

$$m\ddot{x} + kx = F_f = \begin{cases} r & \frac{dx}{dt} < 0 \\ -r & \frac{dx}{dt} > 0 \end{cases}$$

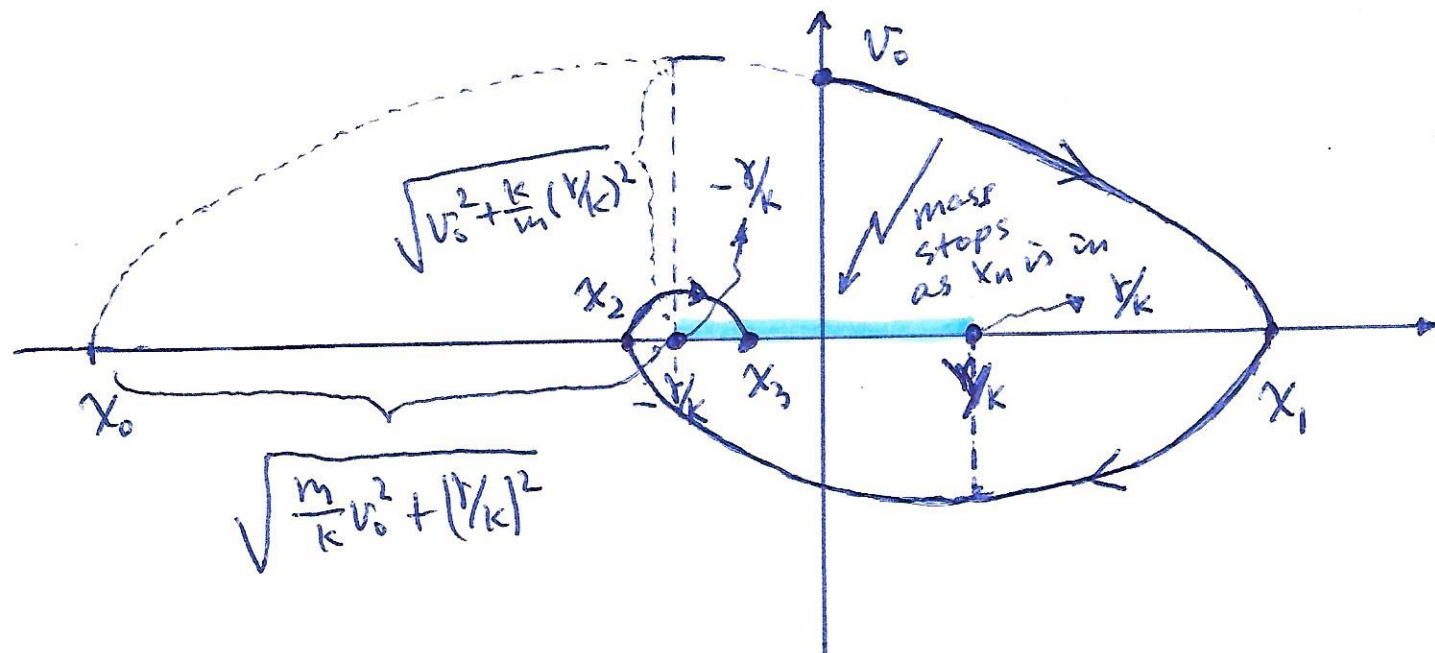
Consider $\frac{dx}{dt}(0) = v_0 > 0$.

$$m\ddot{x} + kx = -r, \quad m\ddot{x} + k(x + r/k) = 0$$

$$E^+(x, \dot{x}) = \frac{1}{2} m(\dot{x})^2 + \frac{1}{2} k(x + r/k)^2$$

$$= \frac{1}{2} m v_0^2 + \frac{1}{2} k (r/k)^2 = \frac{1}{2} k (x_0 + r/k)^2$$

where x_0 is as below



In case $dx/dt < 0$.

$$m \ddot{x} + kx = \delta, \quad m \dot{x} + k(x - \delta/k) = 0$$

$$\begin{aligned} E^-(x, x') &= \frac{1}{2} (\dot{x})^2 + \frac{1}{2} k(x - \delta/k)^2 \\ &= \frac{1}{2} k(x_1 - \delta/k)^2 \end{aligned}$$

Where $x_1 > 0$ is as before on the figures

We have : $x_0 + x_1 = -2\delta/k$

$$x_1 + x_2 = 2\delta/k,$$

$$x_2 + x_3 = -2\delta/k$$

$$x_3 + x_4 = 2\delta/k$$

⋮
stops as $|x_n| \leq \delta/k$.

Notice $x_1 = -(2\delta/k + x_0)$

$$x_2 = 4\delta/k + x_0$$

$$x_3 = -(6\delta/k + x_0)$$

$$x_4 = 8\delta/k + x_0 \quad - - -$$

Now we have

$$|x_n| = \left| \frac{2hr}{k} + x_0 \right|$$

$$\text{and } x_0 = -\frac{\delta}{k} - \sqrt{\frac{m}{k} v_0^2 + \left(\frac{\delta}{k}\right)^2} < -\frac{2\delta}{k}.$$

Criteria to stop:

$$|x_n| \leq \frac{\delta}{k}.$$