Section 3.1

1. (a) $0.7P$ gallons of high-octane are in P gallons of premium, and $0.4R$ gallons of high-octane are in R gallons of regular, for a total of $0.7P + 0.4R$ gallons of high-octane. This must be $\leq$ 250,000 gallons high-octane available. The constraint is $0.7P + 0.4R \leq 250{,}000$.gallons of high-octane.
   (b) $R \leq 100{,}000$ gallons regular
2. (a) $C \leq 30$
   (b) Making T tables uses $4T$ hours of labor, and making C chairs uses $5C$ hours of labor. A total of $4T + 5C$ hours of labor are used in T tables and C chairs, and this must be $\leq$ 240 hours available. Constraint: $4T + 5C \leq 240$ hours of labor.
3. Constraints:
   (1) $20x + 50y \leq 600$
   (2) $40x + 30y \leq 800$
   (3) $35x \leq 400$
   $x \geq 0,\ y \geq 0$

   Points: (a) (15, 6)
   $20(15) + 50(6) = 600$ which is ≤ 600 True
   $40(15) + 30(6) = 780$ which is ≤ 800 True
   $35(15) = 525$ which is not ≤ 400 False; Point is not feasible.

   (b) (10, 8)
   $20(10) + 50(8) = 600 \leq 600$ True Slack $= 600 - 600 = 0$
   $40(10) + 30(8) = 640 \leq 800$ True Slack $= 800 - 640 = 160$
   $35(10) = 350 \leq 400$ True Slack $= 400 - 350 = 50$

   (10, 8) is feasible.
4. Constraints:
   $250x + 400y \leq 32{,}500$
   $350x + 200y \leq 28{,}000$
   $500y \leq 25{,}000$ $x \geq 0,\ y \geq 0$

   Points: (a) (60, 40)
   $250(60) + 400(40) = 31{,}000 \leq 32{,}500$ True Slack $= 32{,}500 - 31{,}000 = 1{,}500$
   $350(60) + 200(40) = 29{,}000$ which is not $\leq 28{,}000$

   Point is not feasible.

   (b) (50, 50)
   $250(50) + 400(50) = 32{,}500 \leq 32{,}500$ True Slack $= 32{,}500 - 32{,}500 = 0$
   $350(50) + 200(50) = 27{,}500 \leq 28{,}000$ True Slack $= 28{,}000 - 27{,}500 = 500$
   $500(50) = 25{,}000 \leq 25{,}000$ True Slack $= 25{,}000 =- 25{,}000 = 0$

   Point is feasible.
5. Substitute the values for the production variables into the constraints. If the values make each constraint true, then the set of values is feasible.
6. The slack in a constraint is the amount of the resource available that exceeds the amount used by the given values of the production variables.
7. The non-negativity constraints state that all the production variables must be non-negative.
8. The objective function describes how the quantity to be maximized (usually profit) can be calculated from the values of the production variables.
9. The product-resource chart is:

	Regular Gasoline	Premium Gasoline	Available
low-octane fuel	0.75 gal	0.5 gal	60,000 gallons
high-octane fuel	0.25 gal	0.5 gal	50,000 gallons

Let R = # gallons of regular gasoline to produce
P = # gallons of premium gasoline to produce.

Maximize $\quad Profit = \$0.20R + \$0.30P$

Subject to: $\quad 0.75R + 0.5P \leq 60{,}000$ gallons of low-octane fuel
$0.25R + 0.5P \leq 50{,}000$ gallons of high-octane fuel
$R \geq 0,\ P \geq 0$

Is $R = 70{,}000$, $P = 40{,}000$ feasible? Test:
Is $0.75(70{,}000) + 0.5(40{,}000) \leq 60{,}000$?
$52{,}500 + 20{,}000 = 72{,}500 > 60{,}000$ **NO**. There is not enough low-octane fuel.

Section 3.1 11. The product-resource chart is:

	Each Shawl	Each Afghan	Available
spinning	1 hours	2 hours	14 hours
dying	1 hour	1 hour	11 hours
weaving	1 hour	6 hours	30 hours

Let $x = \#$ shawls to be made, $y = \#$ afghans to be made
Maximize $P = \$25x + \$40y$
Subject to:
(1) $x + 2y \le$ 14 hours spinning
(2) $x + y \le$ 11 hours dying
(3) $x + 6y \le$ 30 hours weaving
$x \ge 0, y \ge 0$

Is $x = 6, y = 4$ feasible? Test: $6 + 2(4) = 14 \le 14$? Yes Slack $= 14 - 14 = 0$
$6 + 4 = 10 \le 11$? Yes Slack $= 11 - 10 = 1$
$6 + 6(4) = 30 \le 30$? Yes. Slack $= 30 - 30 = 0$ So $x = 6, y = 4$ is feasible

13. Let S be the number of Standard tents, D be the number of Deluxe tents to be made this week.
The product-resource chart is:

	Standard	Deluxe	Available
Cut & Assemble	2 hrs/tent	1 hr/tent	420 hours
Fabric	2 yds/tent	2 yds/tent	500 yards
Finishing	2 hrs/tent	3 hrs/tent	660 hours

Reading the per tent profits listed in the problem leads to the formulation:
Maximize $P = \$30S + \$50D$
subject to (1) $2S + 1D \le 420$ hours of cutting and assembly time
(2) $2S + 2D \le 500$ yards of fabric
(3) $2S + 3D \le 660$ hours of finishing time
$S \ge 0, D \ge 0$

Is $S = 100, D = 150$ feasible? Test:
$2(100) + 1(150) = 350 \le 420$? True Slack $= 420 - 350 = 70$ hours of C&A time unused
$2(100) + 2(150) = 500 \le 500$? True Slack $= 500 - 500 = 0$ yards of fabric unused
$2(100) + 3(150) = 650 \le 660$? True Slack $= 660 - 650 = 10$ hours of finishing time unused
Yes it is feasible.

15. Let $x =$ number of boxes of Rick Pitino mix, $y =$ number of boxes of Denny Crum mix.
The product-resource chart is:

	Boxes of RP	Boxes of DC	Available
chocolate	0.4 lbs/box	0.2 lbs/box	44 lbs
nuts	0.2 lbs/box	0.2 lbs/box	26 lbs
fruit	0.4 lbs/box	0.6 lbs/box	72 lbs

(a) Profit on a box of RP: \$12.95 - 0.4(\$6) - 0.2(\$4) - 0.4(\$3) = \$8.55
Profit on a box of DC: \$9.95 - 0.2(\$6) - 0.2(\$4) - 0.6(\$3) = \$6.15
(b) So total profit is $P = \$8.55x + \$6.15y$
Formulation:
Maximize $P = \$8.55x + \$6.15y$
subject to (1) $0.4x + 0.2y \le$ 44 lbs chocolate
(2) $0.2x + 0.2y \le$ 26 lbs nuts
(3) $0.4x + 0.6y \le$ 72 lbs fruit
$x \ge 0, y \ge 0$

Feasibility Test: $0.4(60) + 0.2(70) = 38 \le 44$ True Slack $= 44 - 38 = 6$ lbs chocolate
$0.2(60) + 0.2(70) = 26 \le 26$ True Slack $= 26 - 26 = 0$ lbs. nuts
$0.4(60) + 0.6(70) = 66 \le 72$ True Slack $= 72 - 66 = 6$ lbs. fruit
So (60, 70) is feasible.

Section 3.1 17. Let x = Number of first type of table to be made,
y = Number of second type of table to be made, z = Number of third type of table to be made

The product-resource chart is:

	Table #1	Table #2	Table #3	Amount Available
Assembly	1 hour	2 hour	3 hour	500 hours
Painting	2 hour	1 hour	2 hour	400 hours
Finishing	1 hour	3 hour	1 hour	600 hours

Then the constraints are:

$$\begin{aligned} 1x + 2y + 3z &\le 500 \text{ hours Assembly} \\ 2x + 1y + 2z &\le 400 \text{ hours Painting} \\ 1x + 3y + 1z &\le 600 \text{ hours Finishing} \\ x \ge 0, y &\ge 0, z \ge 0. \end{aligned}$$

and the profit function is $P = 40x + 50y + 90z$ dollars

Feasibility: Test: $1(100) + 2(100) + 3(100) = 600$ which is not ≤ 500 hours Assembly. Not feasible.

19. Let C = number of cotton-filled jackets to make, W = number of wool-filled jackets to make, and D = number of down-filled jackets to make.

Convert pounds to ounces and hours to minutes. Since all the cotton needed is available, there is no constraint on the amount of cotton. The product-resource chart is:

	Cotton-filled	Wool-filled	Down-filled	Available
Sewing time	50 min	50 min	50 min	40,000 minutes
stuffing time	8 min	9 min	6 min	4,804 minutes
wool	0	12 ounces	0	3600 ounces
down	0	0	12 ounces	2160 ounces

Maximize $P = 24C + 32W + 36D$ dollars

Subject to:

$$\begin{aligned} 50C + 50W + 50D &\le 40000 \text{ minutes of sewing time} \\ 8C + 9W + 6D &\le 4804 \text{ minutes of stuffing time} \\ 12W &\le 3600 \text{ ounces of wool} \\ 12D &\le 2160 \text{ ounces of down.} \quad C \ge 0,\ W \ge 0,\ D \ge 0 \end{aligned}$$

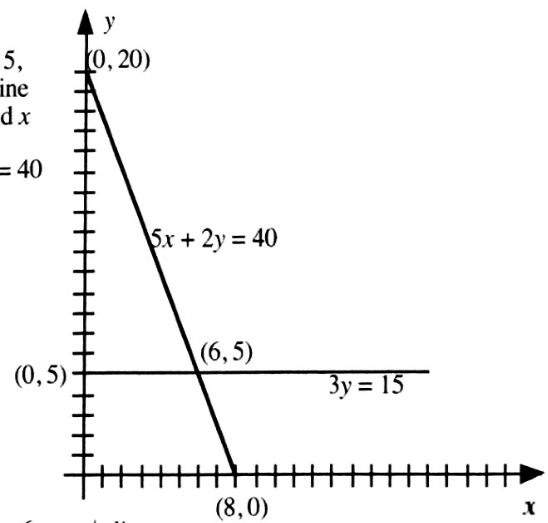
Section 3.2

1. $5x + 2y = 40$
$3y = 15$

To find the point of intersection substitute $y = 5$ into the first equation:
$5x + 2(5) = 40$ simplifies to
$5x + 10 = 40$
which simplifies to
$5x = 30$ so $x = 6$
So the intersection point is $x = 6$, $y = 5$

The second line is $y = 5$, which is a horizontal line since y is fixed at 5 and x is unrestricted.
The first line $5x + 2y = 40$ has intercepts shown below:

x	y
0	20
8	0

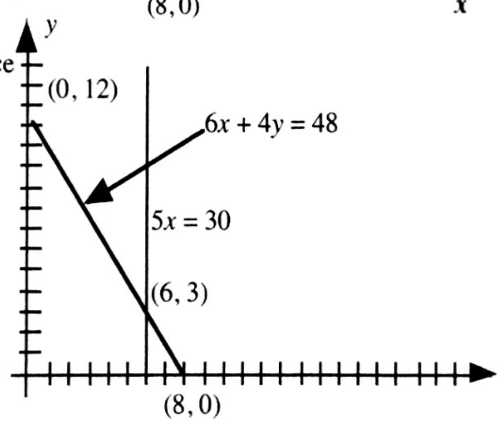


2. $6x + 4y = 48$
$5x = 30$

From second equation $x = 6$
Substitute in first equation:
$6(6) + 4y = 48$ which simplifies to
$4y = 12$, so $y = 3$.
The intersection point is (6, 3)

The second line is $x = 6$, which is a vertical line since x is fixed at 6 and y is unrestricted. The first line $6x + 4y = 48$ has intercepts shown below:

x	y
0	12
8	0



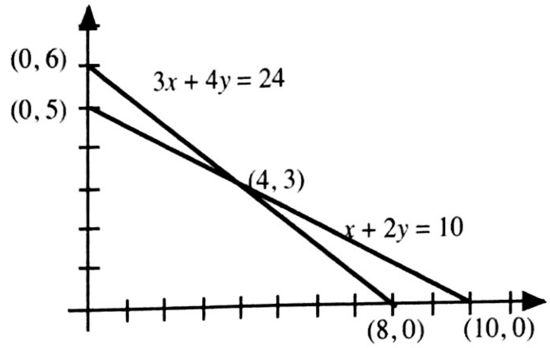
Section 3.2 3. $3x + 4y = 24$
$x + 2y = 10$

$3x + 4y = 24$

x	y
0	6
8	0

$x + 2y = 10$

x	y
0	5
10	0



One solution is to multiply the second equation by 2: $3x + 4y = 24$
$\underline{2x + 4y = 20}$
Subtract: $x = 4$.
Substitute $x = 4$ in first equation: $3(4) + 4y = 24$
$12 + 4y = 24$
$4y = 12$, so $y = 3$. The solution is $x = 4, y = 3$.

4. $4x + 4y = 40$
$2x + 6y = 42$
Multiply second by 2:
$4x + 4y = 40$
$4x + 12y = 84$
Subtract: $-8y = -44$
Divide: $y = 5.5$

Substitute in first equation:
$4x + 4(5.5) = 40$
$4x + 22 = 40$
$4x = 18 \quad x = 4.5$
Intersection point is $(4.5, 5.5)$

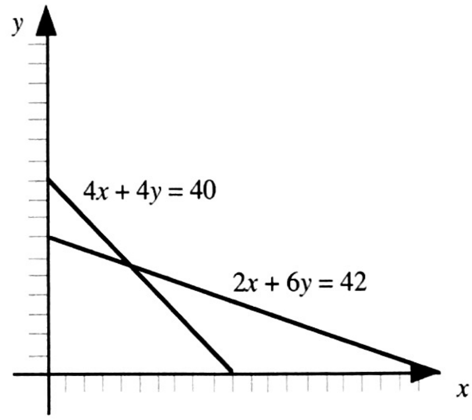
Intercepts:

$4x + 4y = 40$

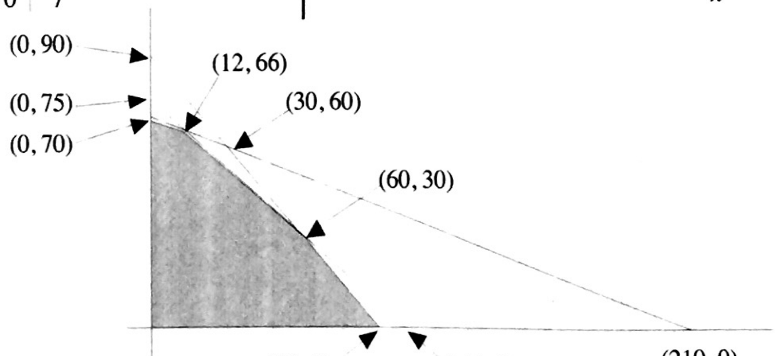
x	y
10	0
0	10

$2x + 6y = 42$

x	y
21	0
0	7



5. (a) The shaded feasible region is shown at right.
(b) $40 + 3(45) = 175 \leq 210$ OK Slack = 35
$40 + 45 = 85 \leq 90$ OK Slack = 5
$3(40) + 4(45) = 300 \leq 300$ OK Slack = 0
$(40, 45)$ is feasible
(c) $(35, 50)$ is **not** feasible; it fails the last constraint:
$3(35) + 4(50) = 305 > 300$



(d) The corners are $(0, 0), (0, 70), (12, 66), (60, 30), (90, 0)$.
(e) and (f) Use the corner point principle; make a chart of the corner points and values of P and Q, as in the table below. Note: Do not use other points, such as $(30, 60)$, which are not corner points.

Corner Point	$P = 60x + 40y$	$Q = 60x + 90y$
$(0, 0)$	0	0
$(0, 70)$	$40(70) = 2800$	$90(70) = 6300$
$(12, 66)$	$60(12) + 40(66) = 3360$	$60(12) + 90(66) = 6660$
$(60, 30)$	$60(60) + 40(30) = 4800$	$60(60) + 90(30) = 6300$
$(90, 0)$	$60(90) = 5400$	$60(90) = 5400$

The maximum of P is 5400 at $(90, 0)$. The maximum of Q is 6660 at $(12, 66)$.

Section 3.2 6. (a) The feasible region is the part of the first quadrant below all the lines, as shown.

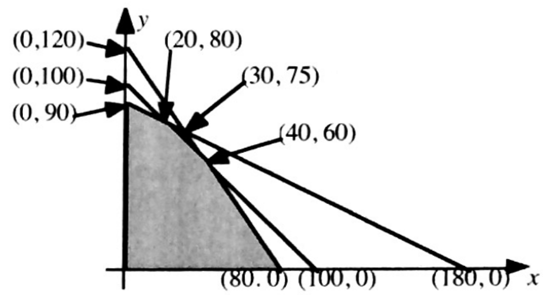
(b) Substitute (50, 40) in all the constraints.

$50 + 2(40) = 130 \le 180$ OK Slack = 50

$2(50) + 2(40) = 180 \le 200$ OK Slack = 20

$3(50) + 2(40) = 230 \le 240$ OK Slack = 10

It satisfies them and hence is feasible.



(c) Substitute (35, 70) in the constraints. It fails the second and third constraints:

$35 + 2(70) = 175 \le 180$ OK

$2(35) + 2(70) = 210$ is not ≤ 200 False

$3(35) + 2(70) = 245$ is not ≤ 240 False

So (35, 70) is not feasible.

(d) Corner points are (0, 90), (20, 80), (40, 60), (80, 0), and (0,0)

(e) and (f) Substitute each corner point into the respective profits:

Corner Point	$P = 60x + 40y$	$Q = 60x + 90y$
(0, 90)	40(90) = 3600	90(90) = 8100
(20, 80)	60(20) + 40(80) = 4400	60(20) + 90(80) = 8400
(40, 60)	60(40) + 40(60) = 4800	60(40) + 90(60) = 7800
(80, 0)	60(80) = 4800	60(80) = 4800
(0, 0)	0	0

(e) For $P = 60x + 40y$ the maximum profit of 4800 is attained at the two corner points (40, 60) and (80, 0) (and along the segment joining them as well).

(f) For $P = 60x + 90y$ the maximum profit of 8400 is attained at (20, 80).

7. Change the $\le$ to = to obtain the equation of the constraint line. Graph this line. (Simplest way is to determine its intercepts. Each is found by setting one variable = 0 and solving for the other variable.) Once the constraint line is graphed, select a test point not on the line. Substitute the test point into the original inequality and determine whether the test point satisfies the inequality. If it does, the graph of the inequality consists of the line and the half-plane containing the point. If the test point does not satisfy the inequality, the graph consists of the line and the half-plane on the other side of the line.

8. The feasible region consists of all the points (x, y) that satisfy all the inequalities. Said another way, it consists of the intersection of all the graphs of the linear inequalities in the system.

9. The Corner Point Principle says that the maximum value of the objective function (usually the profit) must occur at one of the corner points of the feasible region for the system of constraints.

10. First determine the coordinates of all the corner points of the feasible region. Then determine the value of the objective function at each corner point. By the Corner Point Principle, the largest one of these values is the largest value of the objective function at any point in the feasible region. Any point that gives that largest value is a best course of action.

11. If two linear programming problems have the same system of constraints then their feasible regions are the same, and so the corner points are also the same.

12. The objective function is used to choose the best corner point of the feasible region, and that best corner point is the best course of action.

13. The intercepts and the feasible region are shown at right.

The corner points are the origin, the intercepts (0, 9) and (10, 0), and the intersection point of the two lines. One way to find that point is to multiply $x + y = 10$ by 3 and subtract it from $3x + 4y = 36$:

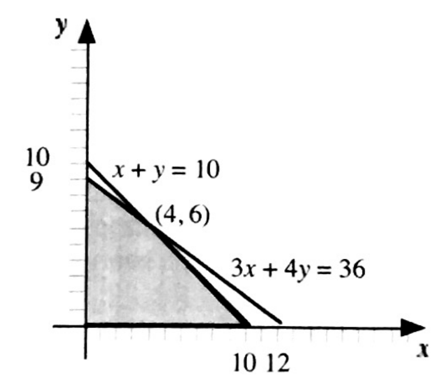
$$\begin{array}{lr} & 3x + 4y = 36 \\ \text{Subtract:} & \underline{3x + 3y = 30} \\ & y = 6 \end{array}$$

Substitute: $x + 6 = 10$, so $x = 4$

This corner point is (4, 6).

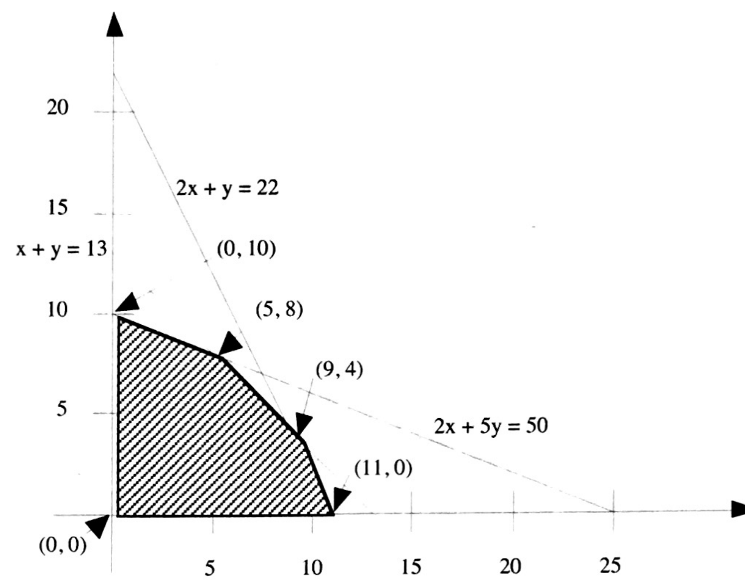
Intercepts

$3x + 4y = 36$		$x + y = 10$	
x	y	x	y
0	9	0	10
12	0	10	0



Section 3.2 15. The intercepts are: For (1) $2x + y = 22$: $(0, 22)$ and $(11, 0)$.
For (2) $x + y = 13$: $(0, 13)$ and $(13, 0)$. For (3) $2x + 5y = 50$: $(0, 10)$ and $(25, 0)$.
The origin makes each inequality true, so we want the region in the first quadrant that is on the same side of all the lines as the origin. See the figure below.

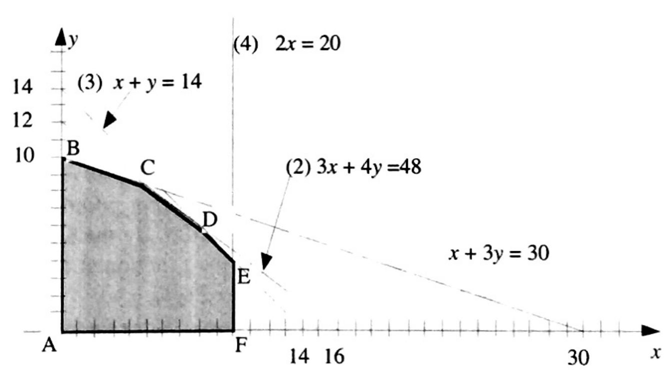
Two of the corners are intercepts of the lines: the y-intercept of $2x + 5y = 50$, which is $(0, 10)$, and the x-intercept of $2x + y = 22$, which is $(11, 0)$.



One of the other corners is the solution of
(2) $x + y = 13$
(3) $2x + 5y = 50$.
Multiply (2) by 2 to make both x terms $2x$: $2x + 2y = 26$
$\underline{2x + 5y = 50}$
Subtract: $-3y = -24$,
so $y = 8$.
Substitute in (2): $x + 8 = 13$, so $x = 5$. This gives $(5, 8)$ as this corner.

The last corner is the solution of
(1) $2x + y = 22$
(2) $\underline{x + y = 13}$
Subtract: $x = 9$ Substitute in (2): $9 + y = 13$, so $y = 4$. This corner is $(9, 4)$.

17. The intercepts are:
For (1) $x + 3y = 30$:
$(30, 0)$ and $(0, 10)$
For (2) $3x + 4y = 48$:
$(16, 0)$ and $(0, 12)$
For (3) $x + y = 14$:
$(14, 0)$ and $(0, 14)$
(4) $2x = 20$ is a vertical line;
$x = 10$.



The origin makes each inequality true, so we want the region in the first quadrant that is on the same side of all the lines as the origin.
Corner A is $(0, 0)$, Corner B is the y-intercept of (1), so B is $(0, 10)$, and
Corner F is the x-intercept of (4), so F is $(10, 0)$.
Corner E is the intersection of (4) $2x = 20$ and (3) $x + y = 14$. Solving (4) gives the value of $x = 10$.
Substituting this into (3) gives $10 + y = 14$, or $y = 4$. So E is $(10, 4)$.
Corner D is the intersection of: (2) $3x + 4y = 48$
(3) $x + y = 14$
Multiply (3) by 3 and subtract the result from (2); (2) $3x + 4y = 48$
3 x (3): $\underline{3x + 3y = 42}$
Subtract: $y = 6$
Substitute this into (3): $x + 6 = 14$, which gives $x = 8$. So D is $(8, 6)$.
Corner C is the intersection of: (1) $x + 3y = 30$ times 3: $3x + 9y = 90$
(2) $3x + 4y = 48$ $\underline{3x + 4y = 48}$
Subtract: $5y = 42$, or $y = 8.4$.
Substitute into (1): $x + 3(8.4) = 30, x + 25.2 = 30$, $x = 4.8$. So C is $(4.8, 8.4)$.

Section 3.2 19. The intercepts are:

For (1) $x + 2y = 38$: $(38, 0)$ and $(0, 19)$
For (2) $2x + y = 34$: $(17, 0)$ and $(0, 34)$
For (3) $5x + 4y = 100$: $(20, 0)$ and $(0, 25)$

The feasible region (at right) consists of all points on the origin side of all three lines.

Corner E is (0, 0). Corner A is the y-intercept of line (1), so A is (0, 19). Corner D is the x-intercept of line (2), so D is (17, 0).

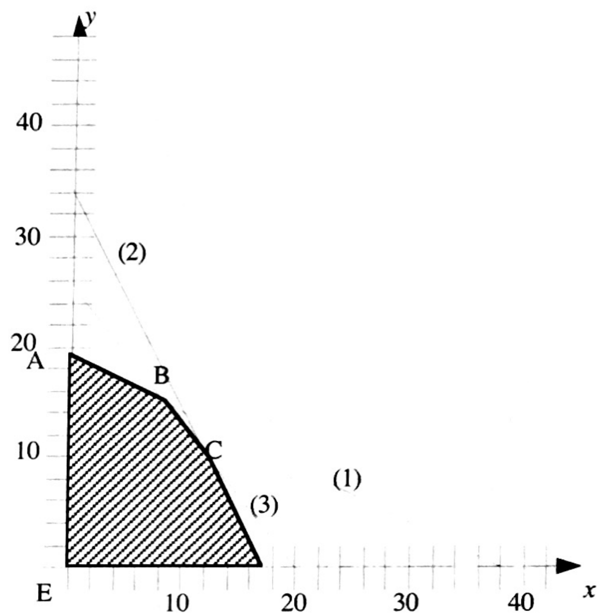
Corner B is the intersection of (1) and (3):

$$\begin{array}{lrl} & (1)\ x + 2y = & 38 \\ & (3)\ 5x + 4y = & 100 \\ \text{2 times (1):} & \underline{2x + 4y =} & \underline{76} \\ \text{Subtract:} & 3x & = 24, \text{ so } x = 8 \end{array}$$

Substitute in (1): $8 + 2y = 38$, so $2y = 30$, $y = 15$. Corner B is (8, 15).

$$\begin{array}{lrl} \text{Corner C:} & (2)\ 2x + y = & 34 \\ & (3)\ 5x + 4y = & 100 \\ \text{4 times (2):} & \underline{8x + 4y =} & \underline{136} \\ \text{Subtract:} & -3x & = -36, \text{ so } x = 12. \end{array}$$

Substitute in (2): $2(12) + y = 34$, so $y = 10$. Corner C is (12, 10).



21. Find the values of (a) $P = 5x + 6y$ and (b) $P = 6x + 4y$ at each corner found in Problem 13.

Corner	$P = 5x + 6y$	$P = 6x + 4y$
(0, 9)	54	36
(4, 6)	20 + 36 = 56	24 + 24 = 48
(10, 0)	50	60
(0, 0)	0	0

(a) Maximum of $P = 5x + 6y$ is 56, which occurs at $(x, y) = (4, 6)$.
(b) Maximum of $P = 6x + 4y$ is 60, which occurs at $(x, y) = (10, 0)$.

23. Find the values of (a) $P = 3x + 4y$ and (b) $P = 2x + 7y$ at each corner found in Problem 15.

Corner	$P = 3x + 4y$	$P = 2x + 7y$
(0, 10)	40	70
(5, 8)	15 + 32 = 47	10 + 56 = 66
(9, 4)	27 + 16 = 43	18 + 28 = 46
(11, 0)	33	22
(0, 0)	0	0

(a) Maximum of $P = 3x + 4y$ is 47 which occurs at $(x, y) = (5, 8)$.
(a) Maximum of $P = 2x + 7y$ is 70 which occurs at $(x, y) = (0, 10)$.

25. Find the values of (a) $P = 8x + 11y$ and (b) $P = 9x + 4y$ at each corner found in Problem 17.

Corner	$P = 8x + 11y$	$P = 9x + 4y$
(0, 10)	110	40
(4.8, 8.4)	38.4 + 92.4 = 130.8	43.2 + 33.6 = 76.8
(8, 6)	64 + 66 = 130	72 + 24 = 96
(10, 4)	80 + 44 = 124	90 + 16 106
(10, 0)	80	90
(0, 0)	0	0

(a) Maximum of $P = 8x + 11y$ is 130.8, which occurs at $(x, y) = (4.8, 8.4)$.
(b) Maximum of $P = 9x + 4y$ is 106, which occurs at $(x, y) = (10, 4)$.

Section 3.2 27. Find the values of (a) $P = 8x + 4y$ and (b) $P = 9x + 4y$ at each corner found in Problem 19.

Corner	$P = 8x + 6y$	$P = 10x + 5y$
(0, 19)	114	95
(8, 15)	64 + 90 = 154	80 + 75 = 155
(12, 10)	96 + 60 = 156	120 + 50 = 170
(17, 0)	136	170

(a) Maximum of $P = 8x + 6y$ is 156, which occurs at $(x, y) = (12, 10)$.
(b) Maximum of $P = 10x + 5y$ is 150, which occurs at both $(x, y) = (12, 10)$ and $(17, 0)$. These corners both lie on constraint (2): $2x + y = 34$. Multiply this equation by 5 on both sides to obtain $10x + y = 170$. Thus all feasible points along constraint line (2) give the maximum profit of 170.

29. The formulation was:

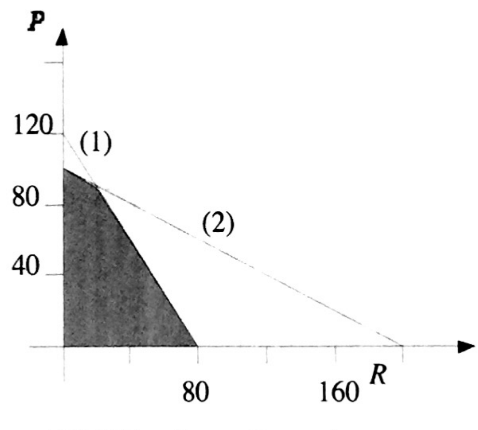
Let R = # gallons of regular gasoline to produce
P = # gallons of premium gasoline to produce.

Maximize $Profit = 0.20R + 0.30P$

Subject to: (1) $0.75R + 0.5P \le 60{,}000$ gallons of low octane
(2) $0.25R + 0.5P \le 50{,}000$ gallons of high octane
$R \ge 0, P \ge 0$

The intercepts are: For (1) $0.75R + 0.5P = 60{,}000$:
If $R = 0$, then $P = 60{,}000 \div 0.5 = 120{,}000$ If $P = 0$, then $R = 60{,}000 \div 0.75 = 80{,}000$
For (2) $0.25R + 0.5P = 50{,}000$:
If $R = 0$, then $P = 50{,}000 \div 0.5 = 100{,}000$ If $P = 0$, then $R = 50{,}000 \div 0.25 = 200{,}000$

The feasible region is shown below (the scales are in thousands).



Two corners are the intercepts $(80{,}000, 0)$ and $(0, 100{,}000)$.
The corner where (1) and (2) meet is the solution of:

(1) $0.75R + 0.5P = 60{,}000$
(2) $0.25R + 0.5P = 50{,}000$

Subtract: $0.5R = 10{,}000$, so $R = 20{,}000$
Substituting: $0.75(20{,}000) + 0.5P = 60{,}000$
$15{,}000 + 0.5P = 60{,}000$ $0.5P = 45{,}000$,
so $P = 90{,}000$. That corner is $(20{,}000, 90{,}000)$

Which corner gives the largest profit $= 0.20R + 0.30P$:
At $(80{,}000, 0)$, profit = \$16,000
At $(0, 100{,}000)$, profit = \$30,000
At $(20{,}000, 90{,}000)$, profit = \$4,000 + \$27,000 = \$31,000

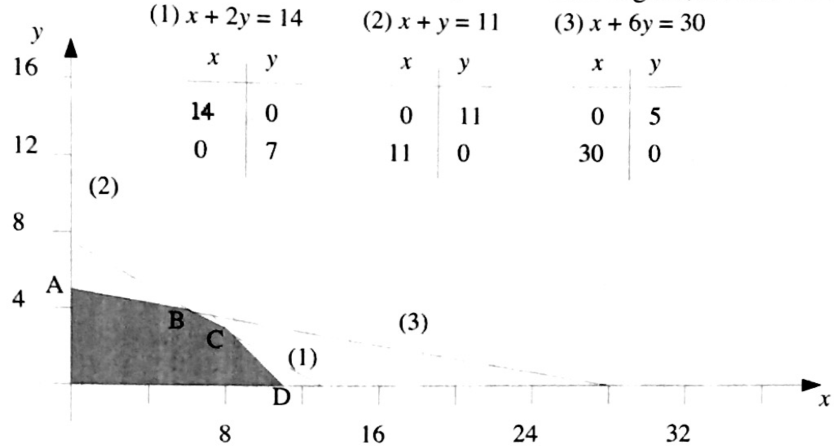
So the best production policy is to produce 20,000 gallons of regular and 90,000 gallons of premium. This produces a profit of \$31,000.

Substitute $(20{,}000, 90{,}000)$ into the constraints to determine the slack in each resource constraint:
$0.75(20{,}000) + 0.5(90{,}000) = 60{,}000 \le 60{,}000$. There will be 0 gallons of low-octane left unused.
$0.25(20{,}000) + 0.5(90{,}000) = 50{,}000 \le 50{,}000$. There will be 0 gallons of high-octane left unused.

Solution: Make 20,000 gallons of regular and 90,000 gallons of premium. This will earn a profit of \$31,000. There will be 0 gallons of low-octane and 0 gallons of high octane left unused (slack).

Section 3.2 31. The intercepts of the constraint lines, and feasible region, are shown below:

(1) $x + 2y = 14$		(2) $x + y = 11$		(3) $x + 6y = 30$	
x	y	x	y	x	y
14	0	0	11	0	5
0	7	11	0	30	0



Corner A is the y-intercept of (3), or (0, 5), and corner D is the x-intercept of (2), or (11, 0).
Corner B is the intersection of: (1) $x + 2y = 14$
(3) $x + 6y = 30$

Subtract (1) from (3): $4y = 16$, so $y = 4$.
Substitute: $x + 2(4) = 14$, so $x = 6$. So B is (6, 4).

Corner C is the intersection of: (1) $x + 2y = 14$
(2) $x + y = 11$

Subtract: (2) from (1): $y = 3$
Substitute: $x + 2(3) = 14$, so $x = 8$. So C is (8, 3).

Make a chart of the corner points and the value of P at each.

Corner	$P = \$25x + \$40y$
A (0, 5)	$\$40(5) = \$200$
B (6, 4)	$\$25(6) + \$40(4) = \$310$
C(8, 3)	$\$25(8) + \$40(3) = \$320$
D(11, 0)	$\$25(11) = \$275$

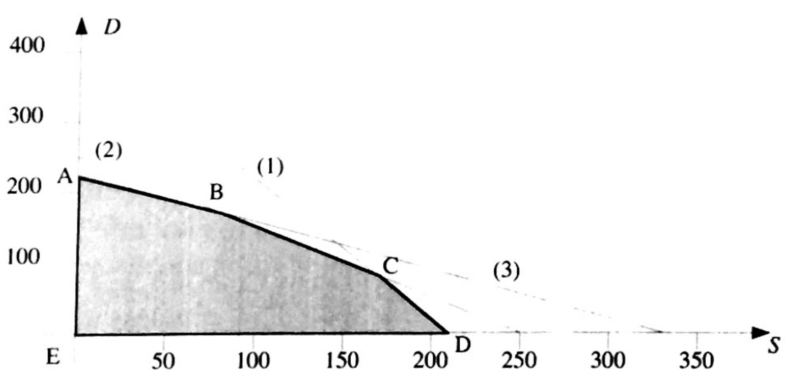
So the best production policy is to make 8 shawls and 3 afghans, for a profit of \$320.
Substitute (8, 3) into constraints to find slack: (1) $8 + 2(3) = 14 \le 14$; 0 hours spinning time unused. (2) $8 + 3 = 11 \le 11$; 0 hours of dying time unused. (3) $8 + 6(3) = 26 \le 30$; 4 hours weaving unused.

33. The mathematical formulation was: Let S = number of Standard tents,
D = number of Deluxe tents to be made.

Maximize $P = \$30S + \$50D$

subject to
(1) $2S + 1D \le 420$ hours of cutting and assembly time
(2) $2S + 2D \le 500$ yards of fabric
(3) $2S + 3D \le 660$ hours of finishing time $S \ge 0, D \ge 0$

The intercepts are: For (1): (210, 0) and (0, 420) For (2): (250, 0) and (0, 250)
For (3): (330, 0) and (0, 220)



Section 3.2 33. (Continued) Corners A and D are intercepts listed above: A is (0, 220) and D is (210, 0).

Corner B is the intersection of: (2) $2S + 2D = 500$

and (3) $2S + 3D = 660$

Subtract (2) from (3): $D = 160$

Substitute into (2): $2S + 320 = 500$ $\quad 2S = 180$, so $S = 90$. Corner B is $(S, D) = (90, 160)$.

Corner C is the intersection of: (1) $2S + D = 420$

and (2) $2S + 2D = 500$

Subtract (1) from (2): $D = 80$

Substitute this value into (1): $2S + 80 = 420$ $\quad 2S = 340$, so $S = 170$. Corner C is $(S, D) = (170, 80)$.

Corner	$P = \$30S + \$50D$
A (0, 220)	\$50(220) = \$11,000
B (90, 160)	\$30(90) + \$50(160) = 10,700
C (170, 80)	\$30(170) + \$50(80) = 9,100
D (210, 0)	\$30(210) = 6,300

The maximum profit of \$11,000 occurs if no Standard tents and 220 Deluxe tents are made.
Substitute (0, 220) into resources constraints: (1) $2(0) + 1(220) = 220 \le 420$; 200 hours of c&a time unused
(2) $2(0) + 2(220) = 440 \le 500$; 60 yards of fabric unused.
(3) $2(0) + 3(220) = 660 \le 660$; 0 hours of finishing time unused.

35. Let x = # boxes of Rick Pitino mix to make, y = # boxes of Denny Crum mix to make.

Maximize $P = \$8.55x + \$6.15y$

subject to (1) $0.4x + 0.2y \le 44$ lbs chocolate

(2) $0.2x + 0.2y \le 26$ lbs nuts

(3) $0.4x + 0.6y \le 72$ lbs fruit $\qquad x \ge 0, y \ge 0$

The intercepts are:

For (1): (110, 0) and (0. 220) For (2): (130, 0) and (0, 130) For (3): (180, 0) and (0, 120)

The constraint lines and feasible region are shown below.

Corner B is the intercept (0, 120); Corner E is the intercept (110, 0).

Corner C is the intersection of:

(2) $0.2x + 0.2y = 26$

and (3) $0.4x + 0.6y = 72$

Multiply (2) by 2: $\underline{0.4x + 0.4y = 52}$

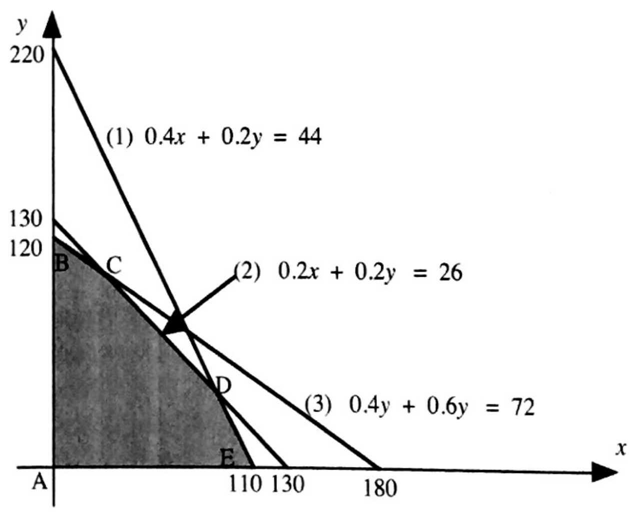
Subtract this from (3): $0.2y = 20$,

so $y = 20 / 0.2 = 100$.

Substitute this value into (2): $0.2x + 20 = 26$

$0.2x = 6$, so $x = 6 / 0.2 = 30$.

So Corner C is (30, 100).



Corner D is the intersection of:

(1) $0.4x + 0.2y = 44$

and (2) $0.2x + 0.2y = 26$

Subtract (2) from (1): $0.2x = 18$,

so $x = 18 / 0.2 = 90$

Substitute this value into (1): $0.4(90) + 0.2y = 44$

$36 + 0.2y = 44$ $\quad 0.2y = 8$, so $y = 8 / 0.2 = 40$. So Corner D is (90, 40).

Corner	$P = \$8.55x + \$6.15y$
B(0, 120)	\$738
C(30, 100)	\$256.50 + \$615 = \$871.50
D(90, 40)	\$769.50 + \$246 = \$1015.50
E(110, 0)	\$940.50

The factory should make 90 boxes of Rick Pitino mix and 40 boxes of Denny Crum mix, and earn a profit of \$1015.50. Substitute to find slack: (1) $0.4(90) + 0.2(40) = 44 \le 44$; 0 lbs. chocolate unused
(2) $0.2(90) + 0.2(40) = 26 \le 26$; 0 lbs nuts unused. (3) $0.4(90) + 0.6(40) = 60 \le 72$; 12 lbs. fruit unused.