MATH 562-01 MATHEMATICAL STATISTICS

Exam 3 (11/21/16, Monday) Name:________________________

1. (10 points) A random sample of size 9 from a normal distribution $N(\mu,\sigma^2)$ yielded the values 7, 14, 8, 13, 9, 12, 6, 11 and 10. Use the t-statistic to find the one-sided upper 97.5% confidence limit for μ.

2. (10 points) If a symmetric 95% confidence interval for μ of a normal distribution $N(\mu,16)$ is to be constructed, what is the smallest sample size n required so that the maximum error of the estimate will be no more than ¼ of the population standard deviation?

3. (10 points) Let p be the proportion of the Americans who select jogging as one of their recreational activities. If $y = 1974$ out of a random sample of $n = 7557$ selected jogging, find an approximate 99% confidence interval for p.

4. (10 points) Let X and Y be the blood volumes in milliliters for a male who is a paraplegic and participates in vigorous physical activities, and a male who is able-bodied and participates in normal activities, respectively. Assume that X is of $N(\mu_X, \sigma_X^2)$, and Y is of $N(\mu_Y, \sigma_Y^2)$. Using the following n=7 observations of X:

   1612, 1352, 1456, 1222, 1560, 1456, 1924

   and m=10 observations of Y:

   1082, 1300, 1092, 1040, 910, 1248, 1092, 1040, 1092, 1288

   to find a point estimate, and a 95% confidence interval for $\theta = \mu_X - \mu_Y$. (NOTE: The variances of the two populations may not be equal. Explain your choice of method.)

5. (10 points) Let X be a single observation from an exponential distribution $EXP(\theta)$. If a test of hypotheses $H_0 : \theta = 2$ versus $H_a : \theta = 4$ is to reject H_0 whenever $X > 2.6$, find the power function $\Pi(\mu) = P[C_T]$, $\alpha = P[\text{Type I error}]$ and $\beta = P[\text{Type II error}]$, where C_T is the critical region.

6. (10 points) Let $X_1, X_2, \cdots, X_{400}$ be a random sample from $N(\mu,4)$. To test the hypotheses $H_0 : \mu = 5$ versus $H_a : \mu > 5$, a critical region $\bar{x} > c$ is to be used. What is c such that the probability of a Type I error is 0.01?

7. (10 Points) Let $X_1, X_2, \cdots, X_9$ be a random sample from $N(\mu,\sigma^2)$. Find the critical region for the test of $H_0 : \mu = 50$ versus $H_a : \mu > 50$, with the size of the test $\alpha = 0.025$. If we observe $\bar{x} = 52.53$, $s^2 = 3.3^2$, what will be the conclusion of the test? What is approximately the p-value in this case?

8. (10 points) A single observation X from the distribution with density function $f(x;\theta) = \theta x^{-\theta-1}$, if $x > 1$, and zero elsewhere, is used to test $H_0 : \theta = 2$ versus

$H_a: \theta = 5$. Let the critical region be defined by $X < k$. If the probability of Type I error of this test is ¾, what is k, and what is the probability of Type II error?

9. (10 points) The hypothesis $H_0: \mu = 0$ is tested against the alternative $H_a: \mu = 1$ using a t-statistic when sampling from a normal distribution. If $\alpha = P[\text{Type I error}]$, and $\beta = P[\text{Type II error}]$, which of the following statements are true? Explain why. (No credit without explanation.)
   (1). $\beta = 1 - \alpha$.
   (2). If α is fixed, and n is increased, then β decreases.
   (3). If n is fixed, and α is increased, then β decreases.

10. (10 points) A single observation is taken from a Cauchy distribution with density function $f(x) = \dfrac{1}{\pi\left[1 + (x-\theta)^2\right]}$. For testing $H_0: \theta = 0$ versus $H_a: \theta \neq 0$ at the 0.05 significant level using the generalized likelihood ratio test, find the critical region