M622, HW ω, due Feb. 7.

1. Number 4, page 306, (a), (b), (c) only. Write on other side of paper if you need room.

2. (Due diligence on the very basics of vector spaces): Let F be a field. Suppose $\mathcal{B} = \{v_1, \ldots, v_n\}$ is a basis for V over F.

   (a) That $\mathcal{B} = \{v_1, \ldots, v_n\}$ is a basis implies it is linearly-independent and it spans V. Give a **short, clear** proof that shows that **each** element $v \in V$ is **uniquely represented** as a linear combination $v = \alpha_1 v_1 + \ldots + \alpha_n v_n$ (where $\alpha_1, \ldots, \alpha_n$ are elements of F). (You'll use both that $\mathcal{B}$ is a spanning set and is linearly independent in your proof.)

(b) Consider the F-vector space $F^n = \{(\alpha_1, \ldots, \alpha_n) : \alpha_i \in F,\ i = 1, \ldots, n\}$. Show that V above (having basis $\mathcal{B} = \{v_1, \ldots, v_n\}$) is isomorphic to F^n by **providing** a specific isomorphism $\phi : V \to F^n$. Verify that ϕ is indeed a F-module homomorphism, and that ϕ is a bijection.

3. Let V be the $\mathbb{Q}$ vector space given by $V = \{c_0 + c_1\sqrt{2} : \{c_0, c_1\} \subseteq \mathbb{Q}\}$. Show that $\{1, \sqrt{2}\}$ is a basis for V over $\mathbb{Q}$.