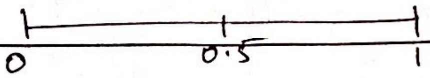
$$DE(\mu, \alpha) = \frac{\alpha}{2} \exp(-\alpha |x-\mu|)$$

$$u = \text{unif}(0,1)$$



Inverse CDF –

$f(x) \leftarrow$ pdf

$$F(x) = \int_{-\infty}^{x} f(x)\,dx \quad , \quad -\infty < x < \infty$$

need to find

pdf $\rightarrow$ cdf $= u \longrightarrow x = (\text{cdf})^{-1}$

① $\frac{\alpha}{2} e^{-\alpha(x-\mu)}$, $x \geq \mu$, $x \geq 0$

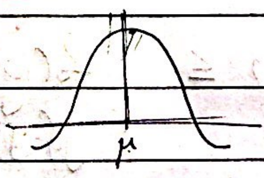
DE

$-\infty < x < \infty$

② $\frac{\alpha}{2} e^{\alpha(x-\mu)}$, $x \leq \mu$, $x \leq 0$

① $$u = 1 - \int_{x}^{\infty} \frac{\alpha}{2} e^{-\alpha(x-\mu)}\,dx$$ $\rightarrow$ as $x > \mu$ ☆[cdf always sums up to 1]

↳ area under the area



$$= 1 - \frac{\alpha}{2} \cdot \frac{e^{-\alpha(x-\mu)}}{\cancel{-\alpha}} \Bigg|_{x}^{\infty}$$

$$= 1 + \frac{1}{2}\left((0) - e^{-\alpha(x-\mu)}\right)$$

↳ $\frac{1}{e^{\infty}} \Rightarrow \frac{1}{\infty} = 0$

$$u = 1 - \frac{1}{2}\left(e^{-\alpha(x-\mu)}\right) \quad , \text{ when } u > 1/2$$

$$e^{-\alpha(x-\mu)} = 2 - u$$

$$-\alpha(x-\mu) = \ln(2-u)$$

$$\boxed{x = \mu - \frac{1}{\alpha}\left(\ln(2-u)\right)} \quad \text{when } u > \frac{1}{2}$$

② $u = \int_{-\infty}^{x} \frac{\alpha}{2} e^{\alpha(x-\mu)} dx$

$= \cancel{\frac{\alpha}{2}} \cdot \frac{e^{\alpha(x-\mu)}}{\cancel{\alpha}} \Big|_{-\infty}^{x}$

$= \frac{1}{2}\left(e^{\alpha(x-\mu)} - 0\right)$

$u = \frac{1}{2}\left(e^{\alpha(x-\mu)}\right)$, when $u \leq 1/2$

$e^{\alpha(x-\mu)} = 2u$

$\alpha(x-\mu) = \ln(2u)$

$\boxed{x = \mu + \frac{1}{\alpha}(\ln(2u))}$ when $u \leq 1/2$

Acceptance / Rejection method –

$Y \sim g(y) \sim DE \leftarrow$ majorizing density

$X \sim f(y) \sim N(0,1) \leftarrow$ target density

$U \sim Unif(0,1)$

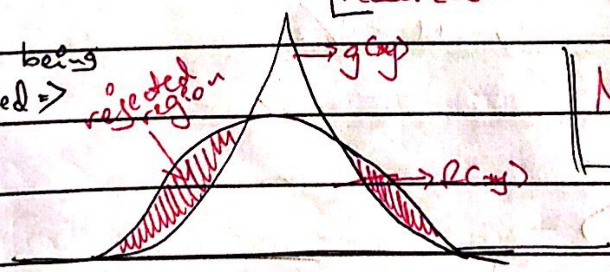
target func.

If $U \leq \frac{f_X(y)}{c\, g_Y(y)}$ $\Rightarrow \frac{N(0,1)}{c(DE(0,1))}$

proposal func.

then $X = Y \rightarrow$ [we take realization y as desired] ★

Before being maximised =>



$\boxed{N = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2}}$

w.k.t.

there exists a constant c such that,

$\forall_y \; c\, g_Y(y) \geq f_X(y)$

$\therefore \quad c \geq \frac{f(y)}{g(y)}$

$f(y) = N(0,1) \longrightarrow$ subst. $\mu = 0$ & $\sigma = 1$

$\therefore \; = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2}$

$\Rightarrow c = \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2}}{\frac{1}{2} e^{-|y|}}$

$c = \sqrt{\frac{2}{\pi}} \, e^{-\frac{1}{2}y^2 + |y|}$

$c' = \frac{\partial c}{\partial y} = \sqrt{\frac{2}{\pi}} \, e^{-\frac{1}{2}y^2 + |y|} \cdot \left(-y + \frac{y}{|y|}\right)$

To get value of y,

Set der. to 0,

$0 = \sqrt{\frac{2}{\pi}} \, \cancel{e^{-\frac{1}{2}y^2 + |y|}} \, e^{-\frac{1}{2}y^2 + |y|} \left(-y + \frac{y}{|y|}\right)$

~~$\ln \sqrt{\frac{\pi}{2}} = -\frac{1}{2}y^2 + |y|$~~ When $y = 1$, $c' = 0$.

Subst. $y = 1$ in c,

$c = \sqrt{\frac{2}{\pi}} \, e^{-\frac{1}{2}} \simeq 1.3$

Random samples (Y) obtained should be of a normal distribution.

When you $\times^{ly}$ $g(y)$ with c, it will get maximised.

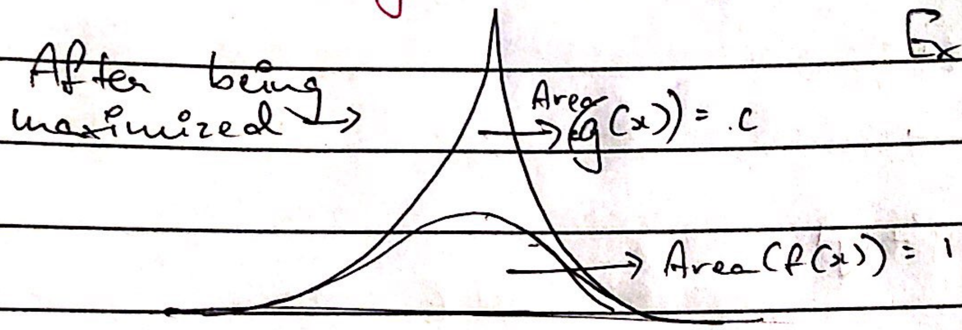
Area under target func. = 1

Because area of prob. = 1

Area under prob. dist. = prob.

Area of maj. func. =

Rej. rate is the prob. that a pt. is outside target distr.



Exp. Rej rate

$= \frac{\text{Rej. region}}{\text{Total region}}$

$\Rightarrow \frac{c-1}{c}$