→ We Know for any L-Smooth function f:

$$f(y) \leq f(x) + \nabla f^T(x)(y-x) + \frac{L}{2}\|y-x\|_2^2$$

Using gradient descent with step size t, where:

$$y = x - t\nabla f(x)$$

$$\boxed{f(x_k - t_k \nabla f(x)) \leq f(x_k) - \left[t_k - \frac{L}{2}t_k^2\right]\|\nabla f(x_k)\|_2^2} \longrightarrow \boxed{1}$$

For the Armijo Line search, the following Condition must be Satisfied to terminate:

$$\boxed{f(x_k - t_i \nabla f(x_k)) \leq f(x_k) - [t_i \gamma]\|\nabla f(x_k)\|_2^2} \longrightarrow \boxed{2}$$

For both $\boxed{1}$ and $\boxed{2}$ to be satisfied, the following Condition must hold:

$$t^* - \frac{L}{2}t^{*2} = \gamma t^* \longrightarrow \boxed{t^* = \frac{2(1-\gamma)}{L}}$$

The decrease in f is then given by $\frac{2\gamma(1-\gamma)}{L}\|\nabla f(x)\|_2^2$

→ We Can then See why the Constraint $\gamma \in [0, 0.5)$ must hold. be Cause we Know that the max decrease a smooth function Could get would be $\frac{1}{2L}\|\nabla f(x)\|_2^2$, using a step $\frac{1}{L}\nabla f(x)$

For any othe Step, the decreas will be less.

$$\therefore \frac{2(1-\gamma)\gamma}{L} < \frac{1}{2L} \longrightarrow \gamma - \gamma^2 \leq \frac{1}{4} \longrightarrow \boxed{\gamma < \frac{1}{2}}$$

To get #iterations to terminate, we solve:

$$\left(\frac{1}{2}\right)^K \leq \frac{2(1-\gamma)}{L}$$

$$\left(1/2\right)^{K+1} \leq \frac{1-\gamma}{L} \longrightarrow -(K+1)\log(0.5) \geq -\log\left(\frac{1-\gamma}{L}\right)$$

$$(K+1)\log 2 \geq \log\left(\frac{L}{1-\gamma}\right)$$

$$\therefore \boxed{K+1 \geq \frac{\log\left(L/(1-\gamma)\right)}{\log 2}}$$