CS146 #3

Code

Create your own probability distribution.

I chose to go with the poisson distribution. the support is all natural numbers N, and 0 for K. i.e.

$k \in \mathbb{N} \cup 0$

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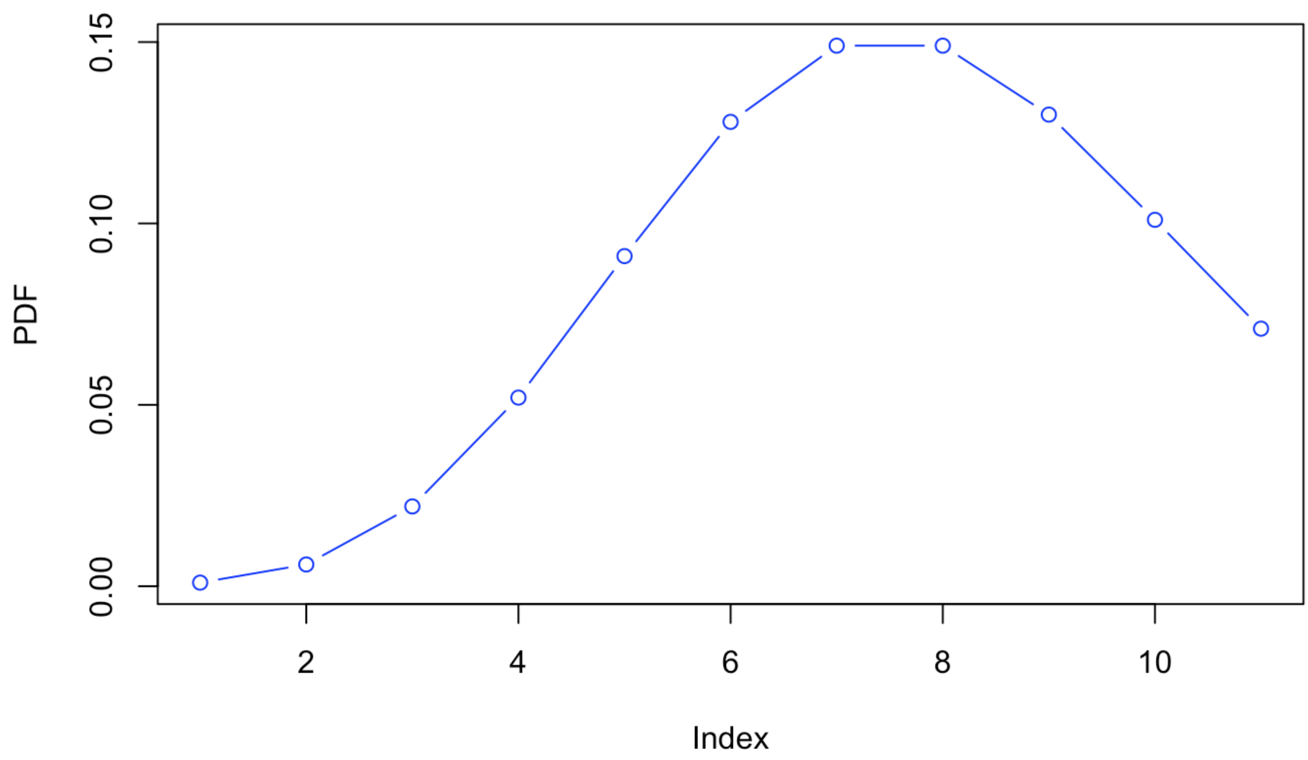
```
Lambda <-7
K = 0:10
PDF<-round(dpois(K,lambda = Lambda),3)
plot(PDF, col="blue", type="b")
```



Will your pdf have one mode or multiple modes?

One, over Lambda.

Will it be skew or symmetric? It is skewed because of the vector I decided to draw (1:20) which is not symmetric in relation to the mean (or Lambda).

Where will most of the probability mass be?

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```
plot(PDF, col="blue", type="b")
most_mass <- qpois(0.51, lambda = Lambda, lower.tail = TRUE)
most_mass
```

```
[1] 7
```

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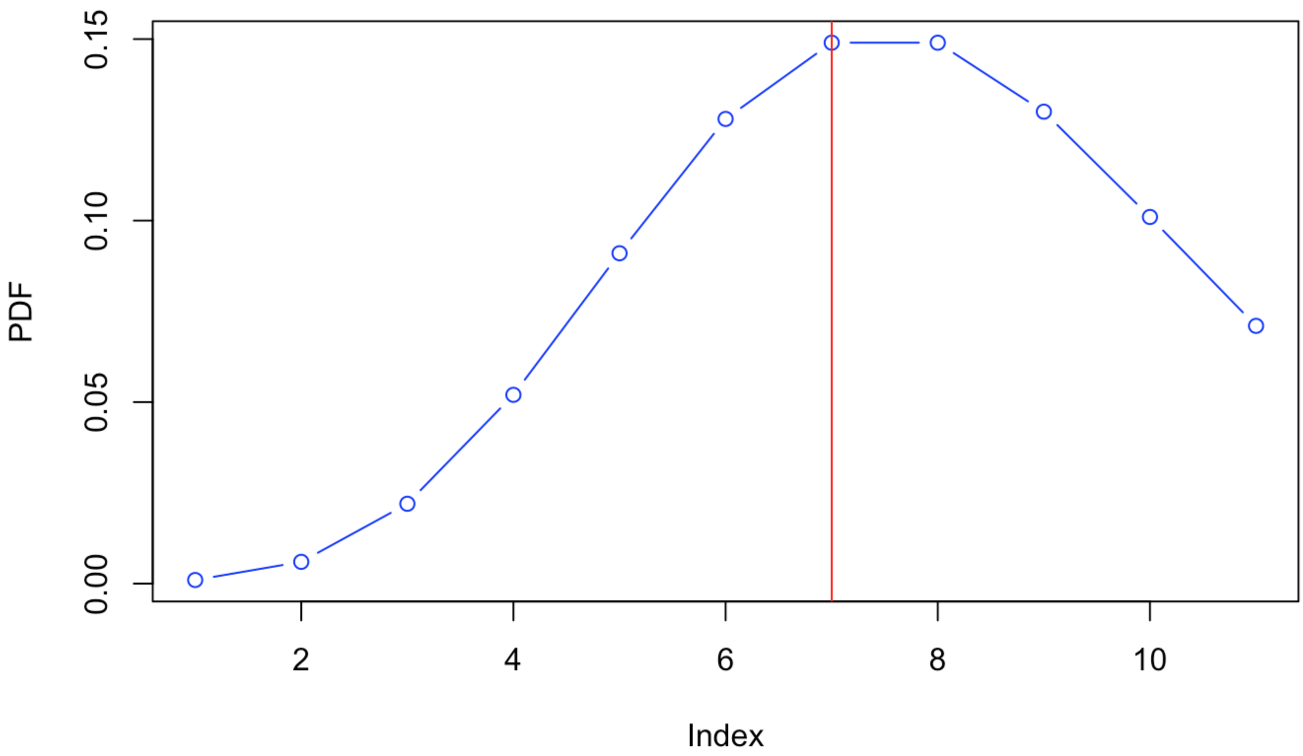
```
abline(v=most_mass,col="red")
```



We can observe that exactly half of the mass is left to the red line, and in interval o to:

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```
most_mass
```

```
[1] 7
```

What are the parameters of your distribution?

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```
# The parameter is K, which describes some known mean, or average rate in a given vol
ume/time.
K
```

```
 [1]  0  1  2  3  4  5  6  7  8  9 10
```

(Optional) Stretch goal: write R functions for the cdf, pdf, qf and random samples from your distribution.

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```
Lambda = 10
K=10
#the CDF function, instead of using ppois:
sum(dpois(0:K,Lambda))
```

```
[1] 0.5830398
```

3.2 Derive the posterior distribution, including its parameter values, from an exponential likelihood function with a gamma prior over the exponential parameter λ, and with data y_i for i = 1, 2, …, n.

Given: posterior is proportional to exponential_pdf_likelihood * gamma_pdf_prior.

exponential_pdf_likelihood =

$$\lambda e^{-\lambda y(i...n)}$$

gamma_pdf_prior. =

$$\frac{\beta^{\alpha}}{\Gamma(\alpha)} y^{\alpha - 1} e^{-\beta y}$$

Therefore the posterior for λ with data y_i for i = 1, 2, …, n. is:

$$\lambda e^{-\lambda(y1+...yn)} \times \frac{\beta^{\alpha}}{\Gamma(\alpha)} \lambda^{\alpha - 1} e^{-\beta\lambda}$$

Which is proportional to:

$$\lambda^{n} e^{-\lambda y(i...n)} \times (\lambda^{\alpha - 1} e^{-\beta\lambda})$$

Further simplification:

$$\lambda^{n+\alpha-1} e^{-\lambda y(i...n)-\beta\lambda}$$

$$\lambda^{n+\alpha-1} e^{-\lambda(y(i...n)+\beta)}$$

which is the Gamma, with parameters:

$$\alpha + n$$

$$\sum_{n=i}^{n} yi + \beta$$