CV Assignment 2

CSE 344 | Winter 25'
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Ans i) $R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$ $R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$

$\Rightarrow \theta = \frac{-\pi}{6} = \begin{bmatrix} \sqrt{3}/2 & 0 & -1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & \sqrt{3}/2 \end{bmatrix}$ $\theta = \frac{\pi}{4} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$

$R' = R_x\left(\frac{\pi}{4}\right) R_y\left(\frac{-\pi}{6}\right) \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & 0 & -1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & \sqrt{3}/2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \sqrt{3}/2 & 0 & -1/2 \\ -1/2\sqrt{2} & 1/\sqrt{2} & -\sqrt{3}/2\sqrt{2} \\ 1/2\sqrt{2} & 1/\sqrt{2} & \sqrt{3}/2\sqrt{2} \end{bmatrix} = R'$

Reflection across XZ plane = $Ref = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R'' = Ref \cdot R' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & 0 & -1/2 \\ -1/2\sqrt{2} & 1/\sqrt{2} & -\sqrt{3}/2\sqrt{2} \\ 1/2\sqrt{2} & 1/\sqrt{2} & \sqrt{3}/2\sqrt{2} \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \sqrt{3}/2 & 0 & -1/2 \\ 1/2\sqrt{2} & -1/\sqrt{2} & -\sqrt{3}/2\sqrt{2} \\ 1/2\sqrt{2} & 1/\sqrt{2} & \sqrt{3}/2\sqrt{2} \end{bmatrix} = R''$

Since we are asked coordinat transformation matrix translation vector can be ignored

ii) $v' = R''v + t$

$$\begin{bmatrix} \sqrt{3}/2\,(3) + 0(-1) + (-1/2)4 \\ 1/2\sqrt{2}\,(3) + (-1/\sqrt{2})(-1) + (\sqrt{3}/2\sqrt{2})4 \\ 1/2\sqrt{2}\,(3) + (1/\sqrt{2})(-1) + (\sqrt{3}/2\sqrt{2})4 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3\sqrt{3}/2 - 2 + 1 \\ 3/2\sqrt{2} + 1/\sqrt{2} + 2\sqrt{3}/\sqrt{2} \\ 3/2\sqrt{2} + (-1/\sqrt{2}) + 2\sqrt{3}/\sqrt{2} - 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1.598 \\ 4.217 \\ 0.803 \end{bmatrix}$$

Mapped origin $\Rightarrow (1, 0, -2)^T$

iii) $\det(R') = \frac{\sqrt{3}}{2}\left(1/\sqrt{2}\,(\sqrt{3}/2\sqrt{2}) - 1/\sqrt{2}\,(-\sqrt{3}/2\sqrt{2}) - 0 \right.$
$\left. + (-1/2)(1/2\sqrt{2})(1/\sqrt{2}) - (1/\sqrt{2})(1/2\sqrt{2}) \right)$

$\Rightarrow \frac{\sqrt{3}}{2}\left(\frac{2\sqrt{3}}{4\sqrt{2}}\right) + (-1/2)\left(\frac{1}{2}/\frac{1}{2}\right)$

$\Rightarrow \frac{3}{4} + \frac{1}{4} = 1$

$R'^T R' = I$

$$\begin{bmatrix} \sqrt{3}/2 & -1/2\sqrt{2} & 1/2\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/2 \\ -1/2 & -\sqrt{3}/2\sqrt{2} & \sqrt{3}/2\sqrt{2} \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & 0 & -1/2 \\ -1/2\sqrt{2} & 1/\sqrt{2} & -\sqrt{3}/2\sqrt{2} \\ 1/2\sqrt{2} & 1\sqrt{2} & \sqrt{3}/2\sqrt{2} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$ [Used calculator]

$\text{tr}(R') = \sqrt{3}/2 + 1/\sqrt{2} + \sqrt{3}/2\sqrt{2} = 0.866 + 0.707 + 0.612 = 2.185$

$$\cos\theta = \frac{tr(R') - 1}{2} = \frac{2.185 - 1}{2} = \frac{1.185}{2} = 0.592$$

$$\theta = \cos^{-1}(0.592) = 53.7^\circ$$

$$n = \frac{1}{2\sin\theta}\begin{bmatrix} R'_{32} - R'_{23} \\ R'_{13} - R'_{31} \\ R'_{21} - R'_{12} \end{bmatrix} \Rightarrow \frac{1}{2\sin 53.7^\circ}\begin{bmatrix} 2+\sqrt{3}/2\sqrt{2} \\ -\sqrt{2}-1/2\sqrt{2} \\ -1/2\sqrt{2} \end{bmatrix} \Rightarrow \frac{1}{1.61}\begin{bmatrix} 1.319 \\ -0.856 \\ -0.354 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0.819 \\ -0.531 \\ -0.219 \end{bmatrix}$$

$$\Rightarrow \hat{n} = \begin{bmatrix} 0.819 \\ -0.531 \\ -0.219 \end{bmatrix}$$

iv) $$N = \begin{bmatrix} 0 & 0.219 & -0.531 \\ -0.219 & 0 & -0.819 \\ 0.531 & 0.819 & 0 \end{bmatrix}$$

$$N^2 = \begin{bmatrix} -0.329 & -0.434 & -0.179 \\ -0.434 & -0.718 & 0.116 \\ -0.179 & 0.116 & -0.952 \end{bmatrix}$$

$$R = I + 0.805\begin{bmatrix} 0 & 0.219 & -0.531 \\ -0.219 & 0 & -0.819 \\ 0.531 & 0.819 & 0 \end{bmatrix} + 0.407\begin{bmatrix} -0.329 & -0.434 & -0.179 \\ -0.434 & -0.718 & 0.116 \\ -0.179 & 0.116 & -0.952 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0.865 & -0.01 & -0.501 \\ -0.354 & 0.707 & -0.613 \\ 0.355 & 0.707 & 0.611 \end{bmatrix} = \text{Rod}$$

$$\begin{bmatrix} \sqrt{3}/2 & 0 & -1/2 \\ -1/2\sqrt{2} & 1/\sqrt{2} & -\sqrt{3}/2\sqrt{2} \\ 1/2\sqrt{2} & 1/\sqrt{2} & \sqrt{3}/2\sqrt{2} \end{bmatrix}$$ Original matrix

$$\begin{bmatrix} 0.866 & 0 & -0.500 \\ -0.354 & 0.707 & -0.612 \\ 0.312 & 0.707 & 0.612 \end{bmatrix}$$

There are small differences in the original and rodrigues rotation matrix because of approximations used above

However both of them would be almost equal

Ans: Given: Two cameras C_1 and C_2 with intrinsic parameters K_1 and K_2.

$$x = K[R|t]X$$

Since C_1 is world camera

$R = I$ and $t = 0$ as it is the reference.

$$x_1 = K_1[I|0]X = K_1(IX + 0) = K_1X$$

$$X = K_1^{-1}x_1 \quad \text{—①}$$

Since C_2 is a pure 3D rotation $t = 0$

$x_2 = K_2 [R|0] X = K_2 (RX + 0) = K_2 RX$ — ②

put ① in ②

$x_2 = K_2 R (K_1^{-1} x_1)$

$x_2 = K_2 R K_1^{-1} x_1$

$x_1 = (K_2 R K_1^{-1})^{-1} x_2$

$x_1 = \underbrace{(K_1 R^{-1} K_2^{-1})}_{H} x_2$ → All three are invertible

$x_1 = H x_2$

Since R is rotation matrix

$R^{-1} = R^T$

$H = K_1 R^T K_2^{-1}$

3.

Part 1.

Focal Length (fx) = 955.983130942084

Focal length (fy) = 956.7749752185975

Skew Parameter = 0.0

Principal Point (cx) = 368.8820613141149

Principal Point (cy)= 648.8286752606324

Part2.

Rotation Matrix Image 1:

0.9969	0.0138	-0.0765
-0.0249	0.9889	-0.1460
0.0736	0.1475	0.9863

Translation Vector Image 1:

-3.9353
-3.5773

14.6556

Rotation Matrix Image 2:

0.9982	0.0539	-0.0224
-0.0507	0.9908	0.1253
0.0290	-0.1239	0.9918

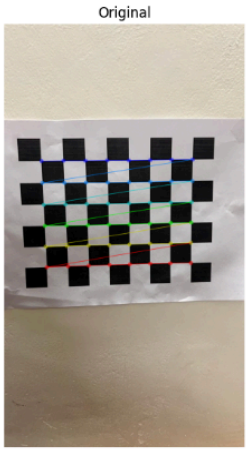
Translation Vector Image 2:

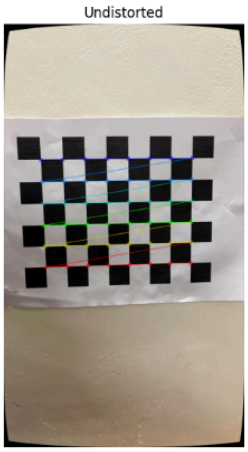
-4.3690
-1.4169
15.8122

Part 3.

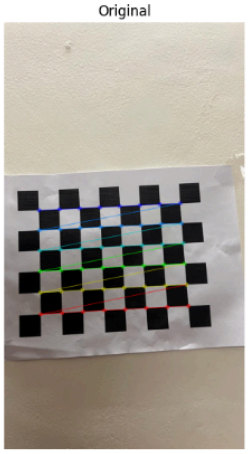
Radial Distortion Coefficients:

0.2156
-1.0390
1.5863





Original



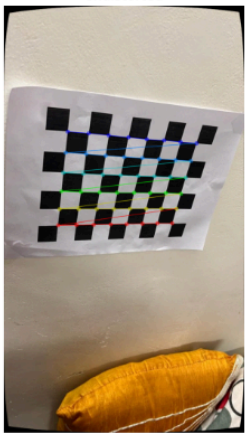
Undistorted

Original

Undistorted

Original

Undistorted



Original

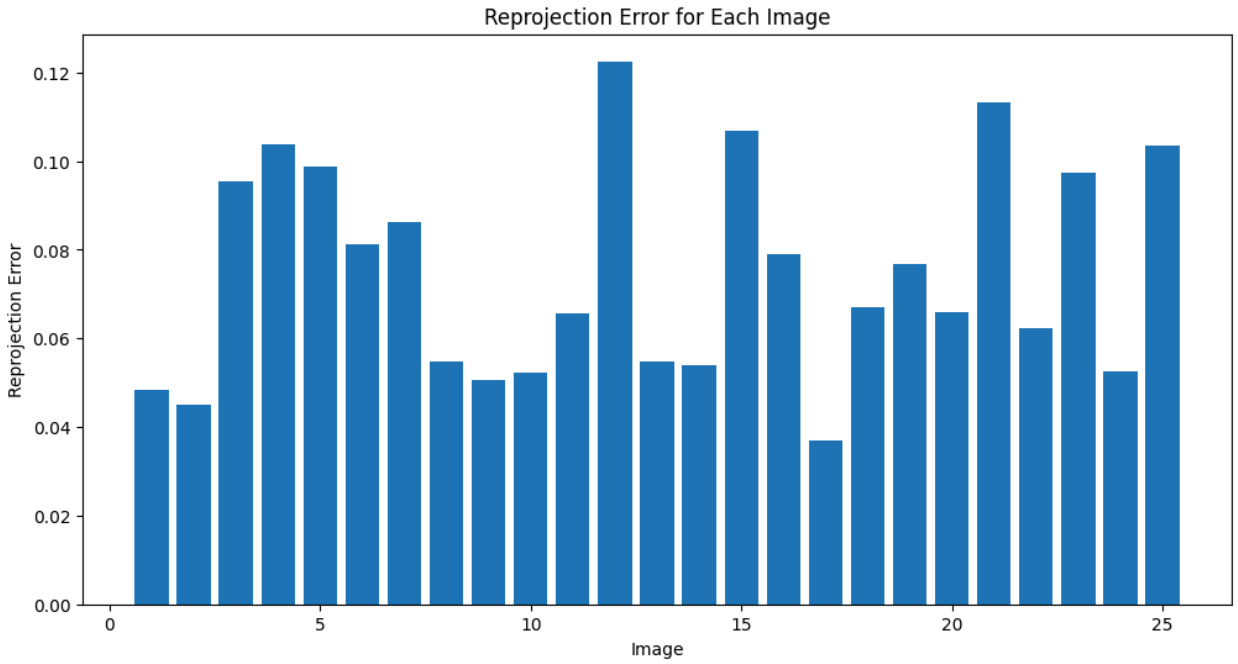
Undistorted

As evident from the 5 images, they display a slight barrel distortion when the radial coefficients are applied. The straight lines at the corners of the image become curved indicating barrel distortion.

Part 4.

Average Reprojection Error: `0.07498802315303951`

Standard Deviation: `0.023956747838967504`



Part 5.

The images aren't visible when I paste them here, please look at the images in the python notebook.

Reprojection error is calculated by projecting the known 3D object points onto the image using the estimated camera parameters and comparing them to the actual detected 2D image points.

Part 6.

```
Image 0 normal: [0.07366454 0.14751179 0.98631324]
Image 1 normal: [ 0.02902482 -0.12396062  0.99186255]
Image 2 normal: [0.39028616 0.20110607 0.8984615 ]
Image 3 normal: [0.3975825  0.48895515 0.77643481]
Image 4 normal: [-0.35645372 -0.40251267  0.84316327]
Image 5 normal: [-0.09063589  0.31180444  0.94581347]
Image 6 normal: [0.09108273 0.51319171 0.85342733]
Image 7 normal: [-0.07651364 -0.21772877  0.97300557]
Image 8 normal: [ 0.09474059 -0.20108647  0.97498126]
Image 9 normal: [ 0.14371054 -0.07803141  0.98653858]
Image 10 normal: [0.10776904 0.08546547 0.99049558]
Image 11 normal: [0.43861018 0.22649536 0.86966716]
Image 12 normal: [0.13576403 0.10450709 0.98521388]
Image 13 normal: [-0.03855277  0.28664423  0.95726108]
Image 14 normal: [ 0.35155881 -0.07742426  0.93295867]
Image 15 normal: [0.24090876 0.31530029 0.91790451]
Image 16 normal: [-0.03398246  0.11049903  0.9932951 ]
Image 17 normal: [ 0.21701635 -0.08703717  0.97228002]
Image 18 normal: [0.27043767 0.09624743 0.95791435]
Image 19 normal: [ 0.171167   -0.15647657  0.97273683]
Image 20 normal: [ 0.42011323 -0.13626374  0.89718285]
Image 21 normal: [0.08096579 0.12640464 0.98866901]
Image 22 normal: [0.10105953 0.39966115 0.91107515]
Image 23 normal: [-0.19166111  0.05394419  0.97997757]
Image 24 normal: [-0.52498406 -0.30521653  0.79450274]
```

Link to the images clicked by me:
https://drive.google.com/drive/folders/1hssOowEGTeZ2o7xHzNfzI584Mm42W4Pn?usp=sharing

Results obtained on these images:
Part 1.

```
Intrinsic Camera Matrix:
[[3.15392746e+03 0.00000000e+00 1.11333065e+03]
 [0.00000000e+00 3.14796429e+03 1.87052901e+03]
 [0.00000000e+00 0.00000000e+00 1.00000000e+00]]
Focal Lengths:
3153.927457499056 3147.9642924899895
Skew Parameter:
0.0
Principal Point:
1113.3306464801728 1870.5290098523756
```

Part 2.

```
---------- First Image ----------
Rotation Matrix:
[[ 3.32065119e-04 -9.57223527e-01 -2.89349284e-01]
 [ 9.99526900e-01 -8.58120760e-03  2.95353827e-02]
 [-3.07549294e-02 -2.89222201e-01  9.56767816e-01]]
Translation Vectors:
[[ 2.48771718]
 [-3.62785088]
 [16.20459226]]
---------- Second Image ----------
Rotation Matrix:
[[ 0.65200992 -0.61537277 -0.44294404]
 [ 0.68745579  0.72622     0.00300711]
 [ 0.31982433 -0.30646511  0.89654422]]
Translation Vectors:
[[ 0.49735577]
 [-3.01215934]
 [16.82658625]]
```

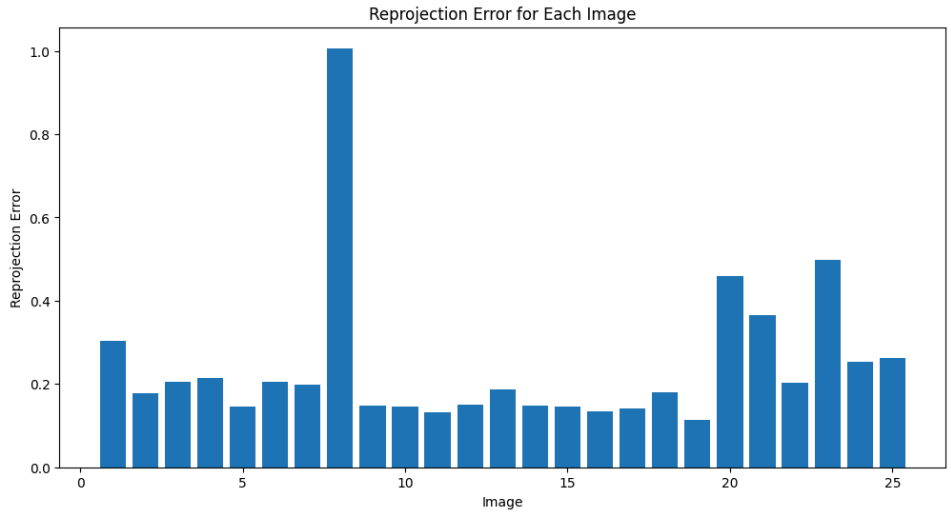
Part 3.

```
Radial Distortion Coefficients:
0.125991394128843 -1.236618736915827 1.1006754269664025
```

There is much more barrel distortion in these images as compared to the previous ones. The straight lines at the corners of the image become curved indicating barrel distortion.

Part 4.

```
Average Reprojection Error: 0.24485286161517938
Standard Deviation of Reprojection Error: 0.18316229263055397
```



Part 5.

The results are in the notebook as they are not visible here.

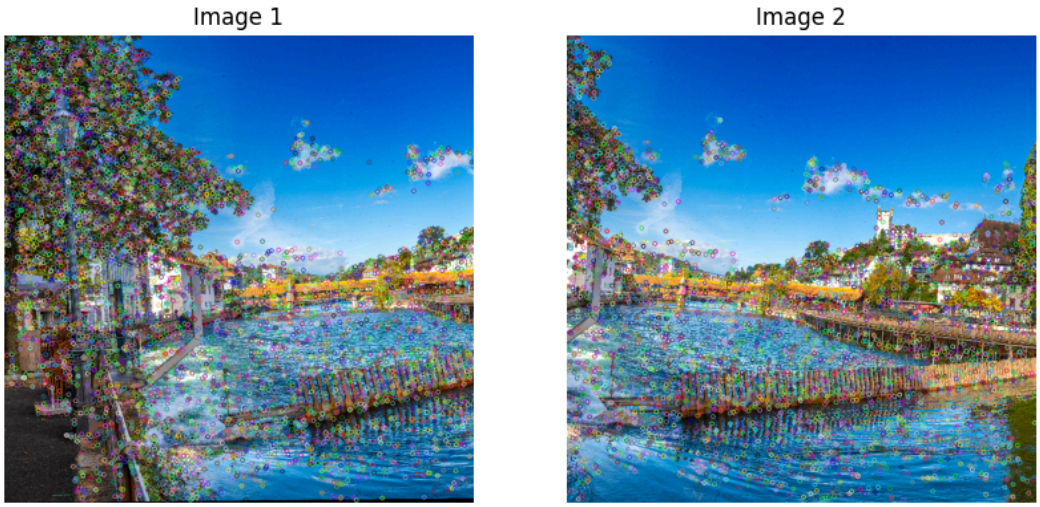
Part 6.

```
Image 0 normal: [-0.03075493 -0.2892222   0.95676782]
Image 1 normal: [ 0.31982433 -0.30646511  0.89654422]
Image 2 normal: [0.35927871 0.34604959 0.86669977]
Image 3 normal: [0.19895068 0.43945268 0.8759566 ]
Image 4 normal: [ 0.63289203 -0.16821464  0.75574567]
Image 5 normal: [0.00958054 0.07086298 0.99744005]
Image 6 normal: [0.67619901 0.07394018 0.73299915]
Image 7 normal: [ 0.45362482 -0.27227615  0.8485813 ]
Image 8 normal: [-0.38769005 -0.22699571  0.89340325]
Image 9 normal: [-0.24506471 -0.41946981  0.87406428]
Image 10 normal: [-0.0185347  -0.50237187  0.86445299]
Image 11 normal: [ 0.28503474 -0.37460723  0.88228375]
Image 12 normal: [-0.25421231  0.06505114  0.96495826]
Image 13 normal: [ 0.24511513 -0.37139968  0.89553384]
Image 14 normal: [ 0.43147688 -0.07228929  0.89922297]
Image 15 normal: [-0.40281155 -0.22000885  0.8884475 ]
Image 16 normal: [-0.15215614 -0.36139732  0.9199133 ]
Image 17 normal: [ 0.24101228 -0.29685035  0.92400917]
Image 18 normal: [-0.31500345 -0.26364993  0.91173545]
Image 19 normal: [-0.51853373 -0.12738602  0.84551498]
Image 20 normal: [ 0.04753156 -0.64733453  0.76072253]
Image 21 normal: [ 0.54821829 -0.30337615  0.7793713 ]
Image 22 normal: [-0.4856868   0.00551682  0.8741155 ]
Image 23 normal: [ 0.08914713 -0.08443207  0.99243338]
Image 24 normal: [ 0.47791095 -0.55505582  0.68081874]
```

4.

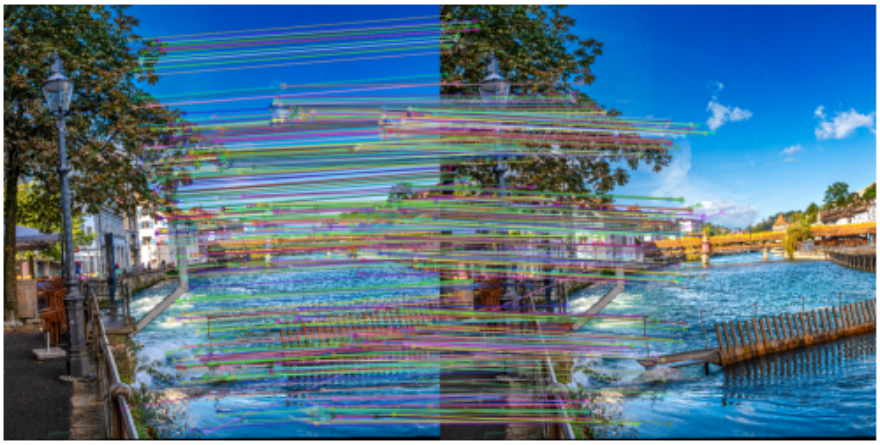
Clusters obtained using color histogram:

Part 1.

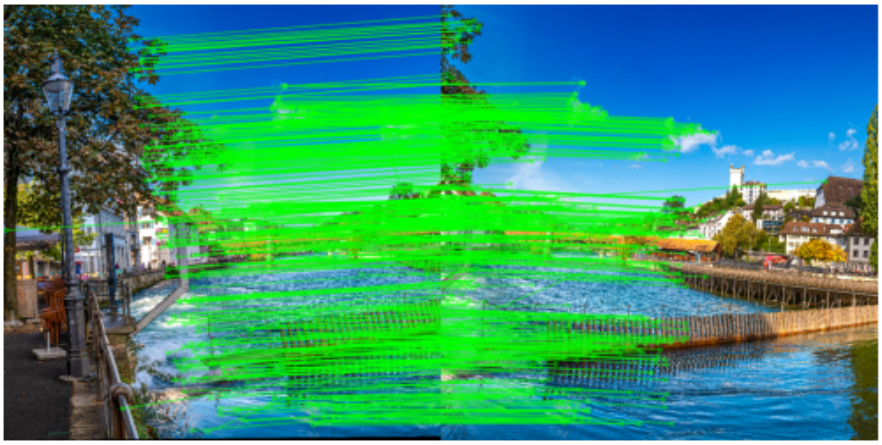


Part 2.

Brute Force



Flann Based



Part 3.

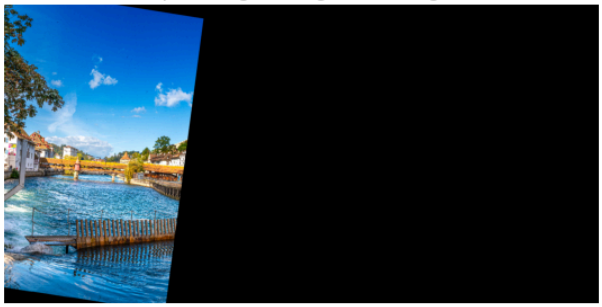
Csv submitted.

```
Homography Matrix (H):
[[ 8.98950781e-01 -1.31348248e-01 -1.64400129e+02]
 [ 1.43082735e-01  9.90633132e-01 -6.93808374e+01]
 [-1.02059934e-07  3.06692828e-07  1.00000000e+00]]
```

Part 4.

Original Image 1

Warped Image 1 (Aligned to Image 2)



Part 5.

Full Panorama (No Cropping)

Cropped

Part 6.

Final Panorama for Cluster 0

Final Panorama for Cluster 1

Final Panorama for Cluster 2

5.

Part 1.

Inlier RMSE: 0.031172835853675338

Fitness: 0.015361117172157976

Estimated Transformation:

0.1924	-0.3481	0.9174	0.1507
0.6786	-0.6280	-0.3806	0.1703
0.7087	0.6959	0.1153	0.2908
0	0	0	1

Part 2.

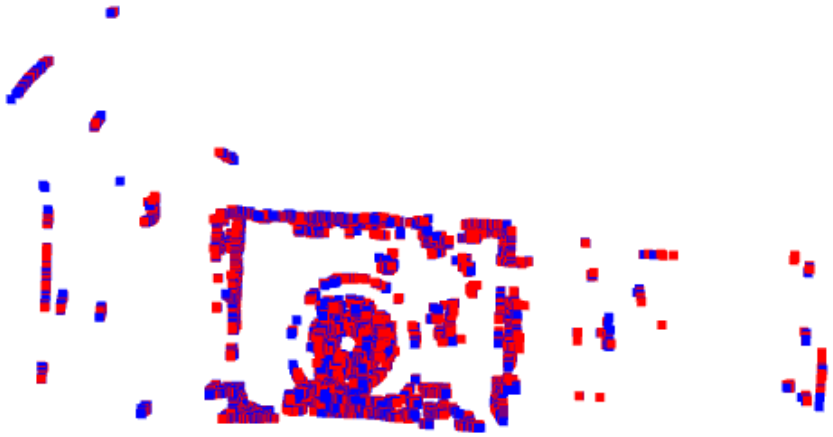
Threshold	Max Iters.	Initialization	Init. Fitness	Ini. RMSE	ICP Fitness	ICP RMSE
0.05	100	orthogonal	0.008161	0.037088	0.011215	0.032619
0.05	100	ransac	0.98573	0.014497	0.986166	0.012468
0.05	200	orthogonal	0.009383	0.034817	0.011783	0.029313
0.05	200	ransac	0.985381	0.01608	0.986166	0.012467
0.05	500	orthogonal	0.037399	0.031518	0.046171	0.031595
0.05	500	ransac	0.986603	0.01272	0.986166	0.012467
0.1	100	orthogonal	0.052324	0.065367	0.062099	0.062406
0.1	100	ransac	0.9976	0.015284	0.997731	0.014403
0.1	200	orthogonal	0.050404	0.070683	0.062055	0.057197
0.1	200	ransac	0.997643	0.020047	0.997731	0.014403
0.1	500	orthogonal	0.067685	0.064109	0.087017	0.055609
0.1	500	ransac	0.997731	0.015739	0.997774	0.014416
0.2	100	orthogonal	0.064761	0.109226	0.077766	0.10427
0.2	100	ransac	0.999607	0.015762	0.999607	0.015427
0.2	200	orthogonal	0.042592	0.12611	0.060964	0.133518
0.2	200	ransac	0.999651	0.018606	0.999607	0.015428

0.2	500	orthogonal	0.140607	0.120251	0.14964	0.117558
0.2	500	ransac	0.999651	0.015888	0.999607	0.015426
0.4	100	orthogonal	0.125376	0.229788	0.123456	0.225212
0.4	100	ransac	0.999956	0.017496	0.999956	0.016136
0.4	200	orthogonal	0.159372	0.233653	0.154571	0.23296
0.4	200	ransac	0.999956	0.020315	0.999956	0.016136
0.4	500	orthogonal	0.201178	0.244707	0.221863	0.241772
0.4	500	ransac	0.999956	0.017269	0.999956	0.016139

T-matrix obtained on best hyperparameters:

9.99999814e-01	-6.09625237e-04	-1.99021302e-05	-9.19777564e-05
6.09625052e-04	9.99999814e-01	-9.31877409e-06	-8.71820512e-04
1.99078075e-05	9.30663952e-06	1.00000000e+00	-5.65106777e-07
0.00000000e+00	0.00000000e+00	0.00000000e+00	1.00000000e+00

Part 3.



Red is pcd 0000 and blue is pcd 0004

Majority of the points have some overlap.

https://docs.opencv.org/4.x/dc/dc3/tutorial_py_matcher.html
https://docs.opencv.org/3.4/d1/de0/tutorial_py_feature_homography.html
https://www.open3d.org/docs/0.18.0/python_api/open3d.geometry.KDTreeSearchParamHybrid.html
https://www.open3d.org/docs/latest/python_api/open3d.visualization.draw_geometries.html
https://courses.cs.washington.edu/courses/cse576/05sp/papers/MSR-TR-2004-92.pdf
https://medium.com/%40paulsonpremsingh7/image-stitching-using-opencv-a-step-by-step-tutorial-9214aa4255ec
References provided in the assignment
Lecture Notes
Co-Pilot (for speeding up repetitive work)
Stanford Lecture Notes (cs231n)