MATH 562-01 MATHEMATICAL STATISTICS

Exam 1 (09/30/16, Friday) Name:___________________________

1. (20 points) Let Z_1, Z_2, Z_3, Z_4 be a random sample from $N(0,1)$, and X_1, X_2, X_3, X_4 a random sample from $N(2,1)$. Determine the sampling distributions of the following statistics. **Explain why.**

   (1). $\dfrac{(X_1 + X_2 - 4)^2}{(Z_1 - Z_2)^2}$ (2). $\dfrac{(X_1 - X_2)^2 + (Z_1 + Z_2)^2 + (X_3 - X_4)^2}{2}$

   (3). $\dfrac{\sqrt{2}(Z_1 - Z_2)}{\sqrt{(X_1 - X_2)^2 + (Z_3 + Z_4)^2}}$ (4). $\dfrac{\sum_{k=1}^{4}(X_k - \bar{X})^2}{\sum_{k=1}^{4}(Z_k - \bar{Z})^2}$

2. (10 points) Let $X_1, X_2, \cdots, X_m$ and $Y_1, Y_2, \cdots, Y_n$ be independent random samples from $N(\mu, \sigma^2)$, and let $\bar{X} = \frac{1}{m}\sum_{i=1}^{m} X_i$, $\bar{Y} = \frac{1}{n}\sum_{j=1}^{n} y_j$, and $S^2 = \frac{1}{m-1}\sum_{i=1}^{m}\left(X_i - \bar{X}\right)^2$. Find the constant c such that the statistic $T = c\dfrac{\bar{Y} - \bar{X}}{S}$ has $t(m-1)$ distribution. Justify your solution.

3. (10 points) Let $X_1, X_2, \cdots, X_6$ and $Y_1, Y_2, \cdots, Y_8$ be independent random samples from a standard normal population $N(0,1)$, and $W = \dfrac{4}{3}\dfrac{\sum_{i=1}^{6} X_i^2}{\sum_{j=1}^{8} Y_j^2}$. Find the 99th percentile of *W*.

4. (10 points) Let $X_1, X_2, \cdots, X_n$ be a random sample from $N(\mu, 10)$, If $P\left[\sum_{i=1}^{n}(X_i - \bar{X})^2 \leq 52.3\right] = 0.05$, find the sample size *n*.

5. (10 points) Let X_1, X_2, X_3 be a random sample from a normal population $N(\mu, \frac{1}{24})$, with $\mu \neq 0$. Find a, b such that the statistic $L = aX_1 + 4X_2 + bX_3$ has standard normal distribution $N(0,1)$.

6. (10 points) Let $X_1, X_2, \cdots, X_{50}$ be 50 integers randomly selected from $\{1, 2, \cdots, n\}$ with replacement. Find the MME $\hat{n}$ and MLE $\tilde{n}$ for n.

7. (15 points) Let $X_1, X_2, \cdots, X_n$ be a random sample from a population with probability density function $f(x;\theta) = \theta e^{-\theta x}$, if $x > 0$, and $f(x;\theta) = 0$ otherwise, where $\theta \in (0, \infty)$. Find the MME $\hat{\theta}$ and MLE $\tilde{\theta}$. If the observed values are 17, 10, 32, and 5, with *n*=*4,* find the method of moment estimate and the maximum likelihood estimate of θ.

8. (15 points) Let $X_1, X_2, \cdots, X_n$ be a random sample from the discrete distribution

$$P[X = x] = \begin{cases} \dfrac{\theta^{2x} e^{-\theta^2}}{x!} & x = 0,1,2,\cdots \\ 0 & otherwise \end{cases}$$

where $\theta > 0$. Find the MME $\hat{\theta}$ and MLE $\tilde{\theta}$. If X_1, X_2, X_3 and X_4 resulted in a data set 17, 10, 32, and 5, find the method of moment estimate and the maximum likelihood estimate of θ.