Fluid Mechanics

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Chapter 1

Fundamental Concepts

1.1 Basic Concepts

1.2 Continuum Hypothesis

- $$\text{Knudsen Number (kn)} = \frac{\text{Mean free path } (\lambda)}{\text{Chateristic Length(L)}}$$

  Continuum hypothesis won't be valid for Kn ≥ 0.01.

- Density $\rho = \frac{\text{mass}}{\text{volume}}$
- Specific volume $\nu = \frac{1}{\rho}$
- Specific weight $\gamma = \frac{mg}{V} = \rho g$
- Specific Gravity SG $= \frac{\rho}{\rho_{\text{water @ } 227K}}$

1.3 Forces on a fluid particle

- Body forces
- Surface forces
- Normal stress,

$$\sigma_n = \lim_{\delta A_n \to 0} \frac{\delta F_n}{\delta A_n}$$

- Shear stress,

$$\tau_n = \lim_{\delta A_n \to 0} \frac{\delta F_t}{\delta A_n}$$

- Shear tensor,

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

1.4 Viscosity

$$\tau_{yx} = \lim_{\delta A_y \to 0} \frac{\delta F_x}{\delta A_y} = \frac{dF_x}{dA_y}$$

And, $$\mathrm{DR} = \lim_{\delta t \to 0} \frac{\delta \alpha}{\delta t} = \frac{d\alpha}{dt}$$

Since, $$\delta l = \delta \alpha \ \delta y \text{ and } \delta l = \delta u \ \delta t$$

we get $$\mathrm{DR} = \frac{du}{dy}$$

For Newtonian fluids,

$$\tau_{yx} = \mu \frac{du}{dy} \qquad \mu \to \text{Viscosity}$$

- Unit: $\frac{\mathrm{N\,s}}{\mathrm{m}^2}$
- Kinematic viscosity: $\nu = \frac{\mu}{\rho}$. Unit of this: $\frac{\mathrm{m}^2}{\mathrm{s}}$
- For non-Newtonian fluids: $\tau_{yx} = k\left(\frac{du}{dy}\right)^n = k|\frac{du}{dy}|^{n-1}\left(\frac{du}{dy}\right) = \eta\frac{du}{dy}$
- $k \to$ consistency index, $n \to$ flow behavior index, $\eta \to$ apparent velocity.

1.5 Description and classification of fluid motion

1.5.1 Steady vs unsteady

$$\frac{\partial \rho}{\partial t} = 0 \implies \rho = \rho(x, y, z) \qquad \frac{\partial \vec{\mathbf{V}}}{\partial t} = 0 \implies \vec{\mathbf{V}} = \vec{\mathbf{V}}(x, y, z) \qquad \frac{\partial p}{\partial t} = 0 \implies p = p(x, y, z)$$

1.5.2 Uniform vs non-uniform

$$\left.\frac{\partial \vec{\mathbf{V}}}{\partial x}\right|_{t=\text{const}} = 0$$

where x is in direction of flow.