Brief comments about Nov. 10 HW

1. pg 147, 13: $|G| = 56 = 2^3 7^1$. If $n_7 > 1$, then $n_7 = 8$, and there 48 nonidentity elements of order 7, leaving only 8 elements. Since a(ny) 2-Sylow subgroup has 8 elements, it follows that $n_2 = 1$.

2. Good problem–everyone got it, which is good.

3. if G is not simple, $n_{11} = 12$, producing 120 elements of order 11; non-simplicity also implies $n_3 \geq 4$, which means that there are at least 8 elements of order 3. We only have 4 elements left to fill out the group. Each 2-Sylow has 4 elements, so there can only be only one 2-Sylow.

4. Everyone got this one—which is good.

5. Let $n \in \mathbb{N}$ with $n \geq 5$. Suppose A_n has a proper subgroup H of index j. Then A_n acts on the left cosets of H by left multiplication–of course there exists j left cosets. There is a homomorphism $\sigma : A_n \to S_j$ associated with the action. That A_n is simple implies that $ker(\sigma)$ is either $\{e\}$ or A_n. But if $g \in A_n - H$, $gH \neq H$, which implies that $g \notin ker(\sigma)$. It follows that $ker(\sigma) \neq A_n$. So $ker(\sigma) = \{e\}$, and σ injects A_n into S_j. Since $|A_n| = n!/2$, and $|S_j| = j!$, we have $j! \geq n!/2$. Since $n \geq 5$, $(n-1)!$ is not an upper bound for $n!/2$. Thus $j \geq n$, from which it follows that $[A_n : H] \geq n$.