MATH 668-01 Homework 2

Due: Tuesday, February 6, 2018

Instructions: *Each of the following problems must be submitted to the instructor on or before the due date. Partial credit may be given for incorrect answers which make some positive progress. Late homework will not be accepted.*

1. (10 points) Suppose that x_1, x_2, and x_3 are random variables such that $E(x_i) = 1$, $\text{var}(x_i) = i$, $\text{cov}(x_1, x_2) = 1$, x_1 and x_3 are independent, and x_2 and x_3 are independent.

(a - 3pts) Compute the expected value of the random vector $\begin{pmatrix} 4x_1 - 3x_2 + 1 \\ x_2 - x_3 + 2 \end{pmatrix}$.

(b - 3pts) Compute $\text{cov}(4x_1 - 3x_2 + 1, x_2 - x_3 + 2)$.

(c - 4pts) Compute the correlation matrix for the random vector $\begin{pmatrix} 4x_1 - 3x_2 + 1 \\ x_2 - x_3 + 2 \end{pmatrix}$.

2. (10 points) Suppose $\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \sim N_3 \left(\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 5 & 1 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 13 \end{pmatrix} \right)$.

(a - 3pts) Find the distribution of $y_3 - y_2 + 1$. Make sure to specify the parameters of the distribution.

(b - 3pts) Compute $P(y_1 > 0 \text{ and } y_3 > 0 | y_2 = 0)$.

(c - 4pts) Compute $P(y_1 > 0 | y_3 - y_2 = 1)$.

3. (10 points) Let α be a positive real number. If $\boldsymbol{y} \sim N_2 \left(\mathbf{0}_2, \begin{pmatrix} 1+\alpha & 1 \\ 1 & 1 \end{pmatrix} \right)$, find a matrix $\mathbf{A}$ such that x_1 and x_2 are independent where x_1 and x_2 are the components of the random vector $\mathbf{A}\boldsymbol{y}$ (that is, where $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \mathbf{A}\boldsymbol{y}$).