Hypothesis Test and Confidence Interval to Confirm Fairness

Becky Desrosiers

2023-11-21

This document reports a hypothesis test conducted with a sample of 100,000 rolls from my Die object with even weights and a confidence interval for the true proportion of Heads. The calculations are done in R.

I import the .csv made from the Die's state data frame without the index column.

```
ht <- read.csv("h-t.csv", header = TRUE)[-1]
head(ht, 5)
```

```
##   X1
## 1  T
## 2  T
## 3  H
## 4  T
## 5  T
```

```
nrow(ht)
```

```
## [1] 100000
```

Then filter for Heads. Heads will be my "success" in terms of proportion.

```
ht %>%
  filter(X1 == "H") %>%
  nrow()
```

```
## [1] 50081
```

$x = 50,081$

$n = 100,000$

The observed proportion of Heads is $\hat{p} = \frac{x}{n} = \frac{50,081}{100,000} =$

```
## [1] 0.50081
```

In order for the proportion to effectively be compared against the normal distribution, $n\hat{p}(1-\hat{p})$ must be greater than 10. In this case, $n\hat{p}(1-\hat{p}) = 25,000 > 10$, so I can proceed.

Hypothesis Test

My null hypothesis is $H_0 : p = 0.5$; the true proportion of Heads rolled is 0.5.

My alternative hypothesis is $H_a : p \neq 0.5$; the true proportion of Heads rolled is not 0.5.

I want to be 99% sure that my coin is fair, so $\alpha = 0.01$.

The sample standard error is found by $se = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{050081(1-0.50081)}{100,000}} = 0.001581137$

The test statistic $z_0 = \frac{\hat{p}-p}{se} = \frac{0.50081-0.5}{0.001581137} =$

```
## [1] 0.5122897
```

The critical value for this test will be $z_{1-\frac{\alpha}{2}} = z_{0.995} =$

```
## [1] 2.575829
```

Since the test statistic does not fall in the critical region, fail to reject the null hypothesis. From the data, I cannot conclude that $p \neq 0.5$.

Confidence Interval

I now construct a 99% confidence interval.

$CI = p_0 \pm z_{0.995} \times se = 0.5 \pm 2.575829 \times 0.001581137 =$

```
## [1] 0.4959273
```

```
## [1] 0.5040727
```

From the confidence interval, I can be 99% certain that the true proportion lies between 0.4959273 and 0.5040727,

Conclusion

I conclude that the die is fair.