MATH 667-01 Homework 7

Due: Thursday, November 30, 2017 at the beginning of class (if it is turned in by Wednesday Nov 29 before 5pm, it will be graded and returned on Thursday Nov 30)

Instructions: *Each of the following problems must be submitted to the instructor on or before the due date. Partial credit may be given for incorrect answers which make some positive progress. Late homework will not be accepted.*

1. (10 points) Suppose the pmf of X under H_0 and H_1 are given in the following table.

	x					
	1	2	3	4	5	6
$f(x\|H_0)$	.01	.03	.05	.07	.08	.76
$f(x\|H_1)$	.12	.04	.06	.50	.04	.24

(a - 5 pts) Use the Neyman-Pearson to find the test function $\phi(x)$ of the UMP test with size .10 for testing H_0 versus H_1. (Hint: To attain this size, you may need a randomized rule which randomly rejects or fails to reject H_0 when x^* is observed with probability $\phi(x^*)$ when $f(x^*|H_1) = kf(x^*|H_0)$ for some value of k.)
(b - 5 pts) Consider the Bayesian setting with prior distribution $\pi(H_0) = .80$. Find the posterior distribution of $\pi(H_0|x)$ for $x = 1, 2, 3, 4, 5, 6$.

2. (10 points) Let X_1 and X_2 be exponential random variables each with pdf $f(x) = \frac{1}{\beta}e^{-x/\beta}I_{(0,\infty)}(x)$ where β is an unknown positive constant.
(a - 7 pts) Using the fact that the solution to $(1+m)e^{-m} = .05$ is $m \approx 4.743865$ (obtained in R with `qgamma(.95,2,1)`), show that the test with critical region $\{(x_1, x_2) : x_1 + x_2 > m\beta_0\}$ is a UMP level .05 test of $H_0 : \beta \leq \beta_0$ vs. $H_1 : \beta > \beta_0$.
(b - 3 pts) Invert the test in part (b) to obtain a 95% confidence interval for β.

3. (10 points) Suppose $X_1, \ldots, X_n$ are iid Uniform$(0, \theta)$ random variables where θ is an unknown positive constant.
(a - 5 pts) Show that $\frac{\max\{X_1, \ldots, X_n\}}{\theta}$ is a pivot. (Hint: Find the cdf of this random variable, then differentiate it to find its pdf.)
(b - 5 pts) Find a $100(1-\alpha)\%$ confidence interval for θ.

4. (10 points) Suppose $X_1, \ldots, X_n$ are iid Poisson(λ) random variables where λ is an unknown positive constant. Let $\bar{X}_n = \frac{\sum_{i=1}^n X_n}{n}$.
(a - 2 pts) Show that $\bar{X}_n$ is a consistent estimator of λ.
(b - 4 pts) Show that $\sqrt{\bar{X}_n}$ is an asymptotically efficient estimator of $\sqrt{\mu}$.
(c - 4 pts) Use your answer to part (b) to construct an approximate 95% confidence interval for $\sqrt{\mu}$. Also, compute the approximate 95% confidence interval for $\sqrt{\mu}$ when $n = 10000$ and $\bar{x} = 4.4$.