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$$\psi_\mu(x) = \sum_{i=1}^n \sqrt{\mu^2 + x_i^2} \quad \mu > 0 \quad x \in \mathbb{R}^n$$

$$\frac{\partial \psi_\mu(x)}{\partial x_i} = \frac{x_i}{\sqrt{\mu^2 + x_i^2}}, \quad \frac{\partial^2 \psi_\mu(x)}{\partial x_i \partial x_j} = \begin{cases} \frac{-\mu^2}{(\mu^2 + x_i^2)^{3/2}} & i=j \\ 0 & i \neq j \end{cases}$$

$$\therefore \nabla^2 \psi_\mu(x) = \text{diag} \left\{ \frac{-\mu^2}{(\mu^2 + x_i^2)^{3/2}} \right\}_{n \times n} \rightarrow \left[\lim_{x \rightarrow \infty} \nabla^2 \psi_\mu(x) = 0 \right] **$$

→ For $\psi_\mu(x)$ to be convex, $\nabla^2 \psi_\mu(x) \geq 0$

We can see from (*) that all eigenvalues of $\nabla^2 \psi_\mu(x) > 0$ which means that $\nabla^2 \psi_\mu(x)$ is positive semi-definite

$$\therefore \psi_\mu(x) \text{ is Convex} \quad \square$$

→ For $\psi_\mu(x)$ to be δ -Strongly convex, $\nabla^2 \psi_\mu(x) \geq \delta I$, $\delta > 0$

We can see from (*) and (**) that there's no positive constant can bound $\nabla^2 \psi_\mu(x)$ from below.

$$\therefore \psi_\mu(x) \text{ is Not Strongly Convex} \quad \square$$