q2

October 11, 2019

0.1 Question 2: is there a "gap" between the practical convergence rate and the theoretical convergence rate? Note that the denoising objective function is strongly convex. Marks: 5

Yes. The practical convergence rate is significantly better than the theoritical convergence rate (i.e. the algorithm takes much less time than the theoritical bound), especially in the first few iterations (as seen on the log-scale curve)

Since the denoising objective function is strongly convex, the theoritical iteration complexity can be proven to be $k \geq \frac{L}{\mu} \log(\frac{f(x_0) - f^*}{\epsilon})$.

Substiuting the values of $L \approx 33, \mu \approx 1, \epsilon = 0.01$ (See question 7 to see how μ is computed, and using the implemetation above to compute $f(x_0)$ and f^*, we can see that $k >\sim 850$ iterations! While in this experiment, the algorithm converges after ~ 350 iterations.

The reason is that, as mentioned in the lecture, these bounds on convergence rates are based on worst-case analysis, which might not necessiarily be the case in the denoising problem with this particular image. A more realistic estimate might be obtained by doing an average case analysis of the algorithm.

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