MATH 668-01 Homework 1

Due: Tuesday, January 23, 2018

Instructions: *Each of the following problems must be submitted to the instructor on or before the due date. Partial credit may be given for incorrect answers which make some positive progress. Late homework will not be accepted.*

1. (10 points) Answer the following questions about the dataset "HeightData.txt".
(a - 1pt) Find the average height of all children.
(b - 1pt) Find the percentage of female children.
(c - 1pt) What is the height of the tallest child?
(d - 1pt) How many families have exactly 4 children?
(e - 2pts) What are the heights of the children in the family with the tallest mother?
(f - 2pts) What percentage of children are taller than both of their parents?
(g - 2pts) What is the average height of females whose mother was at least 68 inches tall?

2. (10 points)
(a - 5pts) Suppose that $\mathbf{H}_1^-, \ldots, \mathbf{H}_m^-$ are generalized inverses of a matrix $\mathbf{H}$ and $\alpha_1, \ldots, \alpha_m$ are scalars. Find a condition on the scalars $\alpha_1, \ldots, \alpha_m$ that makes $\alpha_1\mathbf{H}_1^- + \ldots + \alpha_m\mathbf{H}_m^-$ a generalized inverse of $\mathbf{H}$.
(b - 5pts) Let

$$\mathbf{H} = \begin{pmatrix} 2 & -1 & 3 \\ 2 & 1 & 1 \\ -1 & 1 & -2 \end{pmatrix}.$$

Find a generalized inverse of $\mathbf{H}$ which has no elements equal to 0.

3. (10 points) Consider the symmetric matrix

$$\mathbf{O} = \begin{pmatrix} 255 & 54 & 71 & 87 & 88 & 93 & 255 \\ 54 & 255 & 255 & 255 & 255 & 255 & 97 \\ 71 & 255 & 255 & 255 & 255 & 255 & 101 \\ 87 & 255 & 255 & 255 & 255 & 255 & 103 \\ 88 & 255 & 255 & 255 & 255 & 255 & 108 \\ 93 & 255 & 255 & 255 & 255 & 255 & 115 \\ 255 & 97 & 101 & 103 & 108 & 115 & 255 \end{pmatrix}$$

(it is a grayscale image of the letter "O"; the image can be viewed *right-side up* using the R command `image(t(O[7:1,]),col=gray((0:255)/255))`). Use computer software to find the two largest eigenvalues of $\mathbf{O}$, denoted by λ_1 and λ_2. Letting $\boldsymbol{u}_1$ be the eigenvector of $\mathbf{O}$ associated with λ_1 and $\boldsymbol{u}_2$ be the eigenvector of $\mathbf{O}$ associated with λ_2, compute

$$\mathbf{A} = \lambda_1 \boldsymbol{u}_1 \boldsymbol{u}_1' + \lambda_2 \boldsymbol{u}_2 \boldsymbol{u}_2'.$$