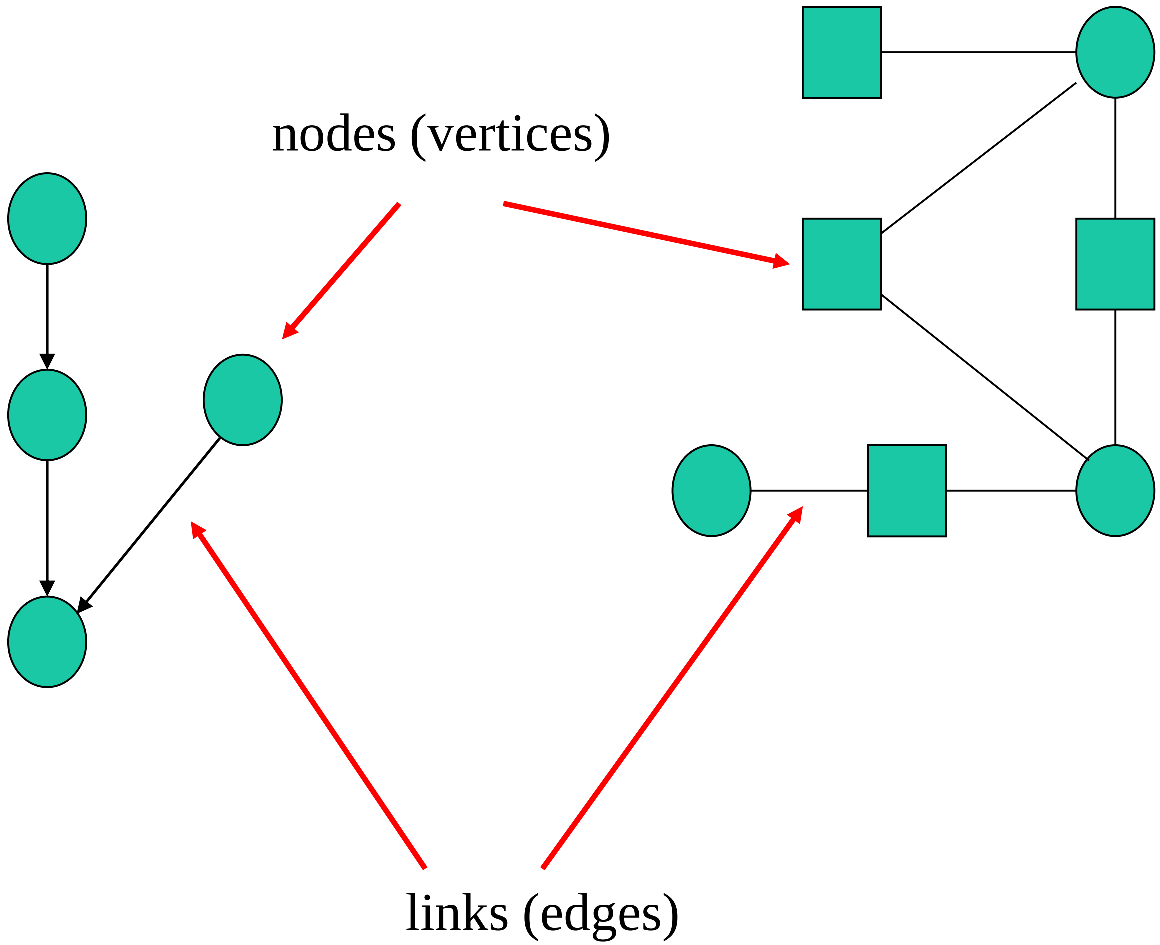
Meeting 13:
Graphical models

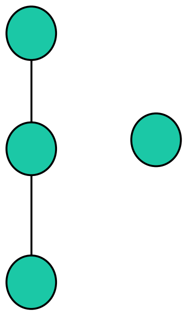
Bayesian networks

Some general graphical model concepts:

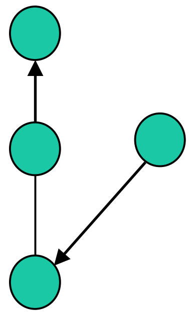


A graph can be

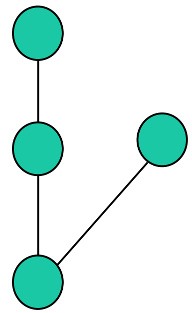
disconnected:



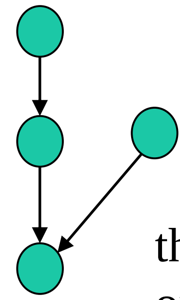
or *connected:*



; *undirected:*

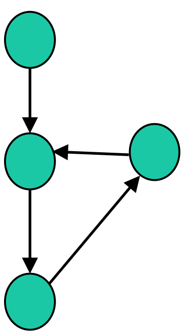


or *directed:*



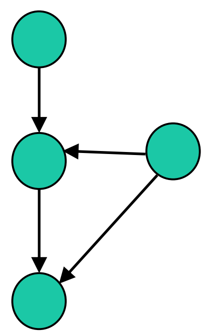
the edges are one-directed arrows

cyclic:



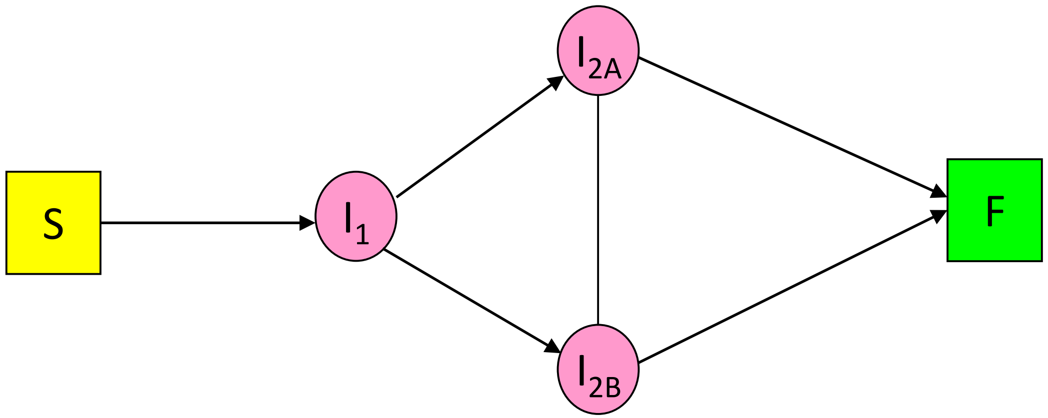
possible to start in one node and "come back"

or *acyclic:*



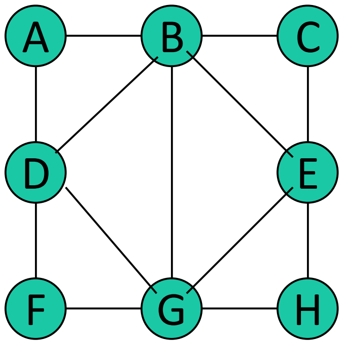
Examples:

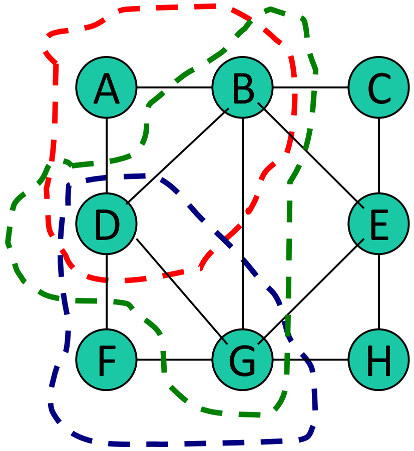
Transport routes:

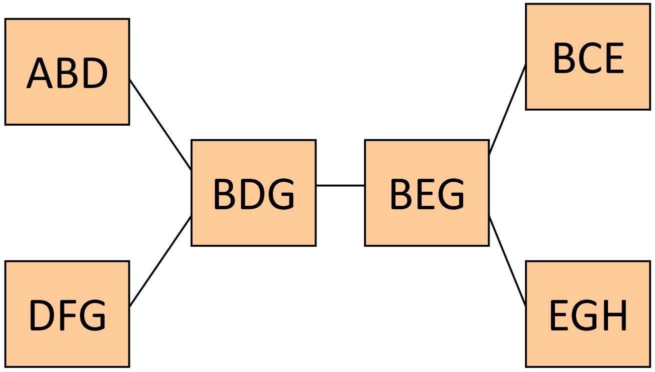


Acyclic, but not completely directed

Junction trees:







From 8 nodes to 6 nodes (Source: *Wikipedia)*

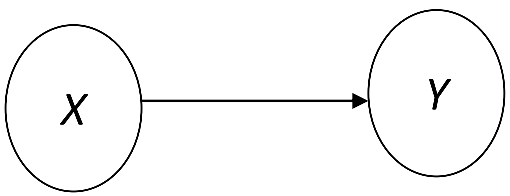
Bayesian (belief) networks

A *Bayesian network* is a connected directed acyclic graph (DAG) in which

• the nodes (vertices) represent *random variables*

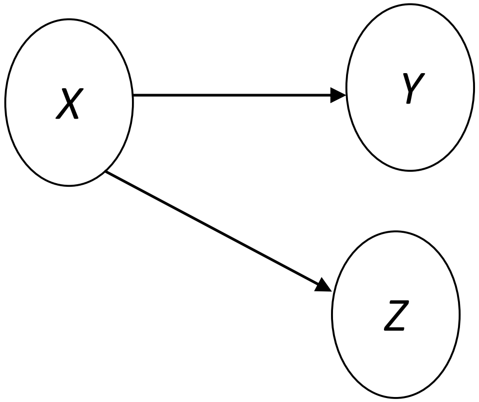
• the links (edges, arcs) represent direct *relevance* relationships among variables

Examples:



This small network has two nodes representing the random variable X and Y.

The directed link gives a relevance relationship between the two variables that means $\Pr(Y = y \mid X = x, I) \neq \Pr(Y = y \mid I)$



This network has three nodes representing the random variables X, Y and Z.

The directed links give relevance relationships that means

$$\Pr(Y = y \mid X = x, I) \neq \Pr(Y = y \mid I)$$

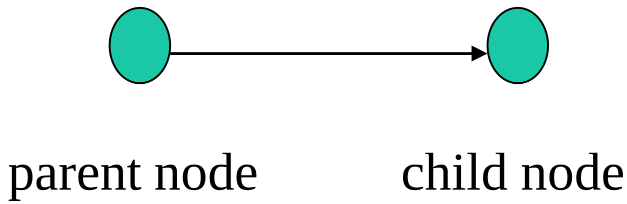
$$\Pr(Z = z \mid X = x, I) \neq \Pr(Z = z \mid I)$$

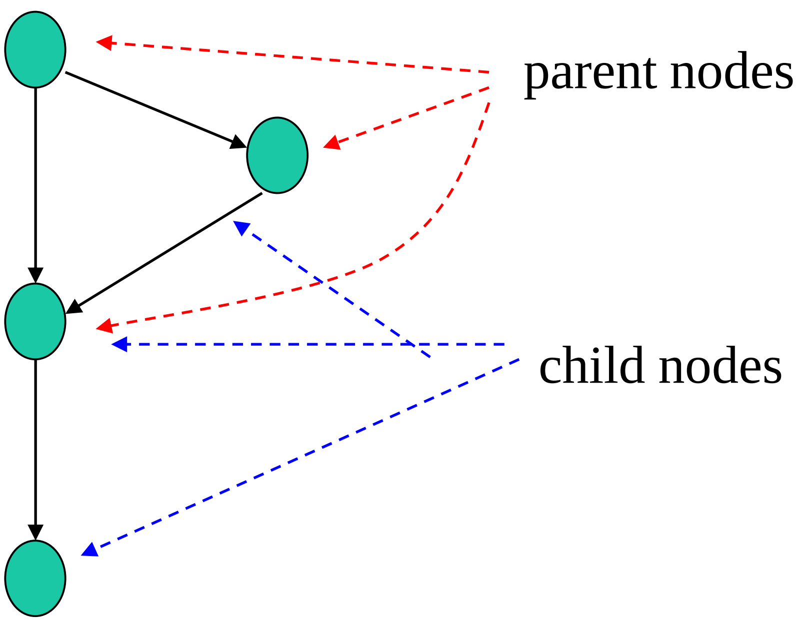
but also (as will be seen below)

$$\Pr(Z = z \mid Y = y, X = x, I) = \Pr(Z = z \mid X = x, I)$$

Structures in a Bayesian network

There are two classifications for nodes: *parent nodes* and *child nodes*





Thus, a node can be solely a parent node, solely a child node *or* both!

Probability “tables”

Each node represents a random variable.

This random variable has *either* assigned probabilities (nominal scale or discrete) or an assigned probability density function (continuous scale) for its states.

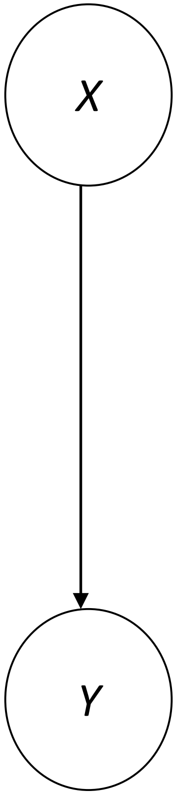
For a node that is *solely* a parent node:

The assigned probabilities or density function are conditional on background information only (may be expressed as unconditional)

For a node that is a child node (solely or joint parent/child):

The assigned probabilities or density function are conditional on the states of its parent nodes (and on background information).

Example:



X has the states x_1 and x_2

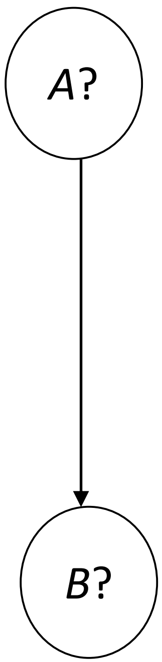
Y has the states y_1 and y_2

Probability tables

X	*Probabilities*
x_1	$\Pr(X = x_1 \mid I)$
x_2	$\Pr(X = x_2 \mid I)$

		Probabilities	
X:		x_1	x_2
Y:	y_1	$\Pr(Y = y_1 \mid X = x_1, I)$	$\Pr(Y = y_1 \mid X = x_2, I)$
	y_2	$\Pr(Y = y_2 \mid X = x_1, I)$	$\Pr(Y = y_2 \mid X = x_2, I)$

Example Dyes on banknotes (from previous lectures)



Two states:

A: "Dye is present"

$\overline{A}$: "Dye is absent"

$A?$	*Probabilities*
A	0.001
$\overline{A}$	0.999

Two states:

B: "Result is positive"

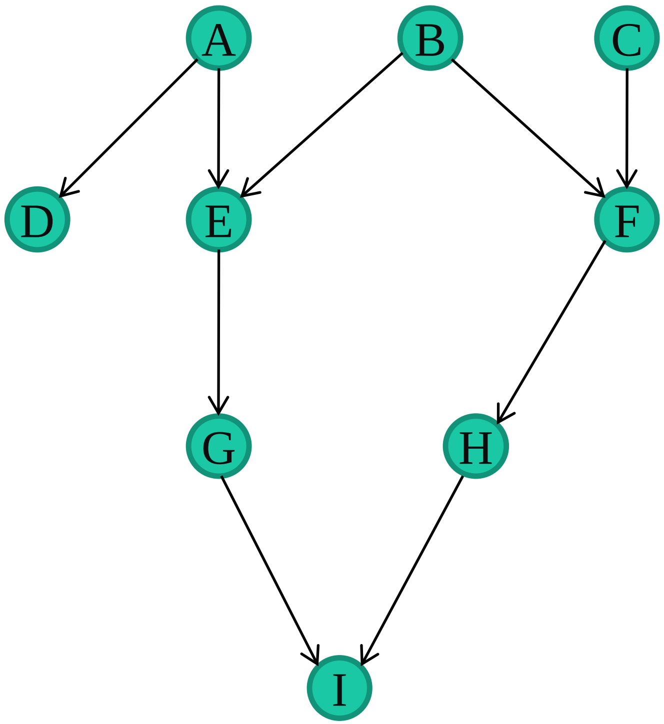
$\overline{B}$: "Result is negative"

		Probabilities	
$A?$:		A	$\overline{A}$
$B?$:	B	0.99	0.02
	$\overline{B}$	0.01	0.98

More about the structure…

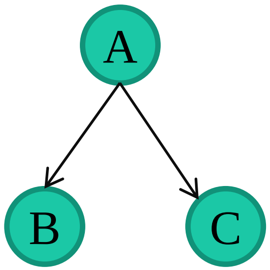
Ancestors and descendants:

A node X is an *ancestor* of a node Y and Y is in turn a *descendant* of X <u>if</u> there is a <u>unidirectional</u> path from X to Y

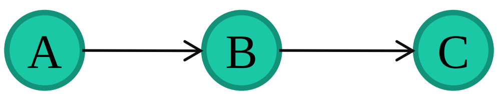


Ancestor	*Descendants*
A	D, E, G, I
B	E, F, G, H, I
C	F, H, I
E	G, I
F	H, I
G	I
H	I

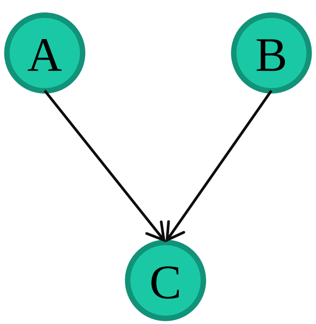
Different connections:



diverging connection



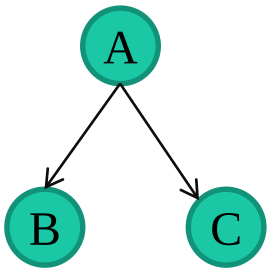
serial connection



converging connection

Conditional independence and *d*-separation

1) Diverging connection



There is a path between B and C even if it not unidirectional

➔ B may be relevant for C (and vice versa)

However, if the state of A is known this relevance is lost.: The path is *blocked*

➔ B and C are *conditionally independent* given A

Example

Let A be a random node with states

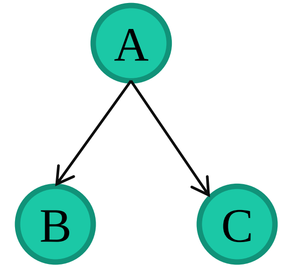
A_1 = "Willie is a cat" 
A_2 = "Willie is a parrot 

Let B be a random node with states

B_1 = "Willie has four legs"
B_2 = "Willie has two legs"

Let C be a random node with states

C_1 = "Willie has a beak"
C_2 = "Willie has no beak"



Given B being equal to B_1 the conditional probability of C_1 is different (lower) than the conditional probability of C_1 given B is equal to B_2.

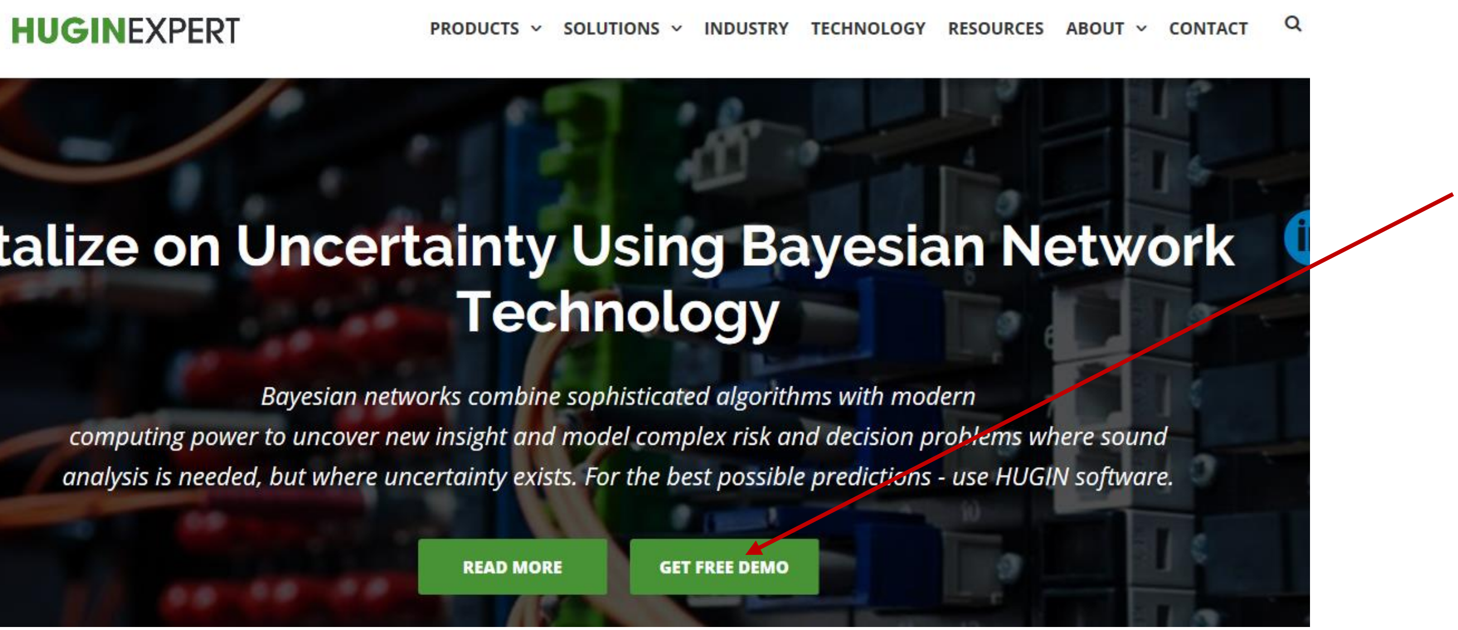
Hence, B is relevant for C and vice versa.

However, if A is instantiated to A_1, i.e. Willie is a cat, B and C are no longer relevant for each other if we reasonably assume that the number of legs a cat has cannot affect whether he has a beak or not.

Software

- Hugin (several types of commercial licenses available), Hugin Lite as demo version free of charge
- GeNIe (https://dslpitt.org/dsl/genie_smile.html) used to be easy download freeware, but today it is more complicated
- Agena Risk (https://www.agenarisk.com/) , trial version can be downloaded, otherwise commercial license needed
- *…several other*

Hugin (www.hugin.com)



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The HUGIN Decision Engine in HUGIN Lite is available with interfaces for four different programming environments: C, C++, .NET, Java, and as an ActiveX-Server for Visual Basic.

It is prohibited to use the free HUGIN Lite for any other purpose than the demonstration of capabilities and proof of concept.

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- Key: 2384

Information to be used during installation

Download URLs:

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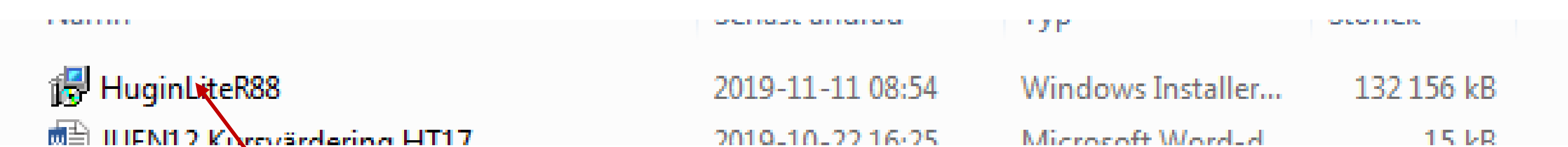
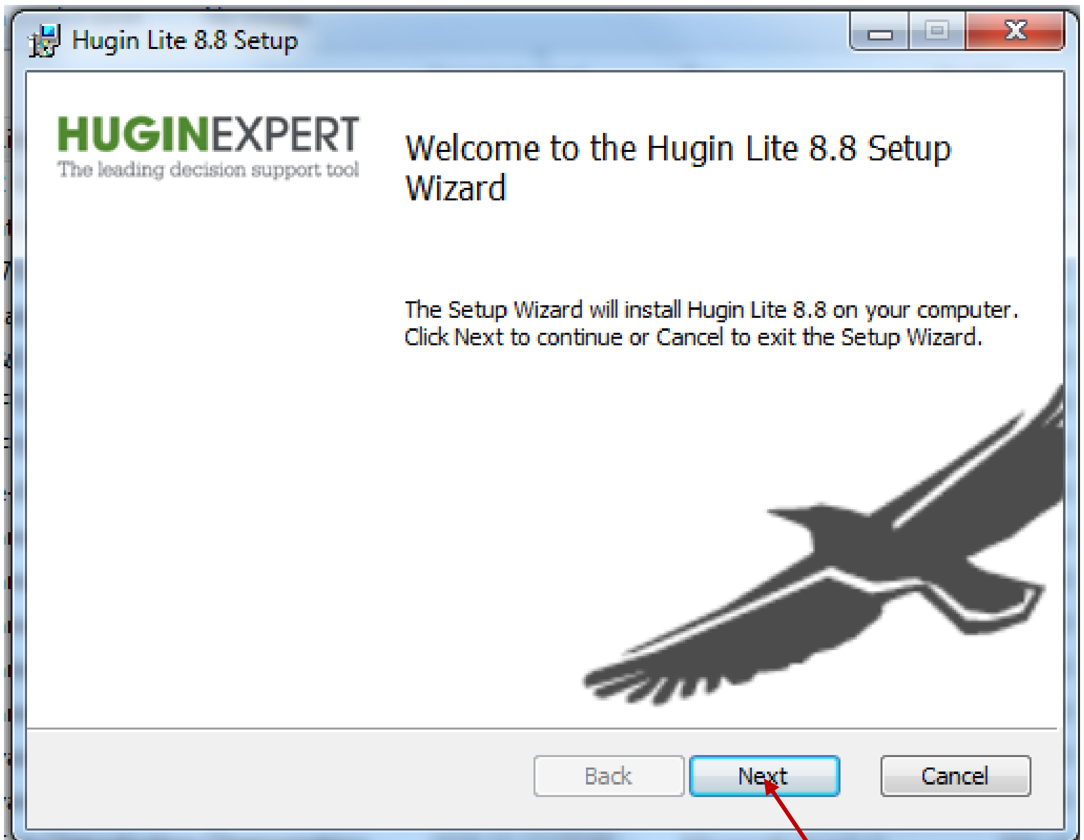

HuginLiteR88
2019-11-11 08:54
Windows Installer...
132 156 kB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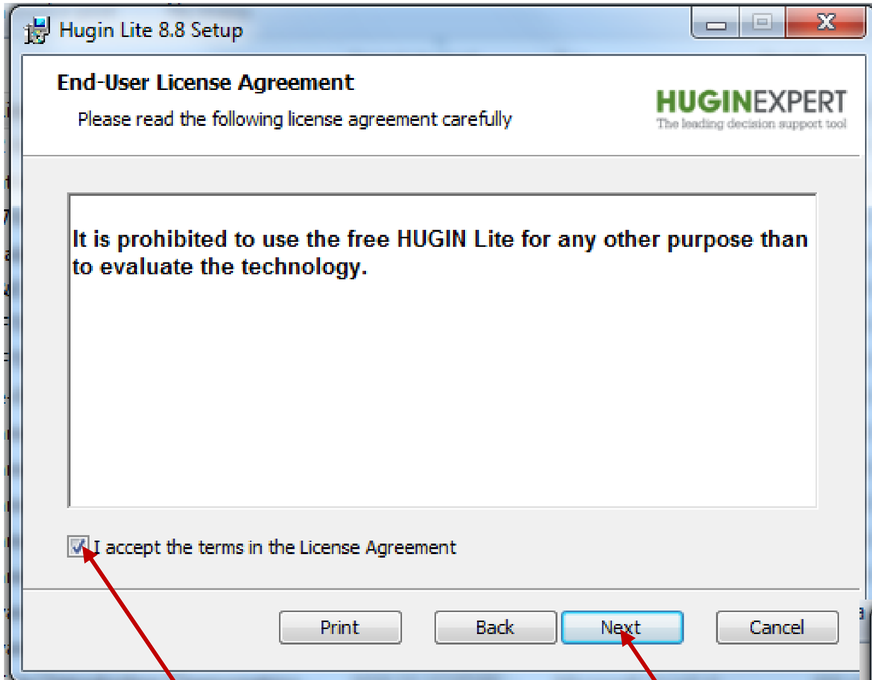
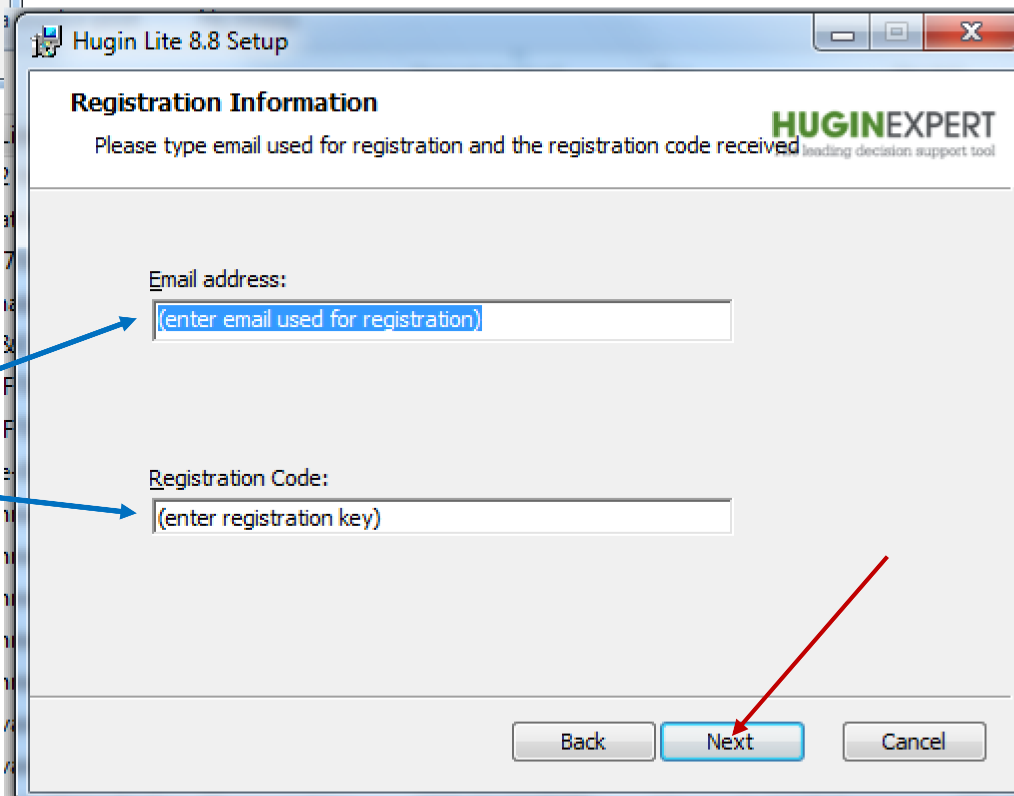
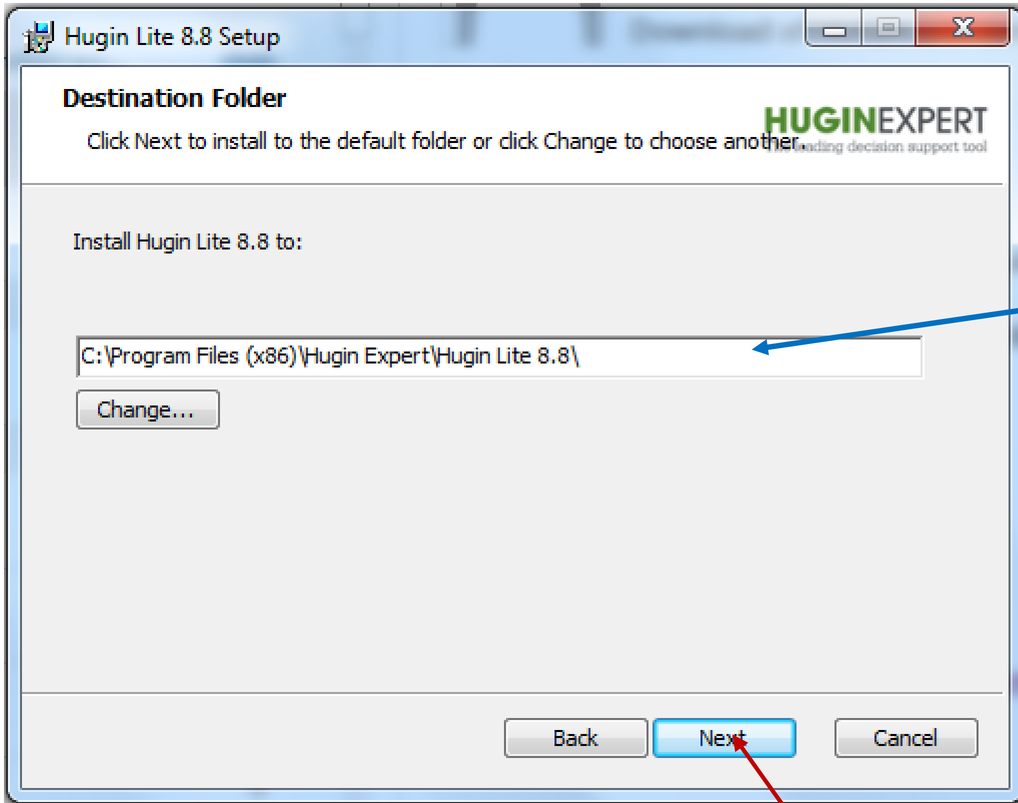
The leading decision support tool
Welcome to the Hugin Lite 8.8 Setup Wizard
The Setup Wizard will install Hugin Lite 8.8 on your computer. Click Next to continue or Cancel to exit the Setup Wizard.
Back
Next
Cancel

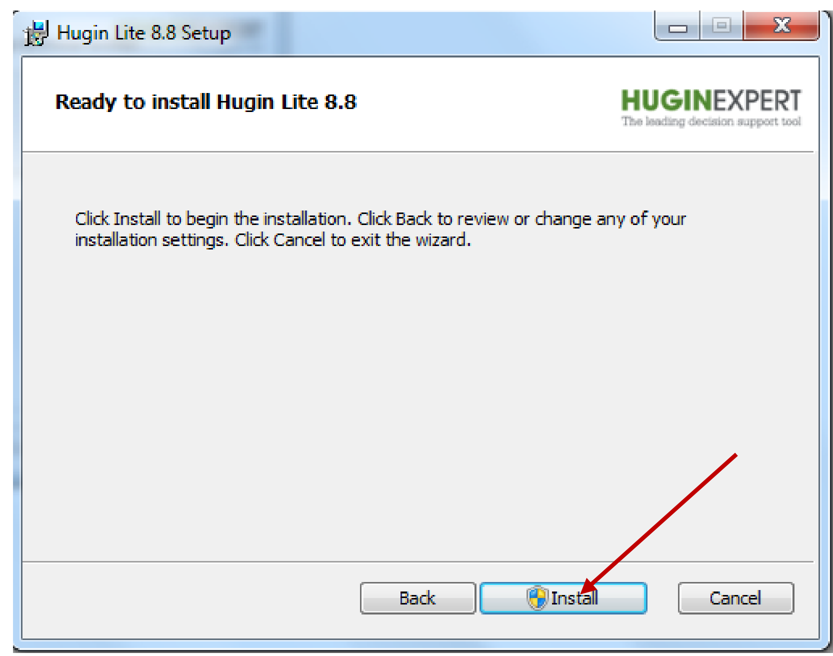


Enter information from your email from Hugin Expert

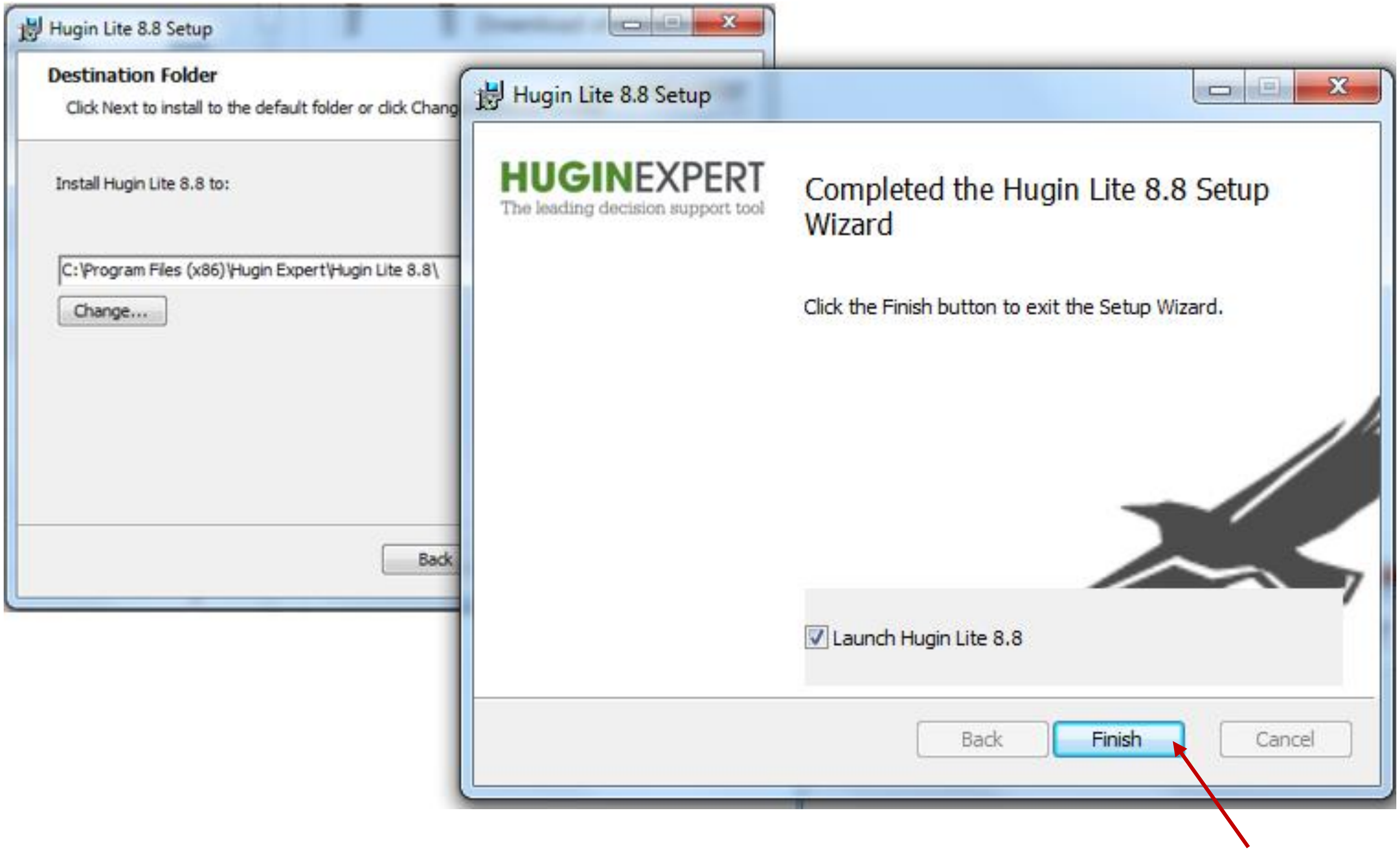




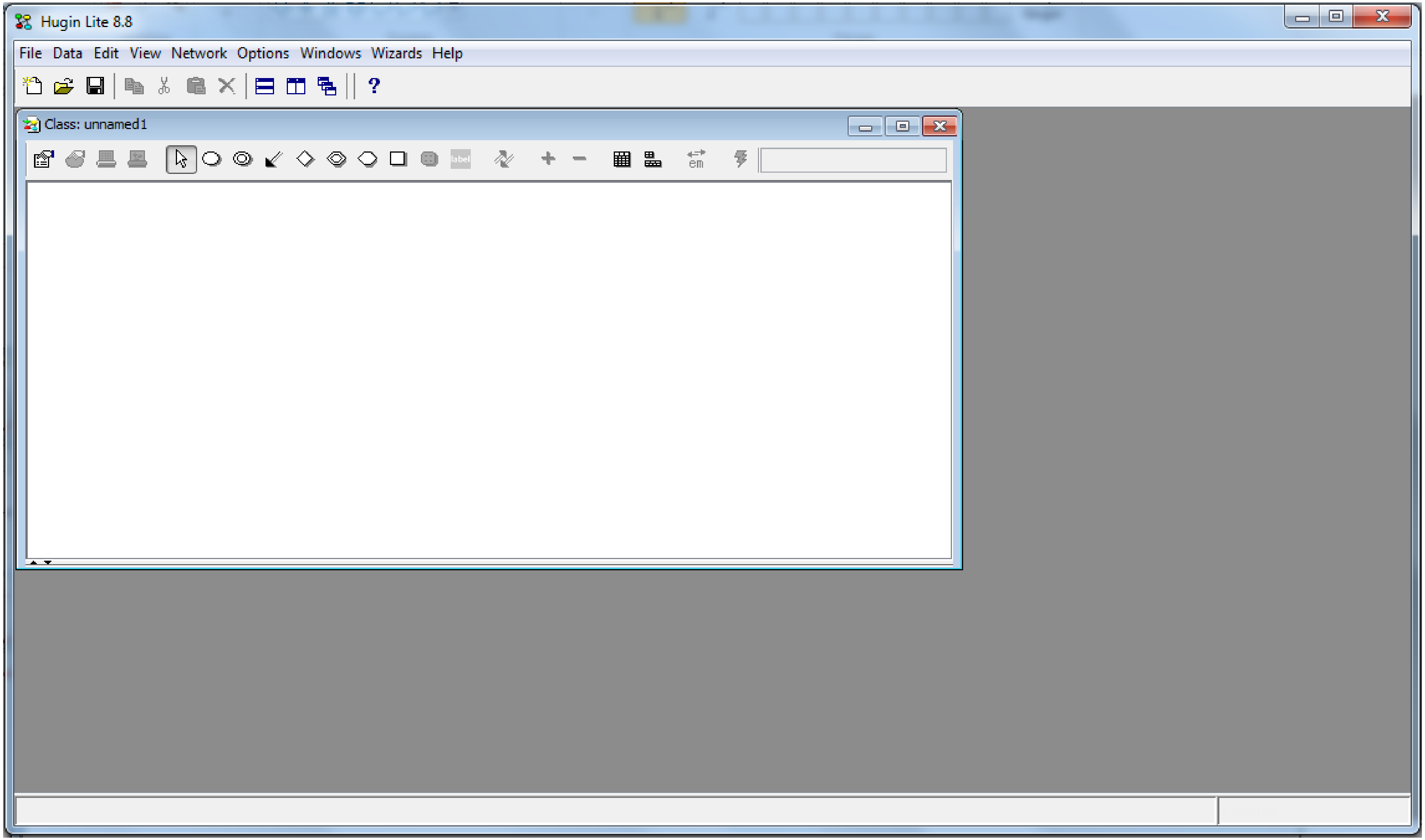
Use this or your own preferred folder



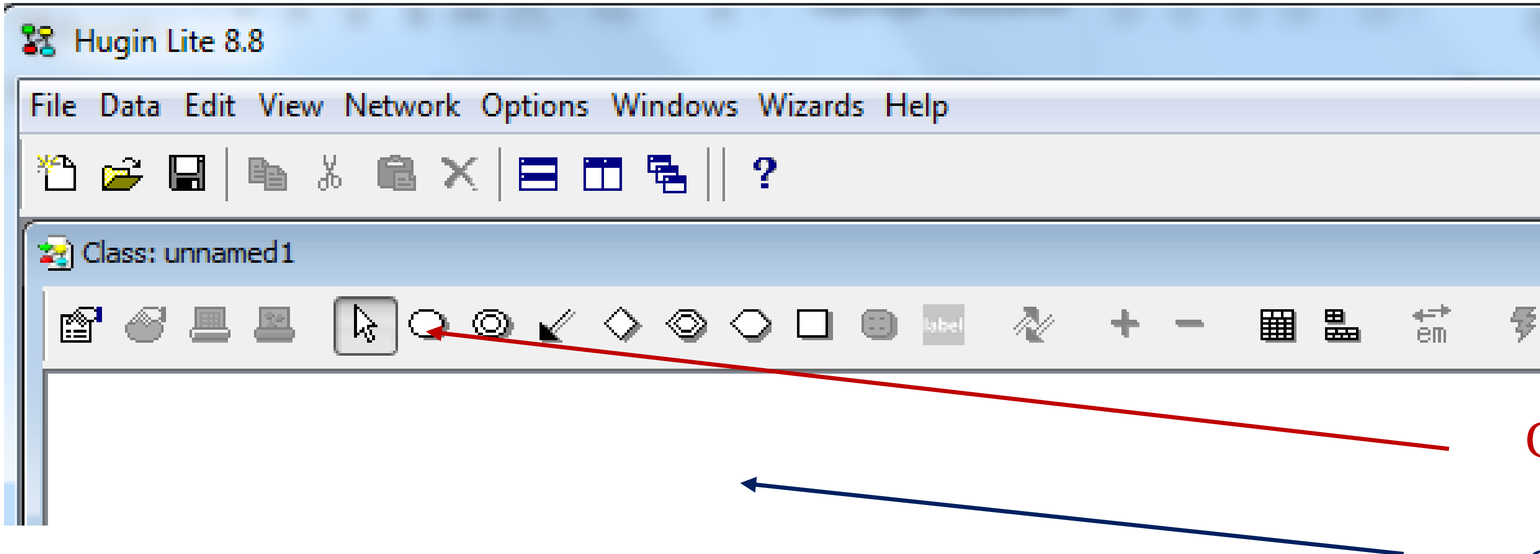
Installation takes less than one minute on most computers.



Upon launching (or opening):



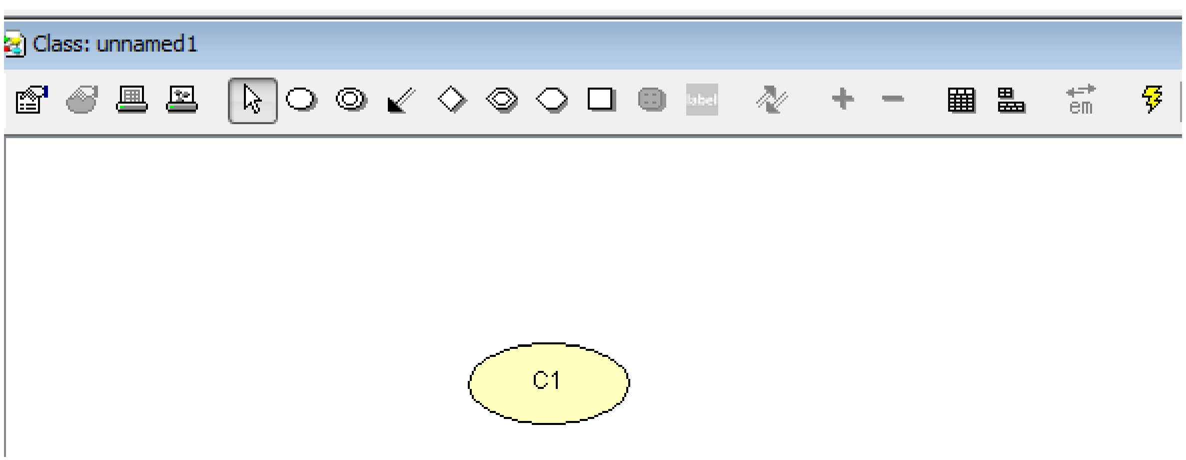
Adding a chance node

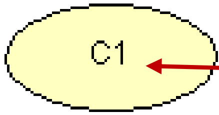


Click here

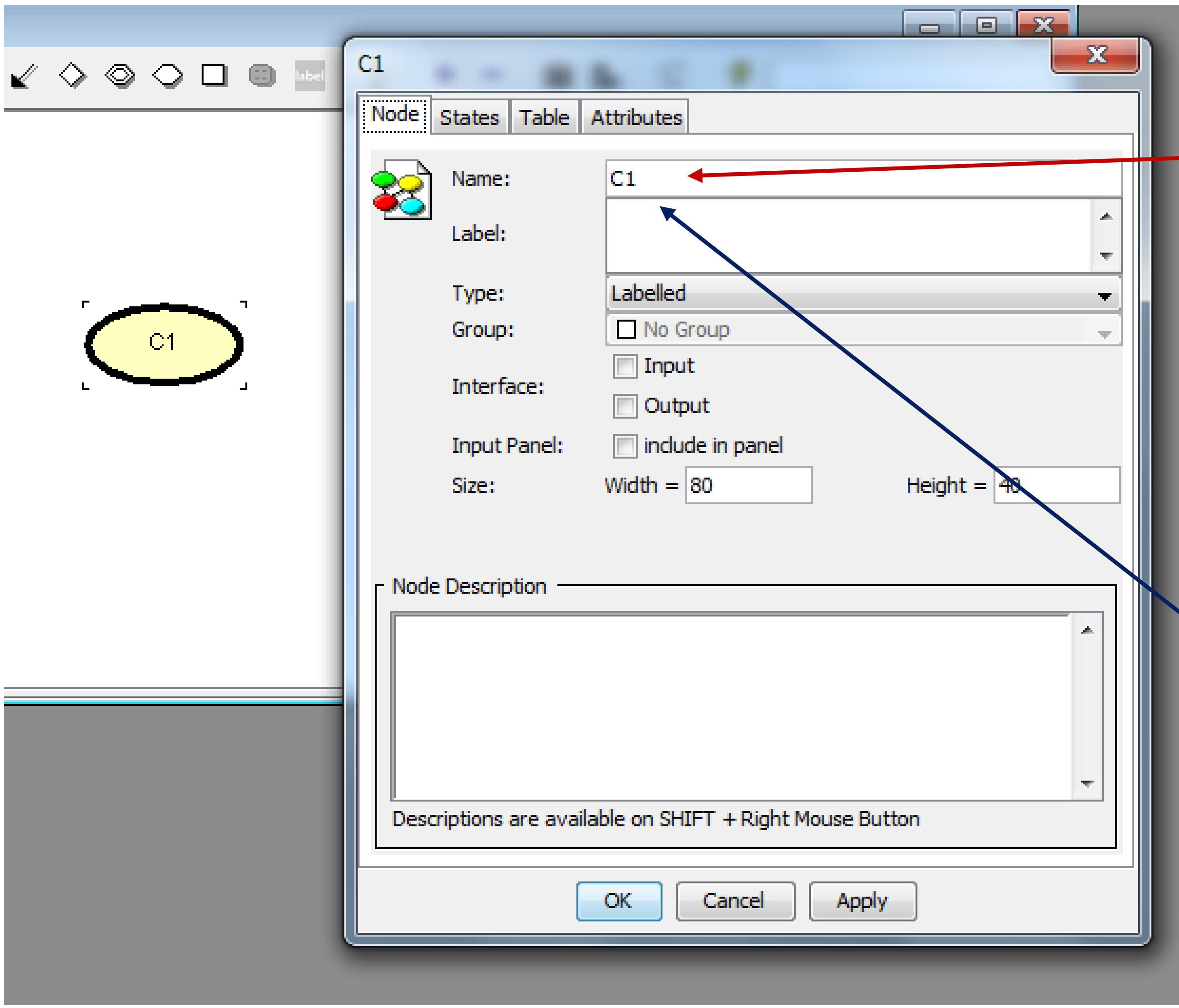
then

Click here





Double-click



Enter a unique name (identifier)

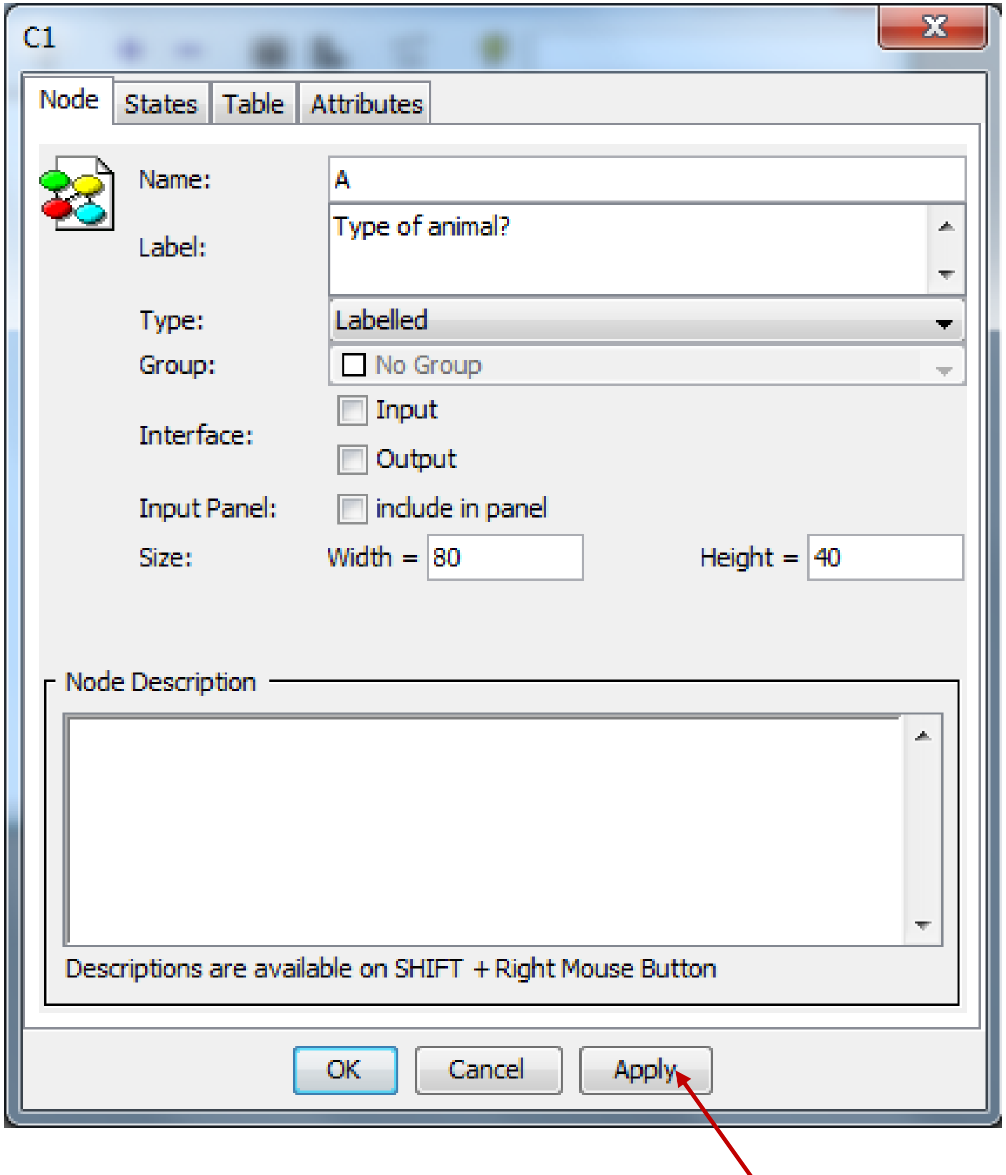
(One single word starting with a letter and otherwise comprised by letters, digits and underscores only)

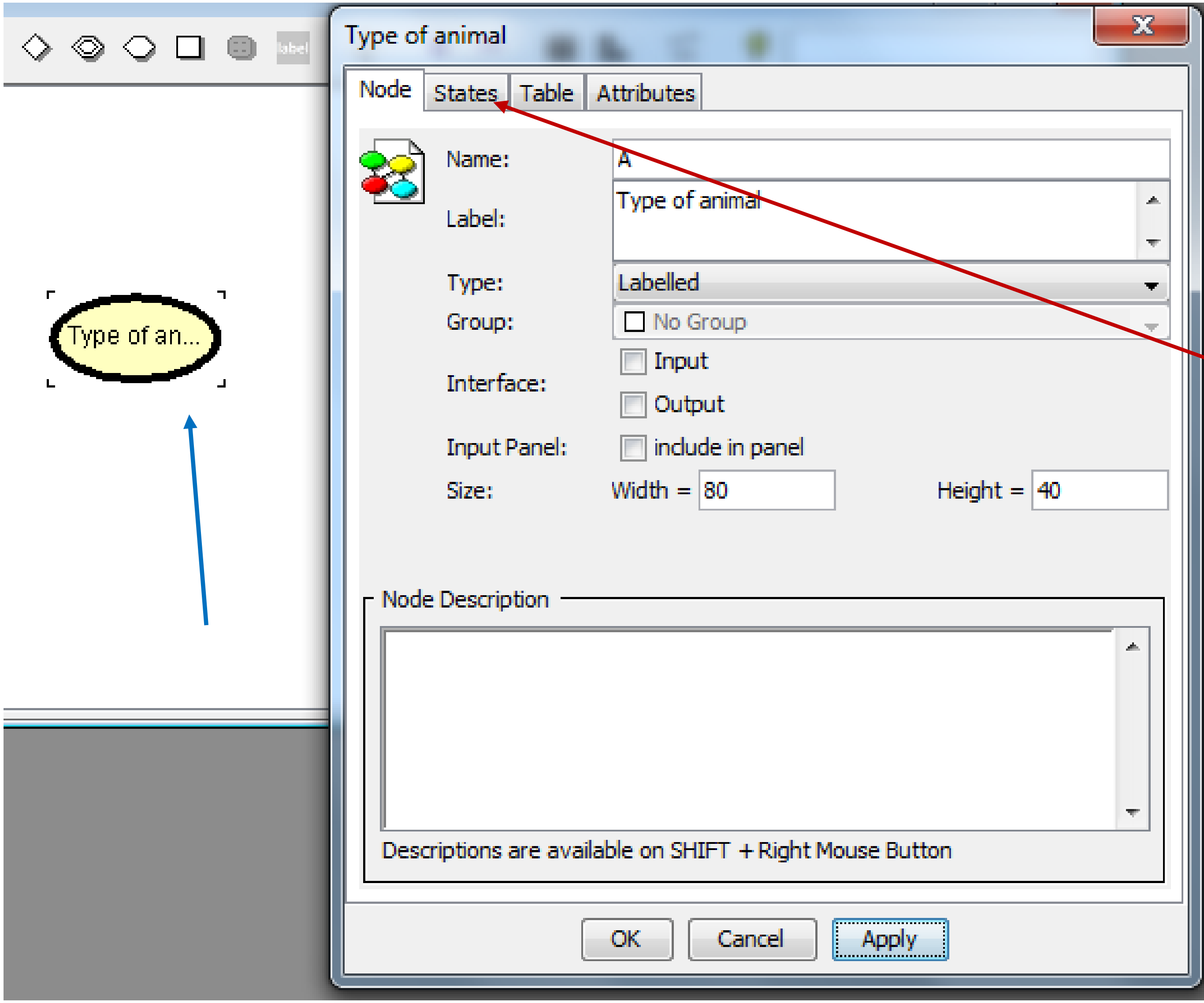
Enter a label

(Free format)

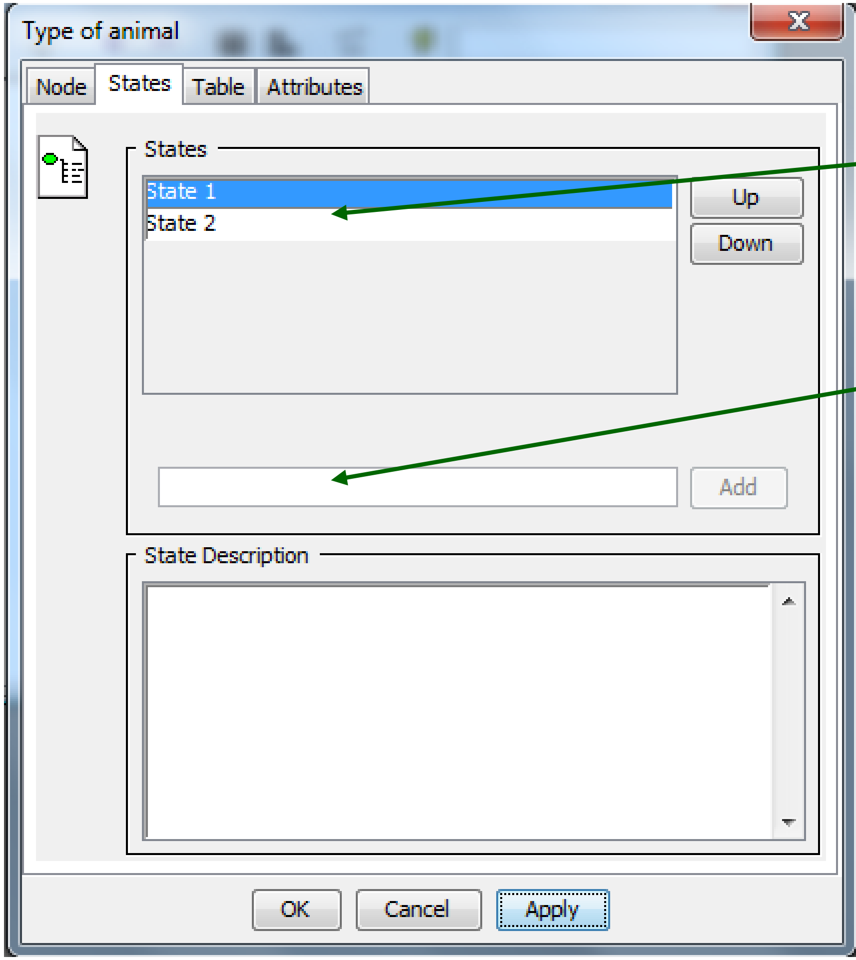
…for instance…

A_1 = "Willie is a cat"
A_2 = "Willie is a parrot





Select tab “States”

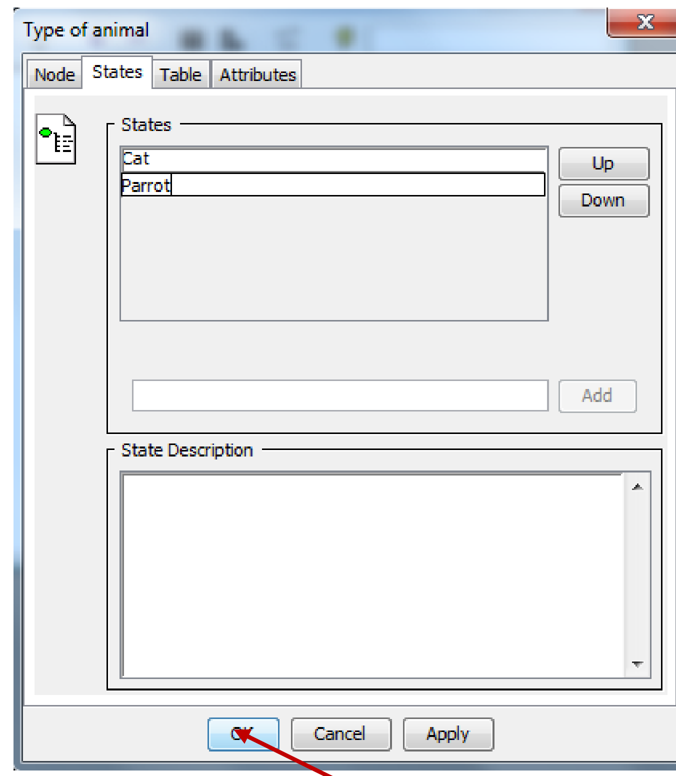


Two states per default

State names can be altered (double-click)

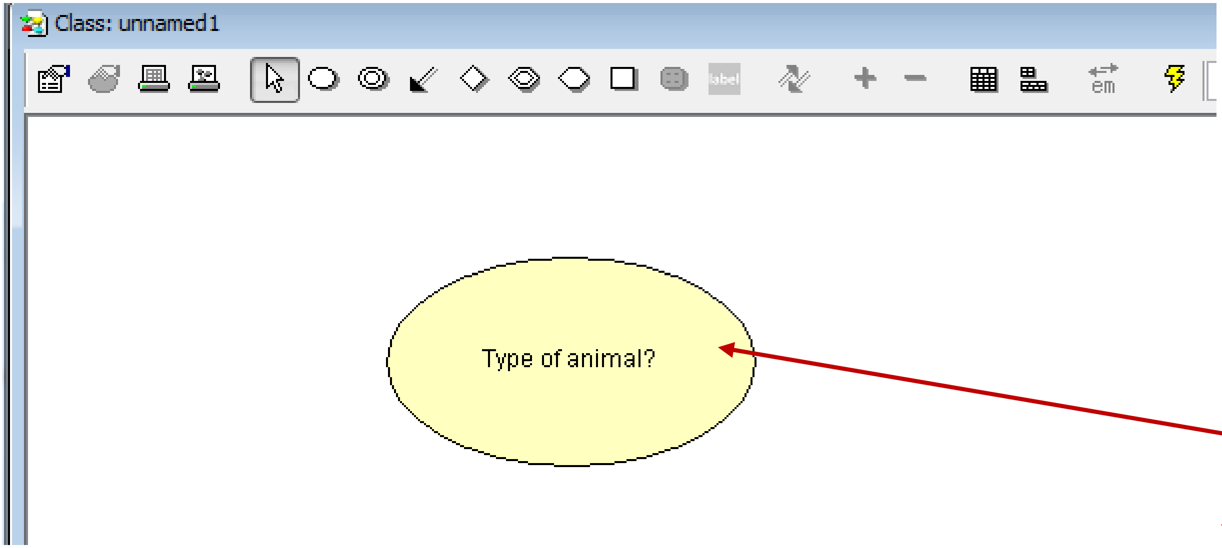
New states can be added

for instance…

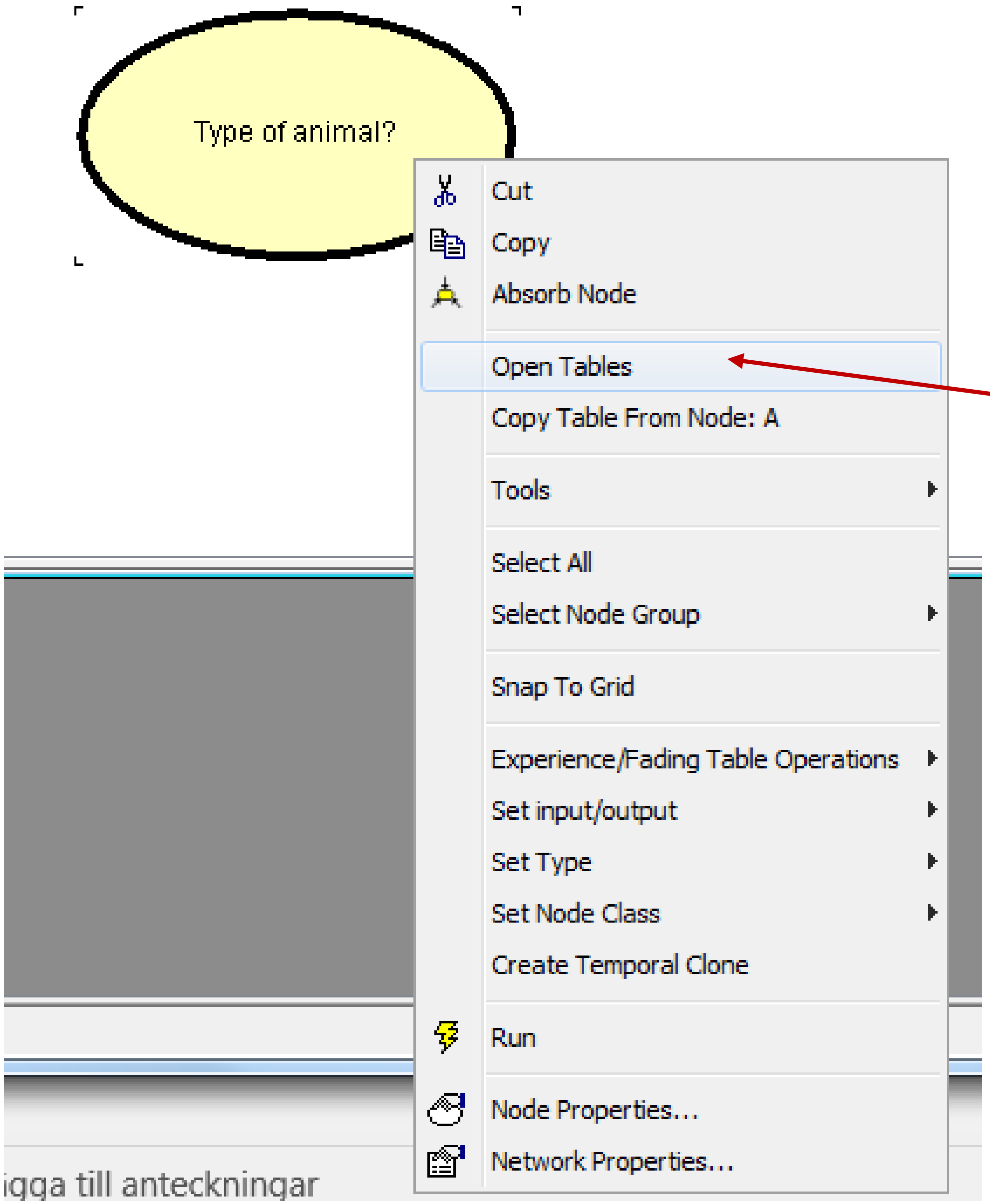


A_1 = "Willie is a cat"
A_2 = "Willie is a parrot

No more selections needed in this dialogue.

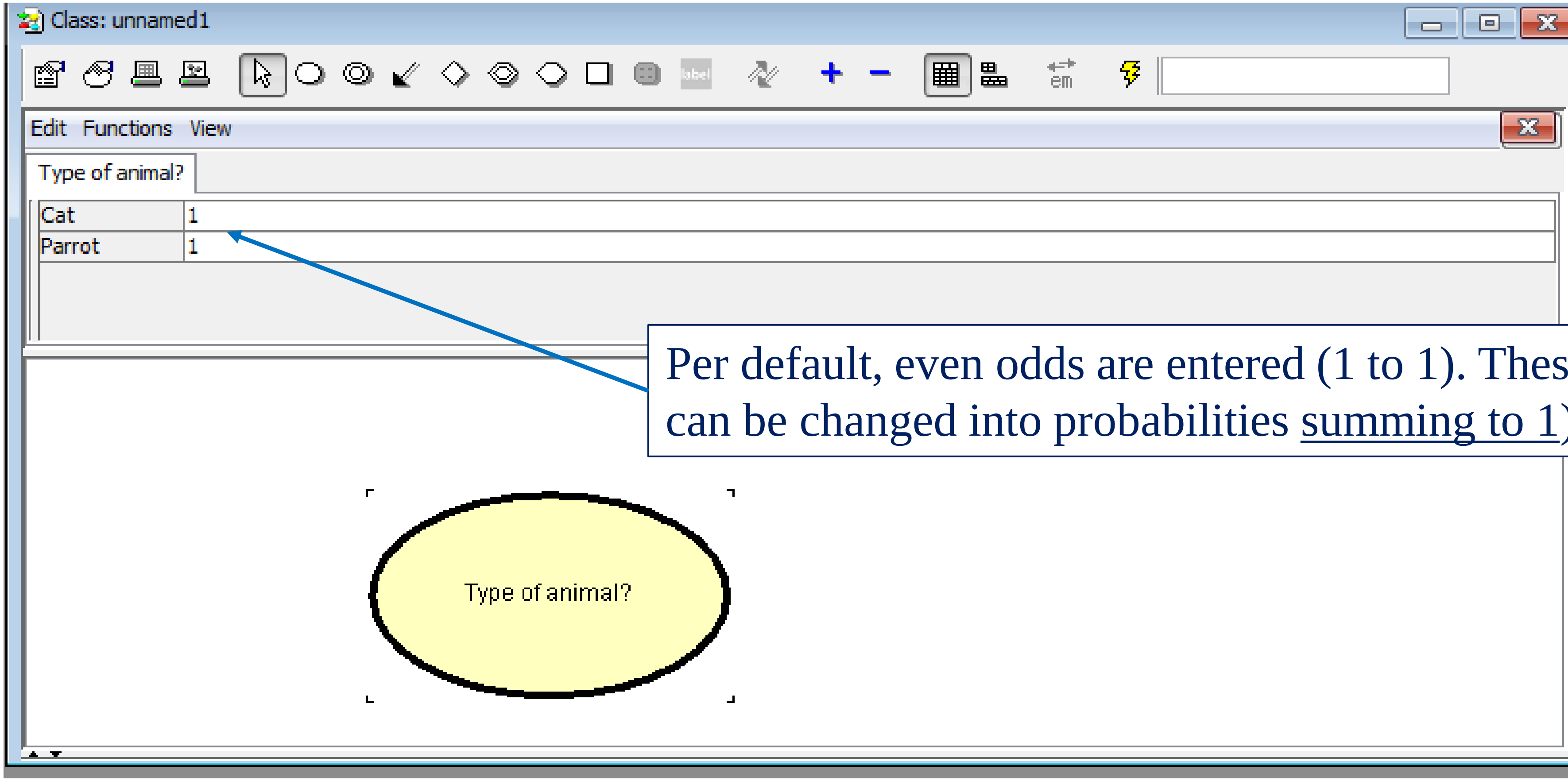


Activate by one left-click (if not active) and right-click inside to open menu

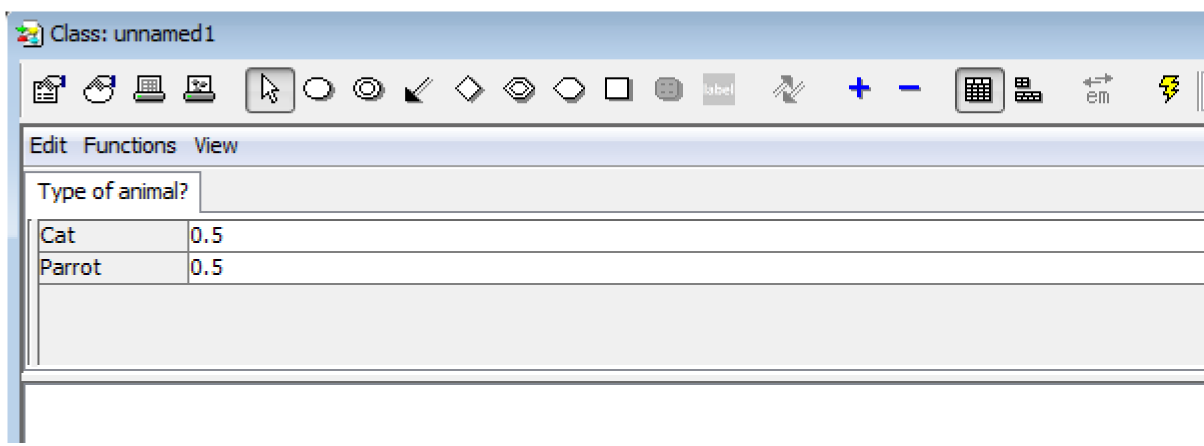


Select "Open Tables"

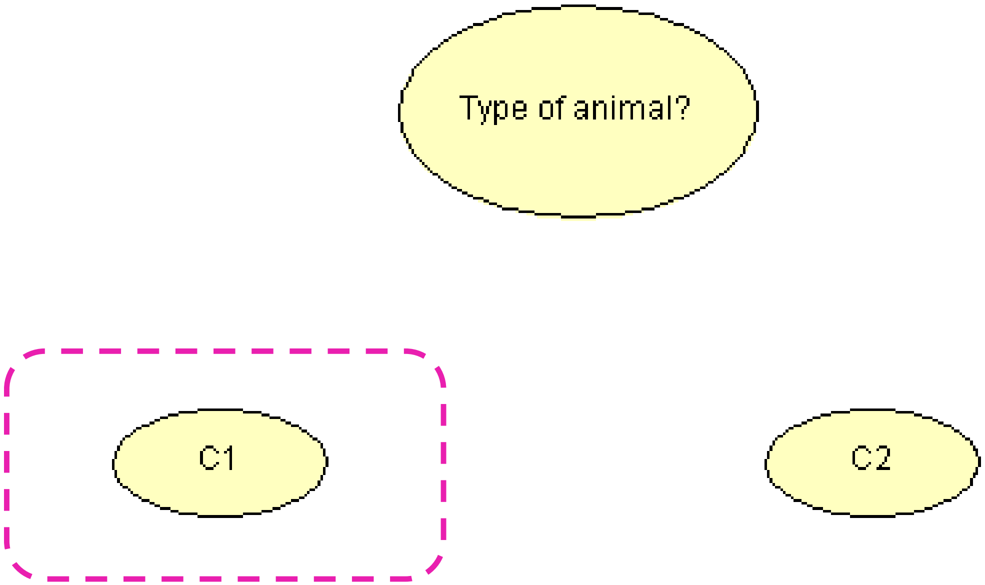
…might be followed by a warning with instructions about how a node table should be visible



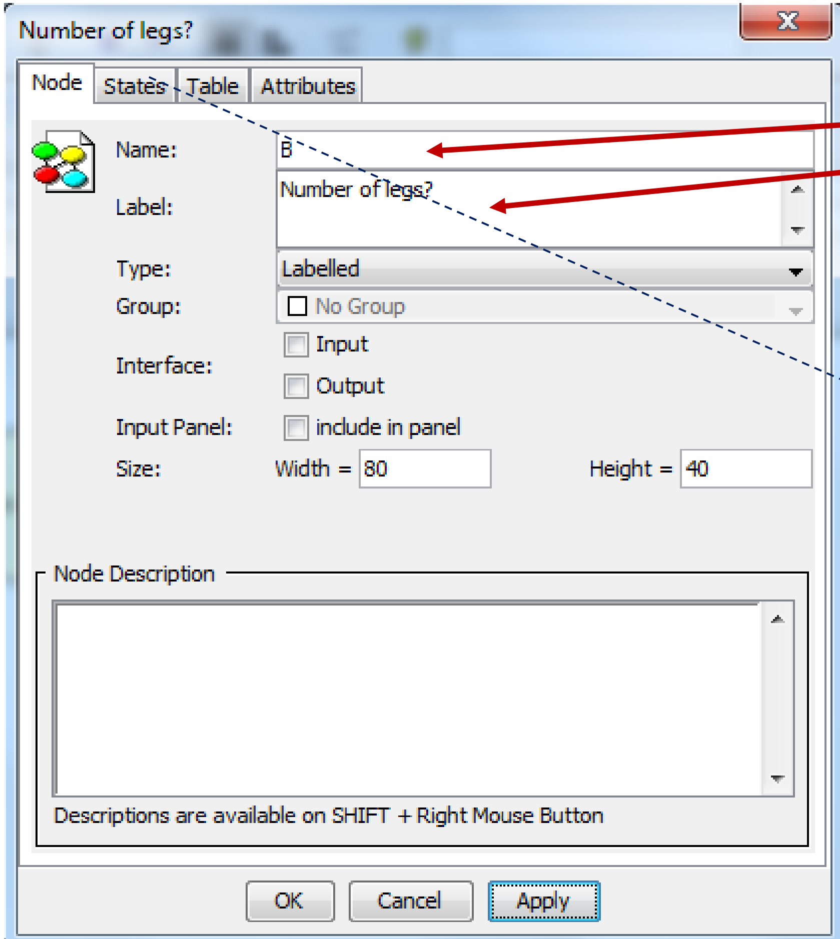
Per default, even odds are entered (1 to 1). These can be changed into probabilities summing to 1).

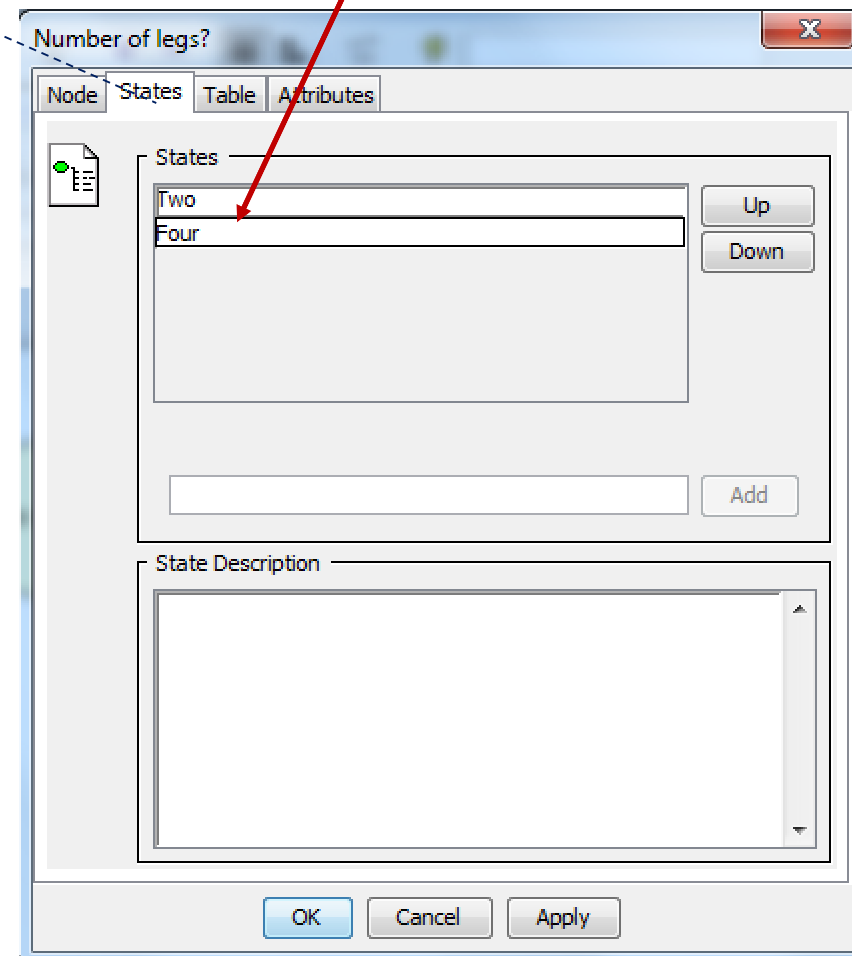


Add two more chance nodes…



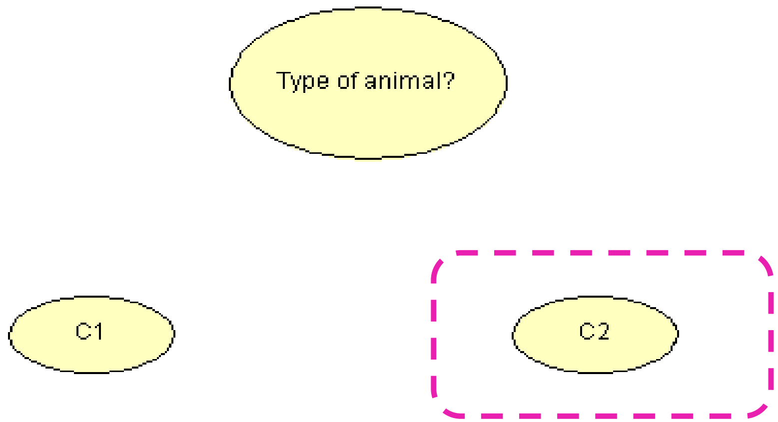
B_1 = “Willie has four legs”
B_2 = “Willie has two legs”

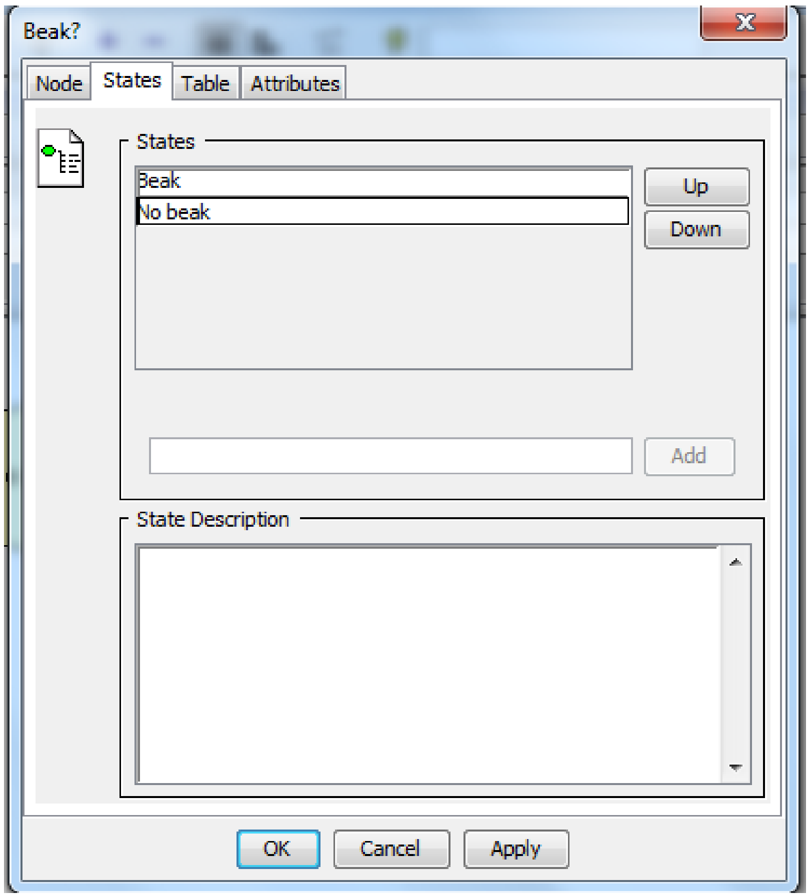


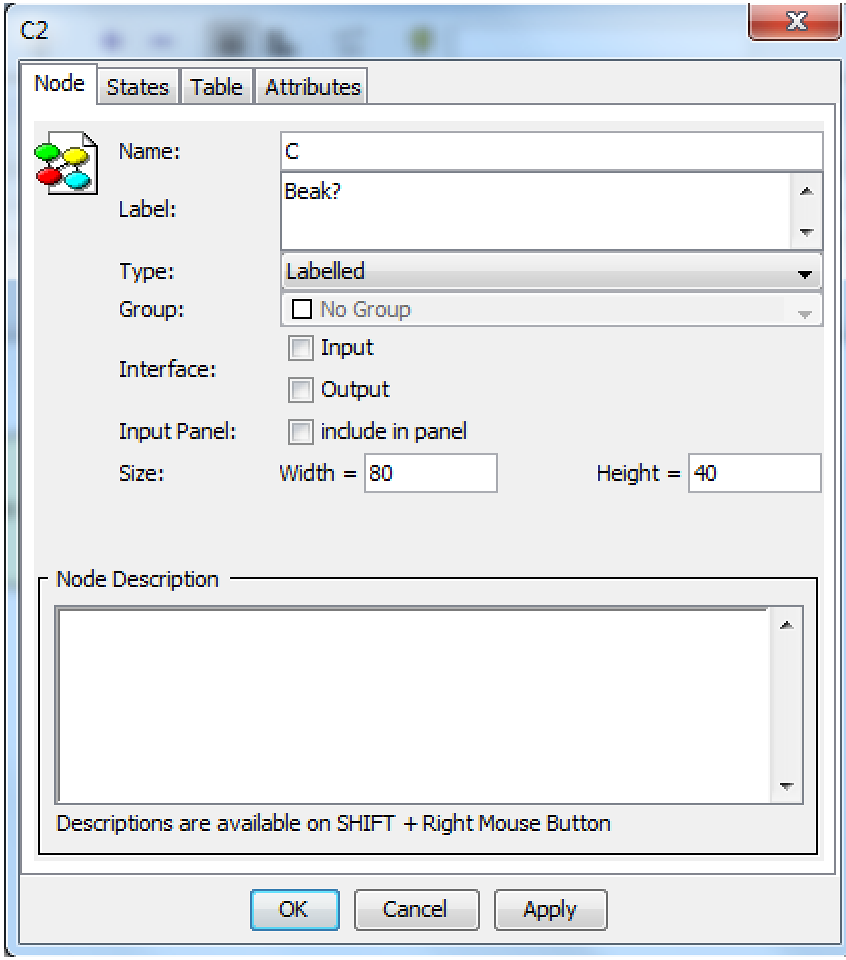


At this moment, do not bother about the probabilities.

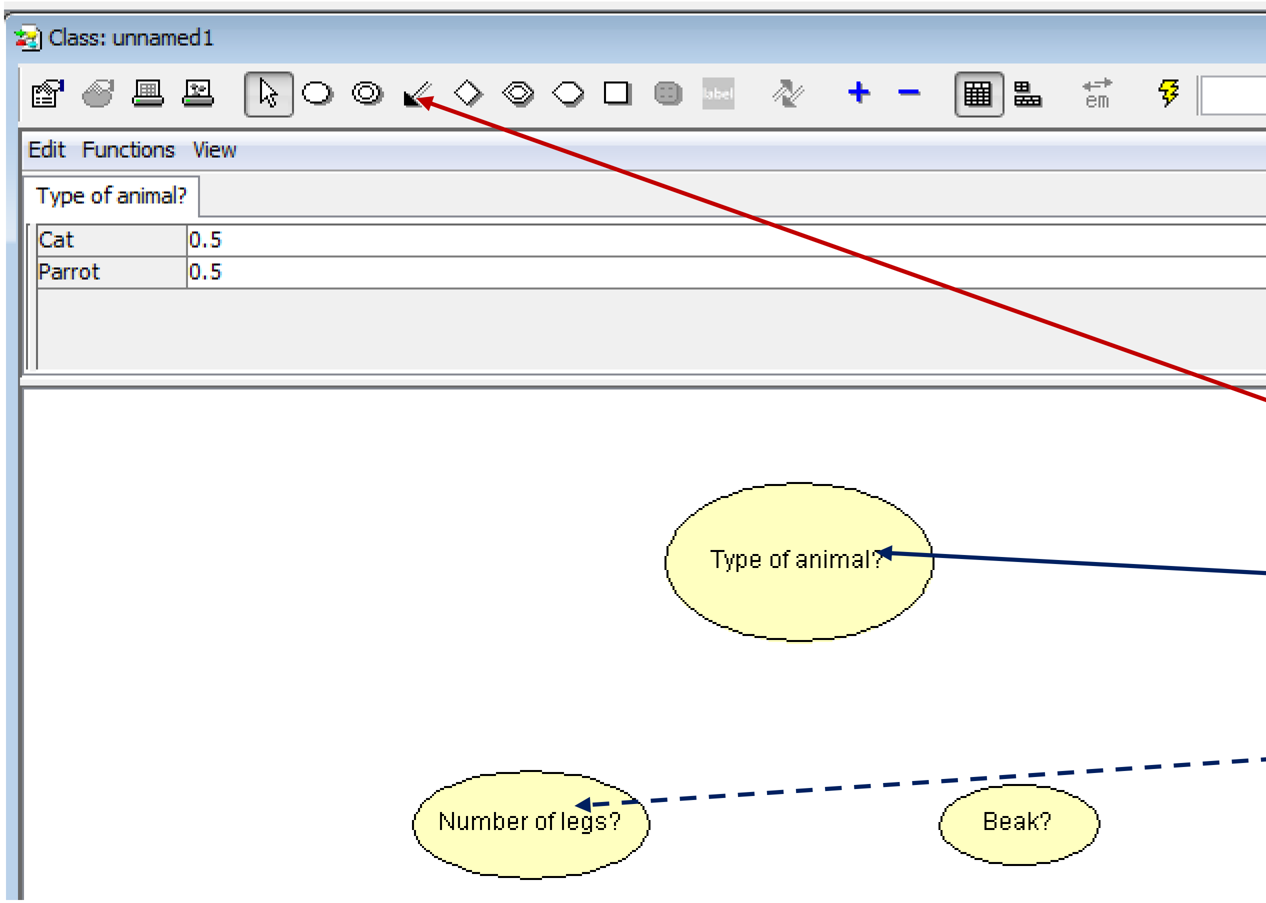
C_1 = “Willie has a beak”
C_2 = “Willie has no beak”







Add links (edges)…

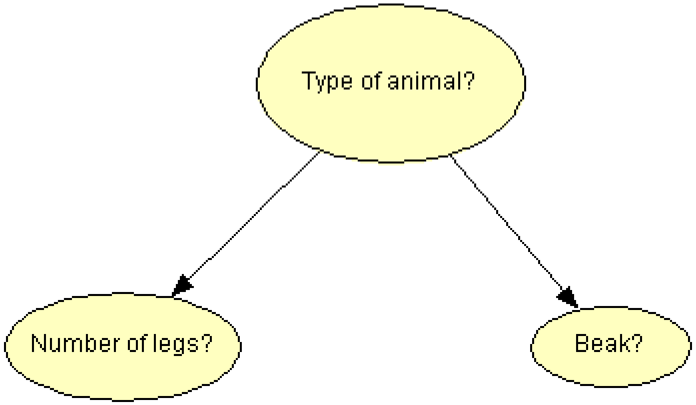


Click here

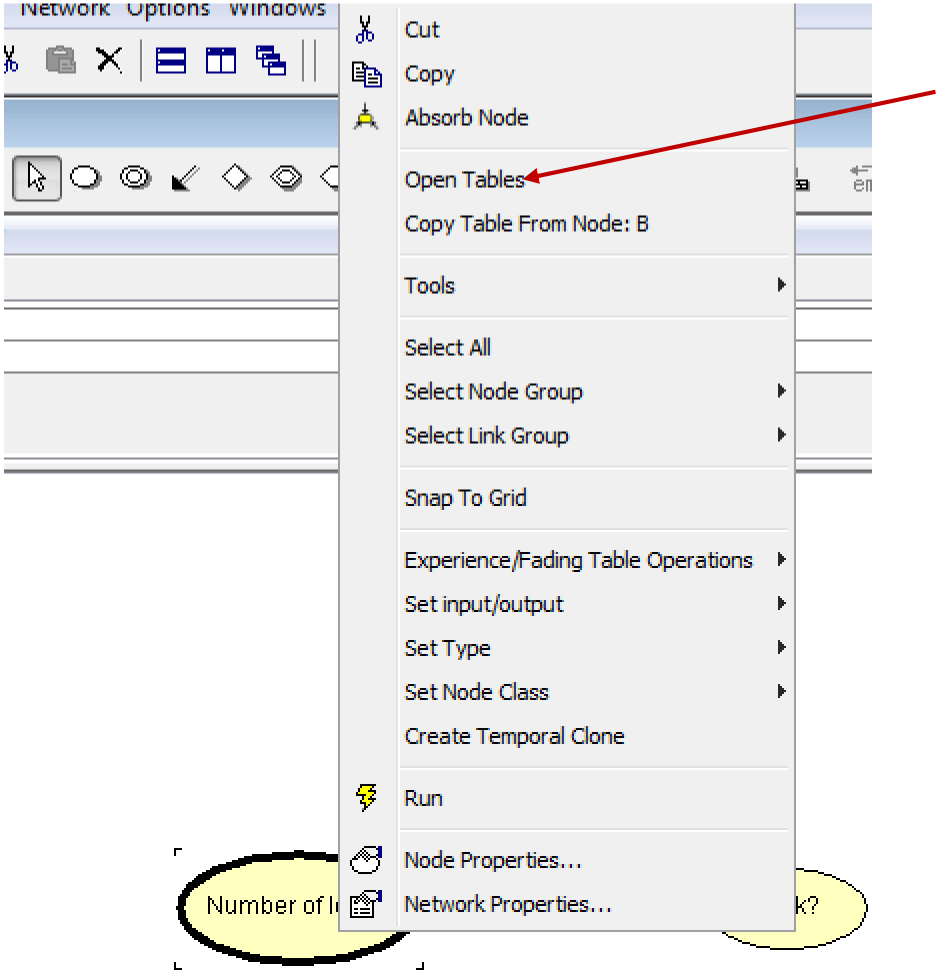
then

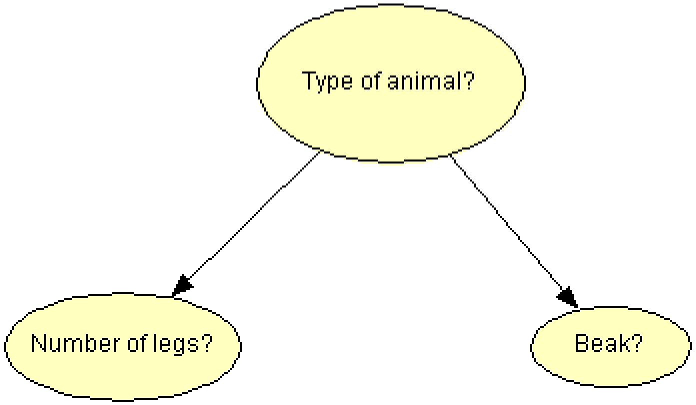
Place cursor in the <u>interior</u> of node A (“Type of animal?”), <u>drag</u> to the interior of node B (“Number of legs?”), and release.

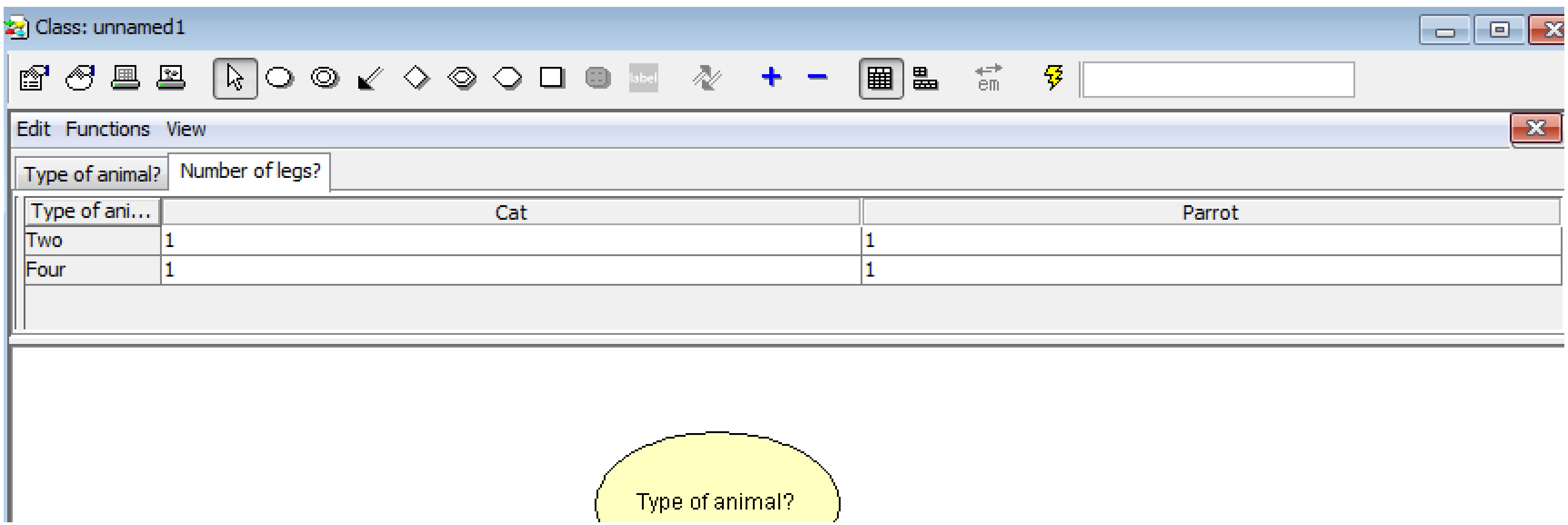
Repeat for link between A and C



Now, activate node B (Number of legs?), right-click and select Open Tables



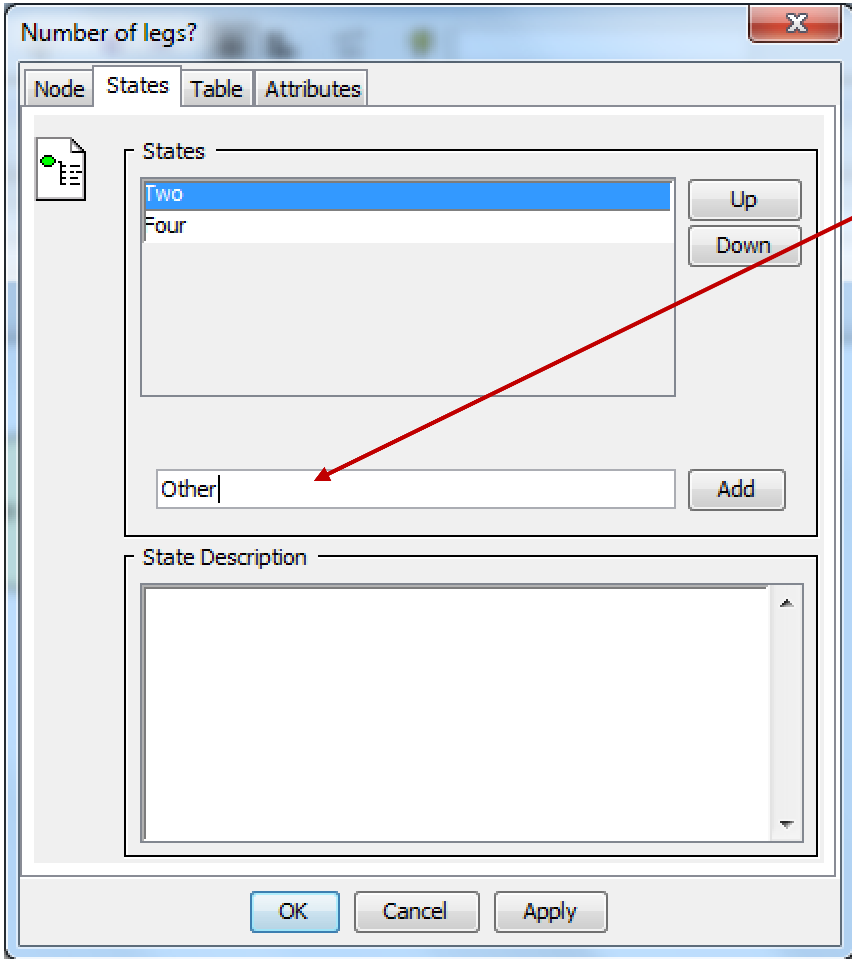




The probability table of node B now has probabilities conditional on the state of node A. The are all set to 1 per default, and will each be treated as 0.25.

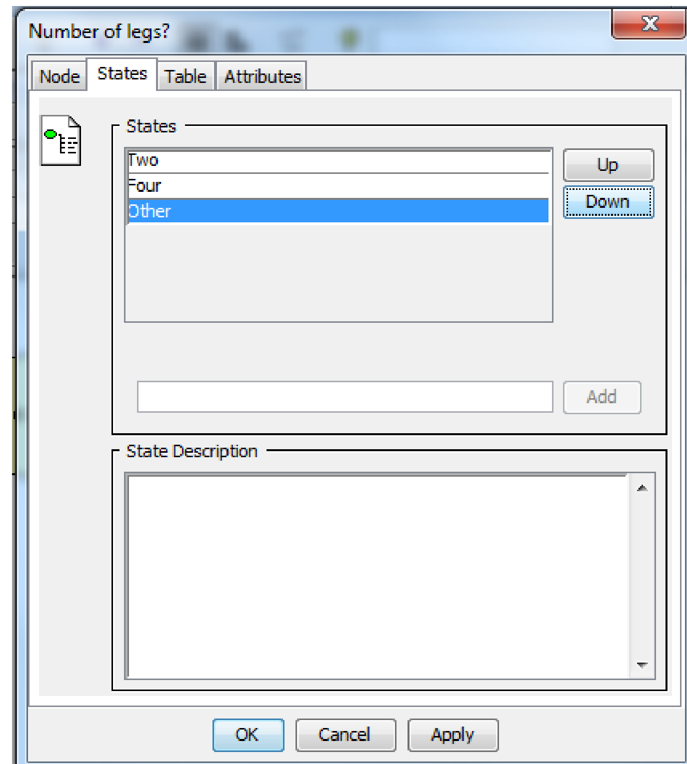
Are the states sufficiently many?

Add a state.
Double-click on node, select tab States



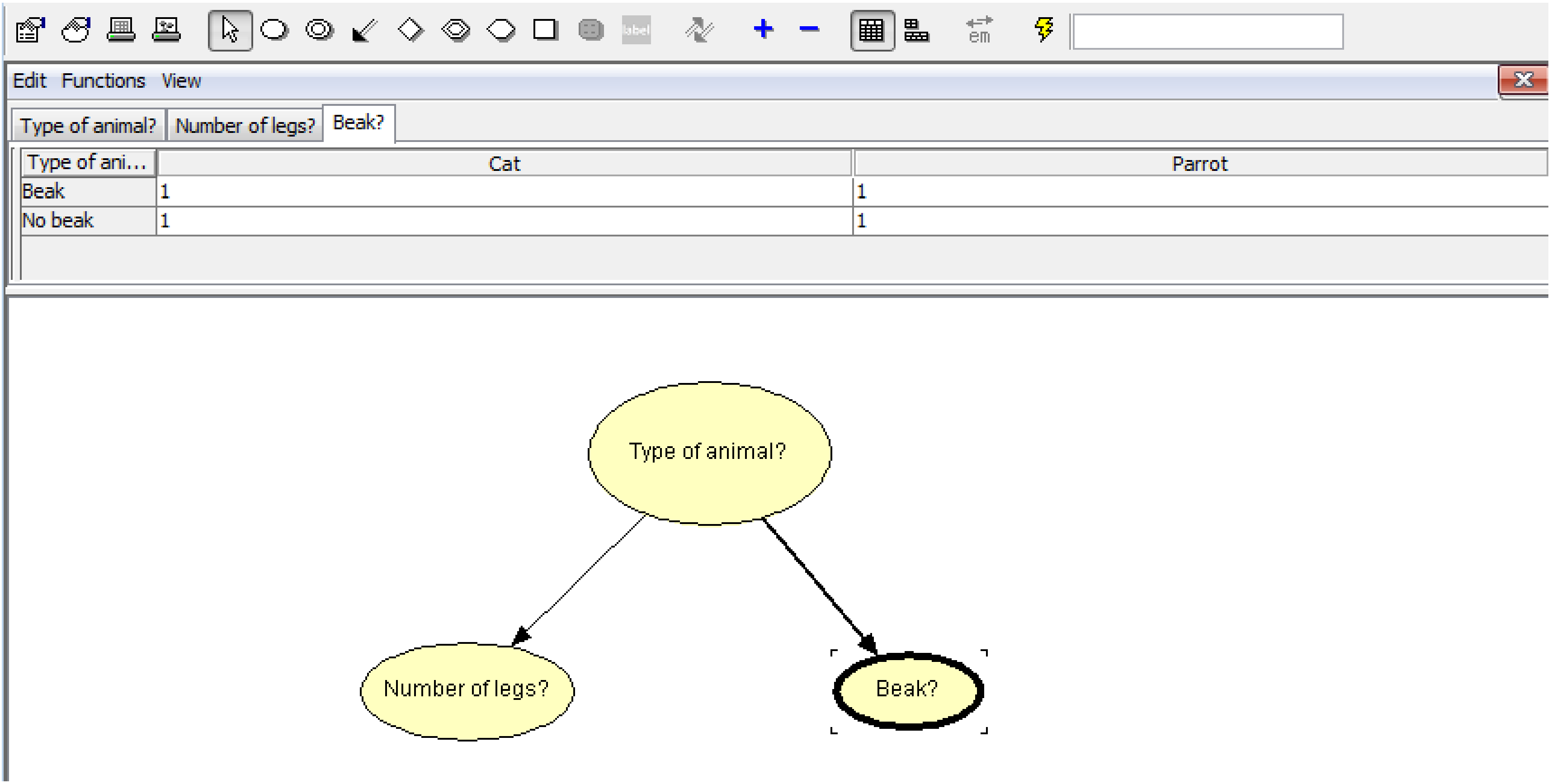
Enter name of new state here, press Add

…and a new state is added



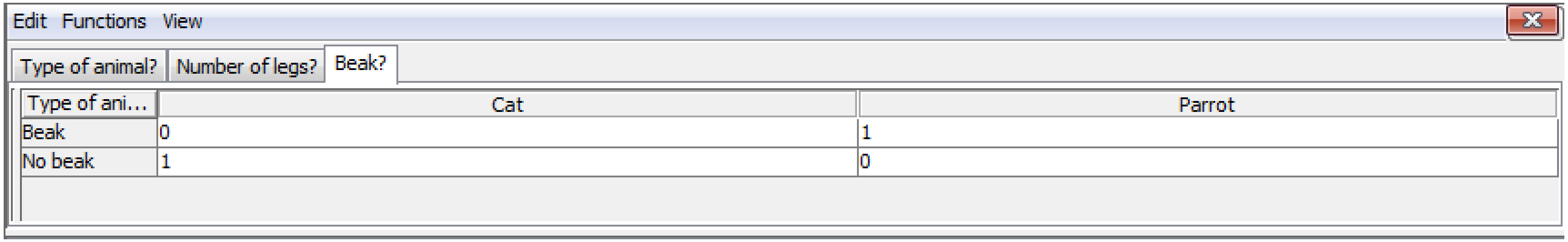
The order of the states can be changed by pressing Up and/or Down

Now, open the table for node C (Beak?) as well.



We can now enter our probabilities into the tables.

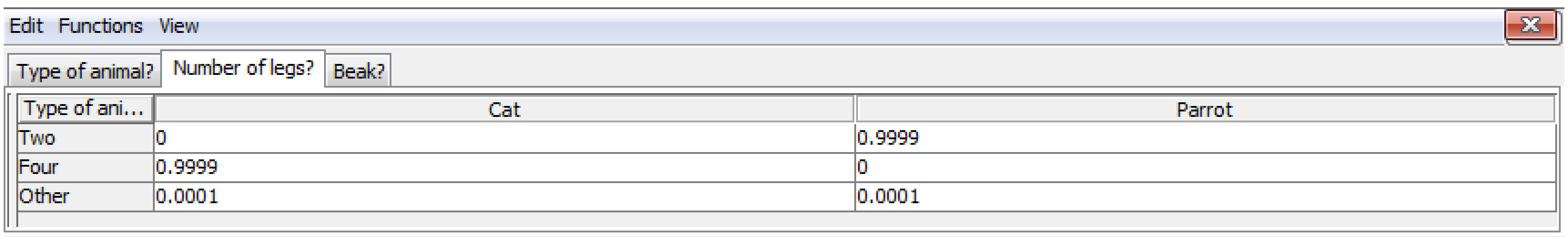
Node C:



Type of ani...	Cat	Parrot
Beak	0	1
No beak	1	0

Reasonable?

Node B:

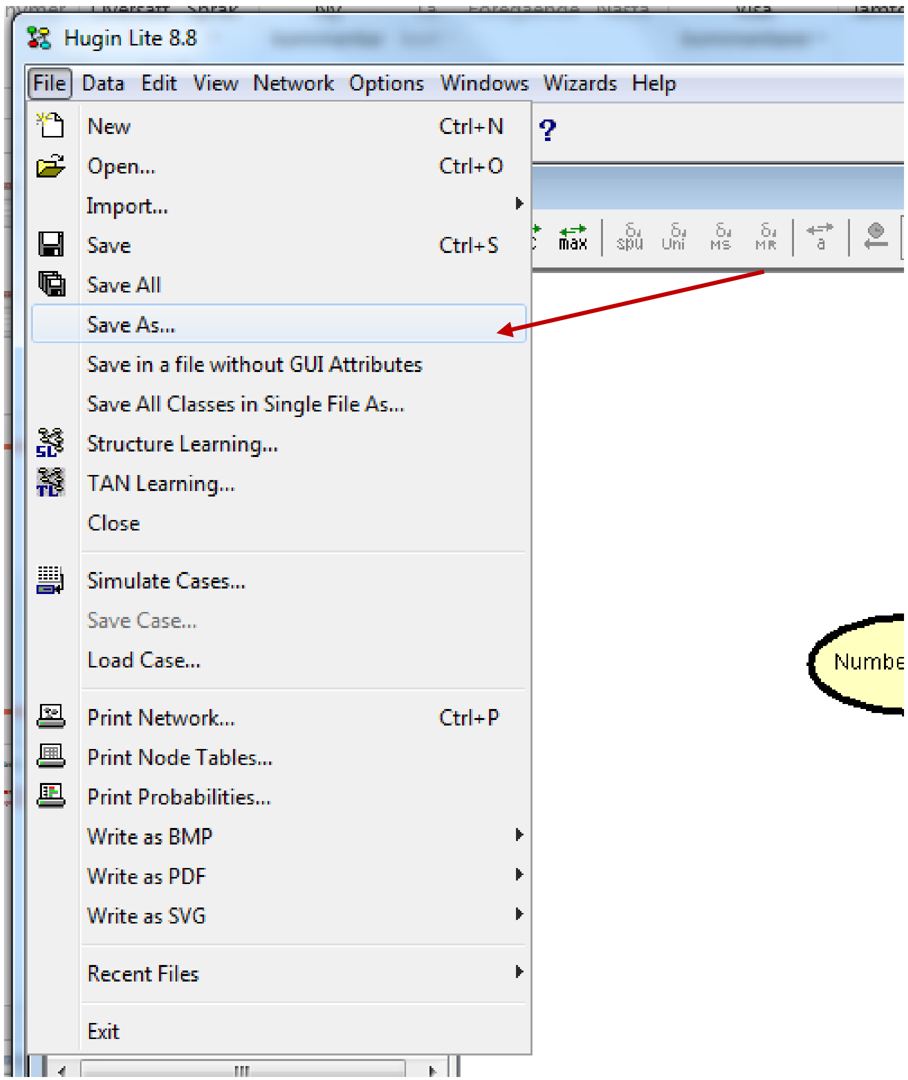


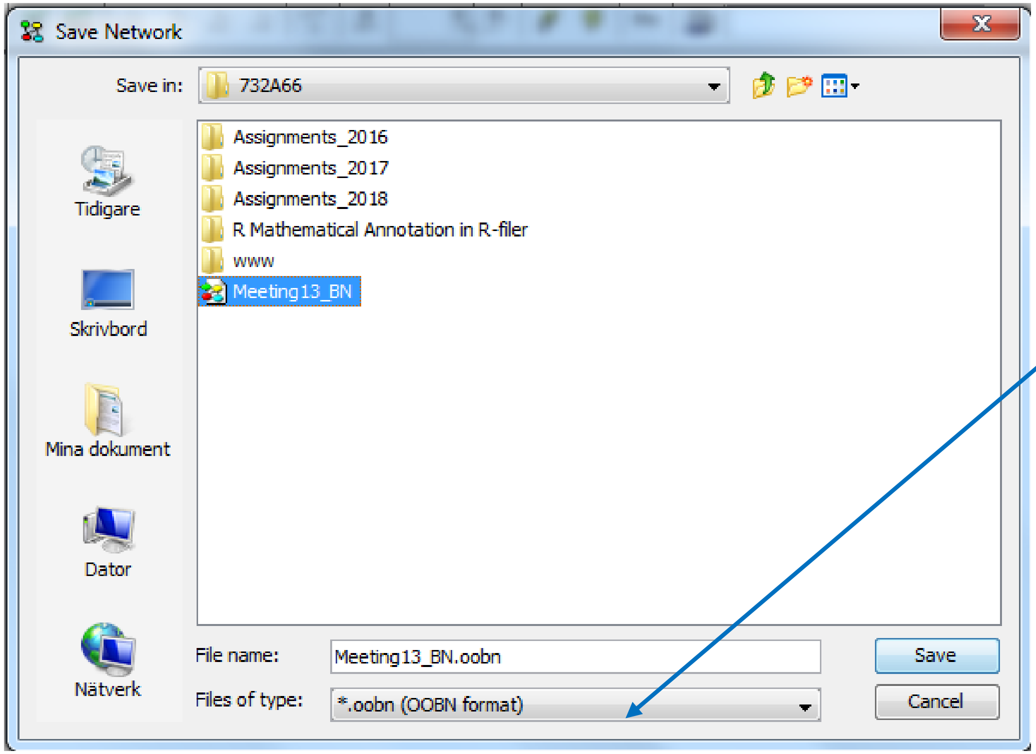
Type of ani...	Cat	Parrot
Two	0	0.9999
Four	0.9999	0
Other	0.0001	0.0001

Reasonable?

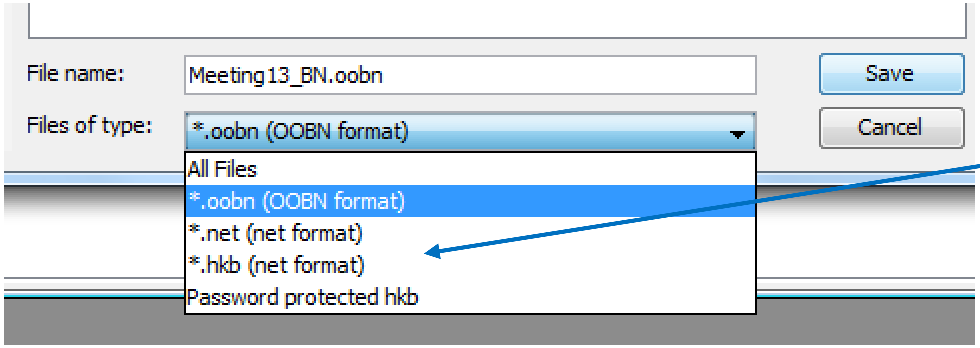
At this moment (or even earlier) it is wise to save the constructed network.

Open the File menu of the GUI and select Save As…





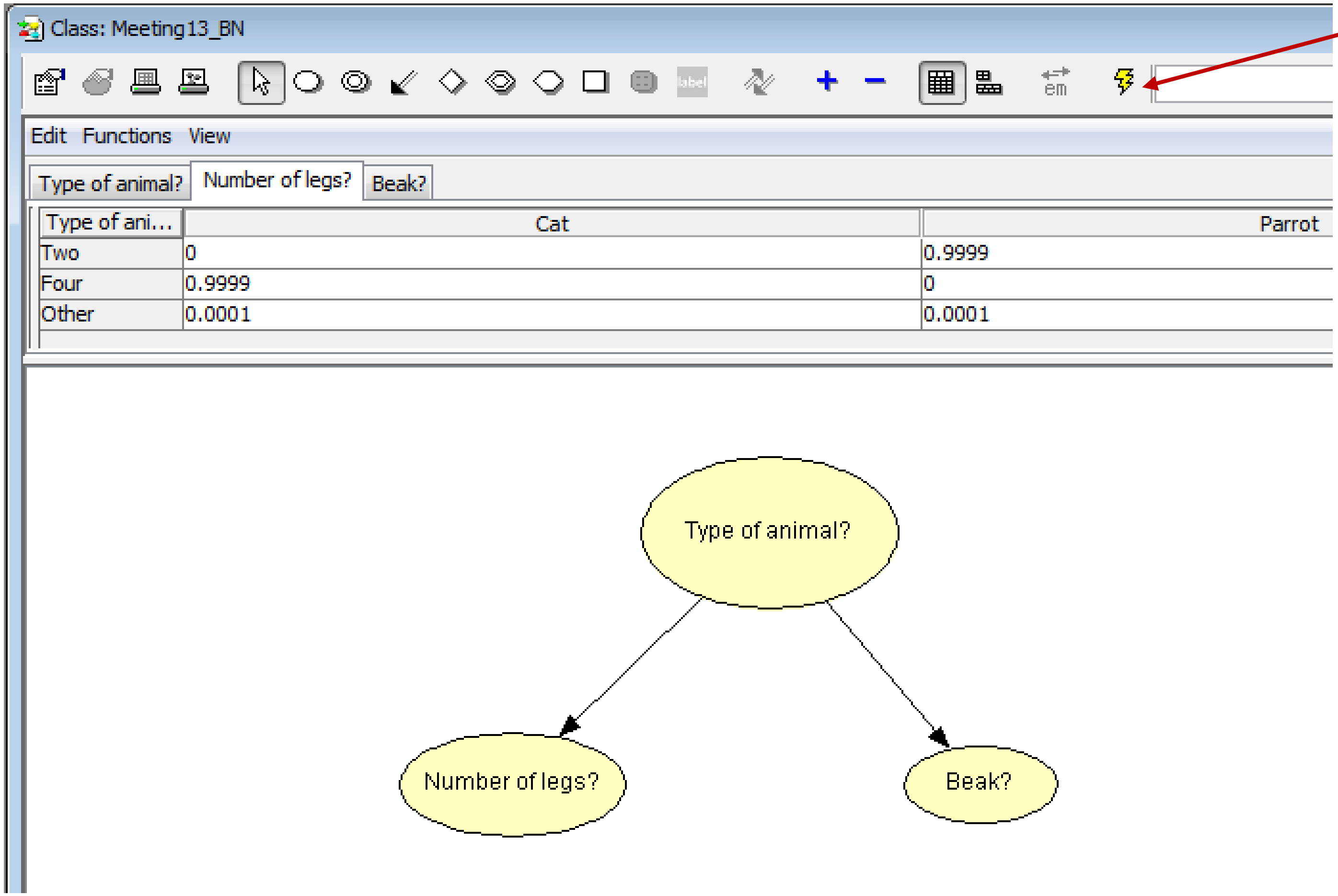
This is the Hugin format (OOBN)

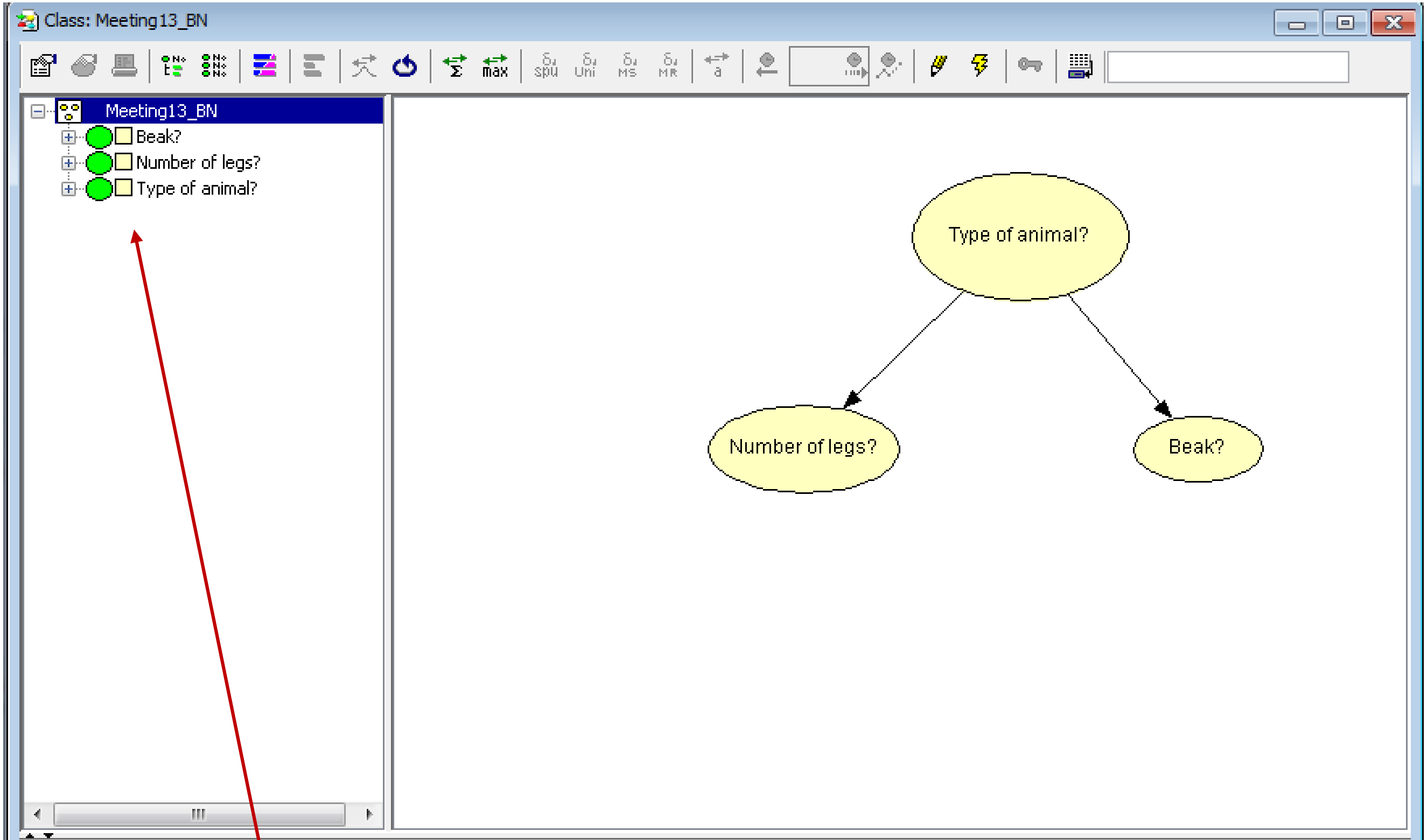


But it is also possible to use the more general NET format for compatibility with other software (like GeNIe)

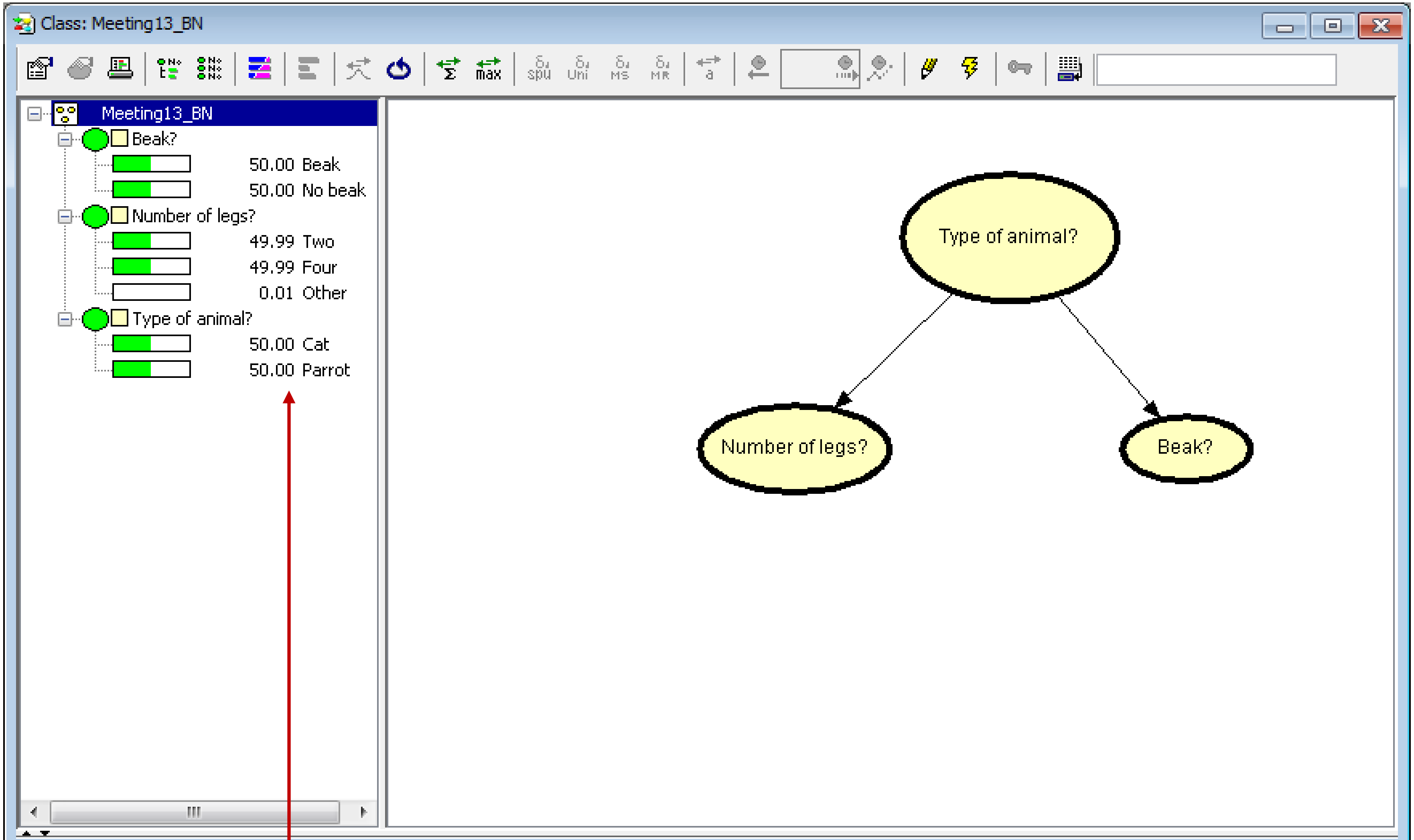
Now, the designed network should be “run”. This means putting the probabilities set into “action”

Click on the flash symbol





Expand this list



Here, we can read of the marginal probabilities (in %) of each state in each node.

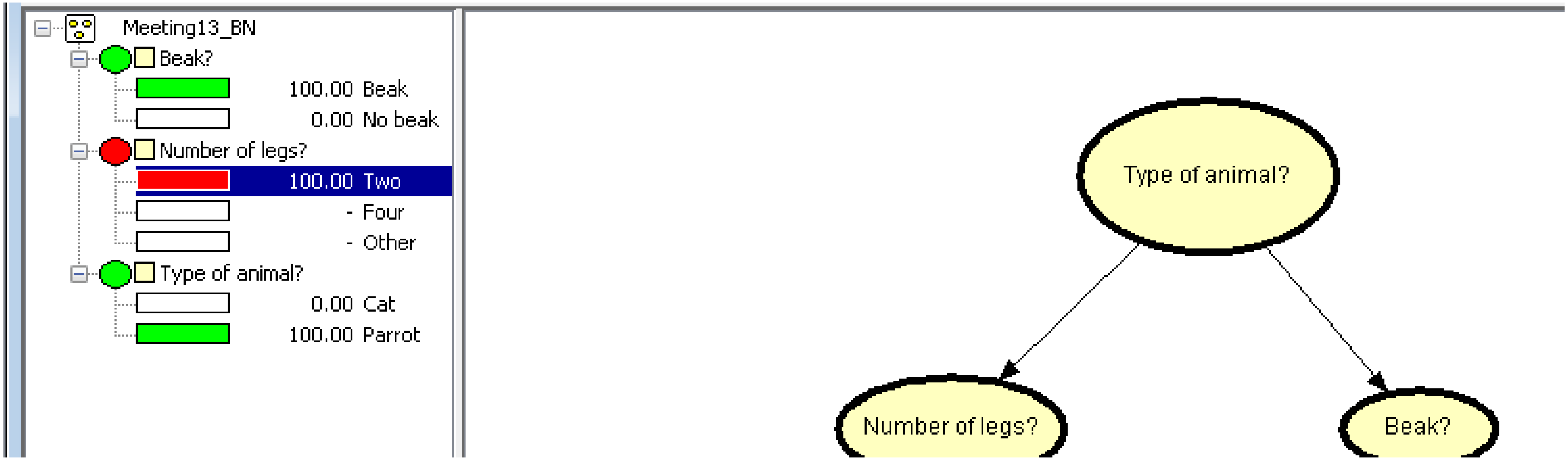
$P(\text{Beak}|I), P(\text{No Beak}|I); P(\text{Two legs}|I), P(\text{Four legs}|I), P(\text{Other}|I)$

and (as previously assigned) $P(\text{Cat}|I)$ and $P(\text{Parrot}|I)$

Entering ”evidence”

It is now possible to calculate updated (conditional) probabilities given a particular state in one or several of the nodes. This is called ” to instantiate” a node to the state of interest and is done in the software by double-clicking on that state.

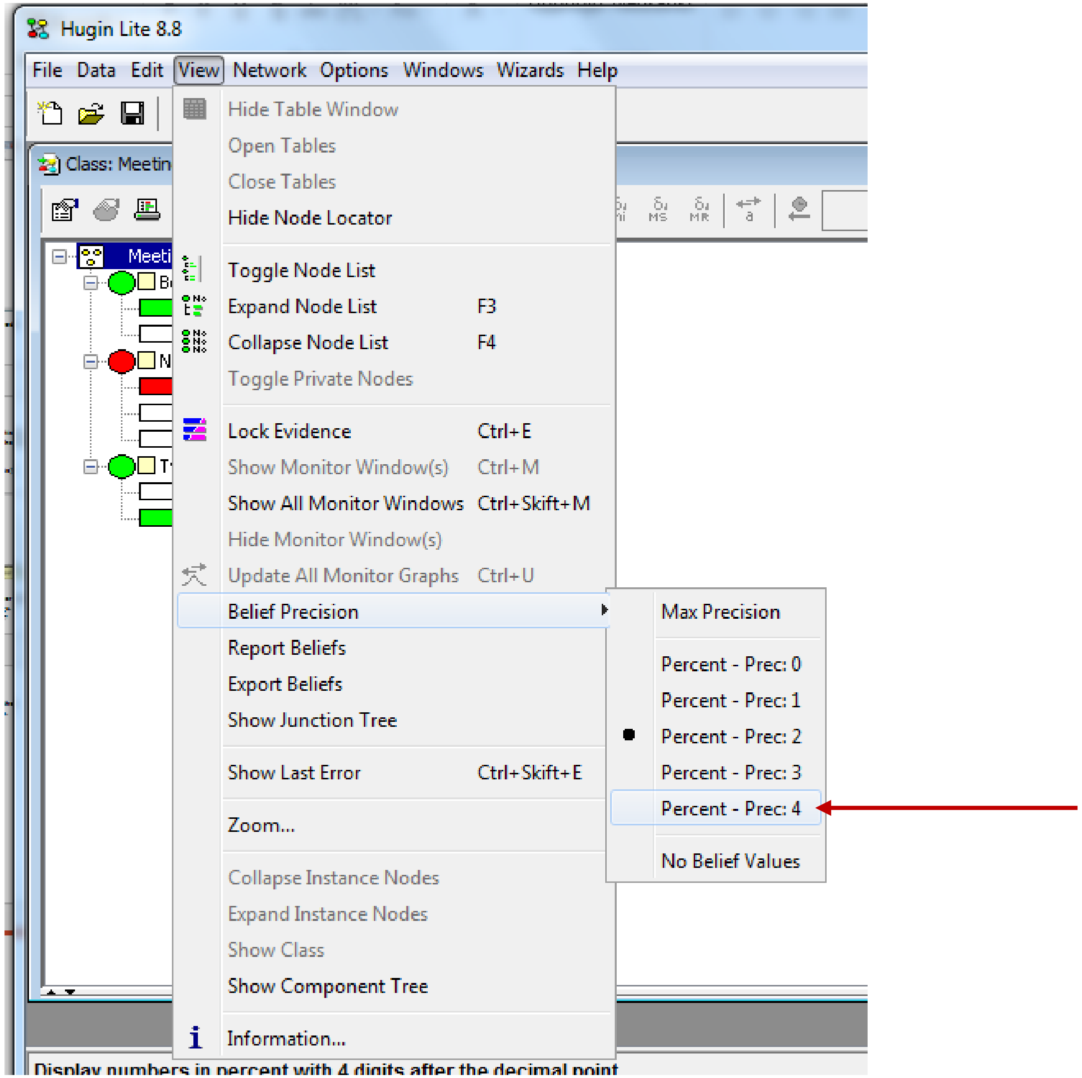
As an example, suppose that we obtain the information that the animal (Willie) has two legs. Then, we double-click on that state in the list.

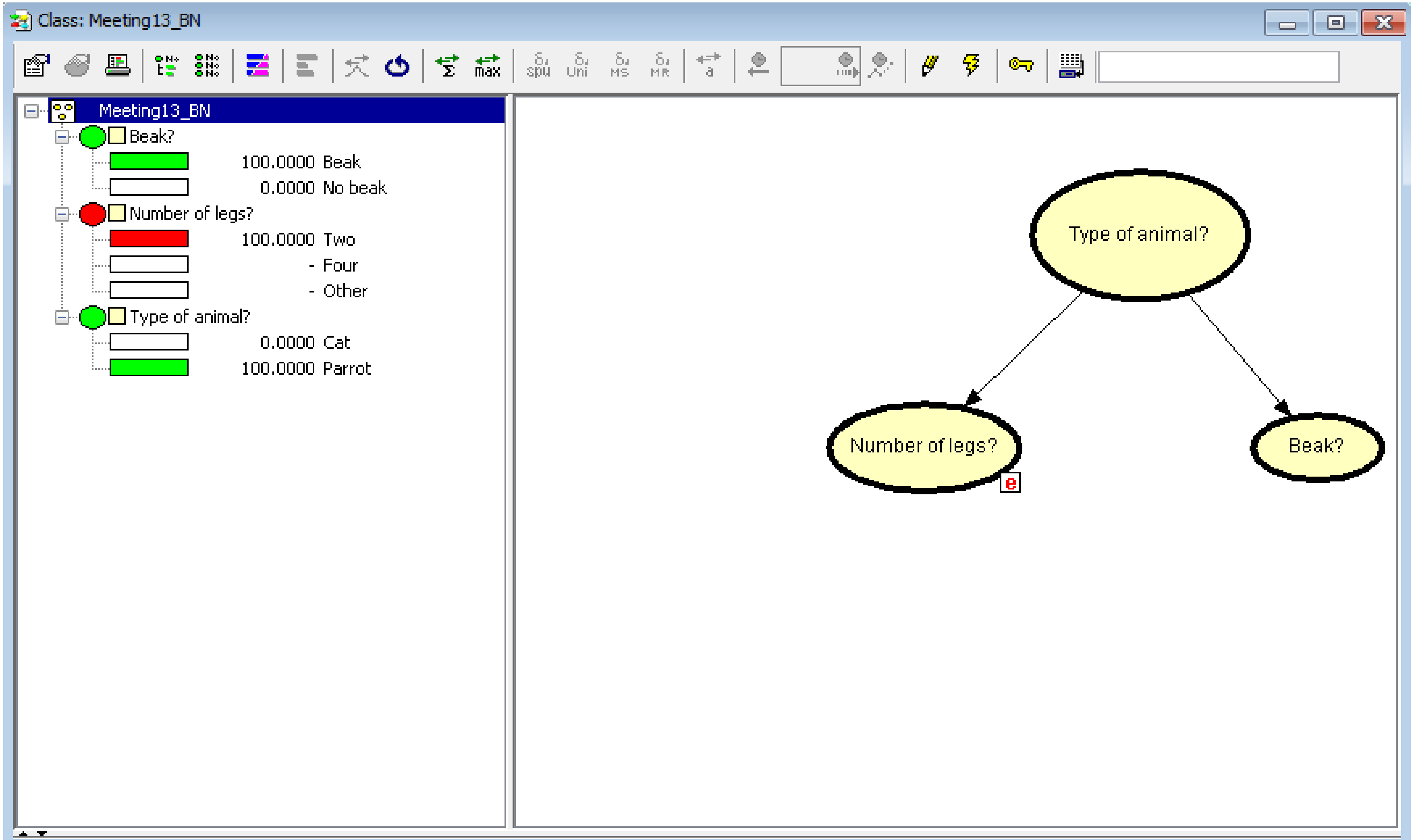


The bar colour of this state changes into read and its value to 100 %. The other bars also change values, i.e. the conditional probabilities given Willie has four legs. However, it seems they are either 100.00% or 0.00%. Could that be correct?

The precision used for displaying the numbers can be changed.

From the GUI menu select View, from the list select Belief Precision and in the following list select Percent – Prec: 4. *Other choice can of course also be made*.





Well, we have now the values 100.0000% and 0.0000% respectively. Trying Max Precision would not give more information.

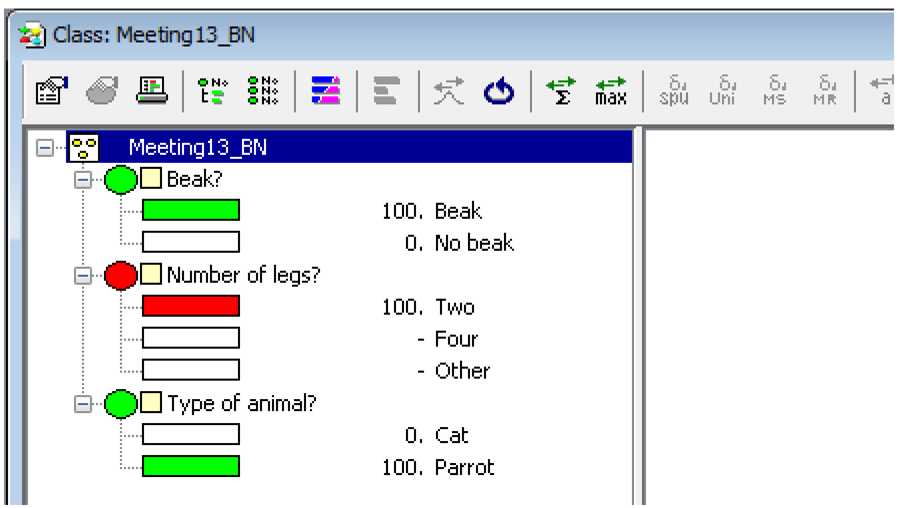
What probabilities are we computing here? Consider for instance the updated probability for A_2, i.e. "Willie is a parrot".

This probability is

$$P(A_2|B_1) = \frac{P(B_1|A_2)\cdot P(A_2)}{P(B_1|A_1)\cdot P(A_1) + P(B_1|A_2)\cdot P(A_2)} = \frac{0.9999\cdot 0.5}{0\cdot 0.5 + 0.9999\cdot 0.5} = 1$$

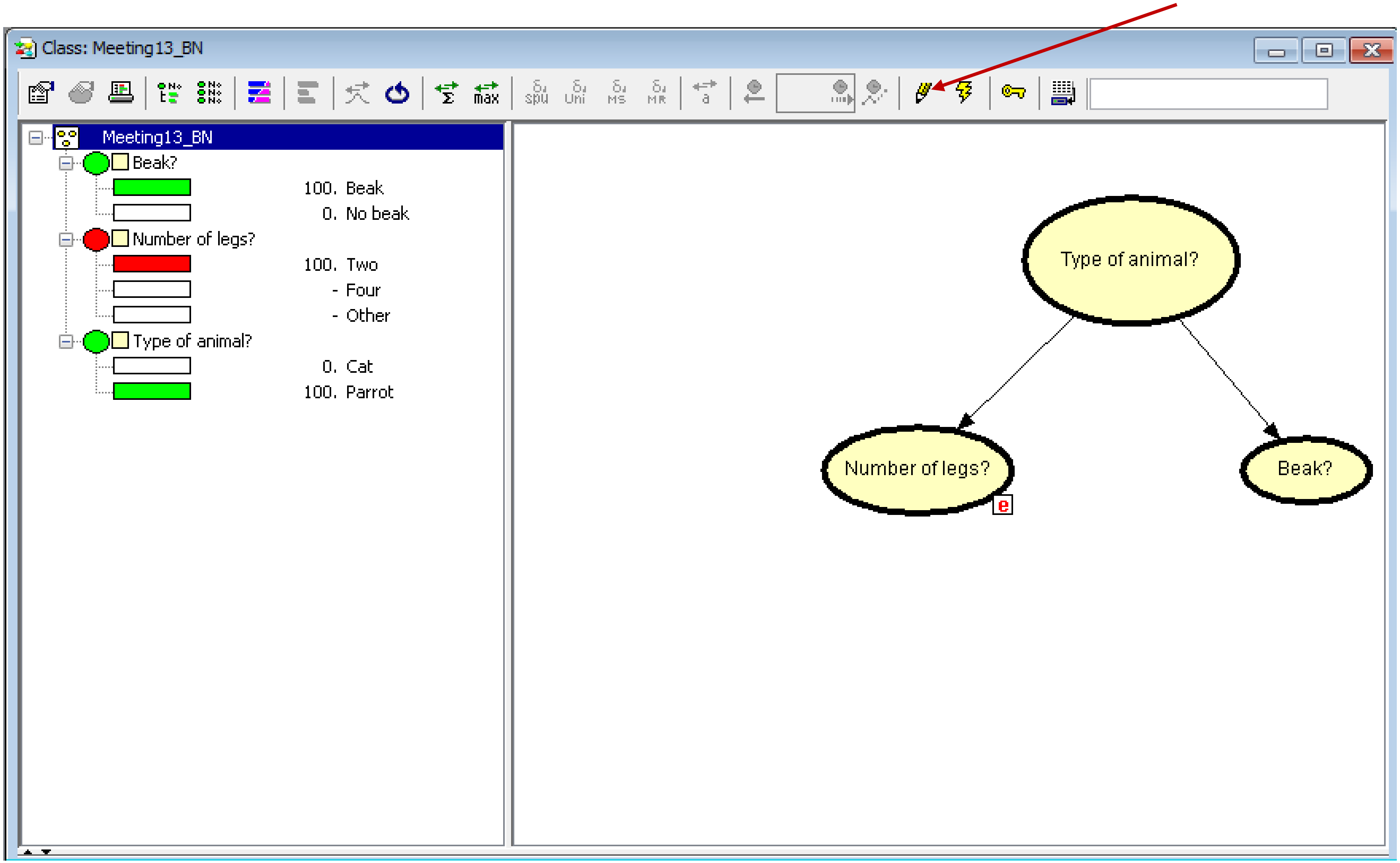
Hence, the probability is (maybe not so unexpected) exactly 100%

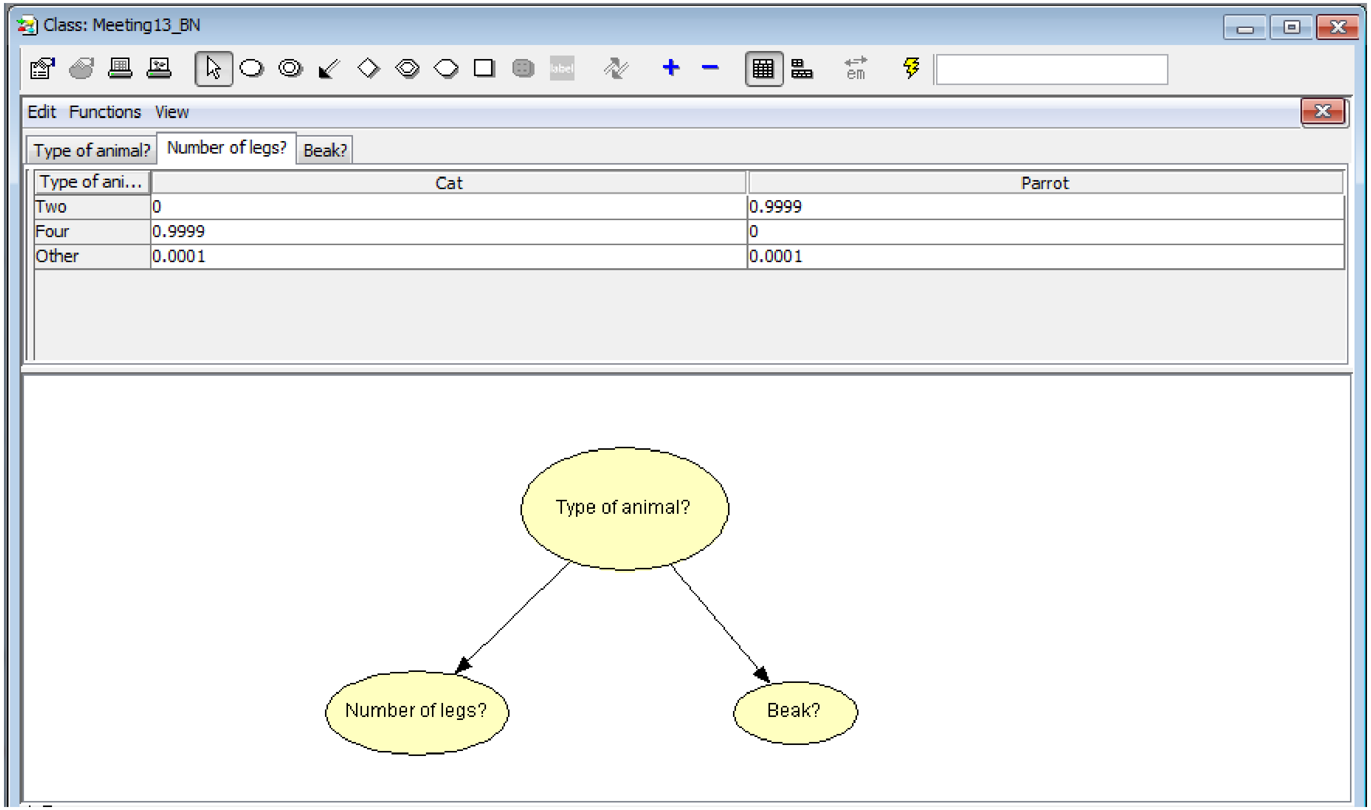
This can also be seen by setting Belief Precision to Max precision



The absence of displayed decimals indicate that the values displayed are exactly 100 and 0 respectively.

If we want to edit the network, e.g. adding nodes and/or changing assigned probabilities, we can return to Edit mode by clicking on the pencil icon.

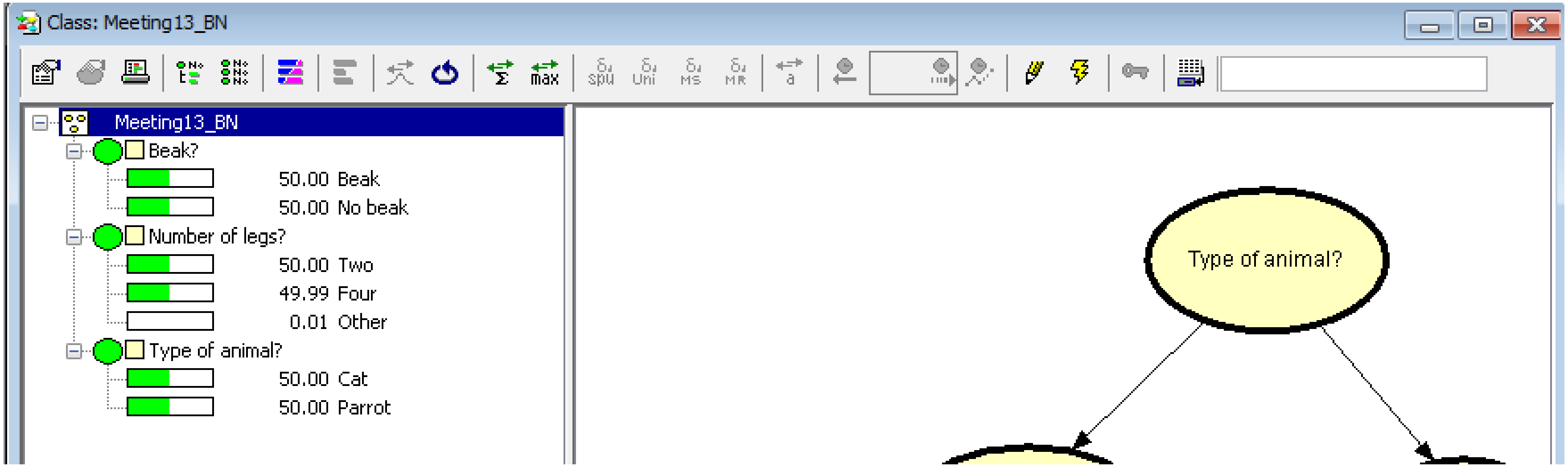




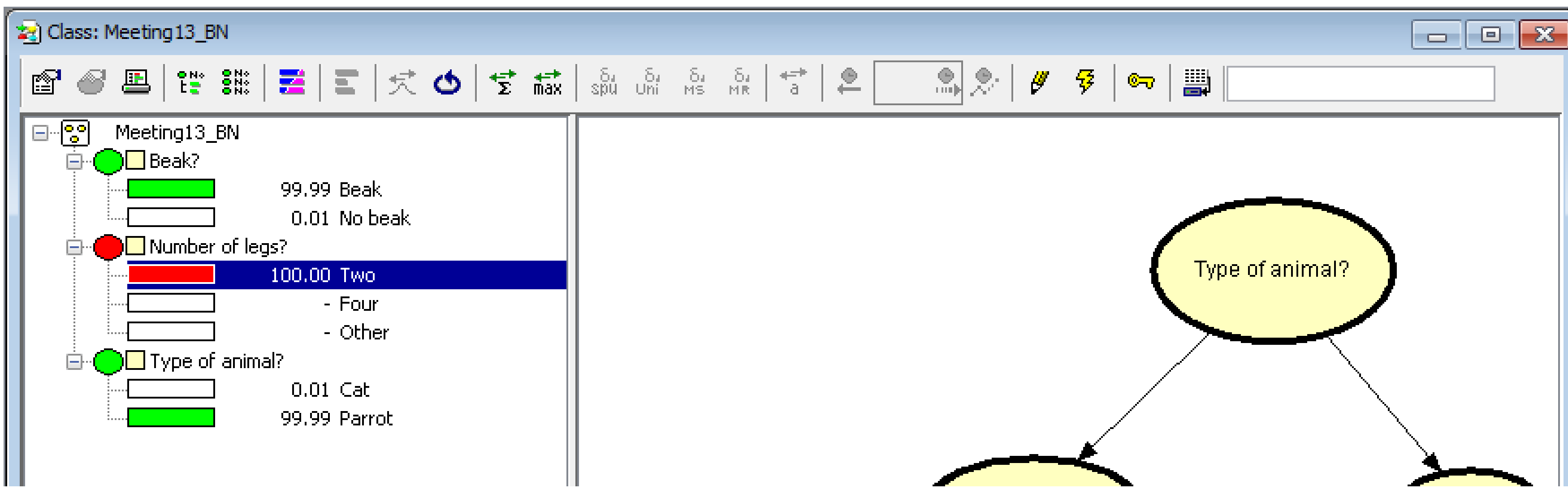
Now, change the conditional distribution of the number of legs given the type of animal is a cat to 0.0001, 0.9998 and 0.0001 respectively, and run network again.

Type of animal? | Number of legs? | Beak?

Type of ani...	Cat	
Two	~~0~~	0.0001
Four	~~0.9999~~	0.9998
Other	0.0001	0.0001



Again, instantiate state ”Two” in node Number of legs?



…and we can see that the updated probability for Willie being a parrot is now 0.9999.