LINKÖPINGS UNIVERSITET
Department of Computer and
Information Science
Division of Statistics and Machine learning/ANd

732A66 Decision theory
Fall semester 2019
Assignment 2

Assignment 2

Below are two tasks that you shall try to solve. All questions put should be answered. Prepare your solutions in a nice format that can be easily read. You can help each other but there must be individual submissions (that are not just copies of one submission).

Your solutions should be submitted at latest on **Friday 10 January 2020**.

1. (This is to some parts Exercises 12,13 and 14 in Chapter 5 of "Winkler: An Introduction to Bayesian inference and decision, 2nd ed.")
   One nonprobabilistic decision-making criterion involves the consideration of a weighted average of the highest and lowest payoffs for each action. The weights, which must sum to 1, can be thought of as an optimism-pessimism index. The action with the highest weighted average of the highest and lowest payoffs is the action chosen by this criterion.

   a) Comment on this decision-making criterion and use it for payoff table (i) below with the highest payoff in each row receiving a weight of 0.4 and the lowest payoff receiving a weight of 0.6

Payoff table (i)

Action	State of the world				
	A	*B*	*C*	*D*	*E*
1	–50	80	20	100	0
2	30	40	70	20	50
3	10	30	–30	10	40
4	–10	–50	–70	–20	200

   b) Use the decision-making criterion described above for payoff table (ii) below, with the highest payoff in each row receiving a weight of 0.8 and the lowest payoff receiving a weight of 0.2. For payoff table (ii) the *ER* criterion would also involve a weighted average of the two payoffs in each row. Compare the criterion described above with the *ER* criterion.

Payoff table (ii)

Action	State of the world	
	I	*II*
1	10	4
2	7	9

   c) Consider payoff table (i) above. Assume the utility function of a person is $U(R) = \log(R+71)$, where R is the payoff (and log is the natural logarithm with base e). Moreover, assume that person's prior probabilities for the five states of nature are $P(\text{A}) = 0.10$; $P(\text{B}) = 0.20$; $P(\text{C}) = 0.25$; $P(\text{D}) = 0.10$ and $P(\text{E}) = 0.35$. Find the optimal action for this person according to the *EU* criterion.

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2. (This task is constructed from Exercises 18 and 29 in Chapter 6 of "Winkler: An Introduction to Bayesian inference and decision, 2nd ed.")

   Consider a big box filled with an enormous amount of poker chips. You know that either 70% of the chips are red and the remainder blue, or 70% are blue and the remainder red. You must guess whether the big box has 70% red / 30% blue or 70% blue / 30% red. If you guess correctly, you win US$5. If you guess incorrectly, you lose US$3. Your prior probability that the big box contains 70% red / 30% blue is 0.40, and you are risk neutral in your decision making (i.e. your utility is linear in money).

   a) If you could purchase sample information in the form of one draw of a chip from the big box, how much should you be willing to pay for it?

   Assume now that the cost of sampling is US$0.25 (i.e. 25 US cents) per draw.

   b) What is the ENGS for a sample of 10 chips using a single-stage sampling plan.

   c) Model this decision problem using an influence diagram showing the contents of the node tables.

3. (This is in essential Exercise 38 in Chapter 6 of "Winkler: An Introduction to Bayesian inference and decision, 2nd ed.")

   A firm is contemplating the purchase of 500 printer cartridges. One supplier, A, offers the cartridges at US$ 15 each, guarantees each cartridge, and will replace all defective cartridges free. A second supplier, B, offers the cartridges at US$ 14 each with no guarantee. However, supplier B will replace defective cartridges with good cartridges for US$ 10 per cartridge. Let the proportion of defective cartridges produced by supplier B be $\tilde{p}$ and suppose your prior distribution for $\tilde{p}$ is a beta distribution with parameters a = 2 and b = 48, i.e. the pdf is $p^{2-1} \cdot (1-p)^{48-1}/B(2,48)$.

   a) What should the firm do on the basis of the prior distribution?
   b) How much is it worth to the firm to know the proportion of defective cartridges for certain?
   c) Suppose that supplier B ca provide a randomly chosen sample of 10 cartridges. What is the expected value of information of this sample?
   d) In the sample of 10 cartridges, 1 is defective. Find the posterior distribution of $\tilde{p}$ and use this distribution to determine which supplier the firm should deal with.

4. Assume some budget calculations depend on whether a certain cost will be at least SEK 120 000 or lower than this amount. A fairly good model for this cost is a normal distribution with standard deviation SEK 12 000 and a mean that can be modelled as

normally distributed with mean 115 000 (SEK) and standard deviation 9 000 (SEK). No trend is anticipated for this cost and for the 6 previous periods the average cost was SEK 121 000.

a) What are the prior odds for the hypothesis that the cost will exceed SEK 120 000 against the alternative that it will not?
b) Calculate the Bayes factor in light of the average cost for the previous 6 periods for the hypothesis that the cost will exceed SEK 120 000 against the alternative that it will not.
c) If the loss of accepting the hypothesis that the cost will be lower than SEK 120 000 while the opposite will be true is SEK 4 000, and the loss of accepting the hypothesis that the cost will be at least SEK 120 000 while the opposite will be true is SEK 6 000, which decision should be made for the budget (according to the rule of minimizing the expected loss)?