M621 HW 3, due Sept 15

1. Recall that if $(G, *) = G$ and $H = (H, \circ)$ are groups, then a map $\Gamma : G \to H$ is a homomorphism if Γ is compatible with the operations—more formally, Γ is a homomorphism if for all $y, z \in G$, $\Gamma(y * z) = \Gamma(y) \circ \Gamma(z)$.

   (a) True or false? If "false", provide a specific counterexample: If G is a group and $b \in G$, then $l_b : G \to G$ given by $l_b(h) = b * h$ for all $h \in H$, is a homomorphism. (You can use "bh" in place of the more cumbersome "$b * h$".)

   (b) True or false? If "false", provide a specific counterexample: If G is an Abelian group, and $g \in G$, then l_b (given above), is a homomorphism.

   (c) Let e_G be the identity of G, and let e_H be the identity of H. Prove that if $\Gamma : G \to H$ is a homomorphism, then $\Gamma(e_G) = e_H$.

   (d) The *kernel of* Γ, $ker(\Gamma)$, is the set of all $g \in G$ such that $\Gamma(g) = e_H$. So $ker(G) = \{g \in G : \Gamma(g) = e_H\}$. Prove that $ker(\Gamma)$ is a subgroup of G.

2. page 23, problem 33.

3. **Short answer.** page 28, problem 15: This is not difficult since $\mathbb{Z}/n\mathbb{Z}$ is cyclic—that is, “1-generated”—so a presentation that involves only one generator (and one relation, for that matter) can be given.

4. page 28, problem 17. Be sure to read the discussion on page 26-27.

5. page 45, problem 18.

6. Let G be a group, and let A be a set. Suppose that G acts on A. You can use "ga" in place of "$g \cdot a$" for the action of $g \in G$ on an element $a \in A$.

   (a) Let $a \in A$. The *stabilizer of* a, $St(a)$, is the set $\{g \in G : ga = a\}$. Provide a short proof that $St(a)$ is a subgroup of G.

   (b) Let B be a non-empty subset of A. The *stabilizer of* B, $St(B)$, is the set $\{g \in G : gB \subseteq B\}$. Provide a short proof that $St(B)$ is a subgroup of G.

   (c) True or false? If "false", provide a specific counterexample: Let a and b are distinct elements of A, then $St(a) \cap St(b) = St(\{a, b\})$.