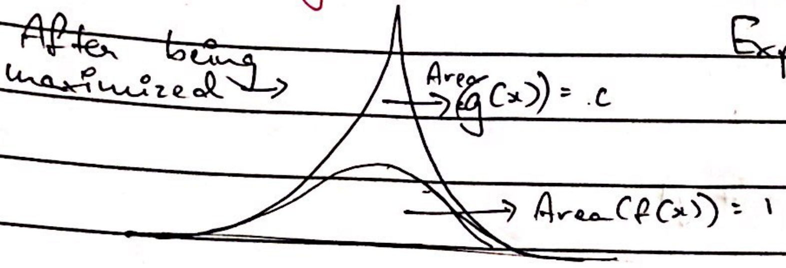
Area of maj. func. =

Rej. rate is the prob. that a pt. is outside target distr.



Exp. Rej rate

$$= \frac{\text{Rej. region}}{\text{Total region}}$$

$$\Rightarrow \frac{c-1}{c}$$

~~Day 4:-~~

Lab 2-

Q2)

$$P(x \mid \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-1}{2\sigma^2}(x_i-\mu)^2}$$

when no particular distribution is mentioned, take normal distr.

Likelihood:

$$L(\mu, \sigma \mid X) = \prod_{n=1}^{N} P(x_n \mid \mu, \sigma)$$

$$= \prod_{n=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\left(\frac{(x_n-\mu)^2}{2\sigma^2}\right)}$$

Log Likelihood:

$$\log L(\mu, \sigma \mid X) = \sum_{n=1}^{N}\left(\log(1) - \log(2\pi\sigma^2)^{1/2}\right) - \sum_{n=1}^{N}\frac{(x_n-\mu)^2}{2\sigma^2}$$

$$= -\frac{1}{2}\sum_{n=1}^{N}\log(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{n=1}^{N}(x_n-\mu)^2 \quad \text{—— (1)}$$

Partially diff. (1) w.r.t μ & set to 0

$$\frac{\partial \log L(\mu, \sigma \mid X)}{\partial \mu} = -\frac{1}{\cancel{2}\sigma^2} \cdot (-\cancel{2})\sum_{n=1}^{N}(x_n-\mu) \Rightarrow \underbrace{\sum_{n=1}^{N}\frac{(x_n-\mu)}{\sigma^2}}_{\text{Gradient of } \mu} = 0$$

$$\sum_{n=1}^{N}(x_n-\mu)=0$$

$$\sum_{n=1}^{N}x_n = n\mu$$

$$\therefore \boxed{\mu = \frac{\sum_{n=1}^{N}x_n}{n}}$$

Partially diff. (1) w.r.t. σ & set to 0,

$$\frac{\partial}{\partial\sigma}\log L(\mu,\sigma|x) = -\frac{1}{\cancel{2}}\cdot\frac{n\times\cancel{(2)}\cdot\cancel{2\pi}\sigma}{\cancel{2\pi}\sigma^{\cancel{2}}}\left(\frac{-\cancel{2}}{\cancel{2}}\frac{1}{\sigma^3}\sum_{n=1}^{N}(x_n-\mu)^2\right)$$

$$\underbrace{0 = \frac{-n}{\sigma} + \sum_{n=1}^{N}\frac{(x_n-\mu)^2}{\sigma^3}}_{\text{Gradient of } \sigma}$$

$$0 = -n\sigma^2 + \sum_{n=1}^{N}(x_n-\mu)^2$$

$$\sigma^2 = \sum_{n=1}^{N}\frac{(x_n-\mu)^2}{n}$$

$$\boxed{\sigma = \sqrt{\sum_{n=1}^{N}\frac{(x_n-\mu)^2}{n}}}$$