M621 Short HW 5 due Sept 29. G is a group, $H \leq G$, and $b \in G$.

1. Show that $bHb^{-1} \leq G$, and the map $F : H \to bHb^{-1}$ given by $F(h) = bhb^{-1}$, all $h \in H$, is an isomorphism. **Do this—don't turn in.**

2. Let A be the set of left cosets of H in G. So $A = \{bH : b \in G\}$. Let G act on A as follows: For all $g \in G$, and all $bH \in A$, let $g \cdot bH = gbH$. This does indeed define a group action—you don't have to prove it. As you know, a group action determines a homomorphism $\sigma : G \to S_A$, where $g \to \sigma_g$, and $\sigma_g : A \to A$ is the permutation of A given by $a \to g \cdot a$, all $a \in A$. **Prove** $ker(\sigma) = \cap\{bHb^{-1} : b \in G\}$.

3. Let $A = \{bHb^{-1} : b \in G\}$, the set of all *conjugates* of H in G. Define an action of G on A as follows: For all $g \in G$, all $bHb^{-1} \in A$, we let $g \cdot bHb^{-1} = g(bHb^{-1})g^{-1}$ (which is $(gb)H(gb)^{-1}$, another conjugate of H). This defines an action of G on A. Show G_H, the stabilizer of H in G under the action, is $N_G(H)$, the normalizer of H in G.