Glacier volume-area relation for high-order mechanics and transient glacier states

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[1] Glacier volume is known for less than 0.1% of the world's glaciers, but this information is needed to quantify the impacts of glacier changes on global sea level and regional water resources. Observations indicate a power-law relation between glacier area and volume, with an exponent $\gamma \approx 1.36$. Through numerical simulations of 3D, high-order glacier mechanics, we demonstrate how different topographic and climatic settings, glacier flow dynamics, and the degree of disequilibrium with climate systematically affect the volume-area relation. We recommend more accurate scaling relations through characterization of individual glacier shape, slope and size. An ensemble of 280 randomly-generated valley glaciers spanning a spectrum of plausible glaciological conditions yields a steady-state exponent $\gamma = 1.46$. This declines to 1.38 for glaciers that are 100 years into a sustained retreat, which corresponds exceptionally well with the observed value for present-day glaciers. **Citation:** Adhikari, S., and S. J. Marshall (2012), Glacier volume-area relation for high-order mechanics and transient glacier states, *Geophys. Res. Lett.*, *39*, L16505, doi:10.1029/2012GL052712.

1. Introduction

[2] Glacier retreat is one of the strongest signals of climate warming, and is testament to the cumulative effects of a global warming of ca. 0.8°C since the late 19th century [*Hansen et al.*, 2006]. Glacier changes impact on global sea level and regional water resources, and they also provide a natural gauge to understand and quantify climate change in remote parts of the planet [e.g., *Oerlemans*, 2005]. There is therefore tremendous interest and value in assessing global glacier changes.

[3] Glacier areas are well-observed and documented through aerial photos, maps, and satellite imagery [e.g., *Williams and Ferrigno*, 1988; *Paul et al.*, 2004; *Bolch et al.*, 2010], and the global area of glaciers and ice caps is known. Excluding the glacier ice in Antarctica and Greenland, estimates vary from 0.52 to 0.54×10^6 km^2, with an additional $0.22 \pm 0.07 \times 10^6$ km^2 in the glaciers and ice caps peripheral to the Antarctic and Greenland ice sheets [*Radić and Hock*, 2010].

[4] While glacier area is an essential variable for evaluating glacier changes and modeling cryosphere-climate processes, glacier volume is the primary variable of interest for glacier mass balance and for quantifying the impacts of glacier changes on global sea level and regional water resources. Unfortunately, thickness and volume are known for less than 0.1% of the estimated global population of more than 200,000 glaciers [e.g., *Bahr et al.*, 1997]. For this reason, indirect estimates of ice thickness are necessary based on various theoretical and statistical approaches [e.g., *Nye*, 1952; *Chen and Ohmura*, 1990; *Bahr et al.*, 1997; *Clarke et al.*, 2009; *Farinotti et al.*, 2009]. Of these indirect methods, volume-area (*V-A*) scaling is the most widely applied approach for global-scale glacier inventories [e.g., *van de Wal and Wild*, 2001; *Radić et al.*, 2008; *Radić and Hock*, 2010; *Slangen and van de Wal*, 2011], and it is also commonplace in assessment of regional glacier volume response to climate change [e.g., *Kotlarski et al.*, 2010; *Marshall et al.*, 2011].

[5] *V-A* scaling has theoretical support [*Bahr et al.*, 1997], based on constraints on ice dynamics imposed by ice rheology and the climatic and topographic conditions that typify glacierized regions. Variations in these conditions influence glacier morphology, however, as do ice dynamics (e.g., sliding vs. deformational flow). A glacier's *V-A* relation also depends on the degree of disequilibrium with climate; advancing vs. retreating glaciers can be expected to differ, based on the nature of mass balance and ice dynamical adjustments and associated time lags. *van de Wal and Wild* [2001] and *Radić et al.* [2007] explore the effects of transient glacier evolution on the *V-A* scaling relation. *V-A* relations are not universal, but are expected to be statistically representative for a regional or global ensemble of glaciers. *Meier et al.* [2007] estimate that errors in volume estimation for individual glaciers can exceed 50% under *V-A* scaling, but these uncertainties reduce to 25% for an ensemble of glaciers.

[6] Such deviations from the global *V-A* relation are not random, and it may be possible to constrain these through knowledge of glacier-specific geometric and climatic settings. We posit that appropriate class-specific values of *V-A* scaling parameters can be introduced based on minimal and readily-available information about glacier morphology. We derive these parameters through examination of *V-A* relations with a finite-element, Stokes model of glacier dynamics [*Adhikari and Marshall*, 2012] under different physiographic conditions, including the effects of glacier disequilibrium. This study is an extension of the flowline modeling analysis of *Radić et al.* [2007], using a high-order 3D flow model and with a comprehensive examination of different glaciological conditions. Our primary aims are (i) to revisit the equilibrium scaling parameters, (ii) to understand the effects of glacier disequilibrium, and (iii) to explore geometry-specific scaling

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Table 1. Summary of Power-Law Relation Between Ice Volume and Glacier Area[a]

	Best-Fit		Constrained (c = 0.027)		
	c	γ	γ	R^2	Area (km^2) $\in$
		Geometric Parameters			
Lateral aspect ratio	0.132	0.969	1.517	0.64	[7.7, 34.7]
Bedrock slope	0.037	1.510	1.632	0.99	[2.1, 30.9]
Roughness elements	0.003, 0.020	2.52, 1.78	1.66, 1.65	0.85, 0.98	[10.8, 13.4]
Valley meander	0.008	2.169	1.664	0.95	[10.8, 16.0]
Variable width elements	0.108, 0.127	1.05, 0.99	1.47, 1.48	0.83, 0.75	[10.8, 43.6]
		Basal Flow Parameters			
Sliding vs. deformation	0.002	2.701	1.591	0.83	[7.4, 10.8]
Stick/slip scenario	0.005	2.371	1.618	0.89	[9.7, 10.8]
		Climatic Parameters			
Accumulation zone	0.071	1.231	1.631	0.88	[4.5, 19.0]
Mass at the glacier head	0.004	2.503	1.623	0.89	[8.6, 11.9]

[a]R^2 > 0.99 in best-fit regressions for all control variables. Values for roughness and variable width elements represent results from perturbations of two different variables in each case (auxiliary material).

parameters that can be recommended for glacier volume estimates and glacier-climate modeling.

2. *V-A* Relation in Steady States

[7] We design numerical experiments covering a broad range of valley geometries, basal conditions and mass balance profiles. We isolate the key parameters and vary them one at a time, initially, allowing control experiments for *V-A* relations in steady states as a function of different glaciological conditions. We then randomly vary free parameters in conjunction for a series of 280 experiments. The glaciological settings are described in detail in the auxiliary material and are summarized in Table 1.[1]

[8] We grow glaciers by simulating the ice-flow model long enough to achieve a steady state. Steady-state ice volume, V, and glacier surface area, A, are recorded in each case. For an ensemble of glaciers, we plot V vs. A and fit the data through regression analysis to establish the power-law relation [e.g., *Macheret et al.*, 1988; *Bahr et al.*, 1997],

$$V = cA^{\gamma}, \tag{1}$$

where c and γ are the power-law coefficient and exponent, respectively. Coefficient c characterizes the magnitude of V for a unit area of glacier and γ determines the degree by which V scales with A. Studies to date recommend $\gamma \in [1.3, 1.6]$ [e.g., *Chen and Ohmura*, 1990; *Radić et al.*, 2007], indicating a nonlinear change in V with A. Note that c has units – km$^{(3-2\gamma)}$ – that vary with associated values of γ, so we do not mention them explicitly.

2.1. Role of Glaciological Parameters

[9] In our control experiments, we consider an ensemble of at least ten glaciers for each glaciological variable and calculate *V-A* scaling parameters (Table 1). These experiments yield $c \in [0.002, 0.132]$ and $\gamma \in [0.97, 2.70]$. There is considerable play and tradeoff between the free parameters c and γ; where one is low in the regression, the other tends to be high. This can be understood by considering the change of V as a function of A,

$$\frac{dV}{dA} = c\gamma A^{\gamma-1}. \tag{2}$$

The product $c\gamma$ indicates the magnitude of volume change resulting from a change in glacier area. This product exhibits less variability than either parameter on its own, because variations in c and γ largely compensate for each other. Perhaps more insightful is the exponent ($\gamma - 1$), which indicates the sensitivity of dV/dA to glacier area. Where $\gamma \approx 1$, $dV/dA \approx c\gamma$ (a constant). This is the case for geometric parameters associated with glacier width, implying that if the glaciological settings of two glaciers differ only with respect to width characteristics, glacier volume is linearly proportional to area.

[10] Most other glaciological parameters indicate a stronger dependence of V on A, indicating comparatively large changes in volume per unit change in glacier area. Bed roughness elements and basal flow have particularly high exponents. This is physically intuitive if one thinks of glacier advance or retreat into a bedrock overdeepening or over a ridge; small changes in area give relatively large changes in ice thickness. Similarly, high rates of basal flow give steady-state glaciers that have moderately less area along with disproportionate reductions in glacier volume (i.e., ice thinning): hence the highest *V-A* sensitivity in our tests, γ = 2.70. Ice volumes are also strongly dependent on glacier area as the magnitude of mass balance varies. This can be interpreted as a stronger volume sensitivity of maritime (vs. continental) glaciers; increases in mass balance increase V in proportion to A. Other parameters can be similarly interpreted, though note that some of the results may be specific to the form of our geometry and mass balance prescription.

[11] Many of the exponents in our experiments (Table 1) are unusual in the context of previous literature [e.g., *Chen and Ohmura*, 1990; *Radić et al.*, 2007]. This is primarily due to the tradeoff between c and γ in the *V-A* regressions. We therefore repeat the regression analysis with the coefficient c constrained to the 'global' value revealed by our random experiments (Section 2.2), 0.027, which is extremely close to the value c = 0.028 found by *Chen and Ohmura*

[1]Auxiliary materials are available in the HTML. doi:10.1029/2012GL052712.

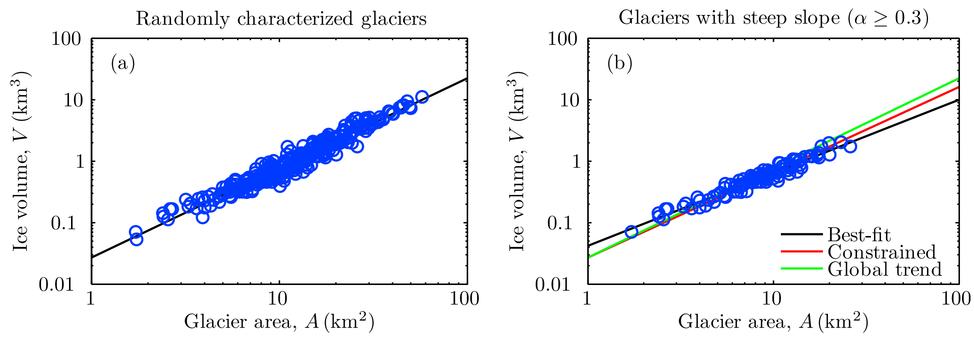

Figure 1. Glacier *V-A* relation in steady states. (a) Ice volume vs. glacier area for randomly characterized 280 glaciers. We find c = 0.027 and γ = 1.458, with R^2 = 0.95. (b) Analogous plot for a subset of 100 glaciers with steep slope. Values of c and γ are listed in Table 2 (slope class C). For comparison, we plot both best-fit and constrained (with c = 0.027) trend lines, along with the one for global sample (Figure 1a).

[1990] for real glaciers. This produces more typical values of $\gamma \in [1.47, 1.66]$, illustrating the same qualitative results in the series of experiments (with respect to low vs. high values of γ). However, in several cases the constrained regressions are very weak, with low R^2 values. This is particularly true when variables exhibiting linear or sublinear *V-A* relations are forced to comply with the global value of c.

2.2. Randomly Characterized Glaciers

[12] We define 280 unique valley glaciers that cover a broad range of glacier size, with $V \in [0.05, 11.11]$ km^3 and $A \in [1.72, 57.45]$ km^2. A plot of V vs. A for the complete set of glaciers (Figure 1a) gives the best-fit 'global' scaling parameters c = 0.027 and γ = 1.458, with R^2 = 0.95. These global values express 'compromise' or median values of the regression results from the control experiments. Variables such as basal sliding promote a high exponent; other variables (e.g., width variations) pull γ to lower values.

[13] When the best-fit parameters are used to estimate V for all of the individual glaciers in our 280–member ensemble, through equation (1), the average error in glacier volume is small (2.8%), with a mean absolute error of 18.3%. The interquartile spread of results is also reasonable, with 50% of errors in the range [−14.7, 15.9] %, but errors for individual glaciers can be high: in the range [−47.6, +99.7] %. This is consistent with previous conclusions that globally-calibrated parameters may be appropriate at the scale of large ensembles of glaciers, but caution is warranted in application to individual glaciers.

[14] Our estimate of γ is larger than γ = 1.357 obtained from the observations of ca. 140 real glaciers [*Macheret et al.*, 1988; *Chen and Ohmura*, 1990], although our value of c = 0.027 is essentially equivalent to that of *Chen and Ohmura* [1990]. Because real glaciers are not in equilibrium, we examine how γ changes in transient states in Section 3.

[15] Our estimate of γ is also larger than the theoretical steady-state value calculated by *Bahr et al.* [1997], γ = 1.375. This difference in γ can be attributed, in part, to the difference in glacier mechanics. In contrast to the Stokes model employed here, *Bahr et al.* [1997] consider the shallow ice approximation (SIA). Because SIA mechanics generally predict thinner ice (i.e., less volume) in valley glaciers compared to Stokes models [e.g., *Adhikari and Marshall*, 2012], V in the latter case scales more strongly with A, giving a larger γ. From an ice dynamics point of view, we therefore argue that the match between γ = 1.375 (SIA, steady state [*Bahr et al.*, 1997]) and γ = 1.357 (real glaciers, transient states [*Chen and Ohmura*, 1990]) is partly coincidence, a result of offsetting biases.

[16] *Radić et al.* [2007] find a value γ = 1.56 based on the results of a numerical (1D, SIA) flowline model for 37 synthetic glaciers with sizes comparable to those in this study. Their value for c = 0.008 is far less than ours, so the tradeoff between c and γ noted above makes it difficult to compare γ values directly. A constrained regression of our 280 glaciers with c = 0.008 gives a higher value of γ = 1.913, but with a weaker fit to the data (R^2 = 0.85). In this constrained regression, the higher γ value may be partially due to the effects of high-order mechanics, as discussed above, and partially due to the fact that our experiments cover a much broader range of glaciological conditions, e.g., 3D geometries (vs. flowline) and glacier sliding (vs. only deformation). More importantly, perhaps, sample size of *Radić et al.* [2007] is not big enough, as we demonstrate (auxiliary material) ca. 200 glaciers are required to produce stable solution of scaling parameters.

2.3. Influence of Glacier Shape, Slope and Size

[17] From the random ensemble of glaciers, we stratify the results as a function of glacier shape, slope and size to examine whether more specific scaling-law recommendations are possible. These represent readily-observed glacier properties, for which an ensemble of real glaciers can be classified. Shape refers to the horizontal (plan-view) aspect ratio of a glacier, $\phi = W/L$, where L is the glacier length measured along the central flowline and W is the mean glacier width, defined as $W = A/L$. For the random ensemble of glaciers, $L \in [1.61, 22.69]$ km, $W \in [0.81, 4.63]$ km, and $\phi \in [0.08, 2.03]$ with a mean value ϕ = 0.40. Low values of ϕ indicate long, narrow valley glaciers, while high values are representative of cirque-style glaciation, with $W > L$ in some cases. Slope, α, is defined as the mean bedrock slope in the principal flow direction; mean surface slope can be considered as a proxy. Size is defined as the glacier surface area. For each of these three characteristics (shape, slope and size), we sort the global glaciers into three classes (Table 2).

[18] We plot V vs. A for each subset of glaciers. A sample plot is shown for glaciers with steep slope (Figure 1b). Scaling parameters c and γ are evaluated for each 'class' of

Table 2. Scaling-Law Parameters for Different Glaciological Settings[a]

	Class A			Class B			Class C		
	c	γ	R^2	c	γ	R^2	c	γ	R^2
Shape, ϕ		≤ 0.25 [100]			0.25–0.67 [148]			≥ 0.67 [32]	
Best-fit	0.0313	1.4486	0.97	0.0353	1.3280	0.94	0.0394	1.1680	0.93
Constrained	0.0271	1.4969	0.97	0.0271	1.4340	0.93	0.0271	1.3474	0.91
Slope, α		≤ 0.15 [45]			0.15–0.3 [128]			≥ 0.3 [100]	
Best-fit	0.0728	1.2162	0.93	0.0336	1.3835	0.94	0.0422	1.1893	0.94
Constrained	0.0271	1.5100	0.87	0.0271	1.4643	0.94	0.0271	1.3881	0.92
Size, A (km^2)		≤ 7.5 [56]			7.5–20 [158]			≥ 20 [60]	
Best-fit	0.0481	1.1439	0.81	0.0237	1.4921	0.79	0.0417	1.3612	0.83
Constrained	0.0271	1.4872	0.74	0.0271	1.4405	0.79	0.0271	1.4887	0.83

[a]Results are listed for both best-fit and constrained regressions. Figures enclosed by brackets indicate the number of glaciers in each class.

glaciers (Table 2), for both best-fit and constrained (c fixed at 0.027) regressions. Exponents from the constrained regressions are more readily compared, as these isolate the nonlinearity of the V-A sensitivity as per equation (2).

[19] Scaling exponents follow consistent trends for the shape- and slope-based classifications, with a decrease in γ as ϕ and α increase. This implies that glacier volume for steep slopes and broad, cirque-type glaciers is less sensitive to changes in glacier area. The best-fit regression parameters give a consistent interpretation for the effects of glacier shape, but are mixed for the slope classifications. Size-based classification does not reveal obvious trends in γ, and also gives low R^2 values for both best-fit and constrained regressions (Table 2). We therefore imbue shape- and slope-based classifications with more significance.

[20] The scaling parameters specific to the corresponding class may provide a refined estimate of glacier ice volume. As an example, the mean and mean absolute errors for individual glacier volume estimates are reduced to 1.5% and 14.4%, respectively, with the shape-based parameters applied to our random ensemble of glaciers (Table 3). This is a moderate but statistically significant improvement relative to the errors associated with the global parameters. Similar gains result from slope-based scaling parameters, while size-based classification produces a slight improvement (Table 3).

3. *V-A* Relation in Transient States

[21] To explore the effects of glacier disequilibrium, we choose a subset of ten steady-state glaciers from the pool of random samples, so that they are consistent with the global values of c and γ. These glaciers also cover the parameter space well: for example, $A \in [4.98, 39.92]$ km^2, $\phi \in [0.17, 0.80]$, and $\alpha \in [0.12, 0.38]$. Mass balance perturbations are imposed to drive each of these glaciers into a transient state. Transient V-A relations are explored under both idealized and realistic climate change scenarios.

3.1. Step Change in Climate

[22] For each glacier, we impose an idealized climatic perturbation through changes in the vertical extent of accumulation zone by one third of its initial magnitude (auxiliary material). Following such a climatic shift, the glacier advances or retreats along an exponential, asymptotic path towards a new steady state. For the given mass balance perturbation, every glacier has its own pace of evolution. We quantify this through the e-folding response time [*Jóhannesson et al.*, 1989] for both the volume, τ_V, and area, τ_A. For these experiments, $\tau_V \in [20.8, 104.8]$ and $\tau_A \in [26.0, 118.9]$ years under glacier advance scenarios, and $\tau_V \in [4.8, 31.5]$ and $\tau_A \in [7.9, 42.5]$ years under glacier retreat.

[23] For systematic exploration of the V-A relation in transient states, it is useful to identify the degree of glacier disequilibrium. Let ΔV_{ss} be the total change in ice volume between the initial and final steady states. On the basis of $\Delta V(t)$, we characterize the degree of glacier disequilibrium at a given time. Degrees of disequilibrium are scaled between zero (initial) and unity (final steady-state), through $V_{adj} = \Delta V(t)/\Delta V_{ss}$. Intermediate values represent transient states; a glacier with $V_{adj} = 0.2$, for example, indicates that the glacier has gone through only 20% of its volume adjustment. Unlike in previous studies [e.g., *Radić et al.*, 2007] where γ is calculated for individual glaciers through a period of transition, we calculate γ for ensemble of glaciers at different stages of disequilibrium.

[24] For $c = 0.027$, evolution of γ through the various stages of glacier transition are shown in Figure 2a. In the glacier advance scenario, γ increases initially, implying that the glacier accumulates ice volume before it undergoes areal expansion. Because τ_A are 29.3 ± 10.4% longer than τ_V, this lag is consistent and expected between glacier volume and area adjustments. When areal growth occurs at a greater pace in the later stage of glacier transition, γ decreases towards the value representing the final steady state, which may differ from the initial γ. The maximum increment in γ, with respect to the initial steady-state value, is found to be 0.014 ± 0.011 at $V_{adj} = 0.3$.

[25] Glacier retreat scenarios follow a similar evolutionary path. The parameter γ decreases initially due to the fact that the glacier thins before it experiences noticeable areal shrinkage. The lag of areal adjustment is even larger in this case; τ_A values are 50.7 ± 10.0% greater than τ_V. We

Table 3. Mean Errors, $\bar{\epsilon}$, and Mean Absolute Errors, $|\bar{\epsilon}|$, for Steady-State Volume Projection[a]

Scaling Parameters	$\bar{\epsilon}$ (%)	$\lvert\bar{\epsilon}\rvert$ (%)	Error Range (%)
Global	2.78	18.29	[−47.58, +99.69]
Shape-based	1.54	14.38	[−38.03, +57.75]
Slope-based	1.35	13.03	[−32.45, +61.64]
Size-based	1.84	15.72	[−41.64, +70.18]

[a]Volume is computed for all 280 glaciers, using global and class-specific best-fit parameters.

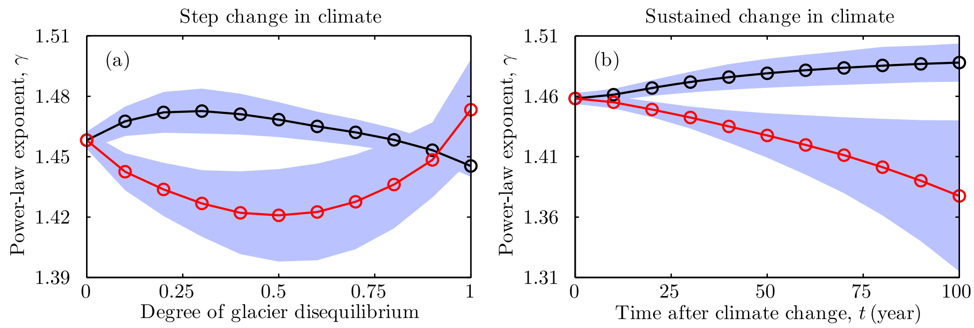

Figure 2. Power-law exponent γ in transient states. (a) Exponent γ at the different stages of glacier growth (black line) and retreat (red). See text for the definition of degree of glacier disequilibrium. (b) Temporal variation of γ in sustained glacier advance (black) and retreat (red) scenarios. The shaded zones define the sample standard deviations.

therefore notice γ decreasing for much of the adjustment period. With respect to the intial steady-state value, the maximum decrement in γ is 0.037 $\pm$ 0.023 at V_{adj} = 0.5, about halfway through the glacier adjustment to its new steady state.

3.2. Sustained Climate Change

[26] Idealized step changes in climate do not provide a realistic representation of recent (20th century) climate change. For the initial steady states, we conduct additional experiments imposing a sustained change in glacier mass balance for 100 years, with balance perturbation $\pm 0.015t$ meters per year, where $t \in [0, 100]$ is the time in years. Unlike before, glaciers do not evolve towards a new steady state. For the ensemble of glaciers, we explore *V-A* relation every ten years after $t = 0$ using the global value of $c = 0.027$.

[27] Due to the shorter timescale of ice volume adjustment, we find a sustained increasing trend of γ in the advance scenario, and a decreasing trend under glacier retreat (Figure 2b). The magnitudes of γ after 100 years are found to be 1.488 $\pm$ 0.016 and 1.377 $\pm$ 0.063 for the glacier advance and retreat scenarios, respectively. The latter magnitude of γ is comparable to γ = 1.357 calculated for real glaciers [*Chen and Ohmura*, 1990]. In the context that the most of the world's glaciers have been retreating for ca. 100 years since their Little Ice Age (LIA) maximum, such a match between γ = 1.377 $\pm$ 0.063 and γ = 1.357 (for equivalent values of c) is notable. This has two major implications: (i) our sample of synthetic glaciers represents the ensemble of real glaciers well, and (ii) the steady-state *V-A* relations discussed in Section 2 may well apply for valley glaciers near the end of the LIA, but overestimate present-day γ values due to glacier disequilibrium. Another implication is that contemporary sea level estimates based on the *V-A* scaling apparently use the correct value of γ = 1.38. As γ evolves, however, one should be careful about using a fixed value for past or future estimates of ice volume.

[28] The net change in power-law exponent, $\Delta\gamma$, can be added into the steady-state γ to roughly account for the effects of a glacier's transient evolution. With respect to the initial steady-state value, γ increases respectively by 0.021 $\pm$ 0.012 and 0.030 $\pm$ 0.016 after 50 and 100 years of climate change in the glacier advance scenario. Corresponding magnitudes of $\Delta\gamma$ in retreat scenarios are -0.031 ± 0.018 and -0.081 ± 0.063. The latter value of $\Delta\gamma$ may characterize the ransient states of the majority of global mountain glaciers that are experiencing sustained retreat, whereas the former value (i.e., $\Delta\gamma \approx -0.03$) is likely to be more suitable for large glacial systems that are in early stages of retreat.

4. Application and Discussion

[29] We test our methods on the scale of a mountain range for a suite of 949 glaciers ($A \in [0.03, 37.98]$ km^2) in the eastern slopes of the Canadian Rocky Mountains [*Marshall et al.*, 2011]. The steady-state parameters, with γ = 1.458, give an ice volume estimate of 48.0 km^3, compared with 40.4 km^3 for γ = 1.375. The latter is the classical equilibrium value [*Bahr et al.*, 1997], but is also close to the value recommended for valley glaciers in a transient state of retreat (Section 3.2). Hence, there is ca. 19% difference in ice volume if one assumes equilibrium. As an illustration of the effects of geometry-specific parameters, shape-based *V-A* scaling (in transient states) yields volume estimate of 43.1 km^3. This produces ca. 7% more ice, a small, but systematic refinement.

[30] The proposed methodology of geometry-specific scaling parameters (Section 2.3) could readily be applied to the global set of mountain glaciers. A multivariate approach should be considered to simultaneously examine the effects of glacier shape, slope and size, all of which can be readily calculated from global glacier inventory and digital elevation data. The parameter space that we cover in this study offers a good representation of the global distribution of valley glaciers, but is not representative of the geometry of ice caps or of tidewater glacier dynamics. This work should be extended to these ice masses before it can be applied to the global glacier inventory, particularly because much of the world's ice volume is stored in the large polar ice caps.

5. Conclusions

[31] We systematically assess the relation between glacier area and volume through synthetic valley glaciers simulated with a 3D Stokes model. These glaciers cover a range of geometric and climatic conditions, including different proportions of sliding vs. high-order deformational flow mechanics. Glaciological parameters have different effects on the *V-A* scaling relation. Volume scales linearly with area in association with variations in glacier width, in contrast with conventional *V-A* scaling relations ($\gamma \in [1.3, 1.6]$) and most other parameters that we tested, for which $\gamma \in [1.2, 2.7]$. In particular, the magnitude of mass balance, presence of bed overdeepenings, and prevalence of basal

sliding give strong *V-A* sensitivities, $\gamma > 2$. Parameters c and γ trade off to give a wider range of values than is seen in the random ensemble of glaciers or in the regressions that are constrained to the global value of $c = 0.027$. We find $\gamma = 1.458$ in steady state for an ensemble of 280 randomly characterized valley glaciers. This value balances the sometimes opposing effects of varying geometric, basal, and climatic conditions.

[32] By imposing climatic perturbations on a subset of glaciers, we also explore the *V-A* relation in transient states. For a chosen value of c, γ increases under glacier advance and decreases for glacier retreat. This is due to the shorter timescale of glacier volume adjustment to a change in climate. Experiments with step-changes in mass balance can be easily interpreted, but *V-A* relations under gradual, ongoing climate change are particularly illuminating. With reference to the initial steady-state value, γ changes by $+0.030 \pm 0.016$ and -0.081 ± 0.063 after 100 years of sustained glacier advance and retreat, respectively. This gives $\gamma = 1.377 \pm 0.063$ after 100 years of glacier retreat, in exceptional agreement with the value $\gamma = 1.357$ that is calculated for the observed sample of real glaciers [*Chen and Ohmura*, 1990] with a comparable value of c. This is consistent with ca. 100 years of sustained warming and glacier retreat experienced by most of the world's glaciers since the end of the LIA. It also indicates that existing *V-A* scaling studies that examine present-day ice volume [e.g., *Radić and Hock*, 2010; *Slangen and van de Wal*, 2011] are fortuitously using an appropriate value of $\gamma = 1.38$, at least for the large host of global valley glaciers that are experiencing sustained retreat. However, we warn that a fixed value of γ may not be a realistic choice for past or future estimates of global or regional ice volume.

[33] Global values of c and γ can give large errors in volume estimates for individual glaciers [e.g., *Meier et al.*, 2007]. Such errors can be reduced systematically through the use of readily-available, glacier-specific morphological information. Shape- and slope-based *V-A* relations yield the greatest improvements. The proposed method of geometry-specific scaling parameters could be applied to estimates of the global ice volume inventory [e.g., *Radić and Hock*, 2010], through characterization of individual glaciers. However, an extension of this method to ice cap geometries is needed prior to the application.

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