

# Testing the importance of explicit glacier dynamics for mountain glacier change projections

Master's Thesis defense

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# Introduction

## 1. Introduction – outlook, motivation and goals

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2. Model definition

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3. Results

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  - ▶ Single glacier test case

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  - ▶ Regional application
  - ▶ Sensitivity analysis
  - ▶ 21st century projections
4. Conclusion

# Introduction

## WHAT did I do?

- ▶ Volume/area scaling model (Marzeion et al. 2012)
- ▶ Flowline model (OGGM Maussion et al. 2019)

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- ▶ Assessing strength and weaknesses of both models

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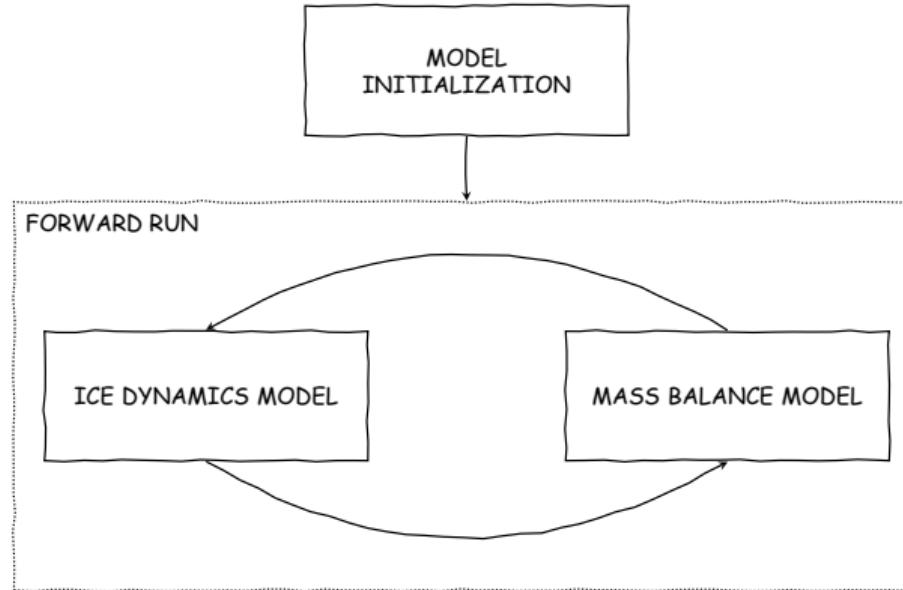
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- ▶ What are the **differences in dynamic response** between the volume/area scaling model and the flowline model?
- ▶ How do those differences **influence ice volume projections for the 21st century** for mountain regions?

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# Volume/area scaling model



# Volume/area scaling model initialization

Hintereisferner - Blick Richtung Weißkugel und Langtaufererspitze  
21.10.20 14:00 3.4°C



[foto-webcam.eu](#)

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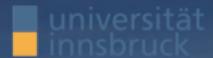
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# Volume/area scaling model initialization

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# Volume/area scaling model initialization

Volume/area scaling relation:

$$V_0 = c_A \cdot A_0^\gamma$$

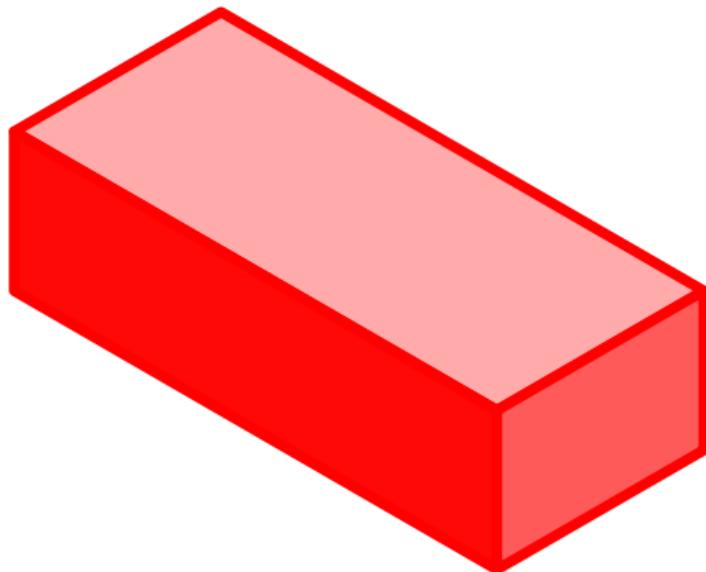
Volume/length scaling relation:

$$V = c_L \cdot L^q$$

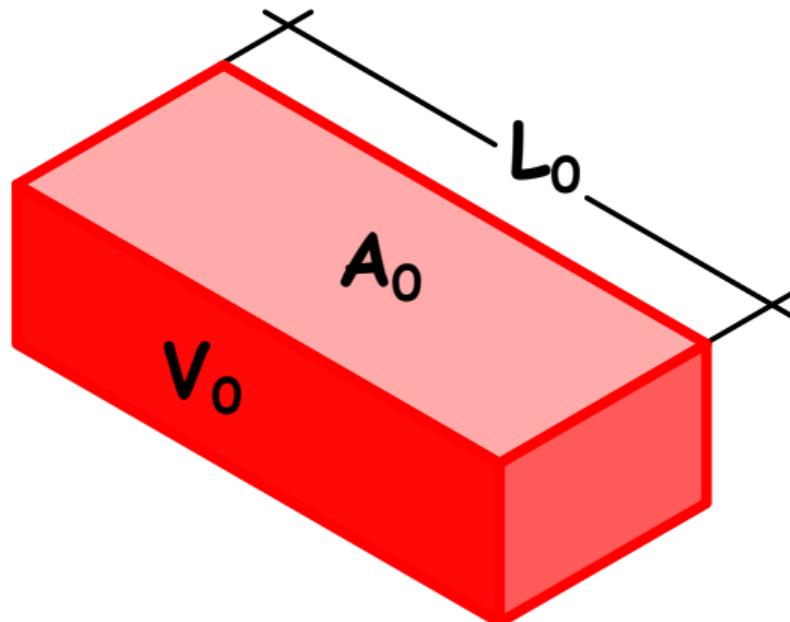
Inverted volume/length scaling relation:

$$L_0 = \left( \frac{V_0}{c_L} \right)^{1/q}$$

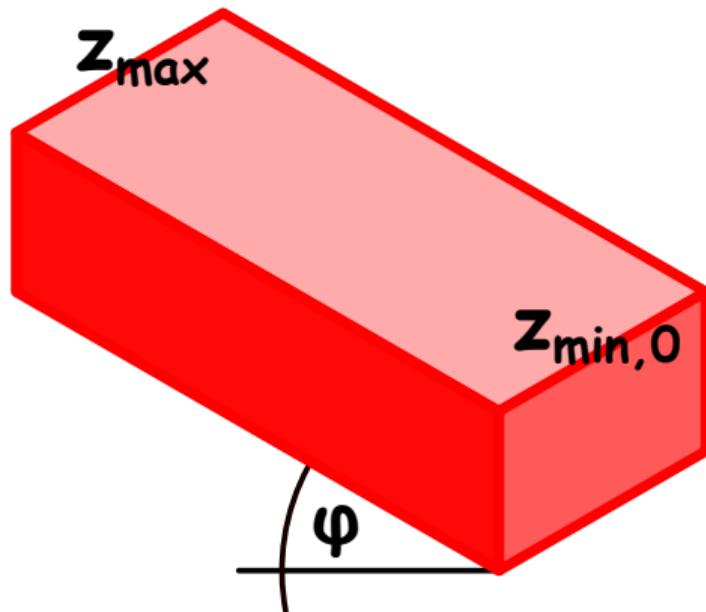
## Volume/area scaling model initialization



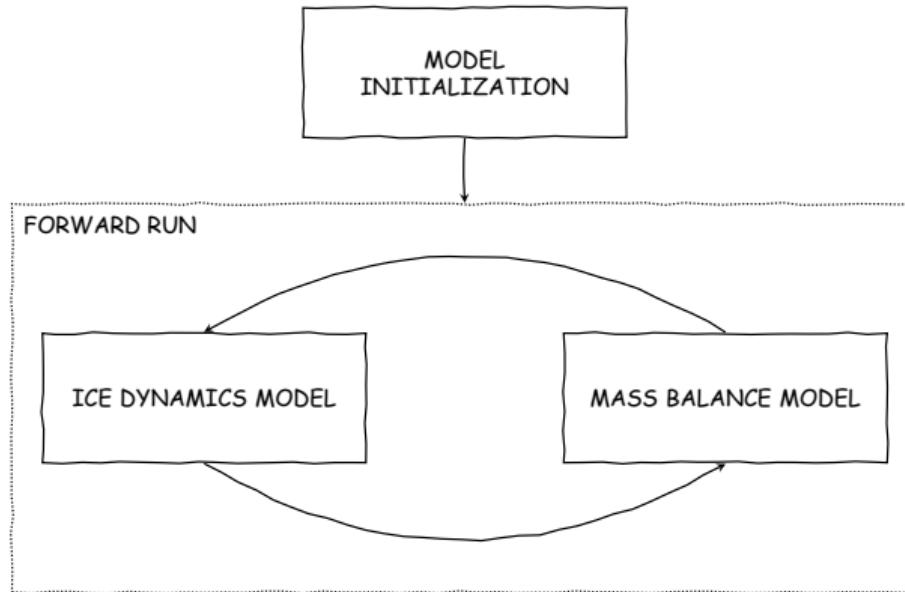
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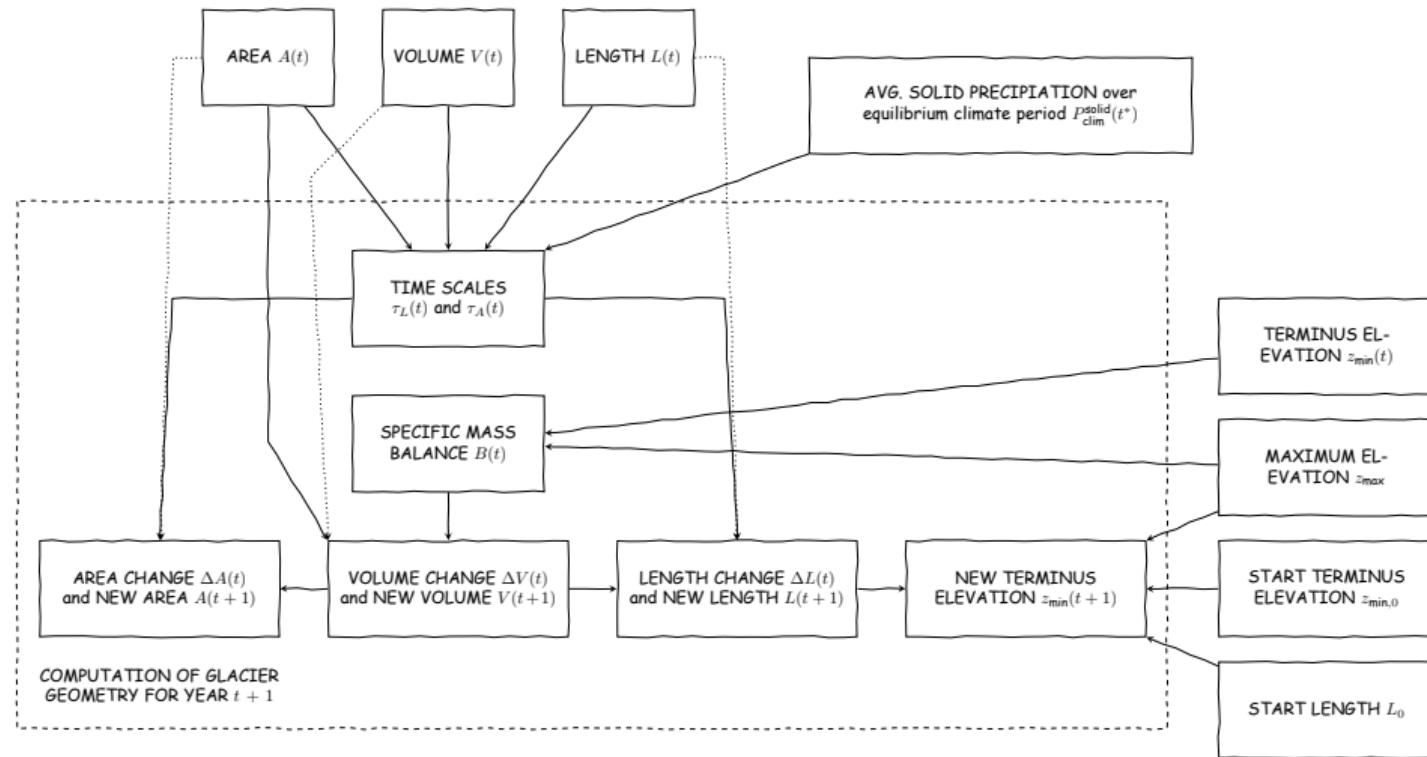
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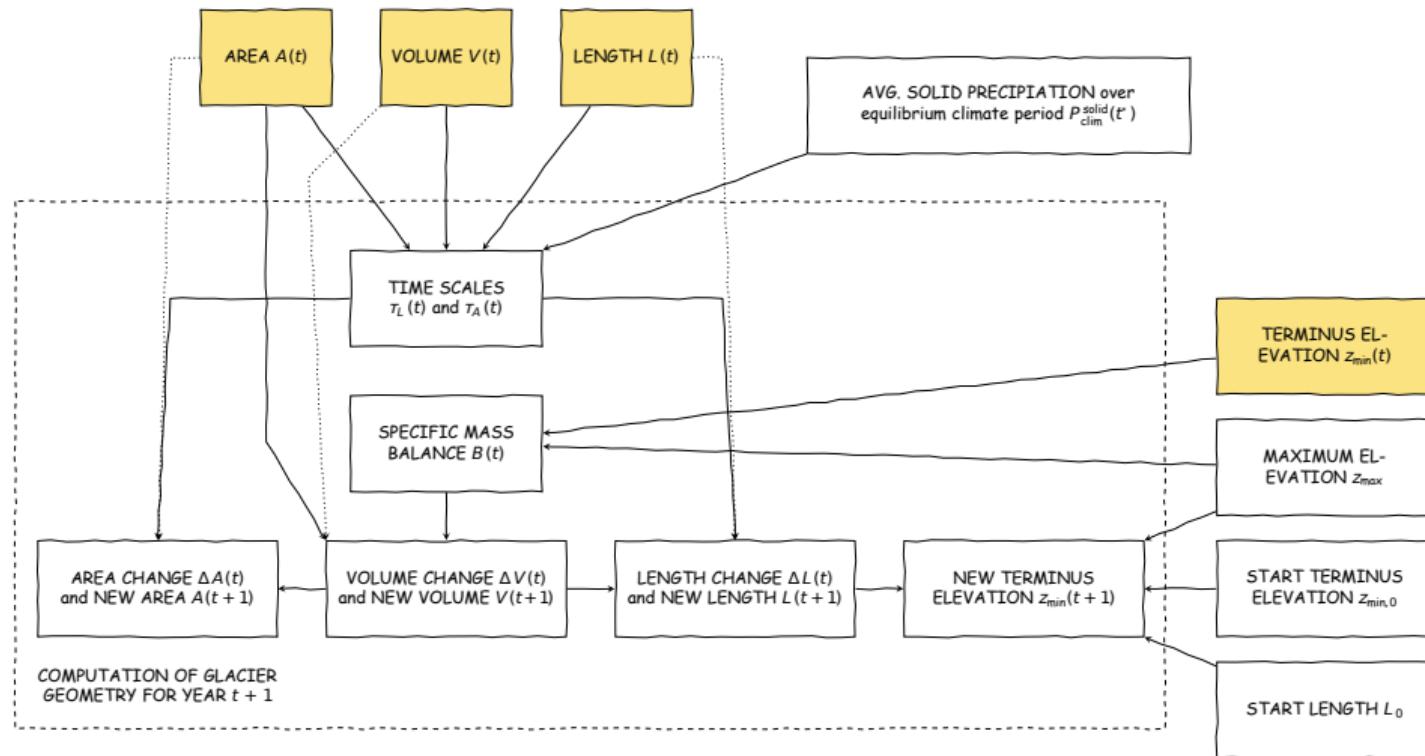
# Forward run



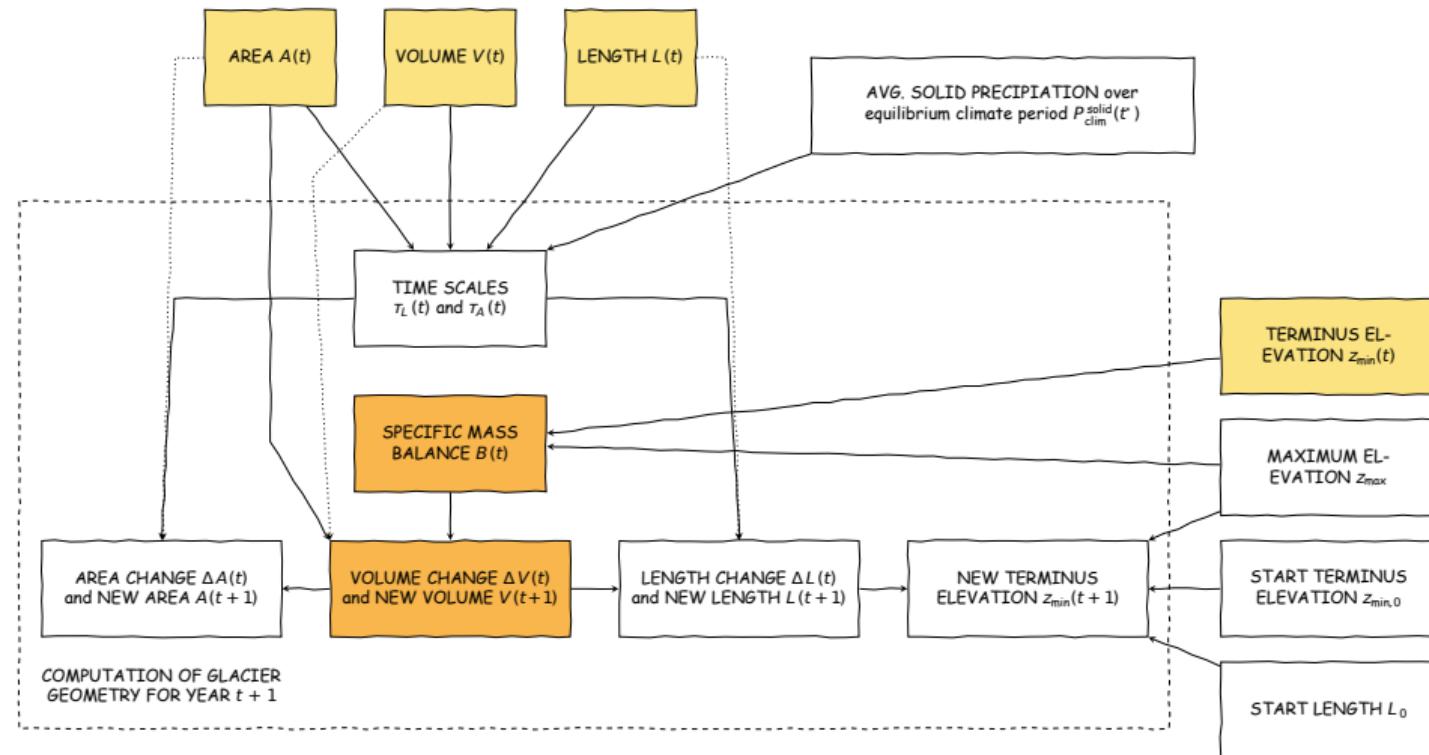
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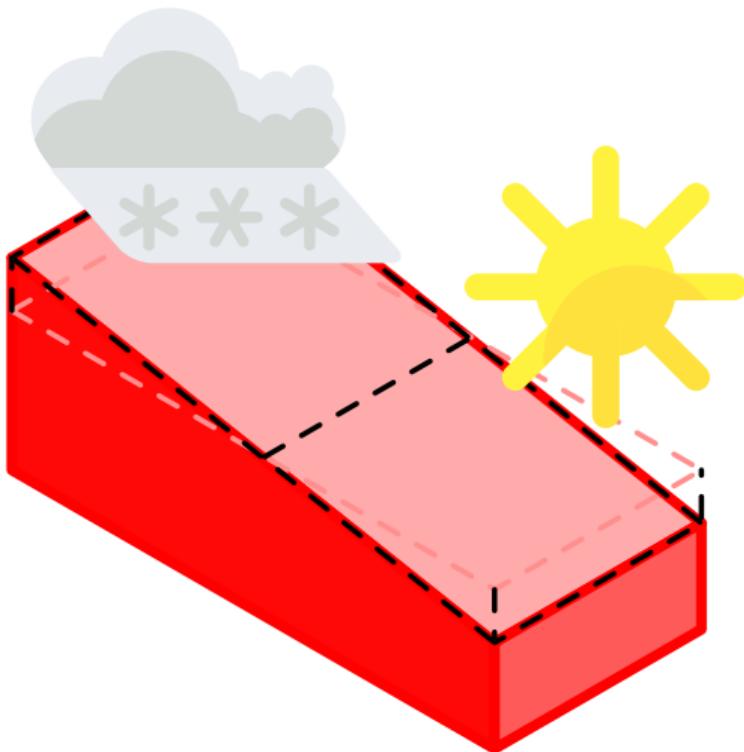
# Forward run



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# Mass balance and volume change



## Mass balance and volume change

Specific mass balance [mm we. yr<sup>-1</sup>] equation from Marzeion et al. (2012)

$$B = \left[ \sum_{i=1}^{12} \left[ P_i^{\text{solid}} - \mu^* \cdot \max(T_i^{\text{terminus}} - T_{\text{melt}}, 0) \right] \right] - \beta^*$$

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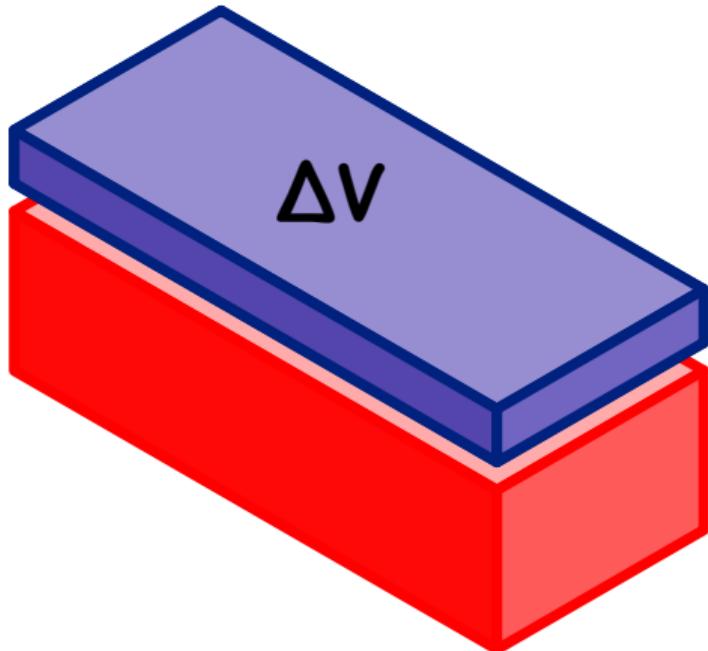
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Volume changes instantaneously

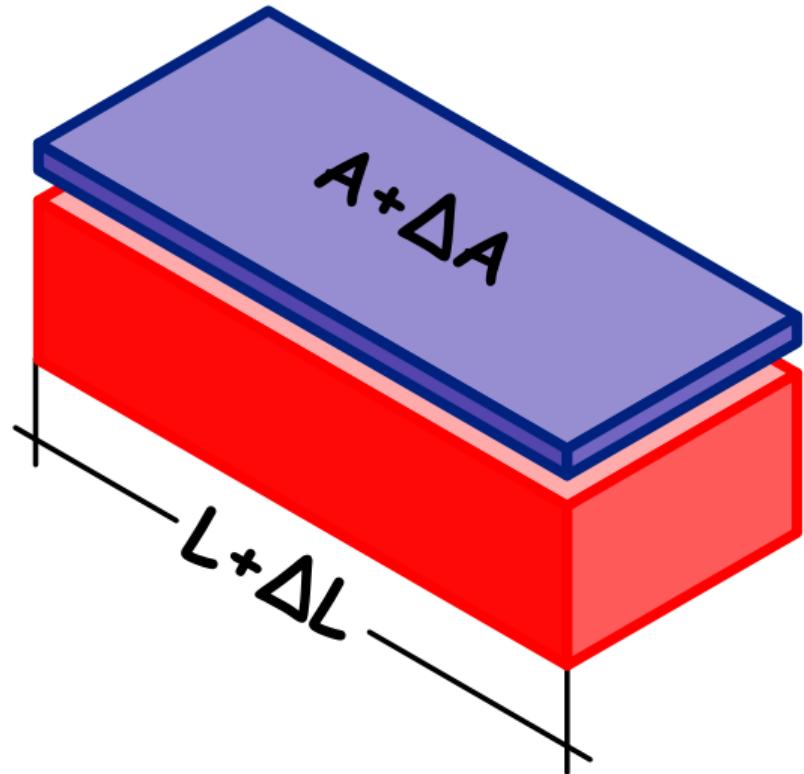
$$\Delta V(t) = \frac{1}{\rho_{\text{ice}}} A(t) \cdot B(t)$$

$$V(t+1) = V(t) + \Delta V(t)$$

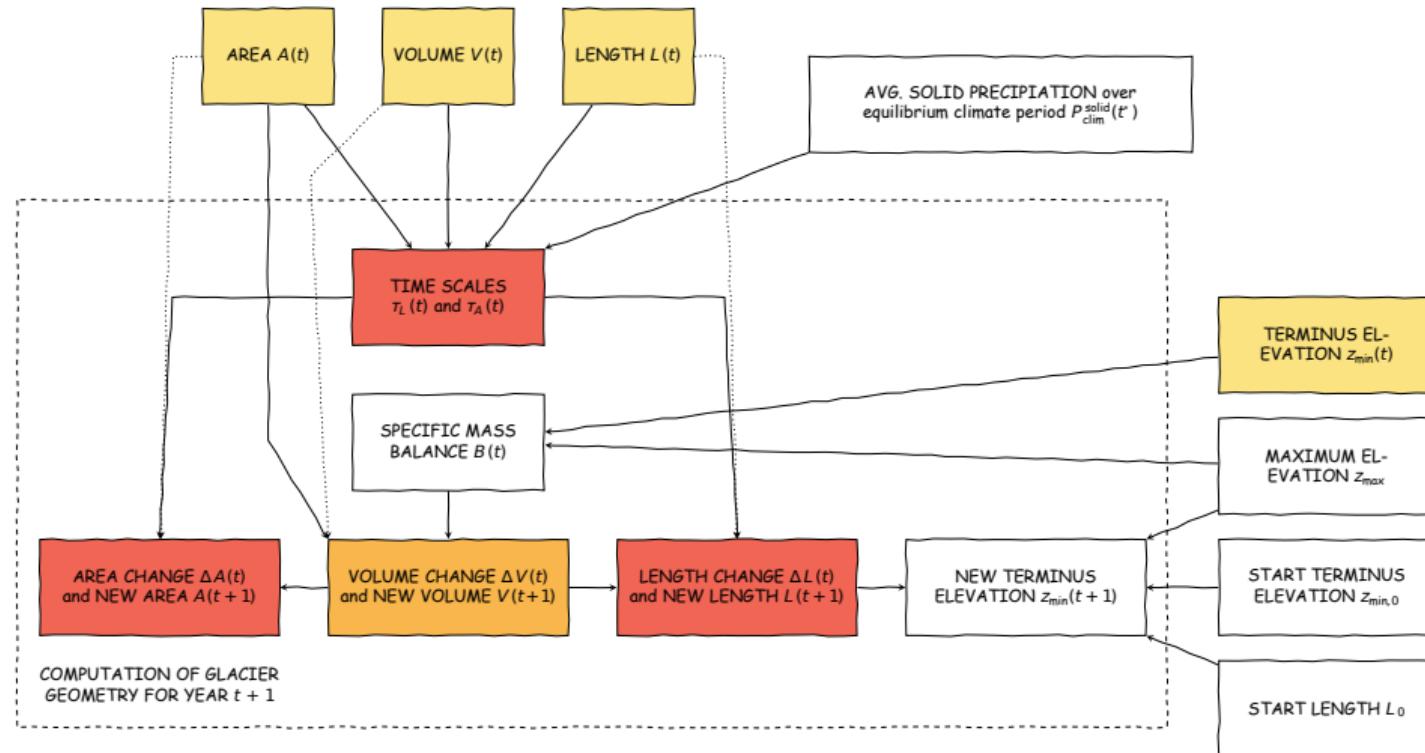
## Mass balance and volume change



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# Area and length changes



# Area and length changes

## Equilibrium changes

$$A_{\text{eq}} = \left( \frac{V(t+1)}{c_A} \right)^{1/\gamma}$$

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Area changes follow **response time scaling**:

$$\Delta A(t) = \frac{1}{\tau_A} \left( \left( \frac{V(t+1)}{c_A} \right)^{1/\gamma} - A(t) \right)$$

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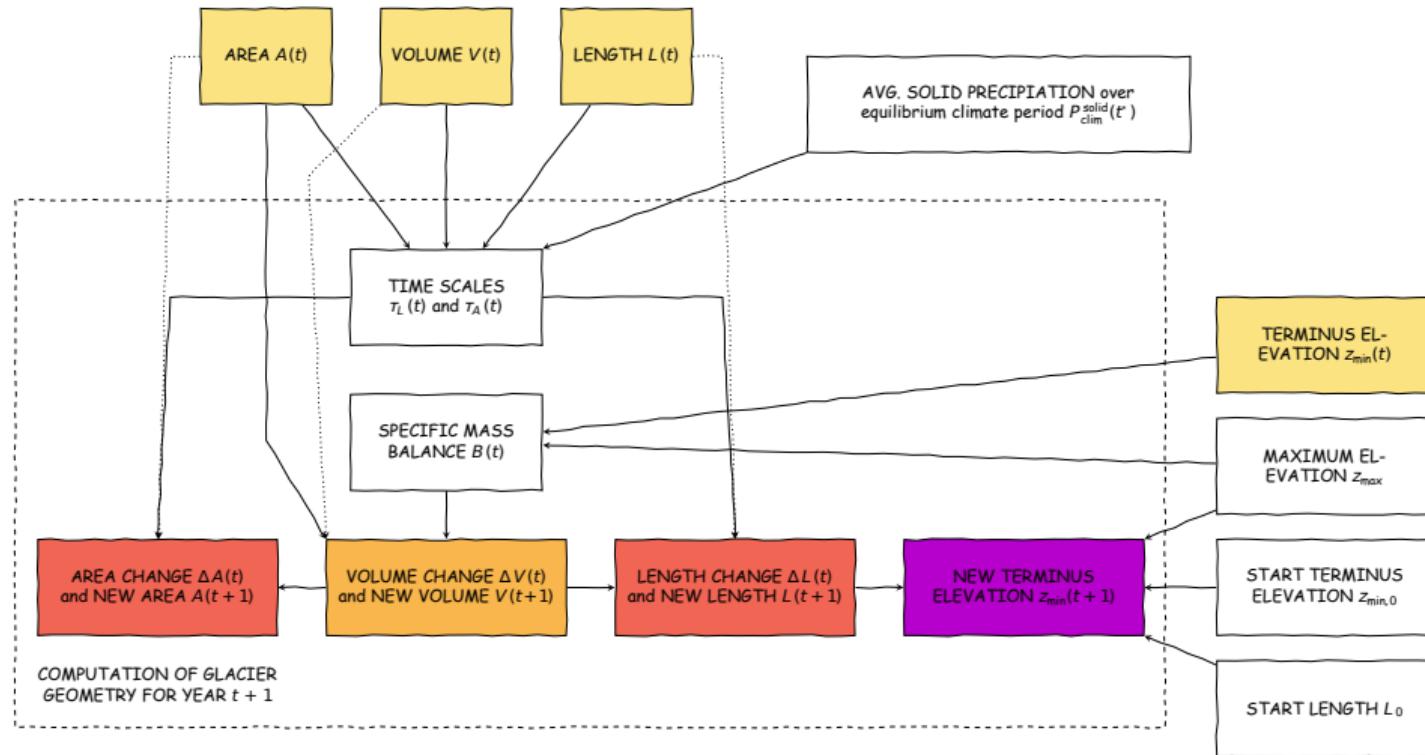
$$L_{\text{eq}} = \left( \frac{V(t+1)}{c_L} \right)^{1/q}$$

Length changes follow **response time scaling**:

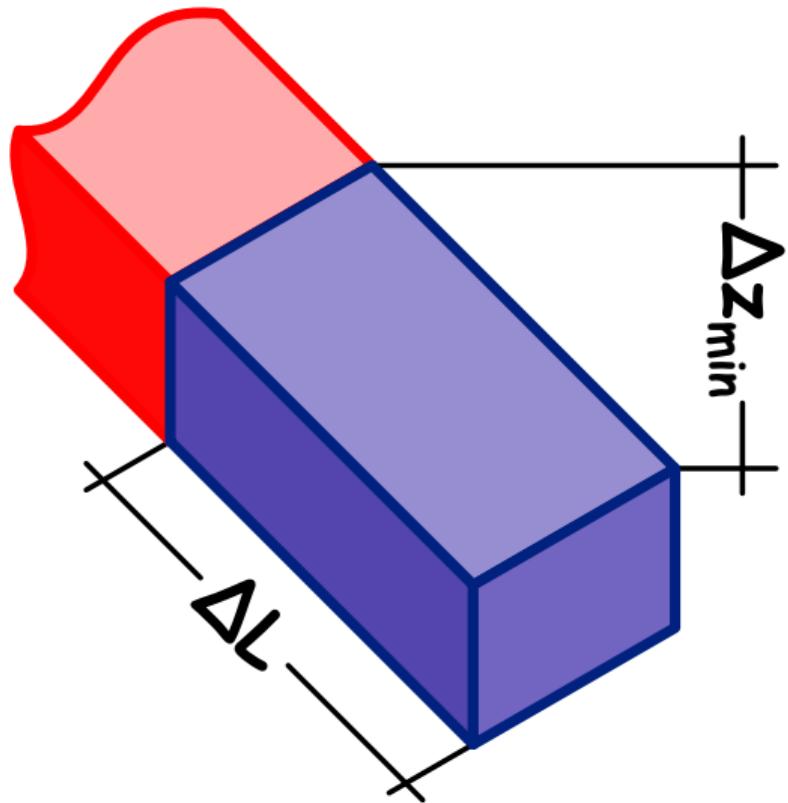
$$\Delta L(t) = \frac{1}{\tau_L} \left( \left( \frac{V(t+1)}{c_L} \right)^{1/q} - L(t) \right)$$

$$L(t+1) = L(t) + \Delta L(t)$$

# Terminus elevation change



## Terminus elevation change



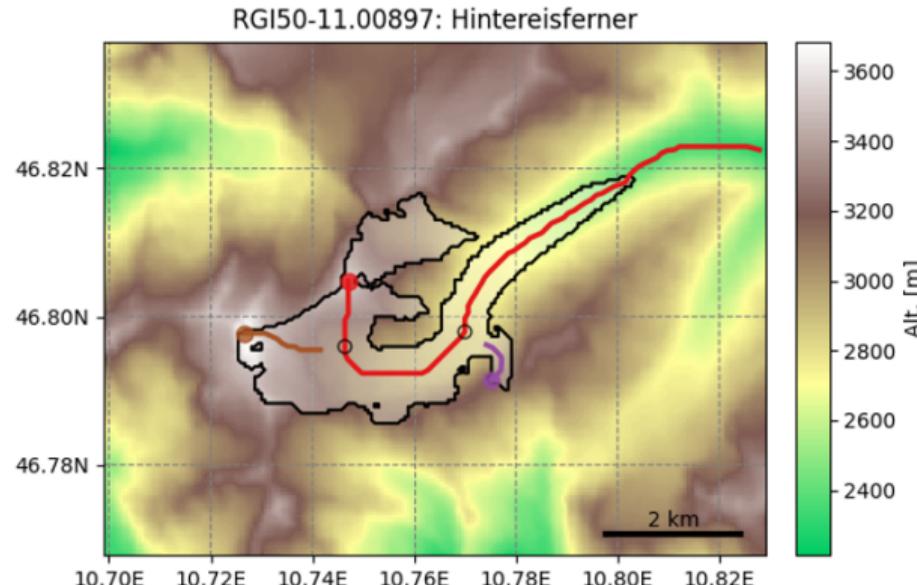
## Flowline model



# Open Global Glacier Model

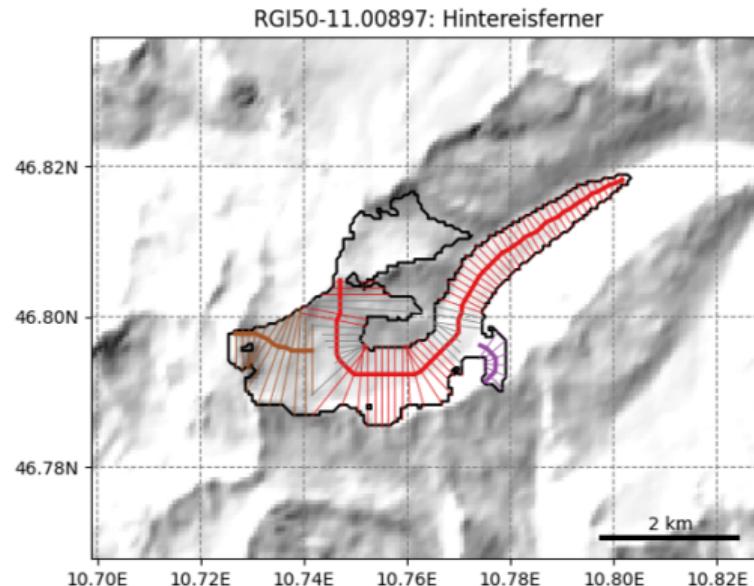
Maussion et al. (2019), documentation under <http://docs.oggm.org>

# Flowline model



Definition of centerlines/flowlines following Kienholz et al. (2014)  
Image source: <http://docs.oggm.org>

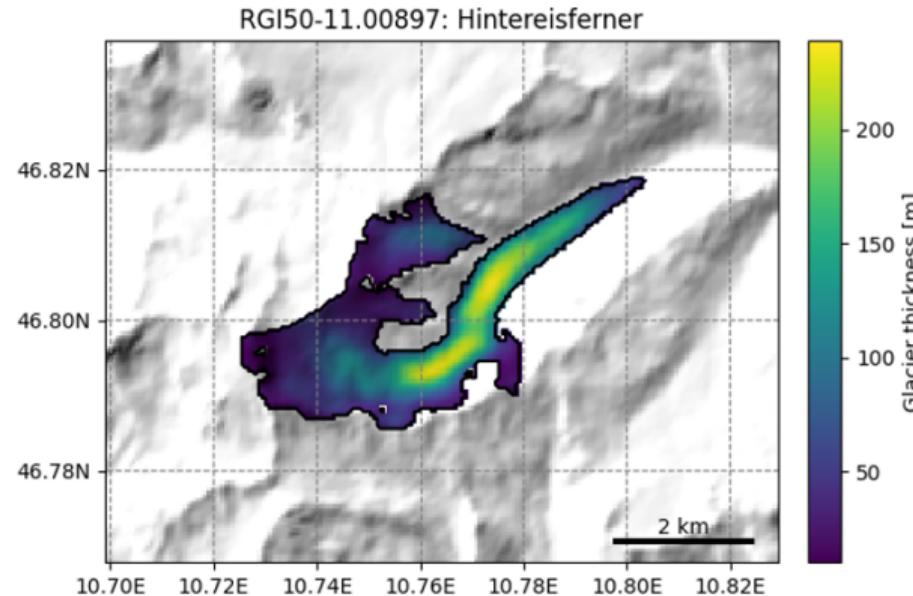
# Flowline model



Flowline width

Image source: <http://docs.oggm.org>

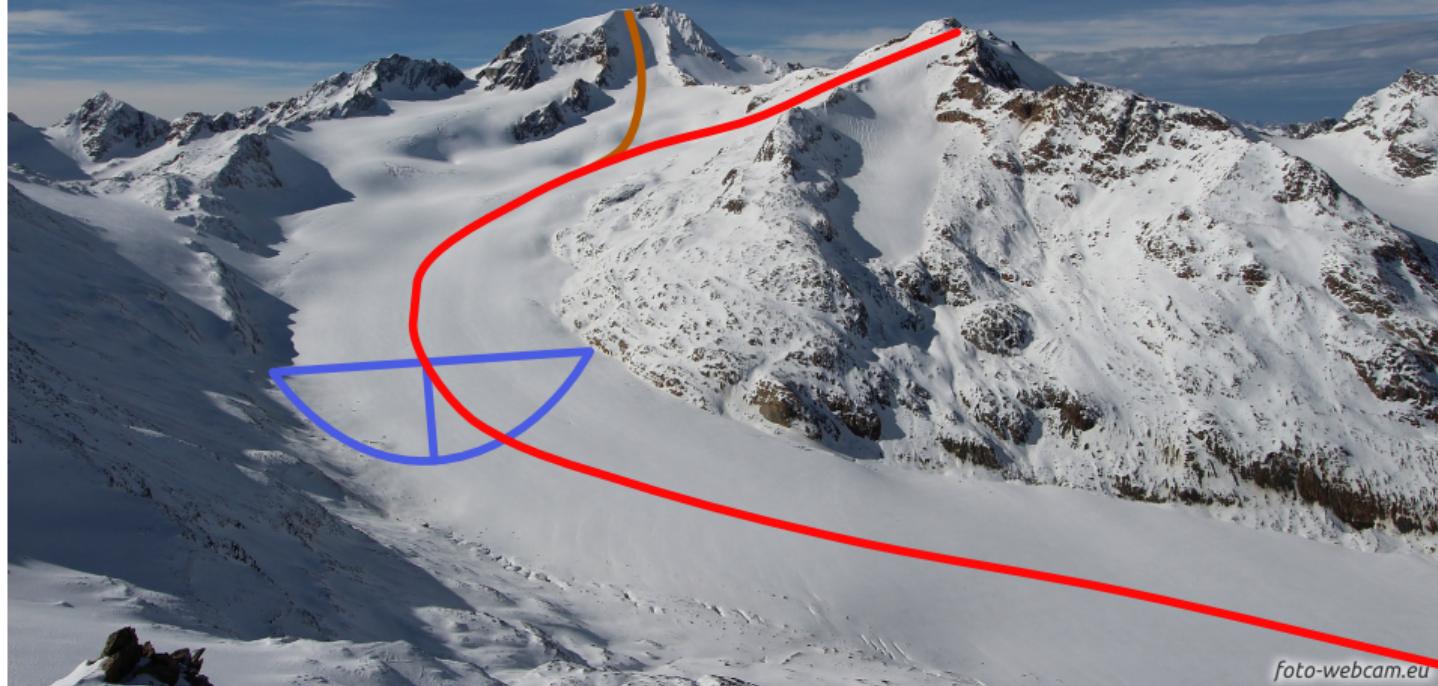
# Flowline model



Ice thickness inversion based on Farinotti et al. (2009)  
Image source: <http://docs.oggm.org>

# Shallow Ice Equations

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21.10.20 14:00 3.4°C



# Shallow Ice Equations

Volume conservation

$$\frac{\partial S}{\partial t} = w\dot{b} - \nabla \cdot q$$

Ice flux

$$q = uS$$

Ice velocity

$$u = f_d h \tau^n + f_s \frac{\tau^n}{h}$$

Deformation parameter

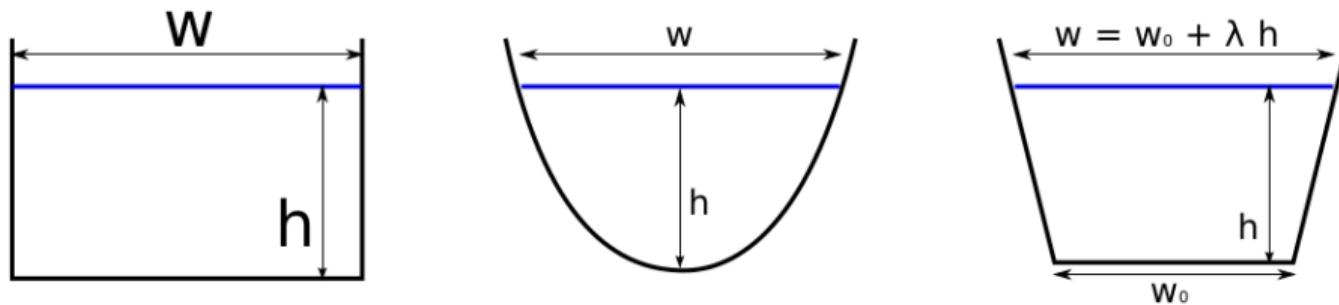
$$f_d = \frac{2A}{n+2}$$

Basal shear stress

$$\tau \approx \tau_d = \alpha \rho_{\text{ice}} g h$$

# Shallow Ice Equations

FluxBasedModel solves for  $\nabla \cdot q$  in the volume conservation equation

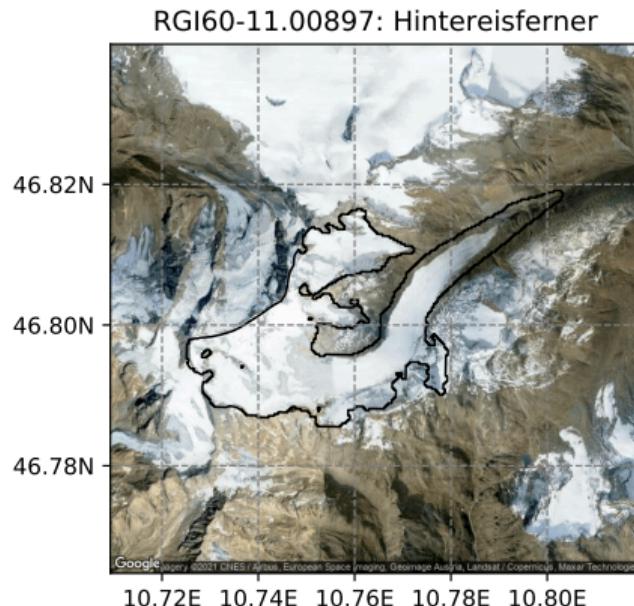


Vertical, parabolic and trapezoidal bed shapes, respectively.

Image source: <http://docs.oggm.org>

# Single glacier test case

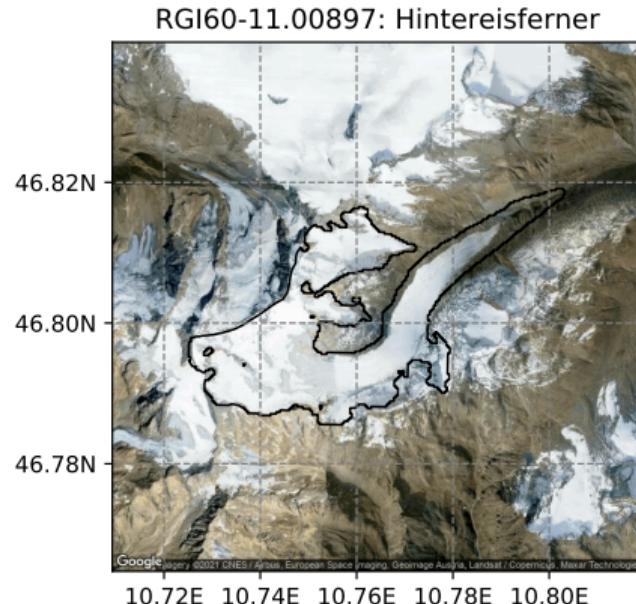
- ▶ Hintereisferner RGI60-11.00897



Satellite image from  
<http://maps.google.com>

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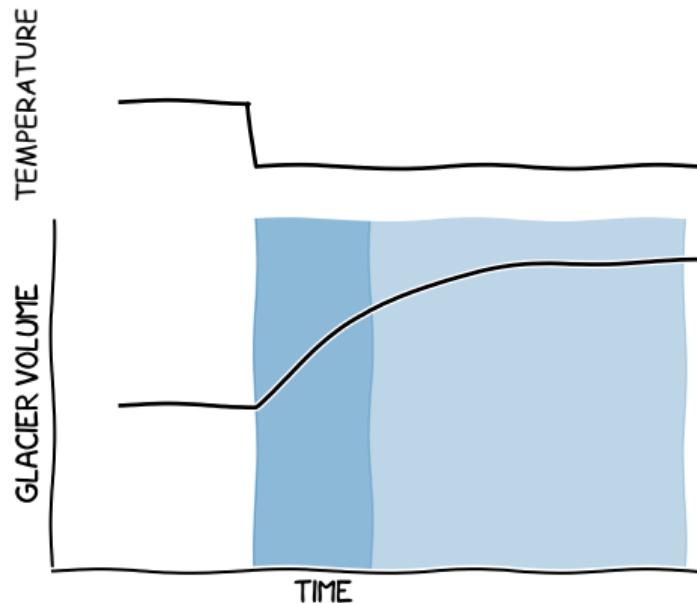
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- ▶ Equilibrium experiments with three temperature biases



Satellite image from  
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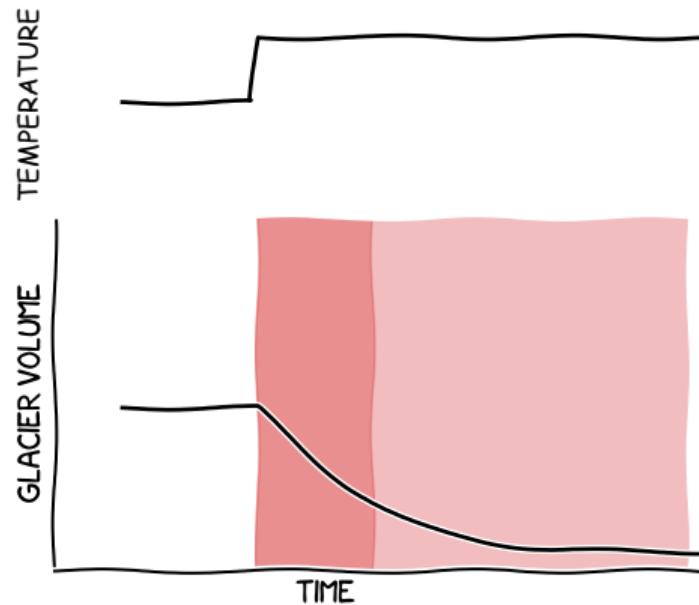
- ▶ Hintereisferner RGI60-11.00897
- ▶ Equilibrium experiments with three temperature biases
  - ▶ Cooling scenario with  $-0.5^{\circ}\text{C}$



Adapted from <http://edu.oggm.org>,  
original author: Fabien Maussion

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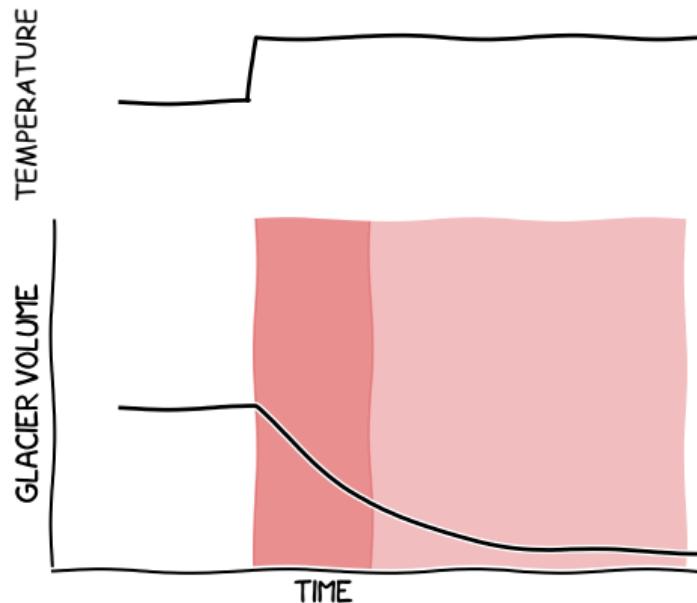
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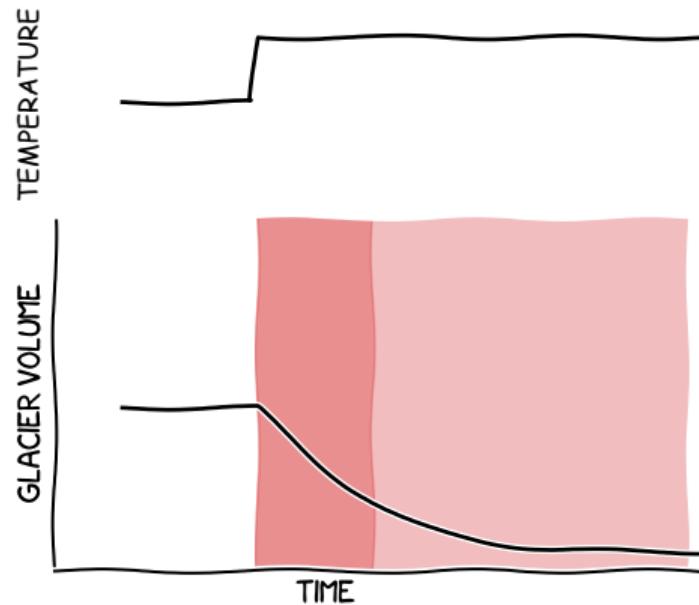
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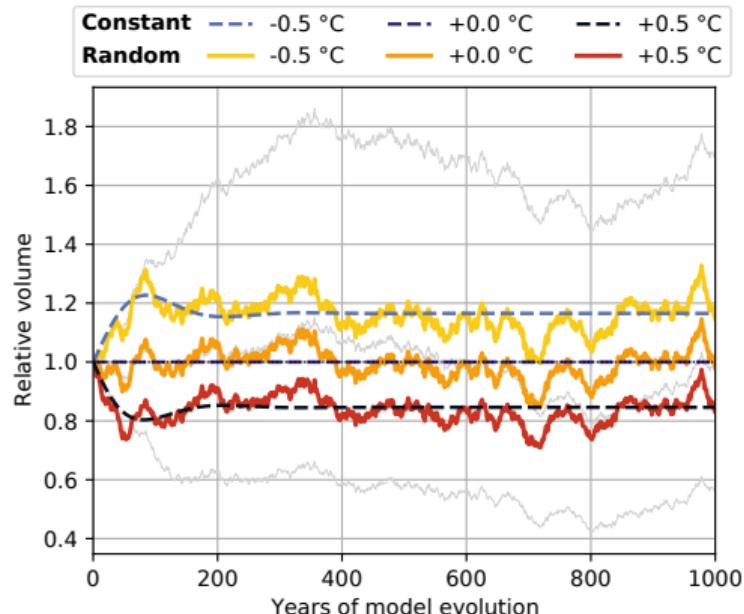
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  - ▶ Equilibrium scenario with  $0^{\circ}\text{C}$
- ▶ Two mass balance models
  - ▶ ConstantMassBalance
  - ▶ RandomMassBalance



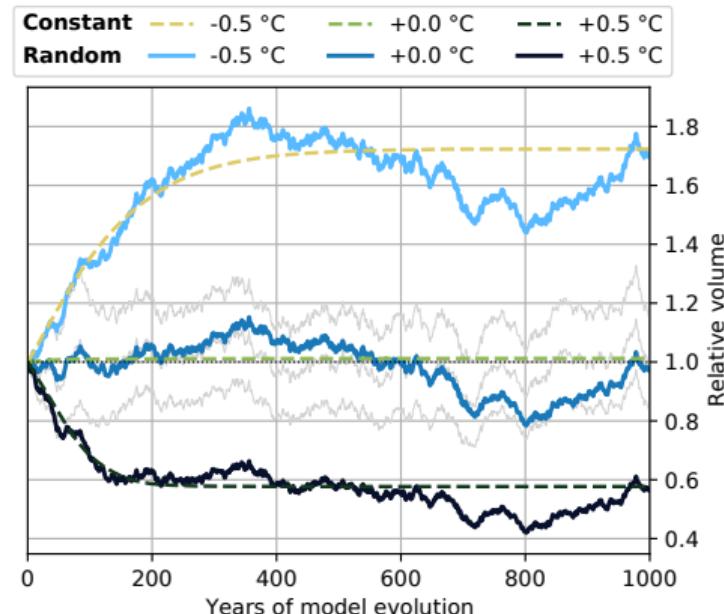
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# Hintereisferner - relative ice volume

Volume/area scaling model

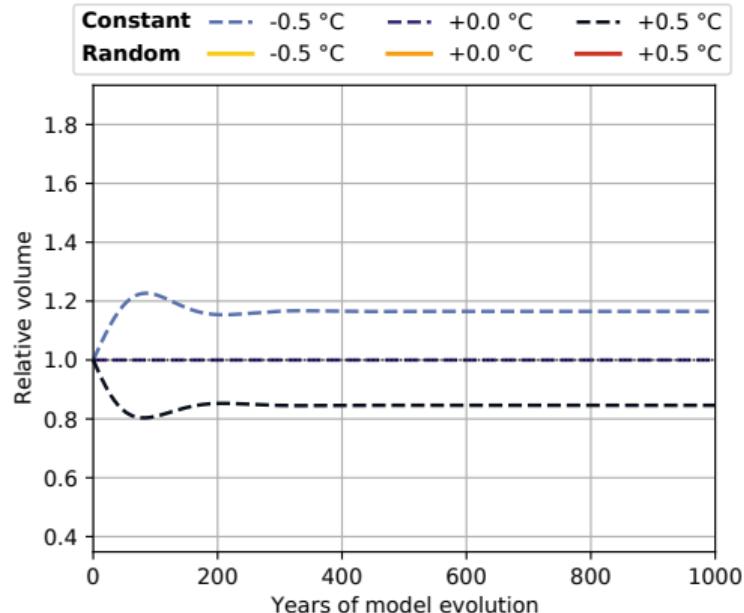


Flowline model

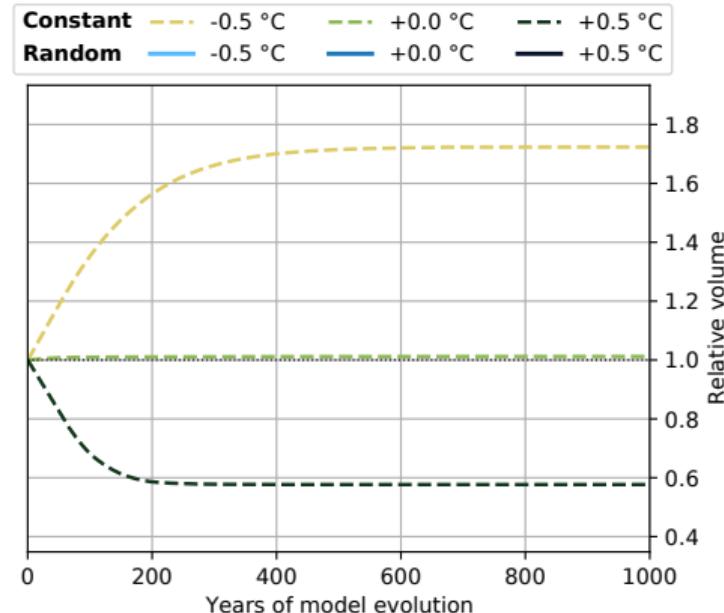


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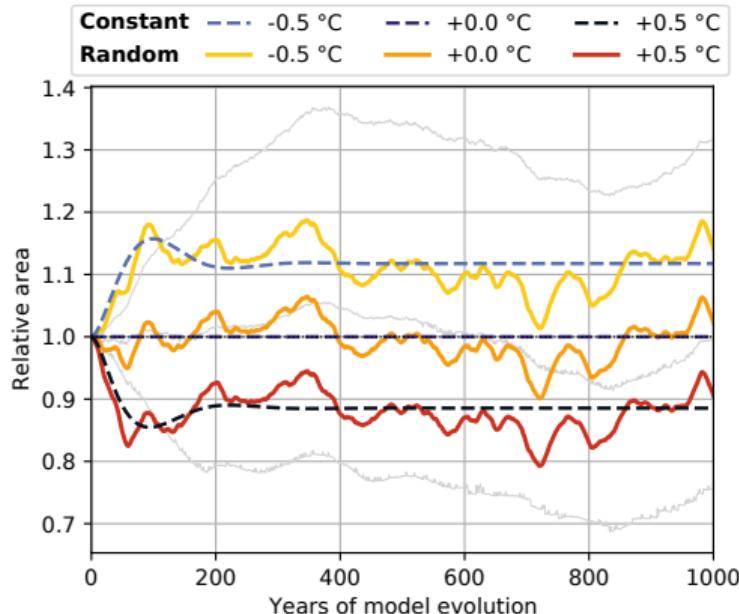


Flowline model

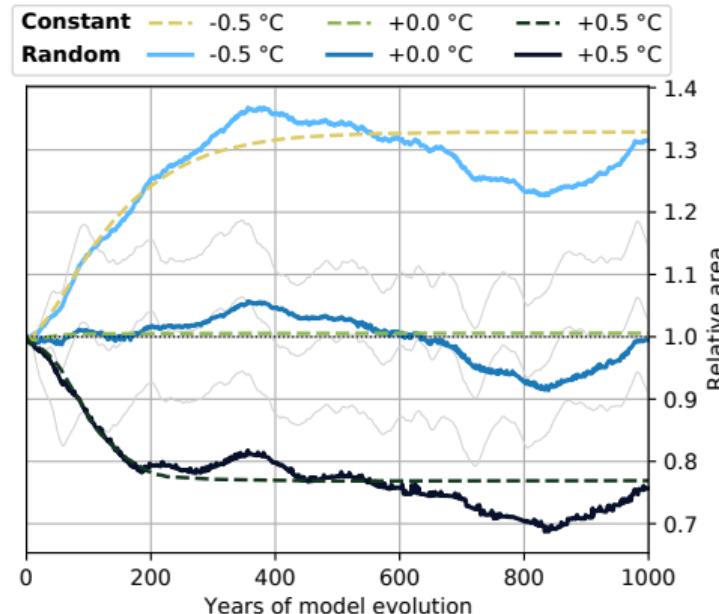


# Hintereisferner - relative surface area

Volume/area scaling model

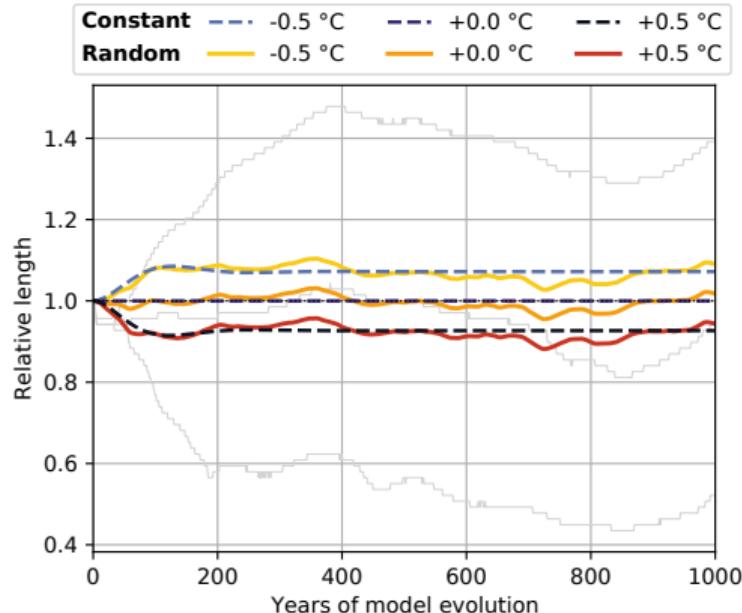


Flowline model

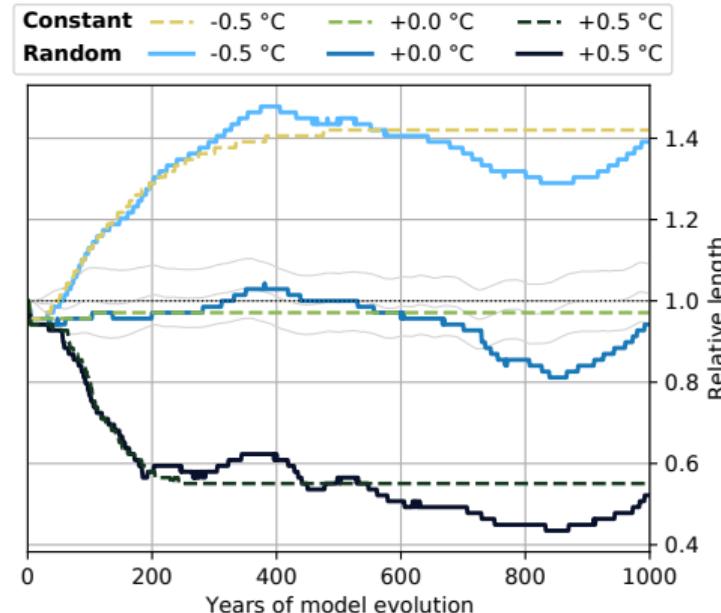


# Hintereisferner - relative glacier length

Volume/area scaling model

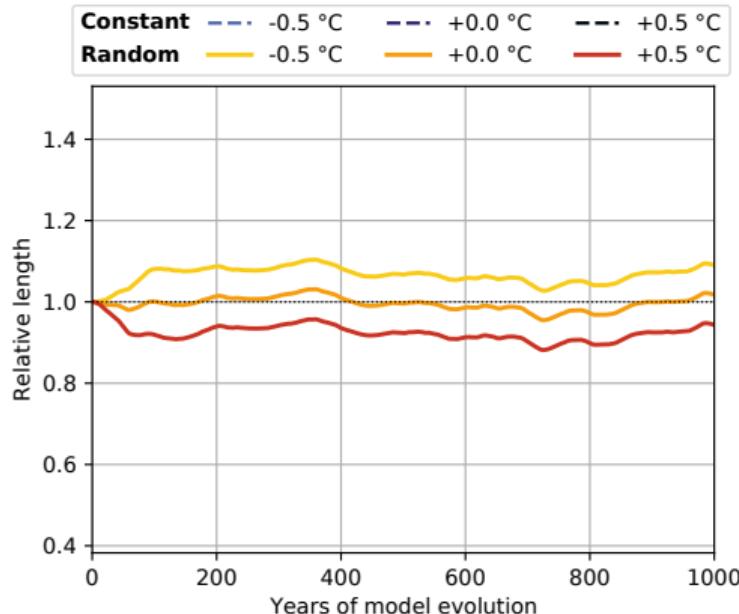


Flowline model

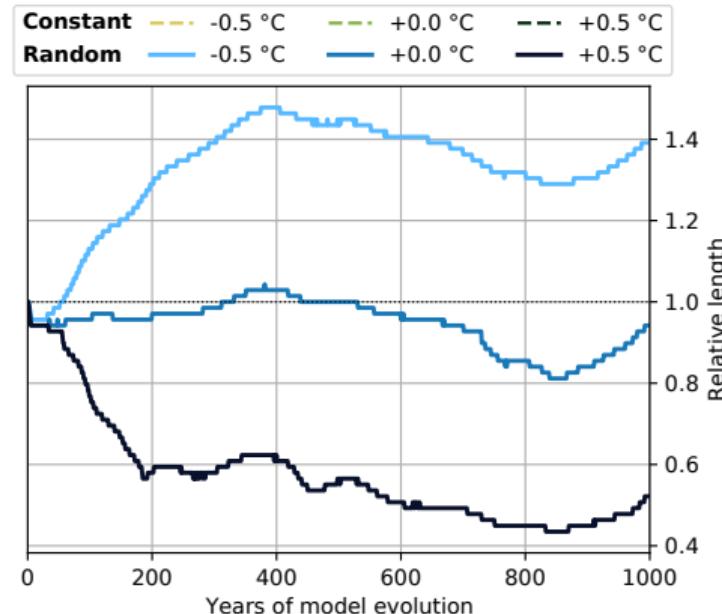


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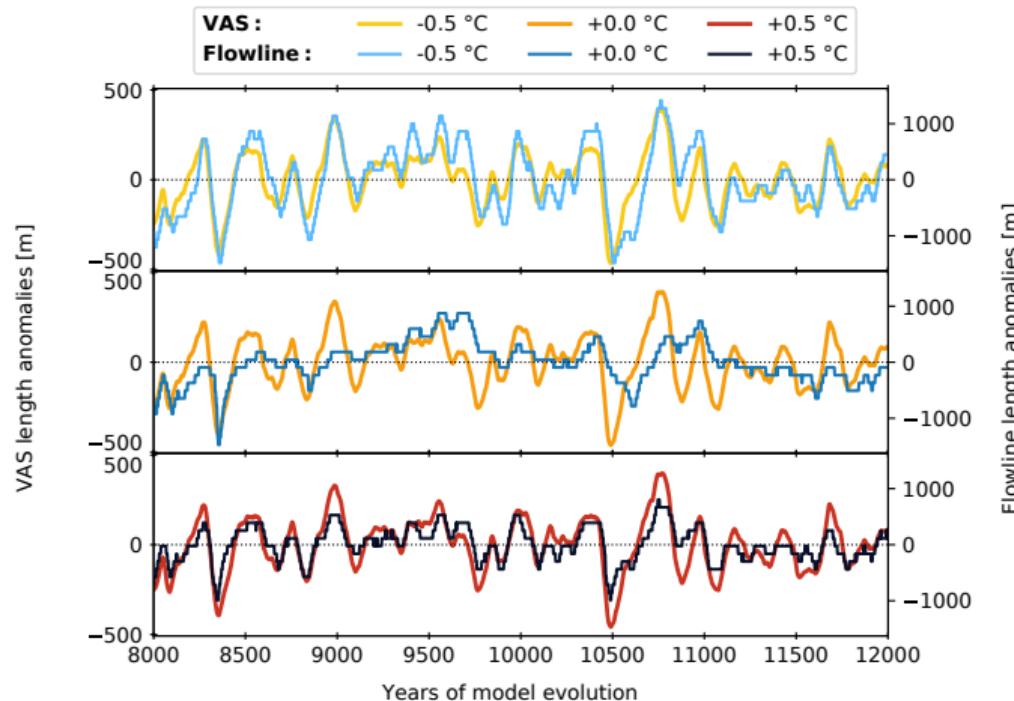
Volume/area scaling model



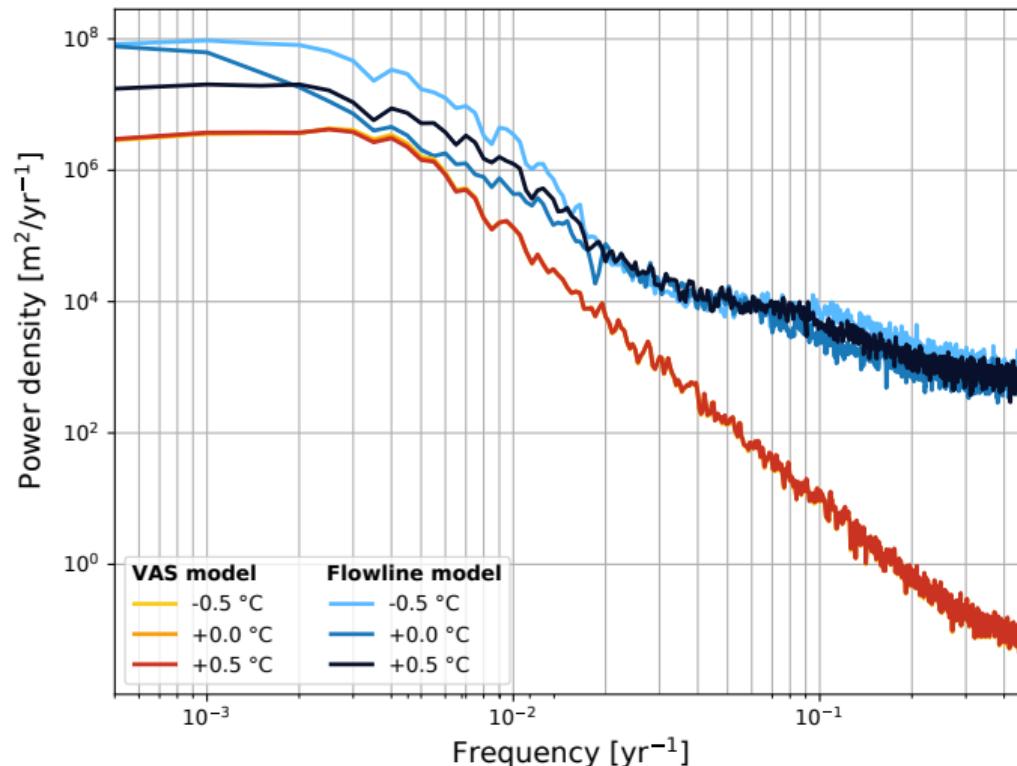
Flowline model



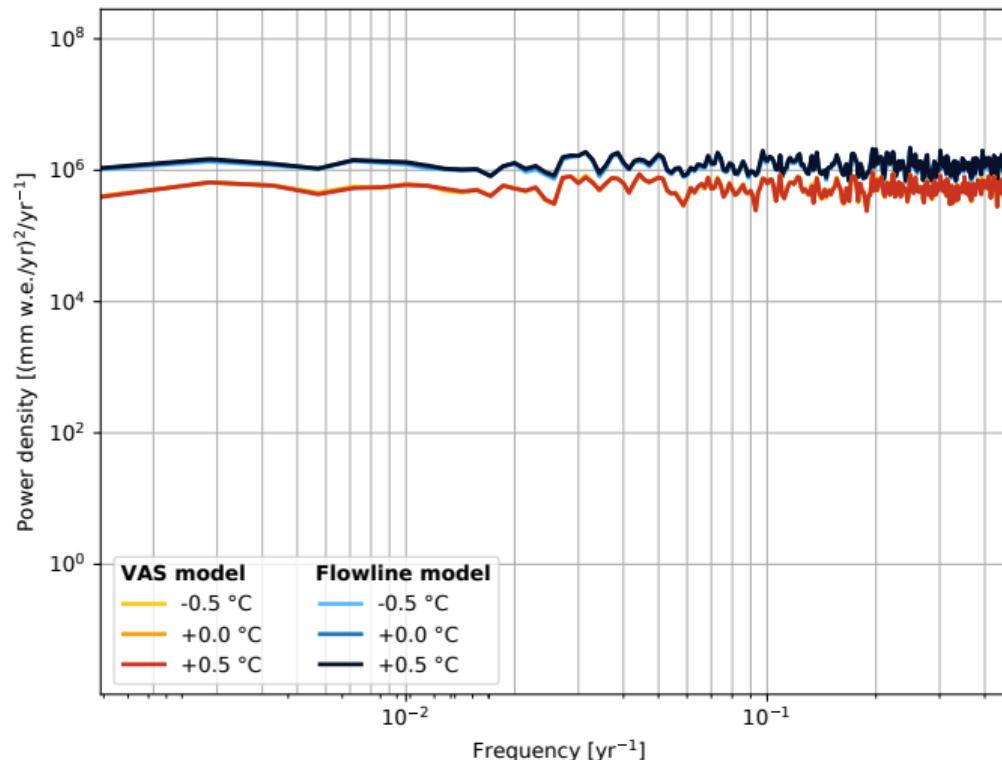
# Hintereisferner - natural length fluctuation



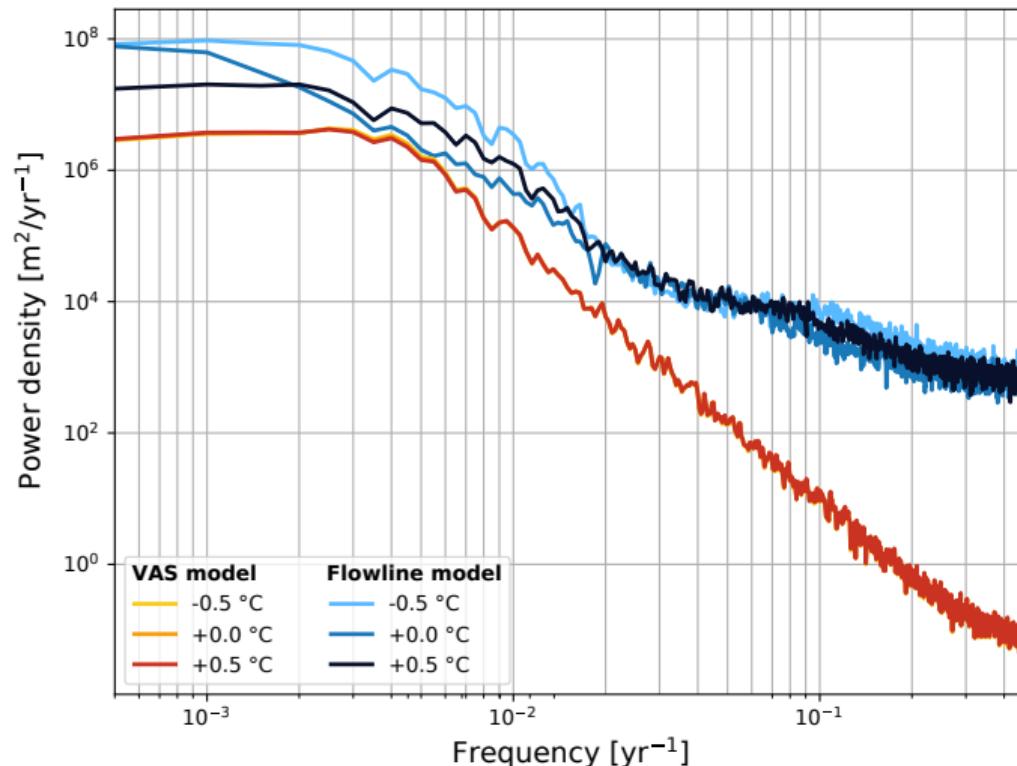
# Hintereisferner - power spectral density of length changes



# Hintereisferner - power spectral density of mass balance

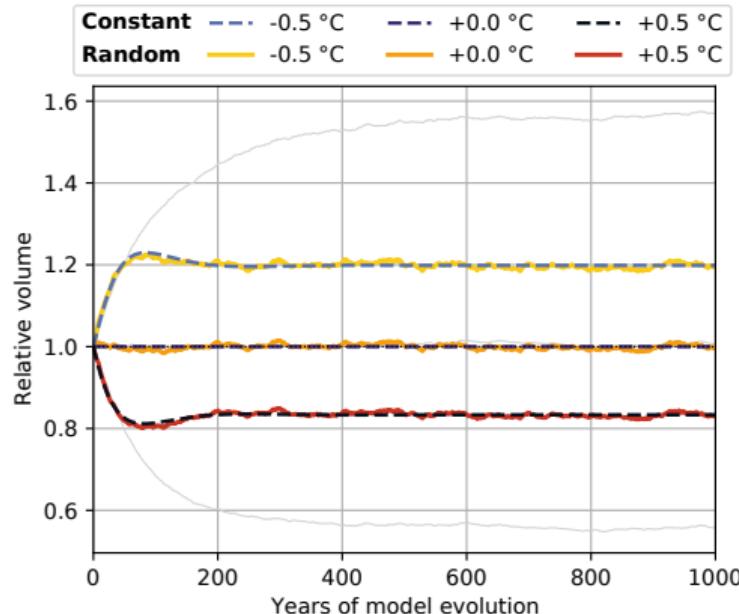


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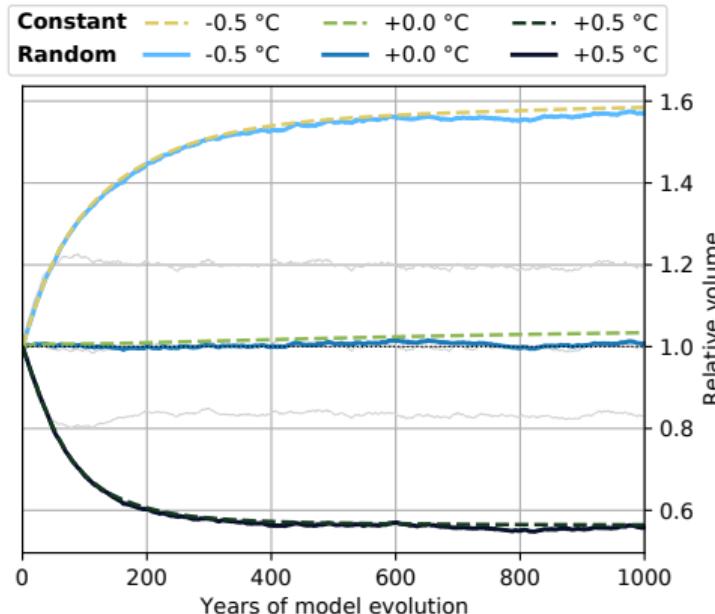


# Alpine glaciers - relative ice volume

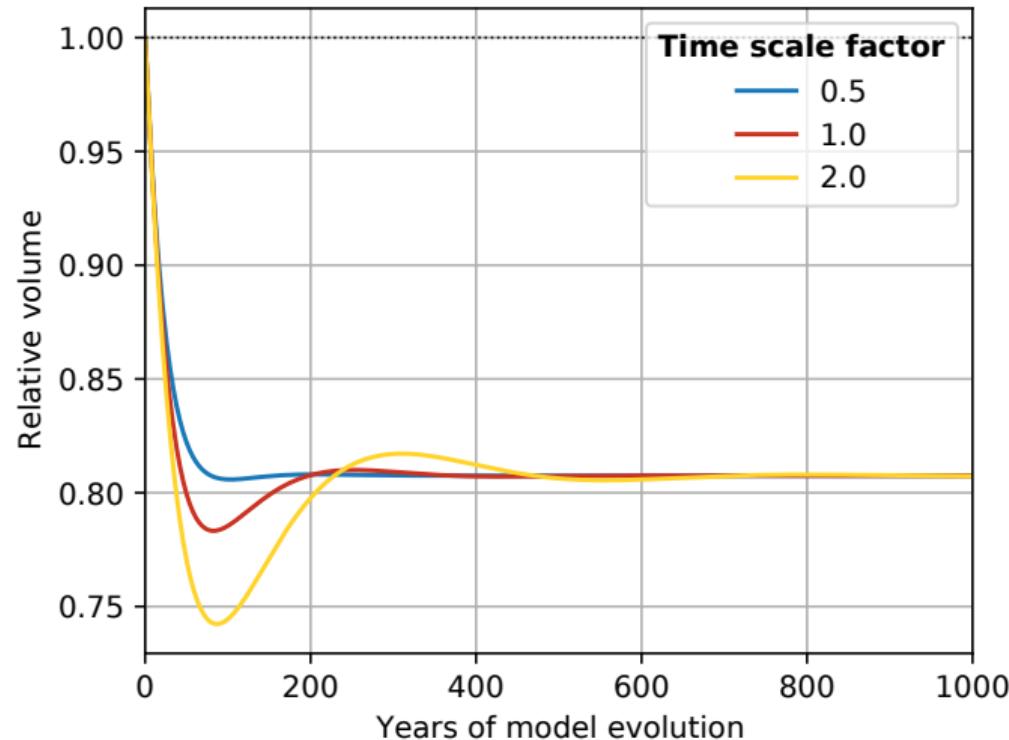
Volume/area scaling model



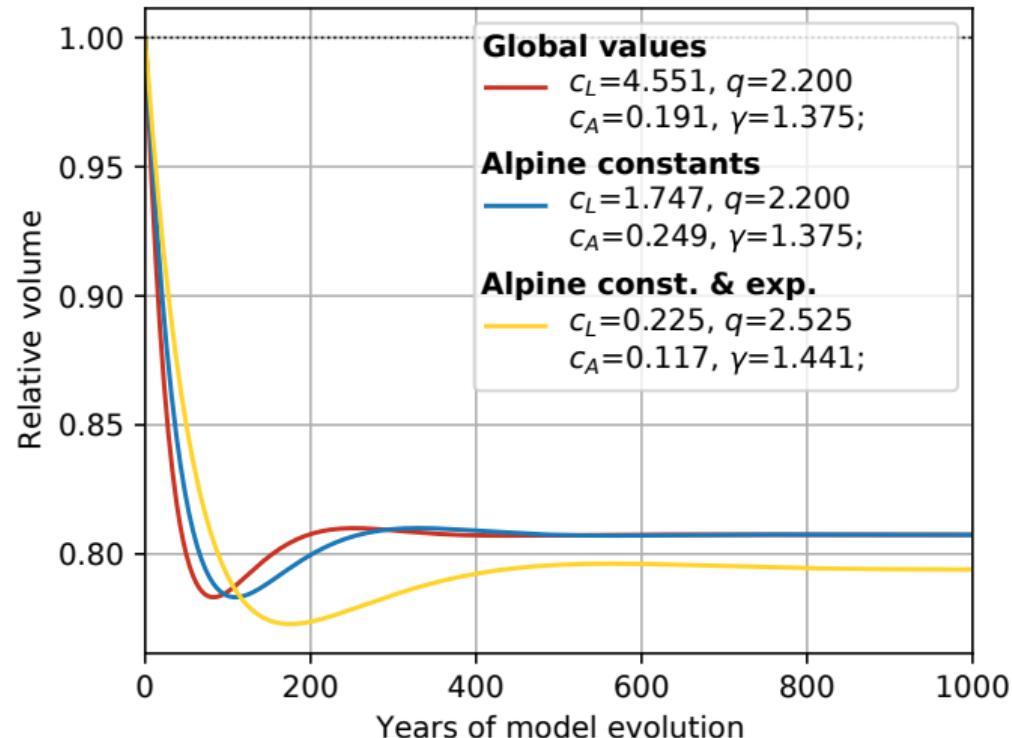
Flowline model



## Sensitivity experiments - response time scaling



# Sensitivity experiments - volume/area and volume/length scaling



## Projections for the 21st century

- ▶ CRU TS (Harris et al. 2020) climate data up to 2020
- ▶ Fifteen GCMs from CMIP6<sup>1</sup>, four different SSPs
- ▶ Calibration match regional observations (Hugonet et al. 2020)
- ▶ Central Europe (RGI Reg. 11), High Mountain Asia (RGI Reg. 13, 14, 15)

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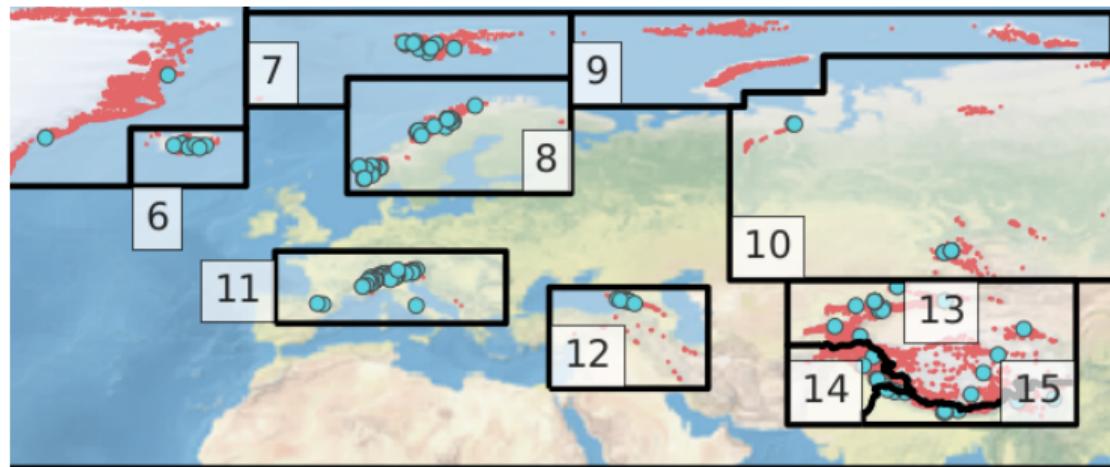
<sup>1</sup>BCC-CSM2MR (Xin et al. 2019), CAMS-CSM1.0 (Rong 2019), CESM2 (Danabasoglu 2019a), CESM2-WACCM (Danabasoglu 2019b), CMCC-CM2-SR5 (Lovato and Peano 2020), EC-Earth3 (EC-Earth), EC-Earth3-Veg (EC-Earth), FGOALS-f3-L (YU 2019), GFDL-ESM4 (John et al. 2018), INM-CM4-8 (Volodin et al. 2019a), INM-CM5-0 (Volodin et al. 2019b), MPI-ESM1.2-HR (Schupfner et al. 2019; Steger et al. 2019), MRI-ESM2.0 (Yukimoto et al. 2019), NorESM2-MM (Bentsen et al. 2019), TaiESM1.0 (Lee and Liang 2020)

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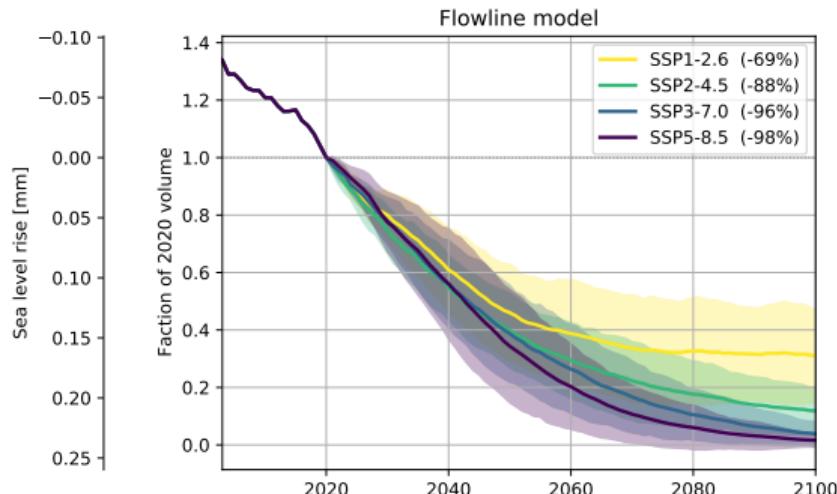
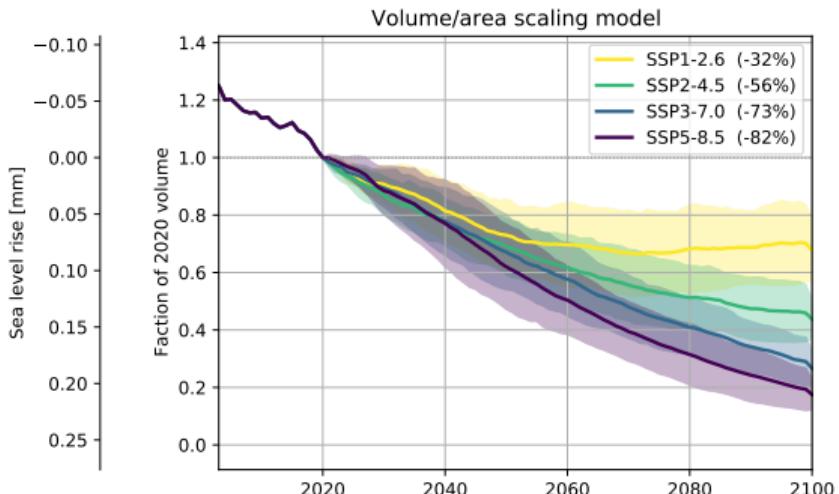
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- ▶ Central Europe (RGI Reg. 11), High Mountain Asia (RGI Reg. 13, 14, 15)

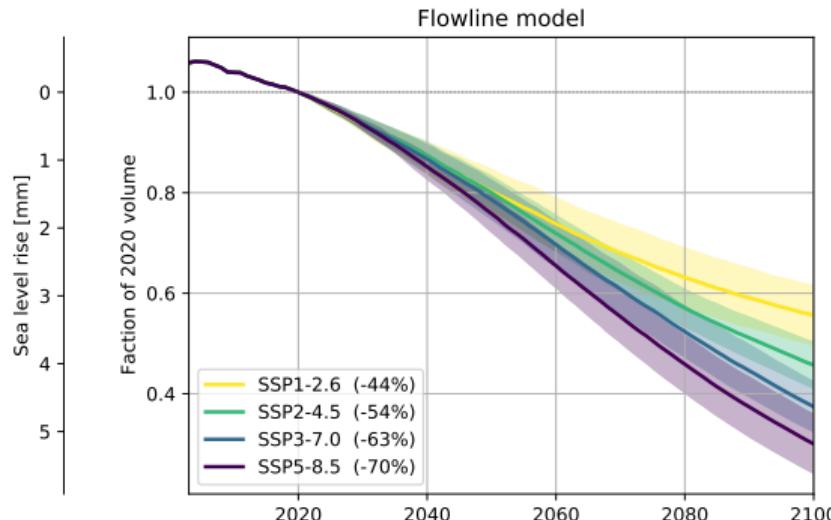
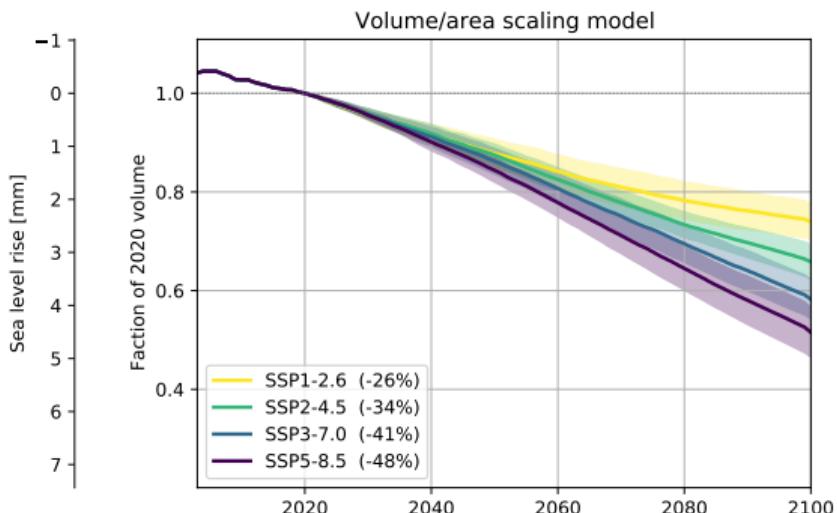


Map section with RGI regions, adapted from (Maussion et al. 2019)

# Central Europe - RGI Region 11



# Central Asia - RGI Region 13



# What are the differences in dynamic response?

**Volume/area scaling model**

**Flowline model**

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- + Computationally cheap

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## Volume/area scaling model

- + Computationally cheap
- Uniform (cuboid) geometry
- ⚠ Estimated changes are small
- Missing mass-balance-elevation feedback
- Oscillating response to step change in climate
- + Relatively insensitive to scaling parameters

## Flowline model

- Computationally (more) expensive
- + Geometry aware (1.5 dimensional)
- ⚠ Estimated changes are bigger
- + Mass-balance-elevation feedback
- + Asymptotical adjustment to step change in climate
- Needs dedicated calibration

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Usage of advanced models is advisable if possible.

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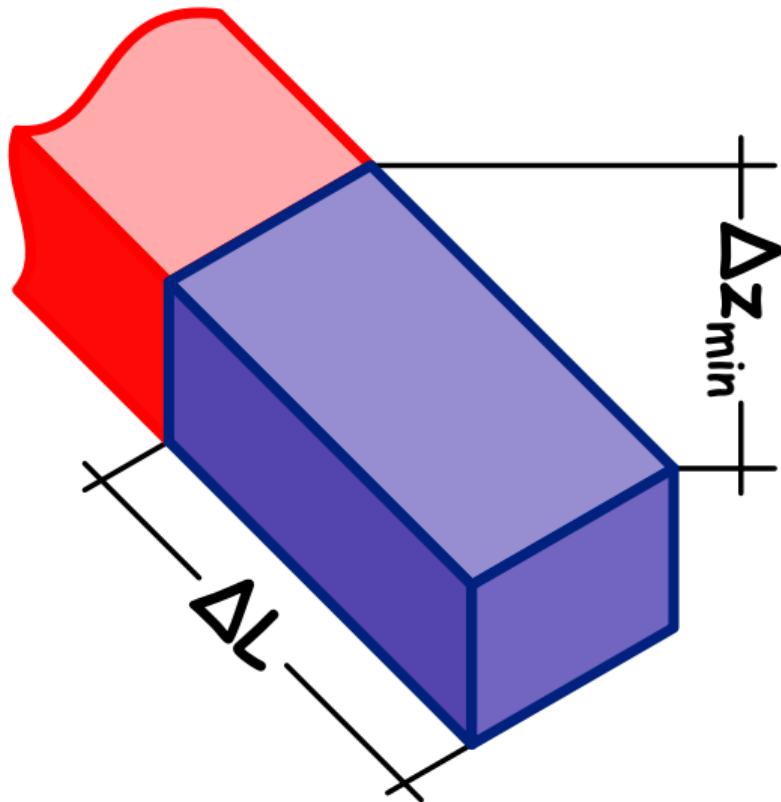
# Mass balance model

$$T_i^{\text{terminus}} = T_i \cdot \gamma_{\text{temp}} (z_{\min} - z_{\text{ref}})$$
$$T_i^{\max} = T_i \cdot \gamma_{\text{temp}} (z_{\max} - z_{\text{ref}})$$

$$P_i^{\text{solid}} = a \cdot P_i \cdot f_{\text{solid}} \cdot (1 + \gamma_{\text{precip}} \cdot (z_{\text{mean}} - z_{\text{ref}}))$$

$$f_{\text{solid}} = \begin{cases} 0 & T_i^{\max} > T_{\text{liquid}} \\ 1 + \frac{T_i^{\text{terminus}} - T^{\text{solid}}}{\gamma_{\text{temp}} \cdot (z_{\max} - z_{\min})} & \forall \text{ other} \\ 1 & T_i^{\text{terminus}} < T_{\text{solid}} \end{cases}$$

## Change of terminus elevation



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Average (initial) gradient

$$\frac{z_{\min,0} - z_{\max}}{L_0}$$

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Terminus elevation as linear function of glacier length

$$z_{\min}(t+1) = z_{\max} + L(t+1) \frac{z_{\min,0} - z_{\max}}{L_0}$$

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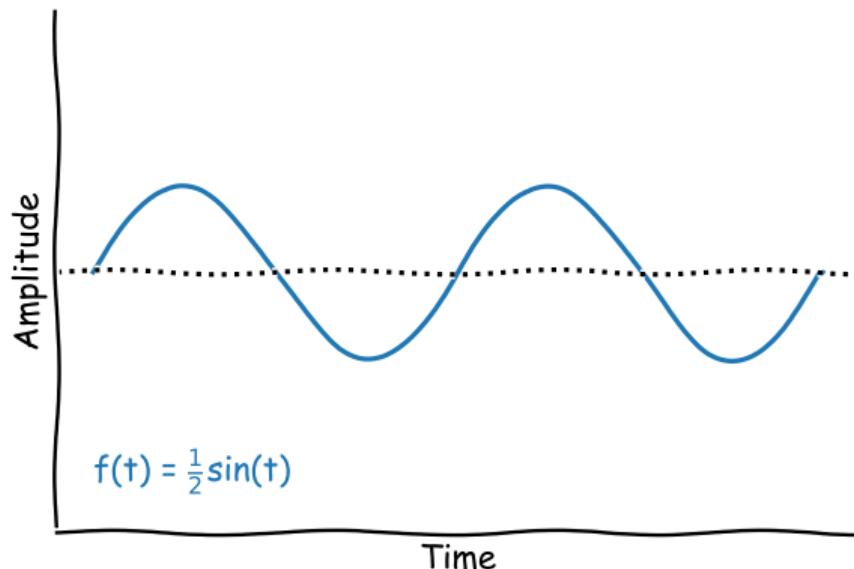
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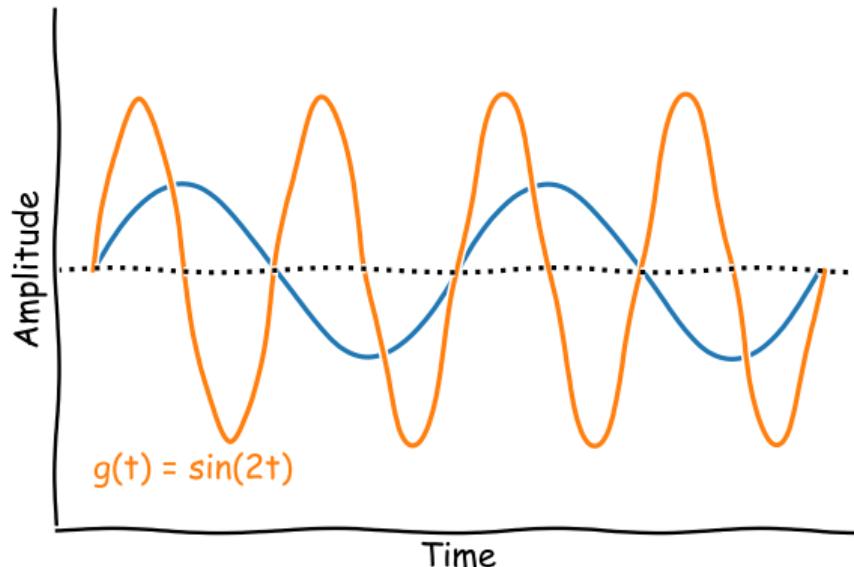
$$z_{\min}(t+1) = z_{\max} + L(t+1) \frac{z_{\min,0} - z_{\max}}{L_0}$$

Only feedback between glacier geometry and specific mass balance

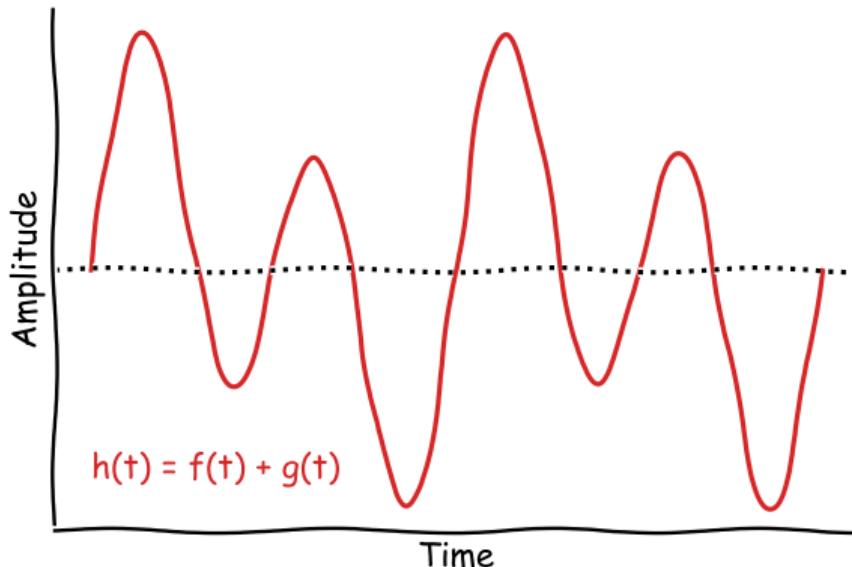
# Time series analysis



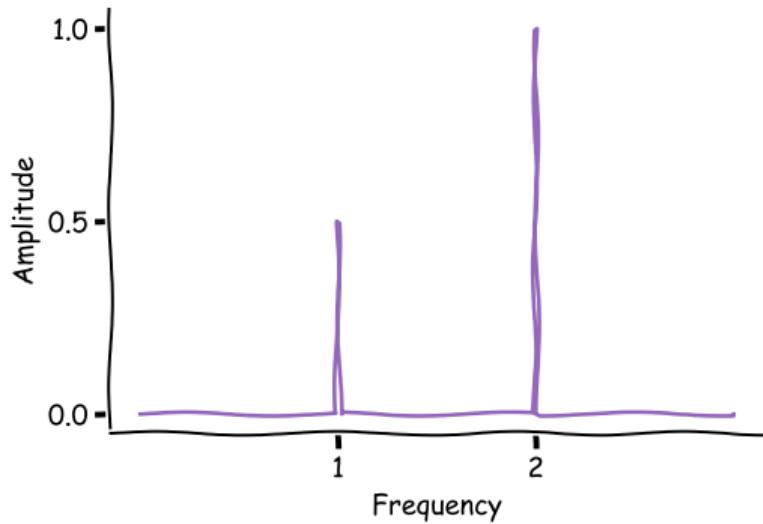
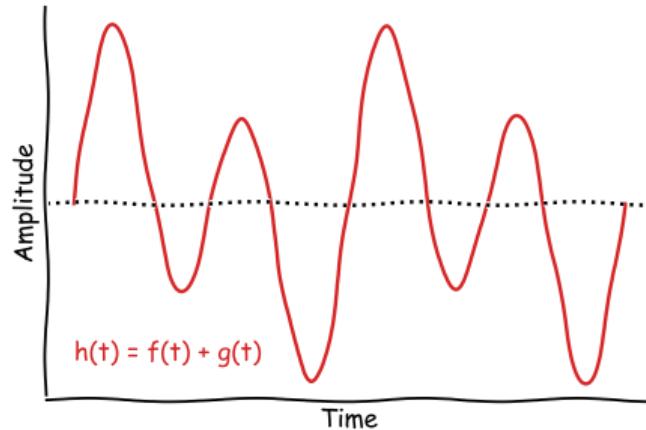
# Time series analysis



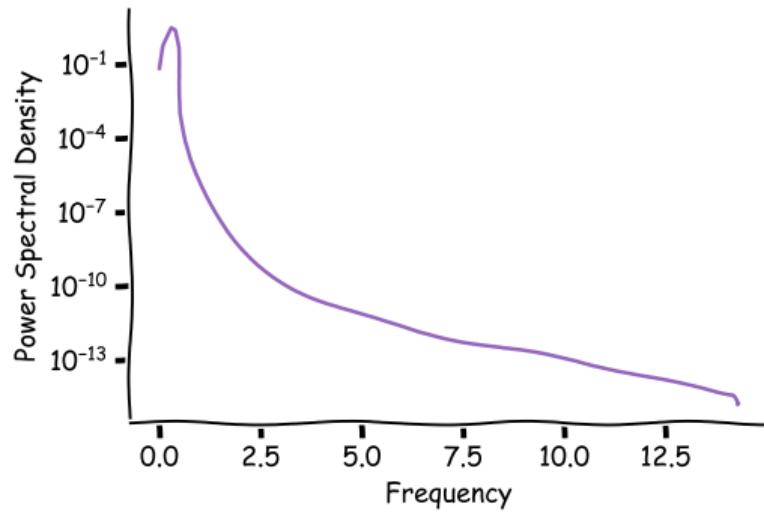
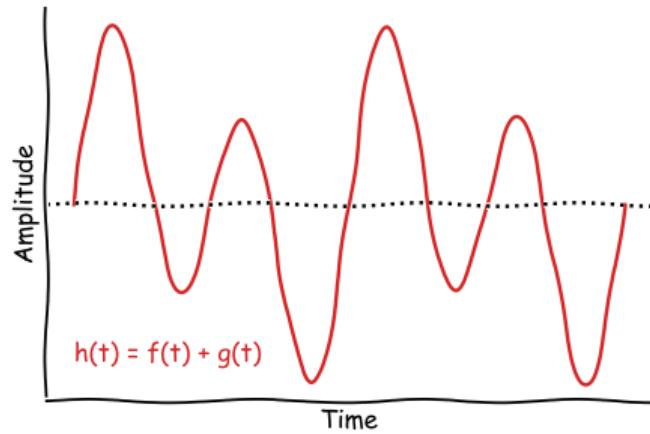
# Time series analysis



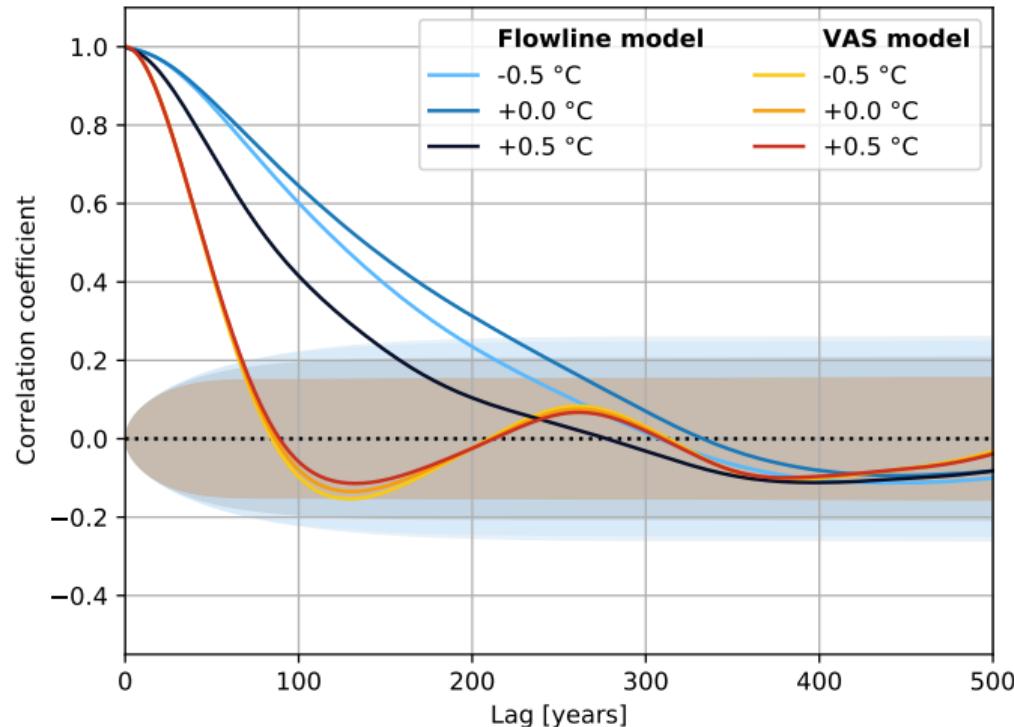
# Time series analysis



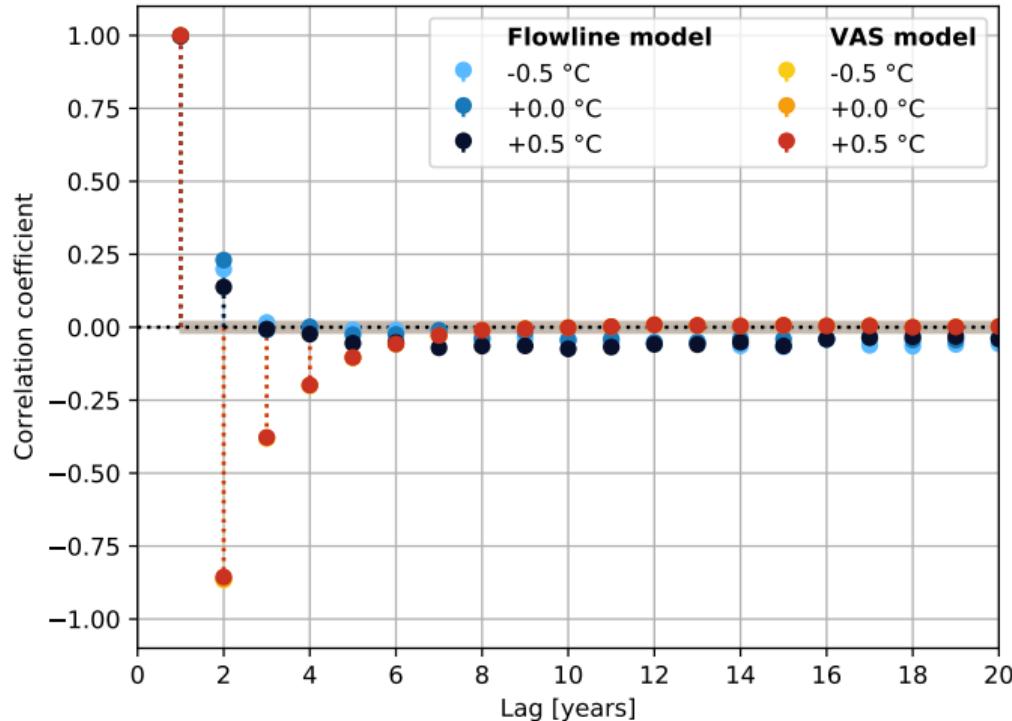
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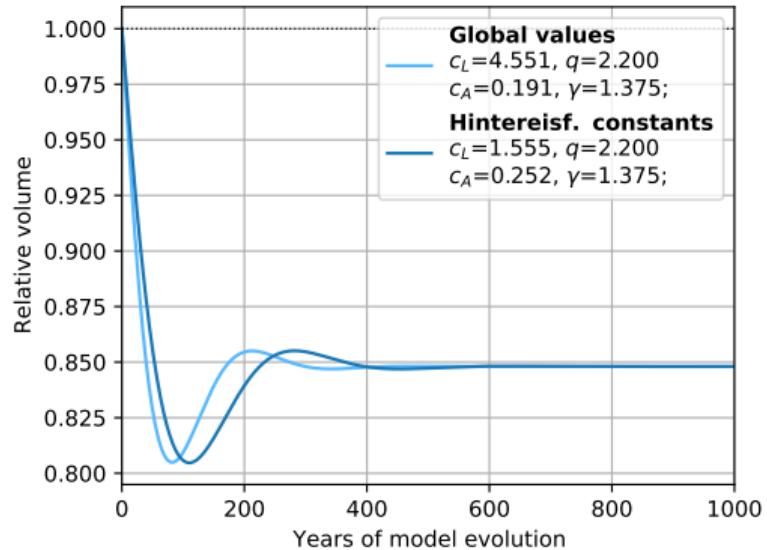
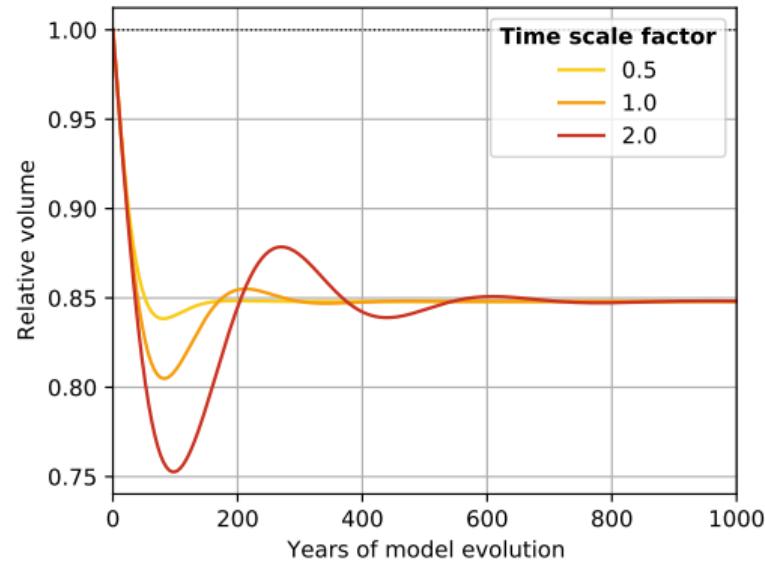
# Hintereisferner - autocorrelation function



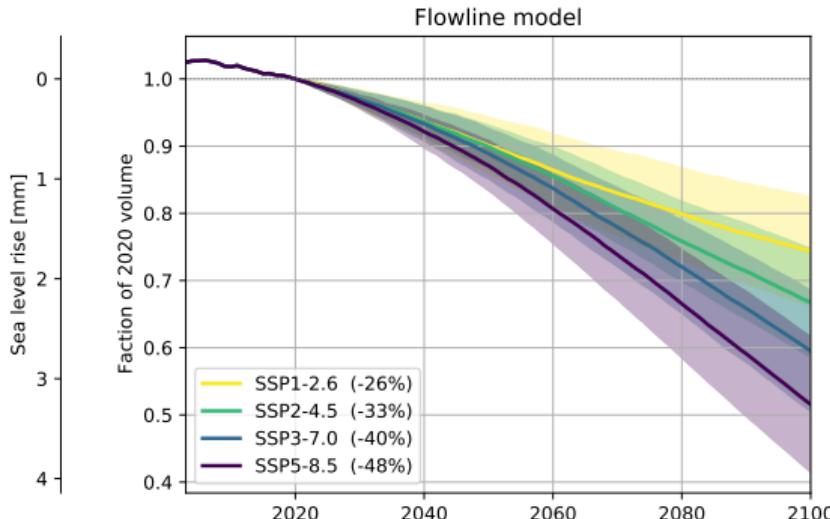
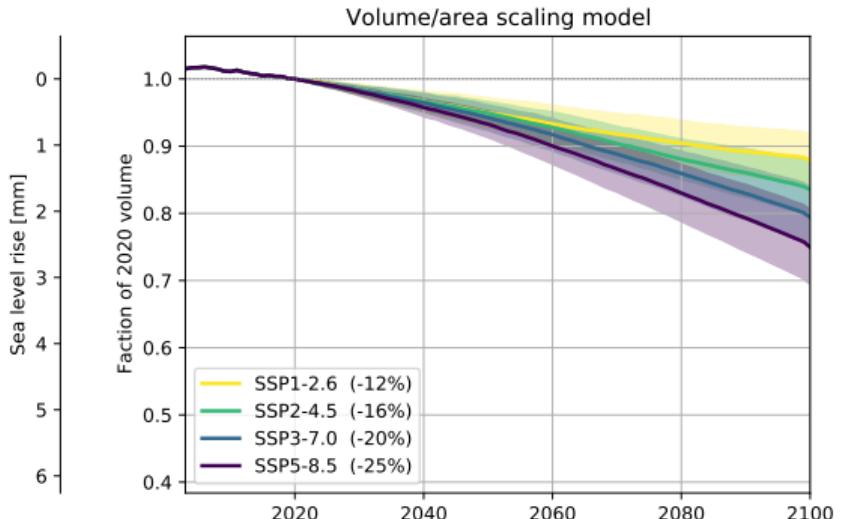
# Hintereisferner - partial autocorrelation function



# Sensitivity experiments - Hintereisferner



# South Asia West - RGI Region 14



# South Asia East - RGI Region 15

