This is a test 2

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This document aims to describe a research proposal for the PhD program in Bioengineering and Robotics, for the curriculum in Advanced and Humanoid Robotics. It is based on the topic "Locomotion planning for multi-legged robotic platforms for traversing unstructured terrains", included in the list of research themes found on the website of the PhD school of the Italian Institute of Technology.

1 Motivation and rationale

There is no doubt, that there has been a growing interest in robotics in the last decades. Catastrophic events, such as the nuclear disaster in Fukushima have made evident, that despite the technological advances achieved in recent years, both theory and practice are still far from developing robots that could face situations and environments which are hazardous to humans, with the same flexibility and adaptability that a human would display. Although robotic platforms can surpass humans and animals on certain tasks (fast computation and decision making, force demanding tasks such as lifting objects), there are some that biological beings perform with extraordinary easiness and elegance. Among these lies the problem of locomotion.

It has been observed that the performance of legged robots surpasses that of wheeled platforms while traversing rough or unstructured terrains. This feature comes at the cost of more difficult ways to model and control the dynamics of the agents involved in the process. This complexity has pushed research to develop advanced techniques in order to tackle the problem of robot locomotion.

Several approaches have been taken to produce locomotion on legged robots. Some take advantage of the underlying mechanic properties of walking, others try to force stability by analyzing quasi-static solutions of the problem online, and the most advanced ones, have taken inspiration from biological systems to emulate walking patterns in robots. From the previously mentioned alternatives, the latter one, has proven to be the most robust against disturbances, and unstructured terrain, by effectively changing gait parameters depending on the situation that the robot is facing. Despite this, there are just a few examples where the agent is able to entirely switch the gait in order to perform a specific task.

All the previously mentioned strategies are based on continuous time models of locomotion. An alternative to this continuous models is the use of Discrete Event Systems (DES), an approach that has been taken in [1], [2]. In particular in [1], the authors make use of a framework based on max-plus algebra systems. This strategy provides several advantages with respect to its continuous counterpart. One of the most notable is that the resulting description is a max-plus linear system, for which linear control techniques can be implemented in order to provide safe switching between different gaits. Due to this max-plus linear property, the computational effort is also reduced, allowing faster reaction times. At current time no paper related to this topic that involves gaits where the robot spends longer times with no support (for example gallop) has been published, although it is mentioned to be studied in the future.

This document proposes the implementation of a locomotion planning strategy based on Discrete Event Systems (DES) on the HyQ robotic platform. This is done by making use of a framework based on max-plus algebra, which offers the possibility to analyze and implement in a max-plus linear way, different gaits for several configurations of legged robots. The main purpose would be to adapt the max-plus algebra framework to generate time references for each leg, using them to design the trajectory of each leg. It is believed that this locomotion planning strategy can be implemented in the current reactive controller framework present in the HyQ robot [3], in substitution of or in combination with the Central Pattern Generator (CPG) already present in the system. Furthermore, other elements of the current framework could be improved, such as the torque control used for each of the joint angles in order to cope with uncertainty. One of the suggested strategies, is to investigate the influence on the performance if adaptive computed torque control is used.

2 State of the art

In robotics, the problem of legged locomotion has proven to be a challenging task. There are three particular areas where most of the issues are encountered: control, mechanics and electronics. Regarding control, the dynamics of a walking robot can be seen as a hybrid system, due to the fact that it involves continuous control of each of the walking limbs, but in coordination with the event triggered movements of the other limbs. More importantly, in most cases, the action of balancing (which humans and animals perform naturally) represents an intrinsically unstable equilibrium point [4]. Considering the mechanics, apart from the underlying properties of the different robotic platforms, modeling and providing proper alternatives to cope with the interactions of the platform with the environment (such as impact and slipping) is still difficult. Finally, performing gaits is a demanding task by itself, which demands high amounts of power, which could limit the autonomy of the robot. This problem is intimately related to the evolution of the mechanics in robotic systems, since the problem has been tackled from the point of view where passive-walking can be achieved without the use of any external source of energy [5], or the development of actuators that provide enough force and torque while optimizing power consumption [6].

Several approaches to achieve locomotion have been researched. One of the first steps towards accomplishing successful walking, was by using the concept of Zero-Moment Point (ZMP), which is defined as "*the point on the ground at which the net moment of the inertial forces and the gravity forces has no component along the horizontal axes*" [7]. Using this concept, traditionally one could design a controller that successfully moves the end effector while maintaining the ZMP inside the support foot, by means of tools like inverse kinematics (which yields several disadvantages such as singularity points and multiple solutions) or by implementing an operational space formulation with constraints [8]. This framework has been applied to some robotic platforms such as Honda's Asimo. Nevertheless, in reality it yields several problems, as it does not represent a natural way of walking, and due to the fact that the resulting equations for the dynamics of the robot can be very complex, it may require a lot of computational power and may not be robust to large perturbations. Furthermore, it considers a quasi-static movement, so running yields a high level of complexity as well. Finally, as the number of legs increases, the problem becomes less tractable.

A different way of tackling the problem is to model the body of the human or animal as a point mass located at the hip joint with two massless legs, equipped with telescopic springs with constant stiffness and length. This model is called the Spring Loaded Inverted Pendulum (SLIP) and it has been shown that it represents both walking and running reliably [9] and it can reproduce the ground reaction forces and hip trajectories observed in gaits. Moreover, it can encode various gaits by defining different model parameters (length of the leg, stiffness of the spring, angle of attack of the foot), and can execute energy efficient passive gaits (in the case of bipeds). A step has been taken further by proposing a Variable Spring Loaded Inverted Pendulum (V-SLIP) model [10]. This is similar to the way humans control the actuation of their limbs via the muscular system, changing their stiffness in order to achieve movement. This strategy makes use of energy-based Hamiltonian mechanics to design a controller to achieve stable walking using different gaits. These two models have been proven to be very suitable to describe walking and running in humans and animals. Despite this, they face the same problem of complexity when it comes to controller design.

One of the most widely utilized tools in robot locomotion, are the so-called CPGs. The problem of animal and human locomotion had been widely studied long before robotics were a relevant subject. The interest on this topic initially arose from the field of biology, achieving great success in understanding how animals such as mammals and insects manage to coordinate in a natural and effortless way the movement of their legs in order to perform specific gaits corresponding to the conditions they face on a daily basis. It was discovered, that complex networks of neurons are in charge of generating the signals to send to each walking limb, using advantageously the periodic nature of the process. CPGs do not actuate each limb independently or are entirely based on the senses of animals/humans. This is because in most of the scenarios, walking constitutes a periodic activity, where the legs move following the same patterns with phase differences. Depending on the situation that the agent is facing, it naturally changes the parameters of the gait (such as the time that the legs remain on the ground or the air or the phase differences between each leg) or even changes the gait completely (such as going from walking to running).

Roboticists around the world, have payed close attention to these neuronal arranges in order to apply them in robot locomotion. This type of structures have been emulated by using simplified models of neurons and their interactions. Using this kind of framework, various types of gaits can be achieved. The way that CPGs are used, differs among all different platforms, and has proven to be efficient to generate different gait motions for several types of robots [11], [12], [3]. Some examples [13], do not use any kind of feedback control in order to generate gaits, although the main purpose of this kind of study is to analyze the animal behavior using robotic models to simulate gait patterns. On the other hand, CPGs can be used to effectively generate angle references for each of the legs in robotic platforms [14], to subsequently be tracked via control strategies (the most widely used in humanoid and animal-like platforms are computed-torque control and feedback linearization techniques).

Despite the elegance that locomotion generation using CPGs displays, roboticists are facing big challenges regarding robustness against unstructured terrain and dealing with disturbances. In most cases, when encountering unfavorable situations, proprioceptive (information coming from the agent itself) and exteroceptive (information coming from the environment) sensory information, is used to change the parameters of the gait such as coupling variables, ground and flight times and velocities of each leg. This framework has proven to efficiently deal with complex types of terrain. In some specific cases, gait switching has been achieved [13], although no control is implemented in the locomotion planning routines. It can be seen that even in the simplest case, a set of nonlinear differential oscillator equations describes the evolution in time of each limb. This is due to the intrinsic periodicity of the process. This provides an extra difficulty in order to change from one gait to another, especially as the number of legs increases.

All previously described strategies, model the locomotion problem as a continuous-time system. Another way to approach the locomotion planning problem, is to model it via Discrete Event Systems [15]. In recent years the use of tools such as petri nets and max-plus algebra systems have been tested and proved to be able to generate locomotion patterns. Max-plus algebra systems have provided not only a simpler and systematic way to generate gait motions but also, due to its linear (in max-plus) properties, provides tools to analyze transitions and velocity of the generated planning.

Max-plus algebra is a type of "tropical algebra" that complies with the axioms of an algebraic semiring [16]. Max-plus algebra represents a very useful tool to model timed events such as railway scheduling [17] or traffic control [18]. Robot locomotion can be modeled as well as a discrete event system, making use of a subclass of petri nets called timed event graphs [15]. Subsequently, one can represent the evolution equations for a subset of timed event graphs as max-plus algebra linear systems.

Using this kind of framework, one could generate time references for each of the leg's touchdown times (time instant when the leg touches the ground) and lift-off times (time instant when the leg leaves the ground). Subsequently, with these time references for each leg, a trajectory can be computed and achieved by means of a controller. It has been proven that this strategy can safely switch between gaits, minimizing the variation in velocity of all the legs touching the ground. It yields the disadvantage that it has been implemented in robots, with rather simple dynamics (RHex-inspired platform), although in theory, complexity in the mechanics of the platform would only represent a difference in the generation of references to be followed by the controller for each of the legs.

In summary, robust and flexible locomotion planning remains an open question in the robotic community. Finding the right balance between complexity and tractability of the studied strategies represents one of the most important trade-offs in the field. The introduction of the concept of ZMP, gave a first glance at the complexity of the locomotion problem, making evident that novel strategies should be developed in order to render the problem tractable. On one hand, accurate models of both human and animal locomotion have been found throughout the past decades, which capture the essential dynamics of the process (such as the SLIP/V-SLIP models). These models have been widely utilized for biped locomotion, achieving great success. But, as the number of legs increases, the underlying non-linearities and the combinatorial space for coordination, render the task more difficult. The use of CPGs has aid regarding coordination of the legs, elegantly modeling the phase differences of each limb in order to achieve different types of gaits. Nevertheless, it faces the same problem regarding non-linearity and combinatorial "gait space". Finally, modeling the problem as a discrete event system using max-plus algebra presents itself as a simple and systematic way to coordinate the legs in order to achieve gaits and, furthermore, can provide safe gait switching. It remains to investigate if this framework can be applied to mammal-like walking limbs, made of several joints.

3 Objectives

The analysis performed on the state of the art of the locomotion problem, has made clear that there are several open questions in the field. This has given important guidelines in order to propose the objectives that can be pursued in the research project on this topic. First of all, the robustness and flexibility of the current strategies has to be improved. Several alternatives have been studied in the previous section, but one that shows systematic implementation, is to use discrete event systems described by making use of max-plus algebra. One of the initial goals, is to prove if this framework can be implemented in a practical manner to robots with topologies different than the RHex inspired platforms.

One subtask of this first objective, is to investigate if this locomotion strategy can be implemented in the reactive framework of the HyQ robot proposed in [3]. This can be done either in tandem with the already implemented CPG trajectory generator, or by implementing a unique discrete max-plus gait scheduler. It is believed that the rest of the elements in this reactive framework, can be used in the same way as it is used making use only of a CPG.

A second objective, would be to test if the use of max-plus algebra systems, indeed provides the alternative to perform gait changes, along with modifying the parameters of the gaits. A subtask of this objective may include the implementation of a supervisory controller, which uses the information from the sensors of the robot, in order to decide the most adequate gait and parameters to face each situation (in a similar way as in [14]).

Furthermore, it is believed that the currently implemented joint controller could be improved if an adaptive scheme is added, which could provide online estimations of the model parameters. This could be helpful to improve robustness, since in most dynamical system the parameters that describe the model could change due to several factors (unmodelled or neglected dynamics, disturbances).

This objectives have to be tested on both simulation and the real system.

4 Methodology

As it has already been mentioned, the goal of this proposal is to use max-plus algebra to execute the locomotion planning task. As an initial step, the basics of this framework applied to locomotion are explained.

Consider the following algebra:

$$(\Re_{max}, \oplus, \otimes, \varepsilon, e) \tag{1}$$

where:

$$\begin{aligned} \Re_{max} &:= \Re \cup \{-\infty\}, \\ x \oplus y &:= max(x, y), \\ x \otimes y &:= x + y, \\ \varepsilon &:= -\infty, \\ e &:= 0. \end{aligned}$$

In place, this algebra is defined by the set of real numbers including the negative infinity; the two binary operations: the maximum of two numbers and the addition; the absorbing element ε given by $-\infty$ and the identity element given by 0. Subsequently, max-plus algebra can be extended to matrices defined by the following structure:

$$(\Re^{n\times m}_{max}, \oplus, \otimes, \mathcal{E}, E) \tag{2}$$

with:

$$[A \oplus B]_{ij} = a_{ij} \oplus b_{ij} := \max(a_{ij}, b_{ij}),$$
$$[A \otimes C]_{ij} = \bigoplus_{k=1}^{m} a_{ik} \otimes c_{kj} := \max_{k=1,\ldots,m} (a_{ik} + c_{kj}),$$

where $A, B \in \Re_{max}^{n\times m}$, $C \in \Re_{max}^{m\times p}$, and the i, j element of A is denoted by $a_{ij} = [A]_{ij}$, and identity and zero matrices defined by:

$$[\mathcal{E}]_{ij} = \varepsilon,$$
$$[E]_{ij} = \begin{cases} e, & \text{if } i = j \\ \varepsilon, & \text{otherwise.} \end{cases}$$

Powers of matrices can also be defined as:

$$D^{\otimes k} := D \otimes D \otimes ... \otimes D. \tag{3}$$

The max-plus algebra structure corresponds to a commutative idempotent semiring. Gaits can be represented and controlled via switching max-plus linear systems of the form:

$$x(k+1) = A \otimes x(k) \tag{4}$$

In the case of locomotion the parameters and states involved in the process are given in Table 1.

Table 1: Parameters for locomotion using max-plus algebra systems[**?**].

Symbol	Parameter
i	Index to indicate each of the legs.
$t_i(k)$	Touchdown time of leg i.
$l_i(k)$	Lift-off time of leg i.
τ	Current time instant.
τ_f	Time that a leg spends in flight (swing).
τ_g	Time that leg spends on the ground (stance).
τ_Δ	Double stance time (this can be adjusted depending on the gait that it is desired to achieve).

Using this parameters one could write the equations of a leg cycle as:

$$t_i(k+1) = l_i(k+1) + \tau_f \tag{5}$$
$$l_i(k+1) = t_i(k) + \tau_g \tag{6}$$

Furthermore, in order to achieve synchronization, one could enforce a leg (i) to lift-off only until some time after (τ_Δ) other leg (j) has touched the ground. Then Equation (6) can be rewritten as:

$$l_i(k+1) = \max(t_i(k) + \tau_g, t_j(k-1) + \tau_\Delta) \tag{7}$$

Then using the max-plus algebra notation:

$$l_i(k+1) = \begin{bmatrix} \tau_g & \tau_\Delta \end{bmatrix} \otimes \begin{bmatrix} t_i(k) \\ t_j(k) \end{bmatrix} \tag{8}$$

There exist several examples of gaits modeled using petri nets or timed event graphs. One could easily go from this kind of representation to a max-plus algebra system in a systematic manner [15] by defining a state vector that contains the touchdown and lift-off times of all the legs at the current time instant. Explaining this method would go beyond the purpose of this proposal.

This max-plus algebra representation, could be used to generate, for each time instant k, the time reference at which each of the legs should leave and touch the ground, depending on the gait parameters. In order to implement these time references, a continuous position reference generator has to be designed. A tentative method to implement this position reference generator could be as follows:

1. As an initial step, one could design a polynomial velocity profile for the leg based on initial (lift-off time l_i) and final (touchdown time t_i) time.

2. A trajectory for each leg could be designed via parametric equations, expressing the position as a function of time. This trajectory could be modeled as an ellipse, in accordance to the current reactive framework.

3. Finally, setting this trajectory as the reference to be tracked, one could implement various type of control strategies. One option is to use workspace PD control, via the transpose geometric Jacobian. Another algorithm that could be used, could be workspace computed torque control, either using the transpose geometric Jacobian (if the number of actuators of each leg is equal to the dimension of the workspace) or by using the pseudo-inverse of the geometric Jacobian.

This would be an initial proposal for locomotion controller design. One could simulate and design this strategy via several tools. Initially, to design this controller one could use MATLAB in combination with Simulink in order to simulate the responses and effects of the modeled system. Furthermore after simulating the results in MATLAB, one could go one step further and implement a graphic simulation via V-REP. One powerful element that could simplify the task to take the leap to a 3D environment, is making use of the Robot Operating System (ROS). One could write the control algorithms in MATLAB as ROS nodes, in order to publish the data into an specific topic. On the V-REP side, there are several libraries to connect this software to ROS, and building a subscriber node, which could be used to read the data from the previously mentioned topic. Furthermore, one could use virtual sensors and actuators to simulate the real life conditions of the robot.

Other programming softwares could be use to implement the control strategy before. Python is one attractive alternative, due to its similarity with MATLAB, and the flexibility that provides as an open-source developing language. There is a vast number of libraries and it can be used to work in connection with ROS.

After extensive testing of the locomotion algorithm, one could make use of the modular structure of ROS, to implement the same algorithms in the HyQ robotic platform. If work has not been performed in building the necessary nodes for the sensors and actuators of the robot, these could be build again by using the ROS framework. This process also involves getting familiar with the platform, understanding the previous work in depth, and adapting the possible new strategies to be implemented.

5 Tentative workplan

In order to achieve the tasks proposed in the methodology a tentative work plan is designed. A brief summary of the plan is shown in Table 2.

Table 2: Tentative plan of work.

Task	Time to fulfill task
Literature research	2-3 months
Analysis of current implementation and platform	4-6 months
Locomotion controller design	7-9 months
Simulation testing	4-6 months
Physical implementation	5-7 months
Unstructured scenario design and testing	3-5 months

Initially, an extensive literature research has to be performed to strongly justify the methods and strategies proposed to achieved the desired objectives. All the suggested time frames include documentation of each of the steps taken, this meaning that the time to write down the results is being considered. Based on previous experiences, a literature research con take between 2 and 3 months, depending on how the topic has been studied. For this topic, the time frame seems reasonable, to both analyze and document the previous research.

Afterwards, an analysis of the previous work performed on the HyQ platform is in place. This analysis includes several subtasks. These can be divided in two main groups, theoretical and practical. Regarding the theoretical tasks, a crucial component is getting a deep understanding of the mechanical modeling of the robot. Furthermore, it is also needed to analyze deeply the current locomotion framework, to give solid reasons to proceed with the research. One should be able to reproduce all previous results. Also, during this part of the research, it has to be considered the need to learn the current tools being used to implement the locomotion controller design, i.e. unfamiliar programming languages and software. It is considered that this task can take between 4 and 6 months.

The next step, is to proceed with the design of the new locomotion strategy based on the previously mentioned methodology. This is one of the most demanding and important tasks of the project. Analyzing the previous section, the max-plus algebra discrete time scheduler has already been implemented in several languages and softwares (python, MATLAB, V-REP) and could be easily adapted to other. The most demanding tasks would be adapting the time scheduler to the configuration of the HyQ robot. Based on the proposal in this last regard, it is considered that this task could take between 5 and 7 months. At this moment, taking into account the information gathered in the literature research and the locomotion strategy, it should be considered if there is time available to improve the motion controller, (i.e., implementing adaptive computed torque control in workspace), or if the current controller will be retained.

Once the controller design has been achieved, the strategy has to be tested via simulation. Several tools can be used. Simulation can be done easily via Simulink and MATLAB, this being convenient since they will be useful as well for controller design. On the other hand, a step further would be to provide with 3D simulations of the robot executing tasks using the previously designed locomotion control. As mentioned in the methodology, V-REP and ROS could be use for this task. Also learning other tools such as Gazebo could be considered. This task could take between 4 and 6 months.

Physical implementation is to be done after testing in simulation. It is considered that this task could be very demanding as well, since it also involves learning how to use the sensors and actuators of the platform. It is considered that this task can take from 5 to 7 months.

Finally, as the end goal of this project is to design and implement a strategy which provides robustness to unstructured terrain as well as disturbances, it is important to carefully design the experiments that are going to be performed in order to fulfill this goal. it is considered that this task can take from 3 to 5 months.

It is considered that the project could be completed in a time frame of 25 to 36 months. If there is time left, additional tasks could be considered.

6 Expected results

The first expected outcome of this research would be, a novel locomotion control strategy that could be adapted to the current reactive framework of the HyQ robot. This in place, is expected to improve robustness and flexibility thanks to its systematic design approach.

On a larger scale, this research could provide, a strategy to systematically design and implement locomotion controllers to a variety of multi-legged robots. The proposed strategy, if proven to be effective for this project, could be easily adapted to several platforms that can design gaits via discrete event systems.

On the practical side, examples where max-plus algebra systems are used for locomotion are scarce. Furthermore, it was not found an example where this framework is implemented for robots with multiple joints in the legs. This would be a first experience using this strategy for this type of legs. This in place, would provide the analytical tools to reproduce this framework. In addition, software tools to implement discrete event systems in locomotion would be a natural consequence of the research process.

It is clear, that if this strategy can manage to provide safe gait switching and parameter variation, robots would be able to overcome more difficult tasks while facing unstructured environments. In an ideal scenario, this strategy may represent a step towards successfully overcoming some of the greatest obstacles while encountering hazardous situations.

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