Functional Equations Presentation on Sine F.E. 2

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Theorem 3.6

Let G be a 2-divisible group and $\mathbb{C}$ the field of complex numbers. Let $[G, G]$ be the 2-divisible commutator subgroup of G. Let $\sigma : G \to G$ be an involution. If $f : G \to \mathbb{C}$ is a solution of (3.1) satisfying $f(x) = f(xy\sigma(y))$ for all $x, y \in G$, then f is a function on the quotient group $G/[G, G]$.

Theorem 3.6 Proof

Replacing x by xu and y by x^{-1} in (3.1) for $u \in [G, G]$, we obtain

$$f(xux^{-1})f(xu\sigma(x^{-1})) = f(xu)^2 - f(x^{-1})^2.$$

Since $u \in [G, G]$ and the latter is a normal subgroup of G, this means that $xux^{-1} \in [G, G]$. Therfore $f(xux^{-1}) = 0$ and hence we have

$$f(xu)^2 = f(x^{-1})^2$$

for all $x \in G$ and $u \in [G, G]$.

Theorem 3.6 Proof cont.

By property (3.5), the above equality can be reduced to

$$f(xu)^2 = f(x)^2$$

for all $x \in G$ and $u \in [G, G]$. Using Lemma 3.5, we find that

$$f(xu^2) = f(xu\sigma(u)).$$

Since $f(x) = f(xy\sigma(y))$ for all $y \in G$, we get that $f(xu^2) = f(x)$. Since $[G, G]$ is 2 -divisible the previous equality can be written as $f(xu) = f(x)$. This means that f is a finction of the quotient group $G/[G, G]$.

- QED