Meeting 12: Even more on the value of information

Sequential analysis

When the value of sample information concerns the entire sample (sometimes referred to as *single-stage sampling*) the expected net gain of sampling can be written

$$ENGS(n) = EVSI(n) - CS(n)$$

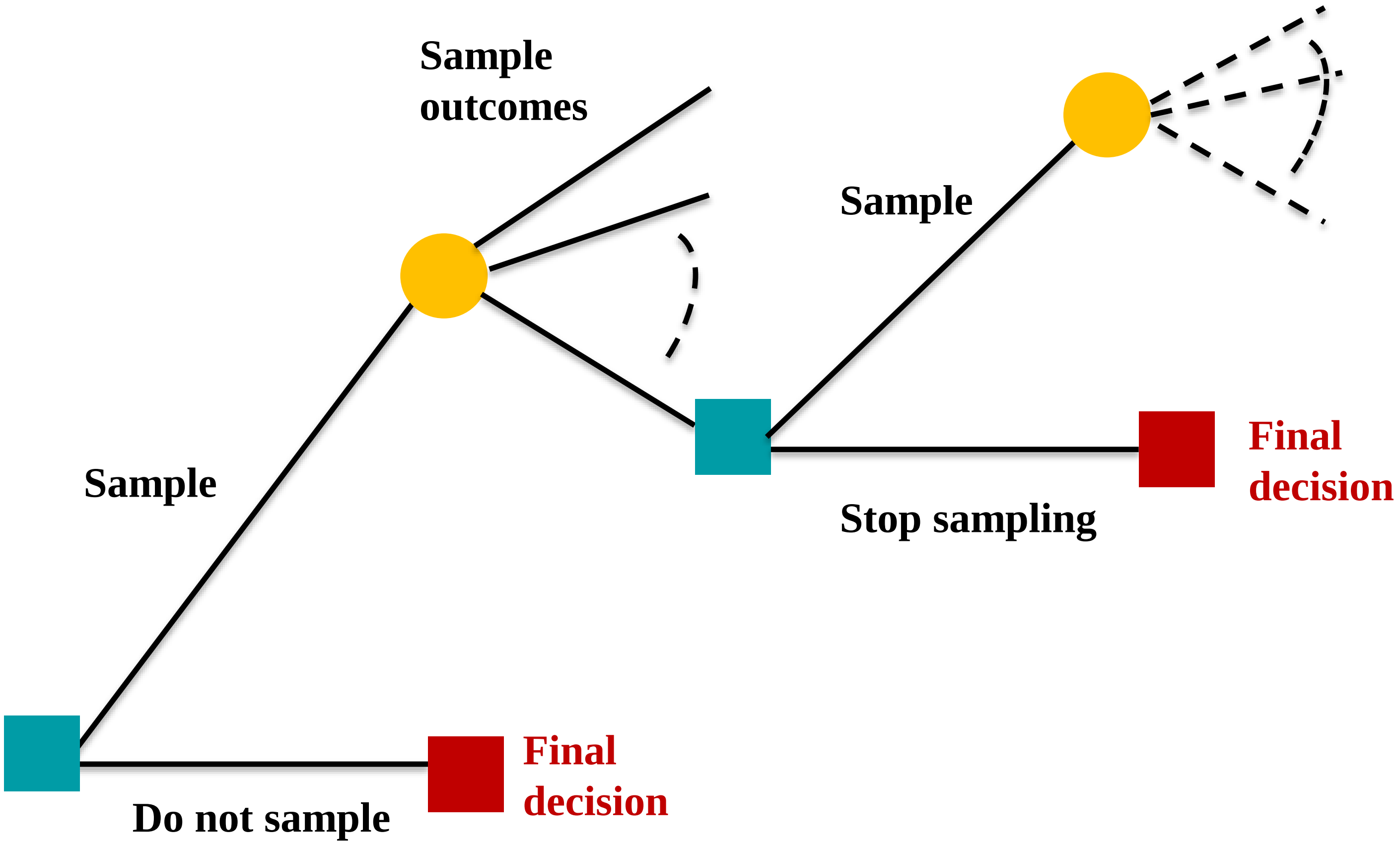
and the optimal sample size n^* satisfies

$$ENGS(n^*) \geq ENGS(n) \quad \text{for} \;\; n = 0,1,2,\ldots$$

However, it is also possible to sample one unit at time and at each step decide whether further sampling should be conducted. This is referred to as *sequential sampling*.

In the textbook there is no attempt to formulate at general description of sequential sampling, since it is a concept closely related to the decision problem at hand.

However, to clarify things it may often be wise to draw decision trees.



Exercise 6.27

27. In Exercise 17, consider a sequential sampling plan with a maximum total sample size of two and analyze the problem as follows.
    (a) Represent the situation in terms of a tree diagram.
    (b) Using backward induction, find the ENGS for the sequential plan.
    (c) Compare the sequential plan with a single-stage plan having $n = 2$.

17. In Exercise 16, suppose that you also want to consider other sample sizes.
    (a) Find EVSI for a sample of size 2.
    (b) Find EVSI for a sample of size 5.
    (c) Find EVSI for a sample of size 10.
    (d) If the cost of sampling is \$0.50 per unit sampled, find the expected net gain of sampling (ENGS) for samples of sizes 1, 2, 5, and 10.

	PROPORTION OF CUSTOMERS BUYING, θ				
DECISION	0.10	0.20	0.30	0.40	0.50
Stock 100	–10	–2	12	22	40
Stock 50	–4	6	12	16	16
Do not stock	0	0	0	0	0

Exercise 6.16 was demonstrated at Meeting 11

16. In Exercise 15, suppose that sample information is available in the form of a random sample of consumers. For a sample of size *one*,
    (a) find the posterior distribution if the one person sampled will purchase the item, and find the value of this sample information;
    (b) find the posterior distribution if the one person sampled will *not* purchase the item, and find the value of this sample information;
    (c) find the expected value of sample information.

(c) EVSI(1) =8.64 – 7.80 = 0.84

17. In Exercise 16, suppose that you also want to consider other sample sizes.

(a) Find EVSI for a sample of size 2.

(b) Find EVSI for a sample of size 5.

(c) Find EVSI for a sample of size 10.

(d) If the cost of sampling is \$0.50 per unit sampled, find the expected net gain of sampling (ENGS) for samples of sizes 1, 2, 5, and 10.

DECISION	PROPORTION OF CUSTOMERS BUYING, θ				
	0.10	0.20	0.30	0.40	0.50
Stock 100	–10	–2	12	22	40
Stock 50	–4	6	12	16	16
Do not stock	0	0	0	0	0

(a)

We need to consider all possible outcomes in a sample of size 2, i.e.

BUY, BUY

BUY, NOT BUY

NOT BUY, BUY

NOT BUY, NOT BUY

However, the second and third outcome are equal by symmetry

DECISION	PROPORTION OF CUSTOMERS BUYING, θ				
	0.10	0.20	0.30	0.40	0.50
Stock 100	–10	–2	12	22	40
Stock 50	–4	6	12	16	16
Do not stock	0	0	0	0	0

BUY, BUY:

$$\text{Posterior distribution}: P(\theta | \text{BUY}, \text{BUY}) = \frac{P(\text{BUY}, \text{BUY} | \theta) \cdot P(\theta)}{P(\text{BUY}, \text{BUY})} = \frac{P(\text{BUY}, \text{BUY} | \theta) \cdot P(\theta)}{\sum_{\lambda} P(\text{BUY}, \text{BUY} | \lambda) \cdot P(\lambda)}$$

$$P(\text{BUY}, \text{BUY} | \theta) = \theta^2$$

$$\Rightarrow$$

$$P(\text{BUY}, \text{BUY}) = 0.10^2 \cdot 0.2 + 0.20^2 \cdot 0.3 + 0.30^2 \cdot 0.3 + 0.40^2 \cdot 0.1 + 0.50^2 \cdot 0.1 = 0.082$$

$$P(0.10 | \text{BUY}, \text{BUY}) = 0.10^2 \cdot 0.2 / 0.082 \approx 0.0244$$

$$P(0.20 | \text{BUY}, \text{BUY}) = 0.20^2 \cdot 0.3 / 0.082 \approx 0.1463$$

$$P(0.30 | \text{BUY}, \text{BUY}) = 0.30^2 \cdot 0.3 / 0.082 \approx 0.3293$$

$$P(0.40 | \text{BUY}, \text{BUY}) = 0.40^2 \cdot 0.1 / 0.082 \approx 0.1951$$

$$P(0.50 | \text{BUY}, \text{BUY}) = 0.50^2 \cdot 0.1 / 0.082 \approx 0.3049$$

$$\mathrm{VSI}(\mathrm{BUY},\mathrm{BUY}) = E''R(a''|\mathrm{BUY},\mathrm{BUY}) - E''R(a'|\mathrm{BUY},\mathrm{BUY})$$

$$a' = \langle = a^* \text{ from exercise } 6.15 \rangle = \text{Stock } 50$$

$$a'' = \arg\max_a ER(a|\mathrm{BUY},\mathrm{BUY})$$

$$ER(a|\mathrm{BUY},\mathrm{BUY}) = \sum_\theta R(a,\theta)\cdot P(\theta|\mathrm{BUY},\mathrm{BUY})$$

$$\Rightarrow$$

$$ER(\text{Stock } 100|\mathrm{BUY},\mathrm{BUY}) = (-10)\cdot 0.0244\ldots + (-2)\cdot 0.1463\ldots + 12\cdot 0.3293\ldots + 22\cdot 0.1951\ldots + 40\cdot 0.3049\ldots \approx 19.90$$

$$ER(\text{Stock } 50|\mathrm{BUY},\mathrm{BUY}) = (-4)\cdot 0.0244\ldots + 6\cdot 0.1463\ldots + 12\cdot 0.3293\ldots + 26\cdot 0.1951\ldots + 16\cdot 0.3049\ldots \approx 12.73$$

$$ER(\text{Do not stock}|\mathrm{BUY},\mathrm{BUY}) = 0\cdot 0.0244\ldots + 0\cdot 0.1463\ldots + 0\cdot 0.3293\ldots + 0\cdot 0.1951\ldots + 0\cdot 0.3049\ldots = 0$$

$$\Rightarrow a'' = \text{Stock } 100$$

$$\Rightarrow$$

$$\mathrm{VSI}(\mathrm{BUY},\mathrm{BUY}) = E''R(\text{Stock } 100|\mathrm{BUY},\mathrm{BUY}) - E''R(\text{Stock } 50|\mathrm{BUY},\mathrm{BUY}) \approx 19.90 - 12.73 = 7.17$$

DECISION	PROPORTION OF CUSTOMERS BUYING, θ				
	0.10	0.20	0.30	0.40	0.50
Stock 100	–10	–2	12	22	40
Stock 50	–4	6	12	16	16
Do not stock	0	0	0	0	0

BUY, NOT BUY or NOT BUY, BUY:

$$\text{Posterior distribution}: P(\theta|\text{BUY, NOT BUY}) = \frac{P(\text{BUY, NOT BUY}|\theta)\cdot P(\theta)}{P(\text{BUY, NOT BUY})} = \frac{P(\text{BUY, NOT BUY}|\theta)\cdot P(\theta)}{\sum_{\lambda} P(\text{BUY, NOT BUY}|\lambda)\cdot P(\lambda)}$$

$$P(\text{BUY, NOT BUY}|\theta) = \theta\cdot(1-\theta)$$

$$\Rightarrow$$

$$P(\text{BUY, NOT BUY}) = 0.10\cdot 0.90\cdot 0.2 + 0.20\cdot 0.80\cdot 0.3 + 0.30\cdot 0.70\cdot 0.3 + 0.40\cdot 0.50\cdot 0.1 + 0.50^2\cdot 0.1 = 0.178$$

$$P(0.10|\text{BUY, NOT BUY}) = 0.10\cdot 0.90\cdot 0.2/0.178 \approx 0.1011$$

$$P(0.20|\text{BUY, NOT BUY}) = 0.20\cdot 0.80\cdot 0.3/0.178 \approx 0.2697$$

$$P(0.30|\text{BUY, NOT BUY}) = 0.30\cdot 0.70\cdot 0.3/0.178 \approx 0.3539$$

$$P(0.40|\text{BUY, NOT BUY}) = 0.40\cdot 0.50\cdot 0.1/0.178 \approx 0.1348$$

$$P(0.50|\text{BUY, NOT BUY}) = 0.50^2\cdot 0.1/0.178 \approx 0.1404$$

$$\mathrm{VSI}(\mathrm{BUY},\mathrm{NOT\ \ BUY}) = E''R(a''|\mathrm{BUY},\mathrm{NOT\ \ BUY}) - E''R(a'|\mathrm{BUY},\mathrm{NOT\ \ BUY})$$

$$a' = \mathrm{Stock\ \ 50} \quad \text{(as before)}$$

$$a'' = \arg\max_a ER(a|\mathrm{BUY},\mathrm{NOT\ \ BUY})$$

$$ER(a|\mathrm{BUY},\mathrm{NOT\ \ BUY}) = \sum_\theta R(a,\theta)\cdot P(\theta|\mathrm{BUY},\mathrm{NOT\ \ BUY})$$

$$\Rightarrow$$

$$ER(\mathrm{Stock\ \ 100}|\mathrm{BUY},\mathrm{NOT\ \ BUY}) = (-10)\cdot 0.1011\ldots + (-2)\cdot 0.2697\ldots + 12\cdot 0.3539\ldots + 22\cdot 0.1348\ldots + 40\cdot 0.1404\ldots \approx 11.28$$

$$ER(\mathrm{Stock\ \ 50}|\mathrm{BUY},\mathrm{NOT\ \ BUY}) = (-4)\cdot 0.1011\ldots + 6\cdot 0.2697\ldots + 12\cdot 0.3539\ldots + 26\cdot 0.1348\ldots + 16\cdot 0.1404\ldots \approx 9.87$$

$$ER(\mathrm{Do\ \ not\ \ stock}|\mathrm{BUY},\mathrm{NOT\ \ BUY}) = 0\cdot 0.1011\ldots + 0\cdot 0.2697\ldots + 0\cdot 0.3539\ldots + 0\cdot 0.1348\ldots + 0\cdot 0.1404\ldots = 0$$

$$\Rightarrow a'' = \mathrm{Stock\ \ 100}$$

$$\Rightarrow$$

$$\mathrm{VSI}(\mathrm{BUY},\mathrm{NOT\ \ BUY}) = E''R(\mathrm{Stock\ \ 100}|\mathrm{BUY},\mathrm{NOT\ \ BUY}) - E''R(\mathrm{Stock\ \ 50}|\mathrm{BUY},\mathrm{NOT\ \ BUY}) \approx 11.28 - 9.87 = 1.41$$

DECISION	PROPORTION OF CUSTOMERS BUYING, θ				
	0.10	0.20	0.30	0.40	0.50
Stock 100	–10	–2	12	22	40
Stock 50	–4	6	12	16	16
Do not stock	0	0	0	0	0

NOT BUY, NOT BUY:

Posterior distribution : $P(\theta|\text{NOT BUY, NOT BUY}) =$

$$\frac{P(\text{NOT BUY, NOT BUY}|\theta)\cdot P(\theta)}{P(\text{NOT BUY, NOT BUY})} = \frac{P(\text{NOT BUY, NOT BUY}|\theta)\cdot P(\theta)}{\sum_{\lambda} P(\text{NOT BUY, NOT BUY}|\lambda)\cdot P(\lambda)}$$

$$P(\text{NOT BUY, NOT BUY}|\theta) = (1-\theta)^2$$

$\Rightarrow$

$$P(\text{NOT BUY, NOT BUY}) = 0.90^2\cdot 0.2 + 0.80^2\cdot 0.3 + 0.70^2\cdot 0.3 + 0.60^2\cdot 0.1 + 0.50^2\cdot 0.1 = 0.562$$

$$P(0.10|\text{NOT BUY, NOT BUY}) = 0.90^2\cdot 0.2/0.562 \approx 0.2883$$

$$P(0.20|\text{NOT BUY, NOT BUY}) = 0.80^2\cdot 0.3/0.562 \approx 0.3416$$

$$P(0.30|\text{NOT BUY, NOT BUY}) = 0.70^2\cdot 0.3/0.562 \approx 0.2616$$

$$P(0.40|\text{NOT BUY, NOT BUY}) = 0.60^2\cdot 0.1/0.562 \approx 0.0641$$

$$P(0.50|\text{NOT BUY, NOT BUY}) = 0.50^2\cdot 0.1/0.562 \approx 0.0445$$

$$\text{VSI}(\text{NOT BUY}, \text{NOT BUY}) = E''R(a'' \mid \text{NOT BUY}, \text{NOT BUY}) - E''R(a' \mid \text{NOT BUY}, \text{NOT BUY})$$

$$a' = \text{Stock 50} \quad \text{(as before)} \qquad a'' = \arg\max_a ER(a \mid \text{NOT BUY}, \text{NOT BUY})$$

$$ER(a \mid \text{NOT BUY}, \text{NOT BUY}) = \sum_\theta R(a, \theta) \cdot P(\theta \mid \text{NOT BUY}, \text{NOT BUY})$$

$$\Rightarrow$$

$$ER(\text{Stock 100} \mid \text{NOT BUY}, \text{NOT BUY}) = (-10) \cdot 0.2883\ldots + (-2) \cdot 0.3416\ldots + 12 \cdot 0.2616\ldots + 22 \cdot 0.0641\ldots + 40 \cdot 0.0445\ldots \approx 2.76$$

$$ER(\text{Stock 50} \mid \text{NOT BUY}, \text{NOT BUY}) = (-4) \cdot 0.2883\ldots + 6 \cdot 0.3416\ldots + 12 \cdot 0.2616\ldots + 26 \cdot 0.0641\ldots + 16 \cdot 0.0445\ldots \approx 5.77$$

$$ER(\text{Do not stock} \mid \text{NOT BUY}, \text{NOT BUY}) = 0 \cdot 0.2883\ldots + 0 \cdot 0.3416\ldots + 0 \cdot 0.2616\ldots + 0 \cdot 0.0641\ldots + 0 \cdot 0.0445\ldots = 0$$

$$\Rightarrow a'' = \text{Stock 50}$$

$$\Rightarrow$$

$$\text{VSI}(\text{NOT BUY}, \text{NOT BUY}) = E''R(\text{Stock 50} \mid \text{NOT BUY}, \text{NOT BUY}) - E''R(\text{Stock 50} \mid \text{NOT BUY}, \text{NOT BUY}) = (5.77 - 5.77) = 0$$

$$\mathrm{EVSI} = \sum_{y} \mathrm{VSI}(y) P(y) =$$

$$= \mathrm{VSI}(\mathrm{BUY}, \mathrm{BUY}) \cdot P(\mathrm{BUY}, \mathrm{BUY}) + 2 \cdot \mathrm{VSI}(\mathrm{BUY}, \mathrm{NOT\ \ BUY}) \cdot P(\mathrm{BUY}, \mathrm{NOT\ \ BUY}) +$$

$$\mathrm{VSI}(\mathrm{NOT\ \ BUY}, \mathrm{NOT\ \ BUY}) \cdot P(\mathrm{NOT\ \ BUY}, \mathrm{NOT\ \ BUY}) =$$

$$= 7.17 \cdot 0.082 + 2 \cdot 1.41 \cdot 0.178 + 0 \cdot 0.652 \approx 1.09$$

(b,c) Tedious to sort out the calculations for sample sizes greater than 2.

Use the fact that the sample outcome is that of binomial sampling:

$$P(\theta|\text{Sample outcome}) \propto P(\text{Sample outcome}|\theta) \times P(\theta) =$$
$$= P(y|\theta) \times P(\theta) = \binom{n}{y} \theta^y \cdot (1-\theta)^{n-y} \times P(\theta)$$

Let

$\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)^{\mathbf{T}}$ *column matrix*

$P(\boldsymbol{\theta}) = \left(P(\theta_1), P(\theta_2), P(\theta_3), P(\theta_4), P(\theta_5)\right)^{\mathrm{T}}$ *column matrix*

$P(y|\boldsymbol{\theta}) = \left(P(y|\theta_1), P(y|\theta_2), P(y|\theta_3), P(y|\theta_4), P(y|\theta_5)\right)^{\mathrm{T}}$ *column matrix*

$$\Rightarrow P(\boldsymbol{\theta}|y) = \frac{P(y|\boldsymbol{\theta}) \odot P(\boldsymbol{\theta})}{P(y|\boldsymbol{\theta})^{\mathrm{T}} \cdot P(\boldsymbol{\theta})} = \frac{\boldsymbol{\theta}^{\circ y} \odot (1-\boldsymbol{\theta})^{\circ(n-y)} \odot P(\boldsymbol{\theta})}{[\boldsymbol{\theta}^{\circ y} \odot (1-\boldsymbol{\theta})^{\circ(n-y)}]^{\mathrm{T}} \cdot P(\boldsymbol{\theta})}$$ *column matrix*

$\boldsymbol{ER}(y) = \left(ER(\text{Stock 100}|y), ER(\text{Stock 50}|y), ER(\text{Do not stock}|y)\right)^{\mathrm{T}}$ *column matrix*

$$\boldsymbol{Rmat} = \begin{pmatrix} -10 & -2 & 12 & 22 & 40 \\ -4 & 6 & 12 & 16 & 16 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ *payoff table as a 3×5 matrix*

$$\Longrightarrow \boldsymbol{ER}(y) = \boldsymbol{Rmat} \cdot P(\boldsymbol{\theta}|y)$$

(b)

y	θ	$P(\theta)$	$P(y \mid \theta)$	$P(\theta \mid y)$	$E''R$(Stock 100)	$E''R$(Stock 50)	$E''R$(Do not stock)	a''	a'	$E''(a')$	VSI	$P(y)$	**EVSI**
0	0.1	0.2	0.591	0.425									
	0.2	0.3	0.328	0.354									
	0.3	0.3	0.168	0.182									
	0.4	0.1	0.078	0.028									
	0.5	0.1	0.031	0.011	-1.716	3.230	0	Stock 50	Stock 50	3.230	0	0.28	
1	0.1	0.2	0.328	0.194									
	0.2	0.3	0.410	0.364									
	0.3	0.3	0.360	0.320									
	0.4	0.1	0.259	0.077									
	0.5	0.1	0.156	0.046	4.703	7.206	0	Stock 50	Stock 50	7.206	0	0.34	

2	0.1	0.2	0.073	0.062									
	0.2	0.3	0.205	0.262									
	0.3	0.3	0.309	0.395									
	0.4	0.1	0.346	0.147									
	0.5	0.1	0.313	0.133	12.169	10.555	0	Stock 100	Stock 50	10.555	1.614	0.23	
3	0.1	0.2	0.008	0.015									
	0.2	0.3	0.051	0.138									
	0.3	0.3	0.132	0.358									
	0.4	0.1	0.230	0.208									
	0.5	0.1	0.313	0.282	19.703	12.893	0	Stock 100	Stock 50	12.893	6.810	0.11	
4	0.1	0.2	5e-04	0.003									
	0.2	0.3	0.006	0.057									
	0.3	0.3	0.028	0.252									
	0.4	0.1	0.077	0.227									
	0.5	0.1	0.156	0.462	26.354	14.373	0	Stock 100	Stock 50	14.373	11.980	0.03	
5	0.1	0.2	0	4e-04									
	0.2	0.3	3e-04	0.019									
	0.3	0.3	0.002	0.147									
	0.4	0.1	0.010	0.206									
	0.5	0.1	0.031	0.628	31.363	15.213	0	Stock 100	Stock 50	15.213	16.150	0.005	**1.63**

(c)

y	θ	$P(\theta)$	$P(y \mid \theta)$	$P(\theta \mid y)$	$E''R$(Stock 100)	$E''R$(Stock 50)	$E''R$(Do not stock)	a''	a'	$E''(a')$	VSI	$P(y)$	**EVSI**
0	0.1	0.2	0.349	0.628									
	0.2	0.3	0.107	0.290									
	0.3	0.3	0.028	0.076									
	0.4	0.1	0.006	0.005									
	0.5	0.1	0.001	9e-04	-5.785	0.245	0	Stock 50	Stock 50	0.245	0	0.11	
1	0.1	0.2	0.387	0.389									
	0.2	0.3	0.268	0.404									
	0.3	0.3	0.121	0.182									
	0.4	0.1	0.040	0.020									
	0.5	0.1	0.010	0.005	-1.868	3.457	0	Stock 50	Stock 50	3.457	0	0.20	

2	0.1	0.2	0.194	0.180									
	0.2	0.3	0.302	0.420									
	0.3	0.3	0.234	0.325									
	0.4	0.1	0.121	0.056									
	0.5	0.1	0.044	0.020	3.306	6.916	0	Stock 50	Stock 50	6.916	0	0.22	
3	0.1	0.2	0.057	0.062									
	0.2	0.3	0.201	0.326									
	0.3	0.3	0.267	0.432									
	0.4	0.1	0.215	0.116									
	0.5	0.1	0.117	0.063	9.002	9.768	0	Stock 50	Stock 50	9.768	0	0.19	
4	0.1	0.2	0.011	0.017									
	0.2	0.3	0.088	0.197									
	0.3	0.3	0.200	0.447									
	0.4	0.1	0.251	0.187									
	0.5	0.1	0.205	0.153	15.024	11.911	0	Stock 100	Stock 50	11.911	3.112	0.13	
5	0.1	0.2	0.002	0.004									
	0.2	0.3	0.026	0.095									
	0.3	0.3	0.103	0.369									
	0.4	0.1	0.201	0.240									
	0.5	0.1	0.246	0.294	21.217	13.509	0	Stock 100	Stock 50	13.509	7.709	0.08	

6	0.1	0.2	1e-04	6e-04									
	0.2	0.3	0.006	0.037									
	0.3	0.3	0.037	0.249									
	0.4	0.1	0.112	0.251									
	0.5	0.1	0.205	0.462	26.922	14.621	0	Stock 100	Stock 50	14.621	12.301	0.04	
7	0.1	0.2	0.0000	1e-04									
	0.2	0.3	8e-04	0.013									
	0.3	0.3	0.009	0.143									
	0.4	0.1	0.043	0.225									
	0.5	0.1	0.117	0.620	31.428	15.302	0	Stock 100	Stock 50	15.302	16.126	0.02	
8	0.1	0.2	0.0000	0.000									
	0.2	0.3	1e-04	0.004									
	0.3	0.3	0.001	0.073									
	0.4	0.1	0.011	0.180									
	0.5	0.1	0.044	0.743	34.555	15.669	0	Stock 100	Stock 50	15.669	18.886	0.006	

y	θ	$P(\theta)$	$P(y\,\vert\,\theta)$	$P(\theta\vert\,y)$	$E''R$(Stock 100)	$E''R$(Stock 50)	$E''R$(Do not stock)	a''	a'	$E''(a')$	VSI	$P(y)$	**EVSI**
9	0.1	0.2	0.0000	0.000									
	0.2	0.3	0.0000	0.001									
	0.3	0.3	1e-04	0.035									
	0.4	0.1	0.002	0.134									
	0.5	0.1	0.010	0.830	36.566	15.849	0	Stock 100	Stock 50	15.849	20.717	0.001	
10	0.1	0.2	0.0000	0.0000									
	0.2	0.3	0.0000	3e-04									
	0.3	0.3	0.0000	0.016									
	0.4	0.1	1e-04	0.095									
	0.5	0.1	0.001	0.888	37.820	15.933	0	Stock 100	Stock 50	15.933	21.888	0.0001	**2.05**

(d) $n = 1$: ENGS(1) = EVSI(1) – CS(1) ≈ 0.84 – 0.50·1 = 0.34

$n = 2$: ENGS(2) = EVSI(2) – CS(2) ≈ 1.09 – 0.50·2 = 0.09

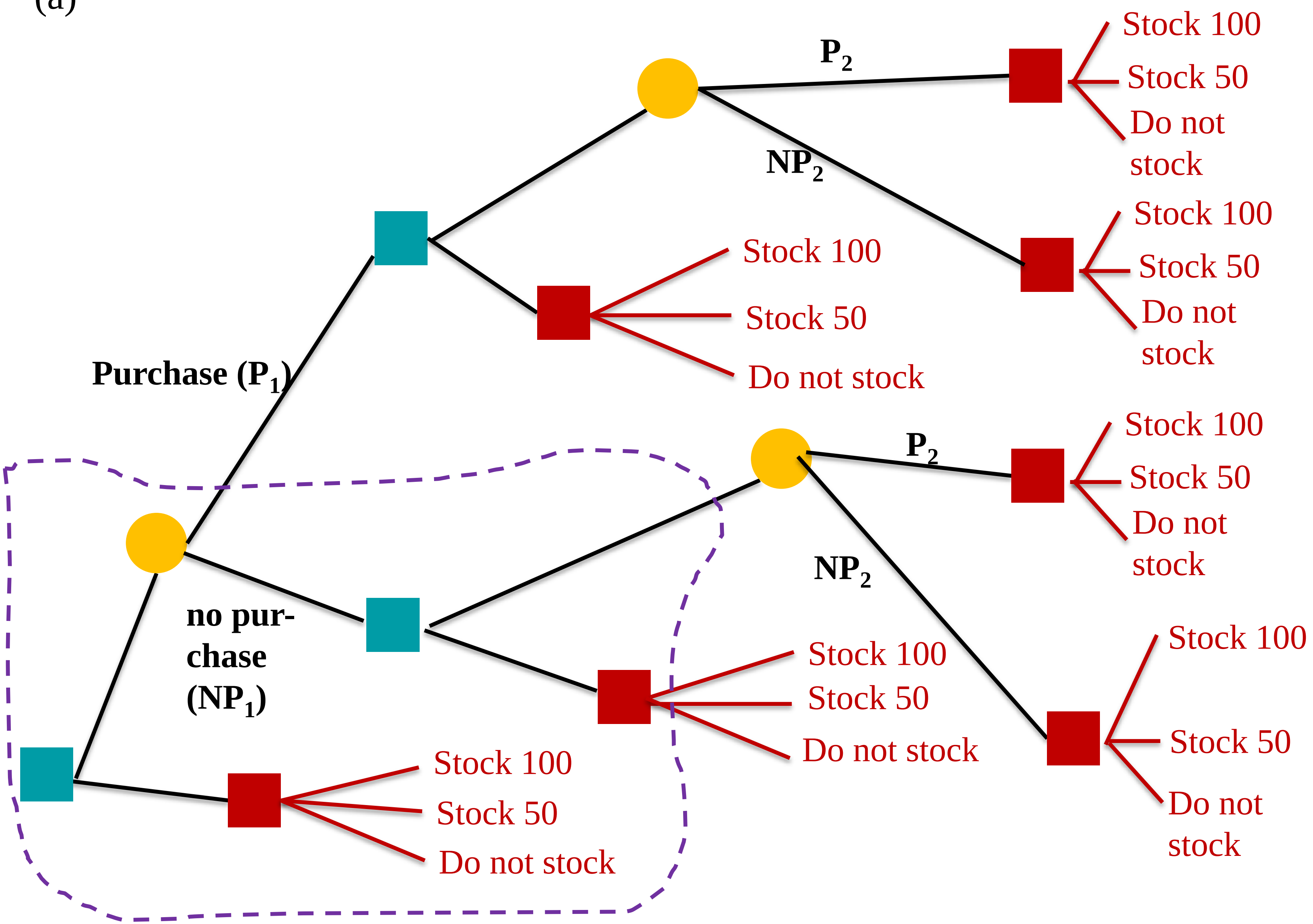
$n = 5$: ENGS(5) = EVSI(5) – CS(5) ≈ 1.63 – 0.50·5 = –0.87

$n = 10$: ENGS(10) = EVSI(10) – CS(10) ≈ 2.05 – 0.50·10 = –2.95

Finally, Exercise 6.27

27. In Exercise 17, consider a sequential sampling plan with a maximum total sample size of two and analyze the problem as follows.
    (a) Represent the situation in terms of a tree diagram.
    (b) Using backward induction, find the ENGS for the sequential plan.
    (c) Compare the sequential plan with a single-stage plan having $n = 2$.

(a)

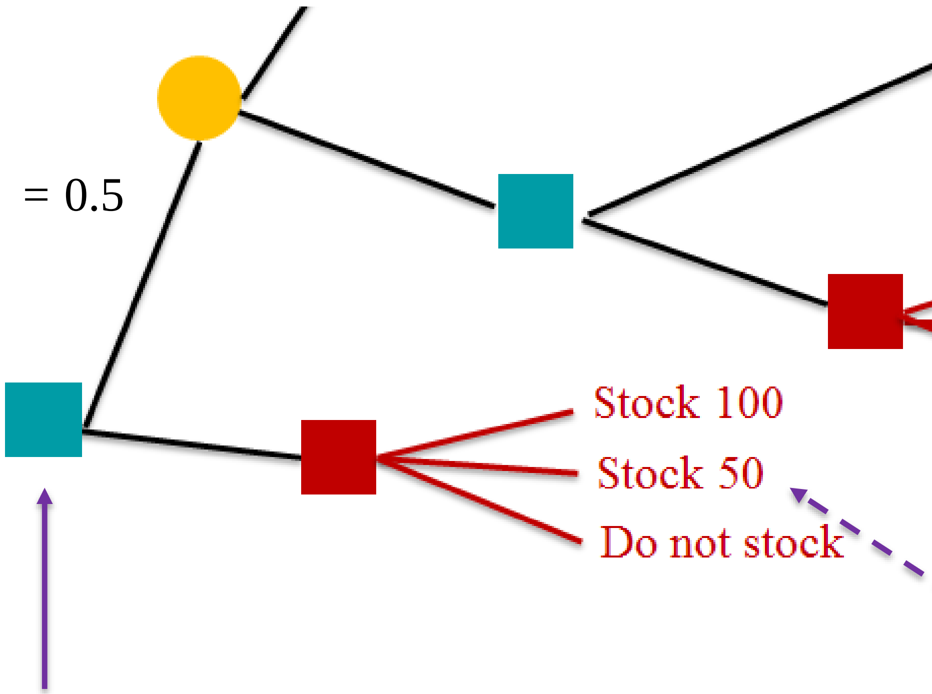


Purchase (P_1)
no pur-
chase
(NP_1)
P_2
NP_2
P_2
NP_2
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock

θ

		PROPORTION OF CONSUMERS PURCHASING				
		0.10	0.20	0.30	0.40	0.50
DECISION	*Stock 100*	-10	-2	12	22	40
	Stock 50	-4	6	12	16	16
	Do not stock	0	0	0	0	0

$CS_1 = 0.5$



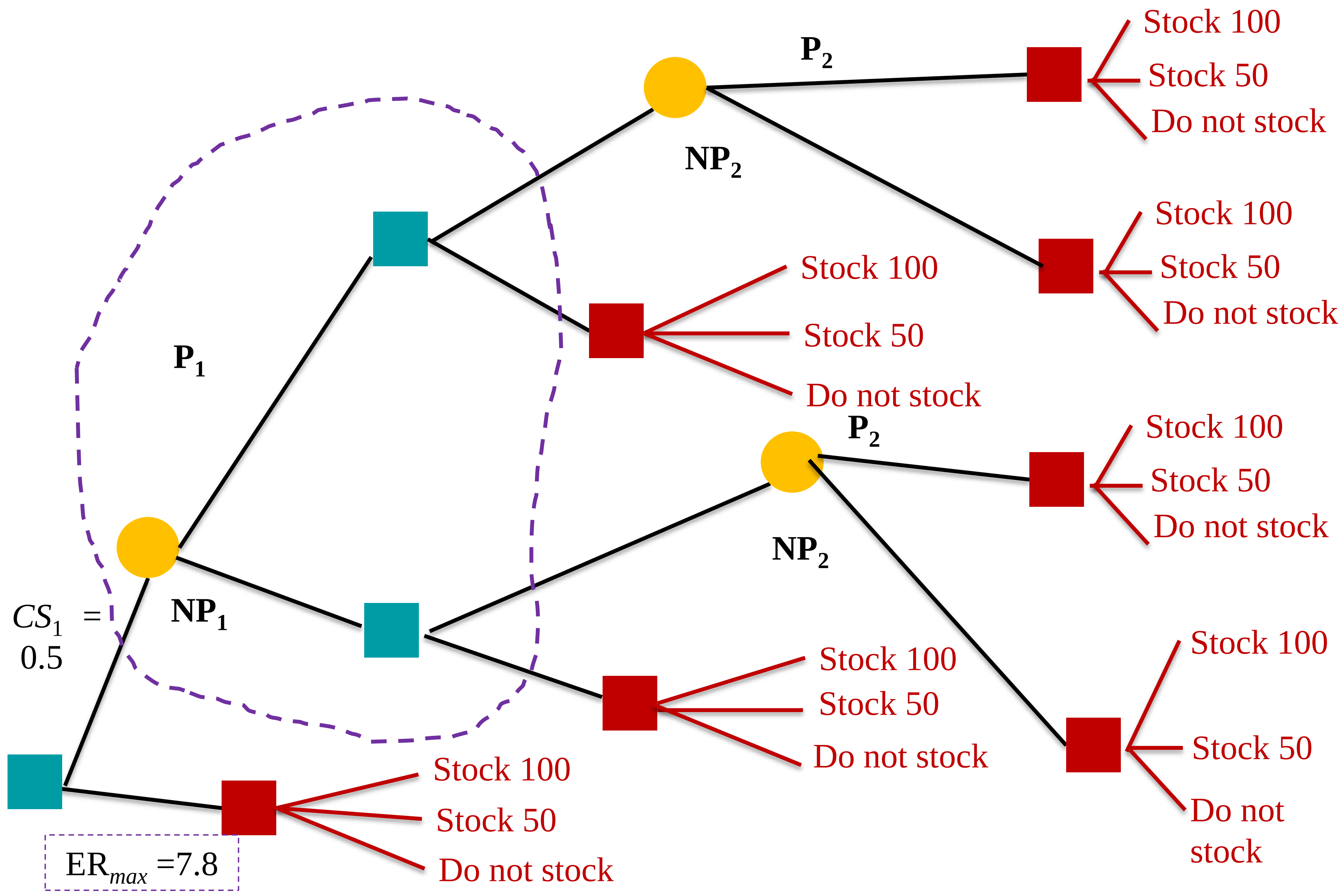
Prior distribution:

θ	$P(\tilde{\theta} = \theta)$
0.10	0.2
0.20	0.3
0.30	0.3
0.40	0.1
0.50	0.1

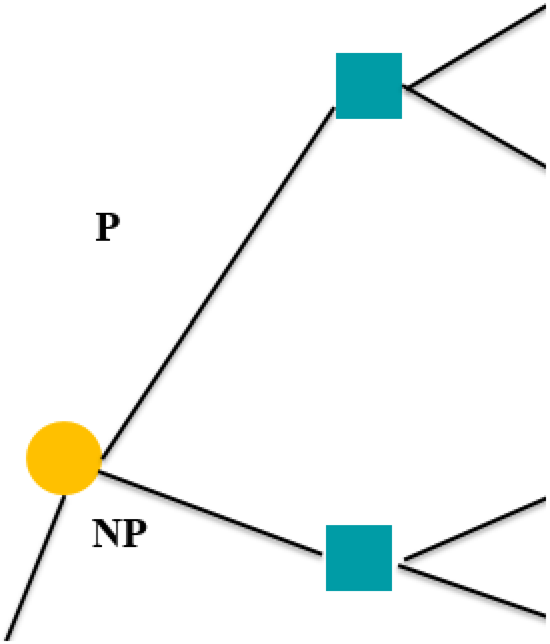
$$ER(\text{Stock } 100) =$$
$$(-10)\cdot 0.2 + (-2)\cdot 0.3 + 12\cdot 0.3 + 22\cdot 0.1 + 40\cdot 0.1 = \underline{\underline{7.2}}$$
$$ER(\text{Stock } 50) =$$
$$(-4)\cdot 0.2 + 6\cdot 0.3 + 12\cdot 0.3 + 16\cdot 0.1 + 16\cdot 0.1 = \underline{\underline{7.8}}$$

Max

$$ER(\text{Do not stock}) =$$
$$0\cdot 0.2 + 0\cdot 0.3 + 0\cdot 0.3 + 0\cdot 0.1 + 0\cdot 0.1 = \underline{\underline{0}}$$


P2
NP2
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
P1
Stock 100
Stock 50
Do not stock
P2
NP2
Stock 100
Stock 50
Do not stock
NP1
CS1 = 0.5
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
ERmax =7.8

First sampled consumer



If outcome = Purchase

Posterior probabilities of θ:

$$P\left(\tilde\theta=\theta\middle|\mathrm{P}_1\right)=\frac{P\left(\mathrm{P}_1\middle|\tilde\theta=\theta\right)\cdot P\left(\tilde\theta=\theta\right)}{\left[\begin{array}{l}P\left(\mathrm{P}_1\middle|\tilde\theta=0.1\right)\cdot P\left(\tilde\theta=0.1\right)+P\left(\mathrm{P}_1\middle|\tilde\theta=0.2\right)\cdot P\left(\tilde\theta=0.2\right)+\\ P\left(\mathrm{P}_1\middle|\tilde\theta=0.3\right)\cdot P\left(\tilde\theta=0.3\right)+P\left(\mathrm{P}_1\middle|\tilde\theta=0.4\right)\cdot P\left(\tilde\theta=0.4\right)+\\ P\left(\mathrm{P}_1\middle|\tilde\theta=0.5\right)\cdot P\left(\tilde\theta=0.5\right)\end{array}\right]}=$$

$$=\frac{P\left(\mathrm{P}_1\middle|\tilde\theta=\theta\right)\cdot P\left(\tilde\theta=\theta\right)}{0.1\cdot0.2+0.2\cdot0.3+0.3\cdot0.3+0.4\cdot0.1+0.5\cdot0.1}=\frac{P\left(\mathrm{P}_1\middle|\tilde\theta=\theta\right)\cdot P\left(\tilde\theta=\theta\right)}{0.26}$$

$$\Rightarrow P\left(\tilde\theta=0.1\middle|\mathrm{P}_1\right)=0.1\cdot0.2/0.26=2/26\,;\ P\left(\tilde\theta=0.2\middle|\mathrm{P}_1\right)=6/26\,;$$

$$P\left(\tilde\theta=0.3\middle|\mathrm{P}_1\right)=9/26\,;\ P\left(\tilde\theta=0.4\middle|\mathrm{P}_1\right)=4/26\,;\ P\left(\tilde\theta=0.2\middle|\mathrm{P}_1\right)=5/26\,;$$

If outcome = No purchase

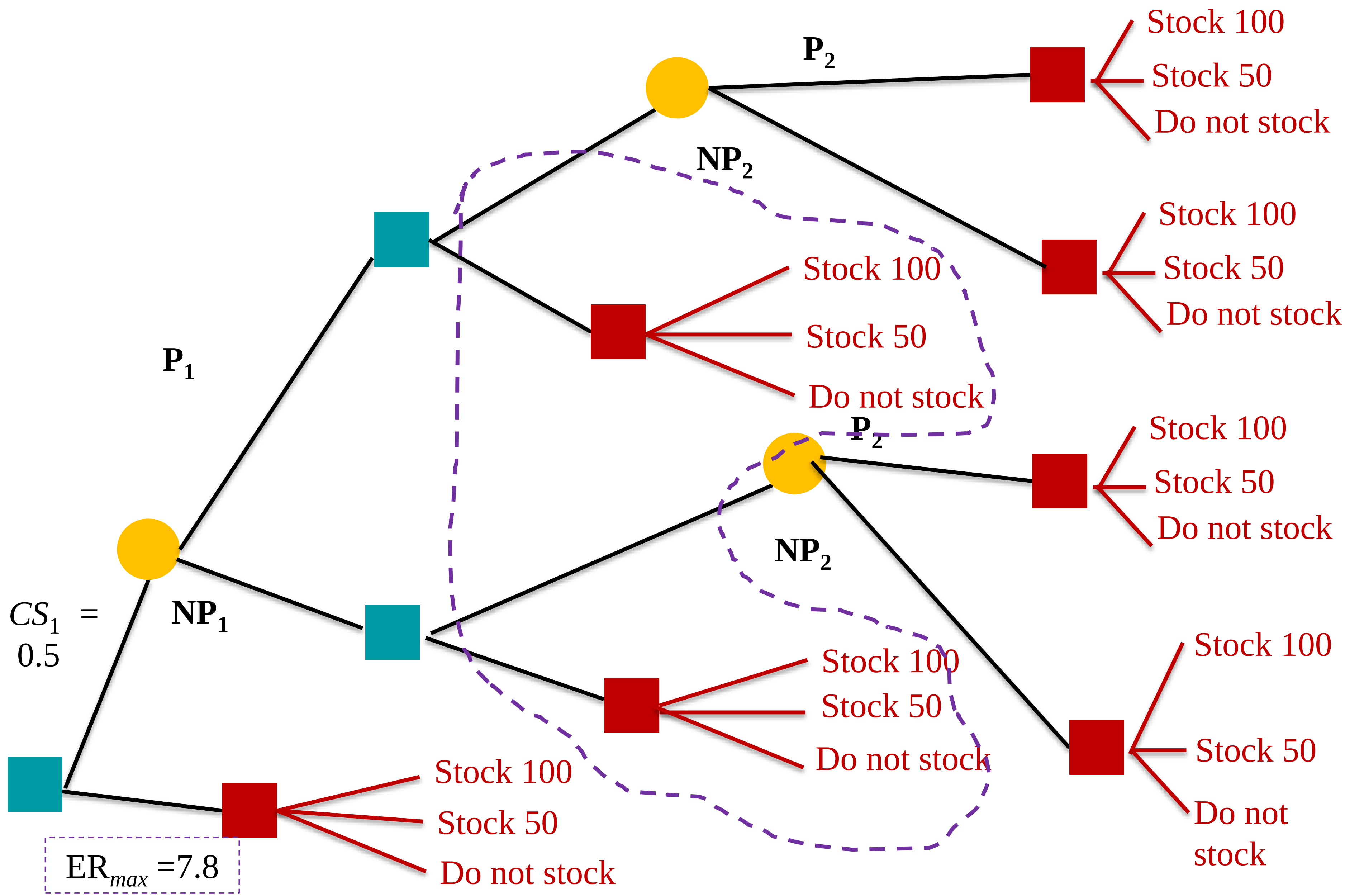
Posterior probabilities of θ:

$$P\left(\tilde\theta=\theta\middle|\mathrm{NP}_1\right)=\frac{P\left(\mathrm{NP}_1\middle|\tilde\theta=\theta\right)\cdot P\left(\tilde\theta=\theta\right)}{\left[\begin{array}{l}P\left(\mathrm{NP}_1\middle|\tilde\theta=0.1\right)\cdot P\left(\tilde\theta=0.1\right)+P\left(\mathrm{NP}_1\middle|\tilde\theta=0.2\right)\cdot P\left(\tilde\theta=0.2\right)+\\ P\left(\mathrm{NP}_1\middle|\tilde\theta=0.3\right)\cdot P\left(\tilde\theta=0.3\right)+P\left(\mathrm{NP}_1\middle|\tilde\theta=0.4\right)\cdot P\left(\tilde\theta=0.4\right)+\\ P\left(\mathrm{NP}_1\middle|\tilde\theta=0.5\right)\cdot P\left(\tilde\theta=0.5\right)\end{array}\right]}=$$

$$=\frac{P\left(\mathrm{NP}_1\middle|\tilde\theta=\theta\right)\cdot P\left(\tilde\theta=\theta\right)}{0.9\cdot0.2+0.8\cdot0.3+0.7\cdot0.3+0.6\cdot0.1+0.5\cdot0.1}=\frac{P\left(\mathrm{NP}_1\middle|\tilde\theta=\theta\right)\cdot P\left(\tilde\theta=\theta\right)}{0.74}$$

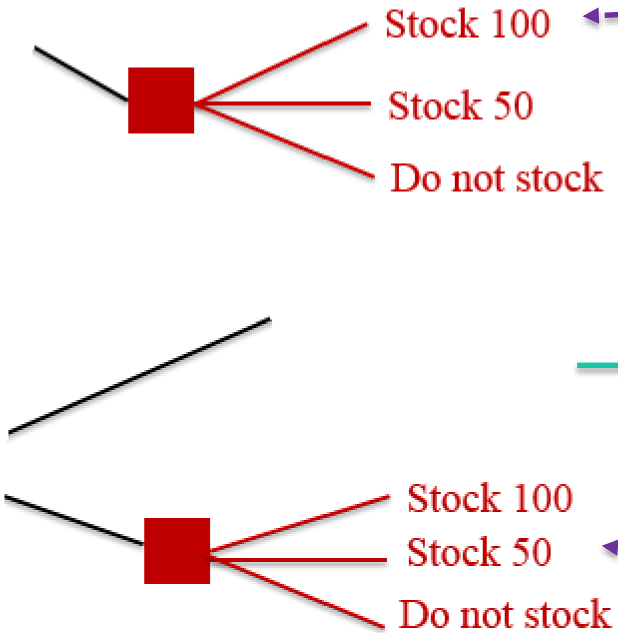
$$\Rightarrow P\left(\tilde\theta=0.1\middle|\mathrm{NP}_1\right)=0.9\cdot0.2/0.74=18/74\,;\ P\left(\tilde\theta=0.2\middle|\mathrm{NP}_1\right)=24/74\,;$$

$$P\left(\tilde\theta=0.3\middle|\mathrm{NP}_1\right)=21/74\,;\ P\left(\tilde\theta=0.4\middle|\mathrm{NP}_1\right)=6/74\,;\ P\left(\tilde\theta=0.2\middle|\mathrm{NP}_1\right)=5/74\,;$$



P2
Stock 100
Stock 50
Do not stock
NP2
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
P1
P2
Stock 100
Stock 50
Do not stock
NP2
CS1 = 0.5
NP1
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
ERmax =7.8

θ	0.10	0.20	0.30	0.40	0.50
$P(\tilde{\theta} = \theta \mid \mathrm{P}_1)$	2/26	6/26	9/26	4/26	5/26



$$ER(\text{Stock } 100 \mid \mathrm{P}_1) =$$

$$(-10)\cdot\tfrac{2}{26} + (-2)\cdot\tfrac{6}{26} + 12\cdot\tfrac{9}{26} + 22\cdot\tfrac{4}{26} + 40\cdot\tfrac{5}{26} \approx \underline{\underline{14.0}}$$ Max

$$ER(\text{Stock } 50 \mid \mathrm{P}_1) =$$

$$(-4)\cdot\tfrac{2}{26} + 6\cdot\tfrac{6}{26} + 12\cdot\tfrac{9}{26} + 16\cdot\tfrac{4}{26} + 16\cdot\tfrac{5}{26} \approx \underline{\underline{10.8}}$$

$$ER(\text{Do not stock} \mid \mathrm{P}_1) = \underline{\underline{0}}$$

$$ER(\text{Stock } 100 \mid \mathrm{NP}_1) =$$

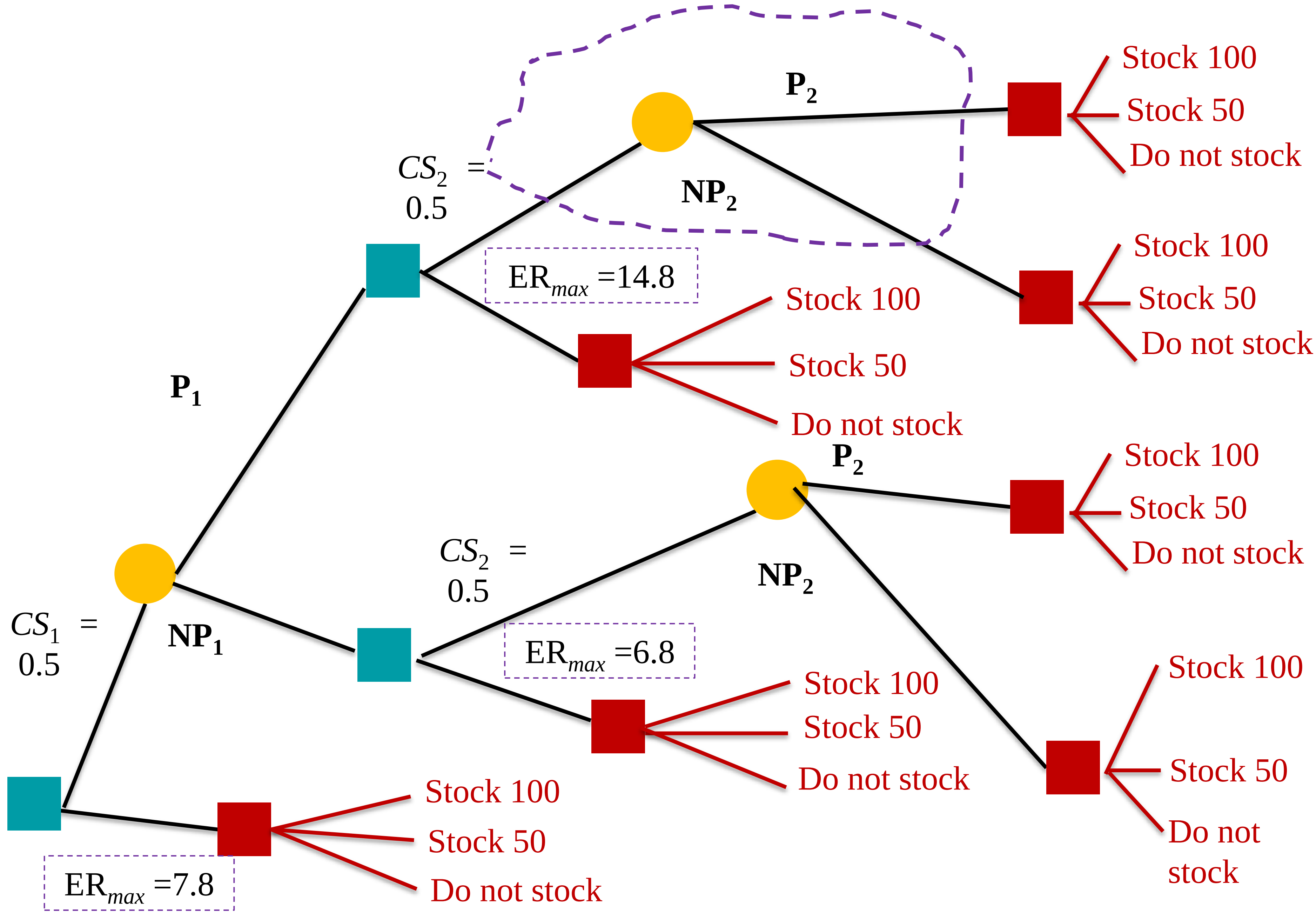
$$(-10)\cdot\tfrac{18}{74} + (-2)\cdot\tfrac{24}{74} + 12\cdot\tfrac{21}{74} + 22\cdot\tfrac{6}{74} + 40\cdot\tfrac{5}{74} \approx \underline{\underline{4.8}}$$

$$ER(\text{Stock } 50 \mid \mathrm{NP}_1) =$$

$$(-4)\cdot\tfrac{18}{74} + 6\cdot\tfrac{24}{74} + 12\cdot\tfrac{21}{74} + 16\cdot\tfrac{6}{74} + 16\cdot\tfrac{5}{74} \approx \underline{\underline{6.8}}$$ Max

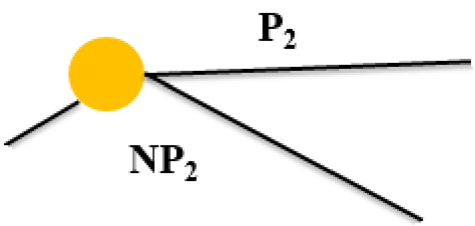
$$ER(\text{Do not stock} \mid \mathrm{NP}_1) = \underline{\underline{0}}$$

θ	0.10	0.20	0.30	0.40	0.50
$P(\tilde{\theta} = \theta \mid \mathrm{NP}_1)$	18/74	24/74	21/74	6/74	5/74



P₂
Stock 100
Stock 50
Do not stock
CS₂ =
0.5
NP₂
Stock 100
Stock 50
Do not stock
ER_max =14.8
Stock 100
Stock 50
Do not stock
P₁
P₂
Stock 100
Stock 50
Do not stock
CS₂ =
0.5
NP₂
CS₁ =
0.5
NP₁
ER_max =6.8
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
ER_max =7.8

Second sampled consumer, case 1



<u>If outcome = Purchase</u>

Posterior probabilities of θ:

$$P\left(\tilde{\theta}=\theta \middle| \mathrm{P}_2,\mathrm{P}_1\right)=\frac{P\left(\mathrm{P}_2,\mathrm{P}_1\middle|\tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{\begin{bmatrix} P\left(\mathrm{P}_2,\mathrm{P}_1\middle|\tilde{\theta}=0.1\right)\cdot P\left(\tilde{\theta}=0.1\right)+P\left(\mathrm{P}_2,\mathrm{P}_1\middle|\tilde{\theta}=0.2\right)\cdot P\left(\tilde{\theta}=0.2\right)+ \\ P\left(\mathrm{P}_2,\mathrm{P}_1\middle|\tilde{\theta}=0.3\right)\cdot P\left(\tilde{\theta}=0.3\right)+P\left(\mathrm{P}_2,\mathrm{P}_1\middle|\tilde{\theta}=0.4\right)\cdot P\left(\tilde{\theta}=0.4\right)+ \\ P\left(\mathrm{P}_2,\mathrm{P}_1\middle|\tilde{\theta}=0.5\right)\cdot P\left(\tilde{\theta}=0.5\right) \end{bmatrix}}=$$

$$=\frac{P\left(\mathrm{P}_2,\mathrm{P}_1\middle|\tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{0.1^2\cdot 0.2+0.2^2\cdot 0.3+0.3^2\cdot 0.3+0.4^2\cdot 0.1+0.5^2\cdot 0.1}=\frac{P\left(\mathrm{P}_2,\mathrm{P}_1\middle|\tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{0.082}$$

$$\Rightarrow P\left(\tilde{\theta}=0.1\middle|\mathrm{P}_2,\mathrm{P}_1\right)=0.1^2\cdot 0.2/0.26=2/82\,;\; P\left(\tilde{\theta}=0.2\middle|\mathrm{P}_2,\mathrm{P}_1\right)=12/82\,;$$

$$P\left(\tilde{\theta}=0.3\middle|\mathrm{P}_2,\mathrm{P}_1\right)=27/82\,;\; P\left(\tilde{\theta}=0.4\middle|\mathrm{P}_2,\mathrm{P}_1\right)=16/82\,;\; P\left(\tilde{\theta}=0.5\middle|\mathrm{P}_2,\mathrm{P}_1\right)=25/82\,;$$

<u>If outcome = No purchase</u>

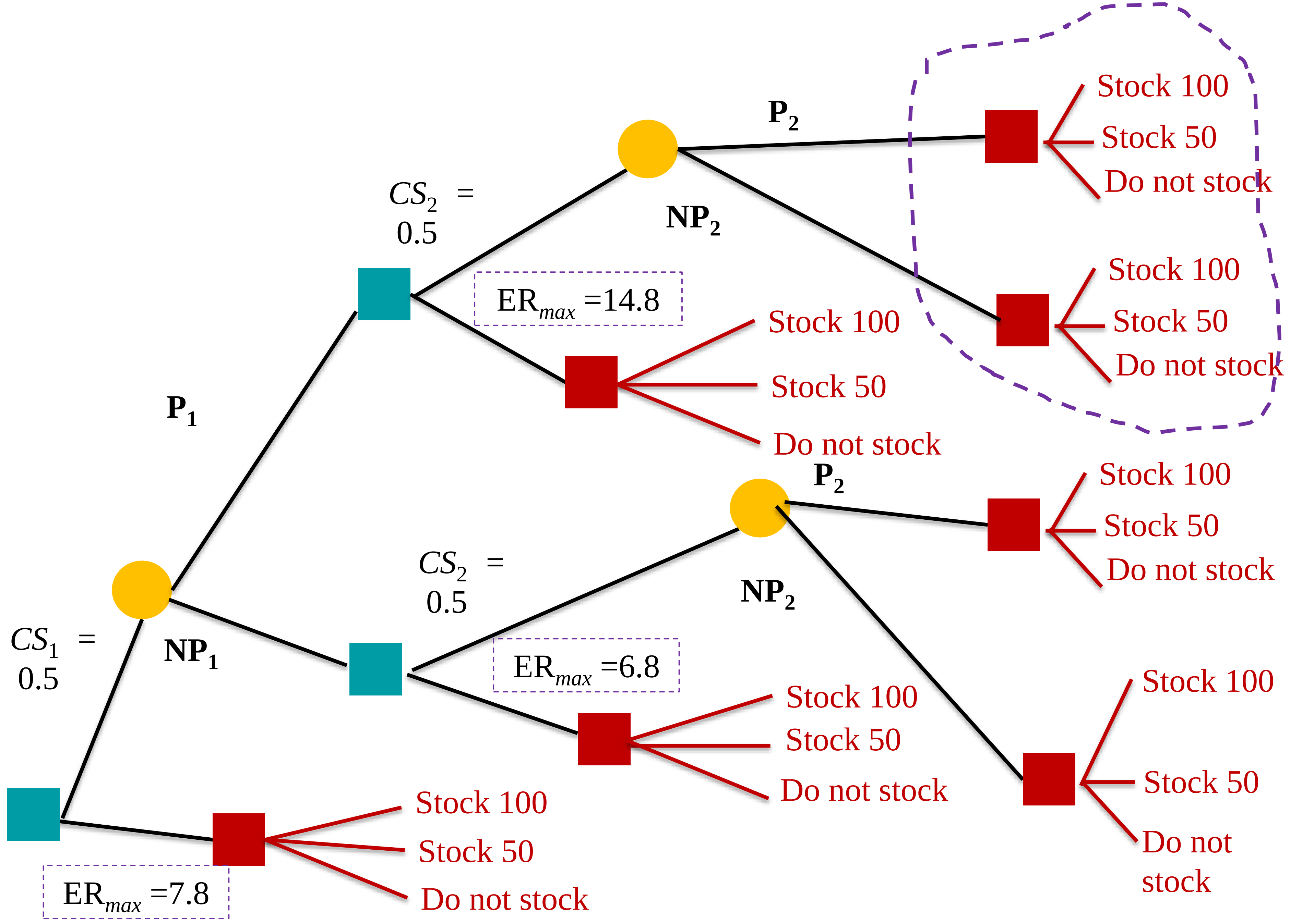
Posterior probabilities of θ:

$$P\left(\tilde{\theta}=\theta \middle| \mathrm{NP}_2,\mathrm{P}_1\right)=\frac{P\left(\mathrm{NP}_2,\mathrm{P}_1\middle|\tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{\begin{bmatrix} P\left(\mathrm{NP}_2,\mathrm{P}_1\middle|\tilde{\theta}=0.1\right)\cdot P\left(\tilde{\theta}=0.1\right)+P\left(\mathrm{NP}_2,\mathrm{P}_1\middle|\tilde{\theta}=0.2\right)\cdot P\left(\tilde{\theta}=0.2\right)+ \\ P\left(\mathrm{NP}_2,\mathrm{P}_1\middle|\tilde{\theta}=0.3\right)\cdot P\left(\tilde{\theta}=0.3\right)+P\left(\mathrm{NP}_2,\mathrm{P}_1\middle|\tilde{\theta}=0.4\right)\cdot P\left(\tilde{\theta}=0.4\right)+ \\ P\left(\mathrm{NP}_2,\mathrm{P}_1\middle|\tilde{\theta}=0.5\right)\cdot P\left(\tilde{\theta}=0.5\right) \end{bmatrix}}=$$

$$=\frac{P\left(\mathrm{NP}_2,\mathrm{P}_1\middle|\tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{0.1\cdot 0.9\cdot 0.2+0.2\cdot 0.8\cdot 0.3+0.3\cdot 0.7\cdot 0.3+0.4\cdot 0.6\cdot 0.1+0.5^2\cdot 0.1}=\frac{P\left(\mathrm{NP}_2,\mathrm{P}_1\middle|\tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{0.178}$$

$$\Rightarrow P\left(\tilde{\theta}=0.1\middle|\mathrm{NP}_2,\mathrm{P}_1\right)=0.1\cdot 0.9\cdot 0.2/0.26=18/178\,;\; P\left(\tilde{\theta}=0.2\middle|\mathrm{NP}_2,\mathrm{P}_1\right)=48/178\,;$$

$$P\left(\tilde{\theta}=0.3\middle|\mathrm{NP}_2,\mathrm{P}_1\right)=63/178\,;\; P\left(\tilde{\theta}=0.4\middle|\mathrm{NP}_2,\mathrm{P}_1\right)=24/178\,;\; P\left(\tilde{\theta}=0.5\middle|\mathrm{NP}_2,\mathrm{P}_1\right)=25/178\,;$$



CS_1 = 0.5
P_1
NP_1
CS_2 = 0.5
CS_2 = 0.5
P_2
NP_2
P_2
NP_2
ER_{max} =14.8
ER_{max} =6.8
ER_{max} =7.8
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock

θ	0.10	0.20	0.30	0.40	0.50
$P(\tilde{\theta} = \theta \mid \mathrm{P}_2, \mathrm{P}_1)$	2/82	12/82	27/82	16/82	25/82

$$ER(\text{Stock } 100 \mid \mathrm{P}_2, \mathrm{P}_1) =$$

$$(-10)\cdot\frac{2}{82} + (-2)\cdot\frac{12}{82} + 12\cdot\frac{27}{82} + 22\cdot\frac{16}{82} + 40\cdot\frac{25}{82} \approx \underline{19.9} \text{ Max}$$

$$ER(\text{Stock } 50 \mid \mathrm{P}_2, \mathrm{P}_1) =$$

$$(-4)\cdot\frac{2}{82} + 6\cdot\frac{12}{82} + 12\cdot\frac{27}{82} + 16\cdot\frac{16}{82} + 16\cdot\frac{25}{892} \approx \underline{12.7}$$

$$ER(\text{Do not stock} \mid \mathrm{P}_2, \mathrm{P}_1) = \underline{0}$$

$$ER(\text{Stock } 100 \mid \mathrm{NP}_2, \mathrm{P}_1) =$$

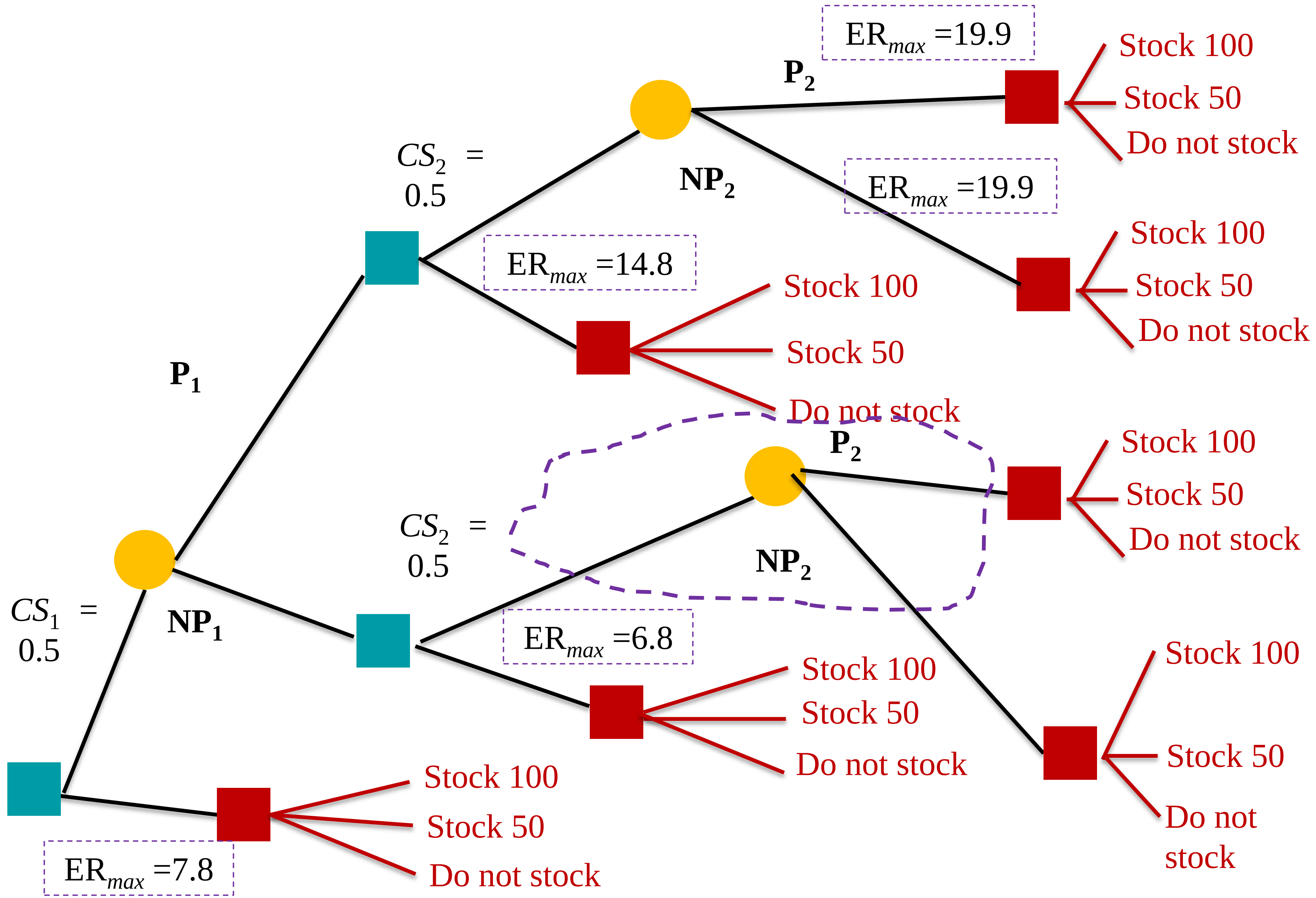
$$(-10)\cdot\frac{18}{178} + (-2)\cdot\frac{48}{178} + 12\cdot\frac{63}{178} + 22\cdot\frac{24}{178} + 40\cdot\frac{25}{178} \approx \underline{\underline{11.3}} \text{ Max}$$

$$ER(\text{Stock } 50 \mid \mathrm{NP}_2, \mathrm{P}_1) =$$

$$(-4)\cdot\frac{18}{178} + 6\cdot\frac{48}{178} + 12\cdot\frac{63}{178} + 16\cdot\frac{24}{178} + 16\cdot\frac{25}{178} \approx \underline{\underline{9.9}}$$

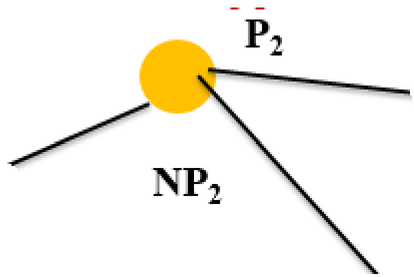
$$ER(\text{Do not stock} \mid \mathrm{NP}_2, \mathrm{P}_1) = \underline{\underline{0}}$$

θ	0.10	0.20	0.30	0.40	0.50
$P(\tilde{\theta} = \theta \mid \mathrm{NP}_2, \mathrm{P}_1)$	18/178	48/178	63/178	24/178	25/178



ER_{max} =19.9
P_2
Stock 100
Stock 50
Do not stock
CS_2 = 0.5
NP_2
ER_{max} =19.9
Stock 100
Stock 50
Do not stock
ER_{max} =14.8
Stock 100
Stock 50
Do not stock
P_1
P_2
Stock 100
Stock 50
Do not stock
CS_2 = 0.5
NP_2
CS_1 = 0.5
NP_1
ER_{max} =6.8
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
ER_{max} =7.8

Second sampled consumer, case 2



If outcome = Purchase

Posterior probabilities of θ:

$$P\left(\tilde{\theta}=\theta \middle| \mathrm{P}_2,\mathrm{NP}_1\right)=\left\langle \begin{array}{l}\text{Independent}\\ \text{samples (Bernoulli}\\ \text{trials)}\end{array}\right\rangle = P\left(\tilde{\theta}=\theta \middle| \mathrm{NP}_2,\mathrm{P}_1\right)=\frac{P\left(\mathrm{NP}_2,\mathrm{P}_1 \middle| \tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{0.178}$$

$$\Rightarrow P\left(\tilde{\theta}=0.1 \middle| \mathrm{P}_2,\mathrm{NP}_1\right)=18/178\,;\; P\left(\tilde{\theta}=0.2 \middle| \mathrm{P}_2,\mathrm{NP}_1\right)=48/178\,;$$

$$P\left(\tilde{\theta}=0.3 \middle| \mathrm{P}_2,\mathrm{NP}_1\right)=63/178\,;\; P\left(\tilde{\theta}=0.4 \middle| \mathrm{P}_2,\mathrm{NP}_1\right)=24/178\,;\; P\left(\tilde{\theta}=0.5 \middle| \mathrm{P}_2,\mathrm{NP}_1\right)=25/178\,;$$

If outcome = No purchase

Posterior probabilities of θ:

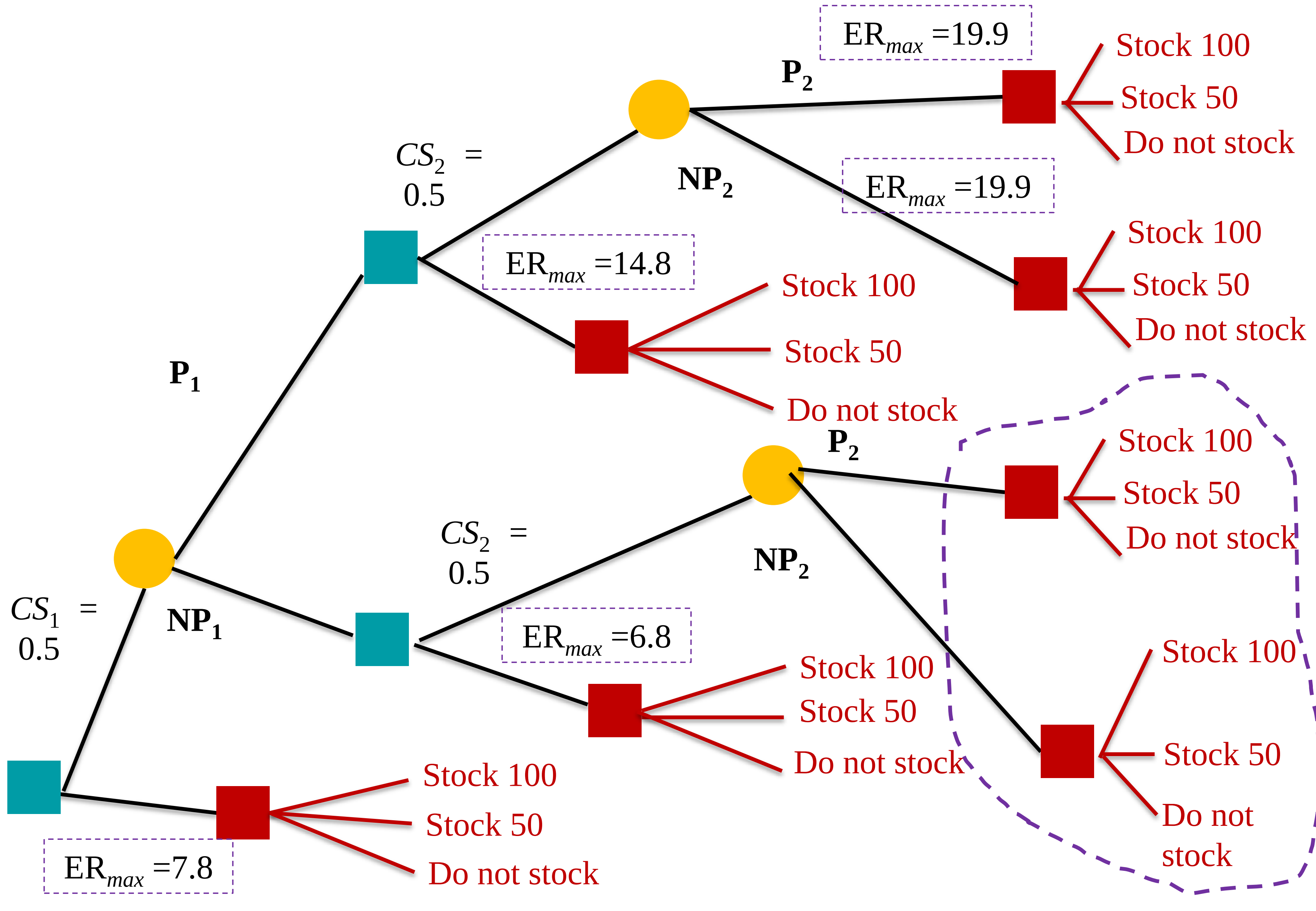
$$P\left(\tilde{\theta}=\theta \middle| \mathrm{NP}_2,\mathrm{NP}_1\right)$$

$$=\frac{P\left(\mathrm{NP}_2,\mathrm{NP}_1 \middle| \tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{\left[\begin{array}{c} P\left(\mathrm{NP}_2,\mathrm{NP}_1 \middle| \tilde{\theta}=0.1\right)\cdot P\left(\tilde{\theta}=0.1\right)+P\left(\mathrm{NP}_2,\mathrm{NP}_1 \middle| \tilde{\theta}=0.2\right)\cdot P\left(\tilde{\theta}=0.2\right)+\\ P\left(\mathrm{NP}_2,\mathrm{NP}_1 \middle| \tilde{\theta}=0.3\right)\cdot P\left(\tilde{\theta}=0.3\right)+P\left(\mathrm{NP}_2,\mathrm{NP}_1 \middle| \tilde{\theta}=0.4\right)\cdot P\left(\tilde{\theta}=0.4\right)+\\ P\left(\mathrm{NP}_2,\mathrm{NP}_1 \middle| \tilde{\theta}=0.5\right)\cdot P\left(\tilde{\theta}=0.5\right)\end{array}\right]}=$$

$$=\frac{P\left(\mathrm{NP}_2,\mathrm{NP}_1 \middle| \tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{0.9^2\cdot 0.2+0.8^2\cdot 0.3+0.7^2\cdot 0.3+0.6^2\cdot 0.1+0.5^2\cdot 0.1}$$

$$=\frac{P\left(\mathrm{NP}_2,\mathrm{NP}_1 \middle| \tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{0.562}$$

$$\Rightarrow P\left(\tilde{\theta}=0.1 \middle| \mathrm{NP}_2,\mathrm{NP}_1\right)=0.9^2\cdot 0.2/0.562=162/562\;\;;\;\; P\left(\tilde{\theta}=0.2 \middle| \mathrm{NP}_2,\mathrm{NP}_1\right)$$
$$=192/562\;\;;$$
$$P\left(\tilde{\theta}=0.3 \middle| \mathrm{NP}_2,\mathrm{NP}_1\right)=147/562\;\;;\;\; P\left(\tilde{\theta}=0.4 \middle| \mathrm{NP}_2,\mathrm{NP}_1\right)=36/562\;\;;\;\; P\left(\tilde{\theta}=0.2 \middle| \mathrm{NP}_2,\mathrm{NP}_1\right)$$
$$=25/562\;\;;$$



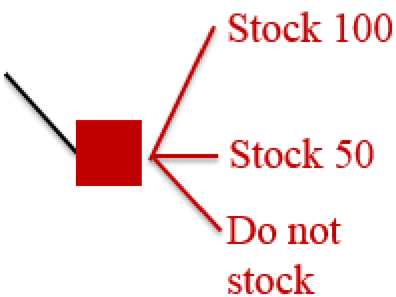
ER_{max} =19.9
P_2
Stock 100
Stock 50
Do not stock
CS_2 = 0.5
NP_2
ER_{max} =19.9
Stock 100
Stock 50
Do not stock
ER_{max} =14.8
Stock 100
Stock 50
Do not stock
P_1
P_2
Stock 100
Stock 50
Do not stock
CS_2 = 0.5
NP_2
CS_1 = 0.5
NP_1
ER_{max} =6.8
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
ER_{max} =7.8

θ	0.10	0.20	0.30	0.40	0.50
$P(\tilde{\theta} = \theta \mid \mathrm{P}_2, \mathrm{NP}_1)$	18/178	48/178	63/178	24/178	25/178

$$ER(\text{Stock } 100 \mid \mathrm{P}_2, \mathrm{NP}_1) = ER(\text{Stock } 100 \mid \mathrm{NP}_2, \mathrm{P}_1) \approx \underline{\underline{11.3}}$$ Max

$$ER(\text{Stock } 50 \mid \mathrm{P}_2, \mathrm{NP}_1) = ER(\text{Stock } 50 \mid \mathrm{NP}_2, \mathrm{P}_1) \approx \underline{\underline{9.9}}$$

$$ER(\text{Do not stock} \mid \mathrm{P}_2, \mathrm{NP}_1) = \underline{\underline{0}}$$



$$ER(\text{Stock } 100 \mid \mathrm{NP}_2, \mathrm{NP}_1) =$$

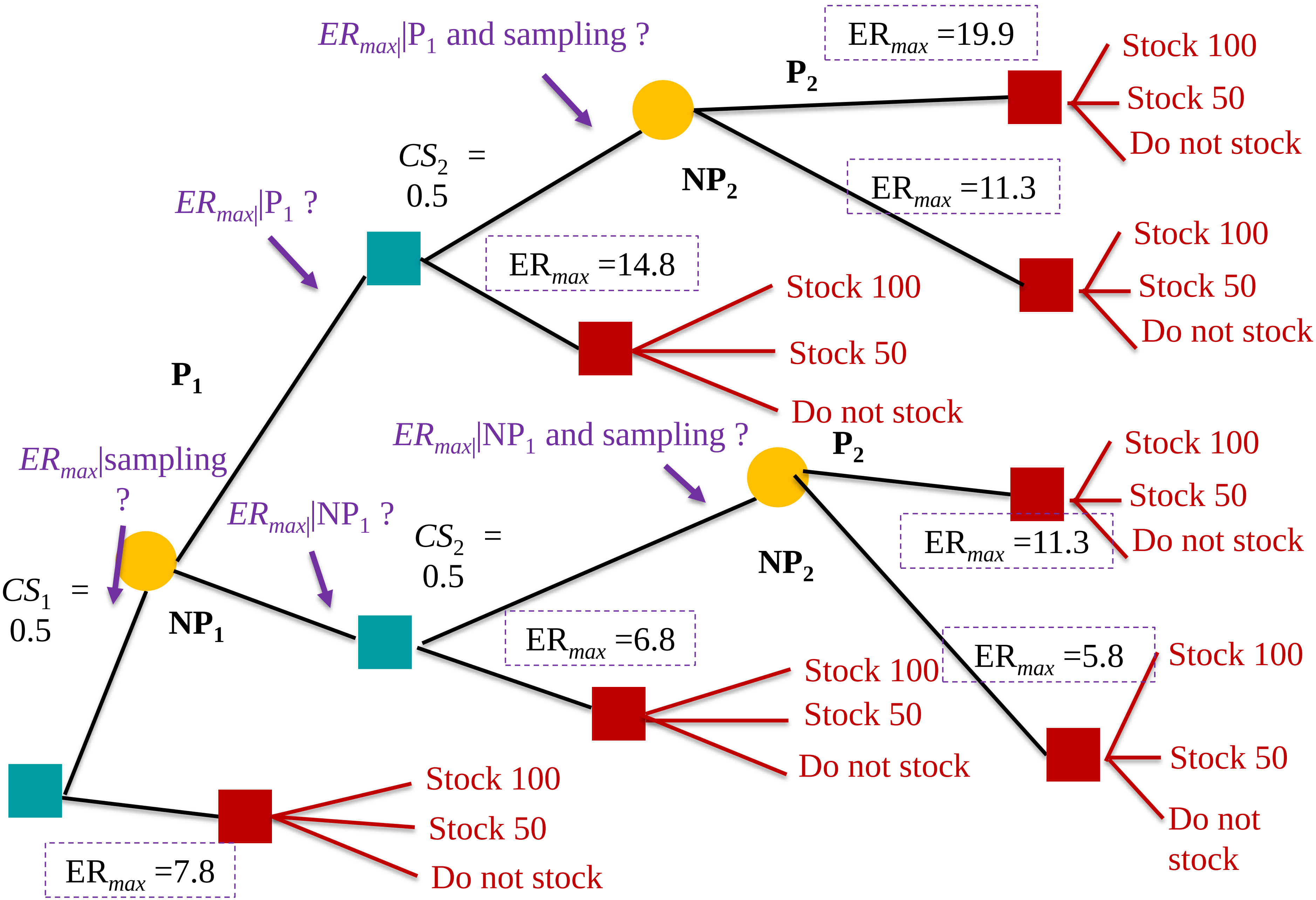
$$(-10)\cdot\frac{162}{562} + (-2)\cdot\frac{192}{562} + 12\cdot\frac{147}{562} + 22\cdot\frac{36}{562} + 40\cdot\frac{25}{562} \approx \underline{\underline{2.8}}$$

$$ER(\text{Stock } 50 \mid \mathrm{NP}_2, \mathrm{NP}_1) =$$

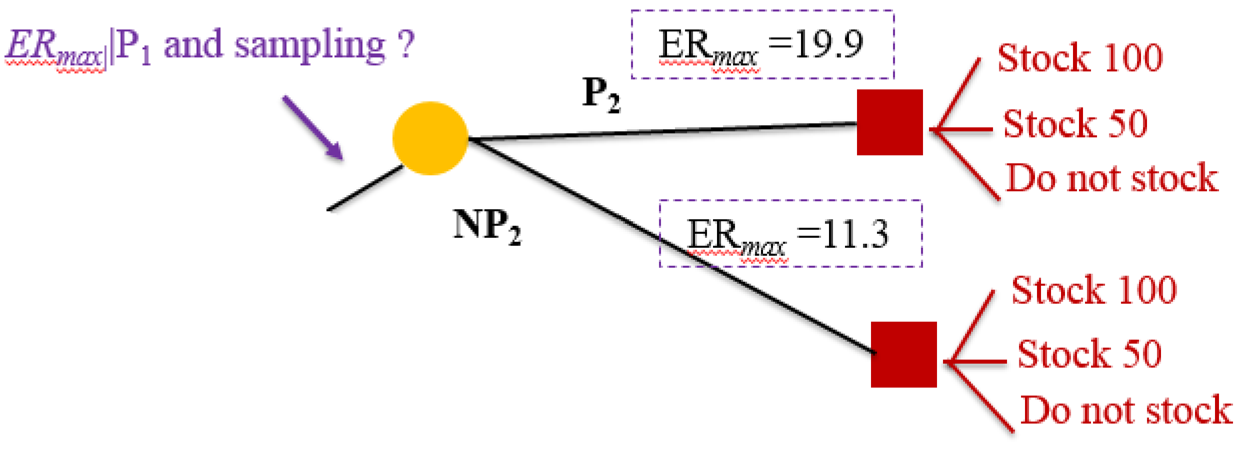
$$(-4)\cdot\frac{162}{562} + 6\cdot\frac{192}{562} + 12\cdot\frac{147}{562} + 16\cdot\frac{36}{562} + 16\cdot\frac{25}{562} \approx \underline{\underline{5.8}}$$ Max

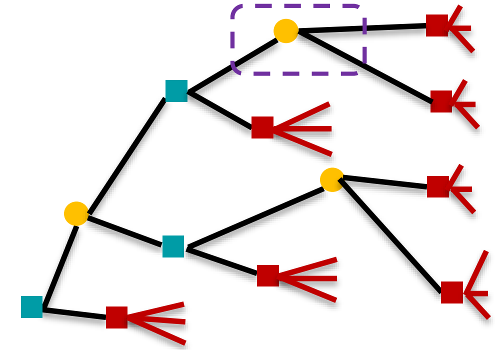
$$ER(\text{Do not stock} \mid \mathrm{NP}_2, \mathrm{NP}_1) = \underline{\underline{0}}$$

θ	0.10	0.20	0.30	0.40	0.50
$P(\tilde{\theta} = \theta \mid \mathrm{NP}_2, \mathrm{NP}_1)$	162/562	192/562	147/562	36/562	25/562



$ER_{max}|P_1$ and sampling ?
ER_{max} =19.9
P_2
Stock 100
Stock 50
Do not stock
CS_2 = 0.5
NP_2
ER_{max} =11.3
$ER_{max}|P_1$?
Stock 100
Stock 50
Do not stock
ER_{max} =14.8
Stock 100
Stock 50
Do not stock
P_1
$ER_{max}|NP_1$ and sampling ?
P_2
Stock 100
Stock 50
Do not stock
$ER_{max}|$sampling ?
$ER_{max}|NP_1$?
CS_2 = 0.5
ER_{max} =11.3
NP_2
CS_1 = 0.5
NP_1
ER_{max} =6.8
ER_{max} =5.8
Stock 100
Stock 100
Stock 50
Do not stock
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
ER_{max} =7.8

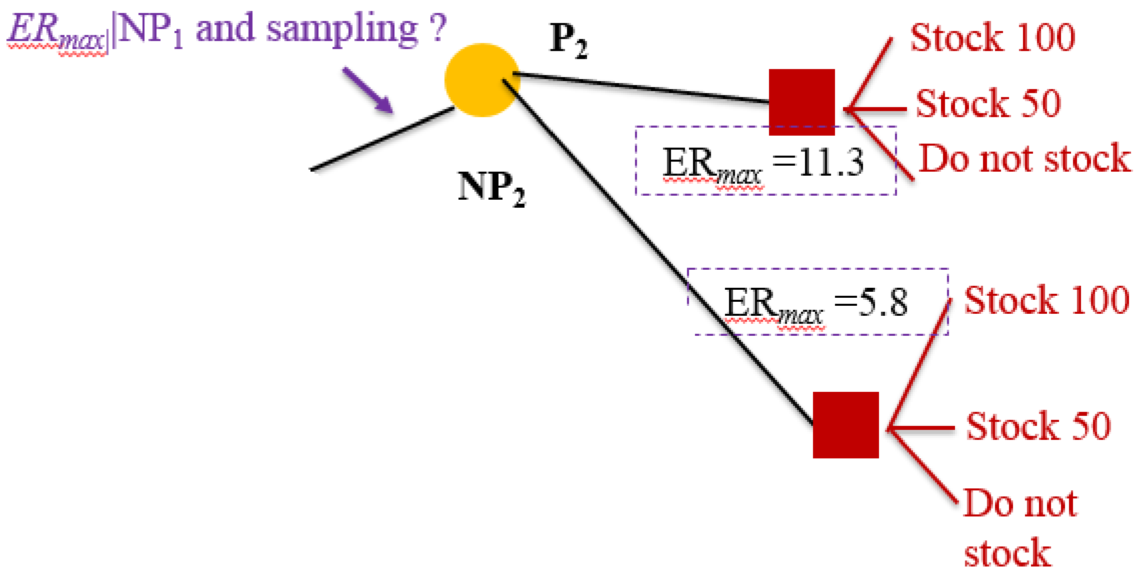


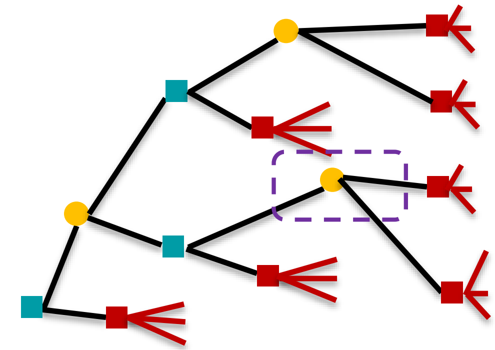


$$ER_{\max}\big|\mathrm{P}_1 \ \text{ and sampling} = 19.9\cdot P(\mathrm{P}_2|\mathrm{P}_1)+11.3\cdot P(\mathrm{NP}_2|\mathrm{P}_1) = \left\langle \begin{array}{l}\text{Sampling in stage 2}\\ \text{is assumed to be}\\ \text{independent of}\\ \text{sampling in stage 1}\end{array}\right\rangle =$$

$$= 19.9\cdot P(\mathrm{P}_2)+11.3\cdot P(\mathrm{NP}_2) = 19.9\cdot \sum_{\theta} P\left(\mathrm{P}_2\middle|\tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)+$$

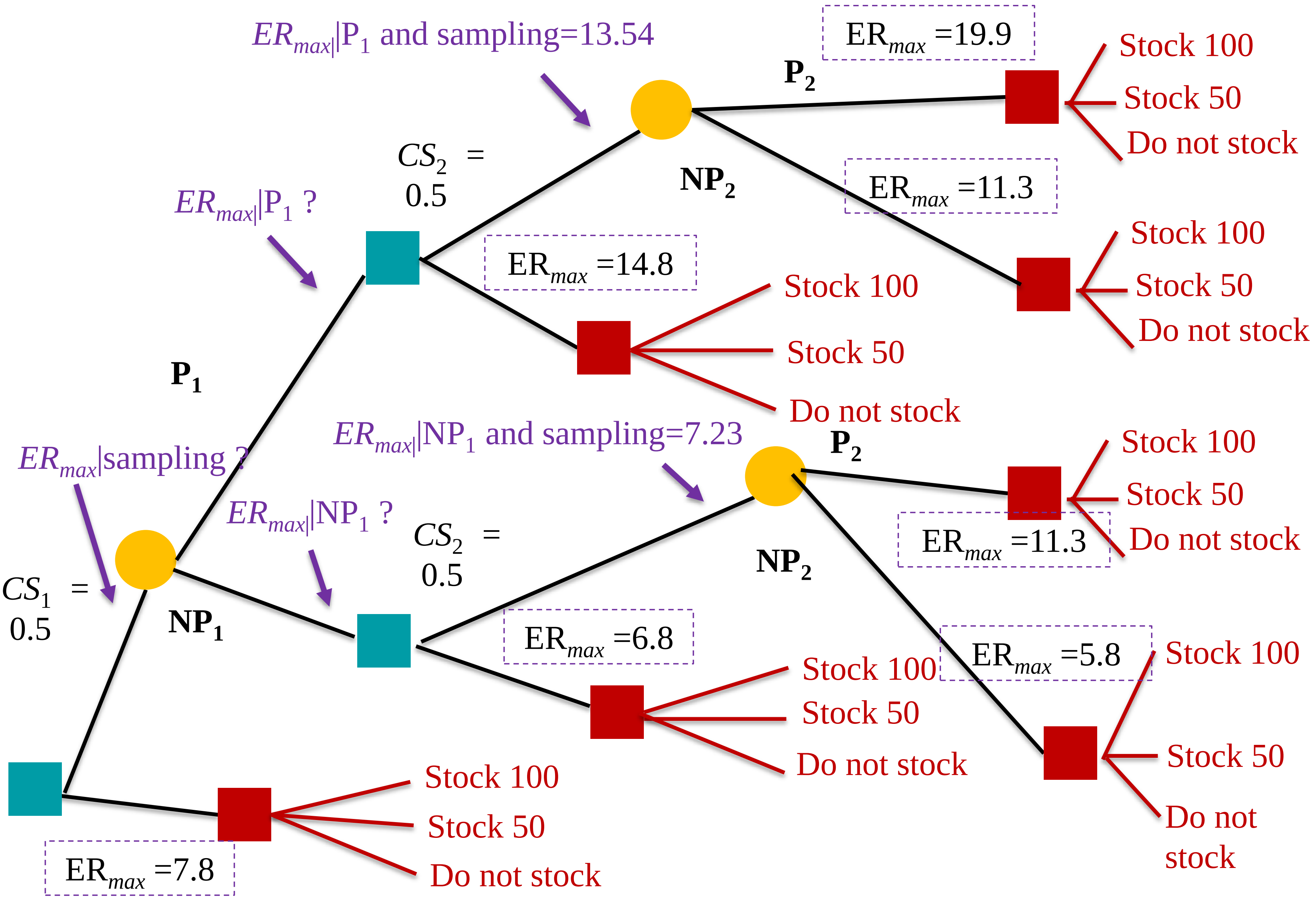
$$+11.3\cdot \sum_{\theta} P\left(\mathrm{NP}_2\middle|\tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right) = 19.9\cdot 0.26+11.3\cdot 0.74 \approx 13.54$$

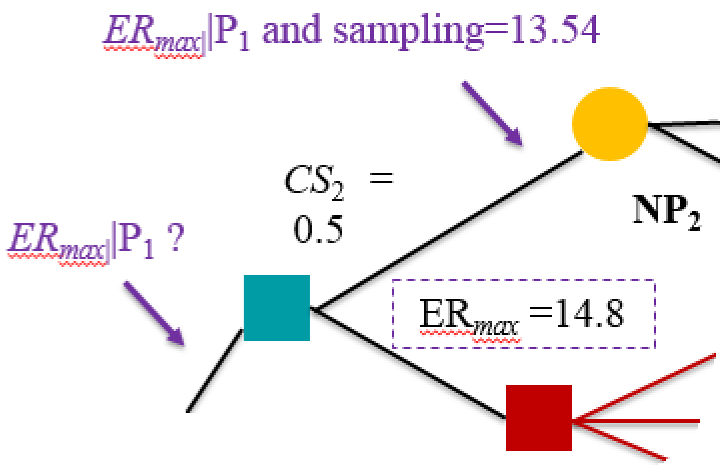


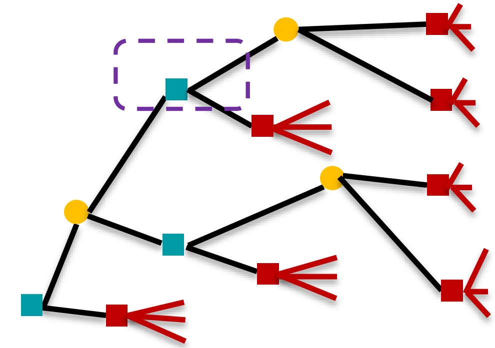


$$ER_{\max}\left|\mathrm{NP}_1 \text{ and sampling}\right. = 11.3 \cdot P\left(\mathrm{P}_2 \middle| \mathrm{NP}_1\right) + 5.8 \cdot P\left(\mathrm{NP}_2 \middle| \mathrm{NP}_1\right) =$$

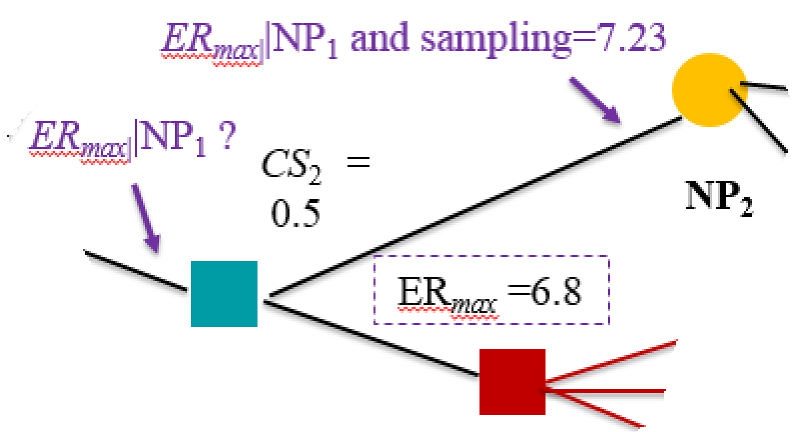
$$= 11.3 \cdot P\left(\mathrm{P}_2\right) + 5.8 \cdot P\left(\mathrm{NP}_2\right) = 11.3 \cdot 0.26 + 5.8 \cdot 0.74 \approx 7.23$$

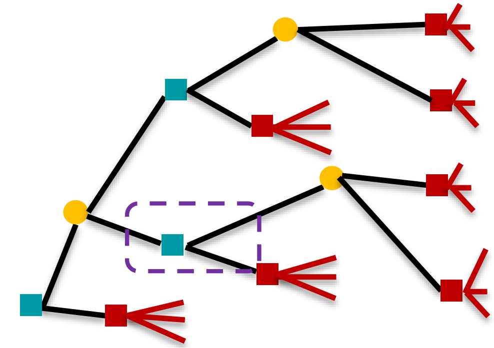

$ER_{max|}|P_1$ and sampling=13.54
ER_{max} =19.9
Stock 100
Stock 50
Do not stock
P_2
CS_2 = 0.5
NP_2
ER_{max} =11.3
$ER_{max|}|P_1$?
ER_{max} =14.8
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
P_1
$ER_{max|}|NP_1$ and sampling=7.23
P_2
Stock 100
Stock 50
Do not stock
ER_{max} =11.3
$ER_{max}|$sampling ?
$ER_{max|}|NP_1$?
CS_2 = 0.5
NP_2
CS_1 = 0.5
NP_1
ER_{max} =6.8
ER_{max} =5.8
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
ER_{max} =7.8



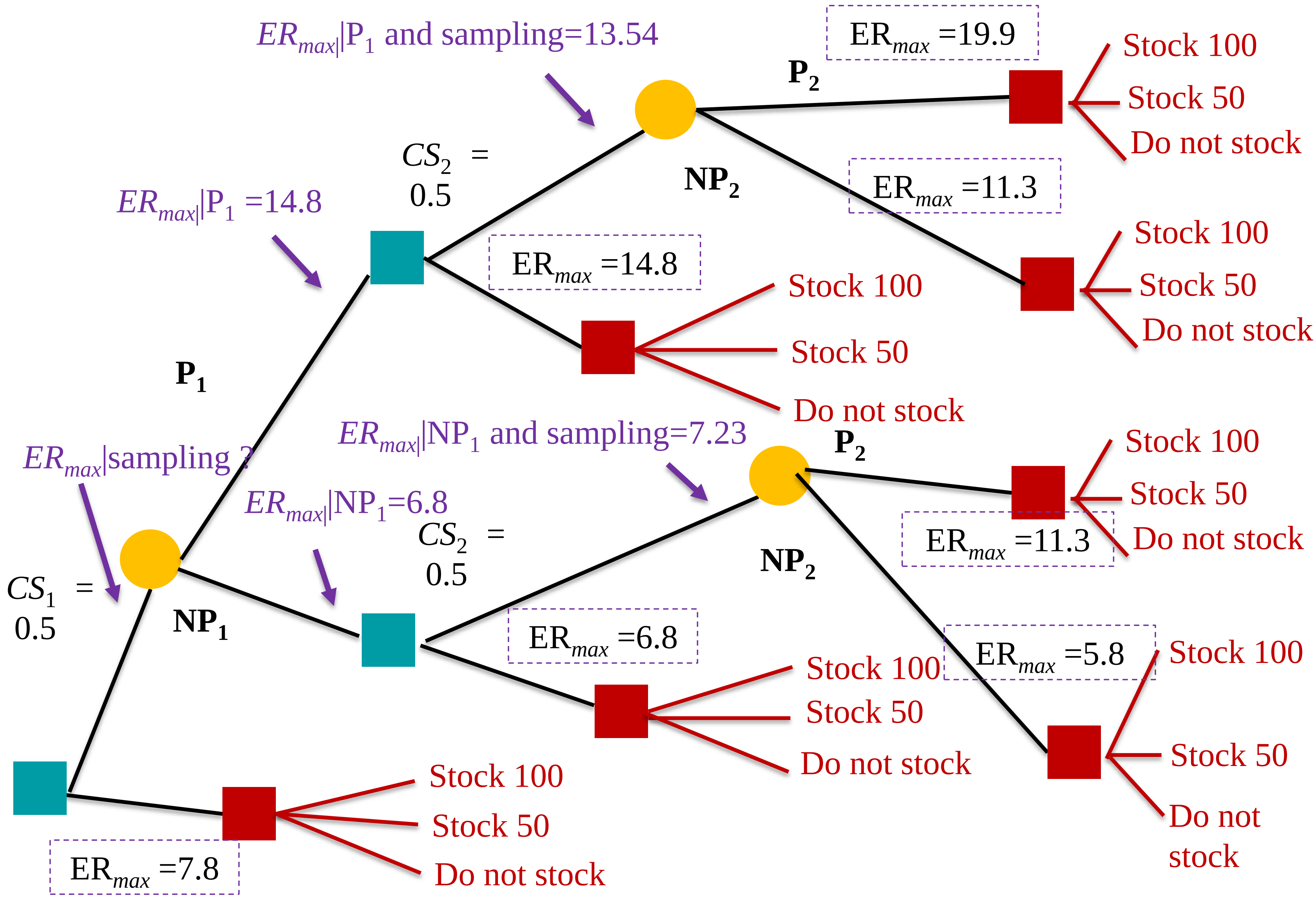


$$ER_{\max}\,\middle|\,P_1 = \max(13.54 - 0.5, 14.8) = 14.8$$

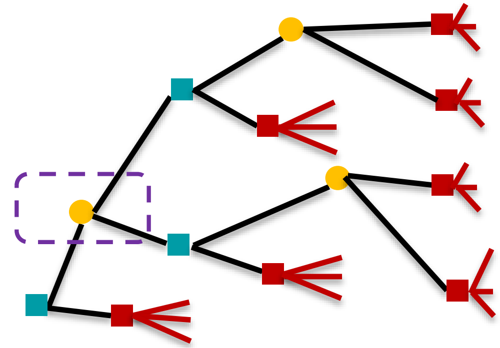


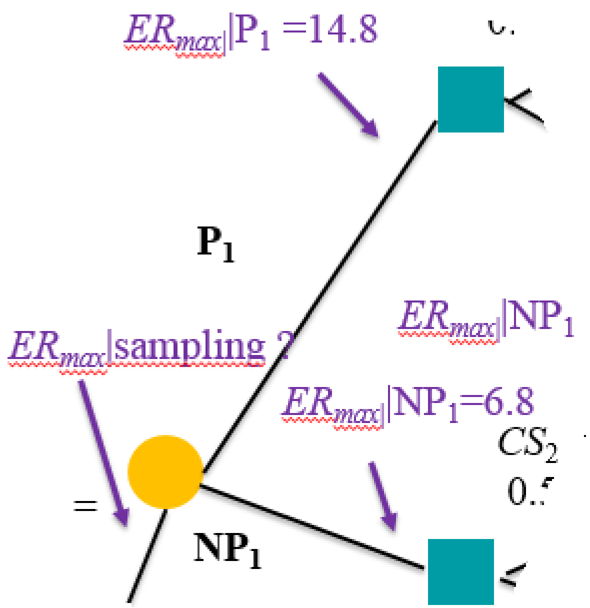


$$ER_{\max}\,\middle|\,NP_1 = \max(7.23 - 0.5, 6.8) = 6.8$$



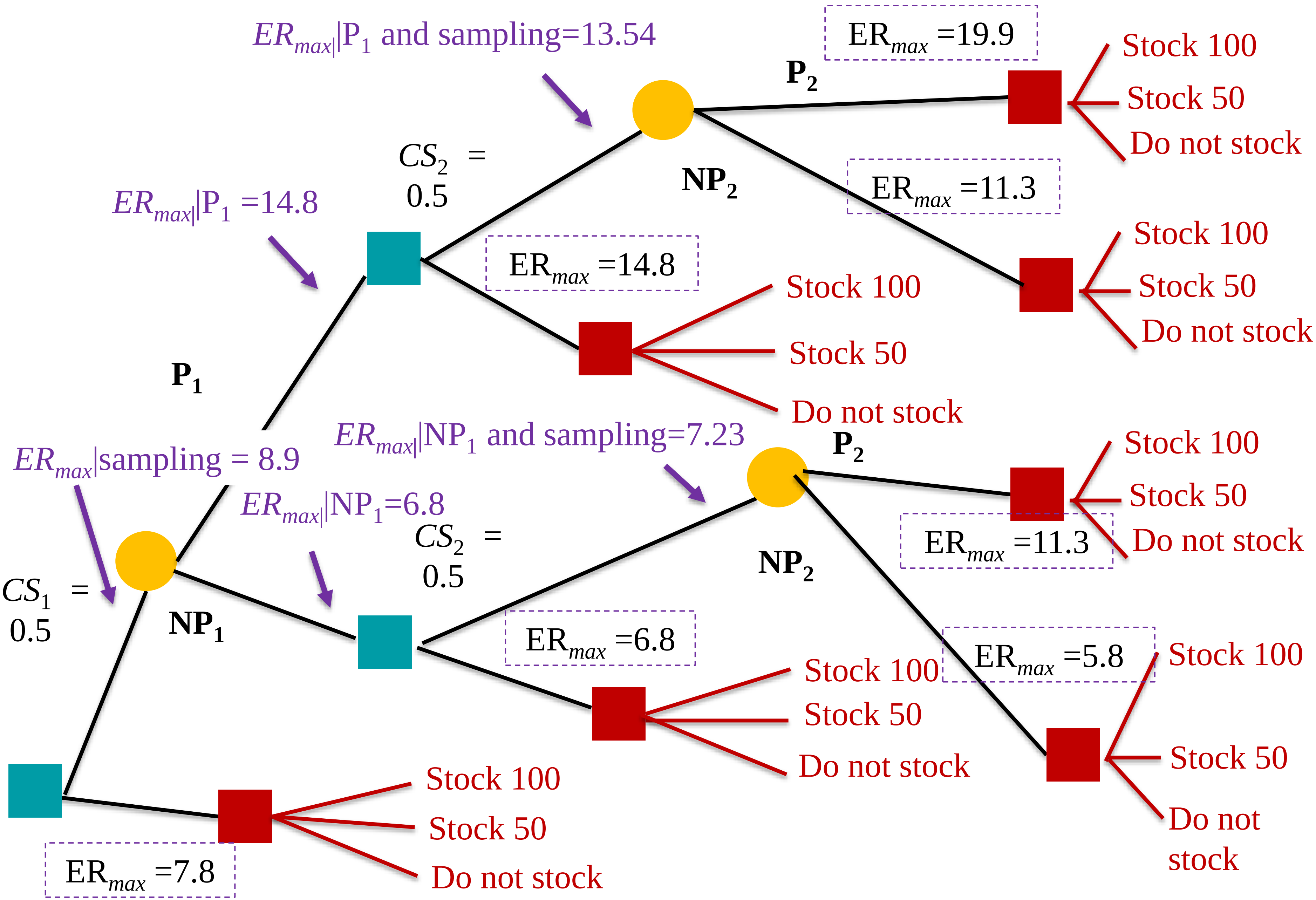
$ER_{max|}|P_1$ and sampling=13.54
ER_{max} =19.9
Stock 100
Stock 50
Do not stock
P_2
CS_2 = 0.5
NP_2
ER_{max} =11.3
$ER_{max|}|P_1$ =14.8
Stock 100
Stock 50
Do not stock
ER_{max} =14.8
Stock 100
Stock 50
Do not stock
P_1
$ER_{max|}|NP_1$ and sampling=7.23
P_2
Stock 100
Stock 50
Do not stock
$ER_{max}|$sampling ?
$ER_{max|}|NP_1$=6.8
CS_2 = 0.5
NP_2
ER_{max} =11.3
CS_1 = 0.5
NP_1
ER_{max} =6.8
ER_{max} =5.8
Stock 100
Stock 100
Stock 50
Do not stock
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
ER_{max} =7.8



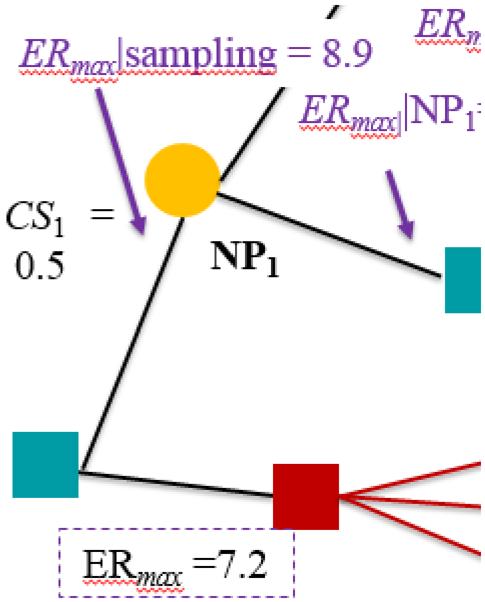


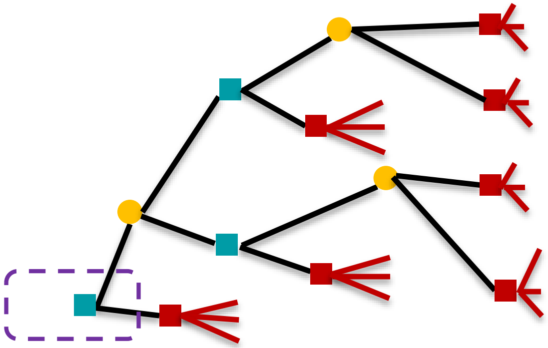
$$ER_{\max}\left|\text{sampling} = 14.8 \cdot P(\text{P}_1) + 6.8 \cdot P(\text{NP}_1) =\right.$$

$$= 14.8 \cdot 0.26 + 6.8 \cdot 0.74 \approx 8.9$$



$ER_{max|}|P_1$ and sampling=13.54
ER_{max} =19.9
P_2
Stock 100
Stock 50
Do not stock
CS_2 = 0.5
NP_2
ER_{max} =11.3
$ER_{max|}|P_1$ =14.8
ER_{max} =14.8
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
P_1
$ER_{max|}|NP_1$ and sampling=7.23
P_2
$ER_{max}|$sampling = 8.9
Stock 100
Stock 50
Do not stock
$ER_{max|}|NP_1$=6.8
CS_2 = 0.5
ER_{max} =11.3
NP_2
CS_1 = 0.5
NP_1
ER_{max} =6.8
ER_{max} =5.8
Stock 100
Stock 100
Stock 50
Do not stock
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
ER_{max} =7.8





$$ER_{\max} \mid \text{optimal} = ER_{\max} \mid \text{sampling} - CS_1 = 8.9 - 0.5 = 8.4$$

$$\Rightarrow \text{ENGS} = 8.4 - 7.8 = 0.6$$

Single-stage plan (from Exercise 6.17): $\text{ENGS}(2) = 0.09$