

Binary to Hexadecimal (1 of 3)

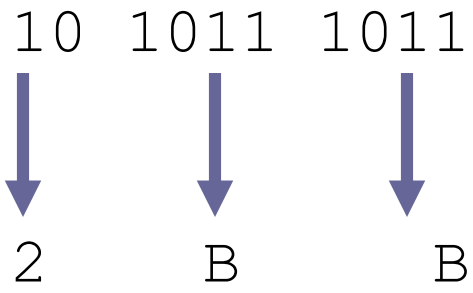


Binary to Hexadecimal (2 of 3)

- Technique
  - Group bits in fours, starting on right
  - Convert to hexadecimal digits

Binary to Hexadecimal (3 of 3)

$1010111011_2 = ?_{16}$



$1010111011_2 = 2BB_{16}$

Exercise - Convert ...

- Each student try to fill in the black
- Write how to calculate each number in the paper
- Random student will be picked to show the answer

Decimal	Binary	Hexa-decimal
33		
	1110101	
		1AF

Common Powers (1 of 2)

- Base 10

Power	Preface	Symbol	Value
10^{-12}	pico	p	.000000000001
10^{-9}	nano	n	.000000001
10^{-6}	micro	μ	.000001
10^{-3}	milli	m	.001
10^{3}	kilo	k	1000
10^{6}	mega	M	1000000
10^{9}	giga	G	1000000000
10^{12}	tera	T	1000000000000

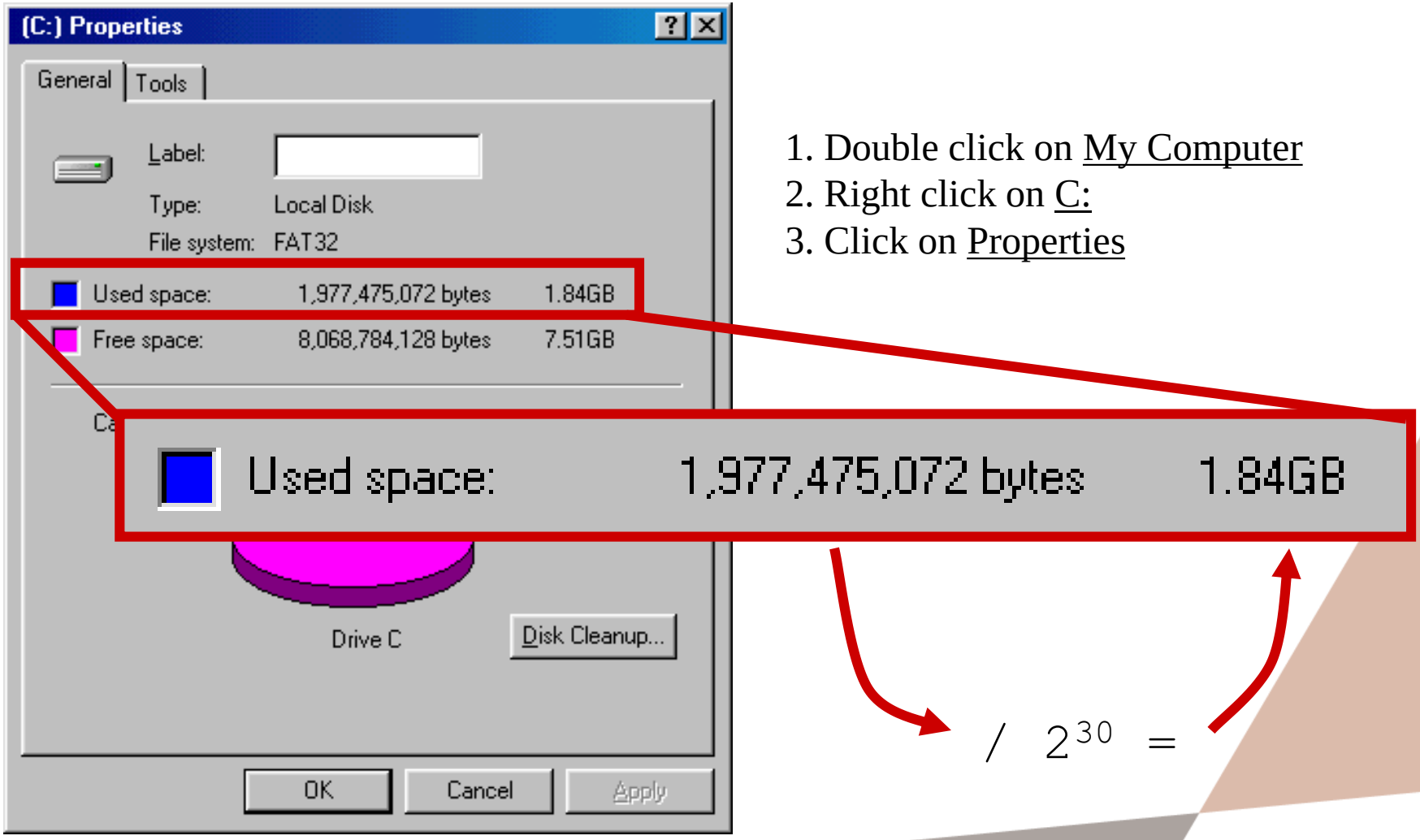
Common Powers (2 of 2)

- Base 2

Power	Preface	Symbol	Value
2^{10}	kilo	k	1024
2^{20}	mega	M	1048576
2^{30}	Giga	G	1073741824

- What is the value of “k”, “M”, and “G”?
- In computing, particularly w.r.t. memory, the base–2 interpretation generally applies

Example



1. Double click on My Computer
2. Right click on C:
3. Click on Properties

Quick Exercise

- Convert 415_{10} to base 8

Quick Exercise

- Convert $FFFF_{16}$ to base 8

Quick Exercise

- Base 60 - sexagesimal system is another base system commonly used in our daily life,

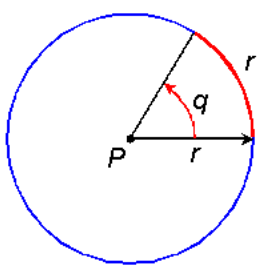
  ... name some of its applications

Quick Exercise

- Base 60 - sexagesimal system is another base system commonly used in our daily life,

  … name some of its applications

- Base 60 is used as the basis of our modern circular coordinate and time measuring system
  - Degrees, minutes, and seconds
  - Minutes and hours



Quick Exercise

- Convert 32949 seconds into HH:MM:SS

Observation

- Converting between bases we are familiar with seems to be simpler than that of we are not.

Quick Exercise

- What about converting 123_5 to base 7

Q&A