MATH 667-01 Homework 4

Due: Thursday, October 12, 2017

Instructions: *Each of the following problems must be submitted to the instructor on or before the due date. Partial credit may be given for incorrect answers which make some positive progress. Late homework will not be accepted.*

1. (10 points) Let $X_1, \ldots, X_n$ be independent identically distributed Poisson random variables with mean λ, and let λ be a random variable with an exponential prior distribution which has mean μ. Assume μ is known and fixed.
(a - 5 pts) Let $Y = \sum_{i=1}^{n} X_i$. Find the posterior distribution of λ given that $Y = y$.
(b - 5 pts) Write the Bayes estimator (with respect to squared error loss) of λ as a linear combination of $\bar{X}$ and μ having the form $a_n(\mu)\bar{X} + b_n(\mu)\mu$. What happens to $a_n(\mu)$ and $b_n(\mu)$ as $n \to \infty$?

2. (10 points) Suppose X is a random variable with probability mass function

$$P(X = x) = \frac{1}{6} I_{\{1,2,3,4,5,6\}}(x).$$

Express E[$|X - a|$] as a piecewise function of a. What value(s) of a maximizes E[$|X - a|$]?

3. (10 points) Suppose $X_1, \ldots, X_n$ is a random sample from a Normal population with mean μ and variance σ^2.
(a - 5 pts) Using the fact that $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$, show that $\mathrm{Var}[S^2] = \frac{2\sigma^4}{n-1}$.
(b - 5 pts) Consider estimating σ^2 with estimators of the form cS^2. Show that the mean squared error of cS^2 is minimized when $c = \frac{n-1}{n+1}$.