Problems for Section 3.1

Sharpen Your Skills (Answers in back of text.)

In Problems 1 and 2 use the given variables to translate the statements into constraints.

1. Let P be the number of gallons of premium gasoline and R be the number of gallons of regular gasoline a refinery produces in a month.
   (a) Each gallon of premium takes 0.7 gallons of high-octane fuel, and each gallon of regular takes 0.4 gallons of high-octane fuel, and there are 250,000 gallons of high-octane fuel available.

   (b) The refinery can produce at most 100,000 gallons of regular gasoline in a month.

2. Let T be the number of tables and C be the number of chairs that a furniture manufacturer will make this week.
   (a) The manufacturer can make at most 30 chairs this week.

   (b) Each table takes 4 hours of labor, and each chair takes 5 hours of labor. There are 240 hours of labor available this week.

In Problems 3 and 4 decide whether each of the given points is feasible for the given system of constraints. If the point is feasible, give the slack in each constraint.

3. Constraints:

$$20x + 50y \le 600$$
$$40x + 30y \le 800$$
$$35x \le 400$$
$$x, y \ge 0$$

Points: (a) (15, 6)

(b) (10, 8)

4. Constraints:

$$250x + 400y \le 32{,}500$$
$$350x + 200y \le 28{,}000$$
$$500y \le 25{,}000$$
$$x, y \ge 0$$

Points: (a) (60, 40)

(b) (50, 50)

Communicate the Concepts: (Answers in back of text.)

5. Describe how to decide whether a given set of values for the production variables is feasible for a system of constraints involving those variables.

6. What is meant by the "slack" in a constraint for a given set of values of the production variables?

7. What are the "non-negativity" constraints on a linear programming problem?

8. What is the "objective function" of a linear programming problem?

Apply Your Knowledge: (Answers to odd-numbered problems in back of text.)

Obtain a mathematical formulation for each linear programming problem below. Follow the steps outlined in the text. Define all the variables, give their units, and label each constraint (inequality) with the name and units of the quantity that causes the constraint. Answer any questions about feasibility and slack.

9. A refinery mixes high-octane and low-octane fuels to make regular and premium gasoline. One gallon of regular is produced by mixing 3/4 gallon of low-octane and 1/4 gallon of high-octane fuel. One gallon of premium is produced by mixing 1/2 gallon of low octane and 1/2 gallon of high-octane fuel. The refinery makes \$0.20 profit on regular and \$0.30 profit on premium. The refinery has 50,000 gallons of high-octane and 60,000 gallons of low-octane available today. Formulate mathematically the problem of determining how many gallons of each type of gasoline the refinery should make today to maximize its profit. Is it feasible to make 70,000 gallons of regular and 40,000 gallons of premium gasoline today? If so, what is the slack in each constraint?

Solution:

Production variables:

Profit Function:

Product-Resource Chart:

Constraints:

Feasibility Test:

Slack?

10. John makes regular and deluxe birdhouses. A store has contracted to purchase his birdhouses, up to 8 regular and up to 6 deluxe each day. He will work on them up to 9 hours a day, but it will take him 3/4 hour to make a regular birdhouse and 1 hour to make a deluxe birdhouse. He makes a $10 profit on each regular birdhouse and $16 profit on each deluxe birdhouse. Formulate mathematically the problem of determining how many of each type he should make per day to maximize his profit. Is it feasible for him to make 6 regular and 5 deluxe birdhouses in one day? If so, what is the slack in each constraint?

Solution:

Production variables:

Profit Function:

Product-Resource Chart: Constraints:

Feasibility Test: Slack?

11. Kentucky Weavers make shawls and afghans. They spin yarn, dye yarn, and weave yarn for each. A shawl requires 1 hour of spinning, 1 hour of dyeing, and 1 hour of weaving. An afghan requires 2 hours of spinning, 1 hour of dyeing, and 6 hours of weaving. In a week, there are at most 14 spinning hours, 11 dyeing hours, and 30 weaving hours available. Formulate mathematically the problem of determining how many shawls and how many afghans Kentucky Weavers should make in one week to maximize profit if a shawl bring a profit of $25 and an afghan brings a profit of $40. Is it feasible for them to make 6 shawls and 4 afghans in one week? If so, what is the slack in each constraint?

Solution:

Production variables:

Profit Function:

Product-Resource Chart: Constraints:

Feasibility Test: Slack?

14. A bottler uses pineapple, orange, and grapefruit juice to make two juice mixtures, orange-pineapple and orange-grapefruit. The mixtures are sold in quart bottles, and the bottler makes a profit of $0.50 per bottle on orange-pineapple and $0.40 per bottle on orange-grapefruit. Each juice mixture is made by mixing equal amounts of the two juices in its name. Today there are 250 gallons of orange juice, 175 gallons of pineapple juice, and 100 gallons of grapefruit juice available. Formulate mathematically the problem of determining how many quart bottles of each juice mixture the bottler should produce today to maximize profit? (1 gallon = 4 quarts)

Solution:

Production variables:

Profit Function:

Product-Resource Chart: Constraints:

15. Cardinal Candy makes a Rick Pitino mix and a Denny Crum mix. A box of the Rick Pitino mix takes 0.4 pounds of chocolate, 0.2 pounds of nuts, and 0.4 pounds of fruit, and sells for $12.95. A box of the Denny Crum mix takes 0.2 pounds of chocolate, 0.2 pounds of nuts, and 0.6 pounds of fruit, and sells for $9.95. Chocolate costs $6.00 per pound, nuts cost $4.00 per pound, and fruit costs $3.00 per pound. This week Cardinal Candy has 44 pounds of chocolate, 26 pounds of nuts, and 72 pounds of fruit.
   (a) How much profit does Cardinal Candy make on a box of each type of candy mix?
   (b) Formulate the problem of finding the number of each type of candy mix Cardinal Candy should make to maximize their profit. (Assume they can sell all they make.)
   (c) Is it feasible to make 60 boxes of Rick Pitino and 70 boxes of Denny Crum? If so, what is the slack in each constraint?

Solution:

Production variables:

Profit on a box of Rick Pitino:

Profit on a box of Denny Crum:

Profit Function:

Product-Resource Chart: Constraints:

Feasibility Test: Slack?

Graph the feasible region, identify each corner point and find its coordinates, and use the Corner Point Principle to solve the problem. Complete the sentence below.

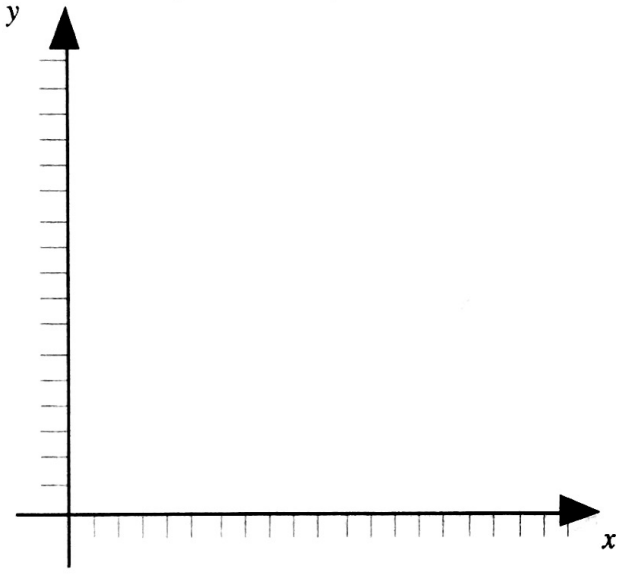
32. Problem 12 of Section 3.1. Let x = number of tables to be made
y = number of chairs to be made
Maximize: $P = \$60x + \$30y$
Subject to: (1) $50x + 20y \le 3300$ feet lumber
(2) $3x + 4y \le 380$ hours of labor
$x \ge 0, y \ge 0$

Corner Points:

Intercepts

x	y

x	y



Choose Best Corner Point:

Solution: Make _____ tables and _____ shawls. This will earn a profit of _____.

There will be _____ feet of lumber and _____hours of labor left unused (slack).

Graph the feasible region, identify each corner point and find its coordinates, and use the Corner Point Principle to solve the problem. Complete the sentence below.

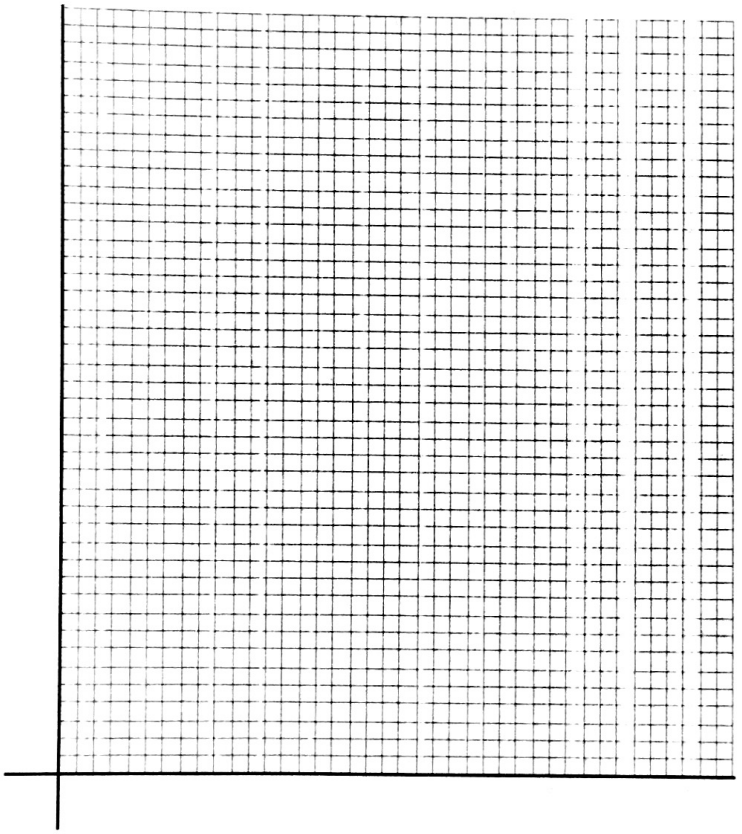
33. Problem 13 of Section 3.1. Let S = number of Standard tents, D = number of Deluxe tents to be made.

Maximize $P = \$30S + \$50D$

Subject to
(1) $2S + 1D \leq 420$ hours of cutting and assembly time
(2) $2S + 2D \leq 500$ yards of fabric
(3) $2S + 3D \leq 660$ hours of finishing time
$S \geq 0, D \geq 0$

Intercepts:

Corner Points:



Corner Points:

Best Corner Point:

Solution: Make _____ standard and _____ deluxe tents. This will earn a profit of _____. There will be _____ hours of cutting and assembly, _____yards of fabric, and _____hours of finishing time left unused (slack).

Graph the feasible region, identify each corner point of the feasible region and find its coordinates, and use the Corner Point Principle to solve the problem. Complete the sentence below.

34. Problem 14 of Section 3.1.

Let x = # **1 qt** bottles of orange-pineapple, y = # **1 qt** bottles of orange-grapefruit

Maximize Profit $P = \$0.50x + \$0.40y$

Subject to: (1) $0.5x + 0.5y \le 1000$ quarts orange juice

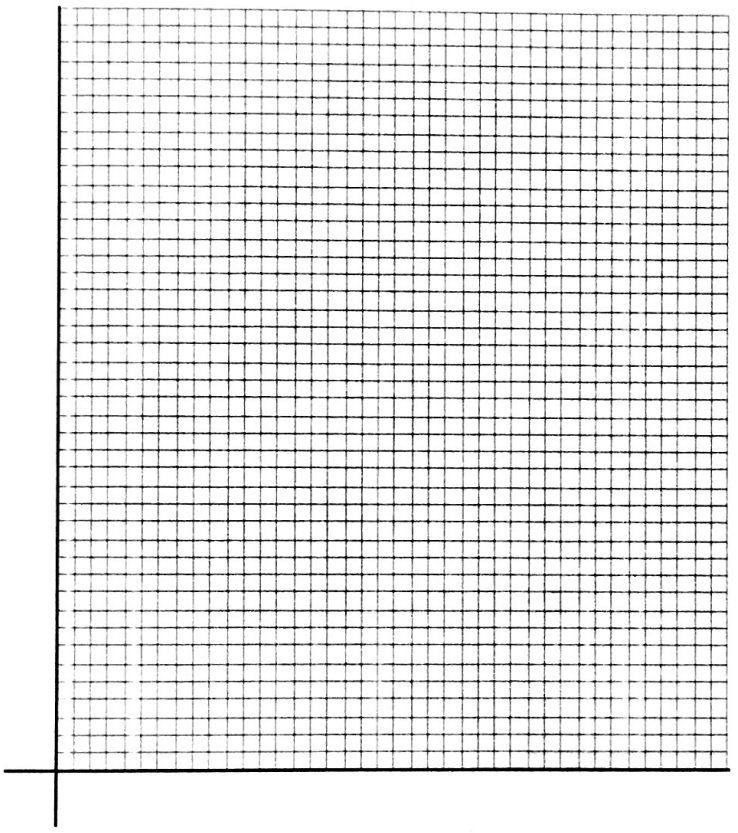
(2) $0.5x \le 700$ quarts pineapple juice

(3) $0.5y \le 400$ quarts grape juice

$x \ge 0, y \ge 0$

Intercepts:

Corner Points:



Corner Points:

Best Corner Point:

Solution: Make _____ bottles of orange-pineapple and _____ bottles of orange-grapefruit.

This will earn a profit of _____. There will be _____ quarts of orange juice,

_____quarts of pineapple juice, and _____quarts of grape juice left unused (slack).

Graph the feasible region, identify each corner point of the feasible region and find its coordinates, and use the Corner Point Principle to solve the problem. Complete the sentence below.

35. Problem 15 of Section 3.1.

Let x = # boxes of Rick Pitino mix to make, y = # boxes of Denny Crum mix to make.

Maximize $P = \$8.55x + \$6.15y$

subject to (1) $0.4x + 0.2y \leq 44$ lbs chocolate

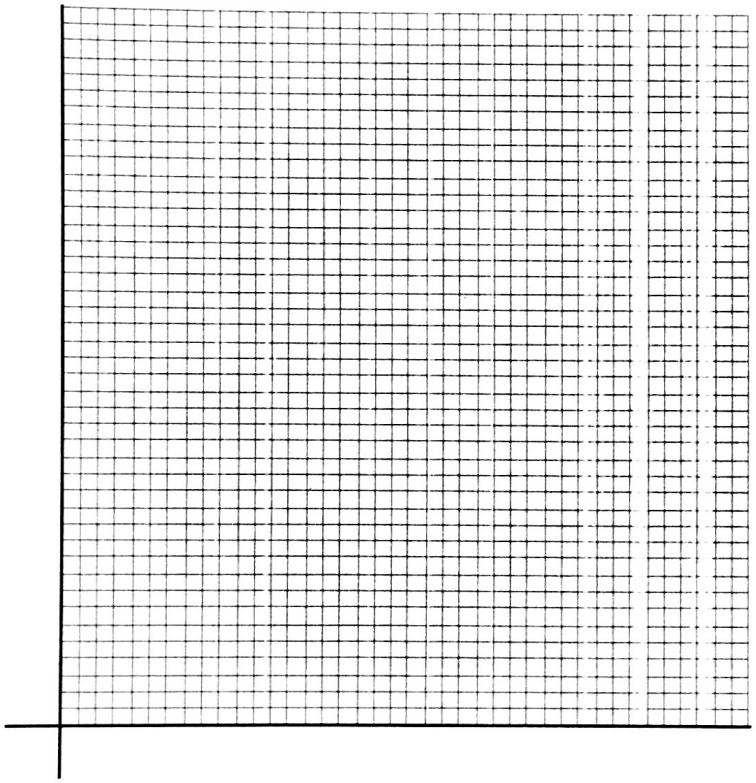
(2) $0.2x + 0.2y \leq 26$ lbs nuts

(3) $0.4x + 0.6y \leq 72$ lbs fruit

$x \geq 0, y \geq 0$

Intercepts:

Corner Points:



Corner Points:

Best Corner Point:

Solution: Make _____ boxes of Rick Pitino mix and _____ boxes of Denny Crum mix. This will earn a profit of _____. There will be _____ lbs. of chocolate, _____ lbs. of nuts, and _____ lbs. of fruit left unused (slack).