Fluid Mechanics

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Chapter 1

Fundamental Concepts

1.1 Basic Concepts

1.2 Continuum Hypothesis

- $$\text{Knudsen Number (kn)} = \frac{\text{Mean free path } (\lambda)}{\text{Chateristic Length(L)}}$$

  Continuum hypothesis won't be valid for Kn ≥ 0.01.

- Density $\rho = \frac{\text{mass}}{\text{volume}}$
- Specific volume $\nu = \frac{1}{\rho}$
- Specific weight $\gamma = \frac{mg}{V} = \rho g$
- Specific Gravity SG $= \frac{\rho}{\rho_{\text{water @ } 227K}}$

1.3 Forces on a fluid particle

- Body forces
- Surface forces
- Normal stress,

$$\sigma_n = \lim_{\delta A_n \to 0} \frac{\delta F_n}{\delta A_n}$$

- Shear stress,

$$\tau_n = \lim_{\delta A_n \to 0} \frac{\delta F_t}{\delta A_n}$$

- Shear tensor,

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

1.4 Viscosity

$$\tau_{yx} = \lim_{\delta A_y \to 0} \frac{\delta F_x}{\delta A_y} = \frac{dF_x}{dA_y}$$

And,
$$\mathrm{DR} = \lim_{\delta t \to 0} \frac{\delta \alpha}{\delta t} = \frac{d\alpha}{dt}$$

Since,
$$\delta l = \delta\alpha \ \delta y \text{ and } \delta l = \delta u \ \delta t$$

we get
$$\mathrm{DR} = \frac{du}{dy}$$

For Newtonian fluids,

$$\tau_{yx} = \mu \frac{du}{dy} \qquad \mu \to \text{Viscosity}$$

- Unit: $\frac{\mathrm{N\,s}}{\mathrm{m}^2}$
- Kinematic viscosity: $\nu = \frac{\mu}{\rho}$. Unit of this: $\frac{\mathrm{m}^2}{\mathrm{s}}$
- For non-Newtonian fluids: $\tau_{yx} = k\left(\frac{du}{dy}\right)^n = k|\frac{du}{dy}|^{n-1}\left(\frac{du}{dy}\right) = \eta\frac{du}{dy}$
- $k \to$ consistency index, $n \to$ flow behavior index, $\eta \to$ apparent velocity.

1.5 Description and classification of fluid motion

1.5.1 Steady vs unsteady

$$\frac{\partial \rho}{\partial t} = 0 \implies \rho = \rho(x, y, z) \qquad \frac{\partial \vec{\mathbf{V}}}{\partial t} = 0 \implies \vec{\mathbf{V}} = \vec{\mathbf{V}}(x, y, z) \qquad \frac{\partial p}{\partial t} = 0 \implies p = p(x, y, z)$$

1.5.2 Uniform vs non-uniform

$$\left.\frac{\partial \vec{\mathbf{V}}}{\partial x}\right|_{t=\text{const}} = 0$$

where x is in direction of flow.

1.5.3 Compressible vs incompressible

- $\rho =$ constant.
- At high pressures,

$$\text{Bulk compressibility modulus } E_v = \frac{dp}{\left({}^{d\rho}/_{\rho}\right)}$$

- Water hammer, cavitation
- Gases are taken to be incompressible if Mach number (Ma) ≤ 0.3.

$$\text{Mach number } Ma = \frac{\text{flow speed, } V}{\text{local speed of sound, } c}$$

- Compressible $\to$ Supersonic ($Ma > 1$) or subsonic ($Ma < 1$).

1.5.4 Inviscid vs viscid flows

- Inviscid flows → zero viscosity (frictionless flow).
- Viscid flows → finite viscosity.

1.5.5 Laminar vs turbulent

- Laminar → smooth layers
- Turbulent → fluid particles randomly mix.
- $$\text{Reynold's Number } Re = \frac{\text{Inertial force}}{\text{Viscous force}} = \frac{\rho \bar{V} L}{\mu}$$
- Laminar → $Re \leq 2100$. Turbulent → $Re \geq 4000$.

1.6 Surface tension

- Pressure difference in a droplet:

$$\text{Tensile force due to surface tension } = \sigma \pi d$$
$$\text{Pressure force on the area } = (P_i - P_o)\frac{\pi d^2}{4}$$
$$\text{Force balance: } \sigma \pi d = (P_i - P_o)\frac{\pi d^2}{4}$$
$$\implies \boxed{P_i = P_o + \frac{4\sigma}{d}}$$

- For a soap bubble: $P_i = P_o + \frac{8\sigma}{d}$
- Capillary effect force balance:

$$\text{Weight of fluid} = (\text{Area} \times \text{height})\rho g$$
$$\text{Verticle component of surface tension} = \sigma \pi d \cos\theta$$
$$\implies \frac{\pi d^2 h}{4}\rho g = \sigma \pi d \cos\theta$$
$$\implies \boxed{h = \frac{4\sigma \cos\theta}{\rho g d}}$$

Chapter 2

Fluid statics

2.1 Pressure formula

Imagine a small cube of fluid with sides dx, dy, dz. Assume this fluid is stationary relative to the stationary rectangular coordinates. Let $\vec{\mathbf{O}}$ denote the center of the cube and the pressure there be $P(\vec{\mathbf{O}})$. We want to do a force balance on this cube. There are two types of forces acting on this: Body and surface forces. The body force is just gravity:

$$\vec{\mathbf{F}}_B = \vec{\mathbf{g}}\ dm = \rho\vec{\mathbf{g}}\ dV = \rho\vec{\mathbf{g}}\ dx\ dy\ dz$$

For surface forces we need pressure difference. For pressure across the y direction, let on the face on the left we write the taylor series of p centered around $\vec{\mathbf{O}}$. Thus,

$$\begin{aligned} p_L &= p + \frac{\partial p}{\partial y}(y_L - y) \\ \implies \quad &= p + \frac{\partial p}{\partial y}\left(-\frac{dy}{2}\right) \\ \implies \quad &= p - \frac{\partial p}{\partial y}\frac{dy}{2} \end{aligned}$$

Similarly,

$$\begin{aligned} p_R &= p + \frac{\partial p}{\partial y}(y_R - y) \\ \implies \quad &= p + \frac{\partial p}{\partial y}\frac{dy}{2} \end{aligned}$$

Thus, force on the left face is $(p_L\ dx\ dz\hat{\mathbf{j}}) = \left(p - \frac{\partial p}{\partial y}\frac{dy}{2}\right)\ dx\ dz(\hat{\mathbf{j}})$ and that on the right face is $(p_R dx dz\,(-\hat{\mathbf{j}})) = \left(p + \frac{\partial p}{\partial y}\frac{dy}{2}\right)\ dx\ dz\,(-\hat{\mathbf{j}})$. Force different in this direction gives us: $(p_R\,(-\hat{\mathbf{j}}) - p_L\,(\hat{\mathbf{j}}))\ dx\ dz = -\left(\frac{\partial p}{\partial y}\ dx\ dy\ dz\right)$ Doing the same for other directions we get,

$$\begin{aligned} d\vec{\mathbf{F}}_S &= -\left(\frac{\partial p}{\partial x}\hat{\mathbf{i}} + \frac{\partial p}{\partial y}\hat{\mathbf{j}} + \frac{\partial p}{\partial z}\hat{\mathbf{k}}\right)\ dx\ dy\ dz \\ &= -\nabla p\ dx\ dy\ dz \end{aligned}$$

Ultimately:

$$\begin{aligned}\vec{\mathbf{F}} &= d\vec{\mathbf{F}}_B + d\vec{\mathbf{F}}_S \\ &= (-\nabla p + \rho\vec{\mathbf{g}})\, dV\end{aligned}$$

For a static fluid, $\vec{\mathbf{F}} = m\vec{\mathbf{a}} = 0$. Thus, we simplify the previous equation. Since $\vec{\mathbf{g}}$ points in the $-\hat{\mathbf{j}}$ direction, all terms cancel but

$$\frac{dp}{dz} = -\rho g$$