Université d'Ottawa
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University of Ottawa
Faculty of Engineering

School of Information
Technology and Engineering

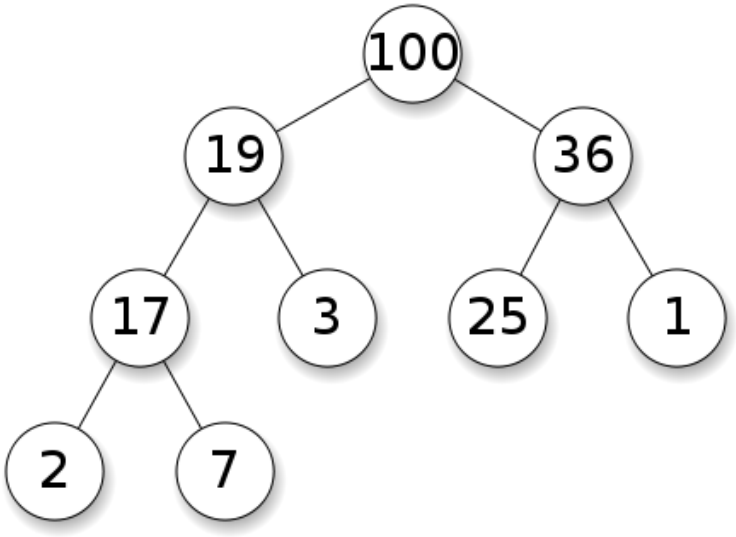
Assignment 3 (5%, 30 marks)

CSI2110

Please submit by 23:55, Oct 20 via virtual campus as a single PDF file.
Late assignments are accepted up to 24hs late with 30% penalty.

Question 1. **[10 points]**

Consider the following tree :



a) List (in order) the nodes that will be visited by a pre-order traversal.
b) List (in order) the nodes that will be visited by a post-order traversal.
c) This tree is a max-heap. Insert nodes 200 and 55 (in that order). Demonstrate your answer by drawing the tree after each insertion.
d) Remove node 19 from the tree drawn above (do not consider nodes inserted in the previous question).

Question 2. **[5 points]**

Consider a a min heap. What would be the exact number of comparisons needed to obtain the three smallest keys without having to remove the nodes? Justify your answer.

Question 3. **[12 points]**

Consider a min-heap complete tree consisting of 5000 nodes.

a) What is the exact height of this tree?
b) How many nodes are at the deepest level?
c) How many nodes are at the second deepest level?
d) How many swaps, in the worst case, are required in order to insert a new element in this tree? Explain.
e) How many swaps, in the best case, are required in order to insert a new element in this tree? Explain.
f) If you want to check if element 10000 is present in this heap, how many comparisons, in the worst case, will be needed?

Question 4. **[10 points]**

Here is the heapsort algorithm .

```
void Heapsort (int [] heap, int N) {
     // Organize the array in a heap (buildheap)
     for (k = N/2; k > = 1; k--) {
          downheap (heap, N, k);
     }
// remove each node in turn
While (N > 1) {
          int tempo = heap [1];
          heap [1] = heap [N];
          heap [N] = tempo;
          N--;
          downheap(heap,N, 1);
     }
}
```

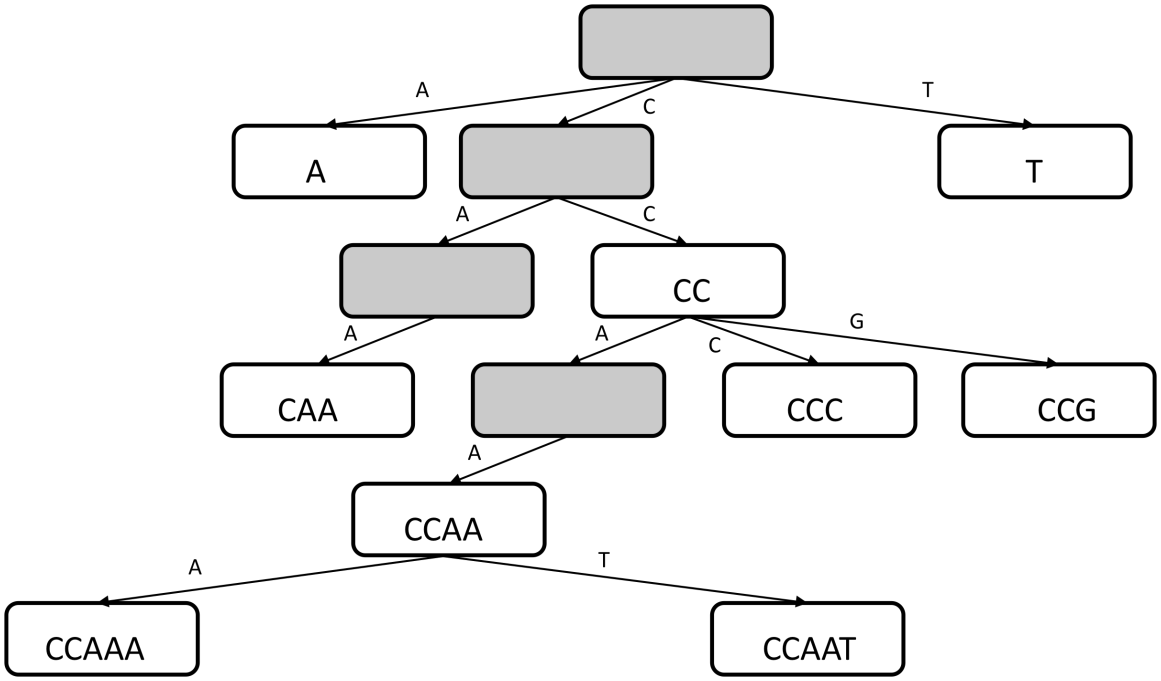
a) Give the algorithm of the method `downheap` in the case of a max-heap.

b) Sort the arrays below using this heapsort. Give the State of the table after each call to the function `downheap`.

	4	11	7	12	32	8	12	21	2

Question 5. **[8 points]**

A nucleic acid sequence in a DNA sequence consists of a sequence of 4 possible letter: A C G T. This sequence is represented using the so called Trie data structure. In this case, it is a Quaternary tree in which each letter of a sequence is used as the key. Thus an 'A' in the sequence will create a branch to the leftmost-left, a 'C' a branch to the middle-left, a 'G' to the middle right and a 'T' to the rightmost-right. Here is an illustration of such a tree containing nine sequences (A, T, CC, CAA, CCC, GCC, CCAA, CCAAA, CTAC).



The branches are always created in the order ACGT (null children references are not shown) The grey nodes are nodes that contain no sequence.

(a) Knowing that the longest sequence has a length of N, show that the search for a sequence in this tree will be O(N).

b) Knowing that the longest sequence has a length of N, give the maximum number of nodes that such a tree can have.

c) Knowing that the longest sequence has a length of N, give the minimum number of nodes that such a tree can have.

d) Print the non-empty nodes in this tree using post-order traversal.