M621 HW 7, due Oct 7

1. Using pg. 85, number 4 (which is an obvious enough fact and exercise– don't turn it in), do pg. 85, number 5, which gives you a useful fact, and a reasonably interesting exercise. Be sure to use the definition of "order of an element" in your explanation.

2. page 86, 11(a) and (b) This one keeps you in contact with $\mathbb{GL}_2(\mathbb{F})$, and helps assure that you understand "fibers".

3. page 89, 41. Good exercise. Note that the *commutator subgroup* N is the subgroup **generated** by $\{xyx^{-1}y^{-1} : x, y \in G\}$—the set at the left is not a subgroup since it is not in general not closed under operation (though it is closed under inverses).

   0. Elements of the form $xyx^{-1}y{-}1$ are themselves called *commutators*.

   1. Note that x and y commute if and only if $xyx^{-1}y^{-1} = e$.

   2. Thus, a group G is Abelian if and only if its commutator subgroup N is the trivial subgroup of G.

   3. Later, you'll prove that if A is a normal subgroup, then G/A is Abelian if and only if $N \subseteq A$. I'll give one ;point extra credit if you can prove that here.