$$Q(\Theta, \Theta^k) = E\left[\text{loglik}(\Theta \mid Y, Z) \mid \Theta^k, Y\right]$$

$$X \sim \text{Exp}(\lambda) \Rightarrow \lambda e^{-\lambda x}$$

$$\therefore Y_i \sim \text{Exp}\left(\frac{X_i}{\lambda}\right) \rightarrow \frac{X_i}{\lambda} e^{-\frac{X_i}{\lambda} \cdot Y_i}$$

$$Z_i \sim \text{Exp}\left(\frac{X_i}{2\lambda}\right) \rightarrow \frac{X_i}{2\lambda} e^{-\frac{X_i}{2\lambda} \cdot Y_i}$$

$$\underbrace{P(x, y, z}_{\text{as dataset comprises of these 3}} \mid \lambda) = \frac{X_i}{\lambda} e^{-\frac{X_i}{\lambda} \cdot Y_i} \cdot \frac{X_i}{2\lambda} e^{-\frac{X_i}{2\lambda} \cdot Z_i}$$

Likelihood :

$$L(\lambda) = \prod_{i=1}^{n} P(\lambda \mid x, y, z)$$

$$= \prod_{i=1}^{n} \frac{X_i^2}{2\lambda^2} e^{-\frac{X_i}{\lambda}\left(Y_i + \frac{Z_i}{2}\right)} \Rightarrow \frac{\prod_{i=1}^{n} X_i^2}{2\lambda^{2n}} e^{-\frac{1}{\lambda} \sum_{i=1}^{n} X_i\left(Y_i + \frac{Z_i}{2}\right)}$$

Log Likelihood :

$$\ln(L(\lambda)) = 2 \ln\left(\prod_{i=1}^{n} X_i\right) - 2n \ln(2\lambda) - \frac{1}{\lambda} \sum_{i=1}^{n} \left[X_i\left(Y_i + \frac{Z_i}{2}\right)\right]$$

Converting Z_i into, $\rightarrow$ due to missing values

$\sum_{i=1}^{r} V_i \rightarrow$ observed & $\sum_{i=r+1}^{n} W_i \rightarrow$ unobserved

$$\therefore \ln(L(\lambda)) = 2\ln\left(\prod_{i=1}^{n} X_i\right) - 2n\ln(2\lambda) - \frac{1}{\lambda}\left(\sum_{i=1}^{n} X_i Y_i + \sum_{i=1}^{r} \frac{X_i V_i}{2} + \sum_{i=r+1}^{n} \frac{X_i W_i}{2}\right)$$

For $Q(\Theta, \Theta^k) = E\left[\text{loglik}(\Theta \mid Y, Z) \mid \Theta^k, Y\right]$

$$\therefore E\left[\ln(\lambda \mid x, y, z) \mid \lambda^k, x, y, z\right]$$

$$= 2\ln\left(\prod_{i=1}^{n} X_i\right) - 2n\ln(2\lambda) - \frac{1}{\lambda} E\left[\sum_{i=1}^{n} X_i Y_i + \sum_{i=1}^{n} \frac{X_i Z_i}{2} \,\Big|\, \lambda^k, x, y, z\right]$$

$Z_i \rightarrow W \rightarrow$ missing

as it is unobserved

$$= 2\ln\left(\prod_{i=1}^{n} x_i\right) - 2n\ln(2\lambda) - \frac{1}{\lambda}\left(\sum_{i=1}^{n} x_i y_i + \sum_{i=1}^{r} \frac{x_i V_i}{2} + \sum_{i=r+1}^{n} \frac{x_i}{2} E\left[w_i \mid \lambda^k, x, y, z\right]\right)$$

$$= 2\ln\left(\prod_{i=1}^{n} x_i\right) - 2n\ln(2\lambda) - \frac{1}{\lambda}\left(\sum_{i=1}^{n} x_i y_i + \sum_{i=1}^{r} \frac{x_i V_i}{2} + \sum_{i=r+1}^{n} \cancel{\frac{x_i}{2}} \cdot \frac{\cancel{2}\lambda^k}{\cancel{x_i}}\right)$$

$\cancel{w_i \sim Exp\left(\frac{x_i}{2\lambda^k}\right)}$ $\quad w_i \sim Exp\left(\frac{x_i}{2\lambda^k}\right)$

$\therefore E[x] = \frac{1}{\lambda} \Rightarrow E\left[w_i \mid x, y, z, \lambda^k\right] = \frac{1}{\frac{x_i}{2\lambda^k}} \Rightarrow \frac{2\lambda^k}{x_i}$

$$E[L(\lambda)] = 2\ln\left(\prod_{i=1}^{n} x_i\right) - 2n\ln(2\lambda) - \frac{1}{\lambda}\left(\sum_{i=1}^{n} x_i y_i + \sum_{i=1}^{r} \frac{x_i V_i}{2} + \sum_{i=r+1}^{n} \lambda^k\right)$$

To ~~s~~ solve for λ,

$$\frac{\partial E[L(\lambda)]}{\partial \lambda} = -\frac{2n}{2\lambda}\cdot 2 + \frac{1}{\lambda^2}\left(\sum_{i=1}^{n} x_i y_i + \sum_{i=1}^{r} \frac{x_i V_i}{2} + (n-r)\lambda^k\right)$$

$$0 = -\frac{2n}{\cancel{2}\lambda} + \frac{1}{\lambda^2}\left(\sum_{i=1}^{n} x_i y_i + \sum_{i=1}^{r} \frac{x_i V_i}{2} + (n-r)\lambda^k\right) \quad \times\lambda^2$$

$$0 = -2n\lambda + \sum_{i=1}^{n} x_i y_i + \sum_{i=1}^{r} \frac{x_i V_i}{2} + (n-r)\lambda^k$$

$$\lambda = \frac{1}{2n}\left(\sum_{i=1}^{n} x_i y_i + \sum_{i=1}^{r} \frac{x_i V_i}{2} + \underbrace{(n-r)}_{m}\lambda^k\right)$$