PROTON – A Python Framework for Physics-Based Electromigration Assessment on Contemporary VLSI Power Grids

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***Abstract*—Electromigration (EM) is a significant reliability concern in modern circuit design practices that poses a considerable risk to the long-term reliability of contemporary integrated circuits and attracts attention from the EDA industry. Hence, the development of a robust, industrial-level EM analysis tool is crucial. In order to address this challenge, we present PROTON, an open-source tool that can be straightforwardly integrated into industrial design flows covering a wide spectrum of EM analysis needs. On top of this, it offers an intuitive graphical user interface with a high level of automation that allows the visualization of EM stress analysis on power grid designs. The core of PROTON incorporates state-of-the-art methodologies for physics-based EM stress analysis which provide robustness and scalability in handling large-scale power grid designs. These are experimentally verified in comparison with the industrial tool COMSOL on multiple benchmarks where PROTON demonstrated a speedup of $\times 685$ with negligible loss in accuracy.**

***Index Terms*—Electromigration, Longterm Reliability, Korhonen's Model, Physical Design, Industrial Design Flow, EDA tool**

I. Introduction

Electromigration (EM) is rapidly becoming a key reliability issue that the EDA industry is facing nowadays. EM analysis is crucial for determining the long-term reliability of the chip, especially in advanced technology nodes. EM failures are caused by the increased currents that are flowing through wires for long periods of time. To deal with these challenges, early-stage EM verification of the corresponding physical structures is required to provide a starting point for eliminating long-term reliability hazards. As a result, EM transient analysis has become an integral part of modern VLSI design flows [1].

In the past, EM analysis was solely based on empirical methods such as the application of the Blech criterion [2] followed by Black's equation [3]. The first one was applied to distinguish the potential "immortal" wires from the rest ones and the latter to check the current density in the potentially mortal wires in order to predict the mean time to failure. However, besides the heuristic nature of these approaches, they can lead to inaccurate results, especially in multi-segment interconnects where physics-based models are required.

A significant contribution was made by Korhonen et al. [4] who formed an exact physics-based model as diffusion-like Partial Differential Equations (PDEs), that can calculate the EM stress in each wire segment. More specifically, the computation methodologies for this model can be divided into two main categories. Numerical methods, such as [5], are well-established since they can be straightforwardly implemented and have already been integrated into academic [6] and commercial tools such as COMSOL [7]. On the other hand, analytical approaches keep both space and time continuous and can be effectively applied to large-scale systems by providing a closed form of the solution [8].

In this paper, we present PROTON, an open-source EDA tool for EM assessment in contemporary VLSI power grids. The main contributions of this paper are summarized hereafter. *First*, our tool can be straightforwardly integrated into any commercial design flow for physical synthesis in order to calculate the EM stress, since it only requires files in SPICE format and the corresponding currents. *Second*, we provide a complete set of accurate numerical and analytical techniques, that can significantly reduce the simulation complexity of large-scale EM problems. In particular, the analytical solution used for the line analysis is exact for the chosen spatial discretization guaranteeing a perfectly accurate and robust solution. For the EM stress equation, we leverage that each segment in an interconnect structure is assumed to carry a constant current density [9]. *Third*, we provide an easy-to-use interface, that allows the user to explore different EM stress scenaria, for multiple time stamps, providing plots for the physical structures of the power grid. Finally, we evaluate our tool on large-scale power grid benchmarks in order to prove the scalability of our methods, while its efficiency and accuracy are validated against COMSOL by achieving great speedups and negligible error.

The remainder of this paper is organized as follows. Section II provides essential background on EM analysis and the two methods that were integrated into our tool. In Section III, we present an extensive analysis of the tool's flow as well as some key implementation details. Finally, the tool's performance is then validated in Section IV, followed by conclusions in Section V.

II. Concept

The proposed tool can perform transient EM stress analysis on contemporary power grids. So, in order to accurately compute the EM stress, it makes use of Korhonen's stress diffusion equation [4]:

$$\frac{\partial \sigma}{\partial t} = \frac{\partial}{\partial x}\left[\kappa\left(\frac{\partial \sigma}{\partial x} + \beta j\right)\right] \tag{1}$$

where $\kappa = D_\alpha B\Omega/(k_B T)$ and $\beta = e\rho Z^*/\Omega$; $D_\alpha = D_o e^{-E_a/kT}$ is the diffusion coefficient and E_a the activation energy; D_o is the diffusivity constant, B is the bulk modulus for the material, k_B is the Boltzmann's constant and T is the temperature; j is the current density through the wire, e is the electron charge, ρ is the electrical resistivity of the wire, Z^* is the effective charge number, Ω is the atomic volume for the metal; t is time and x is the spatial coordinate.

In order to accurately apply the above equation on a power grid line, it must be accompanied by a set of appropriate boundary conditions as stated in [8]. The resulting linear time-variant system is:

$$\dot{\boldsymbol{\sigma}}(t) = \mathbf{A}\boldsymbol{\sigma}(t) + \mathbf{B}\mathbf{j} \tag{2}$$

where n is the number of discretization points and m is the number of segments of the line; $\boldsymbol{\sigma} \in \mathbb{R}^n$ is the vector of the output EM transient stress; $\mathbf{A} \in \mathbb{R}^{n\times n}$ is the system matrix and $\mathbf{B} \in \mathbb{R}^{n\times m}$ is the matrix that corresponds the current densities to the internal states.

PROTON provides two different approaches for EM stress analysis depending on the aim of the user: The first one is an analytical approach that solves in near-linear time the above equation at all discretization points for a specific time point. The second option is a numerical solution that is better suited for transient EM stress analysis at a selected number of discretization points. In the next subsections, we analyze the two options for EM that our tool offers.

A. Analytical Solution

An efficient semi-analytical approach has been proposed in [8], where it can be directly applied on systems that arise from Finite Difference Method (FDM) spatial discretization on a power grid line. This method takes advantage of the specific tri-diagonal form of the resulting system matrix $\mathbf{A}$ and it results in the following closed-form matrix equation:

$$\boldsymbol{\sigma}(t) = \mathbf{V}\mathbf{L}(t)\mathbf{V}^T\mathbf{B}\mathbf{j} \tag{3}$$

where:

$$\mathbf{L}(t) = \mathrm{diag}\left(t, \frac{e^{\lambda_2 t} - 1}{\lambda_2}, \ldots, \frac{e^{\lambda_n t} - 1}{\lambda_n}\right) \tag{4}$$

with $\mathbf{V} \in \mathbb{R}^{n\times n}$ is the matrix containing the eigenvectors of matrix $\mathbf{A}$ and λ_i with $i \in [1, n]$ are the corresponding eigenvalues. This approach takes advantage of the unique structure of matrix $\mathbf{A}$, which can be easily mapped on fast Poisson solvers and thus the eigenvalues and eigenvectors of the matrix can be determined beforehand. In particular, the product of matrices $\mathbf{V}^T$ and $\mathbf{V}$ with a vector $\mathbf{r}$, represented as $\mathbf{V}^T\mathbf{r}$ and $\mathbf{V}\mathbf{r}$, perform a Discrete Cosine Transform of type-II (DCT-II) and an Inverse DCT of type-II (IDCT-II), respectively. This reduces the computational cost of the matrix multiplication from $\mathcal{O}(n^2)$ to $\mathcal{O}(nlogn)$. This solution is straightforward and provides an easily-parallelizable closed-form equation that can be applied independently on each discretization point to compute the EM transient stress for a given t in time in near-linear time.

B. Numerical Approach

The second option for EM analysis is the numerical solution of Eq. 1 using an implicit Backward-Euler (BE) transient analysis method. This option focuses on the cases where the user desires to model the EM stress evolution at specific points. This is a common case since, as shown in previous works [5], voids tend to form at specific points. The transient stresses constitute an output portion of the vector σ that contains all possible points and we can accompany Eq. 2 with a second equation:

$$\mathbf{y}(t) = \mathbf{L}\,\sigma(t) \tag{5}$$

forming a linear-time invariant (LTI) system. In the above equation, q is the number of the selected output points, $\mathbf{y} \in \mathbb{R}^q$ is a vector containing the EM stress at the output points and $\mathbf{L} \in \mathbb{R}^{q\times n}$ is the state-to-output matrix that maps the internal states to the output points. Since the number of discretization points is usually large, we provide the option of performing Model Order Reduction (MOR) on the original system. The process of MOR produces a reduced system as:

$$\dot{\tilde{\sigma}}(t) = \tilde{\mathbf{A}}\tilde{\sigma}(t) + \tilde{\mathbf{B}}\mathbf{j}, \quad \mathbf{y}(t) = \tilde{\mathbf{L}}\,\tilde{\sigma}(t) \tag{6}$$

with r being the reduced order of the LTI ODE system, $\tilde{\mathbf{A}} \in \mathbb{R}^{r\times r}$, $\tilde{\mathbf{B}} \in \mathbb{R}^{r\times p}$ and $\tilde{\mathbf{L}} \in \mathbb{R}^{q\times r}$. The above system is an approximation of the original one. For this purpose, we have chosen to implement a Moment-Matching (MM) method due to its computational efficiency. Moreover, we applied the Extended Krylov Subspace (EKS) [10] whose approximation quality has proven to outperform the standard Krylov subspace.

III. Implementation

The tool has been designed to be integrable into any industrial design flow, ensuring the reliability and lifetime of the power grid designs. It is able to model the behavior of the power grid lines under various stress conditions and identify potential reliability issues before the VLSI design is produced. Fig. 1 illustrates the flow of the tool. In particular, the steps that describe the usage and the automation options of PROTON to perform EM analysis on a power grid are as follows:

1) *Power grid parsing stage* – The first step in using PROTON is to receive the power grid design that needs to be analyzed in SPICE format. It parses the file and extracts all the lines of all different nets and layers. The extracted lines are listed hierarchically under the corresponding net on panel 1, as shown in Fig. 2. In case of an already existing project, it also receives a JSON file with the configurations and changes in the project.
2) *Obtain currents using HSpice* – Once the power grid design has been received, PROTON can be easily integrated into the physical design sign-off stage by communicating with any industry-standard SPICE simulator, e.g. the HSpice simulation software by Synopsys, to solve the power grid and obtain the currents flowing through each line.
3) *Power grid statistics and line selection stage* – After completing the power grid parsing stage, PROTON generates statistics on the power grid such as the maximum current and the line with the maximum number of segments. These statistics are displayed on panel 2 of Fig. 2, on the lower part of the window allowing the user to gain insight into potential vulnerabilities of the power grid lines. Then, the users can select specific lines either by clicking the desired line from the power grid analytics table or by selecting from the provided power grid lines list and the tool will visualize the selected line.
4) *Line configuration and technology options stage* – Once the line is selected, the user can choose the technology for the metal patterning process between aluminum (Al) and copper dual-damascene (CuDD), with the relative parameters already included. The temperature is also user-defined as well as the width of the selected line.
5) *Line discretization stage* – PROTON performs an adjustable FDM discretization on the line, where the user can choose the spatial discretization parameter. In particular, the user can either select either the discretization step or the number of discretization points on the line.

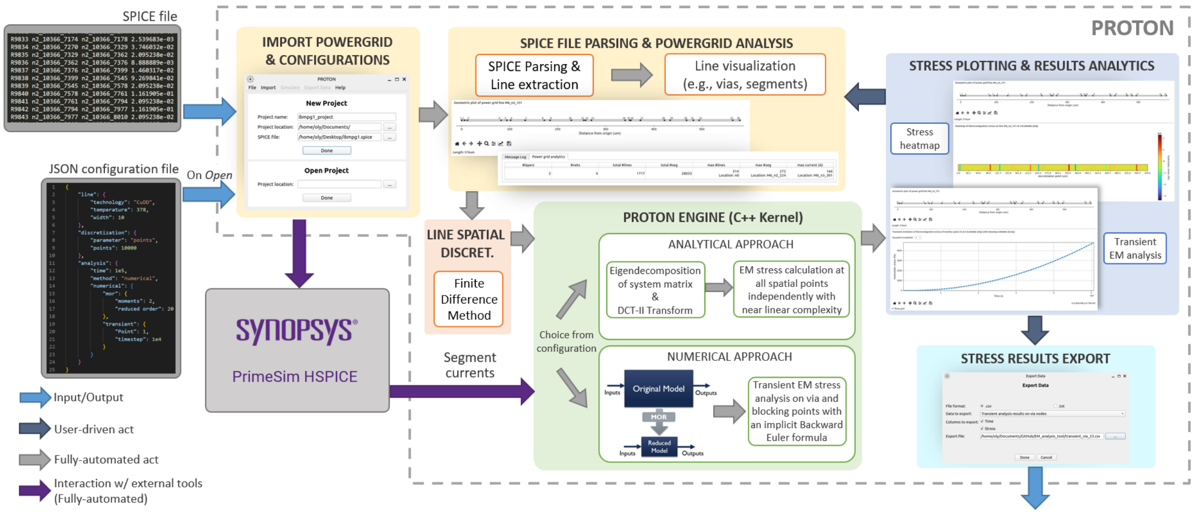


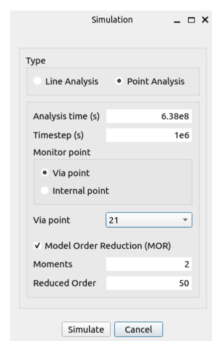
Fig. 1: The PROTON flow.

6) *EM stress line analysis* – The user can choose between two analysis options to perform EM stress analysis on the selected lines. The first one, line analysis, uses analytical methods to perform fast and robust EM stress analysis on the complete line at a specific time. Then, the tool plots a heatmap of the stress distribution on the entire line.
7) *EM transient stress point analysis* – The second option is to analyze the transient behavior of a specific point on the line. The user can choose between a critical via or blocking point or an internal point by directly entering its spatial coordinate. The chosen point will be highlighted in the line figure on the power grid visualization panel, i.e., panel 5 of Fig. 2 and the resulted transient waveform will be displayed.
8) *Data export stage* – Finally, the user can export the calculated data for further processing or analysis. PROTON provides (among other things) the option to export the desired data in either CSV or TXT format, based on the user's preference. The export option is available in the menu bar of the tool.

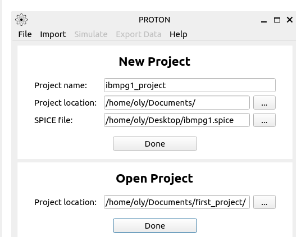
Fig. 2: The PROTON main window.

At this point, we elaborate on some specific details regarding the implementation of the PROTON core technology. PROTON back-end is implemented in C++, providing the computational power needed to efficiently perform complex matrix manipulation tasks. The tool integrates the analytical and numerical approaches described in Sec. II to perform EM stress analysis.

For the line analysis, the efficient formation of Eq. 3, PROTON leverages the state-of-the-art FFTW library [11] to perform the DCT-II required for the analysis. The library is widely recognized as one of the fastest and most optimized solutions for performing fast Fourier transforms (FFTs), ensuring that PROTON can perform the analysis quickly and effectively.



(a) Simulation dialog



(b) New/open project window

Fig. 3: The different options in the simulation dialog and the new/open project window.

The point analysis, on the other hand, involves the EM transient stress analysis of a specific spatial discretization point. The user can choose between full model analysis or reduced order in case the number of discretization points is large. In PROTON, MOR is performed by the EKS MM technique in order to project the system onto a lower-dimensional subspace. Fig. 3a displays the simulation dialog with all different user-defined options. The internal steps of this method utilize the PARDISO solver [12], a highly optimized and widely used library for solving large-scale sparse linear systems. The use of these techniques ensures that MOR is performed quickly and accurately, providing a robust foundation for the transient analysis of a specific point.

The front end of PROTON is implemented in Python. The

TABLE I: Line analysis comparison between PROTON and COMSOL on different benchmarks at 20y

Tools	IBMPG2 runtime (s)	IBMPG2 speedup	IBMPG2 rel. error	IBMPG6 runtime (s)	IBMPG6 speedup	IBMPG6 rel. error
COMSOL	87.09	$\times 279.3$	3e-6	1921	$\times 685$	6e-7
PROTON	0.312			2.803		

tool can be either used through a graphical user interface (GUI) or as a command-line interface (CLI). The GUI for PROTON is built using the PyQT5 library in Python, providing an intuitive and user-friendly environment for conducting the analysis. The CLI version of the tool follows the same flow as the GUI, as illustrated in Fig. 1. In addition, the CLI version can be fully automated, making it highly integrable into any design flow.

IV. Case studies

In this section, we will assess the robustness and performance of our tool. Our tool is designed in order to perform fast and accurate EM analysis for two main case studies.

In case of the line analysis, PROTON employs the analytical approach of Sec. II-A which takes advantage of the specific form of the system matrix $\mathbf{A}$ and calculates the EM stress with $\mathcal{O}(n \log n)$ complexity as explained in [8]. In addition, its robustness is guaranteed due to the analytical nature of the approach. In order to assess the above, we performed some experiments and compared the proposed tool with the industrial FEM-based tool, COMSOL.

To demonstrate the scalability of PROTON, we evaluated its performance on two IBM benchmarks, IBMPG2 and IBMPG6. While IBMPG2 is a relatively small power grid, IBMPG6 is much larger with longer transmission lines, resulting in a smaller system and a much larger one correspondingly. Also, as COMSOL does not offer power grid handling, we inserted as input the already parsed lines produced by our tool and selected a batch of lines from the two aforementioned benchmarks. The mean runtimes with the corresponding speedups and relative errors of the above are illustrated in Table I, where it can be easily seen that PROTON offers very competitive and scalable runtimes with a speedup up to $\times 685$ over COMSOL while its excellent performance is accompanied by small relative errors.

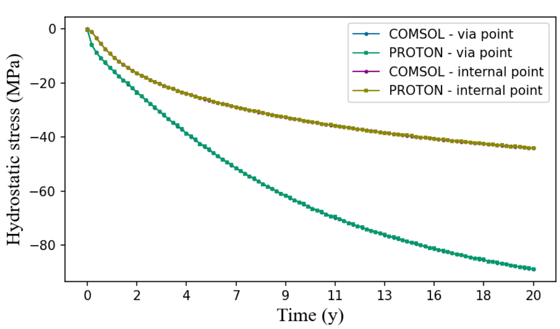


Fig. 4: Point analysis comparison between PROTON and COMSOL on an IBMPG1 via and internal point at 20y.

The second use case involves modeling the transient response of the EM stress at specific discretization points, as described in Section II-B. To demonstrate the effectiveness of our method, we compared it with COMSOL on an IBMPG1 line. Similarly to the line analysis, we provided COMSOL with the parsed power grid lines generated by our tool. In particular, for the experiments, we selected a line with a large number of segments and performed dense discretization so that the built-in MOR technique of PROTON is enabled. We isolated a via and an internal point and performed point analysis using our tool and COMSOL. Again, our tool exhibited impressive runtimes and, as presented in Fig. 4, there is almost a perfect agreement between the two tools.

V. Conclusions

This paper introduces PROTON, an advanced tool for EM analysis of power grid designs. Our implementation emphasizes simplicity and high levels of automation, making it user-friendly and intuitive, while the GUI further enhances the user experience. PROTON integrates state-of-the-art libraries and solvers and outperforms the industrial tool COMSOL in terms of runtime with a speedup of up to $\times 685$ while maintaining comparable accuracy. Finally, the tool's implementation allows for high levels of integration, as it receives the power grid design in SPICE format as input, enabling seamless integration into existing design flows.

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