ISYE_6420_Project

December 2, 2024

1 Abstract

In this project, we use Cricket Test match data from cricsheet to evaluate the performance of national cricket teams. We use multiple approaches to assess a team's performance and state each approach's merits and demerits. We start with a frequentist approach followed by a 1-parameter logistic (1pl) Rasch model followed by a 2-parametric logistic (2pl) Rasch model. In each section, we will evaluate the results of the models and discuss them. In the last section, we will compare results between different models. The code is provided at the end so that the results can be recreated.

Note: to recreate the results in this notebook, please run the "code" section at the end before running the other sections. This is because variables are being populated in that section.

2 Introduction

The dataset for cricket is stored in JSON format where each JSON contains ball-by-ball data for each test match. We parse the json iteratively and store the results in tabular format. The finalized data frame contains: * `Team_x`: The first team in the matchup.
* `Team_y`: The second team in the matchup, i.e. the opposition.
* `num_matchup`: Total number of matchups.
* `wins`: number of wins by `Team_x`.
* `losses`: number of losses by `Team_y`. * `draws`: the number of draws by `Team_x` and `Team_y`.

Here is a subset of the data:

[27]:
```
# to populate First run code cells in the "code section"
# sample data
cross_join_df2[['Team_x',        'Team_y',        'num_matchups', 'wins',␣
↪'losses', 'draws']].head(19)
```

[27]:

	Team_x	Team_y	num_matchups	wins	losses	draws
1	Australia	Bangladesh	4	3	0	0
2	Australia	England	52	27	0	9
3	Australia	India	44	13	0	12
5	Australia	New Zealand	20	17	0	2
6	Australia	Pakistan	21	14	0	3
7	Australia	South Africa	29	15	0	3
8	Australia	Sri Lanka	16	10	0	2
9	Australia	West Indies	20	14	0	4
10	Australia	Zimbabwe	2	2	0	0

```
11  Bangladesh     Australia           4     1        0       0
13  Bangladesh       England           9     1        0       0
14  Bangladesh         India          13     0        0       2
15  Bangladesh       Ireland           1     1        0       0
16  Bangladesh   New Zealand          15     2        0       3
17  Bangladesh      Pakistan          10     2        0       1
18  Bangladesh  South Africa          12     0        0       2
19  Bangladesh     Sri Lanka          22     1        0       5
20  Bangladesh   West Indies          17     4        0       1
21  Bangladesh      Zimbabwe          10     7        0       0
```

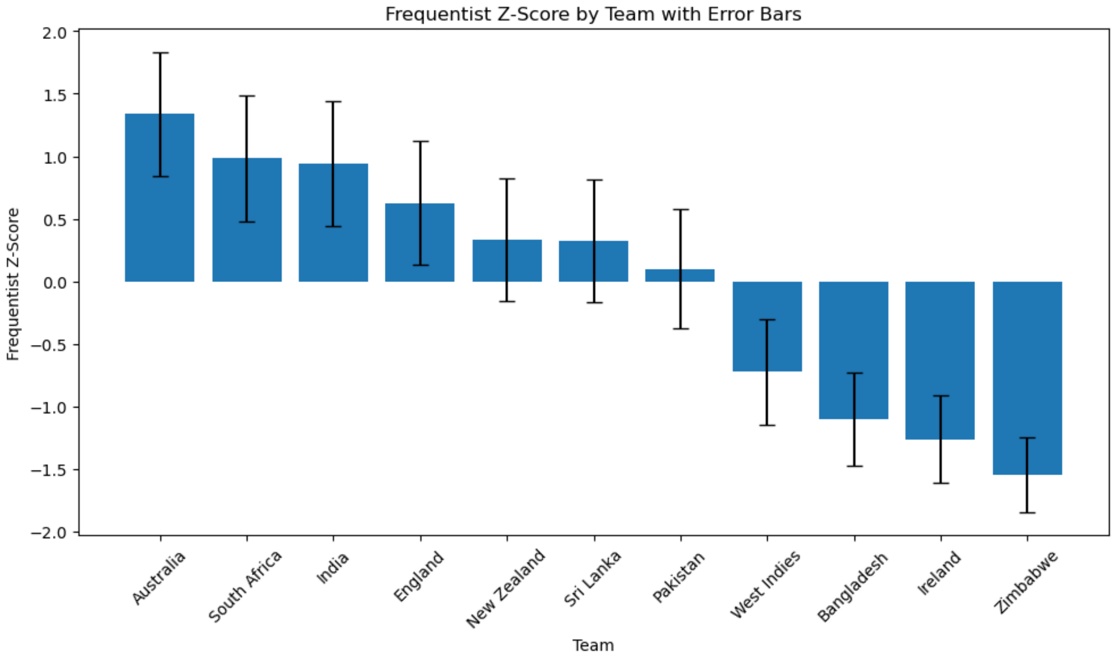
2.1 Frequentist Estimate

[28]:
```
summary_frequentist.sort_values('freqentist_z_score',ascending=False)
```

[28]:
```
    index        Team_x  num_matchups  wins  losses  draws  win_pct  \
0       0     Australia           208   115       0     35     55.3
7       7  South Africa           181    90       0     31     49.7
3       3         India           204   100       0     51     49.0
2       2       England           254   112       0     53     44.1
5       5   New Zealand           157    62       0     32     39.5
8       8     Sri Lanka           168    66       0     36     39.3
6       6      Pakistan           151    54       0     31     35.8
9       9   West Indies           158    36       0     38     22.8
1       1    Bangladesh           113    19       0     14     16.8
4       4       Ireland             7     1       0      0     14.3
10     10      Zimbabwe            41     4       0      3      9.8

    freqentist_z_score   std_dev
0             1.337819  0.497183
7             0.982452  0.499991
3             0.938031  0.499900
2             0.627085  0.496507
5             0.335176  0.488851
8             0.322484  0.488417
6             0.100380  0.479412
9            -0.724580  0.419543
1            -1.105330  0.373866
4            -1.263976  0.350073
10           -1.549539  0.297315
```

[29]:
```
frequentist_summary_chart
```

[29]:



The table above shows results if we employ a frequentist approach. We can see that the team's ability can be ranked in the order above, with Australia being the top team and Zimbabwe being the worst team. The error bars are the standard error associated with each value.

2.2 Bayesian 1pl Rasch model

Next we use a Bayesian approach to model the same data. We use a rash 1 parameter logistic (1pl) model which uses ability and difficulty parameter to model outcomes where the probabilty of outcome is defined as:

$$Pr(out = 1) = \frac{\exp(\alpha_i - \beta_j + \delta)}{1 + \exp(\alpha_i - \beta_j + \delta)}$$

The ability parameter measures a subject's proficiency or skill level. The difficulty parameter indicates how challenging a test item is. More difficult items require a higher ability to achieve a favorable outcome. This model uses the logit-parameterized Bernoulli distribution

Our approach is adapted from the Item Response Models Section of the MC-stan guide and our model is specified as follows:

$$\begin{aligned} \text{delta} \quad &or \quad \delta \sim N(0.75, 1) \\ \text{ability}_i \quad &or \quad \alpha_i \sim \text{Normal}(0, 1) \quad \text{for } i = 1, \dots, N_{\text{teams}} \\ \text{difficulty}_j \quad &or \quad \beta_j \sim \text{Normal}(0, 1) \quad \text{for } j = 1, \dots, N_{\text{difficulty}} \end{aligned}$$

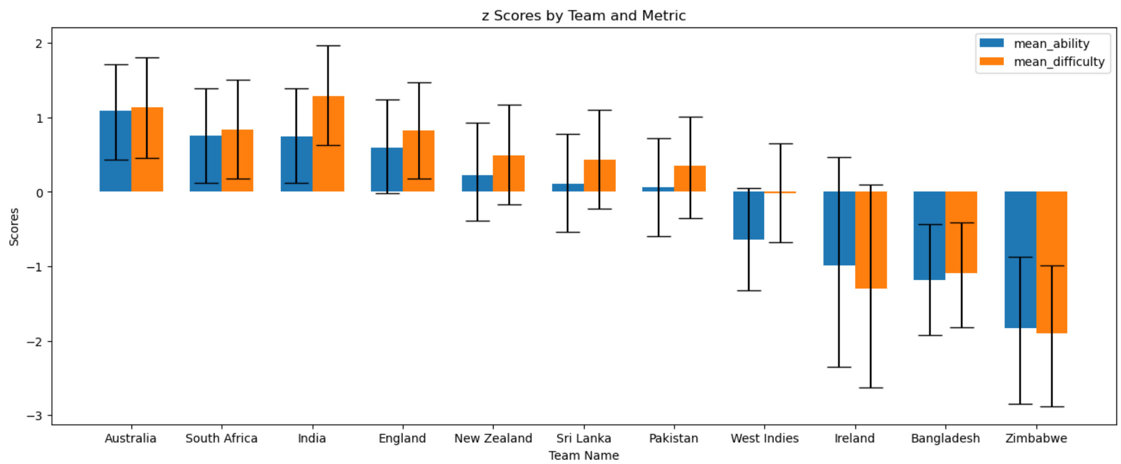
$$\text{logit}_p = \alpha_i - \beta_j + \delta$$

$$\text{out}_n \sim \text{Binomial}(N, \text{logit}_p)$$

[30]:

```
difficulty_ability_plot_1pl
```

[30]:



Ability and difficulty are closely related, but they are not perfectly correlated. For example, India is the most difficult team, but in terms of ability, they rank third. For the top and bottom performers, our 95% HPD credible sets do not contain 0, indicating a high level of certainty that these teams are either underperforming or overperforming. However, for teams in the middle, the HPD interval contains 0, so it is unclear if these teams are above or below average.

Additionally, we can compare the HPD intervals for all the teams to understand how they stack up against one another. For instance, the HPD intervals for the West Indies and Australia do not overlap, indicating a high level of certainty that Australia is a better team than the West Indies. Conversely, the HPD intervals for Pakistan and Sri Lanka overlap significantly, leading to a high degree of uncertainty about which team is better, despite Sri Lanka having a higher mean ability.

The rasch model offers a robust framework for measuring latent traits such as ability and item difficulty. It ensures that comparison of subjects is independent of the specific sample and test items used. However, the Rasch model's assumptions—such as unidimensionality and equal item discrimination—can be limiting. These assumptions may not hold in real-world scenarios where multiple traits influence responses and items vary in their ability to discriminate between different levels of ability. To cater to this limitation we introduce the 2 parameter logistic.

2.3 Bayesian 2pl Rash Model

The Rasch 2-parameter logistic model (2PL) improves upon the 1-parameter logistic model (1PL) by introducing an additional parameter to account for item discrimination. This enhancement addresses one of the primary limitations of the 1PL model by allowing for varying item discrimination.

We add to a 1pl model by adding hierarchy and a discrimination term. The probability of an outcome is modeled by:

$$Pr(out = 1) = \frac{\exp(\gamma_j * (\alpha_i - (\beta_j + \delta)))}{1 + \exp(\gamma_j * (\alpha_i - (\beta_j + \delta)))}$$

The discrimination terms are:

$$\text{standard deviation of discrimination} \quad or \quad \sigma_\gamma \sim \text{HalfCauchy}(0, 3)$$

$$\text{discrimination} \quad or \quad \gamma_j \sim LogNormal(0, \sigma_\gamma)$$

We define the following distribution to model a team's ability:

$$\text{ability} \quad or \quad \alpha \sim Normal(0, 1)$$

To model Opponent Difficulty we define the following distributions:
We recenter difficulty in avoiding fit issues:

$$\text{mean question difficulty} \quad or \quad \delta \sim \text{Cauchy}(0, 5)$$

$$\text{standard deviation of difficulty} \quad or \quad \sigma_\beta \sim \text{HalfCauchy}(0, 5)$$

$$\text{difficulty} \quad or \quad \beta \sim Normal(0, \sigma_\beta)$$

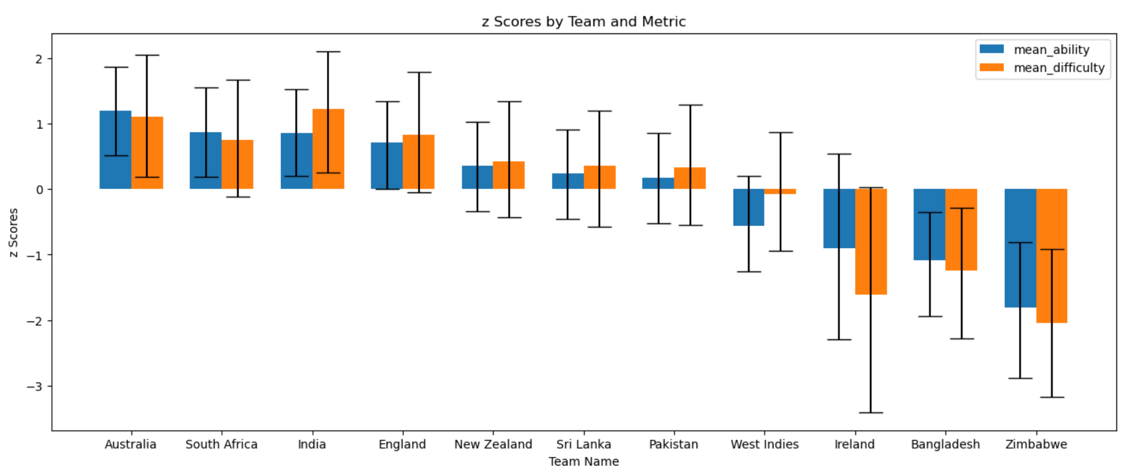
The outcome is defined as:

$$\text{logit}_p = \gamma_j * (\alpha_i - (\beta_j + \delta))$$

$$\text{out}_n \sim \text{Binomial}(N, \text{logit}_p)$$

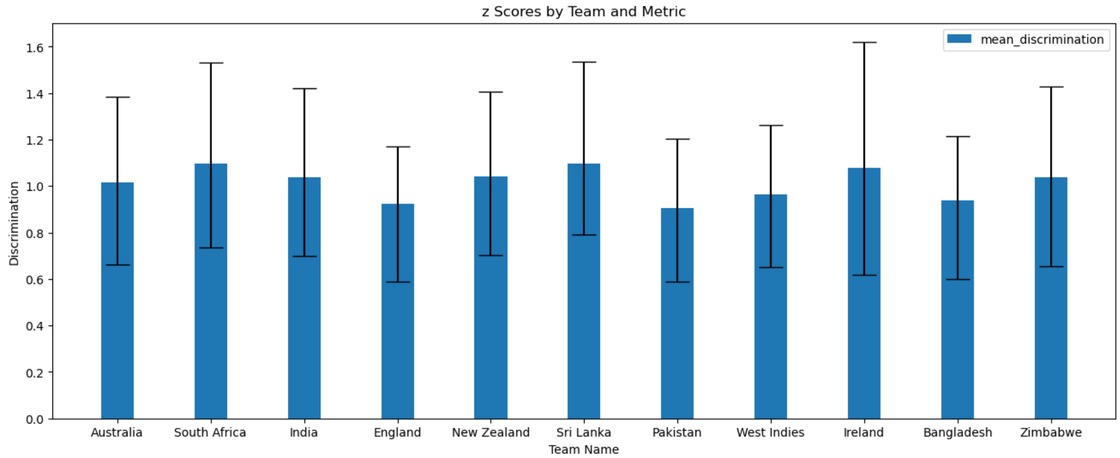
[31]:
```
difficulty_ability_plot_2pl
```

[31]:



[32]:
```
discrimination_2pl
```

[32]:



Our estimates of ability and difficulty are similar to what we get in our 1PL model. However, our 2PL model gives us extra information with the discrimination parameter. It indicates how well an item can differentiate between individuals who have different levels of the underlying trait or ability being measured. A higher discrimination parameter means that the item is better at distinguishing between individuals with slightly different ability levels. This means items with higher discrimination are more sensitive to differences in ability. From the plot above Sri Lanka can have the highest discrimination, however, the HPD around all the values is very wide, which means we cannot say for certain if one team's ability to discriminate is higher than another team's.

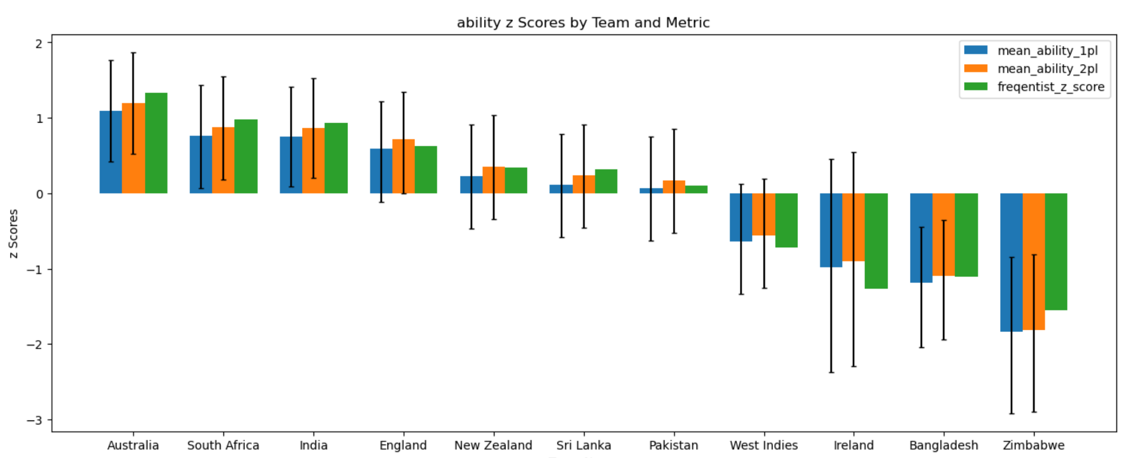
2.4 Compare Models

2.4.1 Estimates of the ability

In this section we compare our frequestist estimates with the estimates from the 1pl model and the estimates from the 2pl model.

[33]:
```
ability_measured_by_each_model
```

[33]:



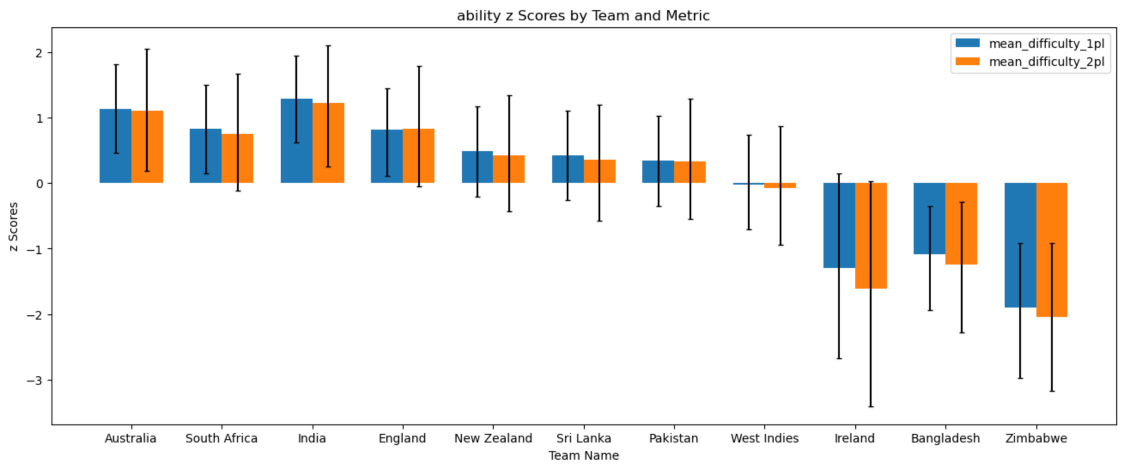
In the above plot, we can see that all 3 methods of estimating ability (win percentage, bayesian 1pl model, and Bayesian 2pl model) give similar means. However, the Bayesian models do a good job of quantifying the uncertainty around the means. Comparing the 1pl with the 2pl model we don't see any major changes however, we do see that the 2pl usually predicts a slightly higher z_score for each team.

2.4.2 Estimate of Difficulty

In this section, we compare the estimates for difficulty from the 1pl model with the estimates for difficulty from the 2pl model.

[34]:
```
difficulty_measured_by_each_model
```

[34]:



Similar results are seen for both 1pl and 2pl models, however, the 95% credible interval for the 2pl is wider indicating lower confidence in our means. Secondly, even though Australia's team has a higher ability, India seems to be the most difficult team to beat and Zimbabwe seems to be the easier team to beat.

3 Conclusion

Based on the results we see that we achieve similar results when using mean values versus when we use a 1-parameter logistic model versus when we use a 2-parameter logistic model. However, the Bayesian models do a good job of quantifying the uncertainty around the means. Moreover, we can get some other informative information from our parameters which can give us information about the difficulty of the opponent or the descriminative power of the opponent. Our results indicate that although Australia has the highest ability, the Indian team is the hardest to beat and that the Sri Lankan team has the highest ability to discriminate between different subjects.

For further exploration, we can explore improvements on our 2pl model by expland also explore 3-parameter logistic models.

4 Code

4.1 Load Libraries

[1]:
```
import numpy as np
import pandas as pd
import pymc as pm
import arviz as az
import os, sys
import json
import re
import matplotlib.pyplot as plt
```

[2]:
```
# Define a function to extract the digit(s) from arviz output
def extract_digit(text):
    match = re.search(r'\[(\d+)\]', text)
    return match.group(1) if match else None
```

[3]:
```
all_teams = ['Australia', 'Bangladesh', 'England', 'India', 'Ireland', 'New␣
 ↪Zealand', 'Pakistan', 'South Africa', 'Sri Lanka',
             'West Indies', 'Zimbabwe']
df = pd.DataFrame({'Team': all_teams})
cross_join_df = df.merge(df, how='cross')
cross_join_df['num_matchups']=0
cross_join_df['wins']=0
cross_join_df['losses']=0
cross_join_df['draws']=0
```

4.2 Parse JSON Files

[4]:
```
files_list = (os.listdir('tests_male_json'))
files_list.remove('README.txt')
for file_name in files_list:
    with open('tests_male_json/'+file_name, 'r') as file:
        data = json.load(file)
        outcome = (data['info']['outcome'])
        teams = data['info']['teams']
        cross_join_df.loc[(cross_join_df['Team_y']==teams[0]) &␣
 ↪(cross_join_df['Team_x']==teams[1]),'num_matchups'] +=1
        cross_join_df.loc[(cross_join_df['Team_x']==teams[0]) &␣
 ↪(cross_join_df['Team_y']==teams[1]),'num_matchups'] +=1
        if 'winner' in outcome.keys():
            cross_join_df.loc[cross_join_df.Team_x.isin(teams) &
                          cross_join_df.Team_y.isin(teams) &
                  (cross_join_df.Team_y != cross_join_df.Team_x) &
                  cross_join_df.Team_x.isin([outcome['winner']])
                  ,'wins']+=1
```

```
        else:
            cross_join_df.loc[cross_join_df.Team_x.isin(teams) &
                              cross_join_df.Team_y.isin(teams) &
                        (cross_join_df.Team_y != cross_join_df.Team_x),'draws'] += 1
```

4.3 Preprocess Dataframe

[5]:
```
cross_join_df2 = cross_join_df[cross_join_df['num_matchups']!=0]

cross_join_df2['Team_x_factor'], unique_teams = pd.
 ↪factorize(cross_join_df2['Team_x'])

# Create a mapping dictionary from unique_teams
team_mapping = {team: idx for idx, team in enumerate(unique_teams)}
reversed_dict = {str(v): k for k, v in team_mapping.items()}

# Apply the same mapping to Team_y
cross_join_df2['Team_y_factor'] = cross_join_df2['Team_y'].map(team_mapping)
teams = cross_join_df2['Team_x'].unique()

pd.set_option('display.max_rows', None)
cross_join_df2.head(20)
```

```
/tmp/ipykernel_50148/1736895967.py:3: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-
docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
  cross_join_df2['Team_x_factor'], unique_teams =
pd.factorize(cross_join_df2['Team_x'])
/tmp/ipykernel_50148/1736895967.py:10: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-
docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
  cross_join_df2['Team_y_factor'] = cross_join_df2['Team_y'].map(team_mapping)
```

[5]:

	Team_x	Team_y	num_matchups	wins	losses	draws	\
1	Australia	Bangladesh	4	3	0	0	
2	Australia	England	52	27	0	9	
3	Australia	India	44	13	0	12	
5	Australia	New Zealand	20	17	0	2	
6	Australia	Pakistan	21	14	0	3	
7	Australia	South Africa	29	15	0	3	
8	Australia	Sri Lanka	16	10	0	2	

```
9    Australia   West Indies          20    14        0       4
10   Australia      Zimbabwe           2     2        0       0
11  Bangladesh     Australia           4     1        0       0
13  Bangladesh       England           9     1        0       0
14  Bangladesh         India          13     0        0       2
15  Bangladesh       Ireland           1     1        0       0
16  Bangladesh   New Zealand          15     2        0       3
17  Bangladesh      Pakistan          10     2        0       1
18  Bangladesh  South Africa          12     0        0       2
19  Bangladesh     Sri Lanka          22     1        0       5
20  Bangladesh   West Indies          17     4        0       1
21  Bangladesh      Zimbabwe          10     7        0       0
22     England     Australia          52    16        0       9

    Team_x_factor  Team_y_factor
1               0              1
2               0              2
3               0              3
5               0              5
6               0              6
7               0              7
8               0              8
9               0              9
10              0             10
11              1              0
13              1              2
14              1              3
15              1              4
16              1              5
17              1              6
18              1              7
19              1              8
20              1              9
21              1             10
22              2              0
```

4.4 Frequentist Estimation

Based on the number of wins, we can calculate the win percentage `win_pct`. We can gauge a team's ability based on their win percentage and rank them in that order. We also create a z score based on the win percentage so that the scale of our experiments is consistent.

[6]:
```
summary_frequentist = cross_join_df2.groupby(['Team_x']).agg({'num_matchups':
 ↪'sum','wins':'sum','losses':'sum','draws':'sum'}).reset_index()
summary_frequentist['num_matchups'] = pd.
 ↪to_numeric(summary_frequentist['num_matchups'], errors='coerce')
```

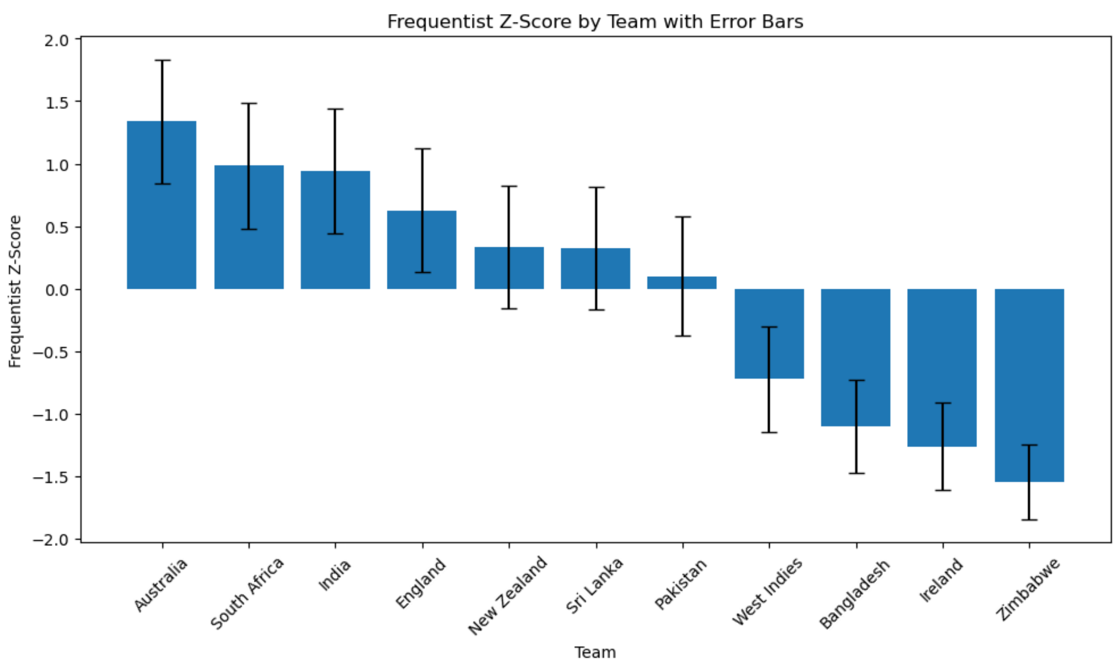
```
summary_frequentist['draws'] = pd.to_numeric(summary_frequentist['draws'], errors='coerce')
summary_frequentist['wins'] = pd.to_numeric(summary_frequentist['wins'], errors='coerce')
summary_frequentist['win_pct'] = round(100*summary_frequentist['wins']/summary_frequentist['num_matchups'],1)
summary_frequentist['win_pct'] = pd.to_numeric(summary_frequentist['win_pct'], errors='coerce')
summary_frequentist_sd = summary_frequentist['win_pct'].std()
summary_frequentist_mean = summary_frequentist['win_pct'].mean()
summary_frequentist['freqentist_z_score'] = (summary_frequentist['win_pct'] - summary_frequentist_mean )/ summary_frequentist_sd
summary_frequentist = summary_frequentist.reset_index()
summary_frequentist['std_dev'] = np.sqrt(summary_frequentist['num_matchups']*(summary_frequentist['win_pct']/100 * (1-summary_frequentist['win_pct']/100))) / np.sqrt(summary_frequentist['num_matchups'])
summary_frequentist = summary_frequentist.sort_values('freqentist_z_score',ascending=False)
summary_frequentist
```

[6]:

```
    index        Team_x  num_matchups  wins  losses  draws  win_pct  \
0       0     Australia           208   115       0     35     55.3
7       7  South Africa           181    90       0     31     49.7
3       3         India           204   100       0     51     49.0
2       2       England           254   112       0     53     44.1
5       5   New Zealand           157    62       0     32     39.5
8       8     Sri Lanka           168    66       0     36     39.3
6       6      Pakistan           151    54       0     31     35.8
9       9   West Indies           158    36       0     38     22.8
1       1    Bangladesh           113    19       0     14     16.8
4       4       Ireland             7     1       0      0     14.3
10     10      Zimbabwe            41     4       0      3      9.8

    freqentist_z_score   std_dev
0             1.337819  0.497183
7             0.982452  0.499991
3             0.938031  0.499900
2             0.627085  0.496507
5             0.335176  0.488851
8             0.322484  0.488417
6             0.100380  0.479412
9            -0.724580  0.419543
1            -1.105330  0.373866
4            -1.263976  0.350073
10           -1.549539  0.297315
```

[7]:
```
frequentist_summary_chart, ax = plt.subplots(figsize=(10, 6))

ax.bar(summary_frequentist['Team_x'],␣
  ↪summary_frequentist['freqentist_z_score'],␣
  ↪yerr=summary_frequentist['std_dev'], capsize=5)
ax.set_xlabel('Team')
ax.set_ylabel('Frequentist Z-Score')
ax.set_title('Frequentist Z-Score by Team with Error Bars')
ax.set_xticklabels(summary_frequentist['Team_x'], rotation=45)
plt.tight_layout()

plt.show()
```

```
/tmp/ipykernel_50148/35629154.py:7: UserWarning: set_ticklabels() should only be
used with a fixed number of ticks, i.e. after set_ticks() or using a
FixedLocator.
  ax.set_xticklabels(summary_frequentist['Team_x'], rotation=45)
```



By just taking the mean of win percentage we can see we can ranks the teams ability as: 1. Australia
2. South Africa
3. India
4. England
5. New Zealand
6. Sri Lanka
7. Pakistan
8. West Indies

9. Bangladesh
10. Ireland
11. Zimbabwe

4.5 Bayesian 1pl Rasch model.

Next, we use a Bayesian model approach for the same data. We use a rash 1-parameter logistic (1pl) model which uses ability and difficulty parameters to model outcomes.

$$Pr(out = 1) = \frac{\exp(\alpha_i - \beta_j + \delta)}{1 + \exp(\alpha_i - \beta_j + \delta)}$$

Our approach is adapted from the Item Response Models Section of the MC-stan guide and our model is specified as follows:

$$\begin{aligned} \text{delta} \quad &or \quad \delta \sim N(0.75, 1) \\ \text{ability}_i \quad &or \quad \alpha_i \sim \text{Normal}(0, 1) \quad \text{for } i = 1, \ldots, N_{\text{teams}} \\ \text{difficulty}_j \quad &or \quad \beta_j \sim \text{Normal}(0, 1) \quad \text{for } j = 1, \ldots, N_{\text{difficulty}} \end{aligned}$$

$$\text{logit}_p = \alpha_i - \beta_j + \delta$$

$$\text{out}_n \sim \text{Binomial}(N, \text{logit}_p)$$

[8]:
```
with pm.Model() as model:
    # Priors for team abilities
    n_pairs = len(cross_join_df2)
    N_teams = len(teams)
    N_difficulty = len(teams)
    team_ix = cross_join_df2['Team_x_factor']
    difficulty_ix = cross_join_df2['Team_y_factor']
    num_matchups = cross_join_df2['num_matchups']
    outcomes = cross_join_df2['wins']

    delta = pm.Normal('delta',mu=0.75,sigma=1)
    ability = pm.Normal('ability',mu=0,sigma = 1, shape = N_teams)
    difficulty = pm.Normal('difficulty',mu=0,sigma = 1, shape = N_difficulty)

    logit_p = ability[team_ix] - difficulty[difficulty_ix] + delta

    out = pm.Binomial('out',n=num_matchups,logit_p = logit_p,observed=outcomes)
    trace_1pl_2 = pm.sample(2000, tune=1000, return_inferencedata=True)
```

```
Auto-assigning NUTS sampler…
Initializing NUTS using jitter+adapt_diag…
```

```
Multiprocess sampling (4 chains in 4 jobs)
NUTS: [delta, ability, difficulty]

Output()

Sampling 4 chains for 1_000 tune and 2_000 draw iterations (4_000 + 8_000 draws
total) took 8 seconds.
```

[9]:

```
s1_1pl_2 = az.summary(trace_1pl_2,hdi_prob=0.95).reset_index()
s1_1pl_2['team_index'] = s1_1pl_2['index'].apply(extract_digit)
s1_1pl_2['team_name']  = s1_1pl_2['team_index'].map(reversed_dict)
s1_1pl_2
```

[9]:

	index	mean	sd	hdi_2.5%	hdi_97.5%	mcse_mean	mcse_sd
0	delta	-0.203	0.405	-0.968	0.638	0.009	0.007
1	ability[0]	1.093	0.327	0.428	1.719	0.007	0.005
2	ability[1]	-1.186	0.381	-1.926	-0.437	0.007	0.005
3	ability[2]	0.595	0.320	-0.016	1.245	0.007	0.005
4	ability[3]	0.748	0.328	0.117	1.388	0.007	0.005
5	ability[4]	-0.986	0.720	-2.354	0.464	0.009	0.007
6	ability[5]	0.229	0.336	-0.383	0.933	0.007	0.005
7	ability[6]	0.068	0.338	-0.596	0.718	0.007	0.005
8	ability[7]	0.761	0.327	0.118	1.388	0.007	0.005
9	ability[8]	0.108	0.335	-0.534	0.775	0.007	0.005
10	ability[9]	-0.642	0.353	-1.318	0.050	0.007	0.005
11	ability[10]	-1.833	0.498	-2.850	-0.877	0.008	0.005
12	difficulty[0]	1.137	0.345	0.460	1.809	0.007	0.005
13	difficulty[1]	-1.087	0.367	-1.823	-0.411	0.007	0.005
14	difficulty[2]	0.823	0.337	0.174	1.475	0.007	0.005
15	difficulty[3]	1.287	0.346	0.628	1.972	0.007	0.005
16	difficulty[4]	-1.295	0.690	-2.627	0.095	0.008	0.006
17	difficulty[5]	0.492	0.346	-0.169	1.173	0.007	0.005
18	difficulty[6]	0.350	0.348	-0.350	1.014	0.007	0.005
19	difficulty[7]	0.836	0.344	0.176	1.512	0.007	0.005
20	difficulty[8]	0.431	0.346	-0.228	1.102	0.007	0.005
21	difficulty[9]	-0.016	0.346	-0.680	0.652	0.007	0.005
22	difficulty[10]	-1.902	0.473	-2.878	-0.986	0.008	0.005

\

	ess_bulk	ess_tail	r_hat	team_index	team_name
0	1872.0	2024.0	1.0	None	NaN
1	2072.0	2702.0	1.0	0	Australia
2	2854.0	3903.0	1.0	1	Bangladesh
3	2016.0	2874.0	1.0	2	England
4	2210.0	3079.0	1.0	3	India
5	7083.0	5624.0	1.0	4	Ireland
6	2275.0	3569.0	1.0	5	New Zealand
7	2357.0	3337.0	1.0	6	Pakistan

8	2219.0	3213.0	1.0	7	South Africa
9	2293.0	3265.0	1.0	8	Sri Lanka
10	2516.0	3512.0	1.0	9	West Indies
11	4295.0	5180.0	1.0	10	Zimbabwe
12	2367.0	3346.0	1.0	0	Australia
13	2720.0	3506.0	1.0	1	Bangladesh
14	2192.0	3253.0	1.0	2	England
15	2334.0	3544.0	1.0	3	India
16	7638.0	5533.0	1.0	4	Ireland
17	2396.0	3533.0	1.0	5	New Zealand
18	2320.0	3592.0	1.0	6	Pakistan
19	2279.0	2936.0	1.0	7	South Africa
20	2356.0	3206.0	1.0	8	Sri Lanka
21	2359.0	3266.0	1.0	9	West Indies
22	3830.0	5003.0	1.0	10	Zimbabwe

4.5.1 Ability Summary

[10]:
```
ability_1pl_1 = s1_1pl_2[s1_1pl_2['index'].str.contains('ability') &␣
↪~s1_1pl_2['index'].str.contains('sigma') ].
↪sort_values('mean',ascending=False)
ability_1pl_1
```

[10]:

	index	mean	sd	hdi_2.5%	hdi_97.5%	mcse_mean	mcse_sd	\
1	ability[0]	1.093	0.327	0.428	1.719	0.007	0.005	
8	ability[7]	0.761	0.327	0.118	1.388	0.007	0.005	
4	ability[3]	0.748	0.328	0.117	1.388	0.007	0.005	
3	ability[2]	0.595	0.320	-0.016	1.245	0.007	0.005	
6	ability[5]	0.229	0.336	-0.383	0.933	0.007	0.005	
9	ability[8]	0.108	0.335	-0.534	0.775	0.007	0.005	
7	ability[6]	0.068	0.338	-0.596	0.718	0.007	0.005	
10	ability[9]	-0.642	0.353	-1.318	0.050	0.007	0.005	
5	ability[4]	-0.986	0.720	-2.354	0.464	0.009	0.007	
2	ability[1]	-1.186	0.381	-1.926	-0.437	0.007	0.005	
11	ability[10]	-1.833	0.498	-2.850	-0.877	0.008	0.005	

	ess_bulk	ess_tail	r_hat	team_index	team_name
1	2072.0	2702.0	1.0	0	Australia
8	2219.0	3213.0	1.0	7	South Africa
4	2210.0	3079.0	1.0	3	India
3	2016.0	2874.0	1.0	2	England
6	2275.0	3569.0	1.0	5	New Zealand
9	2293.0	3265.0	1.0	8	Sri Lanka
7	2357.0	3337.0	1.0	6	Pakistan
10	2516.0	3512.0	1.0	9	West Indies
5	7083.0	5624.0	1.0	4	Ireland
2	2854.0	3903.0	1.0	1	Bangladesh

```
11    4295.0    5180.0    1.0           10      Zimbabwe
```

We can see that the teams ability can be ranked in the following order: 1. Australia 2. South Africa 3. India 4. England 5. New Zealand 6. Sri Lanka 7. Pakistan 8. West Indies 9. Ireland 10. Bangladesh 11. Zimbabwe

4.5.2 Difficulty Summary

[11]:
```python
difficulty_1pl_2 = s1_1pl_2[s1_1pl_2['index'].str.contains('difficulty') & ~s1_1pl_2['index'].str.contains('sigma') ].sort_values('mean',ascending=False)
difficulty_1pl_2
```

[11]:

	index	mean	sd	hdi_2.5%	hdi_97.5%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat	team_index	team_name
15	difficulty[3]	1.287	0.346	0.628	1.972	0.007	0.005	2334.0	3544.0	1.0	3	India
12	difficulty[0]	1.137	0.345	0.460	1.809	0.007	0.005	2367.0	3346.0	1.0	0	Australia
19	difficulty[7]	0.836	0.344	0.176	1.512	0.007	0.005	2279.0	2936.0	1.0	7	South Africa
14	difficulty[2]	0.823	0.337	0.174	1.475	0.007	0.005	2192.0	3253.0	1.0	2	England
17	difficulty[5]	0.492	0.346	-0.169	1.173	0.007	0.005	2396.0	3533.0	1.0	5	New Zealand
20	difficulty[8]	0.431	0.346	-0.228	1.102	0.007	0.005	2356.0	3206.0	1.0	8	Sri Lanka
18	difficulty[6]	0.350	0.348	-0.350	1.014	0.007	0.005	2320.0	3592.0	1.0	6	Pakistan
21	difficulty[9]	-0.016	0.346	-0.680	0.652	0.007	0.005	2359.0	3266.0	1.0	9	West Indies
13	difficulty[1]	-1.087	0.367	-1.823	-0.411	0.007	0.005	2720.0	3506.0	1.0	1	Bangladesh
16	difficulty[4]	-1.295	0.690	-2.627	0.095	0.008	0.006	7638.0	5533.0	1.0	4	Ireland
22	difficulty[10]	-1.902	0.473	-2.878	-0.986	0.008	0.005	3830.0	5003.0	1.0	10	Zimbabwe

Next we can look at which team is the most difficult to face. Our model tells us that India is the model difficult team to face. We can order team inte

1. India
2. Australia
3. South Africa
4. England
5. New Zealand
6. Sri Lanka

7. Pakistan
8. West Indies
9. Bangladesh
10. Ireland
11. Zimbabwe

4.5.3 Compare Difficulty and Ability

[12]:
```
ability_difficulty_1pl_1 = pd.merge(ability_1pl_1[['mean','team_name','hdi_2.
↪5%','hdi_97.5%']],
                                    difficulty_1pl_2[['mean','team_name','hdi_2.
↪5%','hdi_97.5%']],
                                    ␣
↪on='team_name',how='inner',suffixes=('_ability', '_difficulty'))
```

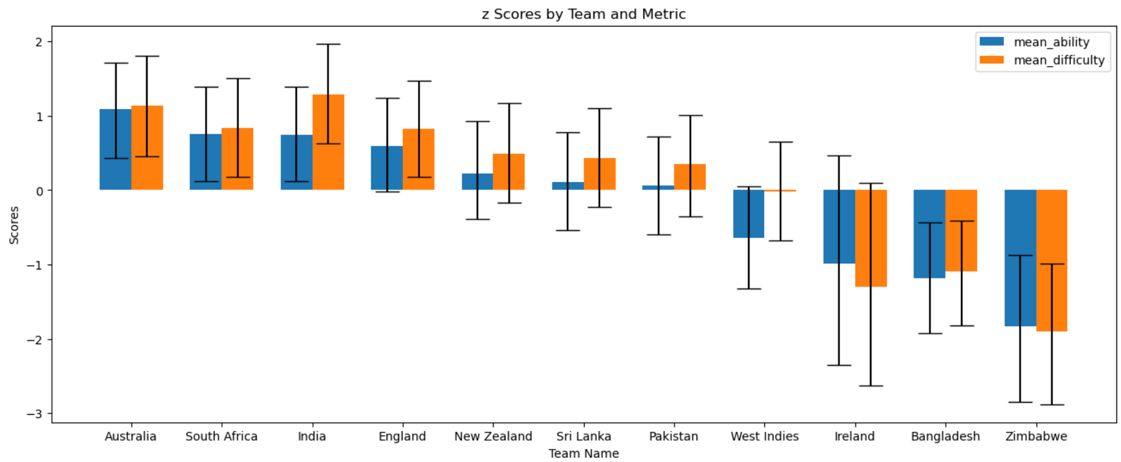
[13]:
```
ability_difficulty_1pl_1
```

[13]:
```
    mean_ability     team_name  hdi_2.5%_ability  hdi_97.5%_ability  \
0          1.093     Australia             0.428              1.719
1          0.761  South Africa             0.118              1.388
2          0.748         India             0.117              1.388
3          0.595       England            -0.016              1.245
4          0.229   New Zealand            -0.383              0.933
5          0.108     Sri Lanka            -0.534              0.775
6          0.068      Pakistan            -0.596              0.718
7         -0.642   West Indies            -1.318              0.050
8         -0.986       Ireland            -2.354              0.464
9         -1.186    Bangladesh            -1.926             -0.437
10        -1.833      Zimbabwe            -2.850             -0.877

    mean_difficulty  hdi_2.5%_difficulty  hdi_97.5%_difficulty
0             1.137                0.460                 1.809
1             0.836                0.176                 1.512
2             1.287                0.628                 1.972
3             0.823                0.174                 1.475
4             0.492               -0.169                 1.173
5             0.431               -0.228                 1.102
6             0.350               -0.350                 1.014
7            -0.016               -0.680                 0.652
8            -1.295               -2.627                 0.095
9            -1.087               -1.823                -0.411
10           -1.902               -2.878                -0.986
```

[14]:
```
ability_difficulty_1pl_1 = pd.merge(ability_1pl_1[['mean','team_name','hdi_2.
↪5%','hdi_97.5%']],difficulty_1pl_2[['mean','team_name','hdi_2.5%','hdi_97.
↪5%']],on='team_name',how='inner',suffixes=('_ability', '_difficulty'))
```

```
error_bars_ability = [ability_difficulty_1pl_1['mean_ability'] -␣
↪ability_difficulty_1pl_1['hdi_2.5%_ability'],␣
↪ability_difficulty_1pl_1['hdi_97.5%_ability'] -␣
↪ability_difficulty_1pl_1['mean_ability']]
error_bars_difficulty = [ability_difficulty_1pl_1['mean_difficulty'] -␣
↪ability_difficulty_1pl_1['hdi_2.5%_difficulty'],␣
↪ability_difficulty_1pl_1['hdi_97.5%_difficulty'] -␣
↪ability_difficulty_1pl_1['mean_difficulty']]

# Set the positions and width for the bars
positions = np.arange(len(ability_difficulty_1pl_1['team_name']))
width = 0.35

# Plotting the bars
fig, ax = plt.subplots(figsize=(16, 6)) # Increase width here
bar1 = ax.bar(positions - width/2, ability_difficulty_1pl_1['mean_ability'],␣
↪width,yerr=error_bars_ability, capsize=10, label='mean_ability')
bar2 = ax.bar(positions + width/2,␣
↪ability_difficulty_1pl_1['mean_difficulty'],width,yerr=error_bars_difficulty,␣
↪capsize=10,  label='mean_difficulty')
# bar
# Adding labels, title, and legend
ax.set_xlabel('Team Name')
ax.set_ylabel('Scores')
ax.set_title('z Scores by Team and Metric')
ax.set_xticks(positions)
ax.set_xticklabels(ability_difficulty_1pl_1['team_name'])
ax.legend()

# Store the figure in a variable
difficulty_ability_plot_1pl = fig
# Later in the code, you can render the stored plot
difficulty_ability_plot_1pl.show()
```

```
/tmp/ipykernel_50148/2040723990.py:26: UserWarning: FigureCanvasAgg is non-
interactive, and thus cannot be shown
  difficulty_ability_plot_1pl.show()
```



- We can see that ability and difficulty are closely related, however, it does not mean that there is a one-to-one correlation. The most difficult team is India, however, in terms of ability, India is third.
- For the bottom performers and the top performers our 95% HPD credible sets do not contain 0 indicating that we can be quite certain that these teams are underperforming or overperforming. however for the teams in the middle, the HPD interval contains 0, so we cannot say for certain if these teams are above average or below average.
- Apart from looking at the mean, we can also compare the HPD intervals for all the teams to gauge how one compares to the others. For example
  - If we compare the ability of West Indies with the ability of Australia, their HPD intervals don't overlap, we can be quite certain that Australia is a better team than West Indies.
  - If we compare the ability of Pakistan with Srilanka, because the HPD intervals have a high degree of overall we have a high degree of uncertainty about our conclusions regarding which team is better even though Sri Lanka has a higher mean ability.

We see that the mean we get from the frequentist approach matches closely with what we observe using the Bayesian 1 pl model.

4.6 Bayesian 2pl Model

We add to a 1pl model by adding hierarchy and a discrimination term. The probability of an outcome is modeled by:

$$Pr(out = 1) = \frac{\exp(\gamma_j * (\alpha_i - (\beta_j + \delta)))}{1 + \exp(\gamma_j * (\alpha_i - (\beta_j + \delta)))}$$

The discrimination terms are:

$$\text{standard deviation of discrimination} \quad or \quad \sigma_\gamma \sim \text{HalfCauchy}(0, 3)$$

$$\text{discrimination} \quad or \quad \gamma_j \sim LogNormal(0, \sigma_\gamma)$$

We define the following distribution to model a team's ability:

$$\text{ability} \quad or \quad \alpha \sim Normal(0,1)$$

To model Opponent Difficulty we define the following distributions: We recenter difficulty in avoiding fit issues:

$$\text{mean question difficulty} \quad or \quad \delta \sim \text{Cauchy}(0,5)$$

$$\text{standard deviation of difficulty} \quad or \quad \sigma_\beta \sim \text{HalfCauchy}(0,5)$$

$$\text{difficulty} \quad or \quad \beta \sim Normal(0,\sigma_\beta)$$

The outcome is defined as:

$$\text{logit}_p = \gamma_j * (\alpha_i - (\beta_j + \delta))$$

$$\text{out}_n \sim \text{Binomial}(N, \text{logit}_p)$$

[15]:
```
rng = np.random.default_rng(1)
with pm.Model() as model:
    # Priors for team abilities
    n_pairs = len(cross_join_df2)
    N_teams = len(teams)
    N_difficulty = len(teams)
    team_ix = cross_join_df2['Team_x_factor']
    difficulty_ix = cross_join_df2['Team_y_factor']
    num_matchups = cross_join_df2['num_matchups']
    outcomes = cross_join_df2['wins']

    a_difficulty = pm.StudentT('a_difficulty',nu=3, mu=0, sigma=5)
    sigma_discrimination = pm.HalfCauchy('sigma_discrimination', beta=3) #␣
 ↪sigma_gamma ~ cauchy(0, 5);
    # sigma_ability = pm.HalfCauchy('sigma_ability', beta=5)
    sigma_difficulty = pm.HalfCauchy('sigma_difficulty', beta=5) #  sigma_beta␣
 ↪~ cauchy(0, 5);

    discrimination = pm.LogNormal('discrimination', mu=0,␣
 ↪sigma=sigma_discrimination, shape=N_difficulty) # gamma ~ lognormal(0,␣
 ↪sigma_gamma);
    ability = pm.Normal('ability', mu=0, sigma=1, shape=N_teams) # alpha ~␣
 ↪std_normal();
    difficulty = pm.Normal('difficulty', mu=0, sigma=sigma_difficulty,␣
 ↪shape=N_difficulty) # beta ~ normal(0, sigma_beta);

    mu_beta =  pm.Cauchy('mu_beta', alpha=0, beta=5) # mu_beta ~ cauchy(0, 5);
    logit_p = discrimination[difficulty_ix] * (ability[team_ix] -␣
 ↪(difficulty[difficulty_ix] + mu_beta))

    out = pm.Binomial('out',n=num_matchups, logit_p=logit_p, observed=outcomes)
```

```
    trace_2pl_2 = pm.sample(2000, tune=1000,␣
↪return_inferencedata=True,random_seed=rng)
```

```
Auto-assigning NUTS sampler…
Initializing NUTS using jitter+adapt_diag…
Multiprocess sampling (4 chains in 4 jobs)
NUTS: [a_difficulty, sigma_discrimination, sigma_difficulty, discrimination,
ability, difficulty, mu_beta]
```

```
Output()
```

```
Sampling 4 chains for 1_000 tune and 2_000 draw iterations (4_000 + 8_000 draws
total) took 18 seconds.
There were 5 divergences after tuning. Increase `target_accept` or
reparameterize.
```

4.6.1 Arviz Summary

[16]:
```
s1_2pl_2 = az.summary(trace_2pl_2,hdi_prob=0.95).reset_index()
s1_2pl_2['team_index'] = s1_2pl_2['index'].apply(extract_digit)
s1_2pl_2['team_name']  = s1_2pl_2['team_index'].map(reversed_dict)
s1_2pl_2
```

[16]:

	index	mean	sd	hdi_2.5%	hdi_97.5%	mcse_mean	\
0	a_difficulty	-0.125	9.018	-16.607	15.330	0.241	
1	ability[0]	1.196	0.347	0.521	1.871	0.008	
2	ability[1]	-1.092	0.402	-1.944	-0.351	0.008	
3	ability[2]	0.714	0.340	0.002	1.339	0.008	
4	ability[3]	0.860	0.341	0.200	1.524	0.008	
5	ability[4]	-0.904	0.726	-2.291	0.541	0.009	
6	ability[5]	0.353	0.351	-0.341	1.036	0.008	
7	ability[6]	0.171	0.353	-0.524	0.854	0.008	
8	ability[7]	0.877	0.347	0.184	1.549	0.008	
9	ability[8]	0.235	0.350	-0.459	0.908	0.008	
10	ability[9]	-0.565	0.368	-1.258	0.198	0.008	
11	ability[10]	-1.809	0.541	-2.892	-0.816	0.009	
12	difficulty[0]	1.103	0.468	0.183	2.060	0.011	
13	difficulty[1]	-1.245	0.504	-2.282	-0.286	0.012	
14	difficulty[2]	0.830	0.462	-0.045	1.795	0.011	
15	difficulty[3]	1.232	0.470	0.251	2.111	0.011	
16	difficulty[4]	-1.615	0.874	-3.409	0.027	0.013	
17	difficulty[5]	0.430	0.446	-0.425	1.350	0.011	
18	difficulty[6]	0.328	0.456	-0.541	1.291	0.011	
19	difficulty[7]	0.748	0.451	-0.117	1.677	0.011	
20	difficulty[8]	0.354	0.441	-0.579	1.196	0.011	
21	difficulty[9]	-0.080	0.448	-0.937	0.872	0.011	
22	difficulty[10]	-2.051	0.582	-3.175	-0.921	0.011	

23	mu_beta	0.375	0.522	-0.630	1.433	0.014
24	sigma_discrimination	0.177	0.131	0.010	0.427	0.005
25	sigma_difficulty	1.332	0.412	0.713	2.208	0.007
26	discrimination[0]	1.014	0.173	0.662	1.386	0.002
27	discrimination[1]	0.939	0.150	0.601	1.215	0.003
28	discrimination[2]	0.923	0.143	0.590	1.172	0.003
29	discrimination[3]	1.038	0.176	0.698	1.421	0.002
30	discrimination[4]	1.078	0.305	0.618	1.621	0.006
31	discrimination[5]	1.043	0.167	0.701	1.405	0.002
32	discrimination[6]	0.904	0.152	0.589	1.205	0.004
33	discrimination[7]	1.096	0.203	0.736	1.531	0.004
34	discrimination[8]	1.097	0.193	0.790	1.535	0.004
35	discrimination[9]	0.964	0.147	0.652	1.263	0.002
36	discrimination[10]	1.037	0.188	0.655	1.430	0.002

	mcse_sd	ess_bulk	ess_tail	r_hat	team_index	team_name
0	0.277	5247.0	2277.0	1.00	None	NaN
1	0.005	2053.0	3441.0	1.00	0	Australia
2	0.005	2721.0	3992.0	1.00	1	Bangladesh
3	0.005	1907.0	2946.0	1.00	2	England
4	0.005	2057.0	3298.0	1.00	3	India
5	0.007	5951.0	5309.0	1.00	4	Ireland
6	0.005	2094.0	3544.0	1.00	5	New Zealand
7	0.005	2157.0	3477.0	1.00	6	Pakistan
8	0.006	1981.0	3376.0	1.00	7	South Africa
9	0.005	2160.0	3584.0	1.00	8	Sri Lanka
10	0.005	2377.0	3528.0	1.00	9	West Indies
11	0.007	3453.0	4230.0	1.00	10	Zimbabwe
12	0.008	1816.0	2708.0	1.00	0	Australia
13	0.008	1846.0	2389.0	1.00	1	Bangladesh
14	0.008	1662.0	2402.0	1.00	2	England
15	0.008	1758.0	2523.0	1.00	3	India
16	0.010	4477.0	3957.0	1.00	4	Ireland
17	0.008	1639.0	2125.0	1.00	5	New Zealand
18	0.008	1678.0	2439.0	1.00	6	Pakistan
19	0.008	1649.0	2337.0	1.00	7	South Africa
20	0.008	1684.0	2463.0	1.00	8	Sri Lanka
21	0.008	1695.0	2366.0	1.00	9	West Indies
22	0.008	2897.0	2569.0	1.00	10	Zimbabwe
23	0.010	1456.0	1988.0	1.00	None	NaN
24	0.004	403.0	394.0	1.01	None	NaN
25	0.005	4035.0	3521.0	1.00	None	NaN
26	0.002	6900.0	3484.0	1.00	0	Australia
27	0.002	2709.0	3431.0	1.00	1	Bangladesh
28	0.002	2801.0	2863.0	1.00	2	England
29	0.002	6782.0	3732.0	1.00	3	India
30	0.005	6517.0	2516.0	1.01	4	Ireland

```
31     0.002    6968.0     3772.0    1.00           5   New Zealand
32     0.003    1999.0     2809.0    1.00           6      Pakistan
33     0.003    3051.0     3816.0    1.00           7  South Africa
34     0.003    3039.0     3079.0    1.00           8     Sri Lanka
35     0.001    6778.0     3291.0    1.00           9   West Indies
36     0.002    6988.0     3633.0    1.00          10      Zimbabwe
```

4.6.2 Ability Summary

[17]:
```
ability_2pl_1 = s1_2pl_2[s1_2pl_2['index'].str.contains('ability') &␣
  ↪~s1_2pl_2['index'].str.contains('sigma') ].
  ↪sort_values('mean',ascending=False)
ability_2pl_1
```

[17]:
```
          index   mean     sd  hdi_2.5%  hdi_97.5%  mcse_mean  mcse_sd  \
1     ability[0]  1.196  0.347     0.521      1.871      0.008    0.005
8     ability[7]  0.877  0.347     0.184      1.549      0.008    0.006
4     ability[3]  0.860  0.341     0.200      1.524      0.008    0.005
3     ability[2]  0.714  0.340     0.002      1.339      0.008    0.005
6     ability[5]  0.353  0.351    -0.341      1.036      0.008    0.005
9     ability[8]  0.235  0.350    -0.459      0.908      0.008    0.005
7     ability[6]  0.171  0.353    -0.524      0.854      0.008    0.005
10    ability[9] -0.565  0.368    -1.258      0.198      0.008    0.005
5     ability[4] -0.904  0.726    -2.291      0.541      0.009    0.007
2     ability[1] -1.092  0.402    -1.944     -0.351      0.008    0.005
11   ability[10] -1.809  0.541    -2.892     -0.816      0.009    0.007

    ess_bulk  ess_tail  r_hat team_index     team_name
1     2053.0    3441.0    1.0          0     Australia
8     1981.0    3376.0    1.0          7  South Africa
4     2057.0    3298.0    1.0          3         India
3     1907.0    2946.0    1.0          2       England
6     2094.0    3544.0    1.0          5   New Zealand
9     2160.0    3584.0    1.0          8     Sri Lanka
7     2157.0    3477.0    1.0          6      Pakistan
10    2377.0    3528.0    1.0          9   West Indies
5     5951.0    5309.0    1.0          4       Ireland
2     2721.0    3992.0    1.0          1    Bangladesh
11    3453.0    4230.0    1.0         10      Zimbabwe
```

We can see that the team's ability can be ranked in the following order:
1. Australia
2. South Africa
3. India
4. England
5. New Zealand
6. Sri Lanka

7. Pakistan
8. West Indies
9. Ireland
10. Bangladesh
11. Zimbabwe

4.6.3 Difficulty Summary

[18]:
```
difficulty_2pl_2 = s1_2pl_2[s1_2pl_2['index'].str.contains('difficulty') &
 ↪~s1_2pl_2['index'].str.contains('sigma') &  ~s1_2pl_2['index'].str.
 ↪contains('a_difficulty') ].sort_values('mean',ascending=False)
difficulty_2pl_2
```

[18]:

	index	mean	sd	hdi_2.5%	hdi_97.5%	mcse_mean	mcse_sd	\
15	difficulty[3]	1.232	0.470	0.251	2.111	0.011	0.008	
12	difficulty[0]	1.103	0.468	0.183	2.060	0.011	0.008	
14	difficulty[2]	0.830	0.462	-0.045	1.795	0.011	0.008	
19	difficulty[7]	0.748	0.451	-0.117	1.677	0.011	0.008	
17	difficulty[5]	0.430	0.446	-0.425	1.350	0.011	0.008	
20	difficulty[8]	0.354	0.441	-0.579	1.196	0.011	0.008	
18	difficulty[6]	0.328	0.456	-0.541	1.291	0.011	0.008	
21	difficulty[9]	-0.080	0.448	-0.937	0.872	0.011	0.008	
13	difficulty[1]	-1.245	0.504	-2.282	-0.286	0.012	0.008	
16	difficulty[4]	-1.615	0.874	-3.409	0.027	0.013	0.010	
22	difficulty[10]	-2.051	0.582	-3.175	-0.921	0.011	0.008	

	ess_bulk	ess_tail	r_hat	team_index	team_name
15	1758.0	2523.0	1.0	3	India
12	1816.0	2708.0	1.0	0	Australia
14	1662.0	2402.0	1.0	2	England
19	1649.0	2337.0	1.0	7	South Africa
17	1639.0	2125.0	1.0	5	New Zealand
20	1684.0	2463.0	1.0	8	Sri Lanka
18	1678.0	2439.0	1.0	6	Pakistan
21	1695.0	2366.0	1.0	9	West Indies
13	1846.0	2389.0	1.0	1	Bangladesh
16	4477.0	3957.0	1.0	4	Ireland
22	2897.0	2569.0	1.0	10	Zimbabwe

The difficulty of the opposition can be ranked as follows 1. India 2. Australia 3. England 4. South Africa 5. New Zealand 6. Sri Lanka 7. Pakistan 8. West Indies 9. Bangladesh 10. Ireland 11. Zimbabwe

4.6.4 Discrimination Summary

[19]:
```
discrimination_2pl_2 = s1_2pl_2[s1_2pl_2['index'].str.
    contains('discrimination') & ~s1_2pl_2['index'].str.contains('sigma') &
    ~s1_2pl_2['index'].str.contains('a_difficulty') ].
    sort_values('mean',ascending=False)
discrimination_2pl_2
```

```
The history saving thread hit an unexpected error (OperationalError('attempt to
write a readonly database')).History will not be written to the database.
```

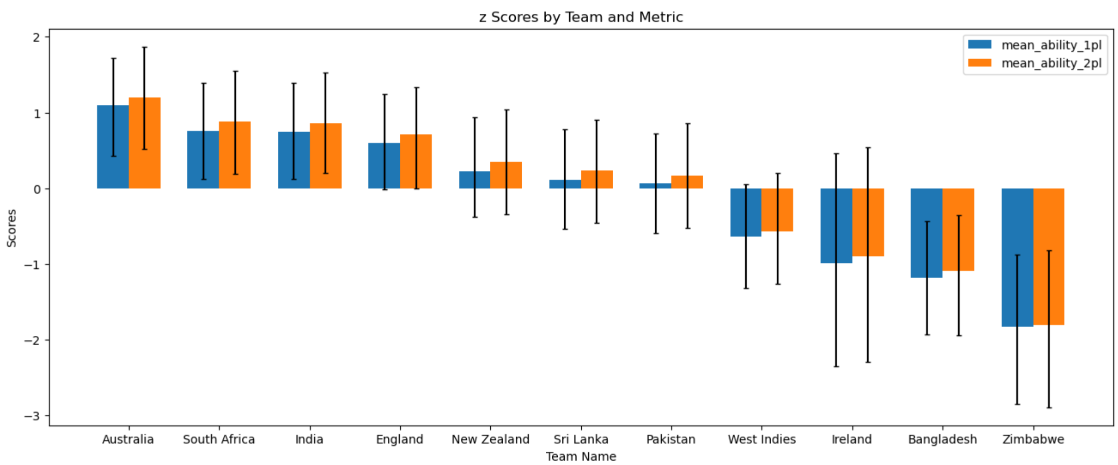
[19]:

	index	mean	sd	hdi_2.5%	hdi_97.5%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat	team_index	team_name
34	discrimination[8]	1.097	0.193	0.790	1.535	0.004	0.003	3039.0	3079.0	1.00	8	Sri Lanka
33	discrimination[7]	1.096	0.203	0.736	1.531	0.004	0.003	3051.0	3816.0	1.00	7	South Africa
30	discrimination[4]	1.078	0.305	0.618	1.621	0.006	0.005	6517.0	2516.0	1.01	4	Ireland
31	discrimination[5]	1.043	0.167	0.701	1.405	0.002	0.002	6968.0	3772.0	1.00	5	New Zealand
29	discrimination[3]	1.038	0.176	0.698	1.421	0.002	0.002	6782.0	3732.0	1.00	3	India
36	discrimination[10]	1.037	0.188	0.655	1.430	0.002	0.002	6988.0	3633.0	1.00	10	Zimbabwe
26	discrimination[0]	1.014	0.173	0.662	1.386	0.002	0.002	6900.0	3484.0	1.00	0	Australia
35	discrimination[9]	0.964	0.147	0.652	1.263	0.002	0.001	6778.0	3291.0	1.00	9	West Indies
27	discrimination[1]	0.939	0.150	0.601	1.215	0.003	0.002	2709.0	3431.0	1.00	1	Bangladesh
28	discrimination[2]	0.923	0.143	0.590	1.172	0.003	0.002	2801.0	2863.0	1.00	2	England
32	discrimination[6]	0.904	0.152	0.589	1.205	0.004	0.003	1999.0	2809.0	1.00	6	Pakistan

[20]:
```
ability_difficulty_1pl_2pl_1 = pd.
    merge(ability_1pl_1[['mean','team_name','hdi_2.5%','hdi_97.
    5%']],ability_2pl_1[['mean','team_name','hdi_2.5%','hdi_97.
    5%']],on='team_name',how='inner',suffixes=('_ability_1pl', '_ability_2pl'))
error_bars_ability_1pl1 = [ability_difficulty_1pl_2pl_1['mean_ability_1pl'] -
    ability_difficulty_1pl_2pl_1['hdi_2.5%_ability_1pl'],
    ability_difficulty_1pl_2pl_1['hdi_97.5%_ability_1pl'] -
    ability_difficulty_1pl_2pl_1['mean_ability_1pl']]
```

```
error_bars_ability_2pl1 = [ability_difficulty_1pl_2pl_1['mean_ability_2pl'] -␣
 ↪ability_difficulty_1pl_2pl_1['hdi_2.5%_ability_2pl'],␣
 ↪ability_difficulty_1pl_2pl_1['hdi_97.5%_ability_2pl'] -␣
 ↪ability_difficulty_1pl_2pl_1['mean_ability_2pl']]

# Set the positions and width for the bars
positions = np.arange(len(ability_difficulty_1pl_2pl_1['team_name']))
width = 0.35

# Plotting the bars
fig, ax = plt.subplots(figsize=(16, 6)) # Increase width here
bar1 = ax.bar(positions - width/2,␣
 ↪ability_difficulty_1pl_2pl_1['mean_ability_1pl'],␣
 ↪width,yerr=error_bars_ability_1pl1, capsize=2, label='mean_ability_1pl')
bar2 = ax.bar(positions + width/2,␣
 ↪ability_difficulty_1pl_2pl_1['mean_ability_2pl'],width,yerr=error_bars_ability_2pl1,␣
 ↪capsize=2,  label='mean_ability_2pl')
# bar
# Adding labels, title, and legend
ax.set_xlabel('Team Name')
ax.set_ylabel('Scores')
ax.set_title('z Scores by Team and Metric')
ax.set_xticks(positions)
ax.set_xticklabels(ability_difficulty_1pl_2pl_1['team_name'])
ax.legend()
```

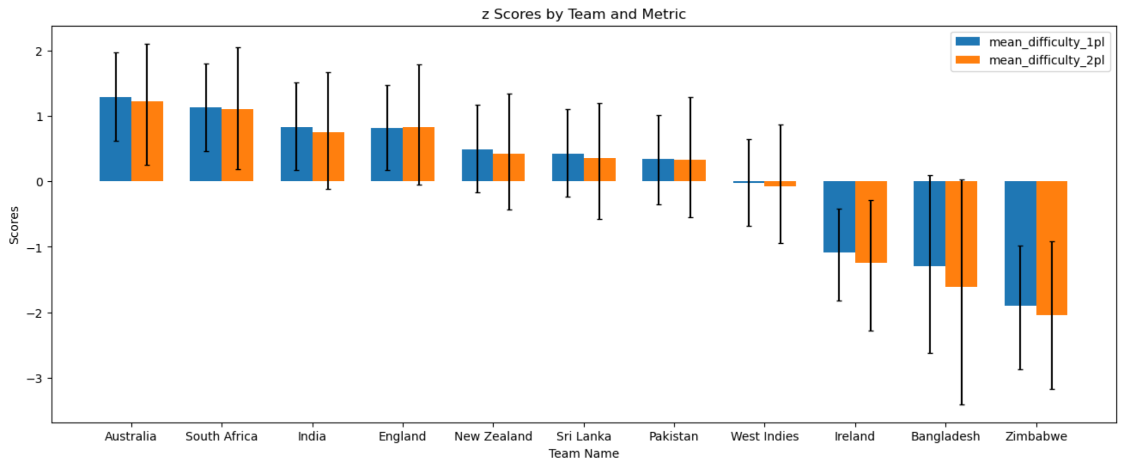
[20]: <matplotlib.legend.Legend at 0x7f1dba092bd0>



[21]:

```
ability_difficulty_1pl_2pl_2 = pd.
 ↪merge(difficulty_1pl_2[['mean','team_name','hdi_2.5%','hdi_97.
 ↪5%']],difficulty_2pl_2[['mean','team_name','hdi_2.5%','hdi_97.
 ↪5%']],on='team_name',how='inner',suffixes=('_difficulty_1pl',␣
 ↪'_difficulty_2pl'))

# Calculate error bars
error_bars_difficulty_1pl1 =␣
 ↪[ability_difficulty_1pl_2pl_2['mean_difficulty_1pl'] -␣
 ↪ability_difficulty_1pl_2pl_2['hdi_2.5%_difficulty_1pl'],␣
 ↪ability_difficulty_1pl_2pl_2['hdi_97.5%_difficulty_1pl'] -␣
 ↪ability_difficulty_1pl_2pl_2['mean_difficulty_1pl']]
error_bars_difficulty_2pl1 =␣
 ↪[ability_difficulty_1pl_2pl_2['mean_difficulty_2pl'] -␣
 ↪ability_difficulty_1pl_2pl_2['hdi_2.5%_difficulty_2pl'],␣
 ↪ability_difficulty_1pl_2pl_2['hdi_97.5%_difficulty_2pl'] -␣
 ↪ability_difficulty_1pl_2pl_2['mean_difficulty_2pl']]

# Set the positions and width for the bars
positions = np.arange(len(ability_difficulty_1pl_2pl_2['team_name']))
width = 0.35

# Plotting the bars
fig, ax = plt.subplots(figsize=(16, 6))  # Increase width here
bar1 = ax.bar(positions - width/2,␣
 ↪ability_difficulty_1pl_2pl_2['mean_difficulty_1pl'], width,␣
 ↪yerr=error_bars_difficulty_1pl1, capsize=2, label='mean_difficulty_1pl')
bar2 = ax.bar(positions + width/2,␣
 ↪ability_difficulty_1pl_2pl_2['mean_difficulty_2pl'], width,␣
 ↪yerr=error_bars_difficulty_2pl1, capsize=2, label='mean_difficulty_2pl')

# Adding labels, title, and legend
ax.set_xlabel('Team Name')
ax.set_ylabel('Scores')
ax.set_title('z Scores by Team and Metric')
ax.set_xticks(positions)
ax.set_xticklabels(ability_difficulty_1pl_2pl_1['team_name'])
ax.legend()

# Store the figure in a variable
difficulty_ability_plot_2pl_vs_1pl = fig
# Later in the code, you can render the stored plot
difficulty_ability_plot_2pl_vs_1pl.show()
```

```
/tmp/ipykernel_50148/1881396008.py:27: UserWarning: FigureCanvasAgg is non-
interactive, and thus cannot be shown
  difficulty_ability_plot_2pl_vs_1pl.show()
```



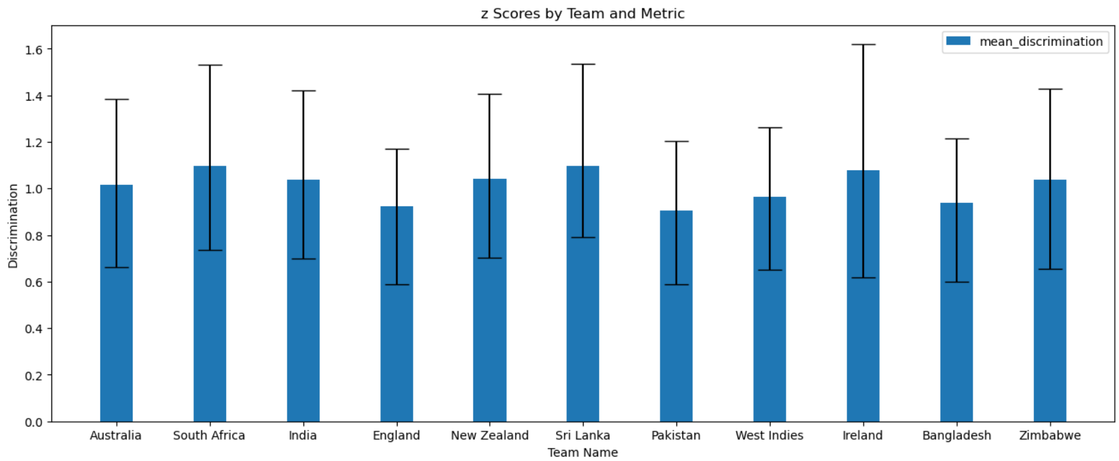
4.6.5 Plot Discrimination

[22]:
```
# Merge data frames
ability_difficulty_discrimination = pd.merge(
    pd.merge(ability_2pl_1[['mean','team_name','hdi_2.5%','hdi_97.5%']],
             difficulty_2pl_2[['mean','team_name','hdi_2.5%','hdi_97.5%']],
             on='team_name',
             how='inner',
             suffixes=('_ability', '_difficulty')),
    discrimination_2pl_2[['mean','team_name','hdi_2.5%','hdi_97.5%']],
    on='team_name',
    how='inner'
).rename(columns={'mean': 'mean_discrimination', 'hdi_2.5%': 'hdi_2.
 ↪5%_discrimination', 'hdi_97.5%': 'hdi_97.5%_discrimination'})

# Calculate error bars
error_bars_ability = [
    ability_difficulty_discrimination['mean_ability'] -␣
 ↪ability_difficulty_discrimination['hdi_2.5%_ability'],
    ability_difficulty_discrimination['hdi_97.5%_ability'] -␣
 ↪ability_difficulty_discrimination['mean_ability']
]
error_bars_difficulty = [
    ability_difficulty_discrimination['mean_difficulty'] -␣
 ↪ability_difficulty_discrimination['hdi_2.5%_difficulty'],
    ability_difficulty_discrimination['hdi_97.5%_difficulty'] -␣
 ↪ability_difficulty_discrimination['mean_difficulty']
]
error_bars_discrimination = [
```

```
    ability_difficulty_discrimination['mean_discrimination'] -␣
 ↪ability_difficulty_discrimination['hdi_2.5%_discrimination'],
    ability_difficulty_discrimination['hdi_97.5%_discrimination'] -␣
 ↪ability_difficulty_discrimination['mean_discrimination']
]

# Set the positions and width for the bars
positions = np.arange(len(ability_difficulty_discrimination['team_name']))
width = 0.35

# Plotting the bars
fig, ax = plt.subplots(figsize=(16, 6))  # Increase width here
# bar1 = ax.bar(
#     positions - width/2,
#     ability_difficulty_discrimination['mean_ability'],
#     width,
#     yerr=error_bars_ability,
#     capsize=10,
#     label='mean_ability'
# )
# bar2 = ax.bar(
#     positions + width/2,
#     ability_difficulty_discrimination['mean_difficulty'],
#     width,
#     yerr=error_bars_difficulty,
#     capsize=10,
#     label='mean_difficulty'
# )
bar3 = ax.bar(
    positions,
    ability_difficulty_discrimination['mean_discrimination'],
    width,
    yerr=error_bars_discrimination,
    capsize=10,
    label='mean_discrimination'
)

# Adding labels, title, and legend
ax.set_xlabel('Team Name')
ax.set_ylabel('Discrimination')
ax.set_title('z Scores by Team and Metric')
ax.set_xticks(positions)
ax.set_xticklabels(ability_difficulty_discrimination['team_name'])
ax.legend()

# Store the figure in a variable
discrimination_2pl = fig
```

```
# Later in the code, you can render the stored plot
discrimination_2pl.show()
```

```
/tmp/ipykernel_50148/2192592902.py:69: UserWarning: FigureCanvasAgg is non-
interactive, and thus cannot be shown
  discrimination_2pl.show()
```



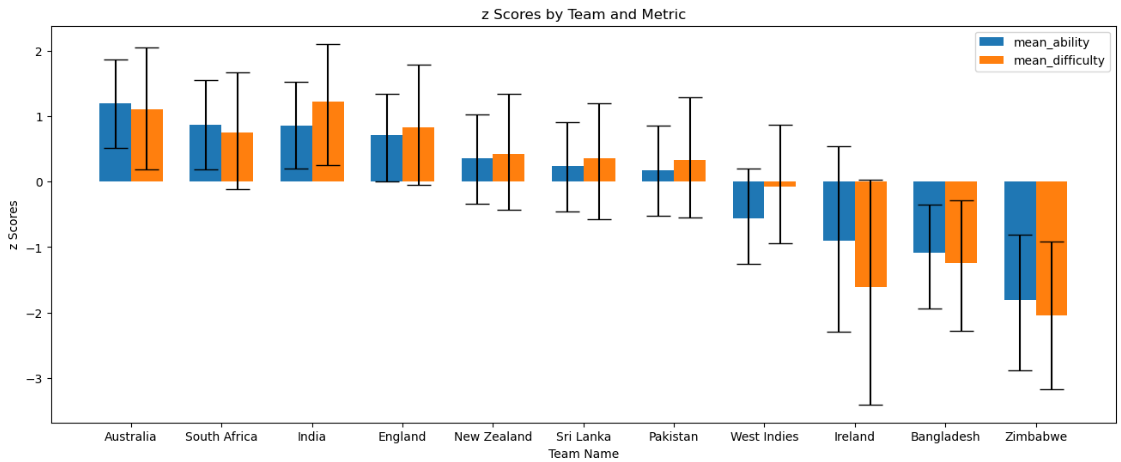
4.6.6 Compare Difficulty and Ability

[23]:
```
# Merge data frames
ability_difficulty_2pl_1 = pd.merge(
    pd.merge(ability_2pl_1[['mean','team_name','hdi_2.5%','hdi_97.5%']],
             difficulty_2pl_2[['mean','team_name','hdi_2.5%','hdi_97.5%']],
             on='team_name',
             how='inner',
             suffixes=('_ability', '_difficulty')),
    discrimination_2pl_2[['mean','team_name','hdi_2.5%','hdi_97.5%']],
    on='team_name',
    how='inner'
).rename(columns={'mean': 'mean_discrimination', 'hdi_2.5%': 'hdi_2.
 ↪5%_discrimination', 'hdi_97.5%': 'hdi_97.5%_discrimination'})

# Calculate error bars
error_bars_ability = [
    ability_difficulty_2pl_1['mean_ability'] - ability_difficulty_2pl_1['hdi_2.
 ↪5%_ability'],
    ability_difficulty_2pl_1['hdi_97.5%_ability'] -␣
 ↪ability_difficulty_2pl_1['mean_ability']
]
error_bars_difficulty = [
```

```
    ability_difficulty_2pl_1['mean_difficulty'] -␣
↪ability_difficulty_2pl_1['hdi_2.5%_difficulty'],
    ability_difficulty_2pl_1['hdi_97.5%_difficulty'] -␣
↪ability_difficulty_2pl_1['mean_difficulty']
]
error_bars_discrimination = [
    ability_difficulty_2pl_1['mean_discrimination'] -␣
↪ability_difficulty_2pl_1['hdi_2.5%_discrimination'],
    ability_difficulty_2pl_1['hdi_97.5%_discrimination'] -␣
↪ability_difficulty_2pl_1['mean_discrimination']
]

# Set the positions and width for the bars
positions = np.arange(len(ability_difficulty_2pl_1['team_name']))
width = 0.35

# Plotting the bars
fig, ax = plt.subplots(figsize=(16, 6))  # Increase width here
bar1 = ax.bar(
    positions - width/2,
    ability_difficulty_2pl_1['mean_ability'],
    width,
    yerr=error_bars_ability,
    capsize=10,
    label='mean_ability'
)
bar2 = ax.bar(
    positions + width/2,
    ability_difficulty_2pl_1['mean_difficulty'],
    width,
    yerr=error_bars_difficulty,
    capsize=10,
    label='mean_difficulty'
)
# bar3 = ax.bar(
#     positions + width,
#     ability_difficulty_2pl_1['mean_discrimination'],
#     width,
#     yerr=error_bars_discrimination,
#     capsize=10,
#     label='mean_discrimination'
# )

# Adding labels, title, and legend
ax.set_xlabel('Team Name')
ax.set_ylabel('z Scores')
ax.set_title('z Scores by Team and Metric')
```

```
ax.set_xticks(positions)
ax.set_xticklabels(ability_difficulty_2pl_1['team_name'])
ax.legend()

# Store the figure in a variable
difficulty_ability_plot_2pl = fig
# Later in the code, you can render the stored plot
difficulty_ability_plot_2pl.show()
```

```
/tmp/ipykernel_50148/1227178939.py:69: UserWarning: FigureCanvasAgg is non-
interactive, and thus cannot be shown
  difficulty_ability_plot_2pl.show()
```



4.6.7 Compare Frequentist vs Bayesian 1pl vs Bayesian 2pl

[24]:
```
ability_difficulty_1pl_2 = ability_difficulty_1pl_1.add_suffix('_1pl')
ability_difficulty_freq_1pl_1 = pd.merge(ability_difficulty_1pl_2,
                                                                    ␣
 ↪summary_frequentist[['Team_x','freqentist_z_score','std_dev']],
                                          left_on='team_name_1pl',
                                          right_on='Team_x',
                                          suffixes=("_1pl", "_frequentist"))

ability_difficulty_2pl_2 = ability_difficulty_2pl_1.add_suffix('_2pl')
ability_difficulty_freq_1pl_1_2pl_1 = pd.merge(ability_difficulty_freq_1pl_1,
                                                                          ␣
 ↪ability_difficulty_2pl_2[['mean_ability_2pl', 'team_name_2pl',
                                                                  'hdi_2.
 ↪5%_ability_2pl', 'hdi_97.5%_ability_2pl',
                                                                          ␣
 ↪'mean_difficulty_2pl',
```

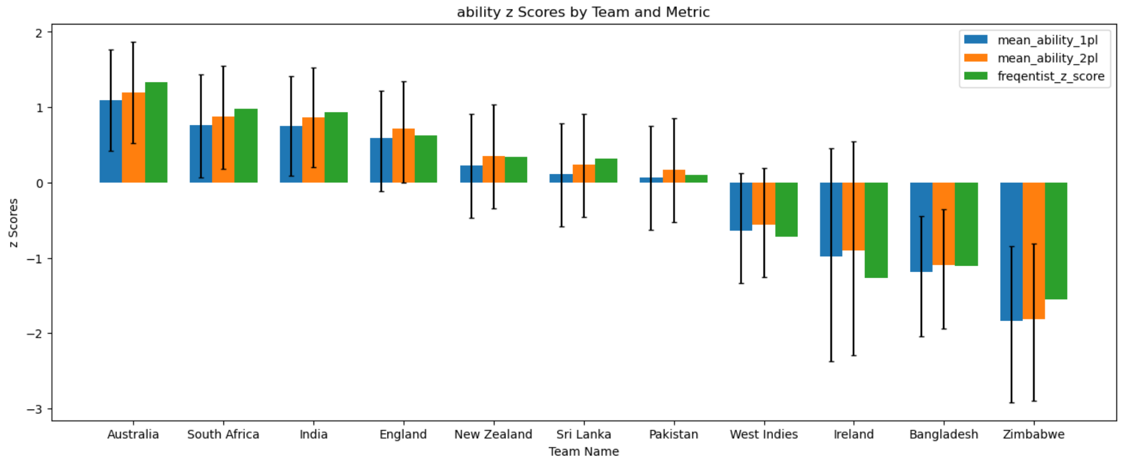
```
                                                                          'hdi_2.
↪5%_difficulty_2pl', 'hdi_97.5%_difficulty_2pl']],
                                        left_on='team_name_1pl',
                                        right_on='team_name_2pl',
                                        suffixes=("", "_2pl"))
```

Plot ability Compare ability estimate of 1pl and 2pl model and also the win percentage seen from the data.

[25]:
```
error_bars_ability_1pl1 =␣
 ↪[ability_difficulty_freq_1pl_1_2pl_1['mean_ability_1pl'] -␣
 ↪ability_difficulty_freq_1pl_1_2pl_1['hdi_2.5%_ability_1pl'],
                           ability_difficulty_freq_1pl_1_2pl_1['hdi_97.
 ↪5%_ability_1pl'] - ability_difficulty_freq_1pl_1_2pl_1['mean_ability_1pl']]
error_bars_ability_2pl1 =␣
 ↪[ability_difficulty_freq_1pl_1_2pl_1['mean_ability_2pl'] -␣
 ↪ability_difficulty_freq_1pl_1_2pl_1['hdi_2.5%_ability_2pl'],
                         ability_difficulty_freq_1pl_1_2pl_1['hdi_97.
 ↪5%_ability_2pl'] - ability_difficulty_freq_1pl_1_2pl_1['mean_ability_2pl']]
error_bars_frequentist =␣
 ↪[ability_difficulty_freq_1pl_1_2pl_1['std_dev'],ability_difficulty_freq_1pl_1_2pl_1['std_de
# error_bars_frequentist

# Set the positions and width for the bars
positions = np.arange(len(ability_difficulty_freq_1pl_1_2pl_1['team_name_2pl']))
width = 0.25

# Plotting the bars
fig, ax = plt.subplots(figsize=(16, 6)) # Increase width here
bar1 = ax.bar(positions - width,␣
 ↪ability_difficulty_freq_1pl_1_2pl_1['mean_ability_1pl'],
              width,yerr=error_bars_ability, capsize=2,␣
 ↪label='mean_ability_1pl')
bar2 = ax.bar(positions,␣
 ↪ability_difficulty_freq_1pl_1_2pl_1['mean_ability_2pl'],
              width,yerr=error_bars_ability_2pl1, capsize=2, ␣
 ↪label='mean_ability_2pl')
bar3 = ax.bar(positions + width,␣
 ↪ability_difficulty_freq_1pl_1_2pl_1['freqentist_z_score'],
              width,
              # yerr=error_bars_frequentist,
              capsize=2,  label='freqentist_z_score')

# Adding labels, title, and legend
ax.set_xlabel('Team Name')
ax.set_ylabel('z Scores')
```

```
ax.set_title('ability z Scores by Team and Metric')
ax.set_xticks(positions)
ax.set_xticklabels(ability_difficulty_freq_1pl_1_2pl_1['team_name_2pl'])
ax.legend()

# Store the figure in a variable
ability_measured_by_each_model = fig
# Later in the code, you can render the stored plot
ability_measured_by_each_model.show()
```

```
/tmp/ipykernel_50148/2369363200.py:34: UserWarning: FigureCanvasAgg is non-
interactive, and thus cannot be shown
  ability_measured_by_each_model.show()
```

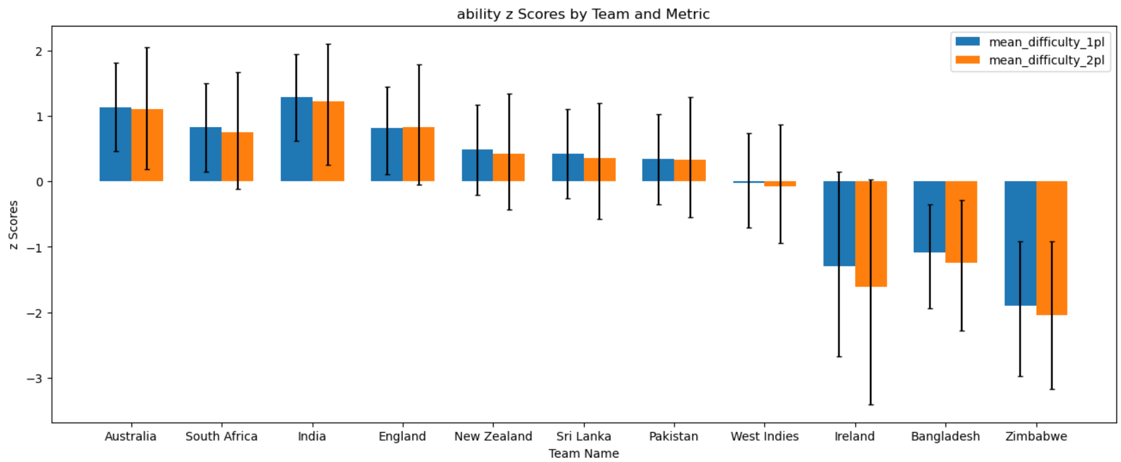


Plot difficulty compare the difficulty estimate of 1pl with 2pl model
means are similar but 2pl model has a wider credible set

[26]:
```
error_bars_ability_1pl1 =␣
↪[ability_difficulty_freq_1pl_1_2pl_1['mean_difficulty_1pl'] -␣
↪ability_difficulty_freq_1pl_1_2pl_1['hdi_2.5%_difficulty_1pl'],
                         ability_difficulty_freq_1pl_1_2pl_1['hdi_97.
↪5%_difficulty_1pl'] -␣
↪ability_difficulty_freq_1pl_1_2pl_1['mean_difficulty_1pl']]
error_bars_ability_2pl1 =␣
↪[ability_difficulty_freq_1pl_1_2pl_1['mean_difficulty_2pl'] -␣
↪ability_difficulty_freq_1pl_1_2pl_1['hdi_2.5%_difficulty_2pl'],
                        ability_difficulty_freq_1pl_1_2pl_1['hdi_97.
↪5%_difficulty_2pl'] -␣
↪ability_difficulty_freq_1pl_1_2pl_1['mean_difficulty_2pl']]
error_bars_frequentist =␣
↪[ability_difficulty_freq_1pl_1_2pl_1['std_dev'],ability_difficulty_freq_1pl_1_2pl_1['std_de
```

```
# error_bars_frequentist

# Set the positions and width for the bars
positions = np.arange(len(ability_difficulty_freq_1pl_1_2pl_1['team_name_2pl']))
width = 0.35

# Plotting the bars
fig, ax = plt.subplots(figsize=(16, 6)) # Increase width here
bar1 = ax.bar(positions - width/2,␣
 ↪ability_difficulty_freq_1pl_1_2pl_1['mean_difficulty_1pl'],
              width,yerr=error_bars_ability, capsize=2,␣
 ↪label='mean_difficulty_1pl')
bar2 = ax.bar(positions + width/2,␣
 ↪ability_difficulty_freq_1pl_1_2pl_1['mean_difficulty_2pl'],
              width,yerr=error_bars_ability_2pl1, capsize=2, ␣
 ↪label='mean_difficulty_2pl')
# bar3 = ax.bar(positions + width,␣
 ↪ability_difficulty_freq_1pl_1_2pl_1['freqentist_z_score'],
#               width,
#               # yerr=error_bars_frequentist,
#               capsize=2,  label='freqentist_z_score')

# Adding labels, title, and legend
ax.set_xlabel('Team Name')
ax.set_ylabel('z Scores')
ax.set_title('ability z Scores by Team and Metric')
ax.set_xticks(positions)
ax.set_xticklabels(ability_difficulty_freq_1pl_1_2pl_1['team_name_2pl'])
ax.legend()

# Store the figure in a variable
difficulty_measured_by_each_model = fig
# Later in the code, you can render the stored plot
difficulty_measured_by_each_model.show()
```

```
/tmp/ipykernel_50148/2797447929.py:34: UserWarning: FigureCanvasAgg is non-
interactive, and thus cannot be shown
  difficulty_measured_by_each_model.show()
```



5 References

- https://mc-stan.org/docs/2_20/stan-users-guide/item-response-models-section.html
- https://areding.github.io/6420-pymc/unit10/Unit10-rasch.html

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