MATH 667-01 Homework 2

Due: Tuesday, September 12, 2017

Instructions: *Each of the following problems must be submitted to the instructor on or before the due date. Partial credit may be given for incorrect answers which make some positive progress. Late homework will not be accepted.*

1. (15 points) Let $X_1, \ldots, X_n$ be independent random variables such that X_i has pdf

$$f(x|\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-i)^2}$$

where $\sigma > 0$.
(a - 10 pts) Find the MLE of σ.
(b - 5 pts) Let $\hat{\sigma}$ denote your answer in part (a). What is the distribution of $Q = \dfrac{n\hat{\sigma}^2}{\sigma^2}$? Justify your answer.

2. (10 points) Let $X_1, \ldots, X_{10}$ be a random sample from a normal population with mean μ and variance σ^2, and let $S^2 = \dfrac{1}{9}\displaystyle\sum_{i=1}^{10}(X_i - \bar{X})^2$. Compute $\mathrm{E}\left[\sqrt{S^2}\right]$. (Your answer should be a function of σ.)

3. (10 points) Suppose that $X_1, \ldots, X_9$ is a random sample from a normal population with mean 1 and variance 16. Compute the probability

$$P\left(\sum_{i=1}^{9} X_i > 12 \text{ or } \sum_{i=1}^{9}\left(X_i - \bar{X}\right)^2 > 64\right).$$