MATH 667-01 Homework 6

Due: Thursday, November 2, 2017

Instructions: *Each of the following problems must be submitted to the instructor on or before the due date. Partial credit may be given for incorrect answers which make some positive progress. Late homework will not be accepted.*

1. (10 points) Let $X_1, \ldots, X_n$ be independent identically distributed random variables uniformly distributed on the set $\{1, 2, \ldots, H\}$ where H is an unknown positive integer.
(a - 3 pts) Show that the probability mass function(pmf) of $\max\{X_1, \ldots, X_n\}$ is

$$f(y) = \begin{cases} \dfrac{y^n - (y-1)^n}{H^n} & \text{for } y = 1, \ldots, H \\ 0 & \text{otherwise} \end{cases}.$$

(b - 4 pts) Show that the family of pmfs for $\max\{X_1, \ldots, X_n\}$ is complete.
(b - 3 pts) Find the UMVUE of H. Justify your answer.

2. (10 points) Suppose that there are ten marbles in a box, M of which are red and $10 - M$ of which are blue. Consider a hypothesis test procedure based on a random sample of four marbles selected with replacement where we reject $H_0 : M \geq 6$ versus $H_a : M < 6$ if and only if we observe no red marbles in the sample.
(a - 1 pt) What is the rejection region for this procedure?
(b - 1 pt) What is the probability of a Type I error?
(c - 1 pt) What is the probability of a Type II error when $M = 5$?
(d - 5 pts) What is the power function of this test?
(e - 1 pt) What is the size of this test?
(f - 1 pt) Is this a level .05 test?

3. (10 points) Suppose $X_1, \ldots, X_n$ is a random sample from a Normal distribution with unknown mean $\mu \in [0, \infty)$ and known variance 1 and an experimenter is interested in testing the null hypothesis $H_0 : \mu = 0$ versus the alternative $H_1 : \mu > 0$.
(a - 2 pts) Show that the likelihood ratio test has a critical region of the form $\left\{(x_1, \ldots, x_n) : \sum_{i=1}^{n} x_i > K\right\}$.
(b - 3 pts) Find the value of K so that the size of the test in part (a) is 0.05.
(c - 3 pts) Compute the power of this test and write it as a function of the standard normal cumulative distribution function $\Phi(t) = \int_{\infty}^{t} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz$ evaluated at an expression involving μ and n.
(d - 2 pts) Find the smallest n such that the power of the test is at least 0.90 for all $\mu \geq 1$.