$\boxed{12}$

f^* is δ-strongly convex iff : $f^*(y) \geq f^*(x) + g^T(y-x) + \frac{\delta}{2}\|y-x\|^2$
$\forall x,y \in \operatorname{dom} f^* \quad \forall g \in \partial f^*(x)$

if $x \in \partial f^*(y)$, $\bar{x} \in \partial f^*(\bar{y})$, we have:

$\rightarrow f^*(\bar{y}) - f^*(y) \geq x^T(\bar{y}-y) + \frac{\delta}{2}\|y-\bar{y}\|_2^2$

$\rightarrow f^*(y) - f^*(\bar{y}) \geq -\bar{x}^T(\bar{y}-y) + \frac{\delta}{2}\|y-\bar{y}\|_2^2$

Adding these inequalities shows:

$$\delta\|y-\bar{y}\|_2^2 \leq (x-\bar{x})^T(y-\bar{y}) \leq \|x-\bar{x}\|\,\|y-\bar{y}\|$$

Cauchy-Schwartz

putting $y = \nabla f(x)$, $\bar{y} = \nabla f(\bar{x})$

$$\delta\|\nabla f(x) - \nabla f(\bar{x})\|_2^2 \leq \|x-\bar{x}\|\,\|\nabla f(x) - \nabla f(\bar{x})\|$$

$$\boxed{\therefore \|\nabla f(x) - \nabla f(\bar{x})\| \leq \frac{1}{\delta}\|x-\bar{x}\|}$$

$\boxed{\therefore \nabla f(x) \text{ is Lip. Cont. with } L = \frac{1}{\delta}}$