Testing the importance of explicit glacier dynamics for future glacier evolution in the Alps

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To Psycho

And all others who try to move their toes individually

Abstract

The abstract is a short summary of the thesis. It announces in a brief and concise way the scientific goals, methods, and most important results. The chapter "conclusions" is not equivalent to the abstract! Nevertheless, the abstract may contain concluding remarks. The abstract should not be discursive. Hence, it cannot summarize all aspects of the thesis in very detail. Nothing should appear in an abstract that is not also covered in the body of the thesis itself. Hence, the abstract should be the last part of the thesis to be compiled by the author.

A good abstract has the following properties: *Comprehensive:* All major parts of the main text must also appear in the abstract. *Precise:* Results, interpretations, and opinions must not differ from the ones in the main text. Avoid even subtle shifts in emphasis. *Objective:* It may contain evaluative components, but it must not seem judgemental, even if the thesis topic raises controversial issues. *Concise:* It should only contain the most important results. It should not exceed 300–500 words or about one page. *Intelligible:* It should only contain widely-used terms. It should not contain equations and citations. Try to avoid symbols and acronyms (or at least explain them). *Informative:* The reader should be able to quickly evaluate, whether or not the thesis is relevant for his/her work.

An Example: The objective was to determine whether … (*question/goal*). For this purpose, … was … (*methodology*). It was found that … (*results*). The results demonstrate that … (*answer*).

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In a nutshell, a glaciers annual specific surface mass balance B is the difference between accumulation and over the course of a year. Hereby, accumulation refers to mass gain by snowfall, avalanches, snow drift, etc., while ablation refers to mass loss via ice melt, sublimation, calving, etc. The temperature index mass balance model used by the volume/area scaling model relies solely on the area average monthly solid precipitation onto the glacier surface P_i^{solid} and the monthly mean air temperature at the glacier's terminus elevation T_i^{terminus} as input. Hereby, the index i denotes the month of the year. The mass balance equation described by Marzeion et al. (2012) reads

$$B = \left[\sum_{i=1}^{12} \left[P_i^{\text{solid}} - \mu^* \cdot \max\left(T_i^{\text{terminus}} - T_{\text{melt}},\ 0\right)\right]\right] - \beta^*. \tag{2.1}$$

The terminus temperature T_i^{terminus} is computed by scaling the monthly average air temperature T_i from the climate file reference elevation z_{ref} to the glacier's terminus elevation z_{min} using the temperature lapse rate γ_{temp}.

$$T_i^{\text{terminus}} = T_i \cdot \gamma_{\text{temp}}(z_{\text{min}} - z_{\text{ref}}) \tag{2.2}$$

The temperature at the maximum glacier elevation $T_i^{\max}$ is computed analogously to terminus temperature: $T_i^{\max} = T_i \cdot \gamma_{\text{temp}}(z_{\max} - z_{\text{ref}})$, whereby $z_{\max}$ represent the maximum glacier surface elevation. The positive melting temperature is computed as the difference between terminus temperature and temperature threshold for ice melt T_{melt}, with an obvious lower bound of 0 °C. The glacier's temperature sensitivity

μ^* relates the positive melting temperature to the actual ice loss and needs to be calibrated for each glacier (as does the potential mass balance residual β^*). The calibration process of these mass balance parameters is described below.

The area average monthly solid precipitation onto the glacier surface P_i^{solid} is computed from the total precipitation P_i (given by the climate file) as

$$P_i^{\text{solid}} = P_i \cdot f_{\text{solid}} \cdot (1 + \gamma_{\text{precip}} \cdot (z_{\text{mean}} - z_{\text{ref}})). \tag{2.3}$$

The total climatic precipitation P_i is scaled from the reference elevation of the climate file z_{ref} to the average glacier surface elevation z_{mean} using the precipitation lapse rate γ_{precip}. The precipitation lapse rate γ_{precip} is given in percentage of precipitation per meters of elevation change [$\%\,\text{m}^{-1}$]. The fraction of solid precipitation f_{solid} depends on the terminus temperature T_i^{terminus}, the temperature at the maximum glacier surface elevation T_i^{max} and the temperature thresholds for solid and liquid precipitation, T^{solid} and T^{liquid}, respectively. For terminus temperatures below the threshold for solid precipitation, all precipitation is solid ($T_i^{\text{terminus}} < T^{\text{solid}} \Rightarrow f_{\text{solid}} = 1$). For temperatures at the maximum glacier surface elevation above the threshold for liquid precipitation, all precipitation is liquid ($T_i^{\text{max}} > T^{\text{liquid}} \Rightarrow f_{\text{solid}} = 0$). For temperatures in between, the fraction of solid precipitation is interpolated linearly as $f_{\text{solid}} = 1 + \frac{T_i^{\text{terminus}} - T^{\text{solid}}}{\gamma_{\text{temp}} \cdot (z_{\text{max}} - z_{\text{min}})}$.

Climate models generally tend to underestimate the precipitation in mountainous regions, hence the monthly precipitation amount is additionally scaled by a factor a. While this scaling factor is implemented in the mass balance models (as `prcp_scaling_factor`), it is not a physical component of the mass balance equation and hence omitted in the Equation 2.3 above. A global mean of $a = 2.5$ is found by Giesen and Oerlemans (2012), whereas Marzeion and Nesje (2012) found a mean of 2.1 for Central Europe and Scandinavia. The sensitivity study by Marzeion et al. (2012) shows the strongest correlation between observed and modeled mass balance for $a \approx 1.3$ and the highest skill score for $a \approx 2.5$. The variability of the modeled mass-balance is quite low for values of $a \leq 2.5$.

The values of the above mentioned hyper parameters (temperature thresholds, lapse rates, scaling factors, ...) can be calibrated, depending on the region and the used baseline climate. For Alpine model runs with the HISTALP baseline climate the following values are recommended (set as default in OGGM) and used this work (Dusch 2018): $\gamma_{\text{temp}} = -6.5\,\text{K}\,\text{km}^{-1}$, $T^{\text{melt}} = -1.75\,^\circ\text{C}$, $\gamma_{\text{precip}} = 0$, $T^{\text{solid}} = 0.0\,^\circ\text{C}$, $T^{\text{liquid}} = 2.0\,^\circ\text{C}$, $a = 1.75$;

Calibration of the mass balance parameters

A complete and thorough description of the mass balance calibration process for this particular temperature index model can be found in Marzeion et al. (2012, Section

2.1.9, 2.1.10) and Maussion et al. (2019, Section 3.3). The following section serves as a summary.

The first step is to estimate the so called *candidates* $\mu(t)$ for all glaciers with available mass balance records (254 glaciers globally, see World Glacier Monitoring Service, Zürich, Switzerland (2017)). This is done by requiring the average mass balance $\overline{B}(t)$ over the 31-year period centered around the year t to be zero and solving for $\mu(t)$.

$$\mu(t) = \frac{P_{\mathrm{clim}}^{\mathrm{solid}}(t)}{\max(T_{\mathrm{clim}}^{\mathrm{terminus(t)}} - T_{\mathrm{melt}}\ 0)}, \tag{2.4}$$

whereby $P_{\mathrm{clim}}^{\mathrm{solid}}(t)$ and $T_{\mathrm{clim}}^{\mathrm{terminus}}(t)$ are the average yearly solid precipitation amount and average yearly air temperature at the glaciers terminus during the climatological period centered around the year t, respectively. The next step is to solve the mass balance equation (Eq. 2.1) for each candidate $\mu(t)$ and compare it to the mass balance observations. The computed difference $\beta(t)$ is a measure of how good the temperature sensitivity candidate $\mu(t)$ approximates the *real* value of μ^*. Hence, μ^* is chosen as the candidate $\mu(t = t^*)$ for which the absolute bias is minimal $\beta^* := \beta(t = t^*) \approx 0$, which in the best case is around zero. Hereby, the *equilibrium year* t^* represents the center of a 31-year climatic period where the given glacier geometry would stay in equilibrium. However, this is more of a model parameter and should not be overinterpreted as a real live value. The same is true for the corresponding temperature sensitivity μ^* and mass balance residual β^*.

For all glaciers without mass balance records, t^* and β^* are interpolated from the ten closest glaciers, inversely weighted with distance. The temperature sensitivity is computed by requiring the mass balance to be zero $\overline{B}(t^*) = 0$ and solving for μ^*. The temperature sensitivity μ^* depends highly on glacier specific factors, such as avalanches from surrounding terrain, topographical shading, etc. Therefore, μ^* can vary drastically from one glacier to another, even between neighboring glaciers. On the other hand, it is intuitively more likely for a glacier to be in equilibrium if its surrounding glaciers are in equilibrium as well. This is one major factor, why the interpolation of t^* instead of μ^* reduces the mass balance error in a leave–one–out cross–validation (cf. Marzeion et al. 2012; Maussion et al. 2019).

Implementation note

The results of the steps above depend on the glacier outlines, the climate data and the mass balance hyper parameters (i.e., the temperature thresholds, lapse rates and the precipitation scaling factor). The equilibrium year t^* and mass balance residual β^* computed for each reference glacier is stored in the `ref_tstars.csv` file. Hence, for a given combination of RGI version, climate data and hyper parameters the calibration for the reference glaciers has to be done only once. Afterwards, it can be read directly from the corresponding file. OGGM comes with reference tables for combinations of RGI v5 and v6 and CRU4 and HISTALP.

Differences between the flowline mass balance model and the volume/area scaling mass balance model

The volume/area scaling mass balance model computes an average mass balance value for the entire glacier. The mass balance model requires only the minimal and maximal glacier elevation as additional input parameters (z_{min}, z_{max}), to compute the monthly terminus temperature T_i^{termiuns} and the area averaged monthly amount of solid precipitation P_i^{solid}. The flowline model, on the other hand, requires a mass balance value for each grid point of the flowline (i.e., for each elevation band). Therefore, the mass balance is a function of elevation $B(z)$ and the elevation of the grid points must be supplied. Solid precipitation and air temperature are then computed for the given points of elevation, resulting in a point mass balance.

2.1.3 Glacier evolution model

Volume/area scaling is derived from the full set of continuum equations with no assumptions of plane strain, shallow ice, perfect plasticity, or steady state conditions. This derivation from the fully time dependent equation of motion allows the volume V, area A and scaling constant c_A to change with time. Especially the scaling constant c_A can incorporate transient behavior, since it depends on closing conditions which show an explicit time dependency. However, to explicitly include a temporal component, volume/area scaling has to be used in conjuncture with proper response time scaling. Response time scaling is a separate but equally valid scaling relation, derived during the same dimensionless analysis. Hence, these two scaling relations cannot be separated and have to be applied together to successfully model glacier evolution (Bahr et al. 2015).

The volume/area scaling model starts with an initial glacier surface area A_0 as input. The initial glacier volume V_0 and the initial glacier length L_0 are computed using the volume/area scaling relation and the inverted volume/length scaling relation, respectively (cf. Section 2.1.1).

$$V_0 = c_A \cdot A_0^{\gamma} \qquad\qquad L_0 = \left(\frac{V_0}{c_L}\right)^{\frac{1}{q}} \tag{2.5}$$

Additionally, only a mass balance model and the initial terminus elevation $z_{\text{min},0}$ and maximal glacier surface elevation z_{max} are needed.

The volume/area scaling model runs with yearly time steps $\Delta t = 1\,\text{yr}$. Each time step from year t to year $t+1$ includes the following steps:

1. Compute the time scale of the glacier's length change response to volume change τ_L and the time scale of the glacier's surface area change response to volume change τ_A as

$$\tau_L(t) = \frac{V(t)}{P_{\text{clim}}^{\text{solid}}(t^*) \cdot A(t)} \qquad\qquad \tau_A(t) = \tau_L(t)\frac{A(t)}{L(t)^2} \tag{2.6}$$

As introduced during the calibration process, $P_{\text{clim}}^{\text{solid}}(t^*)$ is the average solid precipitation during the 31-year period centered around t^*. For more details see Marzeion et al. (2012). The implementation includes lower bounds for both time scales as well as the climatological turnover, for details see Section 2.2.2.

2. Get the specific mass balance $B(t)$ from mass balance model, by solving Equation 2.1. For implementation details see Section 2.2.1

3. Compute the volume change $\Delta V(t) = \frac{1}{\rho_{\text{ice}}} A(t) \cdot B(t)$ as product of specific mass balance and glacier surface area. The volume change happens instantaneously, i.e., over one time step, hence the updated volume equals the sum of current volume and volume change $V(t+1) = V(t) + \Delta V(t)$.

4. The (hypothetical) equilibrium surface area can be computed by inverting the volume/area scaling relation $(V(t+1)/c_A)^{1/\gamma}$. However, the surface area does not change instantaneously, and proper response time scaling must be applied. Hence, the area change is computed as

$$\Delta A(t) = \frac{1}{\tau_A}\left(\left(\frac{V(t+1)}{c_A}\right)^{\frac{1}{\gamma}} - A(t)\right). \tag{2.7}$$

The updated area then equals the sum of current area and area change $A(t+1) = A(t) + \Delta A(t)$.

5. The updated glacier length and length change are computed analogously to the glacier surface elevation. $L(t+1) = L(t) + \Delta L(t)$, with

$$\Delta L(t) = \frac{1}{\tau_L}\left(\left(\frac{V(t+1)}{c_L}\right)^{\frac{1}{q}} - L(t)\right). \tag{2.8}$$

6. Adjust terminus elevation z_{min}, assuming a linear elevation change with changing glacier length (i.e., constant slope):

$$z_{\text{min}}(t+1) = z_{\text{max}} + \frac{L(t)}{L_0}(z_{\text{min},0} - z_{\text{max}}) \tag{2.9}$$

The maximum glacier elevation stays constant during the entire model run $z_{\text{max}} = \text{const}$.

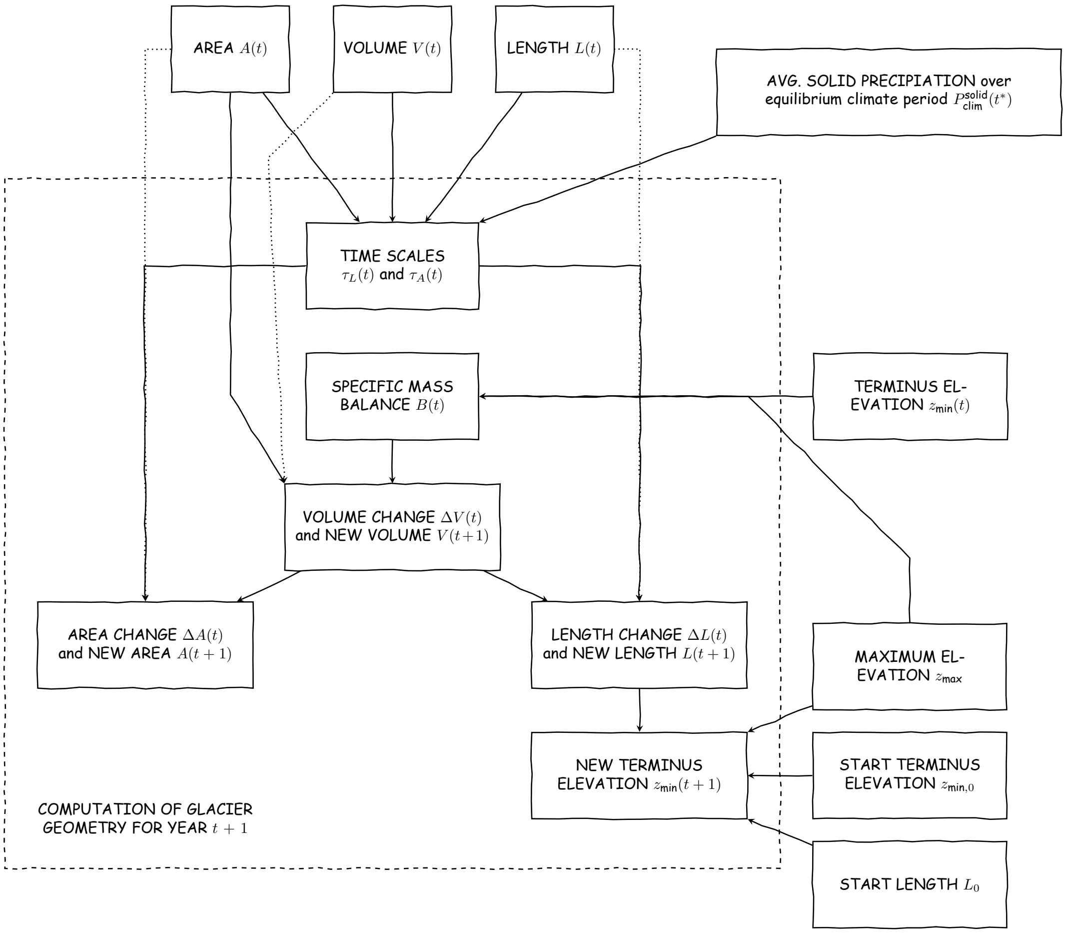


Figure 2.1: Schematic of the glacier evolution model's time stepping.

2.2 Implementation

2.2.1 Mass balance models

Volume/area scaling mass balance model

The `VAScalingMassBalance` model is the implementation of the *original* mass balance model by Marzeion et al. (2012). The model computes the mass balance of a glacier during the climate data period. The general concept is fairly similar to the `oggm.core.massbalance.PastMassBalance` model. The main difference is, that the volume/area scaling mass balance model returns only one glacier wide average mass balance value, instead of point mass balance values for the different elevation bands.

The mass balance model is initialized for a single glacier, denoted by the OGGM specific glacier directory `gdir`. Per default, the model will use the calibrated mass balance parameters μ^* and β^* and read temperature and precipitation records from the preprocessed climate file `climate_historical`. An alternative climate file can be used, by supplying either the filename and/or it's suffix via the parameters `filename` and `input_filesuffix`, repecitvely. It is possible to specify the start year and end year of the climate period (`ys` and `ye`), if not all available data should be used. The parameter `repeat` controlls whether the climate period given by `[ys, ye]` should be repeated indefinitely in a circular way.

The volume/area scaling mass balance model inherits the following methods from the `oggm.core.massbalance.MassBalanceModel` super class:

- `get_annual_climate()` and `get_monthly_climate()` compute and return the mass balance relevant climate information, i.e. positive air temperature at the terminus elevation in °C and solid precipitation amount in $\mathrm{kg\,m^{-2}}$, for the given year and month/year combination, respectively.

- `get_annual_mb()` and `get_monthly_mb()` compute and return the glacier wide average mass balance in $\mathrm{m\,s^{-1}}$, for the given year and month/year combination, respectively. The possible mass balance residual β^* is applied.

- `get_specific_mb()` and `get_monthly_specific_mb()` compute and return the glacier wide average specific mass balance in mmw e /yr, for the given year and month/year combination, respectively. The possible mass balance residual β^* is applied.

All methods need the glacier terminus elevation `min_hgt` and the maximal glacier surface elevation `max_hgt` as parameters. The date is supplied via the `year` parameter, using the hydrological float year convention. Given that the scaling mass balance model computes the glacier wide average mass balance, it is not possible to estimate the equilibrium line altitude. Hence, the the method `get_ela()` is not implemented, in contrast to the `PastMassBalance` model.

Constant climate scenario

The `ConstantMassBalance` model simulates a constant climate based on the observations averaged over a 31-year period centered on a given year `y0`. Hence, the specific mass balance does not change from year to year. The task `run_constant_climate(gdir, ...)` initializes a `ConstantMassBalance` for the given glacier `gdir` and runs for a given number of years `nyears`. The task takes an additional temperature bias as parameters `temp_bias`, to alter the observed climate records.

The same idea of a constant climate is used during the mass balance calibration, solving the mass balance equation (Equation 2.1) for the temperature sensitivity μ^*. So per definition, μ^* is the temperature sensitivity to keep the glacier in equilibrium over the 31-year climate period centered around the *equilibrium year* t^* while neglecting a potential mass balance residual β^*. Consequentially, a `ConstantMassBalance` model with `y0` $= t^*$ keeps the glacier in equilibrium.

Random climate scenario

Similar to the `ConstantMassBalance` model, the `RandomMassBalance` model is based on a 31-year period centered on a given year `y0`. However, the mass balance years are randomly shuffled within that period. More precise, for each simulated year the model computes the specific mass balance using temperature and precipitation records from a randomly selected year within the given period. Hence, the model runs on a synthetic random climate scenario based on actual observations. A seed `seed`' for the random generator can be supplied as parameter, to allow for reproducibility. Additionally, it is possible to choose between draws with and without replacement via the `unique_sample` parameter.

The task `run_random_climate(gdir, ...)` works analogously to the task `run_constant_climate(gdir, ...)`, using an instance of `RandomMassBalance` model instead of the `ConstantMassBalance` model. Hence, using the climatological period centered around `y0` $= t^*$, the model glacier should stay in an equilibrium state while underlying minor fluctuations. Supplying a positive or negative temperature bias will result in a retreating or advancing model glacier, respectively, reaching a new equilibrium after some years.

2.2.2 Glacier evolution model

The `oggm—vas.VAScalingModel` is the implementation of the above describe glacier evolution model (see Section 2.1.3, Marzeion et al. (2012, cf.)) into the OGGM framework. The full source code is publicly available on GitHub.

An instance of the `oggm—vas.VAScalingModel` class is initialized with the initial area `area_m2_0`, the initial glacier terminus elevation `min_hgt` and maximum glacier surface elevation `max_hgt` and an instance of a `oggm—vas.VAScalingMassBalance` model. Additionally, the start year of the simulation `year_0` must be defined. Those initial values are stored as instance variables, since they are needed for later computations. Other than that, the `oggm—vas.VAScalingModel` object stores all model parameters as instance variables for the current year it is in. This includes glacier geometries (V, A, L, z_{min}, z_{max}) and their changes (ΔV, ΔA, ΔL), time scales (τ_A, τ_L), the mass balance model and the specific mass balance B, but also constants like the scaling parameters (c_A, γ, c_L, q) and ice density ρ_{ice}.

To advance the glacier model, there are three different methods. The `step()`

method advances the model by one year, following the above described steps (see Section 2.1.3). The method `run_until(year_end)` runs the model until the specified year and returns the geometric glacier parameters at the end of the model evolution (year, length, area, volume, terminus elevation and specific mass balance). Thereby, the model starts from whatever year it currently is in. It is possible to start the model run from `year_0` with the flag `reset`. The method `run_until_and_store()` works analagous to the previous one, with the difference that all parameters are stored for each time step (i.e., for each year). The resulting data set is returned and possible stored to file, if a file path is give. The method `run_until_equilibrium()` tries to run the glacier model until an equilibrium state is reached. The model runs for a fixed number of iteratrions `max_ite`, the total elapsed time changes with the chosen time step `ystep`. The iteration breaks, either if the glacier volume is below $1\,\mathrm{m}^3$ or an equilibirum is reached. An equilibrium state is reached, if the volume change rate $|V(t) - V(t+\Delta t)|/V(t)$ falls below a given value `rate`. Therefore, the method can only be used with a constant climate scenario (see Section 2.2.1).

2.3 Experimental setup

Implementing the volume/area scaling model is all good and well, but how does it compare to the flowline model?! While successfully passing the unit test is a necessity—or at least good programming practice—unit tests are only testing for coding errors and not for the physicality of the results. Nor do they answer the main research question: "What information is gained from the use of a physical based flowline model?" The following experiments run the newly implemented model in a variety of setting and compare it to the OGGM flowline model. The focus is on the intrinsic model behavior and its comparison to the flowline model, not on absolute ice volume estimations.

The experiments start on the smallest scale possible, namely with a single glacier. This first test case is intended to get a feel for the model and set the stage for the following experiments. Qualitative conclusions are drawn from time series of glacier geometries in response to different equilibrium climates. Some more quantitative results are obtained from an analysis of autocorrelation function and power spectral density of length change signals under a white noise climate, inspired by (Roe and Baker 2014). Both analyses can only be performed on the signal of a single glacier, since mirroring oscillation of different glaciers could cancel each other out. To avoid another N-of-1 experiment, six different Alpine glaciers are investigated during this step. While it may be necessary for the aforementioned experiments, it is not appropriate to apply scaling relations to a single glacier. Volume/area scaling should only be applied to a collection of glaciers (Bahr et al. 2015).

The first regional run looks at the evolution of aggregate ice volume of all Alpine glaciers for different equilibrium climate scenarios. Any differences between the volume/area scaling model and the flowline model found above may be caused by the parameters defining the scaling relations. The parameters in question are the length timescale and area timescale for the response time scaling as well as the scaling constants and scaling exponents for the volume/length and volume/area scaling. A sensitivity analysis of said parameters is performed on a single glacier and on the collective Alpine glaciers. As a final experiment, the volume/area scaling model is used to estimate the potential ice melt over the coming one-hundred years for all Alpine glaciers. This is done by using todays climate in combination with a different positive temperature biases to simulate different climate scenarios.

All the aforementioned experiments can be classified as equilibrium experiments. As most things in nature, glaciers strive toward an equilibrium condition. Such an equilibrium is reached eventually, by adjusting the glacier's geometry in reaction to changes in climatic conditions. Analyzing the behavior of a glacier model subjected to a step change in climate can be used to get insight into the dynamics of the numerical model, e.g. to estimate response times. The OGGM provides two convenient climate scenarios (or rather mass balance models) for such equilibrium experiment: the `ConstantMassBalance` model and the `RandomMassBalance` model. The implementation and workings of both mass balance models are described in Section 2.2.1 (see constant and random climate scenario). The hereafter detailed experiments use the HISTALP dataset (Auer and Böhm 2007) as climate input data, with the corresponding hyper parameters (see Mass-balance model calibration for the Alps (Dusch 2018) on the OGGM blog for more information). This obviously limits the suitable glaciers to the ones inside the HISTALP domain, i.e. the Alps. The HISTALP domain corresponds to the region 11-01 of the Randolph Glacier Inventory (RGI) (RGI Consortium 2017; Pfeffer et al. 2014) (version 6.x) and lists 3892 glaciers.

The following paragraph briefly lists the preprocessing tasks needed for the equilibrium runs. For a detailed description of OGGM workflow see Maussion et al. (2019) and the OGGM documentation.

GIS tasks: using the digital elevation model (DEM) from the Shuttle Radar Topography Mission (SRTM, Jarvis et al. (2008)) and the RGI glacier outlines to compute a local grid, compute a glacier mask, compute centerlines and corresponding catchment areas;

climate tasks: extract the HISTALP time series for the grid point nearest to the glacier and write it in the `climate_historical.nc` file;

mass balance calibration: computing the glacier specific mass balance parameters t^*, μ^* and β^*;

inversion tasks: running the ice thickness inversion to estimate the bed topography (needed only for the flowline model);

The actual model runs are invoked via the `run_constant_climate` and `run_random_climate` tasks (see Section 2.2.1 for implementation details). The used settings depend on the intended experiment and are detailed in the following sections.

2.3.1 Single glacier test case

The first qualitative look at the volume/area scaling performance uses the Hintereisferner (RGI60-11.00897) as test case. This test case is intended to get a feel for the volume/area scaling model and set the stage for the following experiments, by comparing the two mass balance models suitable for equilibrium experiments. It has to be noted that volume/area scaling applied to single glaciers gives only an order of magnitude estimation. The scaling constant c is a globally averaged value, and the relative error in scaling constant is directly proportional to the error in estimated ice volume (Bahr et al. 2015). However, a qualitative comparison between the volume/area scaling model and the flowline model is more practicable and meaningful for a single glacier. The model's sensitivity to its scaling parameters is investigated in Section 3.4

For this first experiment, both evolution models run with the `ConstantMassBalance` model and the `RandomMassBalance` model, for 1000 years each. Both mass balance models emulate an equilibrium climate to keep the glacier in its initial equilibrium state. Therefore, the mass balance models must be initialized with the climatic period centered around the equilibrium year, i.e., `y0` $= t^*$. As explained above, the mass balance calibration depends, among others, on the chosen t^* (Section 2.1.2, see Calibration of the mass balance parameters). Hence, to run both evolution models with the identical climatic forcing, t^* must be equal for both. This is done by computing the temperature sensitivity μ^* for both models using the same t^* reference table (`oggm_ref_tstars_rgi6_histalp.csv`, corresponding to the flowline model). Additionally, the mass balance residual must be omitted during the model run ($\beta^* = 0$, as per the definition of μ^*, see Section 2.1.2). Each mass balance model runs three different climate scenarios defined by the temperature biases of $0\,°\mathrm{C}$, $-0.5\,°\mathrm{C}$ and $+0.5\,°\mathrm{C}$. These runs will be referred to as *equilibrium run*, run with *positive mass balance bias* and run with *negative mass balance bias*, respectively.

2.3.2 Autocorrelation function and Power spectral density

The correlation is a measure for the linear dependency between two (random) variables. A positive correlation coefficient close to $+1$ indicates a strong direct relationship between two variables, while a negative correlation coefficient close to -1 indicates a strong indirect relationship between two variables. A correlation coefficient around zero indicates that the variables are independent. The autocorrelation function (ACF) computes the correlation coefficient between a signal and a lagged copy of itself as a function of lag time k. A lagged copy of a signal is simply the signal shifted by time k. The intuition behind the ACF is the relation between past and present values of a signal. It is used find possible periodicities hidden in noisy signals and gives an estimate on how strong neighboring data points influence each other. A high autocorrelation at lag time k indicates that data points that are a time k apart have similar values. The ACF at lag time $k = 0$ is obviously one, since any signal correlates to 100 % with itself. The partial autocorrelation function (PACF) measures only the direct influence of values with lag time k, eliminating the effects of all shorter lag times (Box et al. 2015). Plots of both functions give a qualitative insight...

On of the simples models to describe a stationary random time series is an autoregressive-moving-average (ARMA) model. It predicts future values of a random variable based on a linear combination of past values and past error terms of said variable. The number of included lag terms is defined by the order p of the autoregressive term and the order q of the moving-average term. The number of statistically significant non-zero terms of the ACF and PACF can be used to estimate the order q and p of an ARMA(p,q) model, respectively (Box et al. 2015). The three stage model by Roe and Baker (2014) is an ARMA(3,3) model. While the goal of this work is not to define an ARMA glacier model, a brief analysis is provided in an attempt to quantify the autocorrelation.

The power spectral density (PSD) is the Fourier transform of the ACF. A Fourier transform decomposes a signal into a spectrum of frequencies, or much rather into a collection of frequency bins for the discrete Fourier transform used. Hence, the PSD is as function of frequency. The signal's power describes its energy per unit time. Thereby, the power is of unit [X^2] if the signal's unit is [X] and must not be actual physical power of unit [W]. The power density is the power normalized with the frequency bin width, hence the power density has the unit [X^2/Hz] if the frequency is measured in [Hz]. The PSD is used to find dominant frequencies in noisy signals.

The ACF, PACF and PSD are computed for the glacier length signal, in analogy to (Roe and Baker 2014). To avoid another N-of-1 experiment, the behavior of the volume/area scaling model is compared to the flowline model for the following six Alpine glaciers:

- Hintereisferner (RGI60-11.00897), Ötztal Alps, Austria

- Pasterze (RGI60-11.00106), Hohe Tauern, Austria
- Mer de Glace (RGI60-11.03643), Mont Blanc massif, France
- Glacier d'Argentière (RGI60-11.03638), Mont Blanc massif, France
- Großer Aletschgletscher (RGI60-11.01450), Bernese Alps, Switzerland
- Rhonegletscher (RGI60-11.01238), Urner Alps, Switzerland

These glaciers are selected because of their size and notoriety, however the choice is somewhat arbitrary. All glaciers are subjected to different random (white noise) climate conditions for 23 000 years. The remaining settings are analogous to the single glacier test case. The **`RandomMassBalance`** model is initialized around the respective *equilibrium year* for each glacier ($\texttt{y0} = t^*$), whereby both evolution model use the same t^* reference table. The mass balance residual is omitted ($\beta^* = 0$). The input climate shows no autocorrelation and a constant PSD curve and can therefore be classified as white noise with non-zero mean. To increase the amount of available data, each glacier is subjected to three different climate scenarios, specified by a temperature bias of $-0.5\,°\mathrm{C}$, $0\,°\mathrm{C}$ and $+0.5\,°\mathrm{C}$. The initial 3000 years during which the glaciers adjust to the changed climate are clipped, which leaves three different sizes of the same glacier, each in equilibrium condition. An ARMA model can only be applied to stationary time series. Stationarity is given if the mean and the variance are time independent and the signal shows no seasonality (for a formal definition Box et al. (e.g., 2015)). While the stationarity is easily determined manually for the glacier length signals, an Augmented Dickey–Fuller test (`statsmodels.tsa.stattools.adfuller`, Cheung and Lai (1995)) results in p-values far below 1 % for all signals.

The ACF is computed using the python function `statsmodels.tsa.stattools.acf` via a fast Fourier transform. The PSD is estimated using Welch's method. Welch's method reduces the variance in estimated power density by time-averaging, at the cost of frequency resolution (e.g., Welch 1967; Proakis and Manolakis 2007). The remaining 20 000 data points are divided into nineteen time windows with a size of 2000 data points and a 50 % overlap. The windows are tapered using the Hann function. For details about additional parameters see the default values of the python function `scipy.signal.welch`, which is used for the PSD computation.

2.3.3 Regional runs with all Alpine glaciers

Volume/area scaling applied to single glaciers gives only an order of magnitude estimation, since the scaling constant c is a globally averaged value. A potential relative error in c will be directly transfered to any volume estimation. Hence, volume/area scaling should only be applied to collections of glaciers (Bahr et al. 2015). This first regional simulations over all Alpine glaciers are based on an equilibrium climate, in analogy to the single glacier test case.

These simulations are intended to reflect the reality as good as possible. Hence, the default OGGM mass balance calibration is used (see Section 2.1.2). This means, each evolution model uses its own t^* reference table (which may result in different t^* for the same glacier depending on the evolution model) and the mass balance residual β^* is used for each run. Both evolution models run with the `ConstantMassBalance` model and the `RandomMassBalance` model, for 1000 years each. The mass balance models are initialized with `y0` $= t^*$ and run with the same three different temperature biases as before (0 °C, −0.5 °C, +0.5 °C).

2.3.4 Sensitivity experiments

... Before moving to the final experiments estimating future ice volume loss, it is necessary to determine the model's sensitivities. The following sensitivity analysis investigates the effects of the model-internal time scales and scaling parameters on the model behavior. Both parameter sets are specific to the scaling model, since they determine the response time scaling as well as the volume/length and volume/area scaling. For consistency, the sensitivity experiments are performed on Hintereisferner (RGI60-11.00897) as a single glacier test case and on all Alpine glaciers inside the HISTALP domain. For simplicity, only the `ConstantMassBalance` model with a temperature bias of +0.5 °C is used for all runs.

The scaling model estimates glacial evolution via the implemented response time scaling. Response time scaling adjusts the yearly change in geometry in relation to the total possible change using the model-internal response time scales for length and area, τ_L and τ_A, respectively (see Section 2.1.3 and 2.2.2 for details). However, the model-internal time scales are only a rough estimate and hence good possible tuning parameters. The sensitivity experiments compare the model output for different time scales, modified by a linear factor $\tau_{\text{sens.}} = f \cdot \tau$. Hereby, the factor f is only applied to τ_L, since τ_A is a linear function of τ_L. The default values with $f = 1$ serves as baseline. From there, the model-internal time scales are halved ($f = 0.5$) and doubled ($f = 2$).

The other obvious choice of tuning parameters are the scaling exponents and scaling constants. The scaling constants for volume/length and volume/area scaling, c_L and c_A, respectively, can be seen as random variables. The randomness stems from the statistically similar but not identical dimensionless parameters for length, area and volume varying from glacier to glacier. Thanks to the law of large numbers, the global scaling constants $c_L = 0.0180\,\mathrm{km}^{3-\mathrm{q}}$ and $c_A = 0.0340\,\mathrm{km}^{3-2\gamma}$ are a reasonable choice for a global ice volume estimation (Bahr et al. 2015). However, those parameters may be a bad fit for certain regions and therefore need calibration. While the scaling constants are not constant, the scaling exponents are. The volume/area scaling exponent was first derived as the slope of the linear regression of volume and area observations in log-log space (e.g., Chen and Ohmura 1990). Bahr et al. (1997) found that their values are fixed by the underlying physics and depend only on a single set of closure conditions. The closure conditions are in turn tightly bound by observations and different common closures lead to the same results. Hence, it is strongly advised to use the global values of $q = 2.2$ and $\gamma = 1.375$ for the volume/length and volume/area scaling. Additionally, even with different closure conditions could be justified, the area scaling exponent is bound $1.1\dot{6} \leq \gamma \leq 1.5$ by simple geometric reasoning (Bahr et al. 2015, Section 8.2).

As for the time scale sensitivity experiments, the global values of the scaling exponents serve as baseline. The next run uses custom scaling constant derived from a linear regression in log-log space but with a fixed slope corresponding to the global scaling exponents. The last run uses full custom scaling constants and exponents, again derived from a linear regression in log-log space. For reasons of simplicity, data points of volume, area and length are taken from the OGGM flowline model glaciers and not from observations. The inversion volume serves as glacier volume, the RGI area as surface area and the longest centerline as glacier length.

Since a linear regression can not be computed from a single data point, the Hintereisferner test case differentiates only between global and custom scaling constants (obtained by solving the scaling relations for c) while using the global default scaling exponents. The following two sets of scaling parameters are used:

(a) global (default) values of $c_L = 4.551\,\mathrm{m}^{3-\mathrm{q}}$, $q = 2.2$ for volume/length scaling and $c_A = 0.191\,\mathrm{m}^{3-2\gamma}$, $\gamma = 1.375$ for volume/area scaling

(b) custom scaling constants $c_L = 1.555\,\mathrm{m}^{3-\mathrm{q}}$ and $c_A = 0.252\,\mathrm{m}^{3-2\gamma}$ with the global and physically based scaling exponents $q = 2.2$ and $\gamma = 1.375$

For comparability, the sensitivity runs on Hintereisferner (RGI60-11.00897) are setup exactly the same as the test case (see Section 2.3.1). This means a fixed *equilibrium year* $t^* = 1927$ and no mass balance residual during the run. The regional Alpine runs are setup as before (see Section 2.3.3) and use the following three sets of scaling parameters:

(a) global (default) values of $c_L = 4.551\,\mathrm{m}^{3-\mathrm{q}}$, $q = 2.2$ for volume/length scaling and $c_A = 0.191\,\mathrm{m}^{3-2\gamma}$, $\gamma = 1.375$ for volume/area scaling

(b) custom scaling constants $c_L = 1.805\,\mathrm{m}^{3-q}$ and $c_A = 0.250\,\mathrm{m}^{3-2\gamma}$ with the global and physically based scaling exponents $q = 2.2$ and $\gamma = 1.375$

(c) custom scaling constants and scaling exponents $c_L = 0.244\,\mathrm{m}^{3-q}$, $q = 2.517$ for volume/length scaling and $c_A = 0.117\,\mathrm{m}^{3-2\gamma}$, $\gamma = 1.441$ for volume/area scaling

Disclaimer: As already mentioned, this work is focused on the model behavior much rather than an absolute ice volume estimation. While scaling constants and exponents based in observations would be preferable, the values of the custom scaling parameters are less important as long as the are different from the global values. In fact, the sensitivity experiment could easily be conducted with a set of fabricated exponents. For the same reason, it is inconsequential that scaling exponents derived from a numerical model may differ depending on the model's closure conditions (Bahr et al. 2015, Section 8.9). It is hereby noted that this potential source for errors is acknowledged, and the flowline model is not tested for its closure conditions. While the custom scaling exponents lie within the range of physical sensible values, finding closure conditions supporting the computed values would go (far) beyond the scope of this work.

2.3.5 Commitment runs

One of the easiest ways of estimating future glacial evolution are so called *commitment runs*. The name stems from the *commitment* to a given climate, which is then held constant for the entire run. For example, applying the current climate to the current glaciers for the next, say, 100 years, would give a—naively optimistic—lower bound of the expected glacial retreat. As for the first regional run, ...

While a climate scenario with yearly fluctuations is more physical than a completely constant climate, the resulting ice volume changes are comparable (see Section **??**). Which is why, the following experiments can be limited to the `ConstantMassBalance` model without any loss of information. For a more tangible experiment, the `ConstantMassBalance` model is initialized with todays climate. Todays climate is assumed to be the average over the most recent 31 years. For the HISTALP dataset this corresponds to the period from 1984 to 2014 with `y0 = 1999`. Since we live in a period of global warming, additional positive temperature biases of 0 °C, +0.5 °C and +1.5 °C are used. Again, the default OGGM mass balance calibration is used assuring the most physical outcome.

Chapter 3

Results

Equilibrium experiments are useful tools to asses the behavior of glacier models. Thereby, an glacier in equilibrium state is subjected to a step change in cliamte and its evolution is modeled until a new equilibrium is reached. The OGGM provides two climate scenarios for such equilibrium experiments, the `ConstantMassBalance` model and the `RandomMassBalance` model (see Section 2.2.1 for implementation details). The experiments are performed on seleceted or all Alpine glaciers, using the HISTALP dataset (Auer and Böhm 2007) as climate input data. The baseline climate for each glacier comes from a 31-year period centered around the *equilibrium year* t^*. An additional temperature bias of 0 °C, −0.5 °C and +0.5 °C results in a neutral, positive and negative step change in mass balance, respectively. The detailed experimental setup can be found in Section...

The first qualitative conclusions are drawn from the temporal evolution of length, surface area and ice volume. We are looking at selected single glaciers as well as at the regional scale, i.e. at the sum over all glaciers in the HISTALP domain. Scaling methods applied to a single give only an order of magnitude estimation (cf. Bahr et al. 2015, Section 8.5), which is accounted for in the following analysis. More quantitative results are drawn from an autocorrelation analysis and a power spectral density analysis, inspired by Roe and Baker (2014).

3.1 Single glacier test case

This first test case is intended to get a feel for the volume/area scaling model and set the stage for the following experiments. The model estimates the evolution of the Hintereisferner (RGI60-11.00897) over 1000 years for three different climate scenarios: an equilibirum climate, a positive and a negavite step change in climate defined by a temperature bias of ±0.5 °C. Additionally, two different mass balance models are used. The `ConstantMassBalance` model simulates a perfectly constant climate and the `RandomMassBalance` introduces random year-to-year variability, as the names may suggest. For details about the experimental setup see Section 2.3.1.

TL;DR: Single glacier test case

- Both evolution models produce the same qualitative results, advancing under colder climates and shrinking under warmer climates. The temporal correlation between both models under a random climate is satisfying.
- The volume/area scaling model drastically underestimates changes in glacier geometry compared to the flowline model (up to four times). For example, the relative volume changes for the run with positive mass balance bias amount to +17 % for the volume/area scaling model and +71 % for the flowline model.
- The volume/area scaling does not account for the mass balance-elevation feedback and therefore produces highly symmetrical results between the positive and negative step change in air temperature. This symmetry can also be seen in the e-folding response times.
- The e-folding response times are much shorter for the volume/area scaling model. For example, the volume response times for the run with positive mass balance bias amount to 39 yr for the volume/area scaling model and 139 yr for the flowline model.
- The volume/area scaling model does not show an asymptotic adjustment but behaves more like a damped harmonic oscillator, whereby the model-internal time scale acts as damping factor.

The volume/area scaling model behaves as expected and produces the same qualitative results as the flowline model. The model glacier stays in an approximate equilibrium using the climate around t^*, decreases and increases in size for a positive and negative temperature bias of $\pm 0.5\,^\circ\mathrm{C}$. However, the volume/area scaling model underestimate changes in geometry compared to the flowline model. This is true for both mass balance models, whereby the `RandomMassBalance` model produces more short term variability (most obviously). Figure 3.1 shows the time series for ice volume, surface area and glacier length for all climate scenrios and both evolution models.

Subjected to the same random climate, glacier advances and retreats correlate nicely between the two evolution models (with correlation coefficients between 0.44 and 0.72). This is no surprise, given that the implementations of the mass balance models are almost identical. Thereby, the ice volume exhibits the highest year-to-year variability, since the volume change happens instantaneously (i.e., over a single time step) as a function of specific mass balance and surface area. The changes in surface area and glacier length are smoother, accounting for the glacier's response time. The flowline model shows less short term and stronger long term variabilities than the volume/area scaling model, indicating shorter response times for the volume/area scaling model. This assumption is backed by the model behavior under the constant climate scenarios. Qualitatively speaking, the flowline model takes longer to reach a new equilibrium (after around 400 years) than the volume/area scaling model (after around 200 years). A quantitative analysis of the response times follows after the evaluation of the equilibrium values.

For the following discussion about the equilibrium values only the constant climate scenarios are considered, if not stated otherwise. It is assumed that the model glacier has reached a new equilibrium after 1000 years of evolution. Hence, the equilibrium values are taken as the final values at year `t = 1000`. This assumption seems valid, given that the fluctuation of volume, area and length are only in the order of 0.01 % over the last 200 years of the simulations. The only exception forms the glacier length of the flowline model subjeceted to a positive mass balance bias. Under that climate scenario, the final equilibrium length oscillates between 9.9 km and 10 km. The glacier jumps back and for one grid cell, due to the spatial resolution of 100 m of the flowline model. Hence, the equilibrium length is assumed to be average between both values. Table 3.1 shows all equilibrium values in response to the positive and negative step change in equilibrium climate.

The most apparent result is that the volume/area scaling model underestimates all changes in glacier geometry when compared to the flowline model. While the volume/area scaling model predicts a volume change of around ±16.5 %, the flowline ice volume increases by 71 % and decreases by 42 %, for the positive and negative mass balance bias, respectively. In other words, the flowline glacier grows more than four times larger and shrinks more than two and a half times smaller than the volume/area scaling glacier. The equilibrium surface area is slightly less underestimated, with a change of ±12 % for the volume/area scaling model versus changes of +33 % and −23 % for the flowline model. The glacier length of the volume/area scaling model does hardly change at all. The maximum year-to-year variation under any climate scenario shows slightly more than six meters, which is about 1 % of the initial value and therefore hardly physical sensible. This results in an length change of ±7.5 % for the volume/area scaling model, which is roughly five to six times less than the changes of +44 % and −39 % for the flowline model. The values proof that the volume/area scaling model cannot, self-evidently, resolve all processes as a dedicated ice physics models can.

The changes in glacier geometry produces by the volume/area scaling model are highly symmetrical. Absolute changes ice volume, surface area and glacier length differ by a maximum of 1 % between positive and negative mass balance bias. This can be explained by the scaling mass balance model. For both implementations of the constant mass balance model, the specific equilibrium mass balance can be approximated as a linear function of the temperature bias through the origin ($r^2 > 99.9\,\%$), for small enough temperature biases between $-1\,^{\circ}\mathrm{C}$ and $+1\,^{\circ}\mathrm{C}$. Thereby, the linear function for the flowline model has a steeper slope than for the volume/area scaling model. The resulting initial specific mass balances are $+306\,\mathrm{mm\,we.\,yr^{-1}}$ and $-322\,\mathrm{mm\,we.\,yr^{-1}}$ for the flowline model and $+210\,\mathrm{mm\,we.\,yr^{-1}}$ and $-218\,\mathrm{mm\,we.\,yr^{-1}}$ for the volume/area scaling model. As can be seen, the initial mass balance values are symmetrical for both evolution models and can therefore not be the cause of the volume/area scaling model's symmetric equilibrium results. However, the question should not be "What makes the volume/area scaling model results symmetric?" but much rather "What allows the flowline model to produces asymmetric results?". And the answer is the mass balance-elevation feedback. The flowline model continuously adjusts the surface elevation of each grid cell and passes the elevation information onto the mass balance model (implementation note: the mass balance feedback can be adjusted via the `mb_elev_feedback` parameter of the `FlowlineModel` class). Suppressing the mass balance-elevation feedback for the flowline model run results in a volume change of +38 % and −34 %. The results are symmetric and lower than with mass balance-elevation feedback in place. The relative changes in ice volume are reduced to about twice the values produces by the volume/area scaling model.

However, the scaling constant c is a random variable which can vary drastically from glacier to glacier. It is possible that the global mean value of $c = 0.034\,\mathrm{km}^{3-2\gamma}$ is a bad fit for the characteristics of Hintereisferner. A detailed look at the model's sensitivity to the scaling constant is provided in Section 3.4.2.

The responses of the volume/area scaling model and the flowline model to a step change in climate are qualitatively similar but do not compare quantitatively. While the absolute equilibrium values are still in the same order of magnitude, they differ substantially. But what about the time domain? The following paragraphs look at temporal characteristics of the glacier model's response.

The implementation of the volume/area scaling model includes the corresponding response time scaling to estimate temporal changes (see Section 2.2.2). For a proper response time scaling, the length response time scale τ_L and the area response time scale τ_A must be estimated. The length response time scale can be estimated as ratio between ice volume and mass turnover (Jóhannesson et al. 1989), the area response time scale than follows from geometric considerations. The time scales computed for the Hintereisferner under a constant equilibrium climate amount to

$\tau_L \approx 52\,\mathrm{yr}$ and $\tau_L \approx 18\,\mathrm{yr}$. Those values are rather low compared to other findings of $\tau_L \approx 100\,\mathrm{yr}$ (Greuell 1992; Schuster 2020). However, it is possible that the used time scales are merely model parameters and do not correspond to the typically used e-folding time scales.

Processes evolving exponentially to an equilibrium can be characterized by their e-folding response time. The assumption that a glacier's geometry changes exponentially is valid for small enough perturbations in climate. The e-folding response time is computed as the time after which the initial difference between a glacier's geometric property (such as ice volume, surface area or glacier length) and its new equilibrium value has decreased by a factor of $1 - \mathrm{e}^{-1} \approx 0.63$. For comparability, e-folding time scales are computed for both evolution models and all geometric properties. The values can be found in Table 3.2. As was to be expected, volume response times τ_V are smallest, followed by τ_A and τ_L. As already qualitatively estimated above, the volume/area scaling model adjust between one and a half times and three and a half times faster to the temperature perturbation of 0.5 °C as the flowline model does. This is especially visible for the growing glacier, where the flowline model takes about 100 yr longer to reach a new equilibrium than the volume/area scaling model does ($\tau_{V,\mathrm{fl}} = 139\,\mathrm{yr}$ vs. $\tau_{V,\mathrm{vas}} = 39\,\mathrm{yr}$).

The results of the volume/area scaling model are again very symmetric between the positive and negative temperature perturbation. The volume/area scaling response time scales range within 9 % of each other, while the flowline response time scales vary up to 55 %. Suppressing the mass balance-elevation feedback for the flowline model runs results in symmetric result, whereby the values for the run with negative mass balance bias do only change by a maximum of five years. It has to be noted, that the e-folding length response time for the volume/area scaling model $\tau_{L,\mathrm{vas}} \approx 80\,\mathrm{yr}$ is about thirty years (≈60 %) longer than the model-internal time scale. However, the volume/area scaling model does not show an asymptotic or exponential adjustment. The adjustment of glacier geometries looks like the signal of an underdamped oscillator, with a strongly discernible overshoot. The damping factor seems to be controlled by the model-internal time scale, which could allow for an additional calibration parameter. A closer look at this oscillatory behavior is provided in Section

3.2 Autocorrelation function and Power spectral density

The autocorrelation function (ACF), partial autocorrelation function (PACF) and power spectral density (PSD) give insight into the periodicity and dominant fre-

Table 3.1: Hintereisferner (RGI60-11.00897) equilibrium values after 1000 years of model evolution in response to a step change in climate of $\Delta T = \pm 0.5\,^{\circ}\mathrm{C}$ relative to the average climate between 1912 and 1942. Percentage values in parenthesis indicate normalized changes in respective to their initial values.

	Length [km]		Area [km^2]		Volume [km^3]	
Initial values						
V/A scaling	4.89		8.04		0.60	
Flowline	6.90		8.04		0.80	
$\boldsymbol{\Delta T = -0.5\,^{\circ}\mathrm{C}}$						
V/A scaling	5.26	(+7 %)	9.02	(+12 %)	0.70	(+17 %)
Flowline	9.95	(+44 %)	10.68	(+33 %)	1.37	(+71 %)
$\boldsymbol{\Delta T = +0.5\,^{\circ}\mathrm{C}}$						
V/A scaling	4.52	(−8 %)	7.08	(−12 %)	0.50	(−16 %)
Flowline	4.20	(−39 %)	6.17	(−23 %)	0.47	(−42 %)

Table 3.2: e-folding time scales for Hintereisferner (RGI60-11.00897) in response to a step change in climate of $\Delta T = \pm 0.5\,^{\circ}\mathrm{C}$ relative to the average climate between 1912 and 1942. Time scales are computed for changes in ice volume, surface area and glacier length, denoted as τ_V, τ_A and τ_L, respectively.

	τ_L [yr]	τ_A [yr]	τ_V [yr]
$\boldsymbol{\Delta T = -0.5\,^{\circ}\mathrm{C}}$			
V/A scaling	85	57	39
Flowline	174	159	139
$\boldsymbol{\Delta T = +0.5\,^{\circ}\mathrm{C}}$			
V/A scaling	80	52	36
Flowline	123	107	79

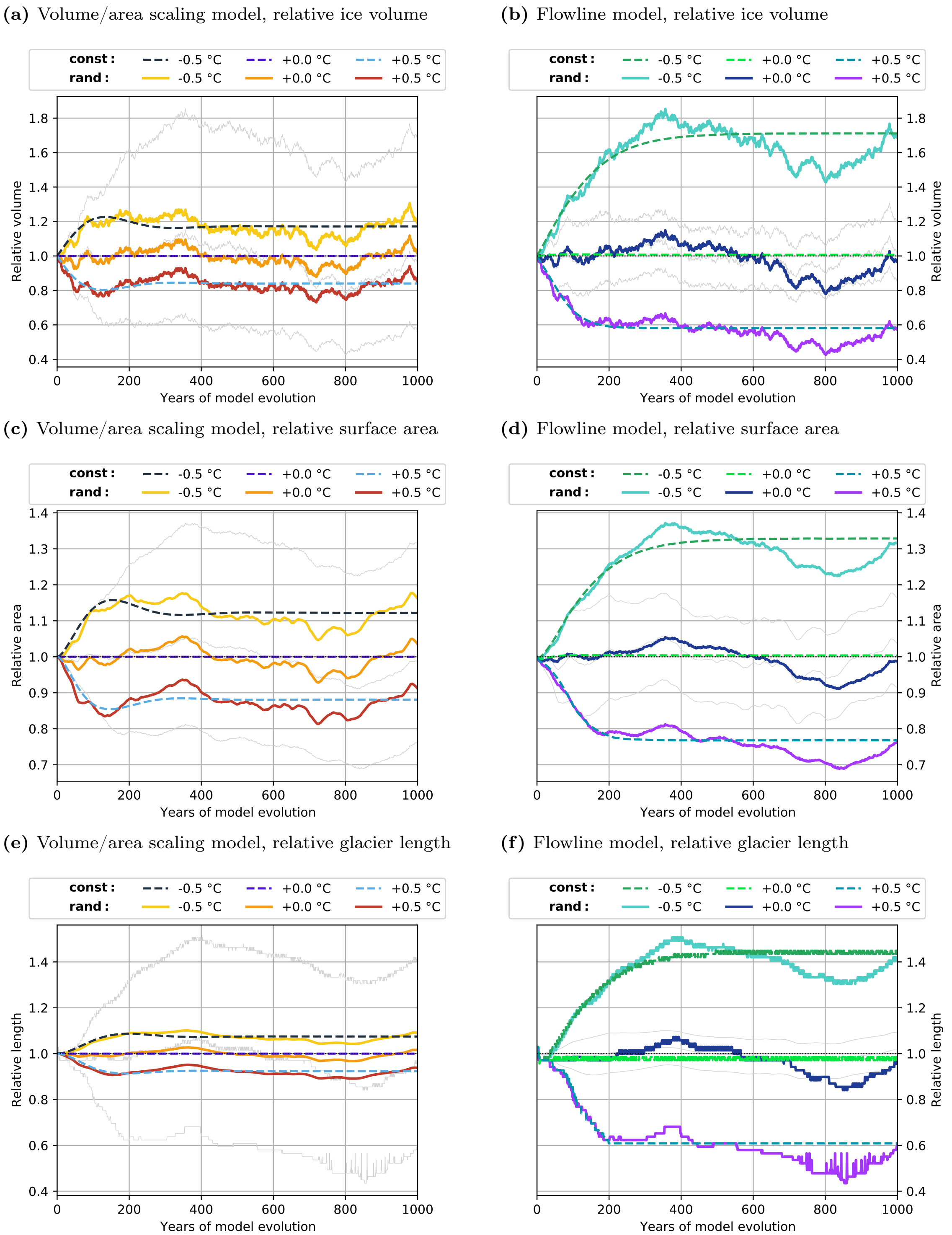


Figure 3.1: Temporal evolution of ice volume in (a) and (b), surface area in (c) and (d) and glacier length in (e) and (f) for Hintereisferner (RGI60-11.00897). The shown values area normalized with their respective initial values. The left panels show the result of the volume/area scaling model, the right panels show the results of the flowline model. Solid lines represent the random climate scenarios, while dashed lines represent the constant climate scenarios. All climate scenarios are based on an equilibrium climate. The applied temperature biases of $-0.5\,^{\circ}$C, $0\,^{\circ}$C and $+0.5\,^{\circ}$C are color coded, see legend for details. The dotted line indicates the initial volume. The light gray lines represent the volume evolutions of the other model, to facilitate comparisons.

quencies of a given signal. Hereafter, the length signals of different model glaciers subjected to a constant climate with random year-to-year variabilities are used. The experiment is setup in analogy to the single glacier test case, since ACF, PACF and PSD should be performed on the time series of a single glacier. To avoid a N-of-1 experiment, five more individual glaciers are investigated, namely the Pasterze, Mer de Glace, Glacier d'Argentière, Aletschgletscher and Rhonegletscher. All glaciers are subjected to different random (white noise) climate conditions for 20 000 years. The spinup period during which the glaciers adjust to the different climate is clipped, with three different sizes of the same glacier. Hence, the temperature bias can be seen as label for glacier size rather than climate. For details about the experimental setup see Section 2.3.2. The ACF for lag times up to 200 years is shown in Figure 3.2, the PACF for lag times up to 20 years in Figure 3.3 and the PSD in Figure 3.5.

Autocorrelation function

As a general observation over all glaciers and climate scenarios, the ACF shows high correlations for the first few lag times before decreasing exponentially. This points at an autoregressive (AR) term and a moving-average (MA) term in the data. An autoregressive-moving-average (ARMA) model predicts future values of a random variable based on a linear combination of past values and past error terms of said variable. This makes intuitive sense for glaciers, since the past and current glacier size and difference to the equilibrium value have a direct influence on next years glacier size. For example, the three-stage model of Roe and Baker (2014) is an ARMA(3,3) model and performs well compared to a flowline model. However, let's look at the differences between volume/area scaling and flowline model first.

The ACF of the volume/area scaling lengths are almost identical between different sizes of the same glacier, indicating that the glacier size has little to no effect on the transient behavior of the model. The volume/area scaling model represents a glacier as a simple cuboid, there is absolutely no information about the form of a glacier. The absolute dimesions of that cuboid seem to have less of an effect than other parameters. Some glaciers, like the Glacier d'Argentière, the Aletschgletscher and the Rhonegletscher show statistically significant negative correlations for higher lag terms between 100 and 300 years, while the others show very little to no significant negative correlation at all. However, no apparent relation between the strength of negative correlations for later lag times and any other glacier parameters, like the average slope or the model-internal lag times, was found.

The flowline model is able to represent different glacial geometries and grasp individual responses under different equilibrium climates, which can be seen in the vastly different ACFs. They differ from glacier to glacier, but also for different sizes of the same glacier. However, there are no discernible patterns, which again confirms the notion that the OGGM flowline model is capable of modeling each glacier's individual response. The following list points to some particular observations:

- for Hintereisferner the ACFs of the flowline model are stronger than those of the volume/area scaling model, while for Mer de Glace and Großer Aletschgletscher they are lower (for all tested climate scenarios, i.e., all different sizes)

- the flowline model of the Pasterze shows a strong autocorrelation under the equilibrium climate, i.e., for its medium size, (>0.7 for lags times between 0 and 95 years, still >0.43 for 200 years lag time, statistically significant up until a lag time of 232 years), while under a warmer and colder equilibrium climate ($\pm 0.5\,°C$) the autocorrelation of all lag times is lower than for the volume/area scaling model

- similarly, the flowline model of the Glacier d'Argentière shows a strong autocorrelation under the warmer equilibrium climate ($+0.5\,°C$), while the autocorrelation under the other two climate scenarios is even lower than the ACF of the volume/area scaling model

The only observation made for all glaciers, it that the volume/area scaling model shows a stronger autocorrelation for shorter lag time (i.e., less than about 20 years) than the flowline model. This is true even for glaciers, where the autocorrelation of the flowline mode is generally stronger than for the volume/area scaling model (e.g., Hintereisferner).

Strong correlations for short lag times influence all following lag terms. For example, an exponentially decaying signal which halves its value with each time step $\Phi(t) = 0.5\Phi(t-1)$ will have an autocorrelation of 0.5 for lag 1, 0.25 for lag 2, 0.125 for lag 3, and so on. Depending on the sample size these values will be statistically significant, even though only one lag term is included in the definition of the signal (i.e., AR(1) process). The partial autocorrelation function (PACF) measures only these direct influences, eliminating the effects of all shorter lag times. All the PACFs of the volume/area scaling model lengths show a strong positive correlation for lag times of 1 year, followed by a strong negative correlation which then decreases slowly towards zero. Correlations for lag times 10 and greater are not or only marginally significant. Again, no discernable differences between different sizes of the same glacier and very little differences from one glacier to another. The PACFs for the flowline model lengths show a high correlation for lag time of 1 year, before decreasing towards zero. The decrease towards statistical insignificant correlations happens faster for some glaciers (e.g., only lag 1 and 2 show a significant correlation for the Pasterze and the Aletschgletscher under the warmer equilibirum) and slower for others (e.g., the Rhonegletscher shows significant correlations for lag times up to 5 and 6 years, depending on the temperature bias). Hereby, there are differences in PACFs between different sizes of the same glacier. For higher lag times between 10 and 20 years there are some negative correlations, even if only marginally significant.

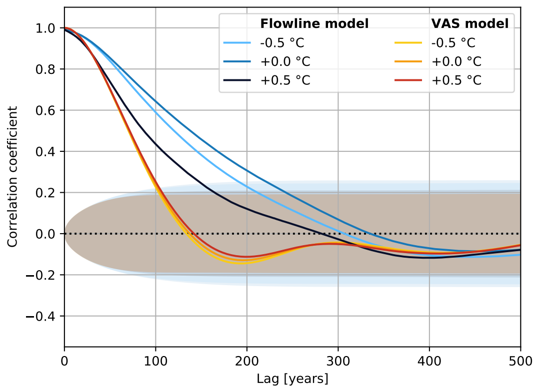
The number of statistically significant terms of the PACF informs on the order p of the AR(p) model. The order specifies the number lag terms are considered. Analogously, the order q of the MA(q) model can be estimated from the number of significant terms of the ACF. By this definition, the volume/area scaling and flowline model length could be modeled with an ARMA(11,103) and ARMA(5,150) model, respectively. Hereby, the orders are taken as average over all glaciers and climate scenarios. While an AR model with 11 lag terms is big but still feasable, a MA model with over 100 lag terms is neither practicable nor does it make sense. Especially, since Roe and Baker (2014) use an ARMA(3,3) model to produce comparable results to a flowline model.

It is not the intent of this work to investigate the relation between a glacier's geometry and its ACF, neither to fit an ARMA model to the data. Hence, this qualitative first look has to suffice. However, it is most notable that the OGGM flowline model behaves differently not only for different glaciers, but also for different sizes of the same glacier. The *one size fits all* approach of the volume/area scaling model produces more homogenous results, the ACFs and PACFs are mostly independent of a glacier's size.

Power spectral density

Glaciers act as low-pass filters for the natural climate variability. A low pass filter passes lower frequencies while attenuating higher frequencies. This can be seen in the PSD for all glaciers under all three equilibrium climates. The power density stays in a constant range for frequencies up to $3 \cdot 10^{-3}\,\mathrm{yr}^{-1}$ (corresponding to signals with a period of over 300 years) and decreases for higher frequencies. This makes intuitive sense, since changes in glacier length are mainly driven by long term climatic trends and less by inter-annual variabilities in the climatic forcing. A single year with low temperatures and high (cold season) precipitation has less impact on the glacier size than a decade of slightly above average temperatures. While the overall shape of the PSDs are similar for the volume/area scaling and flowline model, there are some key differences. The length changes estimated by the volume/area scaling model are small compared to the flowline model. Hence, the overall power of the flowline model is higher across all frequencies. In agreement with the symmetric behavior discussed before, the PSD's of the volume/area scaling model are practically identical for different sizes of the same glacier. The flowline model produces less coherent

(a) RGI60-11.00897 - Hintereisferner



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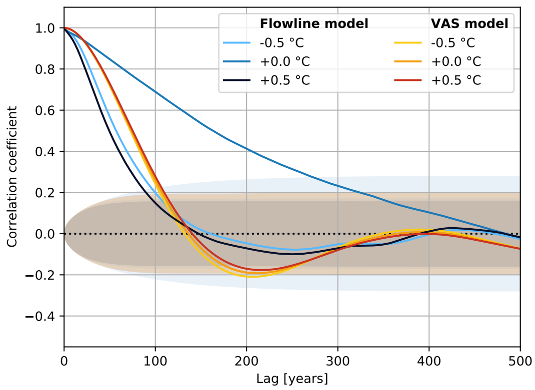
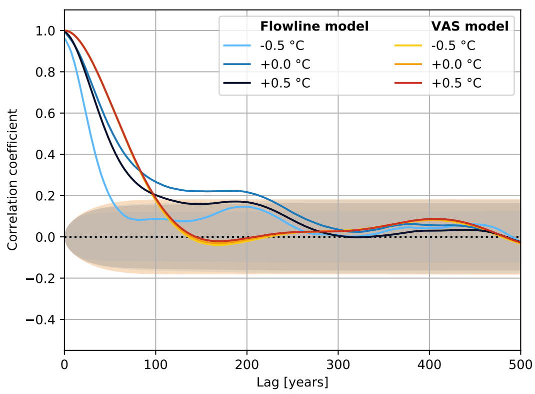
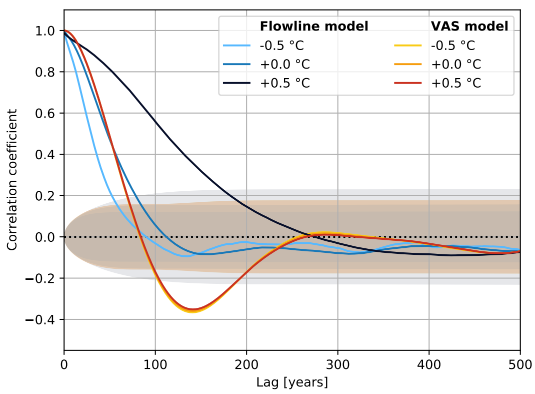


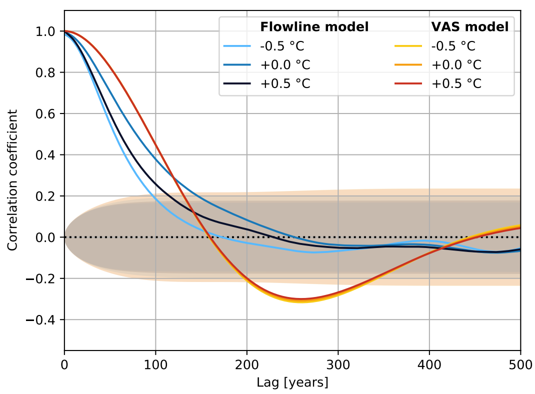
(c) RGI60-11.03643 - Mer de Glace



(d) RGI60-11.03638 - d'Argentière



(e) RGI60-11.01450 - Großer Aletschgletscher



(f) RGI60-11.01238 - Rhonegletscher

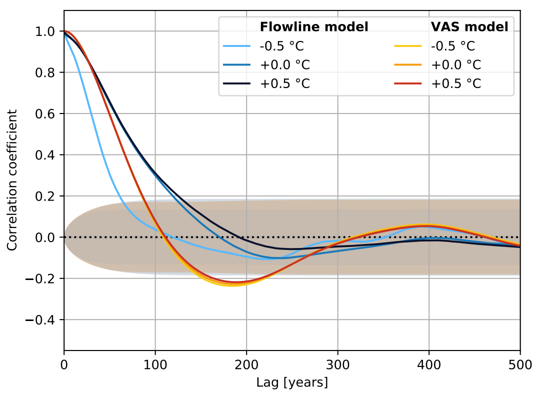
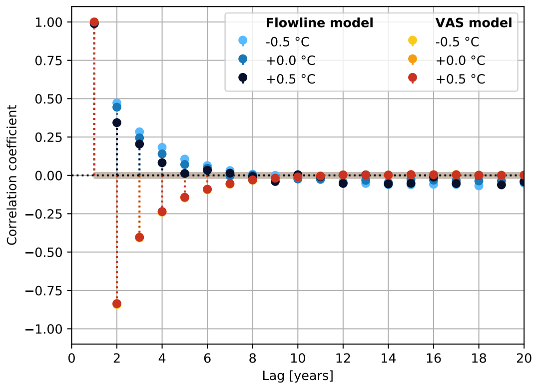


Figure 3.2: Autocorrelation function of modeled length for lag times between zero and 200 years. Different lines represent different combinations of evolution model and climate scenario. The random climate scenario is based on an equilibrium climate, with different temperature biases. Cyan, blue and purple lines represent the flowline model, while yellow, orange and red lines represent the volume/area scaling model, with a temperature bias of $-0.5\,^{\circ}\mathrm{C}$, $0\,^{\circ}\mathrm{C}$ and $+0.5\,^{\circ}\mathrm{C}$, respectively. The 99 % confidence intervals are shaded in the corresponding colors.

(a) RGI60-11.00897 - Hintereisferner



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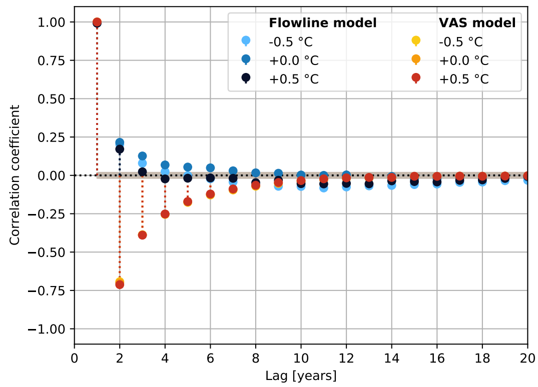

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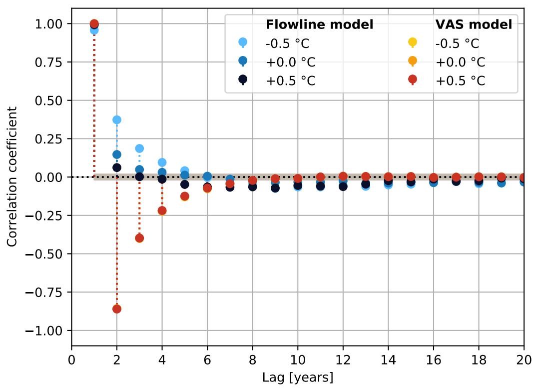

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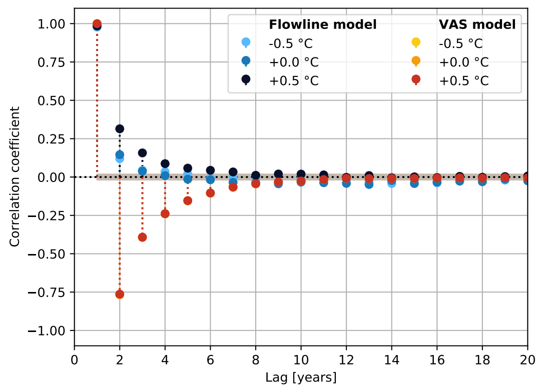
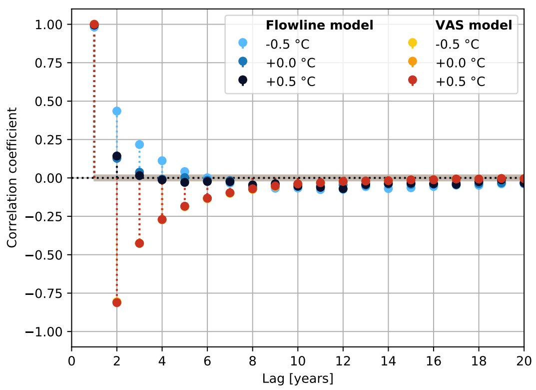


(e) RGI60-11.01450 - Großer Aletschgletscher



(f) RGI60-11.01238 - Rhonegletscher

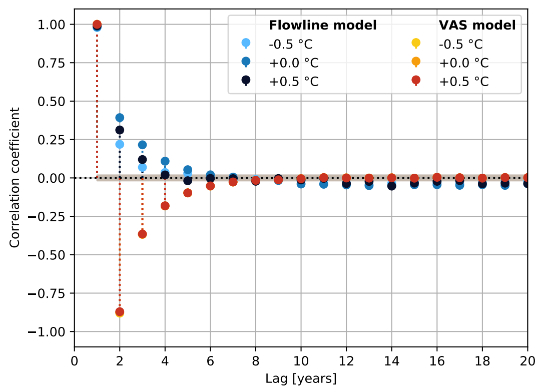


Figure 3.3: Partial autocorrelation function of modeled length for lag times between zero and 200 years. Different lines represent different combinations of evolution model and climate scenario. The random climate scenario is based on an equilibrium climate, with different temperature biases. Cyan, blue and purple lines represent the flowline model, while yellow, orange and red lines represent the volume/area scaling model, with a temperature bias of $-0.5\,^{\circ}\mathrm{C}$, $0\,^{\circ}\mathrm{C}$ and $+0.5\,^{\circ}\mathrm{C}$, respectively. The 99 % confidence intervals are shaded in the corresponding colors.

PSDs between the runs of the same glacier, whereby there is no discernible relation between shape of PSD and glacier size. Additionally, the flowline model shows a rather constant power density for lower frequencies. This is due to the discrete changes in glacier length of the flowline model, the resolution depends on the grid size (100 m in this case). The volume/area scaling model length is a continuous variable, resulting in a constantly decreasing power density. While the absolute values are different, the slope of the PSDs is comparable between volume/area scaling and flowline model. This indicates that both models resolve the same processes.

3.3 Regional runs with all Alpine glaciers

Volume/area scaling should not be applied to individual glaciers but to populations of glaciers (Bahr et al. 2015). This is were the scaling approach shows its strength. The law of large number assures a reasonable estimation of the collective glacier ice volume, since random errors will be canceled out by each other. This section compares the behavior of volume/area scaling model applied to all Alpine glaciers with differnte climate scenarios to the OGGM flowline model, as was done above for a single glacier (see Section 3.1). In contrast to the single glacier test case, the volume/area scaling model and the flowline model each use their respective t^* reference table. Additionally, a possible mass balance residual β^* is applied. This ensure to produce results as physical as possible. Again, both mass balance models simulate different climates with the same three temperature biases as above, based on the equilibirum period centered around `y0` $= t^*$. For more details about the experimenta setup see Section 2.3.3. The time series plots of normalized and absolute volume are shown in Figure 3.6.

Both evolution models run for 1000 years with the `ConstantMassBalance` model and the `RandomMassBalance` model. A random climate with its year-to-year fluctuations is obviously more physical than a completely constant climate. However, the resulting changes in glacier ice volume under both climate scenarios are almost identical. Over the last 200 years of the simulations, the differences in aggragate ice volumes between the constant and random climate scenario average around 0.6 % to 0.7 % for the volume/area scaling model and 0.7 % to 2.3 % for the flowline model, depending on the temperature bias (see 3.6). This makes intuitive sense, considering that glaciers act as natural low-pass filters for climatic variabilities. Short time climatic variability has little to no effect on a glacier's ice volume, and especially not on the aggregate ice volume of an entire region. Therefore, the following discussion is simplified by only considering the constant climate scenarios.

The volume/area scaling model estimates a total Alpine ice volume of 139 km^3 (+6 %), 115 km^3 (−12 %) and 95 km^3 (−27 %), while the flowline model estimates a

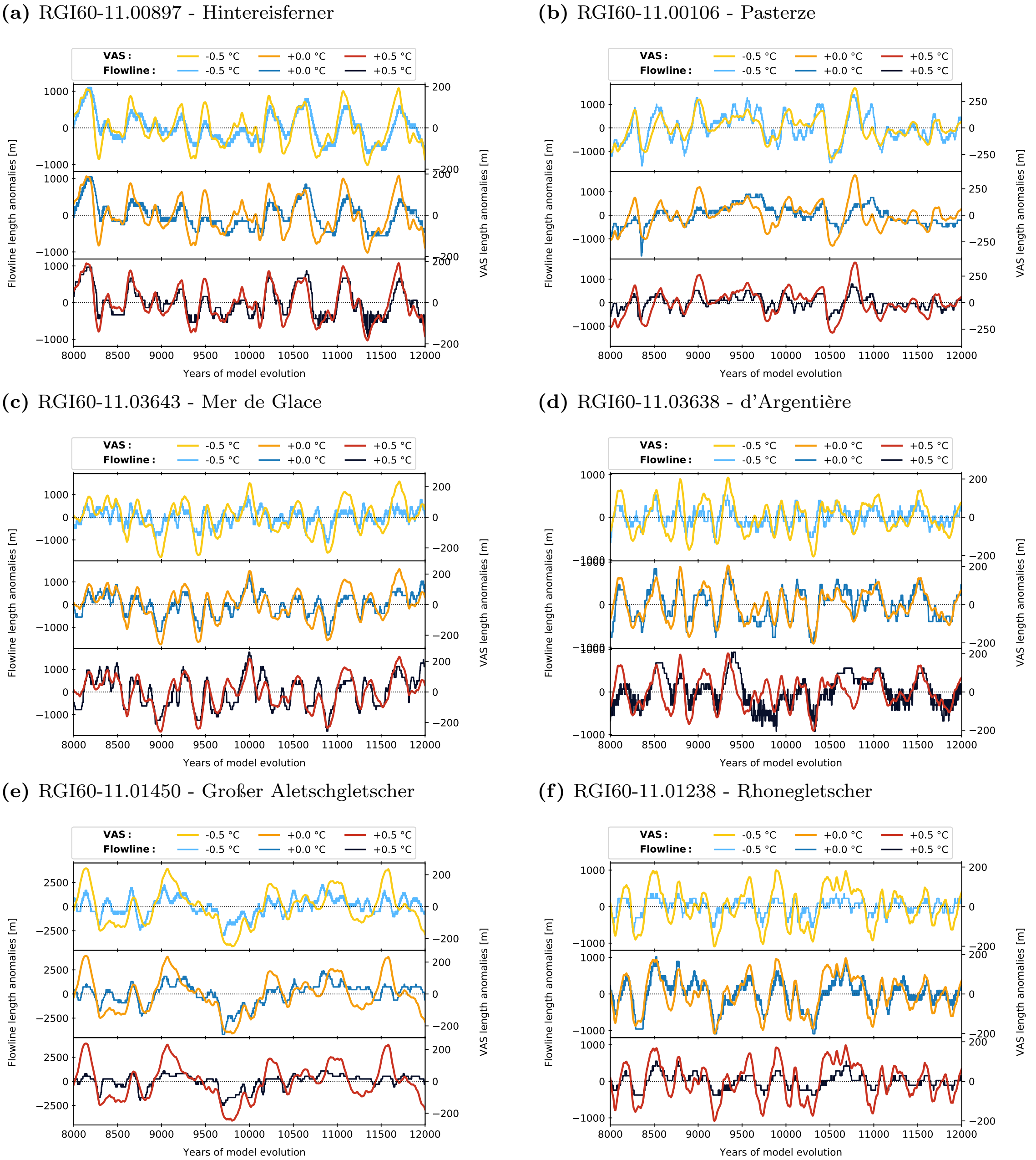
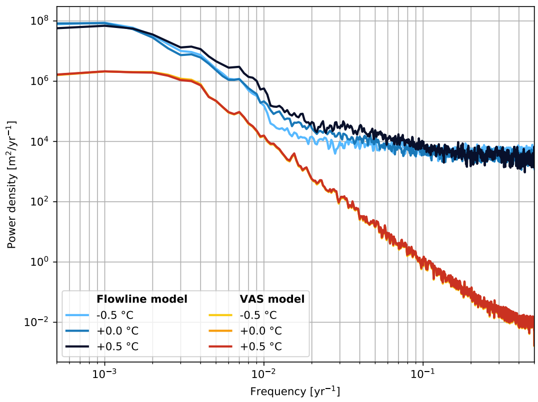
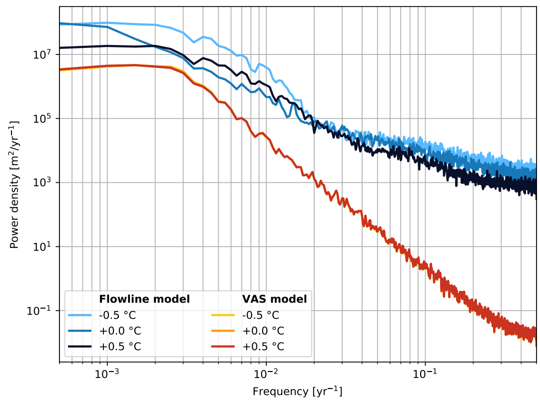


Figure 3.4: Power spectral density of modeled glacier length for different alpine glaciers. Different lines represent different combinations of evolution models and climate scenarios. The climate scenarios are based on a randomized equilibrium climate, with different temperature biases. Cyan, blue and purple lines represent the flowline model, while yellow, orange and red lines represent the volume/area scaling model, with a temperature bias of $-0.5\,^{\circ}$C, $0\,^{\circ}$C and $+0.5\,^{\circ}$C, respectively. Note the differences in y-axis scales.

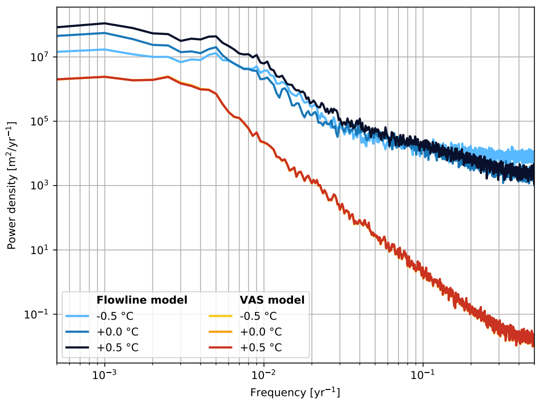
(a) RGI60-11.00897 - Hintereisferner



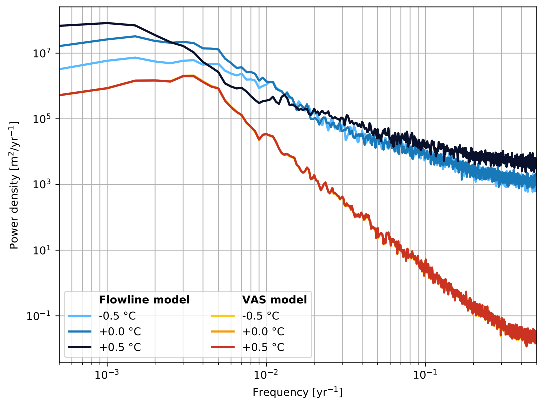
(b) RGI60-11.00106 - Pasterze



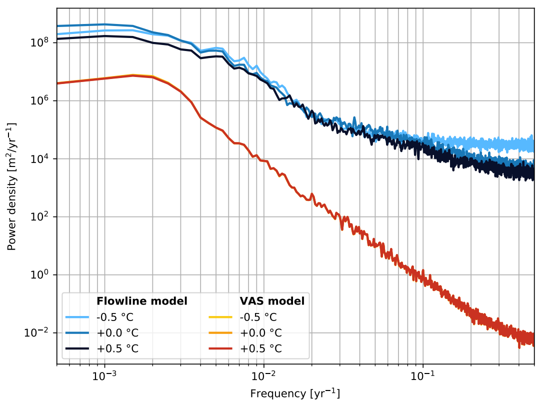
(c) RGI60-11.03643 - Mer de Glace



(d) RGI60-11.03638 - d'Argentière



(e) RGI60-11.01450 - Großer Aletschgletscher



(f) RGI60-11.01238 - Rhonegletscher

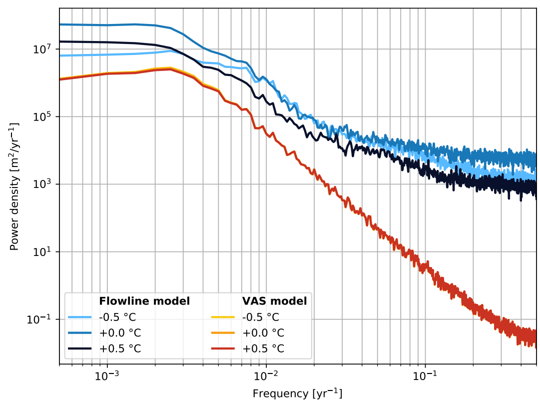


Figure 3.5: Power spectral density of modeled glacier length for different alpine glaciers. Different lines represent different combinations of evolution models and climate scenarios. The climate scenarios are based on a randomized equilibrium climate, with different temperature biases. Cyan, blue and purple lines represent the flowline model, while yellow, orange and red lines represent the volume/area scaling model, with a temperature bias of $-0.5\,^{\circ}$C, $0\,^{\circ}$C and $+0.5\,^{\circ}$C, respectively. Note the differences in y-axis scales.

total Alpine ice volume of 236 km^3 (+45 %), 147 km^3 (−10 %) and 86 km^3 (−47 %), for a temperature bias of −0.5 °C, 0 °C and +0.5 °C, respectively. Both evolution models adjust their initial ice volume downwards under equilibrium climate. This can be explained by the mass balance residual β^* and indicates that either (a) the 2003 RGI geometries are not sustainable by any 31-period in the HISTALP records or (b) the 2003 RGI glaciers are in a transient state and still retracting even after the climate is held constant (Maussion et al. 2019). This sag could be eliminated by a spin-up period, but since the absolute values are of less interest for now it seems unnecessary. Besides that, all characteristics of the volume/area scaling model found for the Hintereisferner test case can be seen here as well. The volume/area scaling scaling model underestimates the changes in ice volume, produces symmetric results and adjusts faster to a given step change in climate. Even the oscillatory behavior is still found on the regional scale. All in all, the volume/area scaling model does a bad job if we consider the flowline model results as the reality. However, The next section explores the possibility of tuning the volume/area scaling model via different parameters of the used scaling realtions. This also provides a range of possible results and serves as uncertainty estimation.

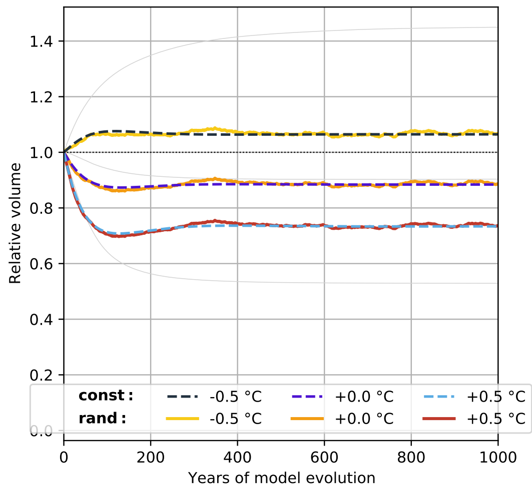
3.4 Sensitivity experiments

All the experiments performed above show quite large differences in projected ice volume change between the volume/area scaling model and the flowline model. However, the "out-of-the-box" scaling model is maybe not the best fit for the Alps and it is definitely not a good fit for any single glacier. Hence, a set of tuning parameters may improve the model performance. The most obvious tuning parameters are the model-internal time scales and the scaling constants and scaling exponents. The following sensitivity experiments run the volume/area scaling model with different values for these parameters. Experiments are perforemd on the Hintereisferner (RGI60-11.00897) and the all Alpine glaciers in the HISTALP domain, in each case with a constant equilibrium climate scenario and a positive temperature bias of +0.5 °C. For details about the experimental setup see Section 2.3.4.

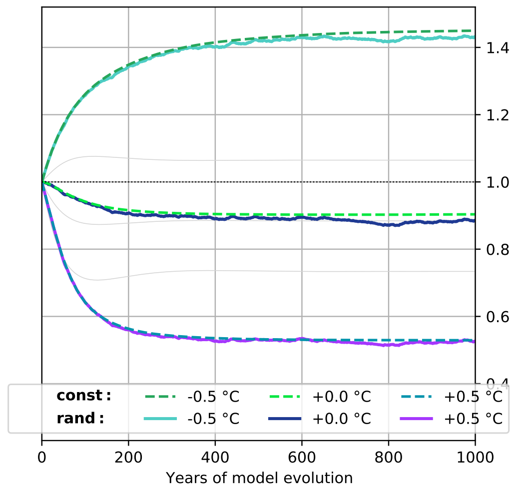
TL;DR: Sensitivity experiments

- the model-internal time scales control the damping ratio of the oscillation, longer time scales correspond to stronger overshoots
- halving the model-internal time scales leads to an asymptotical change in aggregte ice volume of the HISTALP domain, without any oscillations

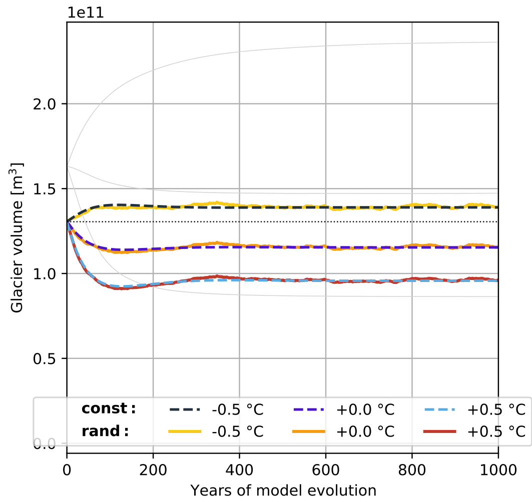
(a) Volume/area scaling model, relative glacier volume



(b) Flowline model, relative glacier volume



(c) Volume/area scaling model, absolute glacier volume



(d) Flowline model, absolute glacier volume

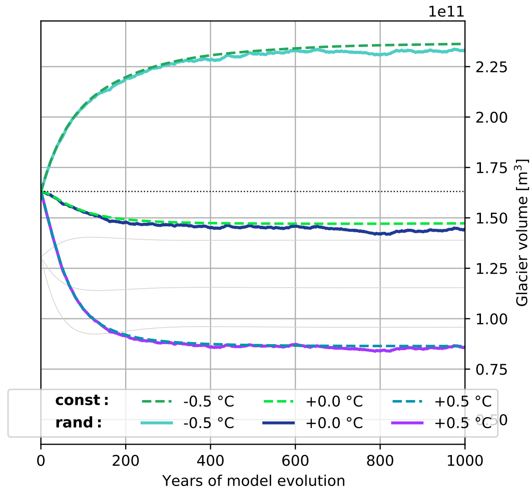
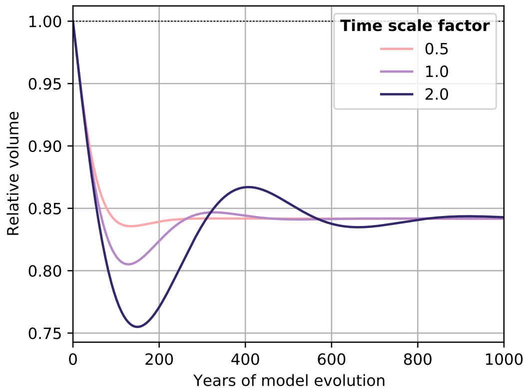


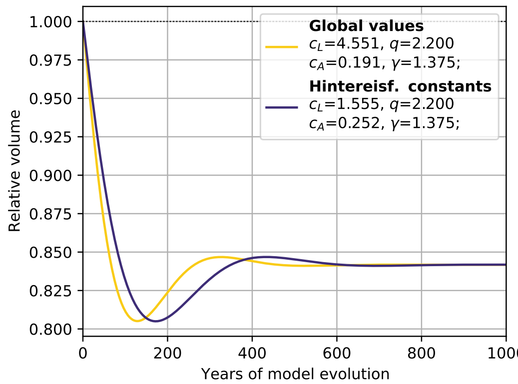
Figure 3.6: Time series of total ice volume for all glaciers in the HISTALP domain. The upper two panels show the relative glacier ice volume, normalized with the initial values, while the lower two panels show the absolute glacier ice volume. The left panels show the result of the volume/area scaling model, the right panels show the results of the flowline model. Solid lines represent the random climate scenarios, while dashed lines represent the constant climate scenarios. All climate scenarios are based on an equilibrium climate, with one of three different temperature biases. Yellow, orange and red solid lines represent the volume/area scaling model, while cyan, blue and purple solid lines represent the flowline model, under a random climate with a temperature bias of $-0.5\,^{\circ}$C, $0\,^{\circ}$C and $+0.5\,^{\circ}$C, respectively. Yellow, orange and red dashed lines represent the volume/area scaling model, while cyan, blue and purple dashed lines represent the flowline model, under a constant climate with a temperature bias of $-0.5\,^{\circ}$C, $0\,^{\circ}$C and $+0.5\,^{\circ}$C, respectively. The dotted line indicate the initial volume. The light gray lines represent the volume evolutions of the other model, to facilitate comparisons.

- different scaling constants lead to a different initial ice volume and a different initial glacier length, which in turn affect the e-folding time scales
- changing the scaling constants has little to no effect on the normalized volume change and normalized equilibrium volume
- custom scaling constants and exponents increase the change in ice volume ever so slightly, but the results are still not comparable to the flowline model

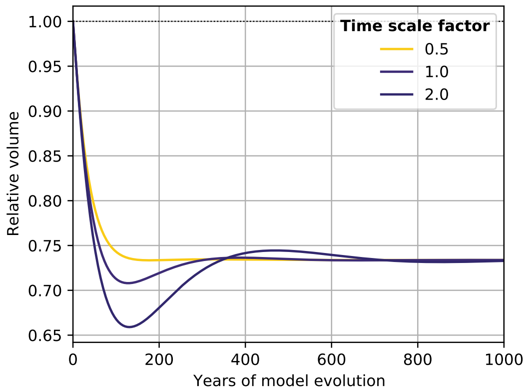
(a) Hintereisferner, different model-internal time scales



(b) Hintereisferner, different scaling constants



(c) HISTALP domain, different model-internal time scales



(d) HISTALP domain, different scaling constants and scaling exponents

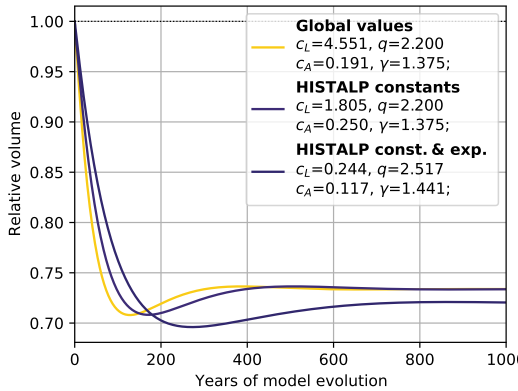


Figure 3.7: Temporal evolution of glacier ice volume under a positive temperature bias of +0.5 °C for the Hintereisferner (RGI60-11.00897) in the two upper panels (a) and (b) and for the entire HISTALP domain in the two lower panels (c) and (d). The right panels show results for different model-internal time scales, scaled by a linear factor (see legend for details). The left panels show results for different scaling constants and scaling exponents (see legend for details). Note the difference in y-axis scales.

3.4.1 Sensitivity to model-internal time scales

Let's again start with the Hintereisferner test case, before moving to the regional scale. As was to be expected, the model-internal time scales do not affect the absolute values but control only the oscillatary behavior. The e-folding response time scales for length and area are directly proportional to the model-internal time scales, changing by approximately 18 years and 8 years, respectively. Interestingly enough, the e-folding response time for volume is indirect proportional and changes only by a maximum of three years. Halving the model-internal time scales leads to an increase of the volume e-folding time scale by three years to $\tau_V = 39\,\mathrm{yr}$, while doubbling the model-internal time scales results in a decrease of the volume e-folding time scale by two years to $\tau_V = 34\,\mathrm{yr}$.

The main change, however, is seen in the oscillation amplitude. The damping ratio seems to be controlled by the model-internal time scales. A higher model-internal time scales lead to a stronger oscillation and vice versa. But even with halved the model-internal time scales, the modeled volume adjustment still shows some oscillations. With the default values, the volume/area scaling model overshoots the volume change estimation by 4 % of the final equilibrium value and it takes 434 years to reach an equilibrium. Hereby, an equilibrium state is (somewhat arbitrarily) defined as the range of ±0.1 % of the equilibrium value at year 1000. Halving and doubling the model-internal time scales changes the overshoot to 1 % and 10 % of the equilibrium value, respectively. The time span until a new equilibrium is reached seems to be almost linearly dependend on the model-internal time scales. By halving and doubling the model-internal time scales it takes 234 years and 832 years for the model to reach a new steady state, respectively.

The same qualitative findings are made for the regional Alpine run. The absolute values do not change for different model-internal time scales, only the oscillatory behavior does. While longer model-internal time scales result again in stronger overshoots, the oscillations seem generally more damped for the regional run. This is most likely a side effect of the summation over all Alpine glacier, whereby small scale oscillations can cancel each other out. The overshoots amount to 0.1 %, 3.5 % and 10.2 % of the equilibrium value for a time scale factor of 0.5, 1 and 2, respectively. Thereby it takes 312 years, 515 years and 941 years to reach a new steady state. When halving the model-internal time scales, the aggregate volume evolution shows no more discernable oscillations and is basically of exponetial (asympotical) nature.

3.4.2 Sensitivity to scaling parameters

As seen above, the model-internal time scale do not change the absolute values of any geometric glacier property. So what about the scaling parameters?! The following paragraph compares the model behavior between the custom Hintereisferner scaling constants and the global scaling constants. The scaling exponents are held constant, since it is not possible to compute a linear regression from a single data point (see Section 2.3.4 for details). Changing the scaling constants leads to different absolute values. As explained in Section 2.2.2, the volume/area scaling model starts by computing the initial glacier volume from the surface area via the volume/area scaling relation. Hence, the initial area stays the same while the initial ice volume increases with the custom scaling constants ($0.787\,\mathrm{km}^3$ vs. $0.596\,\mathrm{km}^3$). Starting with a larger initial ice volume, the absolute change in ice volume increase ($-0.124\,\mathrm{km}^3$ vs. $-0.094\,\mathrm{km}^3$) but still results in a larger equilibrium ice volume ($0.662\,\mathrm{km}^3$ vs. $0.502\,\mathrm{km}^3$). However, when normalized with the respective initial ice volumes, the changes in ice volume, the equilibrium values and the overshoots (i.e., minimum values due to the oscillating behavior) are almost identical (the differences lie far below $0.1\,\%$). This comes as no surprise, since the scaling constants are canceled during the normalization process. In fact, when estimating *changes* in regional or global ice volume the scaling constant c can be eliminated alltogether (Bahr et al. 2015, Section 8.5). While the relative values do no change, the temporal evolution does. As already discusse, the bigger costum scaling constant c_A leads to a bigger initial ice volume. Increasing the glacier's ice volume in turn increases the glacier's response time, since larger glaciers generally react slower to climatic changes. The volume e-folding response time increases to 48 years with the costum scaling constants, compared to 36 years with the global values. The increased response time goes hand in hand with a stronger oscillation. While the amplitude stays the same, the frequency decreases. Hence, the peak of the overshoot shifts by 31 years (to year 178 after the initial climate perturbation) and it takes much longer to reach the new equilibrium state (573 years vs. 435 years). While the glacier length reacts analogously to the costum scaling constants, the surface area does not. This is to be expected, since the initial surface area does not depend on the scaling parameters. While the equilibrium value and the e-folding response time are practically not affected, the oscillation amplifies. In addition to the decreased frequency, as for ice volume and glacier length, the surface area overshoots $12\,141\,\mathrm{m}^2$ more under the costum scaling constants (which corresponds to $\approx 0.2\,\%$ of the equilibrium value).

Again, the results of regional Alpine run are analogous to the Hintereisferner test case. While the Hintereisferner test case compares only the global and custom scaling constants, an additional regional with costum scaling constants and scaling exponents is investigated (see Section 2.3.4 for details). As seen above, changing the scaling constants results in different absolute values (for the initial volume as well as for the final equilibrium volume). However, when normalized with the initial values only the run with costum scaling exponents shows a different (bigger) change in ice volume. The total modeled glacier ice volume shrinks from an initial 229.7 km^3 to a final 165.5 km^3, subjected to a positive temperature bias of +0.5 °C. The change of −64.2 km^3 corresponds to −28 % of the initial value. However, the result does still not compare to the −47 % of the flowline model and is not even significantly different from the −26.5 % for the other two volume/area scaling runs.

3.5 Commitment runs

Chapter 4

Discussion

Chapter 5

Conclusions

Appendix A

Large Quantities of Data

Large quantities of data should be placed in an appendix. They should only be "summarized" in the chapter Results. Another way is to present some representative cases together with some extreme cases in the chapter Results. In any case, there should always appear a reference to the appendix in the main part of the thesis.

Bibliography

Auer, I. and R. Böhm, 2007: HISTALP – historical instrumental climatological surface time series of the Greater Alpine Region. *INTERNATIONAL JOURNAL OF CLIMATOLOGY*, **27**, doi:10.1002/joc.1377.

Bahr, D. B., M. F. Meier, and S. D. Peckham, 1997: The physical basis of glacier volume-area scaling. *Journal of Geophysical Research: Solid Earth*, **102 (B9)**, 20 355–20 362, doi:10.1029/97jb01696.

Bahr, D. B., W. T. Pfeffer, and G. Kaser, 2015: A review of volume-area scaling of glaciers. *Reviews of Geophysics*, **53 (1)**, 95–140, doi:10.1002/2014RG000470, URL http://doi.org/10.1002/2014RG000470.

Box, G. E. P., G. M. Jenkins, G. C. Reinsel, and G. M. Ljung, 2015: *Time Series Analysis: Forecasting and Control (Wiley Series in Probability and Statistics)*. Wiley, URL https://www.xarg.org/ref/a/1118675029/.

Chen, J. and A. Ohmura, 1990: Estimation of Alpine glacier water resources and their change since the 1870s. *Hydrology in mountainous regions I*, **V (193)**, 127–135.

Cheung, Y.-W. and K. S. Lai, 1995: Lag order and critical values of the augmented dickey–fuller test. *Journal of Business & Economic Statistics*, **13 (3)**, 277–280.

Dusch, M., 2018: Mass-balance model calibration for the alps. OGGM Blog, URL "https://oggm.org/2018/08/10/histalp-parameters/", posted on August 10, 2018, URL "https://oggm.org/2018/08/10/histalp-parameters/", posted on August 10, 2018.

Giesen, R. H. and J. Oerlemans, 2012: Calibration of a surface mass balance model for global-scale applications. *Cryosphere*, **6 (6)**, 1463–1481, doi:10.5194/tc-6-1463-2012.

Greuell, W., 1992: Hintereisferner, austria: mass-balance reconstruction and numerical modelling of the historical length variations. *Journal of Glaciology*, **38 (129)**, 233–244, doi:10.3189/S0022143000003646.

Jarvis, A., E. Guevara, H. Reuter, and A. Nelson, 2008: Hole-filled srtm for the globe, version 4. CGIAR Consortium for Spatial Information, University of Twente, published by CGIAR-CSI on 19 August 2008., published by CGIAR-CSI on 19 August 2008.

Jóhannesson, T., C. Rymond, and E. Waddington, 1989: Time-scale for adjustment of glaciers to changes in mass balance. **35 (1)**.

Marzeion, B., A. H. Jarosch, and M. Hofer, 2012: Past and future sea-level change from the surface mass balance of glaciers. *The Cryosphere*, **6 (6)**, 1295–1322, doi:10.5194/tc-6-1295-2012.

Marzeion, B. and A. Nesje, 2012: Spatial patterns of north atlantic oscillation influence on mass balance variability of european glaciers. *The Cryosphere*, **6 (3)**, 661–673, doi:10.5194/tc-6-661-2012, URL https://tc.copernicus.org/articles/6/661/2012/.

Maussion, F., et al., 2019: The Open Global Glacier Model (OGGM) v1.1. *Geoscientific Model Development*, **12 (3)**, 909–931, doi:10.5194/gmd-12-909-2019, URL https://doi.org/10.5194/gmd-12-909-2019.

Pfeffer, W. T., et al., 2014: The randolph glacier inventory: a globally complete inventory of glaciers. *Journal of Glaciology*, **60 (221)**, 537–552, doi: 10.3189/2014JoG13J176.

Proakis, J. and D. Manolakis, 2007: *Digital Signal Processing*. Prentice Hall international editions, Pearson Prentice Hall, URL https://books.google.at/books?id=H_5SAAAAMAAJ.

RGI Consortium, 2017: Randolph glacier inventory - a dataset og global glacier outlines: Version 6.0: Technichal report. Global Land Ice Measurements from Space, Colorado, USA, doi:10.7265/N5-RGI-60.

Roe, G. H. and M. B. Baker, 2014: Glacier response to climate perturbations: an accurate linear geometric model. *Journal of Glaciology*, **60 (222)**, 670–684, doi: https://doi.org/10.3189/2014jog14j016.

Schuster, L., 2020: Response time of glaciers using the Open Global Glacier Model : from simple idealized experiments to an estimate for Alpine glaciers. Ph.D. thesis.

Welch, P., 1967: The use of fast fourier transform for the estimation of power spectra: A method based on time averaging over short, modified periodograms. *IEEE Transactions on Audio and Electroacoustics*, **15 (2)**, 70–73.

World Glacier Monitoring Service, Zürich, Switzerland, 2017: Wgms: Fluctuations of glaciers database. doi:10.5904/wgms-fog-2017-10.

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Curriculum Vitae

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Born on 01 April 1976 in Town, Country

Education and Professional training:

1999–2003	Research assistant and Ph.D. student in the group of Dr. LastName at the Institute of Meteorology and Geophysics, University of Innsbruck.
1998–1999	Diploma thesis under the guidance of Dr. LastName, Institute of Meteorology and Geophysics, University of Innsbruck: *"Title of your diploma thesis"*.
1993–1998	Diploma study at the University of Innsbruck. *Master of Natural Science (Magister rerum naturalium)* in Meteorology.
1989–1993	Highschool, Town. *Matura*.

Meteorological training courses: "Numerical methods and adiabatic formulation of models", ECMWF, 1998; "Data assimilation and use of satellite data", ECMWF, 1998.

Participation in field experiments: Gap flow study (MAP), Austria, 1999.

Epilogue

Here is the place where you may want to tell a little story or a fairy tale which has some relevance for your thesis, such as "Once upon a time, ...". The Epilogue is optional.