HW 6 #2

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Assuming a one-way random effects model $y_i = \mu + a_i + \epsilon_{ij}$ where μ is the mean battery weight, ai $N(0, \sigma_1^2)$ is a random effect for each batch, and ϵ_{ij} $N(0; \sigma^2)$ are independent random errors, compute the restricted maximum likelihood (REML) estimates of $E(y)$ and $var(y)$ where $y^T = (y_{11}, y_{12}, y_{13}, y_{21}, y_{22}, y_{23}, y_{31}, y_{32}, y_{33})$.

Based off of the definition then,

```
yT = c(11.2,11.4,11.6,11.9,11.8,11.3,11.0,11.2,11.3)
y = t(yT)
y = t(y)
```

Then we simply follow the required steps to find the REML!

```
X=rep(1,9)
Z=diag(3)%x%rep(1,3)
C=cbind(diag(8),0)
K=C%*%(diag(9)-1/9*matrix(1,9,9))
sigma=c(1,1)
M=matrix(0,2,2)
q=c(0,0)
ZZt=Z%*%t(Z)
Sigma=sigma[1]*diag(9)+sigma[2]*ZZt
P=t(K)%*%solve(K%*%Sigma%*%t(K))%*%K
M[1,1]=sum(diag(P%*%P))
M[1,2]=sum(diag(P%*%P%*%ZZt))
M[2,1]=sum(diag(P%*%ZZt%*%P))
M[2,2]=sum(diag(P%*%ZZt%*%P%*%ZZt))
q[1]=sum((P%*%y)^2)
q[2]=sum((t(Z)%*%P%*%y)^2)
sigma=solve(M)%*%q
sigma
```

```
##            [,1]
## [1,] 0.05555556
## [2,] 0.04407407
```

Then we perform the next step of the iterative algorithm:

```
Sigma=sigma[1]*diag(9)+sigma[2]*ZZt
P=t(K)%*%solve(K%*%Sigma%*%t(K))%*%K
M[1,1]=sum(diag(P%*%P))
M[1,2]=sum(diag(P%*%P%*%ZZt))
M[2,1]=sum(diag(P%*%ZZt%*%P))
M[2,2]=sum(diag(P%*%ZZt%*%P%*%ZZt))
q[1]=sum((P%*%y)^2)
q[2]=sum((t(Z)%*%P%*%y)^2)
sigma=solve(M)%*%q
sigma
```

```
##             [,1]
## [1,] 0.05555556
## [2,] 0.04407407
```

So there is no change and the algorithm has converged. So the REML estimates are $\hat{\sigma}^2 = 0.0555556$, and $\hat{\sigma}_1^2 = 0.04407407$.