→ We know that for any L-Smooth function f

$$f(y) \leq f(x) + \nabla f^T(x)\,(y-x) + \frac{L}{2}\|y-x\|_2^2 \qquad \boxed{1}$$

using gradient descent with Step size "t", where

$$y = x - t\nabla f(x) \longrightarrow \text{in } \boxed{1}$$

$$\therefore \boxed{f(x - t\nabla f(x)) \leq f(x) - t\|\nabla f(x)\|_2^2 + \frac{Lt^2}{2}\|\nabla f(x)\|_2^2} \qquad \boxed{2}$$

The R.H.S is a simple quadratic in t, minimized by ∇_t R.H.S $= 0$

$$\nabla_t \text{R.H.S} = -\|\nabla f(x)\|_2^2 + Lt\|\nabla f(x)\|_2^2 = 0 \longrightarrow \boxed{t = \frac{1}{L}} \longrightarrow \text{in } \boxed{2}$$

$$\therefore f(x_{k+1}) \leq f(x_k) - \frac{1}{2L}\|\nabla f(x_k)\|_2^2$$

Subtracting f^* from both sides:

$$\boxed{f(x_{k+1}) - f^* \leq f(x_k) - f^* - \frac{1}{2L}\|\nabla f(x_k)\|_2^2} \qquad \boxed{3}$$

→ We know that for μ-Strong Convex function f:

$$\boxed{f(y) \geq f(x) + \nabla f^T(x)\,(y-x) + \frac{\mu}{2}\|y-x\|_2^2} \qquad \boxed{4}$$

R.H.S is quadratic in y for a fixed x, minimized by ∇_y R.H.S $= 0$

$$\nabla_y \text{R.H.S} = \nabla f(x) + \mu(y-x) = 0 \longrightarrow \boxed{y = x - \frac{1}{\mu}\nabla f(x)} \longrightarrow \text{in } \boxed{4}$$

$$\therefore f(y) \geq f(x) - \frac{1}{2\mu}\|\nabla f(x)\|_2^2$$

put $y = x^* \rightarrow f^* - f(x) \geq \frac{-1}{2\mu}\|\nabla f(x)\|_2^2$

$$\boxed{\therefore \|\nabla f(x)\|_2^2 \geq 2\mu\,(f(x) - f^*)} \qquad \boxed{5}$$

Sub $\boxed{5}$ in $\boxed{3}$ $\rightsquigarrow$ $(f(x_{k+1}) - f^*) \leq (f(x_k) - f^*) - \frac{1}{2L} \cdot 2\mu (f(x_k) - f^*)$

$$\therefore (f(x_{k+1}) - f^*) \leq (f(x_k) - f^*) - \frac{\mu}{L}(f(x_k) - f^*)$$

$$\boxed{(f(x_{k+1}) - f^*) \leq \left[1 - \frac{\mu}{L}\right](f(x_k) - f^*)} \quad \boxed{6}$$

it's easy to see that $\boxed{6}$ is a recurrence relation in $\underline{\underline{k}}$

We can easily see that

$$f(x_k) - f^* \leq \left[1 - \frac{\mu}{L}\right]^T (f(x_0) - f^*)$$

by unrolling the recurrence relation T times.

$\rightarrow$ To get iteration Complexity we put $f(x_k) - f^* \leq \varepsilon$

$$\left[1 - \frac{\mu}{L}\right]^T (f(x_0) - f^*) \leq \varepsilon$$

$$\therefore T \cdot \log\left(\frac{L-\mu}{L}\right) + \log(f(x_0) - f^*) \leq \log \varepsilon$$

$$\therefore T \geq \left[\log \varepsilon - \log(f(x_0) - f^*)\right] \cdot \frac{1}{\log\left(\frac{L-\mu}{L}\right)}$$ -ve number

$$T \geq \left[\log \frac{(f(x_0) - f^*)}{\varepsilon}\right] \cdot \frac{-1}{\log\left(\frac{L-\mu}{L}\right)}$$ +ve number

$$\boxed{T = O(\log(1/\varepsilon))}$$