Summation Notation

$$1 + 2 + 3 + \cdots + 10$$

$$\text{code} \begin{cases} s = 0 \\ \text{for } i = 1 \text{ to } 10 \\ \quad \Big| \quad s = s + i \\ \text{end} \\ s = 1 + 2 + 3 + \cdots + 10 \end{cases} \quad \longleftrightarrow \quad \left. S = \sum_{i=1}^{10} i = 1 + 2 + 3 + \cdots + 10 \right\} \text{math}$$

$$S' = \sum_{\odot = 5}^{15} \odot + 1 = (5+1) + (6+1) + (7+1) + \cdots + (15+1)$$

$$A = \begin{bmatrix} 0 & 100 & 50 \\ 0 & 60 & 60 \\ 0 & 200 & 50 \end{bmatrix}$$

$A_{ij} \leftrightarrow$ pixel at row i, col j

$$S = (0 + 100 + 50) + (0 + 60 + 60) + (0 + 200 + 50)$$

$$= (A_{11} + A_{12} + A_{13}) + (A_{21} + A_{22} + A_{23}) + (A_{31} + A_{32} + A_{33})$$

$$= \sum_{i=1}^{3} A_{1i} + \sum_{j=1}^{3} A_{2j} + \sum_{k=1}^{3} A_{3k} = \boxed{\sum_{ij} A_{ij}} \leftrightarrow \text{sum of all pixels in image}$$

Image Similarity

$$A = \begin{bmatrix} 0 & 230 & 75 \\ 0 & 210 & 60 \\ 0 & 200 & 50 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 225 & 70 \\ 0 & 210 & 65 \\ 0 & 200 & 55 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 0 & 5 & 5 \\ 0 & 0 & -5 \\ 0 & 0 & -5 \end{bmatrix}$$

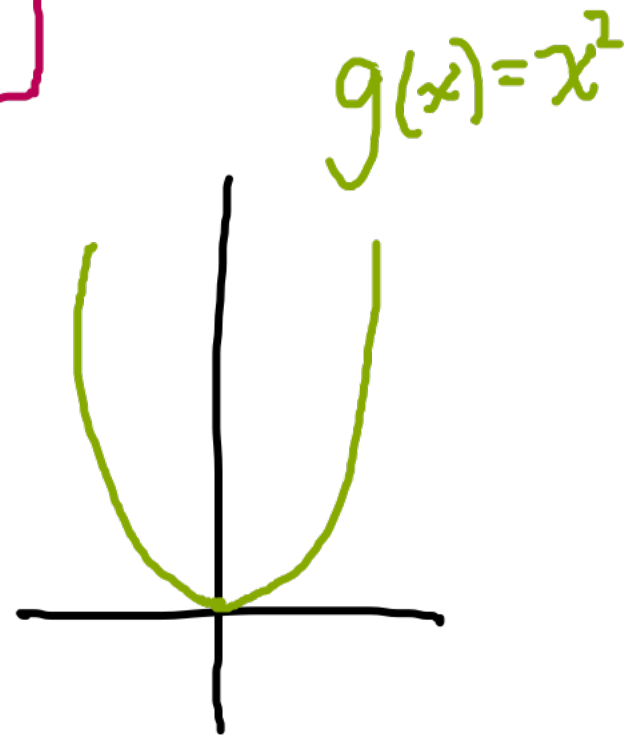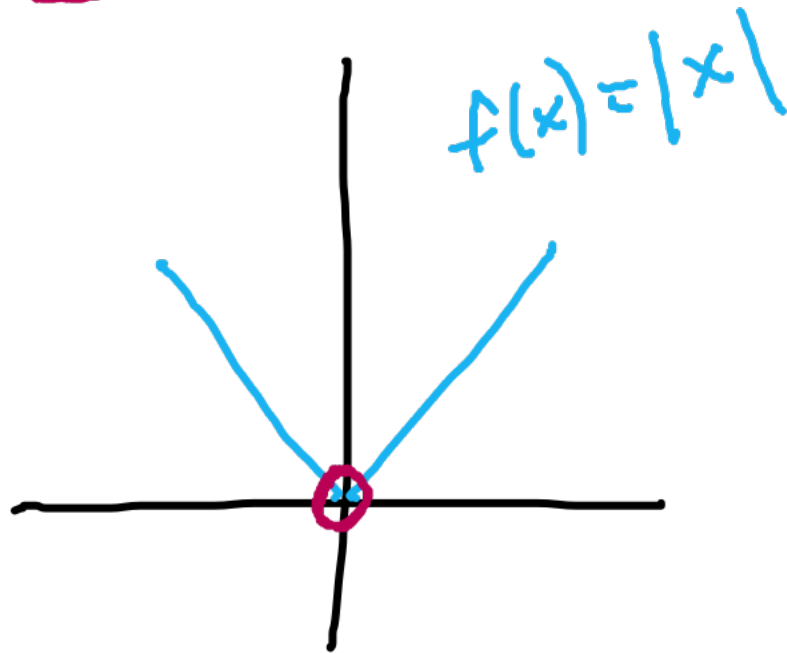$$B - A = \begin{bmatrix} 0 & -5 & -5 \\ 0 & 0 & 5 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\boxed{l_1 = \sum_{i,j} \left| A_{ij} - B_{ij} \right|}$$

$$l_1(A,B) = 20 \neq 0 \rightarrow A \text{ and } B \text{ are different}$$

norm, error

Sum of Squared Errors (SSE)

$$SSE = \sum_{i,j} \left(A_{ij} - B_{ij}\right)^2$$

$f(x) = |x|$

$g(x) = x^2$

Mean Squared Error (MSE)

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad SSE(A,B) = 9$$

$$C = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \Bigg\}^{10}_{=m} \qquad D = \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix} \Bigg\}^{10} \qquad SSE(C,D) = 100$$

$$\underbrace{\qquad\qquad}_{10 = n} \qquad\qquad \underbrace{\qquad\qquad}_{10}$$

$$MSE(A,B) = 1$$

$$MSE(C,D) = 1$$

$$\boxed{MSE = \frac{1}{mn} \underbrace{\sum_{i,j} \left(A_{ij} - B_{ij} \right)^2}_{SSE}}$$

Structural Similarity (SSIM)

MSE / SSE : look at pixels individually

SSIM : groups of pixels

$$SSIM(A,B) = \ell(A,B) \cdot c(A,B) \cdot s(A,B)$$

luminance   contrast   structure

$$-1 \leq SSIM \leq 1$$

perfectly imperfect   perfect