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The Convex Conjugate of L_1 norm is:

$$f^*(y) = \begin{cases} 0 & |y| < 1 \\ \infty & \text{otherwise} \end{cases}$$

$$f(x) = |x| = f^{**}(x) = \max_{y \in R^d} x^T y - f^*(y) = \max_{|y|<1} x^T y$$

Adding $-\frac{1}{2}\|y\|_2^2$ to ensure R.H.S has a unique maximizer

$$\therefore h(x) = \max_{|y|<1} x^T y - \frac{1}{2}\|y\|_2^2$$

$$= \max_{|y|<1} x^T y - \frac{1}{2} y^T y =$$

$$\therefore \nabla_y \left(x^T y - \frac{1}{2} y^T y\right) = x - y$$

$$\nabla_y = 0 \longrightarrow y = x$$

$$\boxed{\therefore h(x) = \begin{cases} \frac{x^T x}{2} = \frac{\|x\|_2^2}{2} & |x| \leq 1 \\ |x| - \frac{1}{2} & |x| > 1 \end{cases}}$$