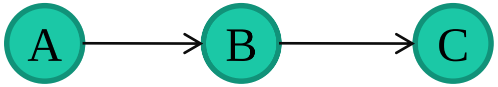
Meeting 14:
More on graphical models

2) Serial connection



There is a path between A and C (unidirectional from A to C).

➔ A may be relevant for C (and vice versa).

If the state of B is known this relevance is lost.: The path is *blocked*.

➔ A and C are *conditionally independent* given (a state of) B.

Example Who smashed the window?

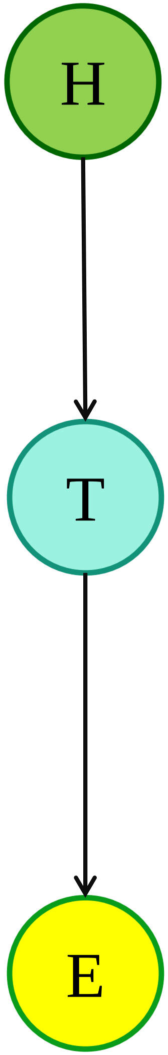
A window (pane) was smashed and a person, Mr G is suspected for having done it. On Mr G's pullover 8 glass fragments were recovered, they all matched the (pane of) the smashed window.


Let

H be a random variable with states H_1= "Mr G smashed the window" and H_2: "Someone (or something) else smashed the window".

T be a random variable for which the state is the number of fragments transferred to Mr G's pullover when the window was smashed. Note that if Mr G's pullover was not sufficiently near the window when it was smashed, then $T = 0$.

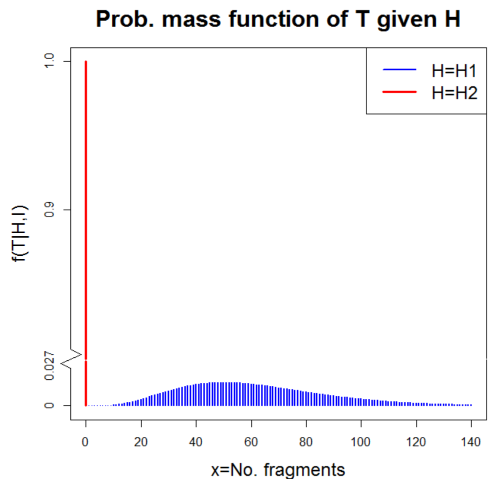
E be a random variable for which the state is the number of fragments that could be (and were) recovered from Mr G's pullover. Note that E is not equal to T since (i) it cannot be assumed that all fragments transferred to Mr G's pullover persisted and (ii) were detectable when analysing it.

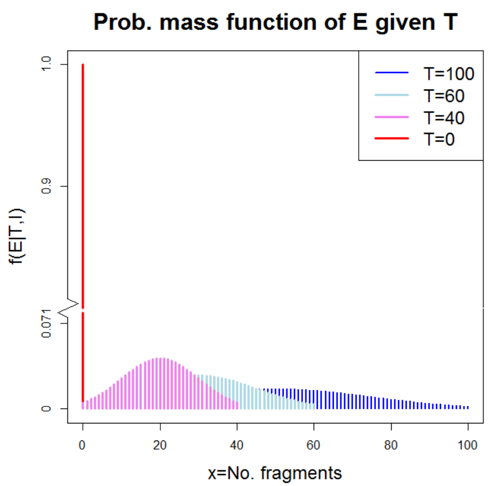


H	Probability
H_1	$P(\mathrm{H} = H_1 \vert I)$
H_2	$P(\mathrm{H} = H_2 \vert I)$

H_1= "Mr G smashed the window"

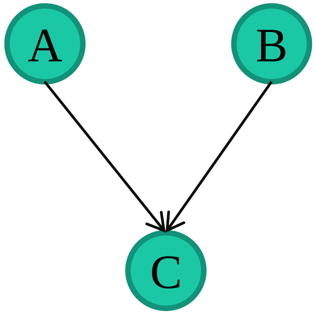
H_2: "Someone (or something) else smashed the window".





Once the value of T is known the state of H is no longer relevant for the state of E.

3) Converging connection



There is a path between A and B (not unidirectional).

➔ A may be relevant for B (and vice versa).

If the state of C is (completely) <u>unknown</u> this relevance does not exist.

If the state of C is known (exactly or by a modification of the state probabilities) the path is opened.

➔ A and C are *conditionally dependent* given information about the states of C, otherwise they are (conditionally) independent.

Example Paternity testing: child, mother and the true father

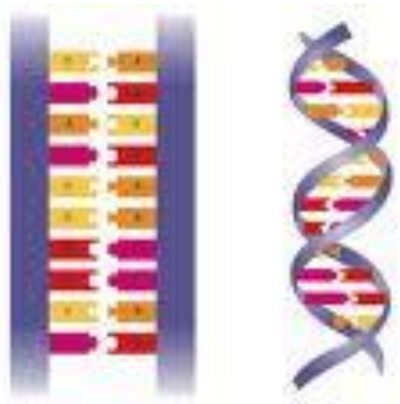
The non-coding genetic information in the DNA double Helix is (simply speaking) divided into *loci* (one *locus*), where a locus consists of one so-called *allele* inherited from the mother and one allele inherited from the father. The two alleles are said to constitute the *genotype* of that locus. For forensic purposes loci with many possible alleles (so-called polymorphic) are used in the DNA analysis. More about DNA will come in later in the course.

Let

A be a random variable representing the mother's genotype in a specific locus.

B be a random variable representing the **true** father's genotype in the same locus.

C be a random variable representing the child's genotype in that locus.



	Alleles	
A:	A_1	A_2
B:	B_1	B_2
C:	C_1	C_2

A:	A_1	A_2
B:	B_1	B_2
C:	C_1	C_2

A: Mother's genotype
B: True father's genotype
C: Child's genotype

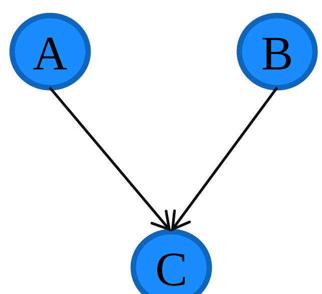
If we know nothing about C (C_1 and C_2 are both unknown) , then

- information about A cannot have any impact on B and vice versa

If we on the other hand know something about the genotype of the child (C_1 and C_2 are both known or one of them is) then

- knowledge of the genotype of the mother has impact on the probabilities of the different genotypes that can be possessed by the true father since the child must have inherited half of the genotype from the mother and the other half from the father

Bayesian network:



Influence diagrams

Decision-theoretic components can be added to a Bayesian network. The complete network is then related to as a *Bayesian Decision Network* or more common ***Influence diagram* (*ID*)**

Method of detection gives a positive result (detection) in 99 % of the cases when the dye is present, i.e. the proportion of false negatives is 1% and a negative result in 98 % of the cases when the dye is absent, i.e. the proportion of false positives is 2%

The presence of dye is rare: prevalence is about 0.1 %

Return to the example with banknotes.

Let

H_0: Dye is present

H_1: Dye is not present

States of the world

and let

E_1: Method gives positive detection

E_2: Method gives negative detection

Data

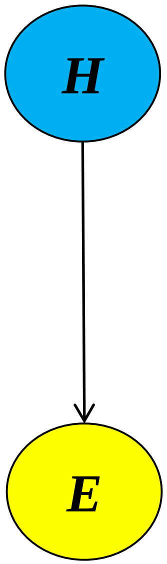
Assume that…

- The banknote is a SEK 100 banknote
- If we deem the banknote to have been contaminated with the dye, we will consider it as useless and it will be destroyed
- If we deem the banknote not to have been contaminated with the dye, we will use it (in the future) for ordinary purchasing
- Upon using the banknote for purchasing, if it is revealed (by other means than our method) that the banknote is contaminated with the dye, there is a fine of SEK 500

Hence, our loss function can be written

Action	State of the world	
	Dye is present (H_0)	Dye is not present (H_1)
Destroy banknote	0	100
Use banknote	500	0

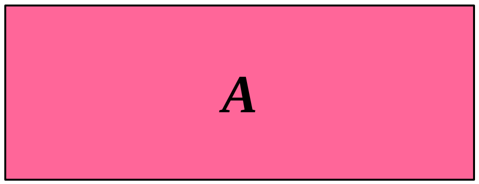
A simple Bayesian network can be constructed for the relevance between the state of nature and data:



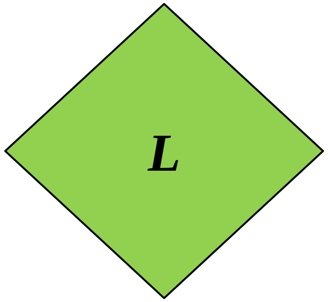
H	*Probabilities*
H_0	0.001
H_1	0.999

		Probabilities	
	H:	H_0	H_1
E	E_1	0.99	0.02
	E_2	0.01	0.98

Now, we will add two nodes to the network, one for the actions that can be taken and one for the loss function



A	
a_1	Destroy banknote
a_2	Use banknote

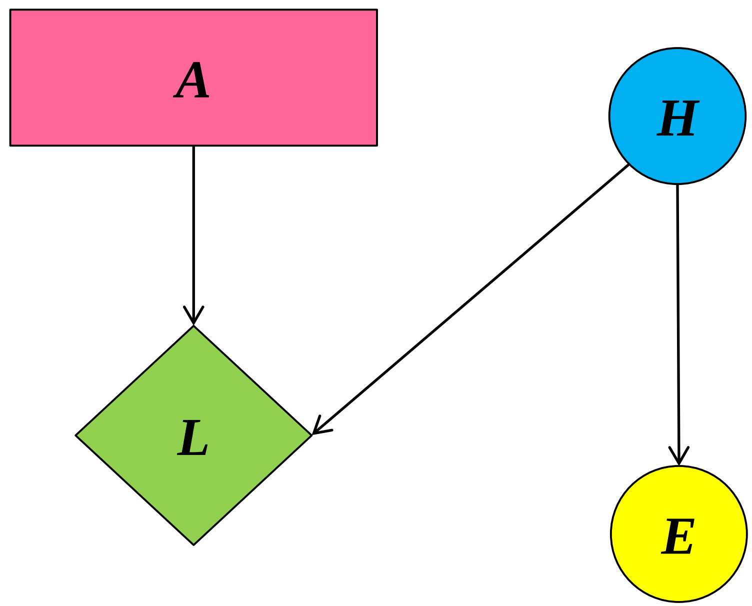


H:	H_0		H_1	
A:	a_1	a_2	a_1	a_2
L	0	500	100	0

H_0: Dye is present
H_1: Dye is not present

Neither of the nodes are random nodes.

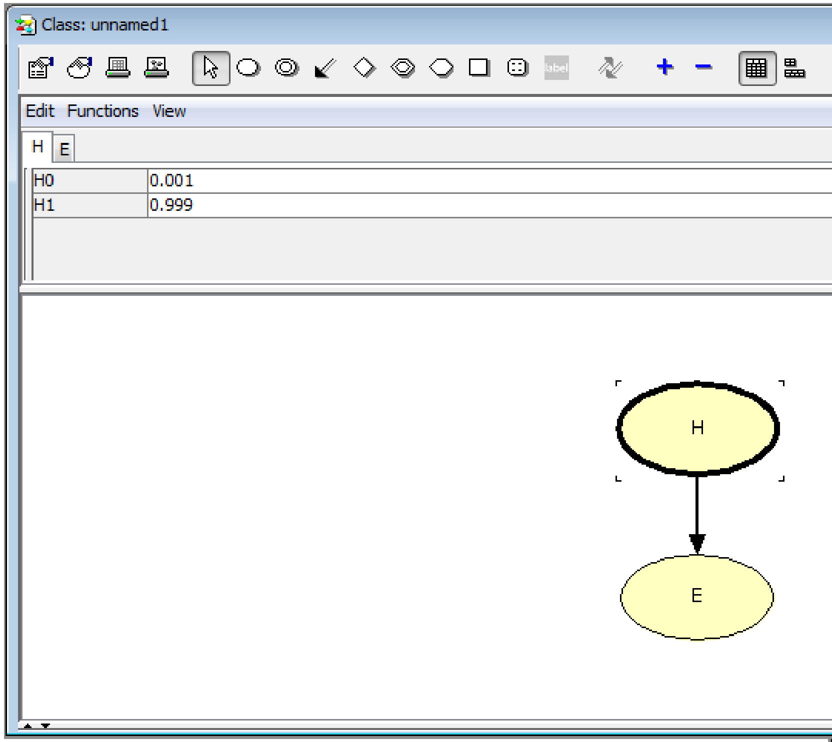
Node ***L*** must be a child node with nodes ***H*** and ***A*** as parents.



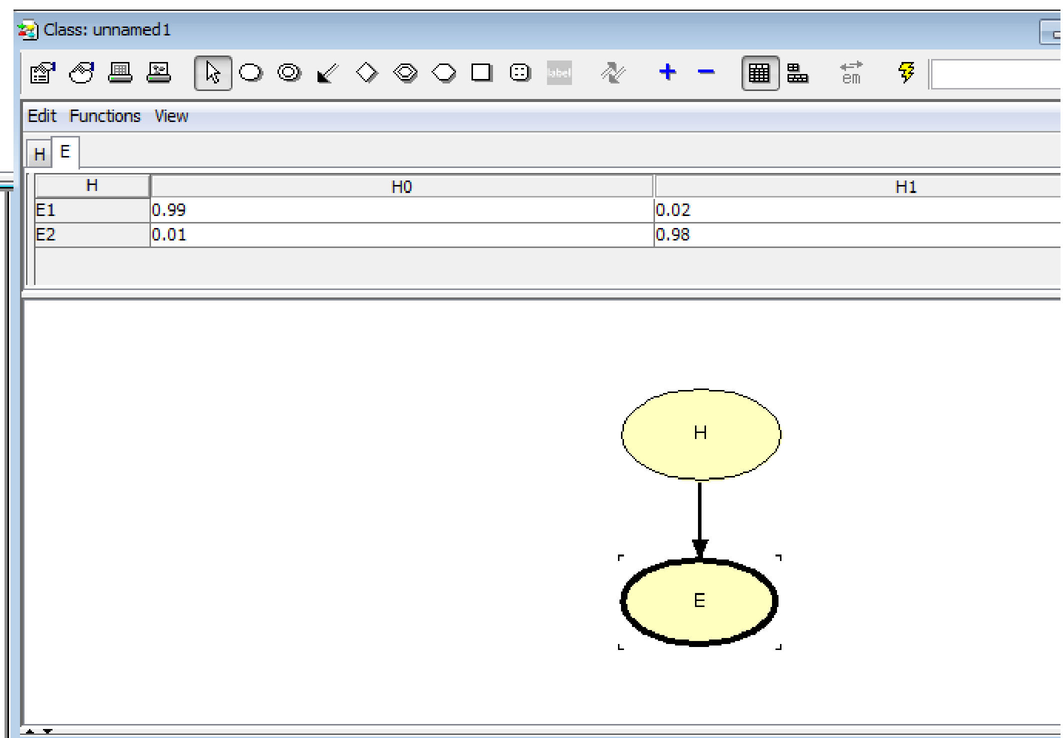
With this network, an influence diagram, we would like to be able to propagate data from node E to a choice of decision in node A.

Hence, in node A the expected posterior loss should be calculated, and the losses should be specified in node L.

Using Hugin:

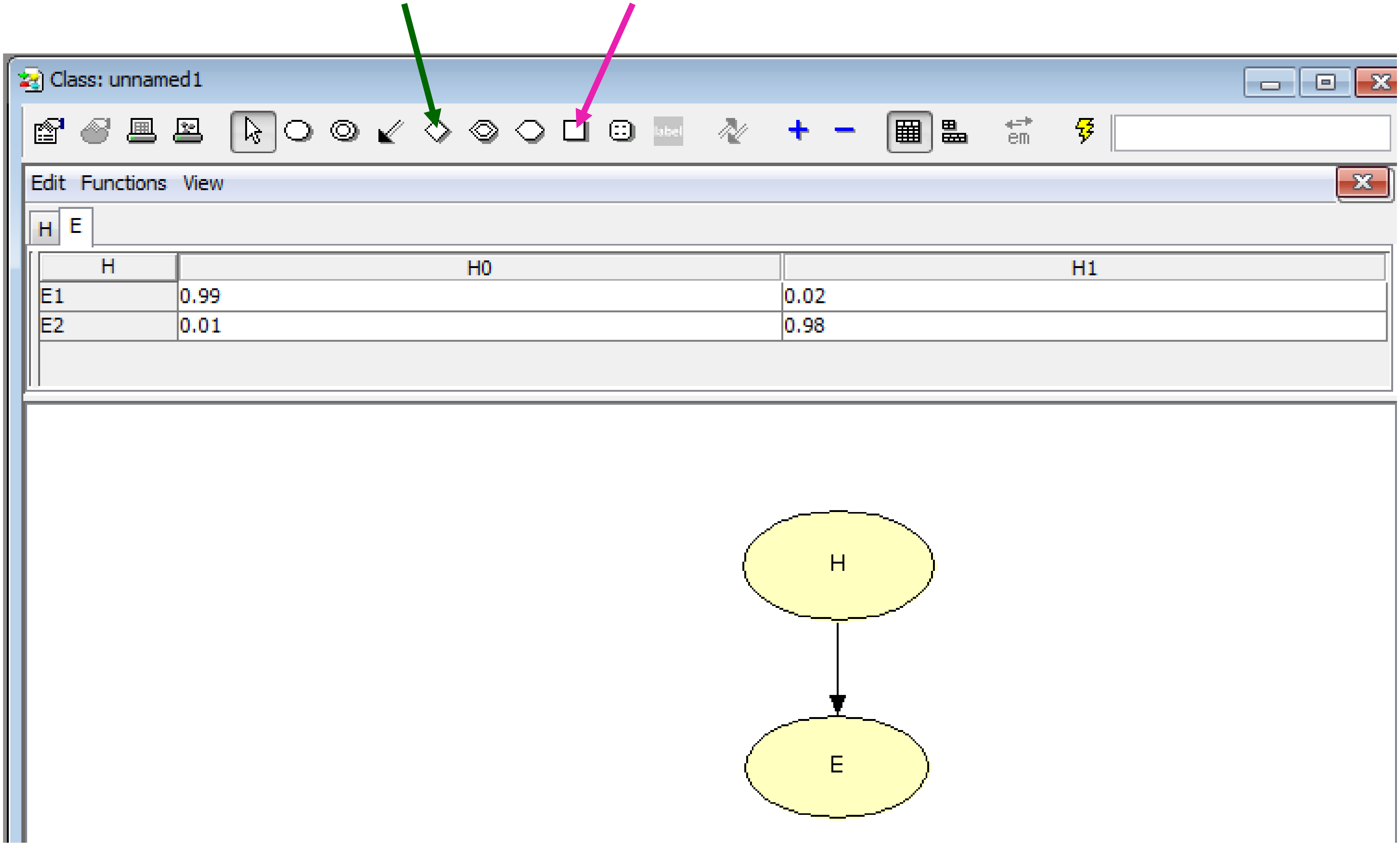


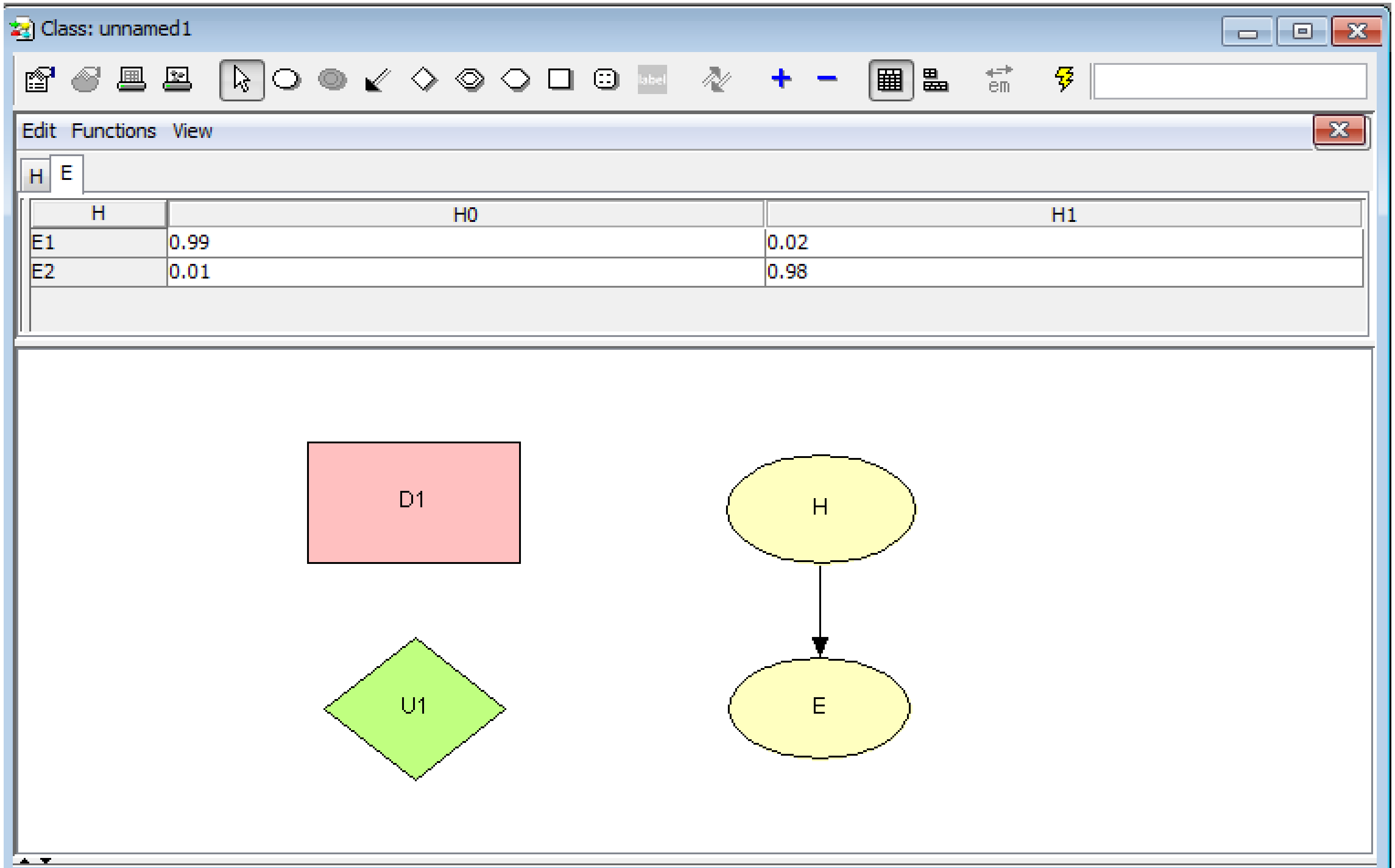
Class: unnamed1
Edit Functions View
H E
H0 0.001
H1 0.999
H
E



Class: unnamed1
Edit Functions View
H E
H H0 H1
E1 0.99 0.02
E2 0.01 0.98
H
E

Now, nodes for action and loss should be added.

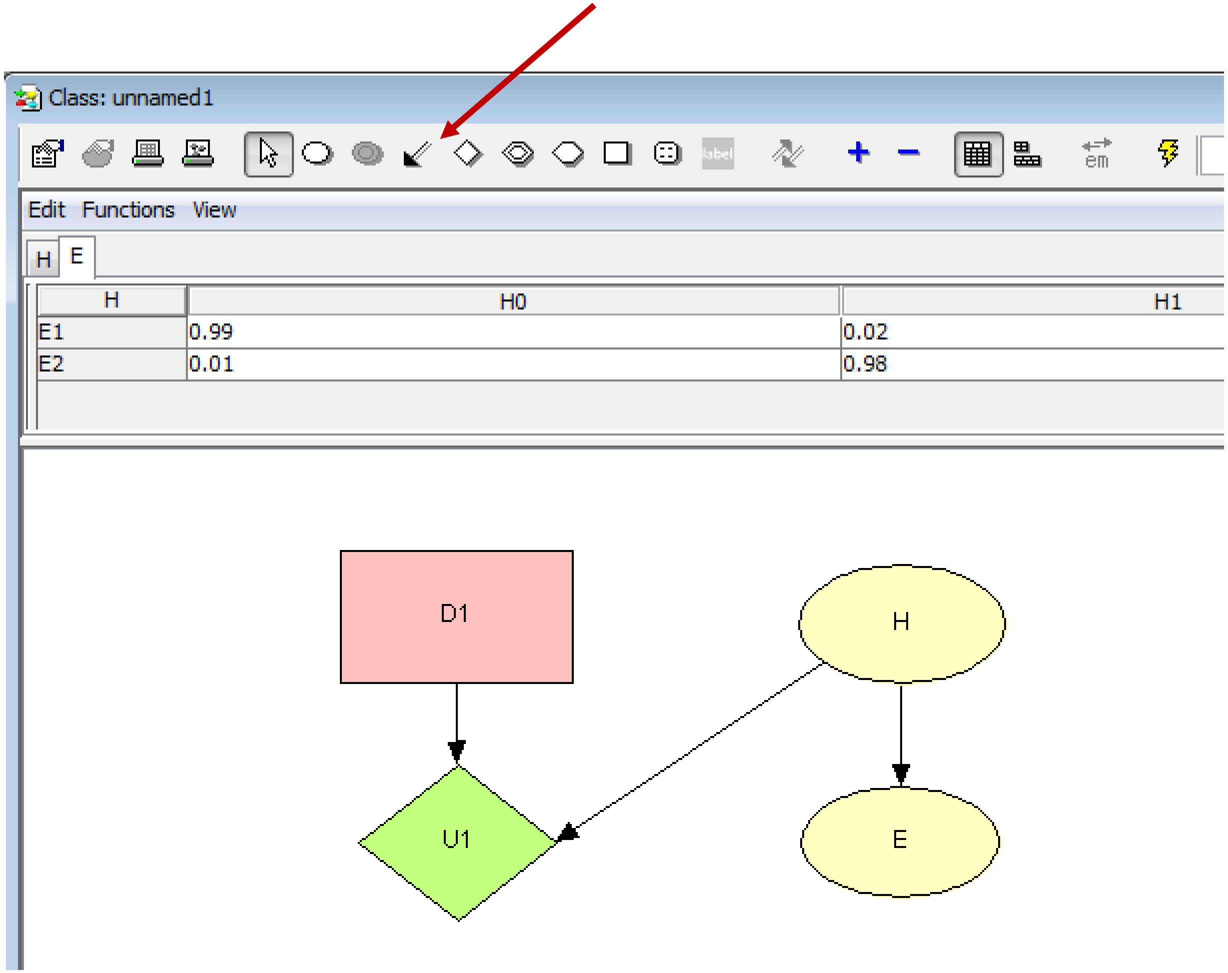




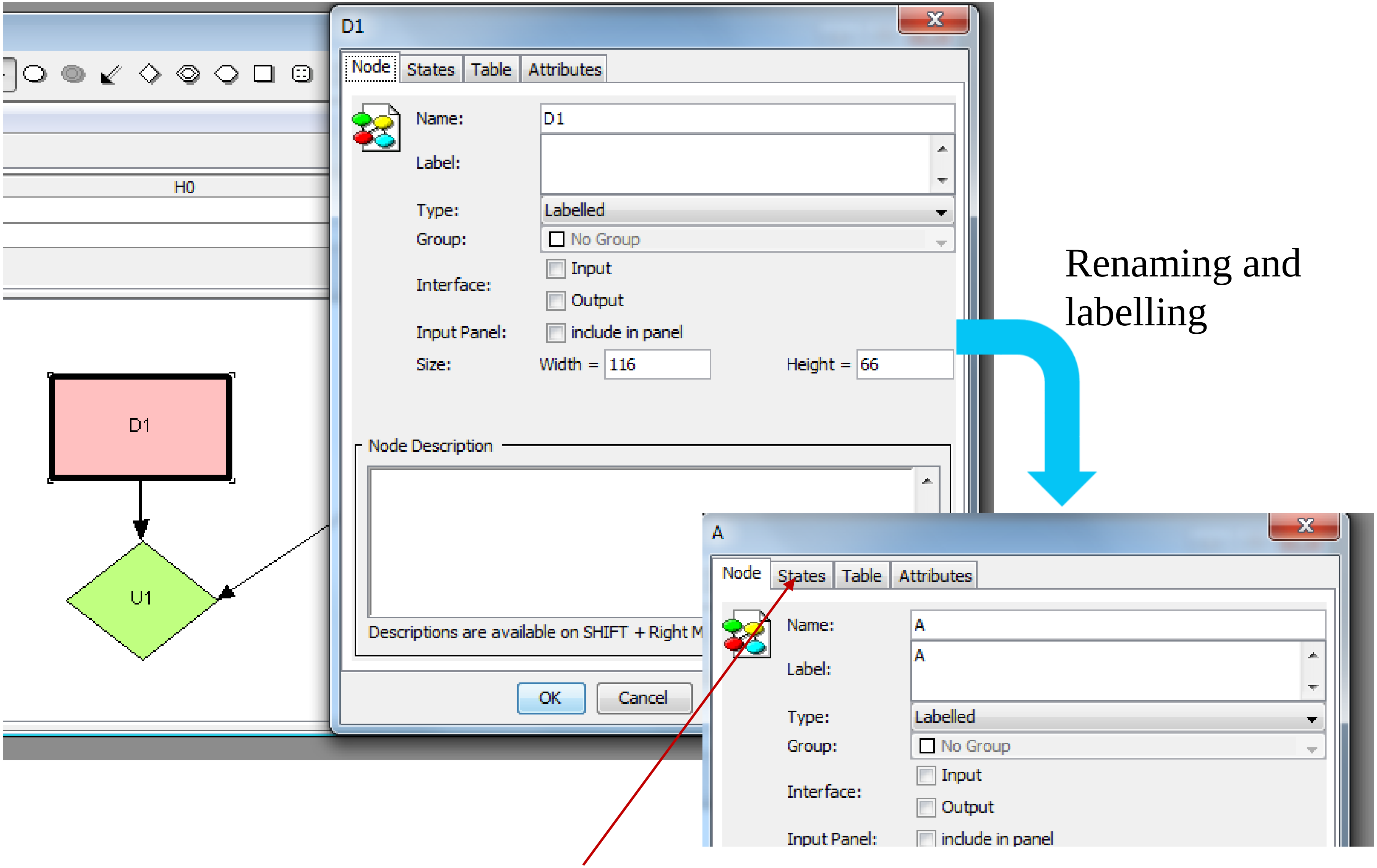
Action nodes by default gets a name with D (for decision) and number.
Utility nodes by default gets a name with U and a number.

We will use negative utilities as losses.

Add the links (drag from interior of parent node to interior of child node).

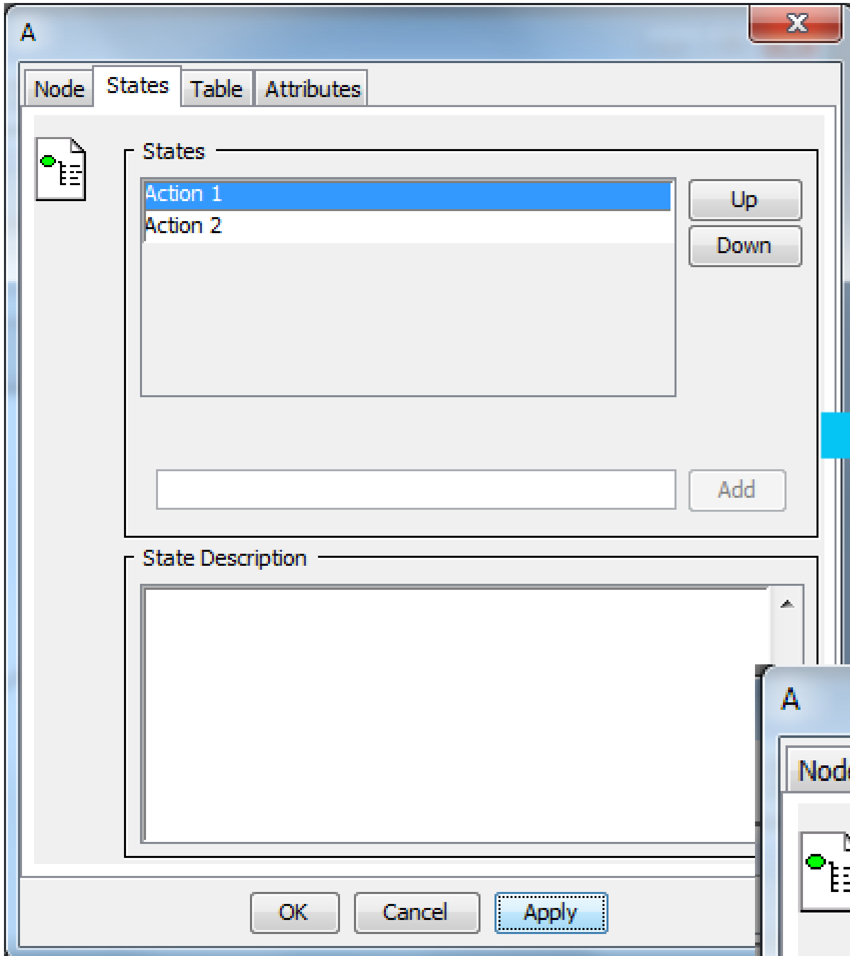


Now, double-click on the action node.

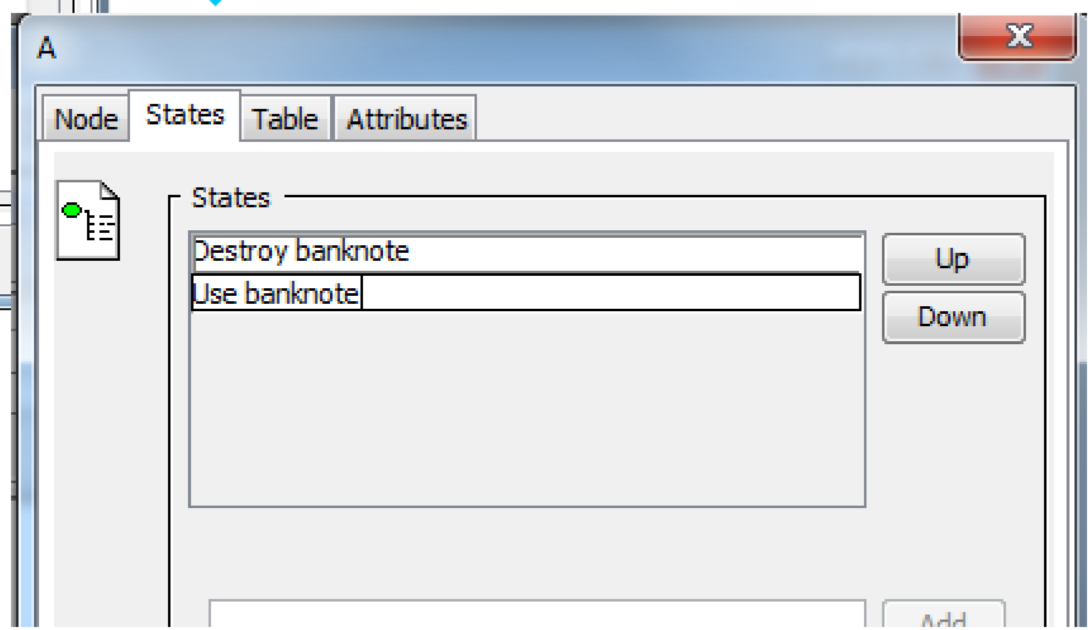


Renaming and labelling

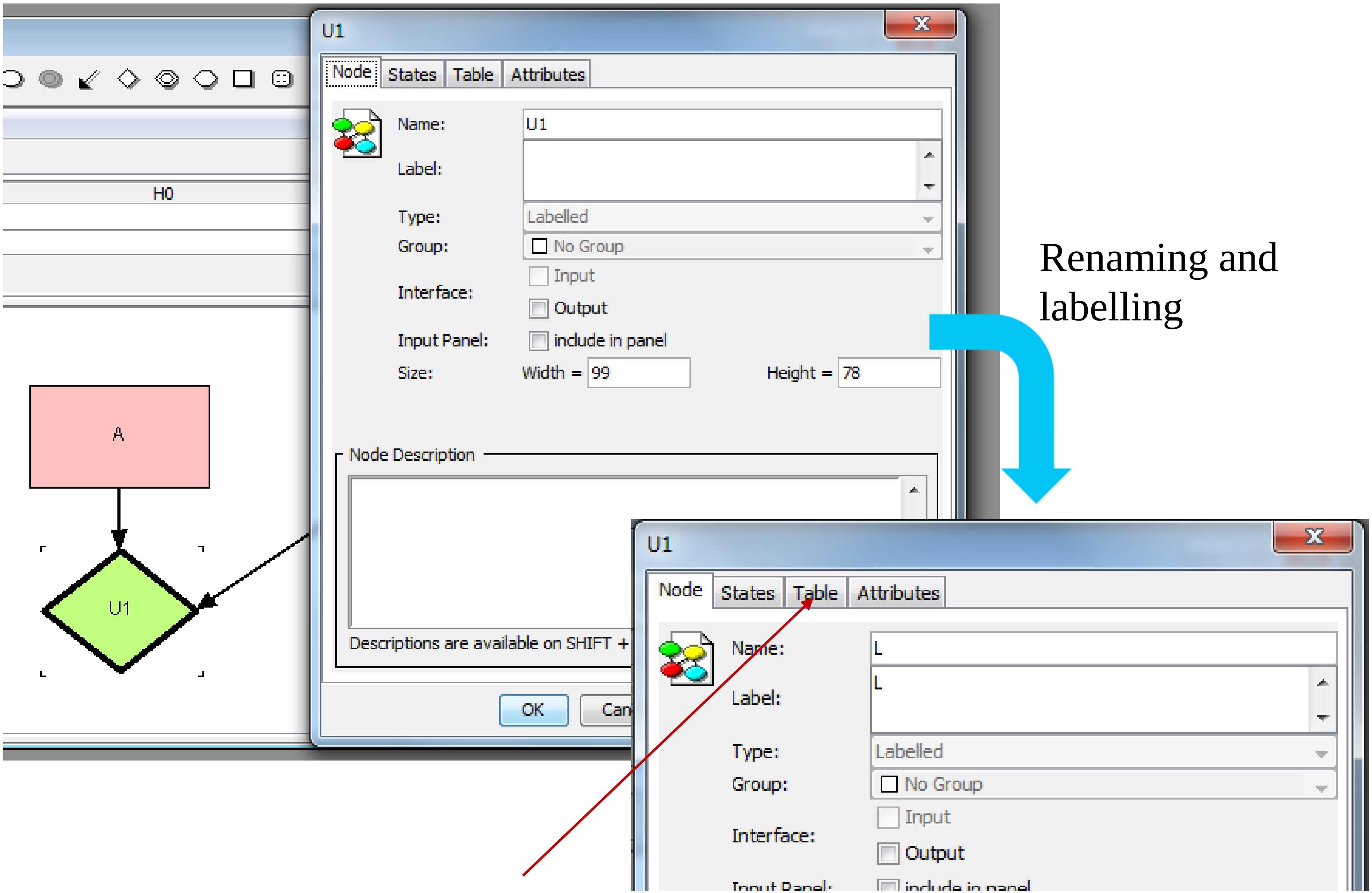
Select tab States.



Renaming the states

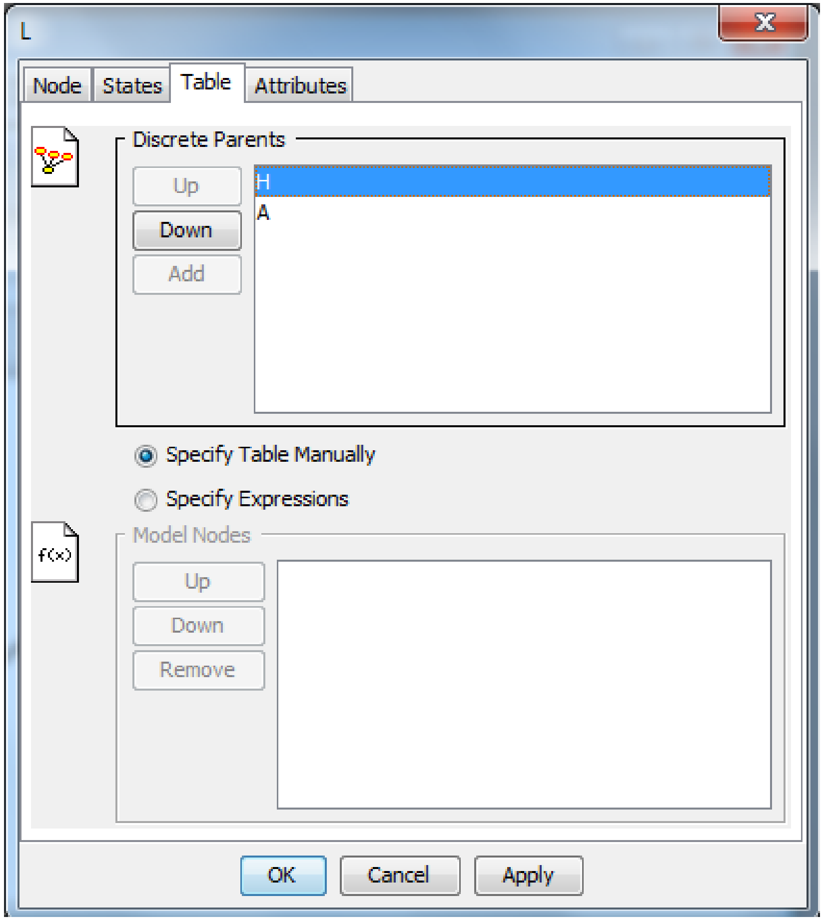


Double-clicking the utility node…



Renaming and labelling

Select tab Table.



Here, we can set the preference order in which the states of H and states of A should appear in the utility table.

The “ordinary” two-way table…

	H_0	H_1
a_1	$U(a_1, H_0)$	$U(a_1, H_1)$
a_2	$U(a_2, H_0)$	$U(a_2, H_1)$

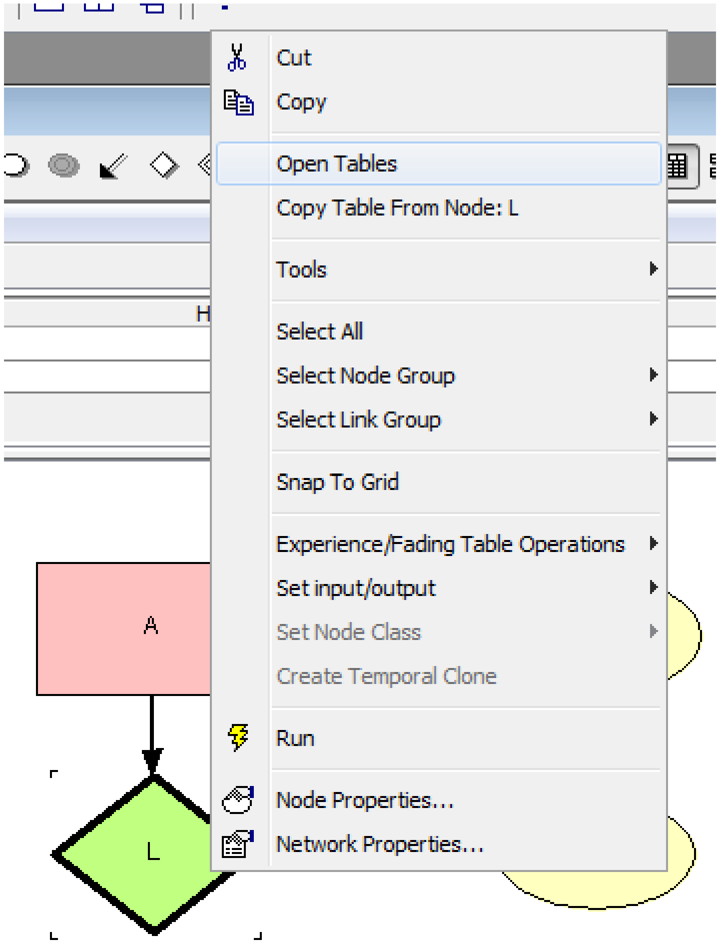
…will in Hugin appear either as… …or as… …depending on the order set.

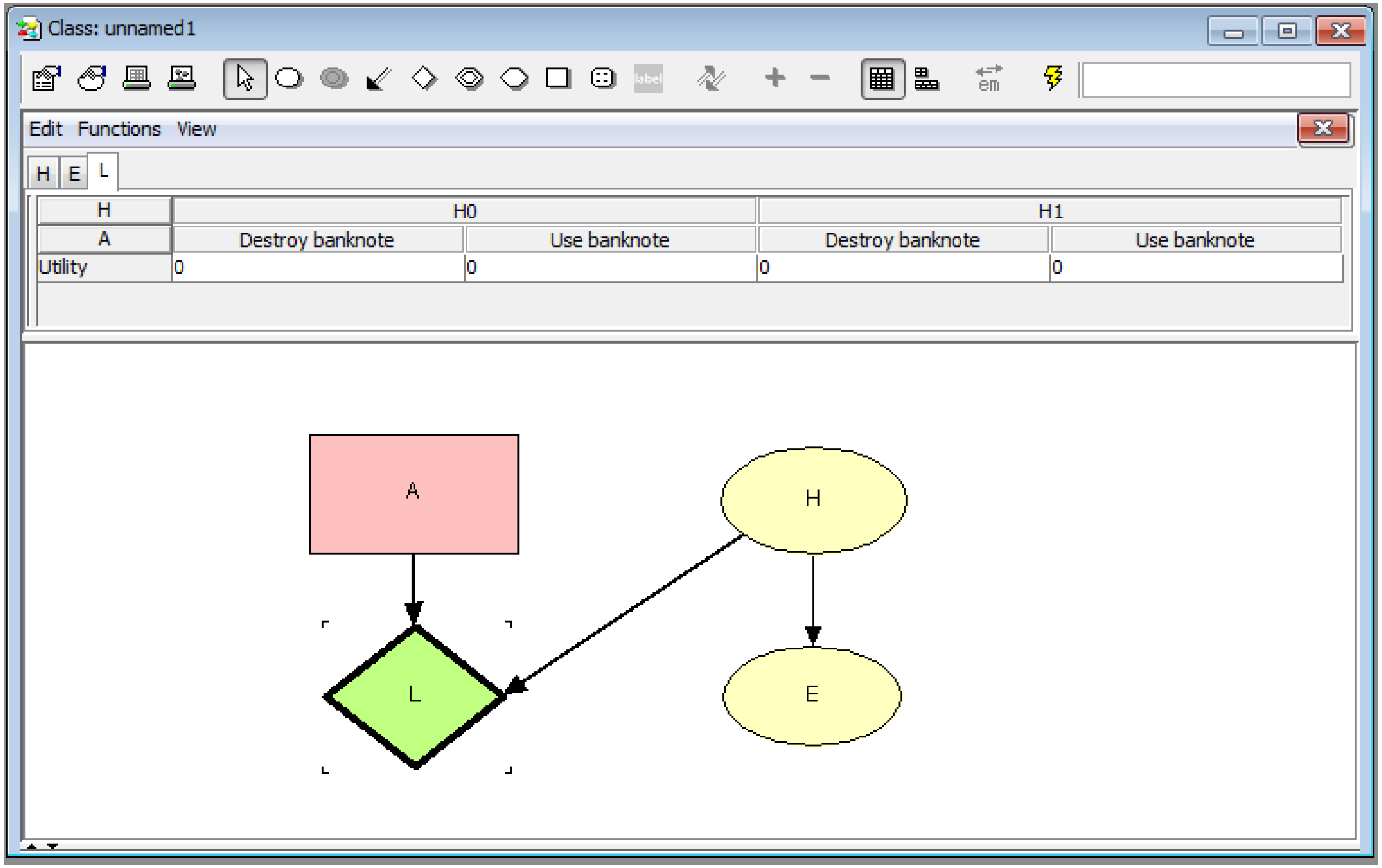
H	H_0		H_1	
A	a_1	a_2	a_1	a_2
U (***L***)	$U(a_1, H_0)$	$U(a_2, H_0)$	$U(a_1, H_1)$	$U(a_2, H_1)$

A	a_1		a_2	
H	H_0	H_1	H_0	H_1
U (***L***)	$U(a_1, H_0)$	$U(a_1, H_1)$	$U(a_2, H_0)$	$U(a_2, H_1)$

Here, we keep the order as given.

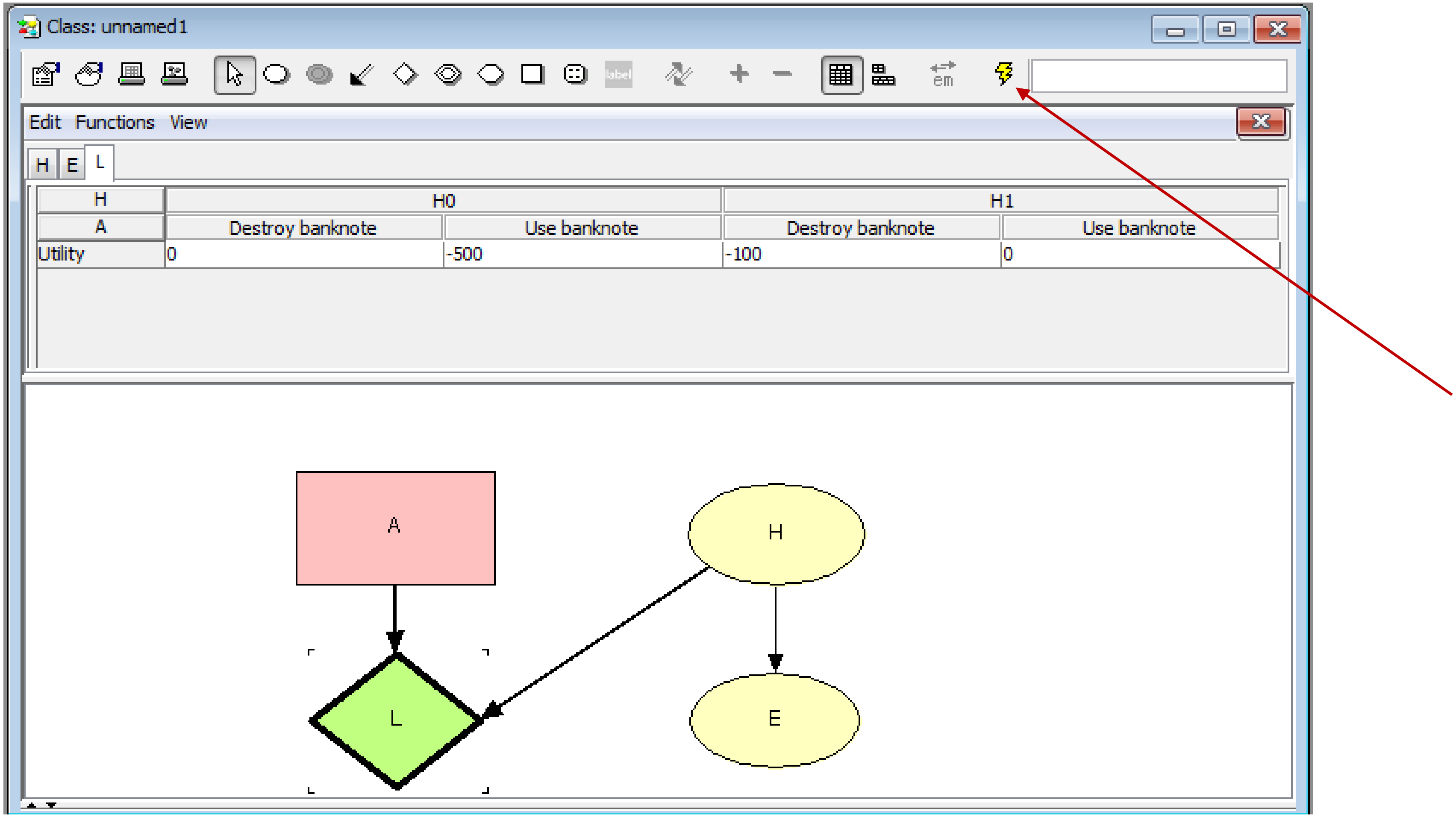
Righ-click on the utility node (L) and select Open Tables.



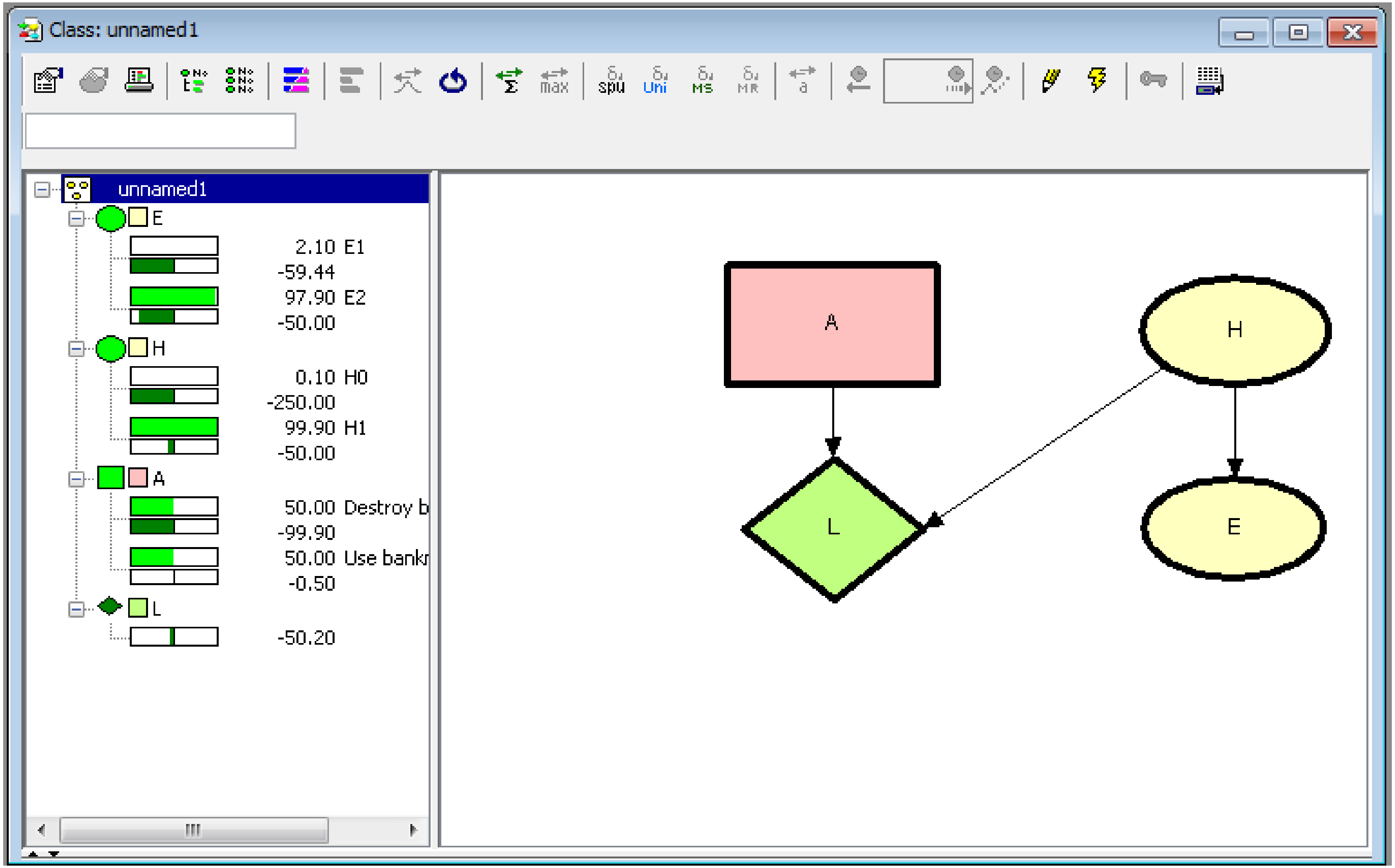


Now, enter manually the utilities as negative losses in the table.

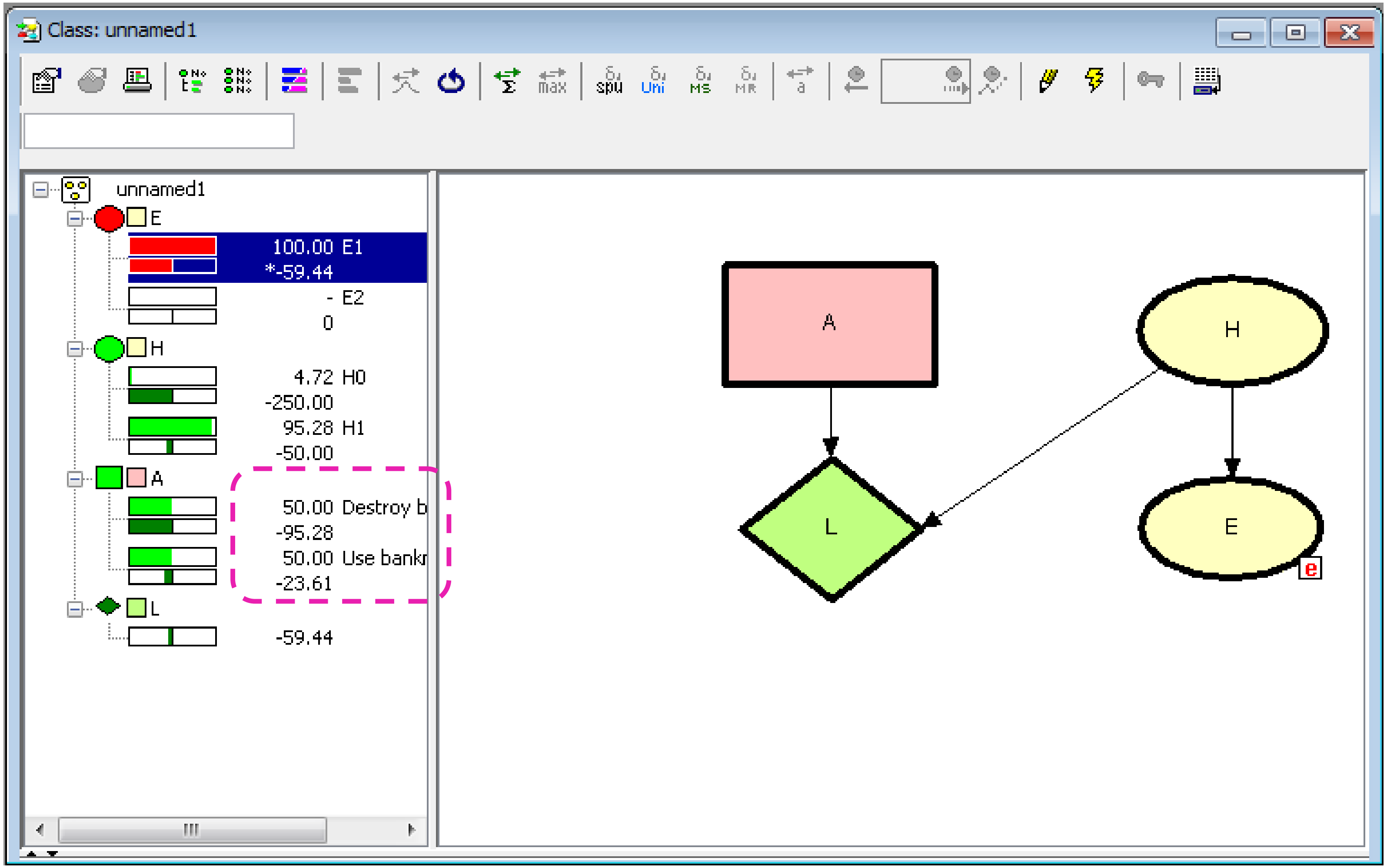
H:	H_0		H_1	
A:	a_1	a_2	a_1	a_2
L	0	500	100	0



Run the network by clicking the flash icon.



Marginal probabilities and averages utilities (losses) are displayed. Now, enter the evidence = “Method gives positive detection” (E_1), by double-clicking on E1.



Here we can read off the calculated expected utilities for each of the two action. Since the expected utility of a_1 (Destroy banknote) –95.28 is lower than the expected utility of a_2 (Use banknote) –23.61, the optimal action is a_2.

Exercise 7.14

14. Suppose that a contractor must decide whether or not to build any speculative houses (houses for which he would have to find a buyer), and if so, how many. The houses that this contractor builds are sold for a price of $30,000, and they cost him $26,000 to build. Since the contractor cannot afford to have too much cash tied up at once, any houses that remain unsold three months after they are completed will have to be sold to a realtor for $25,000. The contractor's prior distribution for $\tilde{\theta}$, the number of houses that will be sold within three months of completion, is:

θ	$P(\tilde{\theta} = \theta)$
0	0.05
1	0.10
2	0.10
3	0.20
4	0.25
5	0.20
6	0.10

If the contractor's utility function is linear with respect to money, how many houses should he build? How much should he be willing to pay to find out for certain how many houses will be sold within three months?

The state of the world is how many houses, θ , that will be sold within tree months of completion. Possible values are 0, 1, 2, 3, 4, 5, 6.

The action to be taken is how many houses to be built,
i.e. a_k = "Build k houses" , k = 0,1,2,3,4,5,6 (the number cannot be higher than the maximum value of θ.

For each house built the payoff is 4 thousand dollars if it is sold and –1 thousand dollars if it has to be sold to a realtor.

The payoff function can then be written

$$R(a_k, \theta) = \begin{cases} 4 \cdot k & \text{if } k \leq \theta \\ 4 \cdot \theta - 1 \cdot (k - \theta) = 5 \cdot \theta - k & \text{if } k > \theta \end{cases}$$

The payoff table then becomes

a_k \ θ	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	–1	4	4	4	4	4	4
2	–2	3	8	8	8	8	8
3	–3	2	7	12	12	12	12
4	–4	1	6	11	16	16	16
5	–5	0	5	10	15	20	20
6	–6	–1	4	9	14	19	24

The (prior) probability distribution of $\tilde{\theta}$ is

θ	0	1	2	3	4	5	6
$P(\tilde{\theta}) = \theta$	0.05	0.10	0.10	0.20	0.25	0.20	0.10

The (prior) expected payoffs for each action are

a_k	$ER(a_k)$
0	0
1	(–1)·0.05 + 4·0.10 + 4·0.10 + 4·0.20 + 4·0.25 + 4·0.20 + 4·0.10 = 3.75
2	(–2)·0.05 + 3·0.10 + 8·0.10 + 8·0.20 + 8·0.25 + 8·0.20 + 8·0.10 = 7
3	(–3)·0.05 + 2·0.10 + 7·0.10 + 12·0.20 + 12·0.25 + 12·0.20 + 12·0.10 = 9.75
4	(–4)·0.05 + 1·0.10 + 6·0.10 + 11·0.20 + 16·0.25 + 16·0.20 + 16·0.10 = 11.5
5	(–5)·0.05 + 0·0.10 + 5·0.10 + 10·0.20 + 15·0.25 + 20·0.20 + 20·0.10 = **12**
6	(–6)·0.05 + (–1)·0.10 + 4·0.10 + 9·0.20 + 14·0.25 + 19·0.20 + 24·0.10 = 11.5

$\Rightarrow$ The optimal action according to the *ER*-criterion is to build 5 houses.

Losses for a_5 ($L(a_k, \theta) = \max_j \left(R\left(a_j, \theta\right)\right) - R(a_k, \theta)$)

θ	0	1	2	3	4	5	6
$\max_k\left(R(a_k, \theta)\right)$	0	4	8	12	16	20	24
$R(a_5, \theta)$	–5	0	5	10	15	20	20
$L(a_5, \theta)$	5	4	3	2	1	0	4

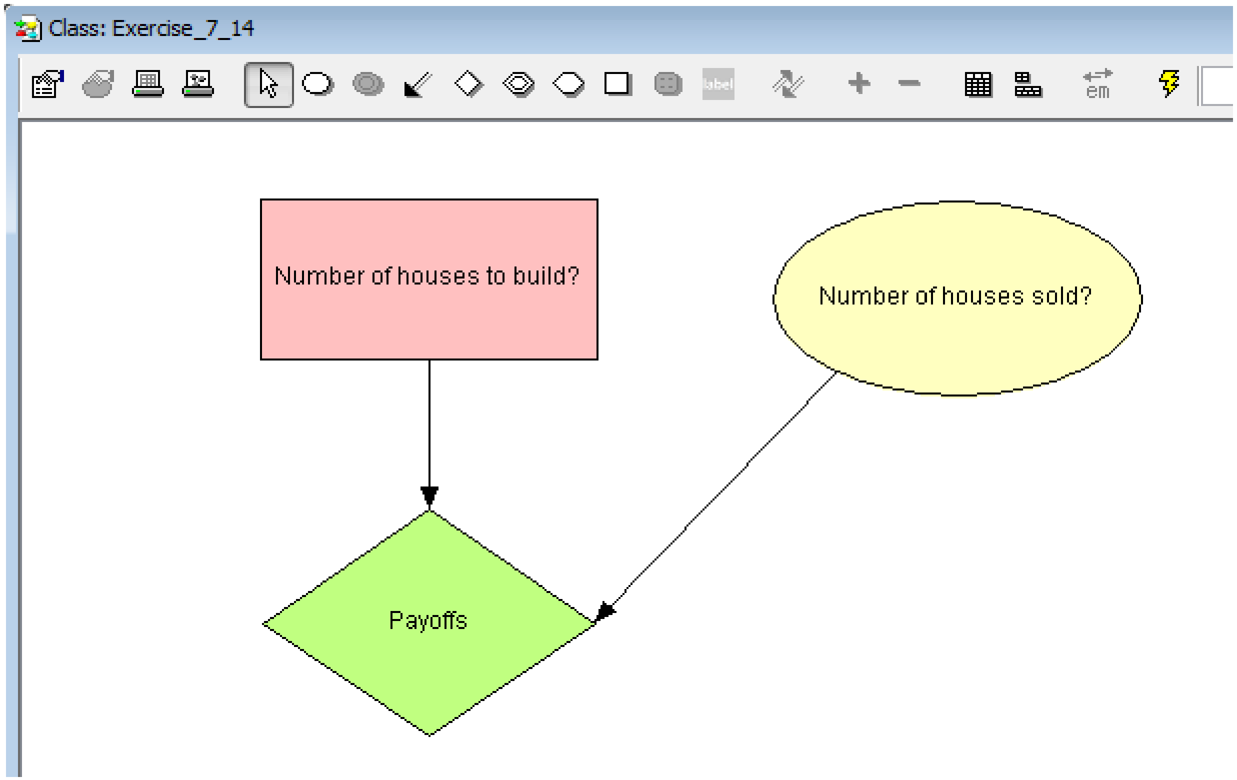
$\Rightarrow$ The expected loss for action a_5 is

$5 \cdot 0.05 + 4 \cdot 0.10 + 3 \cdot 0.10 + 2 \cdot 0.20 + 3 \cdot 0.25 + 0 \cdot 0.20 + 4 \cdot 0.10 = 2$

Since $EVPI = EL(a_{opt})$ the contractor should be willing to pay 2 thousand dollars to find out for certain how many houses will be sold.

With Hugin:

Influence diagram:



Note that no evidence node with observed data is present. The inference is from the prior distribution of θ.

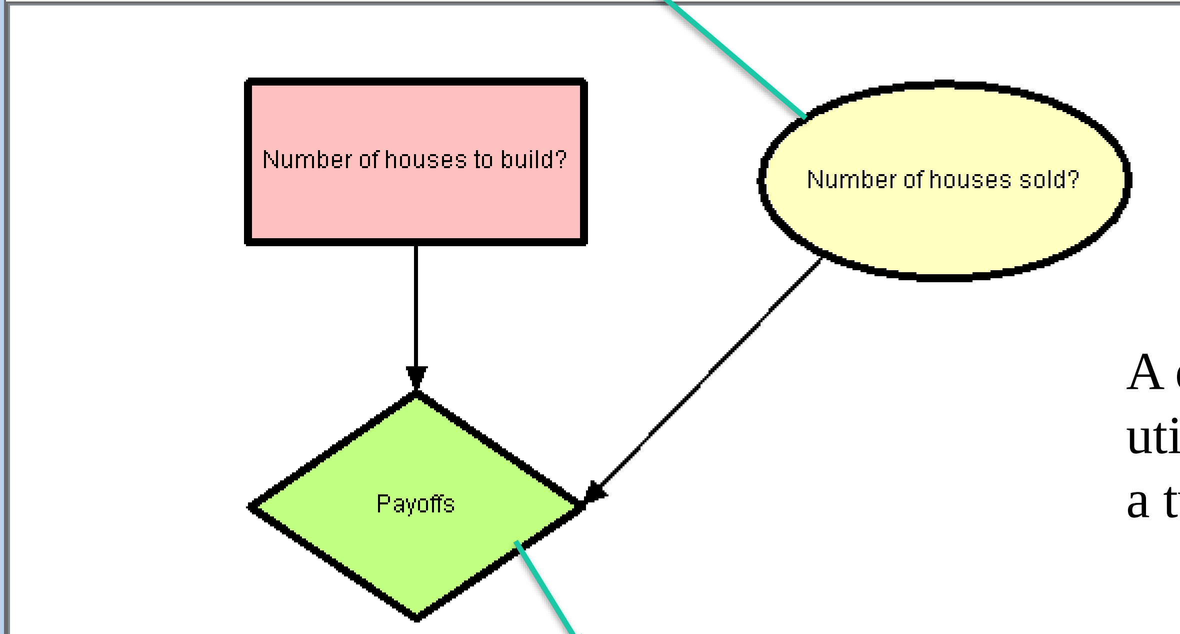
Tables:

Class: Exercise_7_14

Edit Functions View

Number of houses to build? | Payoffs | Number of houses sold?

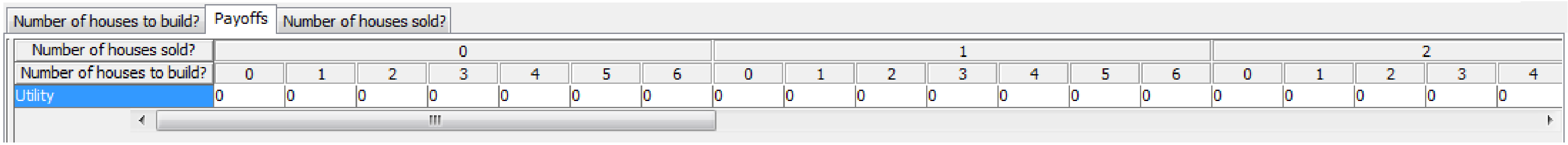
1	0.1
2	0.1
3	0.2
4	0.25
5	0.2
6	0.1



A drawback here is that the utility table cannot be viewed as a two-way table.

Number of houses to build? | Payoffs | Number of houses sold?

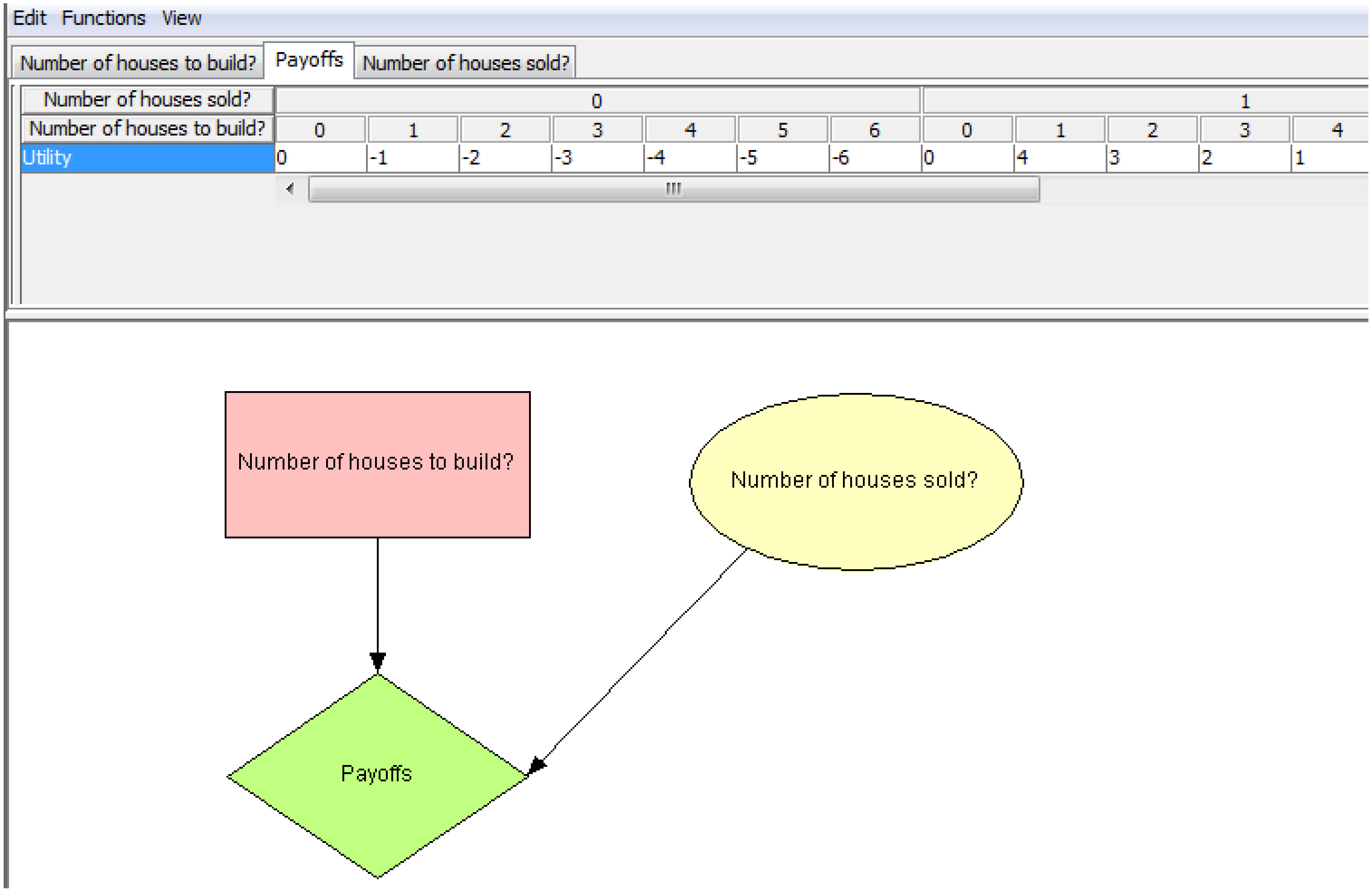
Number of houses sold?	0							1							2				
Number of houses to build?	0	1	2	3	4	5	6	0	1	2	3	4	5	6	0	1	2	3	4
Utility	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0



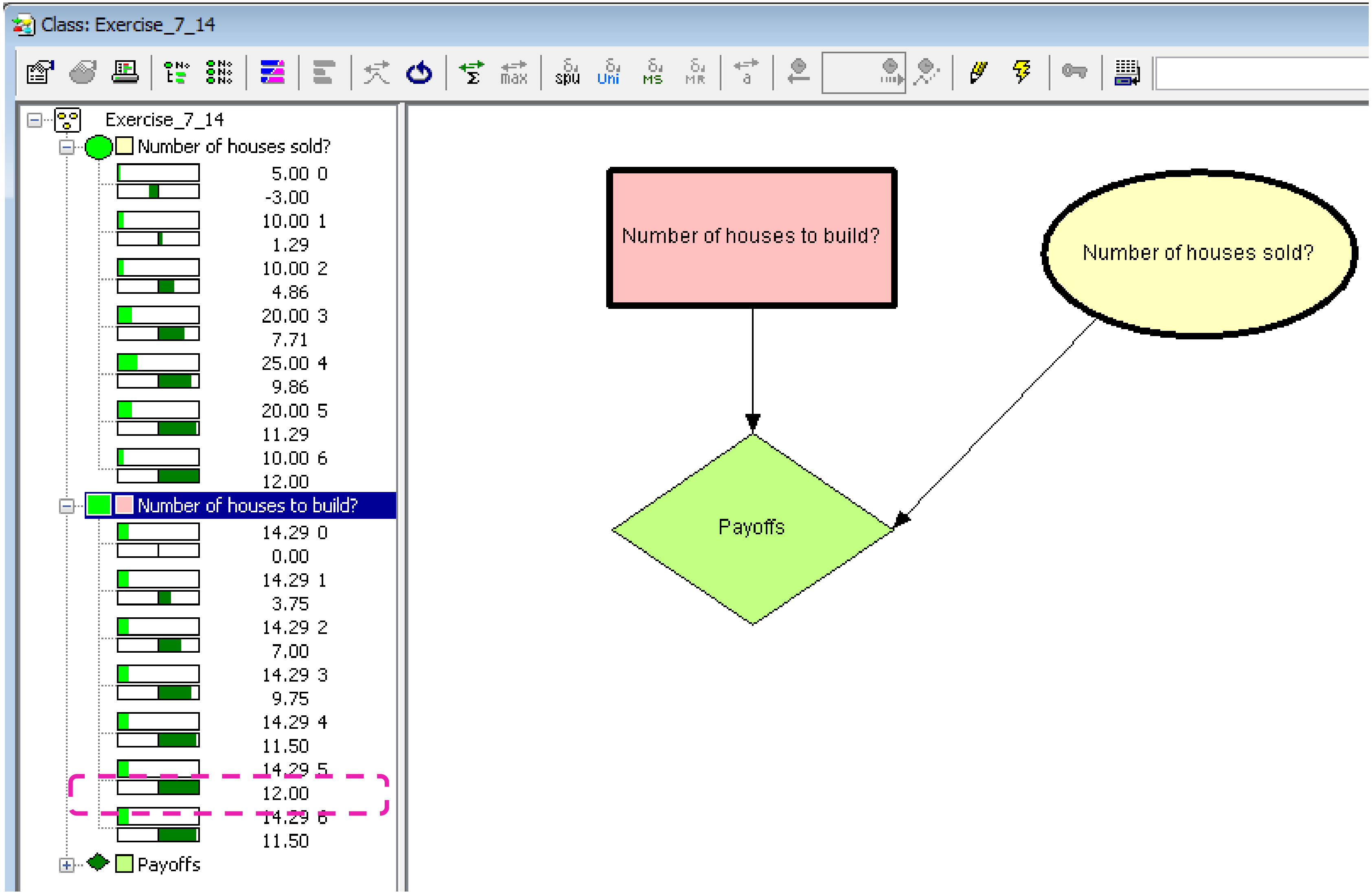
Number of houses sold?	0							1							2				
Number of houses to build?	0	1	2	3	4	5	6	0	1	2	3	4	5	6	0	1	2	3	4
Utility	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Utilities (here payoffs) from the payoff table shall be entered column wise.

a_k \ θ	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	–1	4	4	4	4	4	4
2	–2	3	8	8	8	8	8
3	–3	2	7	12	12	12	12
4	–4	1	6	11	16	16	16
5	–5	0	5	10	15	20	20
6	–6	–1	4	9	14	19	24



Run the network (flash icon).

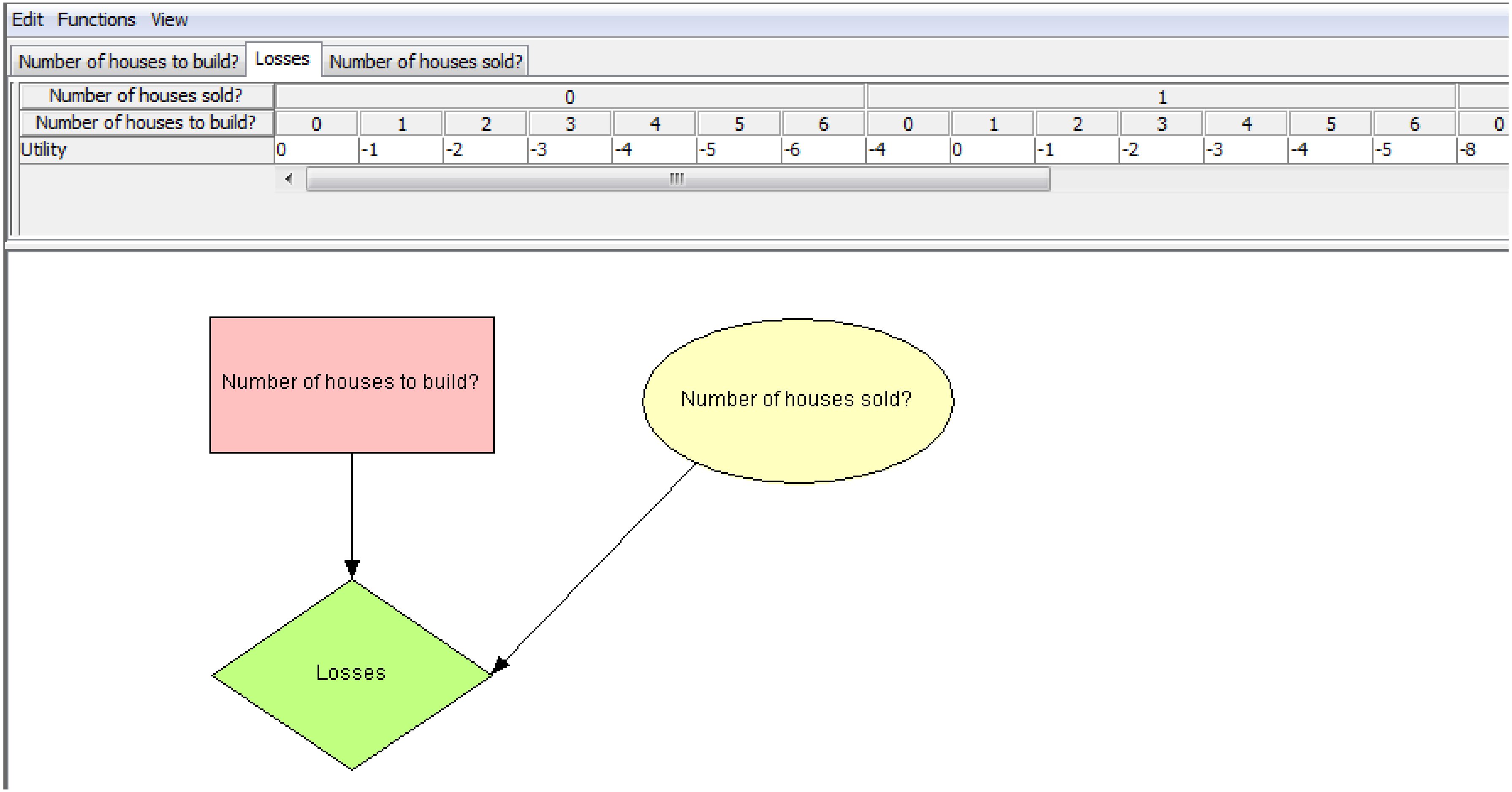


Class: Exercise_7_14
Exercise_7_14
Number of houses sold?
5.00 0
-3.00
10.00 1
1.29
10.00 2
4.86
20.00 3
7.71
25.00 4
9.86
20.00 5
11.29
10.00 6
12.00
Number of houses to build?
14.29 0
0.00
14.29 1
3.75
14.29 2
7.00
14.29 3
9.75
14.29 4
11.50
14.29 5
12.00
14.29 6
11.50
Payoffs
Number of houses to build?
Number of houses sold?
Payoffs

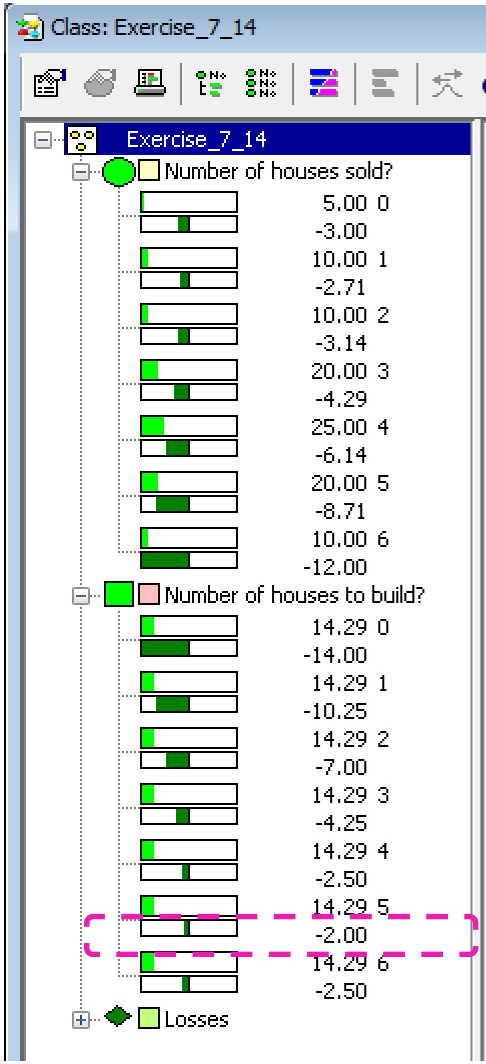
We could also as utilities enter the losses with negative sign.

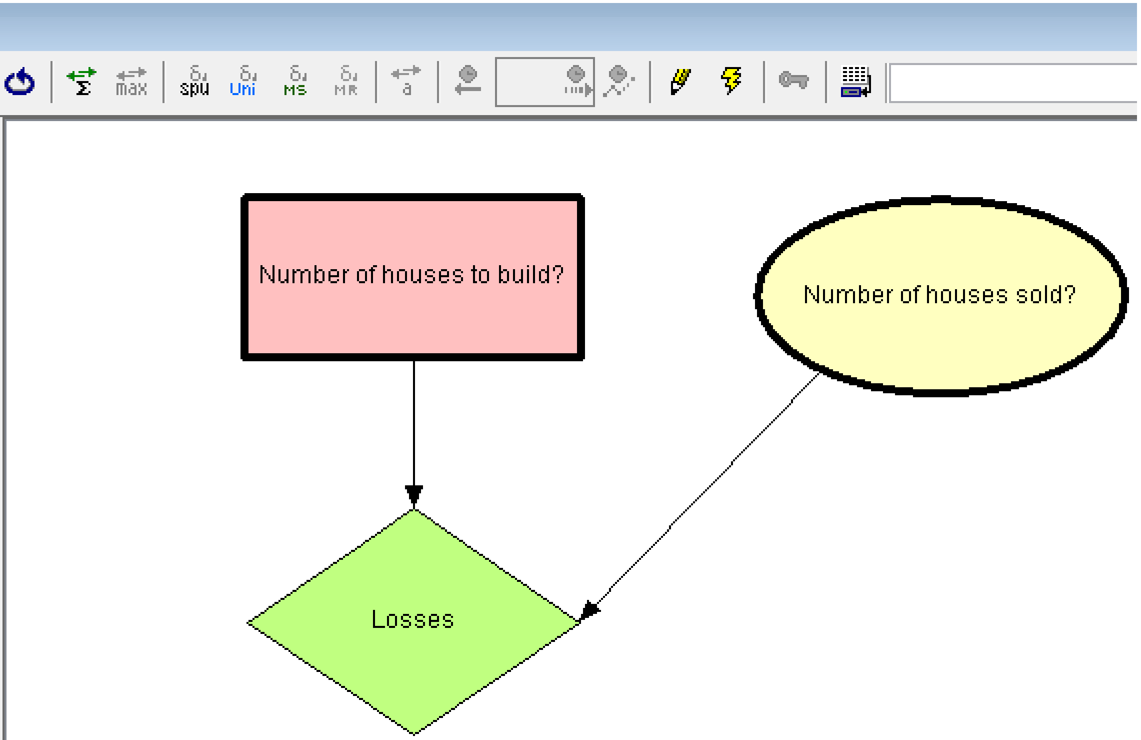
$$L(a_k, \theta) = \max_j \left(R\left(a_j, \theta\right)\right) - R(a_k, \theta)$$

a_k \ θ	0	1	2	3	4	5	6
0	0 – 0 = 0	4 – 0 = 4	8 – 0 = 8	12 – 0 = 12	16 – 0 = 16	20 – 0 = 20	24 – 0 = 24
1	0 – (–1) = 1	4 – 4 = 0	8 – 4 = 4	12 – 4 = 8	16 – 4 = 12	20 – 4 = 16	24 – 4 = 20
2	0 – (–2) = 2	4 – 3 = 1	8 – 8 = 0	12 – 8 = 4	16 – 8 = 8	20 – 8 = 12	24 – 8 = 16
3	0 – (–3) = 3	4 – 2 = 2	8 – 7 = 1	12 – 12 = 0	16 – 12 = 4	20 – 12 = 8	24 – 12 = 12
4	0 – (–4) = 4	4 – 1 = 3	8 – 6 = 2	12 – 11 = 1	16 – 16 = 0	20 – 16 = 4	24 – 16 = 8
5	0 – (–5) = 5	4 – 0 = 4	8 – 5 = 3	12 – 10 = 2	16 – 15 = 1	20 – 20 = 0	24 – 20 = 4
6	0 – (–6) = 6	4 – (–1) = 5	8 – 4 = 4	12 – 9 = 3	16 – 14 = 2	20 – 19 = 1	24 – 24 = 0
$\max_k\left(R(a_k, \theta)\right)$	0	4	8	12	16	20	24


Edit Functions View
Number of houses to build?
Losses
Number of houses sold?
Number of houses sold? 0 1
Number of houses to build? 0 1 2 3 4 5 6 0 1 2 3 4 5 6 0
Utility 0 -1 -2 -3 -4 -5 -6 -4 0 -1 -2 -3 -4 -5 -8
Number of houses to build?
Number of houses sold?
Losses

…and run.





Building 5 houses will give the least expected loss (2 thousand dollars)

…and we can at the same time answer the second question (*EVPI* = 2 000 $)

Exercise 7.15

15. A hot-dog vendor at a football game must decide in advance how many hot dogs to order. He makes a profit of \$0.10 on each hot dog that is sold, and he suffers a \$0.20 loss on hot dogs that are unsold. If his distribution of the number of hot dogs that will be demanded at the football game is a normal distribution with mean 10,000 and standard deviation 2000, how many hot dogs should he order? How much is it worth to the vendor to know in advance exactly how many hot dogs will be demanded?

Let θ = Demand in no. of hot dogs
a_k = Order k hot dogs (action)

The payoff function is (cf. *Exercise 7.14*):

$$R(a_k, \theta) = \begin{cases} 0.10 \cdot k & \text{if } k \leq \theta \\ 0.10 \cdot \theta - 0.20 \cdot (k - \theta) = 0.3 \cdot \theta - 0.2 \cdot k & \text{if } k > \theta \end{cases}$$

The prior distribution of $\tilde{\theta}$ is $N(\mu = 10000, \sigma = 2000)$

$\Longrightarrow$ The pdf of $\tilde{\theta}$ is $f_{\tilde{\theta}}(\theta) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(\theta-\mu)^2}{2\sigma^2}}$ and the cdf of $\tilde{\theta}$ is

$$F_{\tilde{\theta}}(\theta) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \int_{x=\infty}^{\theta} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

The expected payoff with action a_k is

$$E\big(R(a_k,\theta)\big) = \int_{\theta=-\infty}^{\infty} R(a_k,\theta)\cdot f_{\tilde{\theta}}(\theta)d\theta =$$

$$= \int_{\theta=-\infty}^{k} (0.3\cdot\theta - 0.2\cdot k)\cdot f_{\tilde{\theta}}(\theta)d\theta + \int_{\theta=k}^{\infty} 0.1\cdot k\cdot f_{\tilde{\theta}}(\theta)d\theta =$$

$$= 0.3\cdot \int_{\theta=-\infty}^{k} \theta\cdot f_{\tilde{\theta}}(\theta)d\theta - 0.2\cdot k\cdot \int_{\theta=-\infty}^{k} f_{\tilde{\theta}}(\theta)d\theta + 0.1\cdot k\cdot \int_{\theta=k}^{\infty} f_{\tilde{\theta}}(\theta)d\theta =$$

$$= 0.3\cdot \int_{\theta=-\infty}^{k} \theta\cdot f_{\tilde{\theta}}(\theta)d\theta - 0.2\cdot k\cdot F_{\tilde{\theta}}(k) + 0.1\cdot k\cdot \Big(1 - F_{\tilde{\theta}}(k)\Big) =$$

$$= 0.3\cdot \int_{\theta=-\infty}^{k} \theta\cdot f_{\tilde{\theta}}(\theta)d\theta - 0.3\cdot k\cdot F_{\tilde{\theta}}(k) + 0.1\cdot k$$

$$\int_{\theta=-\infty}^{k} \theta \cdot f_{\tilde{\theta}}(\theta) d\theta = \int_{\theta=-\infty}^{k} \theta \cdot \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(\theta-\mu)^2}{2\sigma^2}} d\theta = \langle \text{Form Gaussian integrals} \rangle =$$

$$= \int_{\theta=-\infty}^{k} \left(-\sigma^2 \cdot \left(-\frac{\theta-\mu}{\sigma^2}\right) + \mu\right) \cdot \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(\theta-\mu)^2}{2\sigma^2}} d\theta =$$

$$= -\sigma^2 \cdot \frac{1}{\sigma\sqrt{2\pi}} \cdot \int_{\theta=-\infty}^{k} \left(-\frac{\theta-\mu}{\sigma^2}\right) \cdot e^{-\frac{(\theta-\mu)^2}{2\sigma^2}} d\theta + \mu \cdot \int_{\theta=-\infty}^{k} \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(\theta-\mu)^2}{2\sigma^2}} d\theta =$$

$$= -\sigma^2 \cdot \frac{1}{\sigma\sqrt{2\pi}} \cdot \left[e^{-\frac{(\theta-\mu)^2}{2\sigma^2}}\right]_{\theta=-\infty}^{k} + \mu \cdot F_{\tilde{\theta}}(k) = -\sigma^2 \cdot \frac{1}{\sigma\sqrt{2\pi}} \cdot \left(e^{-\frac{(k-\mu)^2}{2\sigma^2}} - 0\right) + \mu \cdot F_{\tilde{\theta}}(k) =$$

$$= -\sigma^2 \cdot f_{\tilde{\theta}}(k) + \mu \cdot F_{\tilde{\theta}}(k)$$

$$\Longrightarrow E\big(R(a_k, \theta)\big) = 0.3 \cdot \left(-\sigma^2 \cdot f_{\tilde{\theta}}(k) + \mu \cdot F_{\tilde{\theta}}(k)\right) - 0.3 \cdot k \cdot F_{\tilde{\theta}}(k) + 0.1 \cdot k =$$

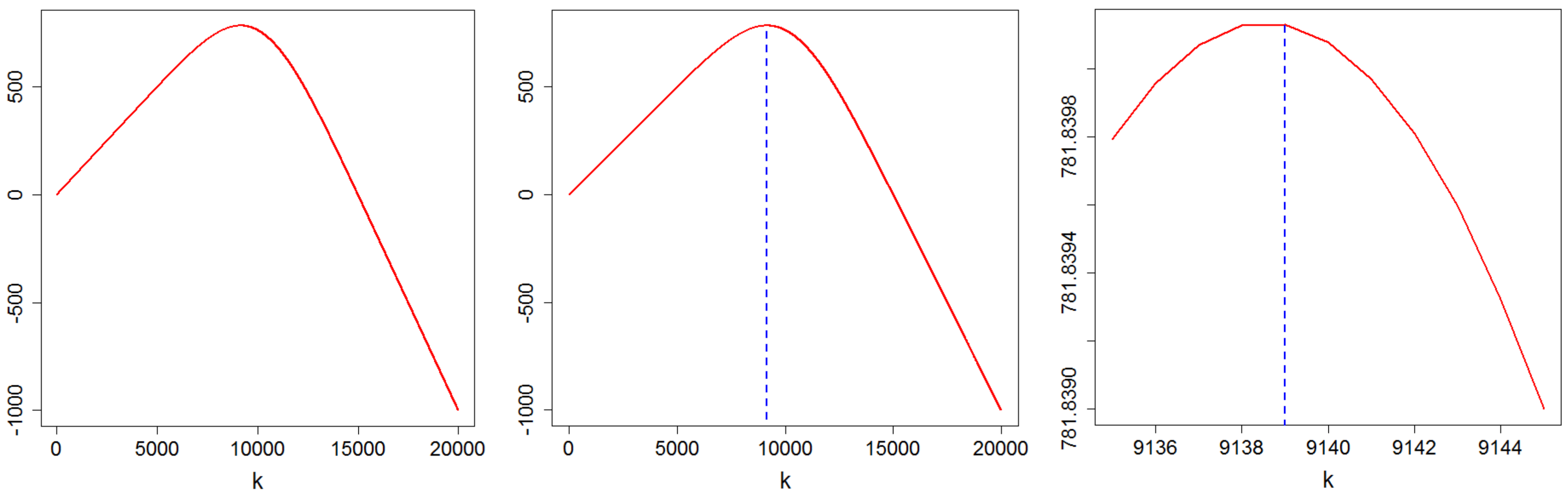
$$= 0.3 \cdot (\mu - k) \cdot F_{\tilde{\theta}}(k) - 0.3 \cdot \sigma^2 \cdot f_{\tilde{\theta}}(k) + 0.1 \cdot k$$

Hence, the optimal action with respect to the *ER*-criterion will be obtained from

$$\max_k \{0.3 \cdot (\mu - k) \cdot F_{\widetilde{\theta}}(k) - 0.3 \cdot \sigma^2 \cdot f_{\widetilde{\theta}}(k) + 0.1 \cdot k\}$$

A bit tricky to find the maximum from differential calculus.

Search maximum by graphing and grid-searching. Note that *k* must be an integer.



The optimal action is to order 9139 hot dogs.

$EVPI = EL(a_{\text{opt}})$

$$L(a_k, \theta) = \max_j \left(R\left(a_j, \theta\right)\right) - R(a_k, \theta)$$

$$R(a_k, \theta) = \begin{cases} 0.1 \cdot k & \text{if } k \le \theta \\ 0.3 \cdot \theta - 0.2 \cdot k & \text{if } k > \theta \end{cases}$$

$$\Longrightarrow \max_j \left(R\left(a_j, \theta\right)\right) \text{ is obtained when } a_j = \theta$$

$$\Longrightarrow L(a_k, \theta) = R(\theta, \theta) - R(a_k, \theta)$$

$$L(a_k, \theta) = \begin{cases} 0.1 \cdot (\theta - k) & \text{if } k \le \theta \\ 0.2 \cdot (k - \theta) & \text{if } k > \theta \end{cases}$$

Could be intuitively found

$$E\left(L(a_k)\right) = \int_{\theta=-\infty}^{\infty} L(a_k, \theta) \cdot f_{\tilde{\theta}}(\theta) d\theta =$$

$$= \int_{\theta=-\infty}^{k} 0.2 \cdot (k - \theta) \cdot f_{\tilde{\theta}}(\theta) d\theta + \int_{\theta=k}^{\infty} 0.1 \cdot (\theta - k) \cdot f_{\tilde{\theta}}(\theta) d\theta =$$

$$= 0.2 \cdot k \int_{\theta=-\infty}^{k} f_{\tilde{\theta}}(\theta) d\theta - 0.2 \cdot \int_{\theta=-\infty}^{k} \theta \cdot f_{\tilde{\theta}}(\theta) d\theta +$$

$$+0.1 \cdot \int_{\theta=k}^{\infty} \theta \cdot f_{\tilde{\theta}}(\theta) d\theta - 0.1 \cdot k \int_{\theta=k}^{\infty} f_{\tilde{\theta}}(\theta) d\theta = \langle \text{from previous calculations} \rangle =$$

$$= 0.2 \cdot k \cdot F_{\tilde{\theta}}(k) - 0.2 \cdot \left(-\sigma^2 \cdot f_{\tilde{\theta}}(k) + \mu \cdot F_{\tilde{\theta}}(k) \right) + 0.1 \cdot \boxed{\int_{\theta=k}^{\infty} \theta \cdot f_{\tilde{\theta}}(\theta) d\theta}$$

$$-0.1 \cdot k \cdot \left(1 - F_{\tilde{\theta}}(k) \right)$$

$$\int_{\theta=k}^{\infty} \theta \cdot f_{\tilde{\theta}}(\theta) d\theta = \langle \text{Analogous to previous calculations} \rangle =$$

$$= -\sigma^2 \cdot \frac{1}{\sigma\sqrt{2\pi}} \cdot \left[e^{-\frac{(\theta-\mu)^2}{2\sigma^2}} \right]_{\theta=k}^{\infty} + \mu \cdot \left(1 - F_{\tilde{\theta}}(k) \right) = -\sigma^2 \cdot \frac{1}{\sigma\sqrt{2\pi}} \cdot \left(0 - e^{-\frac{(k-\mu)^2}{2\sigma^2}} \right) +$$

$$+\mu \cdot \left(1 - F_{\tilde{\theta}}(k) \right) = \sigma^2 \cdot f_{\tilde{\theta}}(k) + \mu \cdot \left(1 - F_{\tilde{\theta}}(k) \right)$$

$$\Rightarrow E\big(L(a_k)\big) = 0.2\cdot k\cdot F_{\tilde{\theta}}(k) - 0.2\cdot\Big(-\sigma^2\cdot f_{\tilde{\theta}}(k) + \mu\cdot F_{\tilde{\theta}}(k)\Big) +$$

$$+0.1\cdot\Big(\sigma^2\cdot f_{\tilde{\theta}}(k) + \mu\cdot\Big(1 - F_{\tilde{\theta}}(k)\Big)\Big) - 0.1\cdot k\cdot\Big(1 - F_{\tilde{\theta}}(k)\Big) =$$

$$= 0.2\cdot k\cdot F_{\tilde{\theta}}(k) - 0.2\cdot\Big(-\sigma^2\cdot f_{\tilde{\theta}}(k)\Big) - 0.2\cdot\mu\cdot F_{\tilde{\theta}}(k) + 0.1\cdot\Big(\sigma^2\cdot f_{\tilde{\theta}}(k)\Big) + 0.1\cdot\mu$$

$$-0.1\cdot\mu\cdot F_{\tilde{\theta}}(k) - 0.1\cdot k + 0.1\cdot k\cdot F_{\tilde{\theta}}(k) =$$

$$= 0.3\cdot k\cdot F_{\tilde{\theta}}(k) - 0.3\cdot\mu\cdot F_{\tilde{\theta}}(k) + 0.3\cdot\sigma^2\cdot f_{\tilde{\theta}}(k) + 0.1\cdot\mu - 0.1\cdot k =$$

$$= 0.3\cdot(k-\mu)\cdot F_{\tilde{\theta}}(k) + 0.3\cdot\sigma^2\cdot f_{\tilde{\theta}}(k) + 0.1\cdot(\mu - k)$$

$$\Rightarrow EVPI = E\left(L\big(a_{\text{opt}}\big)\right) = E\big(L(a_{9139})\big) =$$

$$= 0.3\cdot(9139 - 10000)\cdot F_{\tilde{\theta}}(9139) + 0.3\cdot 2000^2\cdot f_{\tilde{\theta}}(9139) + 0.1\cdot(10000 - 9139)$$

$$= -258.3\cdot F_Z\left(\frac{9139-10000}{2000}\right) + 0.3\cdot 2000^2\cdot\frac{1}{2000}\cdot f_Z\left(\frac{9139-10000}{2000}\right) + 172.2 \approx$$

≈ 218 (dollar)

Alternative means of calculation:

$$E\left(L(a_k,\theta)\right) = E\left(R(\theta,\theta)\right) - E\left(R(a_k,\theta)\right) =$$

$$= E\left(R(\theta,\theta)\right) - 0.3 \cdot (\mu - k) \cdot F_{\tilde{\theta}}(k) + 0.3 \cdot \sigma^2 \cdot f_{\tilde{\theta}}(k) - 0.1 \cdot k$$

$$E\left(R(\theta,\theta)\right) = \int_{\theta=-\infty}^{\infty} R(\theta,\theta) \cdot f_{\tilde{\theta}}(\theta) d\theta =$$

$$= \int_{\theta=-\infty}^{k} (0.3 \cdot \theta - 0.2 \cdot \theta) \cdot f_{\tilde{\theta}}(\theta) d\theta + \int_{\theta=k}^{\infty} 0.1 \cdot \theta \cdot f_{\tilde{\theta}}(\theta) d\theta =$$

$$= \int_{\theta=-\infty}^{k} 0.1 \cdot \theta \cdot f_{\tilde{\theta}}(\theta) d\theta + \int_{\theta=k}^{\infty} 0.1 \cdot \theta \cdot f_{\tilde{\theta}}(\theta) d\theta = \int_{\theta=-\infty}^{\infty} 0.1 \cdot \theta \cdot f_{\tilde{\theta}}(\theta) d\theta =$$

$$= 0.1 \cdot E\left(\tilde{\theta}\right) = 1000$$

$$\Longrightarrow E\left(L\left(a_{\text{opt}}\right)\right) = E\left(L(a_{9139})\right) =$$

$$= 1000 - 0.3 \cdot (10000 - 9139) \cdot F_{\tilde{\theta}}(9139) + 0.3 \cdot 2000^2 \cdot f_{\tilde{\theta}}(9139) - 0.1 \cdot 9139 =$$

$$\approx 218$$