# q7

October 11, 2019

### 0.1 Question 7: prove that the denoising objective function is strongly convex. What is its strong convexity parameter? Marks: 5

From assignment 1 we know that, for the denoising objective function:

$$\nabla^2 f(x) = \lambda_{reg}(D^*.D) + I$$

Where $\lambda_{reg}$ is the regularization parameter.
We know that the defintion of a strongly convex (twice-differentiable) function is:

$$y^T.\nabla^2 f(x).y \geq \mu||y||_2^2$$

For now, let's call $G = \nabla^2 f(x) = \lambda_{reg}(D^*.D) + I$. Which is a constant that does not depend on $x$.

We then rewrite the strong convexity condition as: $y^T.G.y \geq \mu||y||_2^2$

Starting with left-hand side, we replace the product $G.y$ with $\lambda y$, where $\lambda$ is an eigenvalue of the matrix $G$. And noticing that $y^T.y = ||y||_2^2$. We then have:

$$y^T.G.y = y^T.\lambda.y = \lambda.(y^T.y) = \lambda||y||_2^2 \geq \mu||y||_2^2$$

So, the denoising objective function is strongly convex, and the strong convexity parameter is given by the smallest positive eigenvalue of the hessian (Which will always exist given that $G$ is positive semi-definite, so all eigenvalues are greater than or equal to zero).

Using the function *eigsh* with $\lambda = 4.0$ to compute the strong convexity parameter, we find that $\mu = \lambda_{min} \approx 1$

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