CSI2110 Assignment 1
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Question 1

a)

O

$(n+1)(n+8) \in O(n^2) \Leftrightarrow \exists c, n, n_0 \in \mathbb{R} \mid n, c > 0 \;\Rightarrow\; f(n) \leq cg(n) \;\; \forall n \geq n_0$

Let $c=18$ and $n_0=1$, proof by induction.

Hypothesis: $(n+1)(n+8) \leq 18n^2$

Fix $n=0$: $(1+1)(1+8) \leq 18*1^2 \Rightarrow 18 \geq 18$ true

Prove n+1:

$(n+1+1)(n+1+8) \leq 18(n+1)^2$

$\Rightarrow (n+1)(n+9)+(n+9) \leq 18n^2+36n+36$

$\Rightarrow (n+1)(n+8)+(n+9)+(n+1) \leq 18n^2+36n+36$

$\Rightarrow (n+1)(n+8) \leq 18n^2+34n+26$ is true because

$18n^2 \leq 18n^2+34n+26 \,\forall n>0 \wedge (n+1)(n+8) \leq 18n^2$ by hypothesis

Therefore $(n+1)(n+8) \in O(n^2)$

Ω

$(n+1)(n+8) \in \Omega(n^2) \Leftrightarrow \exists c, n, n_0 \in \mathbb{R} \mid n, c > 0 \;\Rightarrow\; f(n) \geq cg(n) \;\; \forall n \geq n_0$

Let $c=1$ and $n_0=1$

$(n+1)(n+8) = n^2+9n+8$

$n^2+9n+8 > n^2 \;\; \forall n \geq 1$ therefore $(n+8)(n+1) \in \Omega(n^2)$

Θ

$(n+1)(n+8) \in \Theta(n^2) \Leftrightarrow (n+1)(n+8) \in O(n^2) \wedge (n+1)(n+8) \in \Omega(n^2)$

The above is true as proven therefore $(n+1)(n+8) \in \Theta(n^2)$

b)

O

$n^2+\log(n) \in O(n^2) \Leftrightarrow \exists c, n, n_0 \in \mathbb{R} \mid n, c > 0 \;\Rightarrow\; f(n) \leq cg(n) \;\; \forall n \geq n_0$

set $c=2$ and $n_0=1$

$2n^2 = n^2+n^2$

$n^2+\log(n) \leq n^2+n^2 \;\forall n \geq 1 \Rightarrow \log(n) \leq n^2 \;\forall n \geq 1 \Rightarrow n^2+\log(n) \in O(n^2)$

Ω

$n^2+\log(n) \in \Omega(n^2) \Leftrightarrow \exists c, n, n_0 \in \mathbb{R} \mid n > n_0, c > 0 \;\Rightarrow\; f(n) \geq cg(n) \;\; \forall n \geq n_0$

set $c=1$ and $n_0=2$

$n^2+\log(n) > n^2 \,\forall n \geq 2 \Rightarrow \log(n) > 0 \,\forall n \geq 2 \Rightarrow n^2+\log(n) \in \Omega(n^2)$

Θ

$n^2+\log(n)\in\Theta(n^2) \Leftrightarrow n^2+\log(n)\in O(n^2)\wedge n^2+\log(n)\in\Omega(n^2)$

The above is true as proven therefore $n^2+\log(n)\in\Theta(n^2)$

c)

O

$(n+8)\log(n) \in O(n^2) \Leftrightarrow \exists c, n, n_0 \in \mathbb{R} \,\middle|\, n, c>0 \;\Rightarrow\; f(n) \le cg(n) \;\; \forall n \ge n_0$

set $c=2$ and $n_0=4$

$(n+8)\log(n)=n\log(n)+8\log(n) \;\wedge\; 2n^2=n^2+n^2$

$n\log(n)+8\log(n)\le n^2+n^2 \,\forall n\ge4 \Rightarrow n\log(n)\le n^2 \wedge 8\log(n)\le n^2 \;\; \forall n\ge4$

$n\log(n)\le n^2 \Leftrightarrow \log(n)\le n$ true

$\Rightarrow \;(n+8)\log(n)\in O(n^2)$

Ω

$(n+8)\log(n) \in \Omega(n^2) \Leftrightarrow \exists c, n, n_0 \in \mathbb{R} \,\middle|\, n, c>0 \;\Rightarrow\; f(n) \ge cg(n) \;\; \forall n \ge n_0$

Assuming the above is true, let $n_0>8$

$n>8 \Rightarrow 2n>(n+8) \wedge n>\log(n) \Rightarrow 2n^2>(n+8)\log(n)$

Contradiction with hypothesis $\forall n_0>8 \wedge c\ge2$

Therefore $(n+8)\log(n) \notin \Omega(n^2)$

Θ

$(n+8)\log(n)\in\Theta(n^2) \Leftrightarrow (n+8)\log(n)\in O(n^2)\wedge(n+8)\log(n)\in\Omega(n^2)$

It was proven that $(n+8)\log(n)\notin\Omega(n^2)$ therefore $(n+8)\log(n)\notin\Theta(n^2)$

d)

O

$10n^3 \in O(n^2) \Leftrightarrow \exists c, n, n_0 \in \mathbb{R} \,\middle|\, n, c>0 \;\Rightarrow\; f(n) \le cg(n) \;\; \forall n \ge n_0$

Assuming the above condition is true

$10n^3\ge cn^2 \Leftrightarrow 10n\ge c \Leftrightarrow n\ge\frac{c}{10}$

Let $n_0=\frac{c}{10}+1$ ...contradiction. Therefore $10n^3 \notin O(n^2)$

Ω

$10n^3 \in\Omega(n^2) \Leftrightarrow \exists c, n, n_0 \in \mathbb{R} \,\middle|\, n, c>0 \;\Rightarrow\; f(n) \ge cg(n) \;\; \forall n \ge n_0$

Let $c=10$ and $n_0=1$

$10n^3\ge10n^2 \,\forall n\ge1 \Leftrightarrow n\ge1 \,\forall n\ge1$

$\Rightarrow 10n^3 \in\Omega(n^2)$

Θ

$10n^3\in\Theta(n^2) \Leftrightarrow 10^3\in O(n^2)\wedge 10n^3\in\Omega(n^2)$

Above it was proven that $10n^3 \notin O(n^2)$ therefore $10n^3\notin\Theta(n^2)$

e)

O

$$\log(n^{10}+n^2) \in O(n^2) \Leftrightarrow \exists c, n, n_0 \in \mathbb{R} \,\middle|\, n, c>0 \;\Rightarrow\; f(n) \le cg(n) \;\; \forall n \ge n_0$$

Let $c=10$ and $n_0=1$

$$\log(n^{10}+n^2)=\log(n^2(n^8+1))=\log(n^2)+\log(n^8+1)=2\log(n)+\log(n^8+1)\le 2\log(n)+\log(n^9)\,\forall n\ge 1$$

$$2\log(n)+\log(n^9)=2\log(n)+9\log(n)=11\log(n)$$

$$11\log(n)\le 11n^2 \Leftrightarrow \log(n)\le n^2 \Rightarrow \log(n^{10}+n^2)\le 11n^2\,\forall n\ge 1$$

$$\Rightarrow \log(n^{10}+n^2)\in O(n^2)$$

Ω

$$\log(n^{10}+n^2) \in \Omega(n^2) \Leftrightarrow \exists c, n, n_0 \in \mathbb{R} \,\middle|\, n, c>0 \;\Rightarrow\; f(n) \ge cg(n) \;\; \forall n \ge n_0$$

Assuming the above to be true $\Rightarrow \log(n^{10}+n^2)\ge c\,n^2$

$$\log(2n^{10})\ge \log(n^{10}+n^2)$$

$$\log(2n^{10})=\log(2)+10\log(n)=10\log(n)+1 \le 11\log(n)$$

$$11\log(n)\ge c\,n^2$$

$n\ge c \Rightarrow 11\log(c)\ge c^3$ ...contradiction

Therefore $\log(n^{10}+n^2)\notin \Omega(n^2)$

Θ

$$\log(n^{10}+n^2)\in\Theta(n^2) \Leftrightarrow \log(n^{10}+n^2)\in O(n^2)\wedge \log(n^{10}+n^2)\in\Omega(n^2)$$

Above it was proven that $\log(n^{10}+n^2)\notin\Omega(n^2)$ therefore $\log(n^{10}+n^2)\notin\Theta(n^2)$

f)

O

$$4n+5 \in O(n^2) \Leftrightarrow \exists c, n, n_0 \in \mathbb{R} \,\middle|\, n, c>0 \;\Rightarrow\; f(n) \le cg(n) \;\; \forall n \ge n_0$$

Let $c=1$ and $n_0=5$

$$4n+5\le n^2 \;\forall\; n\ge 5 \Rightarrow 4n+5 \in O(n^2)$$

Ω

$$4n+5 \in \Omega(n^2) \Leftrightarrow \exists c, n, n_0 \in \mathbb{R} \,\middle|\, n, c>0 \;\Rightarrow\; f(n) \ge cg(n) \;\; \forall n \ge n_0$$

Assuming the above is true and that $n_0>5$

$$n>5 \Rightarrow 4n+n > 4n+5 \Rightarrow 5n > 4n+5$$

$4n+5\ge c\,n^2 \Rightarrow 5\,n>c\,n^2 \Rightarrow 5>c\,n$ ...contradiction for $n>5\wedge c\ge 1$

$$\Rightarrow 4n+5 \notin\Omega(n^2)$$

Θ

$$4n+5\in\Theta(n^2) \Leftrightarrow 4n+5\in O(n^2)\wedge 4n+5\in\Omega(n^2)$$

Above it was proven that $5n+5\notin\Omega(n^2)$ therefore $10n^3\notin\Theta(n^2)$

Question 2

a) The worst case executions is exactly $\sum_{i=0}^{n-1} i$ which equals $\frac{n(n-1)}{2}$

b)

$$\frac{n(n-1)}{2} \in O(n^2) \Leftrightarrow \exists c, n, n_0 \in \mathbb{R} \,\middle|\, n, c > 0 \;\Rightarrow\; f(n) \leq cg(n) \;\; \forall n \geq n_0$$

$$\frac{n(n-1)}{2} = \frac{1}{2}n(n-1) = \frac{1}{2}n^2 - n$$

Let c = $\frac{1}{2}$ and $n_0 = 0$

$$\frac{1}{2}n^2 - n \leq \frac{1}{2}n^2 \Rightarrow -n \leq 0 \Rightarrow n \geq 0$$ which is true because $n \geq n_0 \wedge n_0 = 0$

therefore $\frac{n(n-1)}{2} \in O(n^2)$

c)

```
// Assuming Age has at least 2 values
x = 1;
int minDifference = abs(Age[x]-Age[x-1]);
for (int i = x+1; i < n; i++) {
    int difference = abs(Age[i] - Age[i-1]);
    if (difference < minDifference) {
        minDifference = difference;
        x = i;
    }
}
return x-1, x;
```

d) This algorithm essentially moves through the array once, comparing each adjacent pair of values. Because the number of adjacent pairs is a constant multiple of n, the steps the algorithm performs is therefore order n.

Question 3

A)

```
int mid = (int)(N-1)/2 // Set middle value

for (int i = 0; i < N; i++) { // Loops through every value in the array
   int nums = 0;
   for (int j = 0; j < N; j++) { // Loop once again, finding smaller values
      if (j < i) {
         nums++;
      }
   }
   if (nums == mid) { // Return if value found, ending algorithm
      return i;
   }
}

return 0;
```

B)
The best case running time would be if the first number is the median value, in which case the algorithm would perform N comparisons before determining that it has (N-1)/2 numbers smaller than it. There would therefore be exactly N steps executed, which is precisely order N.

The worst case would be for the median value to be the last value in the array. In this case the algorithm would make N comparisons for each of the N numbers in the array, executing for precisely N^2 steps. Sparing a proof, N^2 steps is precisely order N^2, to the tightest precision.