

1)

**a) Derive  $C \rightarrow B$**

First we can see that  $A \rightarrow BC$  which according to decomposition gives us  $A \rightarrow B$  &  $A \rightarrow C$ . After that we also see that  $C \rightarrow A$  which according to transitivity with  $A \rightarrow B$  gives us  $C \rightarrow B$

**b)  $AE \rightarrow F$**

Using  $DE \rightarrow F$  and pseudo-transitivity we can see that if we find a  $A$  such that  $A \rightarrow D$  we get  $AE \rightarrow F$ . We know that  $A \rightarrow C$  and  $C \rightarrow D$ , using transitivity we get  $A \rightarrow D$  which in turn lets us derive  $DE \rightarrow F \Rightarrow AE \rightarrow F$

2)

**a) Attribute closure of set:  $X = \{ A \}$**

FD1:  $A \rightarrow BC \Rightarrow A \rightarrow B$  &  $A \rightarrow C$

$X^+ = \{ A, B, C \}$

FD2:  $C \rightarrow AD \Rightarrow A \rightarrow AD$  which means  $A \rightarrow D$

$X^+ = \{ A, B, C, D \}$

FD3:  $DE \rightarrow F \Rightarrow$  If  $A \rightarrow D$  then  $AE \rightarrow F$  which means  $A \rightarrow F$

$X^+ = \{ A, B, C, D, F \}$

**b) Attribute closure of set:  $X = \{ C, E \}$**

FD1:  $A \rightarrow BC \Rightarrow$  Because  $C \rightarrow A$  then  $C \rightarrow B$

$X^+ = \{ C, E, B \}$

FD2:  $C \rightarrow AD \Rightarrow C \rightarrow A$  &  $C \rightarrow D$

$X^+ = \{ C, E, B, A, D \}$

FD3:  $DE \rightarrow F \Rightarrow C \rightarrow D$  gives us  $CE \rightarrow F$

$X^+ = \{ C, E, B, A, D, F \}$

3)

**a) Determine the candidate key(s) for  $R(A, B, C, D, E, F)$**

$A$  is the only value not in RHS. We look at  $\{A\}^+$ :

FD1:  $AB \rightarrow CDEF$

FD2:  $E \rightarrow F$

FD3:  $D \rightarrow B$

$\{A\}^+ = \{A, C, D, E, F, B\}$  which means it is a super key and thus the only CK

b) Note that R is not in BCNF. Which FD(s) violate BCNF condition?

FD2 and FD3 violates BCNF.

3) a) **Determine the candidate key(s) for R(A, B, C, D, E, F)**

**Ans:**

FD1: AB → CDEF

FD2: E → F

FD3: D → B

Candidate keys for R:

A is not in RHS; must be present in Ck.

F and C are not in LHS; can not be present in a Ck.

X = {A,B},

X+ = {A,B,C,D,E,F} FD1:AB → CDEF

X+ = {A,D,B,C,E,F} FD2: E → F

X+ = {A,D,B,C,E,F} FD2: D → B

X = {A,D}

X+ = {A,D,B,C,E,F} FD1:AB → CDEF

X+ = {A,D,B} FD3: D → B

X = {A,E}

X+ = {A,E,F} FD2:E → F

Candidate key's are {A,B} and {A,E}

b) Note that R is not in BCNF. Which FD(s) violate the BCNF condition?

**Ans:** FD2 and FD3 do not contain a superkey. Hence, they violate the Boyce Codd Normal Form.

c) Decompose R into a set of BCNF relations, and describe the process step by step (don't forget to determine the FDs and the candidate key(s) for all of the relation schemas along the way).

**Ans:**

Decompose based on FD3: E → F (as it violates BCNF):

R1: (F,E) with all attributes in X and Y

R2: (A,B,C,D,E) with all attributes in R, without the ones in Y and not in X.

FD4: AB → CDE ; FD1: AB → CDEF

FD5: AD → BCE ; FD1: AB → CDEF

Decompose based on FD2: D → B (as it violates BCNF)

R1: (D,B) ; FD3 : D → B ; candidate key: {D}

R2: (A,C,D,E) ; FD9 : AD → CE; candidate key: {AD}

FD6 → AD → AB ; FD3 augmented

FD7:  $AD \rightarrow CDEF$  ; FD1 and FD4 transitivity

FD8:  $AD \rightarrow CDE$ ; Decomposition of FD7

FD9:  $AD \rightarrow CE$ ; Decomposition of FD8

4) Consider the relation schema  $R(A, B, C, D, E)$  with the following FDs

FD1:  $ABC \rightarrow DE$

FD2:  $BCD \rightarrow AE$

FD3:  $C \rightarrow D$

a) Show that R is not in BCNF.

**Ans:** FD3 violates BCNF, C is not a superkey.

The elements that are not in LHS but are in RHS:

$X^+ = \{B, C\}$

$X^+ = \{B, C, D\}$  ; FD1:  $C \rightarrow D$

$X^+ = \{B, C, D, A, E\}$  ; FD2 :  $BCD \rightarrow AE$

Candidate key is  $\{B, C\}$

b) Decompose R into a set of BCNF relations (describe the process step by step).

**Ans:**

R1:  $\{C, D\}$  ; FD3:  $C \rightarrow D$  ; ck =  $\{C\}$

R2:  $\{A, B, C, E\}$  ;

FD4:  $C \rightarrow CD$

FD5:  $BC \rightarrow BCD$  (Augmentation of FD4)

FD6:  $BC \rightarrow AE$  (transitivity of FD5)

Decomposing:

$BC \rightarrow E$  ; (BC is a candidate key)