M621 HW, due Nov. 10

1. pg. 147, 13.

2. pg. 147, 16.

3. pg. 147, 22.

4. Prove that if G is a group with $|G| = 35$, then G is cyclic.

5. Show that if $n \geq 5$, then A_n contains no subgroup of index less than n. (That is, for any subgroup $H \leq A_n$, $[A_n : H] \geq n$.) This is a good problem.

Suggestion: You know A_n is simple. Also, if K is a subgroup of A_n with $[A_n : K] = j \in \mathbb{N}$, then A_n acts by left multiplication on the set of all left cosets of K ($\{gK : g \in A_n\}$). Since A_n is simple, what are the possibilities are there for the kernel of this action? Now explain why $k \geq n$.