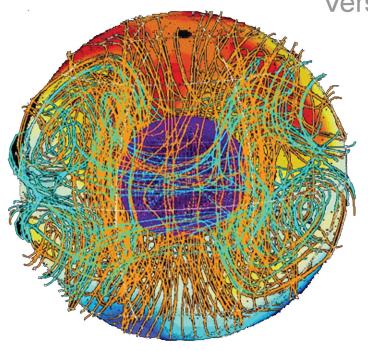
# MAG

User Manual Version 1.0.2



www.geodynamics.org

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## MAG

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# Contents

1	Preface	7
II	Chapters	11
1	Introduction 1.1 About MAG	
2	Installation and Getting Help           2.1 Introduction            2.2 Getting Help            2.3 System Requirements            2.4 Downloading and Unpacking Source            2.5 Installation Procedure            2.5.1 MAG File Structure            2.5.2 Prepare MAG for Running            2.6 Installing from the Software Repository            2.6.1 Tools You Will Need            2.6.2 Download Source from Subversion	15 15 16 16 16 16
3	Running MAG 3.1 Using MAG	
4	Postprocessing and Graphics 4.1 Introduction	21
5	Examples         5.1 Benchmark Cases	28 31 31 32
II	I Appendices	35
$\mathbf{A}$	Variables Used in MAG	37

4	CONTENTS
B MAG Input File Format	49
C MAG Output File Format	53
D License	55

# List of Figures

4.1	Time series plot of energy	22		
4.2	Spectra plot of a time depentant dynamo	23		
4.3	IDL figure with Map option	24		
4.4 $$ Kinetic energy (yellow) and Magnetic energy (blue) plot by the MAGVOL procedure				
5.1	Field Plot for Reversal Dynamo Case	28		
	Field Plot for Reversal Dynamo Case (longer run)			
5.3	Magnetic Field Pole Plot. The top is the pole plot at the beginning of the second field reversal;			
	the bottom is the pole plot at the end of the second field reversal	30		
5.4	Interval to record movie	31		
5.5	Reversal Dynamo Movie. Left: first frame of movie; right: last frame of movie	32		

6 LIST OF FIGURES

# Part I

# Preface

### Preface

#### **About This Document**

This document is organized into three parts. Part I consists of traditional book front matter, including this preface. Part II begins with an introduction to MAG version 1.0.2 and its capabilities and proceeds to the details of implementation. Part III provides appendices and references.

The style of this publication is based on the Apple Publications Style Guide (developer.apple.com/documentation/UserExperience/Conceptual/APStyleGuide/AppleStyleGuide2003.pdf), as recommended by Python.org (www.python.org). The documentation was produced using LyX (www.lyx.org) to facilitate the transformation of files from one format to another. LyX is a document processor that encourages an approach to writing based on the structure of your documents, not their appearance. It is released under a Free Software/Open Source license.

Errors and bug fixes in this manual should be directed to CIG Geodynamo Mailing List (cig-geodyn@geodynamics.org).

#### Who Will Use This Document

This documentation is aimed at scientists who prefer to use prepackaged and specialized analysis tools. Users are likely to be experienced computational Earth scientists and have familiarity with basic scripting, software installation, and programming; but are not likely to be professional programmers. Of those, there are likely to be two classes of users: those who just run models and those who modify the source code.

#### Citation

Computational Infrastructure for Geodynamics (CIG) is making this source code available to you in the hope that the software will enhance your research in geophysics. The underlying Fortran code was donated to CIG in July of 2006. A number of individuals have contributed a significant portion of their careers toward the development of MAG. It is essential that you recognize these individuals in the normal scientific practice by citing the appropriate peer reviewed papers and making appropriate acknowledgements.

The MAG development team asks that you cite the following:

- Olson, P., G.A. Glatzmaier (1993), Highly supercritical thermal convection in a rotating spherical shell: centrifugal vs. radial gravity. *Geophys. Astrophys. Fluid Dyn.*, 70, 113-136.
- Olson, P., G.A. Glatzmaier (1995), Magnetoconvection in a rotating spherical shell: structure of flow in the outer core. *Phys. Earth Planet Int.*, *92*, 109-118.
- Olson, P., G.A. Glatzmaier (1996), Magnetoconvection and Thermal Coupling of the Earth's Core and Mantle. *Phil. Trans. R. Soc. Lond.*, A354, 1413-1424.
- Christensen, U.R., J. Aubert (2006), Scaling properties of convection-driven dynamos in rotating spherical shells and application to planetary magnetic fields. *Geophys J. Int.* 166, 97-114.
- Olson, P., U. Christensen, G.A. Glatzmaier (1999), Numerical Modeling of the Geodynamo: Mechanisms of Field Generation and Equilibration. *J. Geophys. Res.*, 104, 10,383-10,404.

- Christensen, U., P. Olson, G.A. Glatzmaier (1999), Numerical modelling of the geodynamo: a systematic parameter study. *Geophys. J. Int.*, 138, 393-409.
- Christensen, et al. (2001), A numerical dynamo benchmark. *Phys. Earth Planet Int.*, 128, 25-34 (benchmark cases).

[Note: there are more recent papers by the same authors.] The developers also request that in your oral presentations and in your paper acknowledgements that you indicate your use of this code, the authors of this code (G. Glatzmaier, U. Christensen, P. Olson), and CIG (geodynamics.org).

#### Support

MAG development was funded by grants from NASA HPC and NSF Geophysics. Continued support of MAG is made possible under NSF EAR-0406751.

# Part II Chapters

## Introduction

Dynamo codes represent a powerful new tool for the quantitative study of a broad range of geophysical processes, ranging from short time-scale phenomena such as magnetic variations, rotational variations, and flow in the core, to long-term phenomena such as magnetic excursions, reversals, superchrons, and the evolution of the core and its thermal and chemical interaction with the mantle. The primary objective of CIG in this area is to provide the Earth Science community with robust, reliable, efficient, flexible, state-of-the-art numerical codes for modeling dynamo processes in the Earth's core and in the interiors of other planets. Another CIG objective is to support graphical- and user-interfaces for these codes that allow Earth scientists to analyze, display, and interpret dynamo code results, and to compare results from the various codes that we support, as well as with geomagnetic, space magnetic, and paleomagnetic data.

#### 1.1 About MAG

MAG is a serial version of Gary Glatzmaier's rotating spherical convection/magnetoconvection/dynamo code, modified by Uli Christensen and Peter Olson. The code solves the non-dimensional Boussinesq equations for time-dependent thermal convection in a rotating spherical shell filled with an electrically conducting fluid. The equations of motion are: the Navier-Stokes equation including Coriolis, Lorentz, Buoyancy, pressure, viscous, and inertial terms; the heat equation including advection, diffusion, and uniform-density heat sources; the continuity equation for velocity and Gauss' law for magnetic field; and the induction equation for the magnetic field.

All variables are non-dimensional (see Appendix A); time scale is viscous diffusion, length scale is shell thickness, temperature scale is boundary temperature difference, magnetic field and electric currents use Elsasser number scaling. A variety of boundary and initial conditions has been selected as options.

Mag uses toroidal-poloidal decomposition for velocity and magnetic field with explicit time steps. Linear terms are evaluated spectrally (spherical harmonics plus Chebyshev polynomials in radius) and nonlinear terms are evaluated on a spherical grid.

Additional technical information is found in [1]-[8].

#### 1.2 Governing Equations

MAG solves the following non-dimensional Boussinesq magnetohydrodynamics equations for dynamo action due to thermal convection of an electrically conducting fluid in a rotating spherical shell (e.g., Olson et al. 1999)[5].

$$E\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} - \nabla^2 \boldsymbol{u}\right) + 2\hat{\boldsymbol{z}} \times \boldsymbol{u} + \nabla P = Ra\frac{r}{r_o}T + \frac{1}{Pm}\left(\nabla \times \boldsymbol{B}\right) \times \boldsymbol{B}$$
 (1.1)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{Pm} \nabla^2 \mathbf{B}$$
 (1.2)

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \frac{1}{Pr} \nabla^2 T + \epsilon \tag{1.3}$$

$$\nabla \cdot \boldsymbol{u} = 0, \ \nabla \cdot \boldsymbol{B} = 0 \tag{1.4}$$

where u is the velocity, B is the magnetic field, T is temperature, t is time,  $\hat{z}$  is a unit vector in the direction of the rotation axis, P is pressure, and r is the position vector in the spherical coordinates  $r\theta\phi$ .

Four basic non-dimensional parameters in 1.1 - 1.4 control the dynamo action. The Rayleigh number represents the strength of buoyancy force driving the convection

$$Ra = \frac{\alpha g_0 \Delta T D^3}{\nu \kappa} \tag{1.5}$$

where  $\alpha$  is thermal expansivity,  $g_0$  is gravitational acceleration on the outer boundary at radius R,  $\Delta T$  is the temperature difference between the inner and outer boundaries, D is shell thickness,  $\nu$  is kinematic viscosity, and  $\kappa$  is thermal diffusivity. The Ekman number represents the ratio of viscous and Coriolis forces

$$E = \frac{\nu}{\Omega D^2} \tag{1.6}$$

Here  $\Omega$  is rotation rate. The Prandtl number is the ratio of kinematic viscosity to thermal diffusivity

$$Pr = \frac{\nu}{\kappa} \tag{1.7}$$

and the magnetic Prandtl number is the ratio of kinematic viscosity to magnetic diffusivity  $\lambda$ 

$$P_m = \frac{\nu}{\lambda} \tag{1.8}$$

An additional (optional) control parameter is the non-dimensional volumetric heat source (or heat sink) strength  $\epsilon$ .

# Installation and Getting Help

#### 2.1 Introduction

To test run MAG, download the source package (in the form of a compressed tar file) from the Geodynamics Software Packages web page (geodynamics.org/cig/software/packages). After unpacking the source, use the make utility to build MAG from source, and background execute MAG with the provided benchmark input file.

Advanced users and software developers may be interested in downloading the latest MAG source code directly from the CIG source code repository, instead of using the prepared source package. See the 2.6 section later in this chapter. MAG has been tested on Linux, Mac OS X, and Windows.

#### 2.2 Getting Help

For help, send e-mail to the CIG Geodynamo Mailing List (cig-geodyn@geodynamics.org). You can subscribe to the Mailing List and view archived discussion at Geodynamics Mail Lists (geodynamics.org/cig/lists).

#### 2.3 System Requirements

MAG requires the following:

- A Fortran compiler, g77 or gFortran.
- For Windows you need to install cygwin (cygwin.com).

#### 2.4 Downloading and Unpacking Source

Download MAG from the Geodynamics website (geodynamics.org). Click the "software" tab at the top of the page. Then click "Software Packages" and then "Geodynamo." Once you click the MAG link, download the source archive and unpack it using the tar command:

```
$ tar xzf MAG-1.0.2.tar.gz
```

If you don't have GNU Tar, try the following command instead:

```
$ gunzip -c MAG-1.0.2.tar.gz | tar xf -
```

#### 2.5 Installation Procedure

#### 2.5.1 MAG File Structure

After unpacking the source, you will find the following directories:

~/src Contains the set of FORTRAN source code files with suffix ".f." This includes sample grid parameter value files with names like param32s4.f for a coarse grid (up to 32 spherical harmonics, 24 radial grid intervals, and 4-fold symmetry in φ). A makefile named makefile. Sample files with input parameters, par.XXX. The case par.bnch0 is for rotating convection at an Ekman number of 1E-3, starting from a conductive temperature perturbation with imposed perturbation with l=4, m=4, and running for a short time. This is the "benchmark0" test case in Christensen et al., 2001[6]. Another input file is par.bnch1, the dynamo "benchmark1" case in Christensen et al. [6].

~/doc The directory where you will find this manual and other documentation files.

~/bench-data Contains output files ls.benchX, l.benchX, g.benchx, and d.benchx obtained with short runs of benchmark0 and benchmark1 on a Linux workstation. Explanations of the contents of these files are found in Appendix C. These data files can be used for comparison with the result obtained by your local run of MAG.

~/rev-data Contains output files from runs of the reversal dynamo case; movie files are also included.

~/idl This is where the postprocessing IDL (Interactive Data Language) routines reside.

#### 2.5.2 Prepare MAG for Running

1. First you need to create a path for execution of magx (below is an example; use your path):

```
printenv PATH $ PATH=$PATH:/your_mag_dir_path $ export PATH
```

2. Compile the program using make in the source directory, which by default uses the existing param.f grid and symmetry

#### \$ make

Note that makefile uses -g77 or other Fortran compiler, and creates executables, either magx (default) or magxYYsZ, where YY=spherical harmonic truncation and Z=longitudinal symmetry.

3. To delete all the object files and executables, type:

\$ make clean

#### 2.6 Installing from the Software Repository

The MAG source code is available via a Subversion server at the Geodynamics website (geodynamics.org). This allows users to view the revision history of the code, and check out the most recent development version of the software.

**NOTE:** If you are content with the prepared source package, you may skip this section.

#### 2.6.1 Tools You Will Need

In addition to the usual system requirements, you must have a Subversion client installed in order to work with the source from the CIG software repository. To check whether you have a subversion client installed on your machine, type:

\$ svn help

It should return a usage message. For more information on Subversion, visit the Subversion website (subversion.tigris.org).

#### 2.6.2 Download Source from Subversion

To check out the latest version of the software, use the svn checkout command:

\$ svn checkout http://geodynamics.org/svn/cig/geodyn/3D/MAG/trunk MAG

where "MAG" is the directory created with the file structure mentioned in 2.5.1.

# Running MAG

#### 3.1 Using MAG

For test-running the code, perform the following steps:

- 1. Uncompress all files, and create a path (see 2.5.2)
- 2. Link the grid parameter file to param.f, which enters into most subroutines through "include" statements. For example, a grid parameter file named param32f4.f (32 spherical harmonics truncation degree, longitude symmetry is 4) is linked using
  - \$ ln -sf param32s4.f param.f
- 3. Compile the program with:
  - \$ make
  - \$ mv magx magx32s4 (Renaming is optional)
- 4. MAG uses a standard input file. Background execute using par.XXX as the input file and .YYY as the output file's extension:

```
$ magx32s4 <par.XXX >p.YYY &
```

For running with the benchmark input files (par.bnch0 or par.bnch1) , the execution statement should be:

```
$ magx32s4 <par.bnch0 >p.bench0 &
```

- 5. If there is a problem with the input file list, it is often the final three lines; with some systems, a "\$" may be required at the end.
- 6. MAG produces a series of output files. For example, when using input file par.bnch0 (the example in step 4) MAG generates: 1.bench0, ls.bench0, g[i].bench0 and d[i].bench0, where i=0,1,2...9. See Appendix C for details on MAG's output files. Compare your output files with the data provided in the directory ~/bench-data/data\_bench0.

Warning: You must delete, move, or rename all of the output files in the current directory before re-running with the same "output" filename. Retaining same-named output files in the current directory causes MAG to crash.

<sup>&</sup>lt;sup>1</sup>To change grids or symmetry (in param.f), MAG needs to be recompiled. The code looks for param.f, which needs to be changed for remaking. Examples of param.f are param32s6.f and param32s4.f.

## 3.2 Changing Parameters

Physical and time-step parameters can be changed in the par-file namelist without re-compiling MAG. See Appendix B for a list of the input parameter names and definitions. Grid parameters must be changed in param.f and MAG must be then re-compiled. There are some numerical restrictions on the grid parameter combinations, which are given in Appendix A.

# Postprocessing and Graphics

#### 4.1 Introduction

Once you finish running MAG, you should have a series of output data files. To visualize your results, the MAG software package provides a set of IDL routines found in the directory called PREFIX/idl, where PREFIX is the directory under which you installed MAG. We should mention here that IDL is a commercial visualization tool by ITT Visual Information Solutions (www.ittvis.com/idl). A free IDL compatible program called GDL (gnudatalanguage.sourceforge.net) works with MAG's line plot IDL procedure MAGTS.pro, but not with the interactive IDL procedures in MAG.

#### 4.2 Time Series and Spectra Plots

Procedures MAGTS.pro and MAGXY.pro take data from l-files generated by MAG and create time series plots and statistics. This version reads an l-file consisting of 17 time series, the first record being dimensionless time (see Appendix C for details on output file format). Energies and rms magnetic field and velocity are scaled as in MAG; tilt is dipole vector colatitude; pole longitude is dipole vector longitude. Figures 4.1 and 4.2 show the energy time series plot and spectra plot from a time-dependent dynamo.

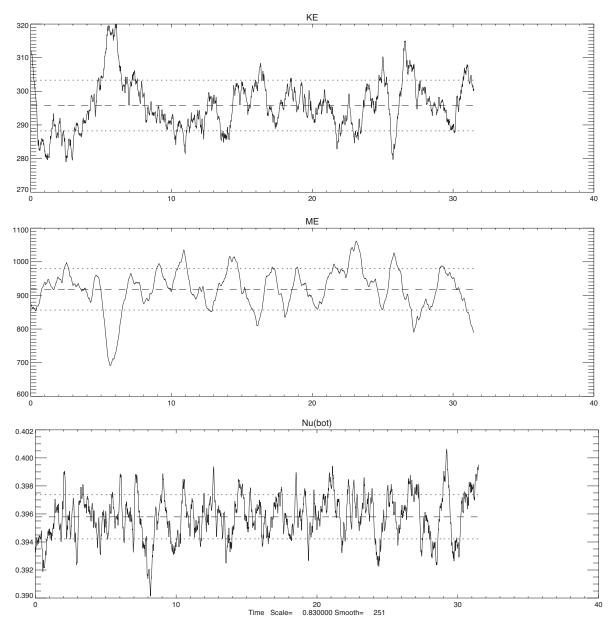


Figure 4.1: Time series plot of energy

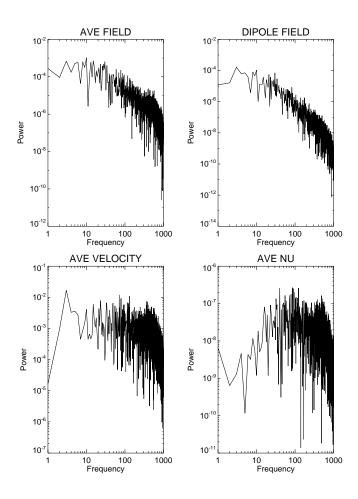


Figure 4.2: Spectra plot of a time depentant dynamo

#### 4.3 Interactive IDL Procedures

MAGSYM.pro is an interactive procedure display which results from a g-file produced by MAG. This version uses modified IDL color tables and assumes formatted input. It creates either postscript .ps or .gif files. If other output file formats are required, you must modify "LABELOUT." MAGSYM has many plot options: map, closeup, equator, slice, etc. Producing each plot is straightforward and accomplished by choosing from the option menu. Figure 4.3 is plotted with the map option.

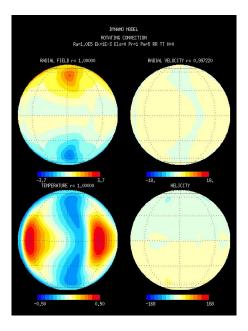


Figure 4.3: IDL figure with Map option

MAGVOL.pro is another interactive IDL procedure to display volume results from rotating convection, magnetoconvection and dynamo calculations (written by P. Olson). It uses g-files produced by MAG (some longitudinal symmetry may be assumed in the g-file). This version uses modified IDL color tables and assumes formatted or unformatted input (it asks for .gif but creates .jpg files); if other output file formats are required, modifications of "labelout" are required. This version assumes x-window screen graphics; for other graphics devices, change the set\_plot,'x' and tvrd() commands accordingly. MAGVOL procedure creates volume-rendered images of temperature, helicity, the z-component of vorticity, kinetic and magnetic energy, joule heating, work by Lorentz forces and buoyancy forces. Figure 4.4 shows a plot of kinetic energy and magnetic energy obtained from a numerical dynamo model.

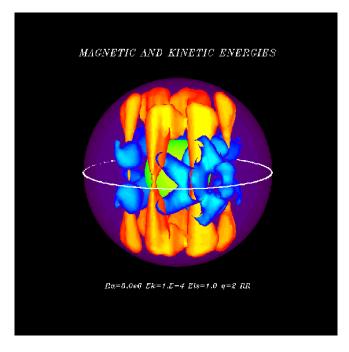


Figure 4.4: Kinetic energy (yellow) and Magnetic energy (blue) plot by the MAGVOL procedure

# Examples

Included in this chapter are two benchmark cases and a reversal dynamo case. We will present the cases with their input parameters and a typical run of the case with output data analysis.

#### 5.1 Benchmark Cases

Historically, these are the cases defined in the benchmark study published in the 2001 paper by Gary Glatzmaier et al. [6]. Case 0 is a benchmark of rotating non-magnetic convection. Case 1 is a dynamo with an insulating inner core co-rotating with the outer boundary. The regions outside the fluid shell are electrical insulators and the magnetic field on the boundaries matches with appropriate potential fields in the exterior that imply no external sources of the field.

In both cases the Ekman number is  $E=10^{-3}$  and the Prandtl number is Pr=1. The Rayleigh number is set to Ra=100000. Note that the definition of the Rayleigh number differs from the one in the published cases [6] by a factor of Ekman number E, i.e.,  $Ra=\frac{Ra}{E}$ .[6] The magnetic Prandtl number is zero in the non-magnetic convection case 0, and is Pm=5 in case 1. The spherical harmonic expansion is truncated at degree lmax=32 and a four-fold symmetry is assumed in the longitudinal direction (param.f should be linked to param32s4.f when you compile MAG). The input parameter files are par.bench0 for case 0 and par.bench1 in case 1; both files reside in the ~/src directory.

The output files of the benchmark cases are stored in the directory ~/bench-data/data\_bench0 and ~/bench-data/data-bench1 respectively. In the following table we see the solutions from MAG agree with the benchmark suggested value with a small difference margin. In both case 0 and case 1, the values listed were obtained with low resolution and a relatively short run of MAG.

	Case 0 Suggested Value	Mag Case 0	Case 1 Suggested Value	Mag Case 1
$E_{kin}$	$58.348 \pm 0.050$	58.35	$30.733 \pm 0.020$	30.72
$E_{mag}$			$626.41 \pm 0.40$	627.15
T	$0.42812 \pm 0.00012$		$0.37338 \pm 0.00040$	
$\mu_{\phi}$	$-10.1571 \pm 0.0020$	-10.80	$-7.6250 \pm 0.0060$	-7.84
$B_{\theta}$			$-4.9289 \pm 0.0060$	
$\omega$	$0.1824 \pm 0.0050$		$-3.1017 \pm 0.0040$	

#### 5.2 Reversal Dynamo Case

In this example, we produce a magnetic field reversal using MAG. The input parameter in the source directory for this case is ~/src/par.Rev. There is no longitudinal symmetry in this case, so when you compile MAG,

use param32s1.f linking to param.f. The Ekman number is E = 0.02, the Prandtl number is Pr = 1 and the magnetic Prandtl number is Pm = 10. The Rayleigh number is Ra = 12000.

#### 5.2.1 Results and Discussions

This case has run on 32-bit and 64-bit Intel processors. Figure 5.1 shows a plot of mean velocity Vrms, mean magnetic field Brms, the axial dipole and the dipole tilt on the outer boundary. It indicated a magnetic field reversal between time steps 25 and 30. Figure 5.2 shows a longer run of MAG, where we see the magnetic field reversed again. At this time, the magnetic field had weakened substantially. In Figure 5.3, the top is the pole plot before the second field reversal and the bottom is the pole plot after the second field reversal.

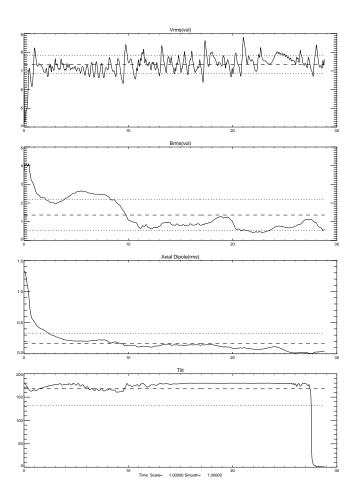


Figure 5.1: Field Plot for Reversal Dynamo Case

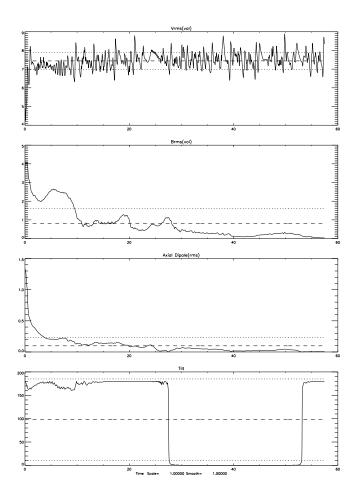
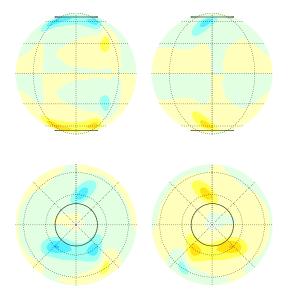


Figure 5.2: Field Plot for Reversal Dynamo Case (longer run)



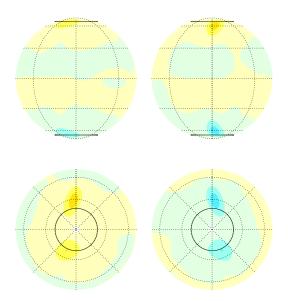


Figure 5.3: Magnetic Field Pole Plot. The top is the pole plot at the beginning of the second field reversal; the bottom is the pole plot at the end of the second field reversal.

#### 5.2.2 Creating a Reversal Dynamo Movie

#### 5.2.2.1 Generating Movie Files

MAG has the function to record a movie. The input paramter imovopt gives the option to write  $B_r$  at the outer surface, the output file is produced with prefix "mm." when imovopt=100 or the digit at the hundreds is larger than zero. For our reversal dynamo case we examine the field plot and decide to record the first field reversal. As shown the figure 5.4, we pick the time interval to generate the movie file. The input parameter for movie recording is in "/src/par.revRmv. We choose restart file d5 as our starting point, and set to record the movie at time t=22. This records 400 frames over an 8 time unit. The sample output files are in "/rev-data.

This version of MAG provides an IDL routine magmovieCIG.pro (~/idl/magmovieCIG.pro), it reads in the movie file mm. and displays the magnetic field at the outer surface. This procedure can also create JPG files of the movie-frame images. Figure 5.5shows the first and the last frames of the reversal dynamo movie.

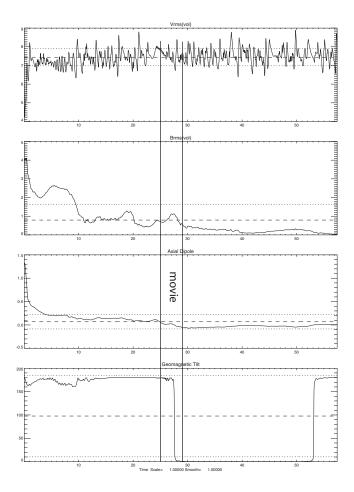


Figure 5.4: Interval to record movie

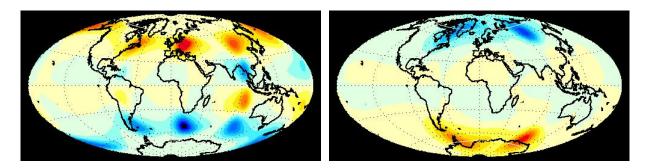


Figure 5.5: Reversal Dynamo Movie. Left: first frame of movie; right: last frame of movie

#### 5.2.2.2 Creating MPEGs with IDL

IDL provides an IDLgrMPEG class that allows the user to save an array of image frames as an MPEG movie. See the IDL Reference Guide (www.rsinc.com/idl/pdfs/refguide.pdf).

The MPEG\_PUT procedure stores the specified image array at a specified frame index in an MPEG sequence. The MPEG\_SAVE procedure encodes and saves an open MPEG sequence. The images can be read using the READ\_IMAGE function.

The module mpg.pro reads a sequence of images (named prefix00001.suffix) and produces an MPEG1 or MPEG2 movie. Supported formats are BMP, GIF, JPEG, PNG, PPM, SRF, TIFF, and DICOM. mpg.pro runs with the following parameters:

suffix string (required). A string specifying the suffix of the images to read, e.g., "jpg"

n start integer (required). An integer specifying the first image in the sequence

n end integer (required). An integer specifying the last image in the sequence

digits integer (required). An integer specifying the number of digits of the number field in the sequence, e.g., image00001.jpg would require digits=5

dims integer (optional). An integer array specifying the size of the output image; if not specified the size of the first image is used

format integer (optional). An integer with values 0 for MPEG1 and 1 for MPEG2 (default is MPEG1)

frame rate integer (optional). An integer with values

```
1 = 23.976 frames/sec: NTSC encapsulated film rate.
```

2 = 24 frames/sec: standard film rate.

3 = 25 frames/sec: PAL video frame rate

4 = 29.97 frames/sec: NTSC video frame rate

5 = 30 frames/sec: NTSC drop frame video rate (the default).

6 = 50 frames/sec: double frame rate/progressive PAL

7 = 59.94 frames/sec: double frame rate NTSC

8 = 60 frames/sec: double frame rate NTSC

mpeg\_file string (optional). A string specifying the output MPEG file (default outfile.mpg)

tmp\_dir string (optional). A string specifying the temporary directory to use for the temporary image files

To run the IDL movie generator, type in a shell:

A movie file (magrev#.mpg) produced by magmovieCIG.pro and mpg.pro is also included in the ~/rev-data directory.

#### 5.3 Gauss Coefficients Conversion

Making the output useful to the wider community is one of MAG's primary goals. In this release MAG includes Gauss coefficients in the output file.

Here is some background: To extrapolate the magnetic field between the core, the Earth's surface, and nearby space requires a spherical harmonic representation. MAG uses fully-normalized, complex-valued spherical harmonics for its magnetic field and other variables. These harmonics are packed into one-dimensional arrays for optimal computation and they are also made non-dimensional in MAG.

In contrast, the geomagnetism scientific community has a completely different standard. They **always** use real-valued, un-normalized spherical harmonics, whose coefficients are called the Gauss coefficients. The Gauss coefficients – denoted by g(l,m) and h(l,m) respectively, where (l,m) are harmonic degree and order, respectively – are real-valued and have dimensional units, usually expressed in nanoTeslas. For example, the present-day axial dipole field has a Gauss coefficient near g(1,0)=-29,000 nT, the minus sign indicating the field points radially inward in the northern hemisphere. Users from that whole community often just want dynamo model output consisting of standardized files of the Gauss coefficients at various times.

# Part III Appendices

## Appendix A

### Variables Used in MAG

This is a list of variables and names used in the program set in MAG. The list is in alphabetical order for ease of reference.

adrke axisymmetric toroidal kinetic energy (diagnostic)

ai imaginary unit = complex(0,1)

aj (nlma,nn+1) poloidal magnetic field potential (spectral form); the second index is either the Chebyshev degree (n) or the radial grid point (kc)

ajmat (nn,nn,lmax) LU-decomposed matrix from Chebyshev collocation of toroidal induction equation. Built in lude, used in amhd.

aleg1 (nlma,ni) Value of associated Legendre function at grid points

aleg2 (nlma,ni) Value of associated Legendre function, multiplied with Gaussian weight, at grid points

aleg3 (nlma,ni) Value of derivative of associated Legendre function multiplied with sin(theta) at grid points

alfilt [INPUT] Filter parameter for B r in graphics output, see nfilt

alffac [INPUT] Controls the contribution of the (modified) Alfven velocity to the Courant time step limit (see under "courfac"). The modified Alfven velocity is given by v\_alfven' =  $(v_a)^2 / (v_a)^2 + [pi*(eta+nu)/delx]^2$  where v\_a = B/sqrt(mu\*rho) and delx is the Courant length (either delxr or delxh)

**alpha** [INPUT] =0 linear terms in the equations are treated fully explicit, =1 linear terms are treated fully implicit, =0.5: Crank-N.

alumn0 Factor for scaling heat flow in output amcke: axisymmetric poloidal kinetic energy

amhd [SUBROUTINE] The "workhorse" of the program: advance solution by nstep time steps

amps [INPUT] can be used to re-scale entropy

ampj [INPUT] can be used to re-scale toroidal magnetic field

ampb [INPUT] can be used to re-scale poloidal magnetic field

ampw [INPUT] can be used to re-scale poloidal velocity

ampz [INPUT] can be used to re-scale toroidal velocity

 $\mathbf{anorm} = \operatorname{sqrt}(2/[\operatorname{nn}+1])$ 

apome axisymmetric poloidal magnetic field energy

atome axisymmetric toroidal magnetic field energy

b (nlma,nn+1) poloidal magnetic field potential (spectral form, see aj)

bleg1 (lmax) auxiliary array for calculation of aleg1

bleg2 (lmax) auxiliary array for calculation of aleg2

bleg3 (lmax) auxiliary array for calculation of aleg3

bmat(nn,nn,lmax) LU-decomposed matrix from Chebyshev collocation of poloidal induction equation. Built in lude, used in amhd.

bnlc1 (nja/2,ni) bnlr1 stored in complex form

bnlc2 (nja/2,ni) bnlr2 stored in complex form

bnlc3 (nja/2,ni) bnlr3 stored in complex form

bnlr1 (nja,ni) nonlinear products for updating b (on grid points)

bnlr2 (nja,ni) nonlinear products for updating aj (on grid points)

bnlr3 (nja,ni) nonlinear products for updating aj (on grid points)

**bots(0:lmax,0:mmax)** [INPUT] harmonic coefficients of prescribed temperature (entropy) on inner boundary

**br** (**nja,ni**) = r^2 \* B\_r on gridpoints

brc br stored as complex array

 $\mathbf{bscl} = \mathrm{dt} * \mathrm{radtop}^2$ 

 $bt (nja,ni) = r * sin(theta) *b_theta$ 

btrdt (ni) used in movmout to calculate j\_phi

bts (ni,3) used in movmout to calculate j phi

btc bt stored as complex array

**bp** 
$$(nja,ni) = r * sin(theta) * b phi$$

**bpc** bp stored as complex array

bpeak [INPUT] maximum value of imposed field on boundaries

bpeakbot maximum value of imposed field on inner boundary

bpeaktop maximum value of imposed field on outer boundary

$$cbr (nja,ni) = r^2 * curl (B) * e r$$

cbrc cbr stored as complex array

cbtc ctr stored as complex array

$$cbp (nja,ni) = r * sin(theta) * curl (B) * e phi$$

cbpc cpb stored as complex array

```
cheb (nn,nn) cheb(i,j) = value of Chebyshev polynomial i at grid point j
chebi [SUBROUTINE] initialize subroutine chebtf
chebtf [SUBROUTINE] multiple fast Chebyshev transform
clm (lmax,mmax) normalization factors of spherical harmonics
cmb [INPUT] integrated conductivity of thin D"-layer
colat (ni) vector of colatitudes (Gauss points), local array in subroutine prep
courfac [INPUT] factor controlling the time step as fraction of courant advection length. The time step is
     limited to dt < \min( dx / [ courfac * v + alffac * v | alfven' ] )
\operatorname{\mathbf{cvr}}(\mathbf{nja,ni}) = r^2 \operatorname{\mathbf{curl}}(v) \operatorname{\mathbf{r}} e r
cvrc cvr stored as complex array
db (nlma,nn+1) radial derivative of poloidal magetic potential (spectral form, see aj)
dbdt (nlma,nn,2) time derivative of poloidal magnetic potential b
dcheb (nn,nn) dcheb(i,j) = 1st derivative of Chebyshev polynomial i at grid point j
d2cheb (nn,nn) d2cheb(i,j) = 2nd derivative of Chebyshev polynomial i at grid point j
d3cheb (nn,nn) d2cheb (i,j) = 3rd derivative of Chebyshev polynomial i at grid point j
ddb (nlma,nn+1) 2nd radial derivative of poloidal magetic potential b
ddj (nlma,nn+1) 2nd radial derivatic of toroidal magnetic potential aj
ddw (nlma,nn+1) 0.25 * 2nd radial derivative of poloidal velocity potential w
ddz (nlma,nn+1) 0.25 * 2nd radial derivative of toroidal velocity potential z
djdt (nlma,nn,2) time derivative of toroidal magnetic potential z
dpdt (nlma,nn,2) time derivative of pressure
dsdt nlma,nn,2) time derivative of temperature (entropy)
dw (nlma,nn+1) 0.50 * radial derivative of poloidal velocity potential w
dwdt (nlma,nn,2) time derivative of poloidal velocity potential w
dz (nlma,nn+1) 0.50 * radial derivative of toroidal velocity potential z
dzdt (nlma,nn,2) time derivative of toroidal velocity potential z
delxh (nn) horizontal Courant length squared
delxr (nn) radial Courant length
difamp [INPUT] amplitude of hyperdiffusivity D=D*(1 + difamp *[(l+1-ldif)/(lmax+1-ldif)]^ldifexp) when
     l>ldif
dipfilt [INPUT] If nfilt>0 multiply axial dipole component of B r on outer surface by dipfilt in graphics
     file
dj (nlma,nn+1) radial derivative of toroidal magnetic potential (spectral form, see aj)
dt current time step
```

```
dtchck [SUBROUTINE] controls time step
dth Courant time based on horizontal velocity + Alfven velocity
dtmax [INPUT] Upper limit on time step (and initial step)
dtmin Lower limit on time step (stop when dt < dtmin)
dtold Time step of previous iterative step
dtr Courant time based on radial velocity + Alfven velocity
dtstart [INPUT] Initial time step. If =0, dtmax, or when beginning from restart file, the old dt is taken
dvpdr (nja,ni) = d [r * sin(theta) * v phi]/dr on gridpoints
dvpdrc dvpdr stored as complex array
dvpdp (nja,ni) = d [r * sin(theta) * v phi]/dphi on gridpoints
dvpdpc dvpdp stored as complex array
dvrdp (nja,ni) = d [r^2 * v_r]/dphi on gridpoints
dvrdpc dvrdp stored as complex array
dvrdr (nja,ni) = d [r^2 * v_r]/dr  on gridpoints
dvrdrc dvrdr stored as complex array
dvrdt (nja,ni) = \sin(theta) * d [r^2 * v_r]/dtheta on gridpoints
dvrdtc dvrdt stored as complex array
dvtdp (nja,ni) = d [r *sin(theta) * v_theta]/dphi on gridpoints
dvtdpc dvtdp stored as complex array
dvtdr (nja,ni) = d [r * sin(theta) * v_theta]/dr on gridpoints
dvtdrc dvtdr stored as complex array
dw (nlma,nn+1) 0.5 * radial derivative of poloidal velocity potential w
dz (nlma,nn+1) 0.5 * radial derivative of toroidal velocity potential z
escale scaling factor for energies in output
ek [INPUT] Ekman number
enb [OUTPUT] magnetic energy
ens [OUTPUT] thermal energy
enscale [INPUT] in output listing, energies are multiplied by enscale
ent [OUTPUT] total energy
env [OUTPUT] kinetic energy
epsc0 [INPUT] internal heating rate
flmb1 (nlma+...) r-component of (v x B) term
flmb2 (nlma+..) theta-component of (v x B) term
```

```
flmb3 (nlma+...) phi-component of (v x B) term
flms1 (nlma+..) r-component of entropy advection term
flms2 (nlma+..) theta-component of entropy advection term
flms3 (nlma+..) phi-component of entropy advection term
flmw1 (nlma+..) r-component of v*grad(v) + Lorentz force term
flmw2 (nlma+...) theta-component of v*grad(v) + Lorentz force term
flmw3 (nlma+..) phi-component of v*grad(v) + Lorentz force term
gauss (ni) vector with Gaussian weighting factors, local array in subroutine prep
gquad [SUBROUTINE] finds zeros and Gaussian weight of associated Legendre function
grafile [CHARACT] file name for data on spatial grid for graphics with prefix "g."; added to outfile set
grav (nn) gravity at radial levels
ib (nn,lmax) pivot array for LU-decomposition of matrix bmat created in sgefa, used in sgesl
ic stepping variable commonly used for steps in colatitude
icour [INPUT] Courant criterion is checked each ICOUR'th time step
idiftype [INPUT] controls radial variation of diffusivity; =0, no variation
ifaxc [13] auxiliary array (factorization) for Chebyshev transform
ifaxf [13] auxiliary array (factorization) for Fourier transform
ifbfrz [INPUT] logical; if .TRUE., do not update magnetic field
ifirst =1 before first call of time-step checking routine, =0 thereafter
iframes [INPUT] write altogether iframes frames on the movie files (see description under imovopt)
ifsfrz [INPUT] logical; if .TRUE., do not update temperature (entropy)
ifvfrz [INPUT] logical; if .TRUE., do not update velocity
```

ij (nn,lmax) pivot array for LU-decomposition of matrix ajmat created in sgefa, used in sgesl

imagcon [INPUT] <0 imposed poloidal field (l=1,m=0) at ICB >=0 imposed toroidal field (l=2,m=0) at ICB >=10 additionally imposed field at CMB, field is of same sign and amplitude if imagcon=10 and of opposite sign if imagcon=11

imovopt [INPUT] four-digit integer number, controls options for generating movie files.

**Last digit**>0 write B\_z, W\_z (vortic) and T in the equatorial plane on file with prefix "me.", imovopt=0001

2nd last digit>0 write longitudinally averaged B\_phi, j\_phi and v\_phi on file with prefix "ma.", imovopt=0010

3rd last digit>0 write B r at outer surface on file with prefix "mm.", imovopt=0100

4th last digit>0 write spherical harmonic coefficients for poloidal field at outer boundary and for velocity potentials at radial level on file with prefix "cc.". (This option works only when any or all of the movie options are turned on, i.e., any of the "m?" files are also produced. imovopt=1000 will not produce a "cc." file.)

imovct counter variable for movie frames

infile [CHARACT INPUT] name of input file for initial values (restart)

init [INPUT] =0 start from dat-file, =1: random initial cond., =-1: hydro. condition from dat-file, magnetic random >=100: initial temperature perturbation in a single mode l,m. Here m is given by the last two digits of init and l by the preceding digits.

ip0 (nn) pivot array for LU-decomposition of matrix p0mat created in sgefa, used in sgesl

iprnt counting blocks in time iteration sequence with printed output created at completion of block

is (nn,lmax) pivot array for LU-decomposition of matrix smat created in sgefa, used in sgesl

is0 (nn) pivot array for LU-decomposition of matrix s0mat created in sgefa, used in sgesl

iscale [INPUT] determines which diffusivity is used for scaling of time, velocity, energy. 1=viscous, 2=thermal, 3=magnetic

istep time step counter (routine amh)

istor counting superblocks in time iteration sequence, upon completion of superblock disk file with data saved

ivfilt [INPUT] Apply filter to v\_r at radial level ivfilt and right into first radial position in graphics file; see nfilt

iwp (nn,lmax) pivot array for LU-decomposition of matrix wpmat created in sgefa, used in sgesl

iz (nn,lmax) pivot array for LU-decomposition of matrix zmat created in sgefa, used in sgesl

**k2k** (nn1) auxiliary array for Chebyshev transform

kc stepping variable commonly used for steps in radius

kcour auxiliary variable for time step checking procedure

**kbotb** [INPUT] magnetic bottom condition; =1 insulating, =2 perfect condition

**kbotv** [INPUT] mechanical bottom condition; =1 free, =2 rigid

**kbots** [INPUT] thermal bottom condition; =1 fixed entropy, =2 flux

kei [SUBROUTINE] calculates kinetic energy

kstep global time step counter

**ktops** [INPUT] thermal top condition; =1 fixed entropy, =2 flux

ktopb [INPUT] magnetic top condition; =1 insulating, =2 perfect condition

**ktopv** [INPUT] mechanical top condition; =1 free, =2 rigid

ldif [INPUT] control parameter for hyperdiffusivity, see difamp

ldifexp [INPUT] control parameter for hyperdiffusivity, see difamp

Im stepping variable used to cover all l and m lm = m\*(lmax+1)/minc - m\*(m-minc)/(2\*minc) + l-m+1

lmax maximum harmonic degree, calculated as (nj-1)/3

logfile [CHARACT] file name for continuous log of energies and other data prefix "l."; added to outfile set

lot [PARAM] = 2 \* nlma (twice the number of harmonic modes)

43 lpfile [CHARACT] file name for continuous log of specified values with prefix "lp."; added to outfile set lsfile: [CHARACT] file name for power spectra of magnetic and kinetic as function of l and m with prefix "ls."; added to outfile set ludc: [SUBROUTINE] Chebyshev collocation mclm (nlma) used to unscramble harmonic order m from variable lm mclma (nlma) = m/minc+1 for given lm (storage order of m) kei [SUBROUTINE] calculates magnetic energy minc [PARAM] if >1, minc-fold symmetry in longitude assumed mmax maximum harmonic order, is the largest integer <= lmax divisible by minc movafile [CHARACT] file name for movie data (longitudinal averages) with prefix "ma."; added to outfile movefile [CHARACT] file name for movie data in equatorial plane with prefix "me."; added to outfile set movmfile [CHARACT] file name for movie data in map views with prefix "mm."; added to outfile set n, nc stepping variables commonly used for steps over Chebyshev polynomial ncp [PARAM] = nja/2 used for storage of points in phi in complex array nfilt [INPUT] Apply filter F(1)=exp(-[1/lfilt]^nfilt) to B r on outer surface in graphics output file (if nfilt>0 and alfilt>0) When nfilt>0, alfilt<0, apply cos-tapered filtered with cutoff at nfilt and taper width alfilt **ngcolat** [INPUT] graphics output on each ngcolat'th point in latitude **ngform** [INPUT] if .ne. 0, graphics output is written each time a restart file is (finally) written. ngform=1 or -1: formatted graphics file, ngform=2: unformatted for ngform=-1 additional comment lines are inserted (this is to look at the file, not for graphics) nglon [INPUT] graphics output for each nglon'th point in longitude ngrad [INPUT] graphics output on each ngrad'th radial level ni [PARAM] # of grid points in colatide; must be even nip1 [PARAM] = ni+1nj [PARAM] # of grid points in longitude; nj/minc must be multiple of four nja = nj/minc, # of actually needed grid points in phi njp1 [PARAM] = nj+1nlaf [PARAM] = lmax + 1nlafp1 [PARAM] = lmax + 2 $\mathbf{nlm} [PARAM] = (mmax+1)*(mmax+2)/2$ nlma [PARAM] # of angular modes employed nlma = mmax\*(lmax+1)/minc - mmax\*(mmax-minc)/(2\*minc)+ lmax-mmax+1.

nlmpa [PARAM] = nlma + mmax/minc + 1

**nlogstep** [INPUT] write data on logfile (prefix l.) after each nlogstep steps.

```
nmaf [PARAM] = mmax+1
nmafa [PARAM] = mmax/minc+1
nn [PARAM] # of radial grid points, nn-1 must be multiple of 4, and contain no prime factors larger than
nn1 [PARAM] = nn-1
nn2 [PARAM] = nn-2
nn3 [PARAM] = nn-3
nnp1 [PARAM] = nn+1
nnp2 [PARAM] = nn+2
nnaf [PARAM] # of radial Chebyshev modes, must be <= nn
nnx2 [PARAM] = 2*nn
nplog [INPUT] if >0 write velocity values at specific points of the grid on separate logfile (prefix "lp.") after
     every nplog steps (for arrays, see vrpoint, vppoint, vtpoint in subroutine amhd for details)
nprnt [INPUT] # of printed output blocks created until next data storage for restart
nps2 [PARAM] = (nn+1)/2
nrp [PARAM] = nja+2 (\# of points in phi +2)
ns2 [PARAM] = (nn-1)/2
nstep [INPUT] # of time steps done until next printed output (total # of time steps is nstep*nprnt*nstor)
nstor [INPUT] # of data storages before program termination
ntf [PARAM] = 3*nja/2+1, used for Fourier transform array trigsf
ocorevol volume of spherical shell (outer core)
oek = 1. / Ekman number
oekpm = 1. / (Ekman number * Magnetic Prandtl number)
\mathbf{oodt} = 1. / dt (inverse time step)
\mathbf{oosscl} = 1. \ / \ \mathrm{dt}
opr = 1. / Prandtl number
outfile [CHARACT INPUT] Name of output files (pre-fixes d.,l.,ls.,g.,me.,ma.,mm., lp. added)
p0mat (nn,nn) LU-decomposed matrix from Chebyshev collocation of pol. equation of motion, l=0-term
     for pressure. Constructed in lude, used in amhd
pbar [SUBROUTINE] Calculates value of associated Legendre function
pscale scaling pressure in output
pr [INPUT] Prandtl number
prmag [INPUT] Magnetic Prandtl number
prnt [SUBROUTINE] print diagnostic data
```

```
pscl = radtop^2
qi (ni,5) array with various coefficients depending on colatitude (look in subroutine prep, loop "do 32" for
     details)
qk (nn,16) array with various coefficients depending on radius (look in subroutine prep for details)
ql (nlma,10) various expressions depending on l and m (look in subroutine prep, loop "do 35" for details)
qn (nn,6) Chebyshev integrals
r (nn) vector with radial levels, r(1)=radtop, r(nn)=radbot
ra [INPUT] Rayleigh number
rapr = Rayleigh number / Prandtl number
radbot radius of inner boundary
radratio [INPUT] ratio of inner radius to outer radius
radtop radius of outer boundary
rderiv [SUBROUTINE] radial derivative
rffti [SUBROUTINE] subroutine called in chebi
rstfile [CHARACT] file name for data in spectral form ('restart data') with prefix "d." or "d0.", "d1." ...;
     added to outfile set
runid [CHAR*64] text identifying the run
rva (nn) auxiliary array used in prep
rvap (nn) auxiliary array used in kei, mei
rvat (nn) auxiliary array used in kei, mei
rvb (nn) auxiliary array used in prep, kei, mei
rvc (nn) auxiliary array used in kei, mei
p (nlma,nn+1) pressure (spectral form)
p00co = 4/sqrt(3)
prep [SUBROUTINE] parameter input, set up auxiliary arrays, set initial conditions, etc.
s (nlma,nn+1) entropy perturbation (spectral form)
sc (ncp,ni) sr stored in complex form
snlc1 (ncp,ni) slnr1 stored in complex form
snlc2 (ncp,ni) slnr2 stored in complex form
snlc3 (ncp,ni) slnr3 stored in complex form
snlr1 (nrp,ni) nonlinear term (radial advection) for updating temperature
snlr2 (nrp,ni) nonlinear term (theta advection) for updating temperature
snlr3 (nrp,ni) nonlinear term (phi advection) for updating temperature
sr (nrp,ni) temperature (entropy) on grid points
```

s0mat (nn,nn) LU-decomposed matrix from Chebyshev collocation of temperature equation, l=0-term. Constructed in lude, used in amhd samp [INPUT] amplitude of initial entropy perturbation smat (nn,nn,lmax) LU-decomposed matrix from Chebyshev collocation of temperature equation. Built in lude, used in amhd sr (nja,ni) entropy on gridpoints **src** sr stored as complex array sscl = dtstor [SUBROUTINE] store data in restart file tei [SUBROUTINE] calculates thermal energy time time timediff time tipdipole [INPUT] rotate poloidal dipole term when beginning from restart file tmovnext auxiliary variable (next output time) for movie file generation tmovstart [INPUT] time at which to start writing movie-frames on m.\*-file tmovstep [INPUT] time increments for writing movie-frames on m.\*-file tops (0:lmax,0:mmax) [INPUT] harmonic coefficients of prescribed temperature (entropy) on outer boundtreset [INPUT; LOGICAL] if true reset time and step counter to zero when starting from a stored dataset trigsc (nn) auxiliary array for Chebyshev transform routine created in chebi, used in chebtf trigsf (ntf) auxiliary array for Fourier transform routine created in fftrig, used in fourtf tscale scaling of time in output up (nja,3) phi-component of velocity in equatorial plane for three consecutive radial levels; used in moveout to calculate vorticity urdp (nja) derivative dv r/dphi in equatorial plane; used in moveout to calculate vorticity  $\mathbf{vr} (\mathbf{nja,ni}) = r^2 * v \text{ r on grid points}$ vrc vr stored as complex array vp (nja,ni) = c \* sin(theta) \* v phi on grid points vpc vp stored as complex array vscale scaling of velocity in output vt (nja,ni) = r \* sin(theta) \* v theta on grid points

wpmat (nn,nn,lmax) LU-decomposed matrix from Chebyshev collocation of poloidal equation of motion; built in lude, used in amhd

vtc vt stored as complex array

w (nlma,nn+1) poloidal velocity potential (spectral form)

```
wnlc1 (nja/2,ni) wnlr1 stored in complex form
wnlc2 (nja/2,ni) wnlr2 stored in complex form
wnlc3 (nja/2,ni) wnlr3 stored in complex form
wnlr1 (nja,ni) nonlinear products for updating w (on grid points)
wnlr2 (nja,ni) nonlinear products for updating z (on grid points)
wnlr3 (nja,ni) nonlinear products for updating z (on grid points)
work (lot,nnp2) work array used in Fourier and Chebyshev transforms
wsave (nn) auxiliary array used for Chebyshev transform
wscl = dt * radtop^2
y00 = 1/sqrt(4*pi)
z (nlma,nn+1) toroidal velocity potential (spectral form)
zscl = dt * radtop^2
zmat (nn,nn,lmax) LU-decomposed matrix from Chebyshev collocation of toroidal equation of motion;
built in ludc, used in amhd
```

## Appendix B

## MAG Input File Format

#### Introduction

This is an overview of the components of the code, input parameters, structure of output files, etc. MAG expects Unix-styled ASCII files (i.e., no carriage-return character following new line character) for all input files. This can be a nuisance in DOS/Windows systems. You may want to find a text editor that can write Unix-style ASCII files. All parameters are in non-dimensional units unless specified.

#### Input Parameters

Parameters have pre-defined (default) values. They are read through a namelist in the subroutine "prep."

#### INPUT, OUTPUT, STEPPING CONTROL, INITIALIZATION OF THE RUN

outfile Name of output files (prefixes d., g., 1., 1s., me., ma., mm., are added)

infile Complete name of file from which initial values are read (restart file)

runid Arbitrary text of up to 64 characters to describe the model

init Set 1 to start from scratch (random noise initial condition); set 0 to start from a previous result obtained on the same grid which has been written into a file named d[0-9].<name> set to a value>= 100 to start from an initial temperature perturbation of one given mode l,m. Here, m is given by the two last digits of init and l by the preceding digits; for example init=606 means that a temperature perturbation of l=6 and m=6 is imposed.

samp Amplitude of initial perturbation (whether random or single mode)

- **nstep** Do one block of nstep time steps before producing a summary printout of some diagnostics standard output; nstep should be an even number
- nprnt Do one "superblock" consisting of nprnt blocks of nstep time steps each, before saving all data in file 'd[0-9].name'. If nstor=1, there is no number added after the 'd'; if nstor>1 the number is incremented by one for each new superblock, starting with zero.
- nstor Do nstor "superblocks" consisting of nstep\*nprnt time steps before terminating the process. The total number of time steps is nstep\*nprnt\*nstor; nstor must be <=10.
- **ngform** Write data at grid points for graphics processing and other post-processing (programs column.f diagnos.f) into file 'g[0-9].<name>' each time a superblock is written.

ngform=2 unformatted file

ngform=1 formatted file

ngform=0 nothing written

ngform=-1 comment lines are included into file for easier reading (cannot be used for graphics processing in this form)

ngrad Output on graphics file for each ngrad'th radial point

ngcolat Output on graphics file every ngcolat'th point in colatitude

nglon Output on graphics file every nglon'th point in longitude

**nfilt** If>0 apply filter of type  $F(l)=\exp[-(l/alfilt)^nfil]$  to the radial component of the magnetic field on the outer radius (kc=1) before writing data into graphics file (for alfilt >0). When alfilt<0 then apply filter  $F(l)=(1+\sin(pi^*(l-nfilt)/alfilt))$  as long as |l-nfilt|<0.5\*alfilt, and F=1 and F=0 respectively for small/large l.

alfilt See under nfilt

ivfilt If >0 apply the same filter as above to the radial velocity at radial level ivfilt and write the result into graphics file at the first radial location (kc=1)

dipfilt If nfilt>0 multiply axial dipole component of B\_r on outer surface by dipfilt in graphics output

nlogstep Writes data on logfile (prefix l.) after each nlogstep step

**nplog** If >0, writes velocity values at specific points of the grid on separate logfile (prefix "lp.") after every nplog step (for arrays, see vrpoint, vppoint, vtpoint in subroutine amhd for details)

iscale Determines which diffusivity is used for scaling of time, velocity and energy. 1=viscous, 2=thermal, 3=magnetic

enscale In output listings, energies are multiplied by enscale

treset [LOGICAL] If true, reset time and step counter to zero when starting from a stored dataset

**tipdipole** When starting calculation without imposed symmetry (minc=1) from a data file with symmetry (minc>1), add an equatorial dipole component with tipdipole times the magnitude of the polar dipole

amps Option for rescaling temperature perturbation (from restart file) by factor amps (if not equal to 1)

ampw Same for poloidal velocity

ampz Same for toroidal velocity

ampb Same for poloidal magnetic field

ampj Same for toroidal magnetic field

ifvfrz [LOGICAL] If true, do not update velocity during iteration

ifbfrz [LOGICAL] If true, do not update magnetic field during iteration

ifsfrz [LOGICAL] If true, do not update temperature during iteration

#### TIME STEP CONTROL

dtmin Minimum time step (in seconds). If the dynamically determined time step becomes less, the program terminates.

dtmax Maximum (and usually initial) time step. This must be less than 0.25\*ek. Between dtmax and dtmin the actual time step is controlled by a Courant criterion (see below).

dtstart Initial time step. If dtmax=0, dtmax is used for the initial time step when init>0 and the last time step used in the previous run (stored in the restart file) is used when init=0.

courfac Controls the contribution of the fluid velocity to the Courant time step limit (a larger value leads to smaller dt)

alffac Controls the contribution of the (modified) Alfven velocity to the Courant time step limit (a larger value leads to smaller dt)

icour Check Courant criterion after each icour time step (even numbers)

#### PHYSICAL CONTROL PARAMETERS

ra Rayleigh number (defined with gravity on outer boundary)

ek Ekman number

pr Prandtl number

prmag Magnetic Prandtl number

radratio Ratio of inner to outer radius

**bpeak** Peak value of magnetic field imposed by boundary conditions at ICB (also when imagcon=0, bpeak controls the initial magnetic field: toroidal when bpeak>0, poloidal dipole when bpeak<0!)

epsc0 Volumetric rate of internal heating

#### BOUNDARY CONDITIONS AT INNER AND OUTER RADII

ktops thermal boundary condition at CMB. 1-fixed temp, 2-fixed radial heat flow. (ktops=2 not tested!).

**kbots** thermal boundary condition at ICB. As above.

ktopv velocity condition at CMB. 1-free, 2-rigid.

**kbotv** velocity condition at ICB. As above.

**kbotb** =1 for insulating inner core =2: ideally conducting inner core

**ktopb** =1 for insulating mantle =2: not implemented! imagcon: <0 imposed poloidal field (l=1,m=0) at ICB >=0 imposed toroidal field (l=2,m=0) at ICB >=10 imposed toroidal field (l=2,m=0) at both CMB and ICB (same amplitude and same sign if =10, opposite sign if =11)

**cmb** If >0, thin conducting layer at bottom of mantle (not tested!)

#### **HYPERDIFFUSIVITIES**

difamp Amplitude of hyperdiffusivities

ldif Hyperdiffusivites applied for harmonic degrees l >= ldif

**ldifexp** Exponent for increase of hyperdiffusivities with 1 (analytical details see definition of ql(lm,11) in prep.f)

#### PARAMETERS FOR GENERATING MOVIE FILES

imovopt Three-digit integer number, options for generating movie files

Last digit>0 Write B\_z, W\_z (vorticity) and T in the equatorial plane on file with prefix "me."

2nd last digit>0 Write longitudinally averaged B\_phi, j\_phi and v\_phi on file with prefix "ma."

3rd last digit>0 Write B\_r at outer surface and B\_r and v\_r at mid- depth on file with prefix "mm."

4th last digit>0 Write spherical harmonic coefficients for poloidal field at outer boundary and for velocity potentials at radial level on file with prefix "cc."

iframes Write altogether iframes frames on the movie files

tmovstart Time at which to start writing movie-frames

tmovstep Time increments for writing movie-frames

## Appendix C

## MAG Output File Format

MAG produces a set of output files for further processing. All outputs are in non-dimensional units unless specified.

l.[outfile] Lists a set of diagnostic values each nlogstep time-steps.

**ls.[outfile]** Spectra of kinetic energy and magnetic field every nprint time steps, sorted for modes with equal l, and additionally sorted for modes with equal m.

**g.[outfile]** or **g[i].[outfile]** where i=0,1,2,..9 (optional, written when ngstep>0). Contains temperature, velocity and magnetic field components for graphics processing (idl-program magts).

**d.[outfile] or d[i].outfile** Restart files with the complete set of variables (stored as spectral values 1,m in the angular coordinates for radial grid-levels).

lp.[outfile] Written when nplog>0. Velocity at specific points written every nplog'th time step.

me.[outfile] Written when first digit of imovopt>0, i.e., imovopt=0001. Values in the equatorial plane for producing movie (idl-program: magmov EQ3.pro).

ma.[outfile] Written when second digit of imovopt>0, i.e, imovopt=0010. Longitudinal averages for producing movie (idl-program, not provided).

mm.[outfile] Written when third digit of imovopt>0, i.e., imovopt=0100. Values on spherical surfaces for producing movie (idl-program: magmovieCIG.pro, magmovCMB.pro).

cc.[outfile] Written when the last (fourth) digit and any of imovopt>0 and any of the other three digits is larger than zero, i.e. imovopt>1000. Record the spherical harmonic coefficients for poloidal field at outer boundary and their converted Gauss coefficients.

**Note:** If one of the above files already exists, the program will not run.

The standard output file contains summaries of grid parameters and all process control and physical parameters that occur in the namelist statements. It lists the values of non-dimensional parameters and of the various diffusive time-scales. Then, at the end of each block, it lists a number of diagnostic values:

17)

Parameters	Definitions		
dt	actual time step		
dtrmin	Courant time calculated with radial velocities		
dthmin	Courant time calculated with horizontal velocities		
cour	maximum inverse Courant time based on radial fluid velocity		
couh	maximum inverse Courant time based on horizontal fluid velocity		
alfr	maximum inverse Courant time based on radial modified Alfven velocity		
alfh	maximum inverse Courant time based on horizontal modified Alfven velocity (in		
	addition, the radial level at which the maximum is reached is indicated)		
ent	total energy		
env	kinetic energy		
enb	magnetic energy		

The meaning of other quantities is obvious.

mean velocity

For the primary variables, the modes for which they assume their absolute maximum and the maximum are printed. Maxima are determined for the toroidal potential multiplied by 1/r, and for poloidal potentials multiplied by  $l(l+1)/r^2$ , in order to find the modes which exhibit the maximum longitudinal toroidal velocity (field strength) and the maximum radial velocity (field strength), respectively.

1.[outfile] printed every nlogstep time step, one record is printed that contains 17 output fields:

- 1) time 2) mean kinetic energy density 3) mean poloidal kinetic energy density 4) mean magnetic energy density 5) mean poloidal magnetic energy density 6) mean axisymmetric toroidal kinetic energy density 7) mean axisymmetric poloidal kinetic energy density 8) mean axisymmetric poloidal magnetic energy density 9) mean axisymmetric toroidal magnetic energy density 10) mean top heatflow (nusselt number) mean bottom heatflow (nusselt number) 11) 12) mean magnetic field strength 13) rms dipole, outer boundary rms axial dipole, outer boundary 14) 15) dipole tilt, outer boundary 16) dipole longitude, outer boundary
- **Is.[outfile]** Printed each nprint time step are four records with time being the first variable followed by the spectral power of kinetic and magnetic energy, respectively, as a function of harmonic degree l, from l=0 to lmax (first two records in a block), and spectral power as a function of harmonic order m in the last two records of a block.

## Appendix D

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