Lab1ML

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Assignment 1

Spam classification with nearest neighbors

```
##
## Call:
   glm(formula = Spam ~ ., family = "binomial", data = train)
##
##
  Deviance Residuals:
##
       Min
                 1Q
                      Median
                                    3Q
                                             Max
   -2.5205
            -0.4402
                      -0.0005
                                0.6584
                                          3.6196
##
## Coefficients:
##
                 Estimate Std. Error z value Pr(>|z|)
## (Intercept)
                1.508e+00
                            2.011e-01
                                        7.499 6.44e-14 ***
## Word1
                            5.015e-01
                                        -1.434 0.151520
               -7.192e-01
## Word2
                3.994e-02
                            3.014e-01
                                        0.133 0.894580
               -3.529e-01
                            1.839e-01
## Word3
                                        -1.919 0.055019
## Word4
                1.370e-01
                            1.117e-01
                                        1.226 0.220230
## Word5
                1.221e-01
                            1.413e-01
                                        0.864 0.387400
## Word6
                2.887e-01
                            4.153e-01
                                        0.695 0.486888
## Word7
               -2.948e-01
                            3.348e-01
                                        -0.880 0.378676
               -1.034e-01
                            3.510e-01
## Word8
                                        -0.295 0.768337
## Word9
               -1.085e-01
                            4.053e-01
                                        -0.268 0.788983
## Word10
               -3.091e-02
                            1.668e-01
                                        -0.185 0.852991
## Word11
               -6.088e-01
                            6.790e-01
                                        -0.897 0.369902
## Word12
                1.614e-01
                            1.073e-01
                                        1.505 0.132378
## Word13
                7.811e-01
                            3.572e-01
                                         2.187 0.028771 *
## Word14
               -4.819e-01
                            3.003e-01
                                        -1.605 0.108598
## Word15
               -1.305e-01
                            3.884e-01
                                        -0.336 0.736861
## Word16
                3.171e-01
                            2.332e-01
                                         1.360 0.173891
## Word17
               -9.201e-02
                            2.823e-01
                                        -0.326 0.744479
## Word18
                2.330e-02
                            2.322e-01
                                        0.100 0.920074
## Word19
                3.694e-04
                            5.746e-02
                                        0.006 0.994871
## Word20
                1.688e-02
                            3.181e-01
                                        0.053 0.957687
## Word21
               -2.821e-02
                            1.072e-01
                                        -0.263 0.792524
               -4.767e-01
## Word22
                            3.200e-01
                                        -1.490 0.136337
## Word23
                2.541e-01
                            3.413e-01
                                        0.745 0.456525
## Word24
               -2.483e-01
                            6.255e-01
                                        -0.397 0.691372
## Word25
               -7.828e-02
                            5.852e-02
                                        -1.338 0.181027
## Word26
                3.733e-03
                            1.358e-01
                                        0.027 0.978071
## Word27
               -2.238e-01
                            1.077e-01
                                       -2.078 0.037749 *
## Word28
                1.320e-01
                            1.880e-01
                                        0.702 0.482776
## Word29
               -8.131e-02
                            9.119e-02
                                        -0.892 0.372564
## Word30
                -1.815e+00
                            6.195e-01
                                        -2.930 0.003391 **
## Word31
               -4.694e+00
                                       -2.533 0.011296 *
                            1.853e+00
## Word32
               -1.194e+02 1.513e+04
                                       -0.008 0.993703
```

```
## Word33
              -2.899e+00 6.794e-01 -4.268 1.98e-05 ***
## Word34
              -3.710e+00 4.352e+00 -0.852 0.394004
## Word35
              -7.033e+00 1.996e+00 -3.522 0.000428 ***
## Word36
              -1.678e+00 3.810e-01 -4.404 1.06e-05 ***
## Word37
              -8.583e-01 2.175e-01
                                     -3.947 7.92e-05 ***
## Word38
              -6.043e-01 1.279e+00 -0.472 0.636575
## Word39
              -1.877e+00 4.282e-01 -4.384 1.16e-05 ***
## Word40
                                      0.217 0.827885
               7.393e-02 3.400e-01
## Word41
              -3.326e+02 1.656e+04 -0.020 0.983978
## Word42
              -5.352e+00
                          1.302e+00 -4.109 3.98e-05 ***
## Word43
              -2.592e+00 7.353e-01
                                     -3.525 0.000423 ***
## Word44
              -2.931e+00
                          6.601e-01
                                     -4.441 8.96e-06 ***
## Word45
              -1.141e+00
                          1.681e-01
                                     -6.785 1.16e-11 ***
## Word46
                                     -6.350 2.15e-10 ***
              -3.288e+00 5.178e-01
## Word47
              -3.741e+00 2.030e+00 -1.843 0.065399 .
## Word48
              -4.390e+00 1.473e+00
                                     -2.980 0.002878 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for binomial family taken to be 1)
##
##
      Null deviance: 1696.82 on 1369 degrees of freedom
## Residual deviance: 928.54 on 1321 degrees of freedom
## AIC: 1026.5
##
## Number of Fisher Scoring iterations: 23
## p_class2
## Not Spam
               Spam
##
       888
                482
##
     p_class2
##
      Not Spam Spam
##
           791 146
    0
##
    1
            97 336
## [1] 0.1773723
## p_class1
## Not Spam
               Spam
##
       884
                486
##
     p_class1
##
      Not Spam Spam
##
           803 142
##
    1
            81 344
## [1] 0.1627737
## p_class3
## Not Spam
               Spam
      1363
                  7
##
```

```
p_class3
##
##
      Not Spam Spam
##
            936
##
            427
                   6
     1
## [1] 0.3124088
## p_class3.1
## Not Spam
##
      1363
##
     p_class3.1
##
      Not Spam Spam
##
            944
##
     1
            419
## [1] 0.3065693
##
## Call:
## kknn(formula = Spam ~ ., train = train, test = test, k = 30)
## Response: "continuous"
## p_class4.1
## Not Spam
                Spam
##
        859
                 511
     p_class4.1
##
##
     Not Spam Spam
##
            672 265
##
            187 246
     1
## [1] 0.329927
##
## Call:
## kknn(formula = Spam ~ ., train = train, test = train, k = 30)
## Response: "continuous"
## p_class4.2
## Not Spam
                Spam
##
        905
                 465
##
     p_class4.2
##
      Not Spam Spam
##
          807 138
##
     1
            98 327
## [1] 0.1722628
```

```
##
## Call:
## kknn(formula = Spam ~ ., train = train, test = test, k = 1)
##
## Response: "continuous"
## p_class5.1
## Not Spam
                 Spam
##
        817
                  553
##
      p_class5.1
##
       Not Spam Spam
##
            640
                  297
##
     1
            177
                  256
## [1] 0.3459854
##
## Call:
## kknn(formula = Spam ~ ., train = train, test = train, k = 1)
## Response: "continuous"
## p_class5.2
## Not Spam
                 Spam
##
        945
                  425
##
      p_class5.2
##
       Not Spam Spam
##
     0
            945
                    0
##
     1
               0
                  425
## [1] 0
```

1.2) For the train dataset: From the coefficients of the linear model we can say that the Intercept,word 3,13,27,30,31,33,35,36,37,39,42,43,44,45,46,47 and 48 are significant. The Confusion matrix showcases the model on the Train data set. The possible values are 'Not spam'=0, and 'Spam'=1. Out of 1370 emails, the classifier has predicted Spam mails 486 times and predicted Not Spam mails 884 times. But the actual 'Spam' and 'Not spam' mails are 425 and 945 emails respectively. There are 344 True positive values and 803 True negative values. And there are 142 False positive values and 81 false negative values. Misclassification rate which is an error rate is a measure to test the accuracy, it stands at 16.27%.

For the test dataset: The Confusion matrix showcases the model on the Train data set. The possible values are 'Not spam'=0, and 'Spam'=1. Out of 1370 emails, the classifier has predicted Spam mails 888 times and predicted Not Spam mails 482 times. But the actual 'Spam' and 'Not spam' mails are 433 and 937 emails respectively. There are 336 True positive values and 791 True negative values. And there are 146 False positive values and 97 false negative values. Misclassification rate which is an error rate is a measure to test the accuracy, it stands at 17.7%.

1.3) With the change in the probability from 0.5 to 0.9 in the classification criterion, the misclassification rate has increased from 16.27% to 31.67% for train data set and 17.7% to 30.6% for the test dataset and the number of spam mails correctly predicted has reduced.

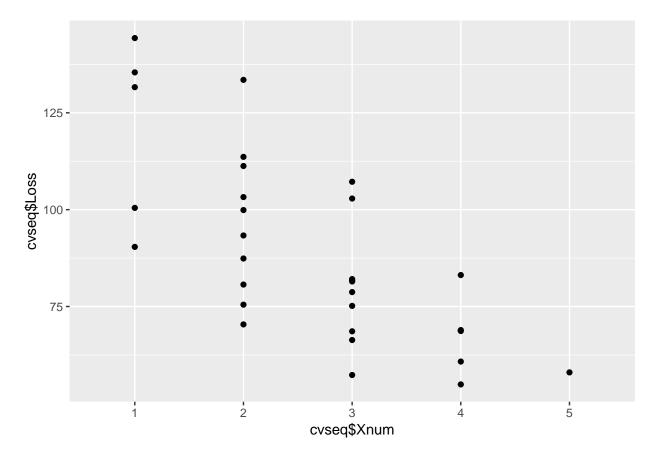
- 1.4) The misclassification rate for train data set is 17.22% and for test data set it is 32.99% whereas in step 2 it was 16.27% and 17.73% for train and test data set respectively.
- 1.5) The misclassification rate using KNN with the training dataset and test dataset is 0% and 34.59% respectively. In step 2, the misclassification rate stood at 16.2% for the training data and 17.7% for the test data. With decrease in the K value, the misclassification rate increases. With the 1-nearest neighbor model, each observation of the training dataset is potentially the center of an area predicting the outcome, with the other neighbors potentially predicting the other possible outcomes and hence making it highly complex. With k=30, the area's predicting each outcome will be more smooth as its the majority of the 30 nearest neighbors that predict the outcome of any observation. The area's are hence in lesser number, larger sizes and less complex. The variance term is simply the variance of an average here, and decreases as the inverse of k. So as k varies, there is a bias-variance tradeoff. More generally, as the model complexity of our procedure is increased, the variance tends to increase and the squared bias tends to decrease. The opposite behavior occurs as the model complexity is decreased. For k-nearest neighbors, the model complexity is controlled by k.

Assignment 3

Feature selection by cross-validation in a linear model

Optimal subset of features: 1,3,4,5

Cross validation score: 54.88725

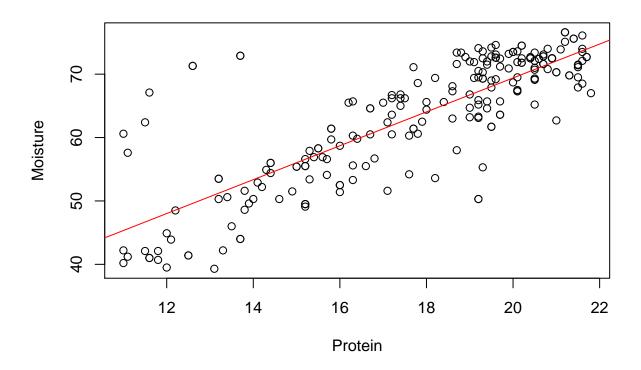


The optimal subset of X variables is Agriculture, Education, Catholic and Infant Mortality. The minimum cv score is 54.88 using a 4 variable subset. It is quite evident that Examination results do not affect the fertility measures. It is reasonable that these subset variable's will have a large impact on the fertility measure.

Assignment 4

Linear regression and regularization

 $\mathbf{Q}\mathbf{1}$



```
## (Intercept)
                   Protein
     15.924560
                  2.673778
## [1] "y = 2.7*x +15.9"
##
## lm(formula = Moisture ~ Protein, data = tecator)
##
## Residuals:
##
       Min
                  1Q
                       Median
                                            Max
                                    ЗQ
## -16.9611 -3.1660
                       0.0255
                               2.5836
                                       21.6858
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
                            2.3405
                                     6.804 1.01e-10 ***
## (Intercept) 15.9246
## Protein
                 2.6738
                            0.1305 20.491 < 2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 5.758 on 213 degrees of freedom
## Multiple R-squared: 0.6634, Adjusted R-squared: 0.6619
## F-statistic: 419.9 on 1 and 213 DF, p-value: < 2.2e-16
## [1] 32.85021</pre>
```

Yes, the data is well described by a linear model as Protein is directly proportional to Moisture values, with only a few outliers with very high values of Moisture.

$\mathbf{Q2}$

```
M_i, i \text{ is } 1, 2, 3, ..., i \text{ y \^{}} \text{ is the expected moisture x is protein } M_i : p(y|x, w) \text{ is } y_{N(aplha}0_{+aplha}1_{.x}1_{+aplha}2_{.x}2_{+\dots+aplha}i_{.x}i \sim \text{,std.dev \^{}} 2)
```

It is appropriate to use the Mean Squared Error criterion when fitting this model to the training data as the model is fitted using the Least Squares Method, where the fitted line is chosen such that the vertical distances from the data are the least. MSE consists of two components, the squared bias and variance. MSE is also used due to the curse of dimensionality, when there is no noise in the target function , the MSE approximates to squared bias. When the target function is constant in all but one dimension, The variance dominates and is approximated by MSE.

There is a big drawback in the simple error metric. This is because the positive and the negative errors cancel out. This can happen in the real scenario too, where the errors across all samples of observed data can cancel out each other. To get around this problem, Mean Squared Error. Now, no matter what the sign of error is, the squaring operation always amplifies the errors in the positive direction.

$\mathbf{Q3}$

```
(Intercept)
                   Protein
     17.685838
                  2.582355
## [1] "v = 2.7*x +15.9"
##
## Call:
## lm(formula = Moisture ~ Protein, data = train, x = TRUE, y = TRUE)
##
## Residuals:
##
        Min
                  10
                       Median
                                    30
                                            Max
                       0.0494
                                2.7753
                                        21.0765
##
  -16.9671
            -2.9550
##
##
  Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                 17.686
                             3.402
                                     5.199 9.95e-07 ***
## Protein
                  2.582
                             0.188 13.739 < 2e-16 ***
##
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.853 on 105 degrees of freedom
## Multiple R-squared: 0.6426, Adjusted R-squared: 0.6391
## F-statistic: 188.7 on 1 and 105 DF, p-value: < 2.2e-16
## [1] 33.61836
```

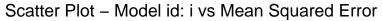
```
## (Intercept)
                  Protein
                             Protein2
## 49.6471841 -1.3920468
                            0.1189638
## [1] "y = -1.4*x + 0.1*x^2 + 49.6"
##
## Call:
## lm(formula = Moisture ~ Protein + Protein2, data = train, x = TRUE,
      y = TRUE)
## Residuals:
       Min
                      Median
                 1Q
                                   3Q
                                           Max
## -16.4747 -2.7866
                      0.1618
                               2.9023 20.3059
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 49.64718
                         18.93534
                                    2.622
              -1.39205
                          2.32448 -0.599
                                            0.5506
## Protein
## Protein2
               0.11896
                          0.06935
                                    1.715
                                           0.0893 .
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 5.8 on 104 degrees of freedom
## Multiple R-squared: 0.6524, Adjusted R-squared: 0.6457
## F-statistic: 97.59 on 2 and 104 DF, p-value: < 2.2e-16
## [1] 32.69342
## (Intercept)
                    Protein
                                Protein2
## 226.91446170 -35.02882584
                              2.19476262 -0.04178493
## [1] "y = -35*x +2.2*x^2 +0*x^2 +226.9"
##
## Call:
## lm(formula = Moisture ~ Protein + Protein2 + Protein3, data = train,
      x = TRUE, y = TRUE)
## Residuals:
       Min
                 1Q
                     Median
                                   3Q
## -17.3893 -2.9224
                     0.4621
                               3.0396 21.3893
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 226.91446
                          97.20177
                                    2.334
                                             0.0215 *
              -35.02883
                          18.24432 -1.920
## Protein
                                             0.0576 .
## Protein2
                2.19476
                           1.11904
                                    1.961
                                             0.0525 .
## Protein3
               -0.04178
                           0.02248 -1.858
                                             0.0660 .
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 5.732 on 103 degrees of freedom
## Multiple R-squared: 0.6637, Adjusted R-squared: 0.6539
## F-statistic: 67.75 on 3 and 103 DF, p-value: < 2.2e-16
```

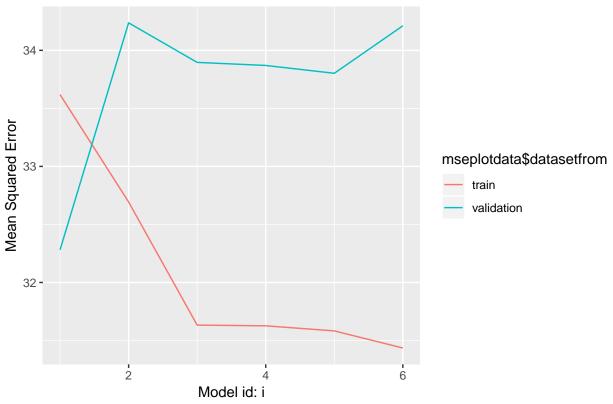
```
## [1] 31.63266
## (Intercept)
                    Protein
                                Protein2
                                              Protein3
                                                           Protein4
## 147.20755297 -14.71009881
                               0.28880722
                                            0.03626647
                                                       -0.00117897
## Call:
## lm(formula = Moisture ~ Protein + Protein2 + Protein3 + Protein4,
       data = train, x = TRUE, y = TRUE)
##
## Residuals:
##
      Min
               1Q Median
                               3Q
                                      Max
## -17.413 -2.961 0.442
                            3.086 21.293
##
## Coefficients:
                Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 147.207553 569.806627
                                      0.258
## Protein
              -14.710099 144.273696
                                     -0.102
                                                0.919
## Protein2
                0.288807 13.470609
                                      0.021
                                                0.983
## Protein3
                0.036266
                            0.550178
                                      0.066
                                                0.948
## Protein4
               -0.001179
                            0.008303 -0.142
                                                0.887
## Residual standard error: 5.76 on 102 degrees of freedom
## Multiple R-squared: 0.6637, Adjusted R-squared: 0.6505
## F-statistic: 50.33 on 4 and 102 DF, p-value: < 2.2e-16
## [1] 31.62641
     (Intercept)
                       Protein
                                   Protein2
                                                 Protein3
                                                                Protein4
   1.504946e+03 -4.491577e+02 5.509783e+01 -3.373110e+00 1.034524e-01
       Protein5
## -1.268213e-03
##
## Call:
## lm(formula = Moisture ~ Protein + Protein2 + Protein3 + Protein4 +
      Protein5, data = train, x = TRUE, y = TRUE)
##
## Residuals:
       Min
                 1Q
                      Median
                                   3Q
                                            Max
## -17.2813 -3.0122
                      0.4175
                               2.8961 21.2493
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.505e+03 3.678e+03
                                     0.409
                                                0.683
## Protein
              -4.492e+02 1.171e+03
                                     -0.383
                                                0.702
## Protein2
               5.510e+01 1.473e+02
                                      0.374
                                                0.709
## Protein3
              -3.373e+00 9.139e+00
                                                0.713
                                     -0.369
## Protein4
              1.035e-01 2.801e-01
                                      0.369
                                                0.713
## Protein5
              -1.268e-03 3.393e-03 -0.374
                                                0.709
##
## Residual standard error: 5.784 on 101 degrees of freedom
## Multiple R-squared: 0.6642, Adjusted R-squared: 0.6476
## F-statistic: 39.95 on 5 and 101 DF, p-value: < 2.2e-16
```

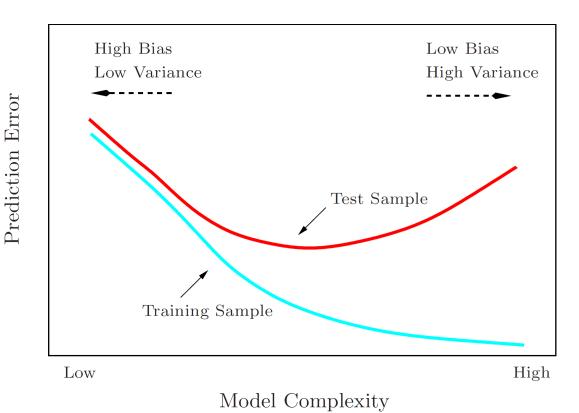
```
##
    (Intercept)
                      Protein
                                   Protein2
                                                 Protein3
                                                               Protein4
   1.478818e+04 -5.554997e+03 8.636619e+02 -7.091387e+01 3.243092e+00
##
       Protein5
                     Protein6
## -7.830290e-02 7.797287e-04
##
## Call:
## lm(formula = Moisture ~ Protein + Protein2 + Protein3 + Protein4 +
      Protein5 + Protein6, data = train, x = TRUE, y = TRUE)
##
## Residuals:
##
                 1Q
                      Median
       Min
                                   ЗQ
                                           Max
## -17.6110 -2.9827
                      0.5657
                               2.9141 21.5539
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.479e+04 1.973e+04
                                     0.749
                                               0.455
## Protein
              -5.555e+03 7.543e+03 -0.736
                                               0.463
## Protein2
               8.637e+02 1.189e+03
                                     0.726
                                               0.469
## Protein3
              -7.091e+01 9.899e+01
                                     -0.716
                                               0.475
## Protein4
               3.243e+00 4.590e+00
                                     0.706
                                               0.482
## Protein5
              -7.830e-02 1.125e-01 -0.696
                                               0.488
               7.797e-04 1.138e-03
## Protein6
                                     0.685
                                               0.495
##
## Residual standard error: 5.8 on 100 degrees of freedom
## Multiple R-squared: 0.6658, Adjusted R-squared: 0.6457
## F-statistic: 33.2 on 6 and 100 DF, p-value: < 2.2e-16
```

[1] 31.43513

[1] 31.58273







The model with 1st order polynomial (Linear model) is the best model as there is the right amount of bias variance tradeoff. The training error decreases at first and then stabilises beyond 3rd order polynomial while the validation error increases at first and stabilises beyond 2nd order polynomial function. Bias-Variance tradeoff: As the more generalised view of the prediction error vs the model complexity is shown in the picture attached. In the Mean Squared Error, the variance is the average of the variances and decreases as the inverse of k. So as the k varies, there is a bias variance tradeoff. For the training sample, As the Model complexity increases, the prediction error decreases. For the testing sample, as the model complexity increases the the prediction error decreases at first but reaches a minima and starts to increase as the model complexity goes higher. Typically we would like to choose our model complexity to trade biasoff with variance in such a way as to minimize the test error. An obvious estimate of test error is the training error, 1/N * sum(i(yi ??? yi)2), But the training error is not a good estimate of the testing error.

$\mathbf{Q4}$

```
## Stepwise Model Path
## Analysis of Deviance Table
##
## Initial Model:
  Fat ~ (Channel1 + Channel2 + Channel3 + Channel4 + Channel5 +
##
       Channel6 + Channel7 + Channel8 + Channel9 + Channel10 + Channel11 +
##
       Channel12 + Channel13 + Channel14 + Channel15 + Channel16 +
       Channel17 + Channel18 + Channel19 + Channel20 + Channel21 +
##
       Channel22 + Channel23 + Channel24 + Channel25 + Channel26 +
##
##
       Channel27 + Channel28 + Channel29 + Channel30 + Channel31 +
##
       Channel32 + Channel33 + Channel34 + Channel35 + Channel36 +
##
       Channel37 + Channel38 + Channel39 + Channel40 + Channel41 +
##
       Channel42 + Channel43 + Channel44 + Channel45 + Channel46 +
##
       Channel47 + Channel48 + Channel49 + Channel50 + Channel51 +
##
       Channel52 + Channel53 + Channel54 + Channel55 + Channel56 +
       Channel57 + Channel58 + Channel59 + Channel60 + Channel61 +
##
##
       Channel62 + Channel63 + Channel64 + Channel65 + Channel66 +
##
       Channel67 + Channel68 + Channel69 + Channel70 + Channel71 +
       Channel72 + Channel73 + Channel74 + Channel75 + Channel76 +
##
##
       Channel77 + Channel78 + Channel79 + Channel80 + Channel81 +
##
       Channel82 + Channel83 + Channel84 + Channel85 + Channel86 +
##
       Channel87 + Channel88 + Channel89 + Channel90 + Channel91 +
##
       Channel92 + Channel93 + Channel94 + Channel95 + Channel96 +
       Channel97 + Channel98 + Channel99 + Channel100 + Protein +
##
##
       Moisture) - Moisture - Protein
##
## Final Model:
##
  Fat ~ Channel1 + Channel2 + Channel4 + Channel5 + Channel7 +
##
       Channel8 + Channel11 + Channel12 + Channel13 + Channel14 +
##
       Channel15 + Channel17 + Channel19 + Channel20 + Channel22 +
       Channel24 + Channel25 + Channel26 + Channel28 + Channel29 +
##
##
       Channel30 + Channel32 + Channel34 + Channel36 + Channel37 +
##
       Channel39 + Channel40 + Channel41 + Channel42 + Channel45 +
##
       Channel46 + Channel47 + Channel48 + Channel50 + Channel51 +
##
       Channel52 + Channel54 + Channel55 + Channel56 + Channel59 +
       Channel60 + Channel61 + Channel63 + Channel64 + Channel65 +
##
##
       Channel67 + Channel68 + Channel69 + Channel71 + Channel73 +
##
       Channel74 + Channel78 + Channel79 + Channel80 + Channel81 +
##
       Channel84 + Channel85 + Channel87 + Channel88 + Channel92 +
```

```
##
       Channel94 + Channel98 + Channel99
##
##
##
              Step Df
                           Deviance Resid. Df Resid. Dev
                                                                AIC
## 1
                                           114
                                                 169.8123 151.27203
  2
##
       - Channel70 1 5.580758e-05
                                           115
                                                 169.8124 149.27210
##
       - Channel89
                    1 6.338934e-04
                                           116
                                                 169.8130 147.27290
##
       - Channel66
                    1 4.350148e-04
                                           117
                                                 169.8135 145.27345
##
   5
      - Channel100
                    1 9.526559e-04
                                           118
                                                 169.8144 143.27466
##
   6
       - Channel57
                    1 1.512331e-03
                                           119
                                                 169.8159 141.27657
##
  7
       - Channel38
                    1 4.235150e-03
                                           120
                                                 169.8202 139.28193
## 8
       - Channel58
                    1 7.141818e-03
                                           121
                                                 169.8273 137.29098
## 9
       - Channel53
                    1 2.509829e-02
                                           122
                                                 169.8524 135.32275
## 10
        - Channel9
                    1 3.771904e-02
                                           123
                                                 169.8901 133.37049
       - Channel91
## 11
                    1 3.178511e-02
                                           124
                                                 169.9219 131.41071
## 12
       - Channel77
                    1 5.501288e-02
                                           125
                                                 169.9769 129.48030
##
   13
       - Channel49
                    1 9.282875e-02
                                           126
                                                 170.0698 127.59769
##
   14
       - Channel33
                    1 1.137405e-01
                                           127
                                                 170.1835 125.74143
  15
##
       - Channel96
                    1 1.838591e-01
                                           128
                                                 170.3674 123.97358
##
   16
       - Channel93
                    1 1.204802e-01
                                           129
                                                 170.4878 122.12557
##
   17
       - Channel82
                    1 2.012906e-01
                                           130
                                                 170.6891 120.37927
       - Channel86
  18
                    1 2.608049e-01
                                           131
                                                 170.9499 118.70753
       - Channel72
                    1 3.340581e-01
## 19
                                           132
                                                 171.2840 117.12725
##
  20
       - Channel35
                    1 4.539629e-01
                                           133
                                                 171.7380 115.69633
##
  21
       - Channel43
                    1 3.667681e-01
                                           134
                                                 172.1047 114.15500
  22
       - Channel44
                    1 3.686336e-01
                                           135
                                                 172.4734 112.61502
   23
##
       - Channel90
                    1 4.430432e-01
                                           136
                                                 172.9164 111.16659
##
   24
       - Channel83
                    1 4.636039e-01
                                           137
                                                 173.3800 109.74225
   25
##
        - Channel3
                    1 4.495464e-01
                                           138
                                                 173.8295 108.29899
                                          139
##
  26
       - Channel23
                    1 4.393963e-01
                                                 174.2689 106.84177
##
  27
        - Channel6
                    1 6.745513e-01
                                           140
                                                 174.9435 105.67238
##
   28
       - Channel62
                    1 6.873639e-01
                                           141
                                                 175.6309 104.51547
##
   29
       - Channel10
                    1 6.770690e-01
                                           142
                                                 176.3079 103.34272
##
   30
       - Channel18
                    1 5.551316e-01
                                           143
                                                 176.8631 102.01861
##
   31
       - Channel27
                    1 8.012085e-01
                                                 177.6643 100.99038
                                           144
##
  32
       - Channel16
                    1 8.124404e-01
                                           145
                                                 178.4767
                                                           99.97132
  33
      - Channel21
                    1 9.726859e-01
                                           146
                                                 179.4494
                                                           99.13987
##
  34
       - Channel95
                    1 8.809590e-01
                                                 180.3304
                                           147
                                                           98.19277
  35
       - Channel97
                    1 6.630855e-01
##
                                           148
                                                 180.9934
                                                           96.98189
##
  36
       - Channel76
                    1 1.451145e+00
                                           149
                                                 182.4446
                                                           96.69882
   37
       - Channel75
                    1 7.506552e-01
                                           150
                                                 183.1952
                                                           95.58160
      - Channel31 1 1.682931e+00
  38
                                           151
                                                 184.8782
                                                           95.54769
##
## Call:
   lm(formula = Fat ~ Channel1 + Channel2 + Channel4 + Channel5 +
##
       Channel7 + Channel8 + Channel11 + Channel12 + Channel13 +
##
       Channel14 + Channel15 + Channel17 + Channel19 + Channel20 +
##
       Channel22 + Channel24 + Channel25 + Channel26 + Channel28 +
##
       Channel29 + Channel30 + Channel32 + Channel34 + Channel36 +
##
       Channel37 + Channel39 + Channel40 + Channel41 + Channel42 +
       Channel45 + Channel46 + Channel47 + Channel48 + Channel50 +
##
##
       Channel51 + Channel52 + Channel54 + Channel55 + Channel56 +
       Channel59 + Channel60 + Channel61 + Channel63 + Channel64 +
##
```

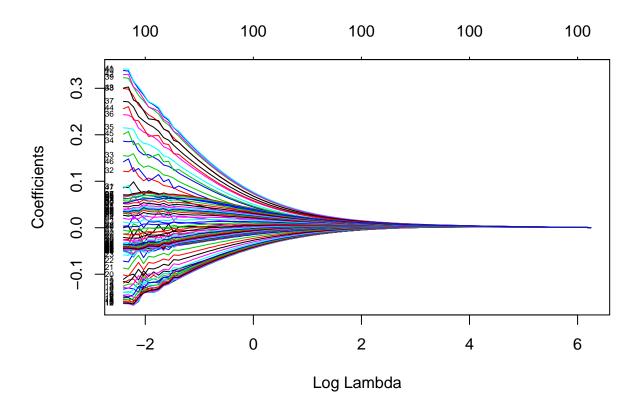
```
##
       Channel65 + Channel67 + Channel68 + Channel69 + Channel71 +
       Channel73 + Channel74 + Channel78 + Channel79 + Channel80 +
##
##
       Channel81 + Channel84 + Channel85 + Channel87 + Channel88 +
       Channel92 + Channel94 + Channel98 + Channel99, data = data.frame(tecator1))
##
##
## Residuals:
        Min
                   10
                        Median
                                      30
                                              Max
   -2.82961 -0.57129 -0.00696 0.58152
                                         2.86375
##
##
   Coefficients:
##
                  Estimate Std. Error t value Pr(>|t|)
##
   (Intercept)
                     7.093
                                1.453
                                         4.882 2.64e-06 ***
                10559.894
                             2333.430
                                         4.525 1.21e-05 ***
##
   Channel1
##
   Channel2
               -12636.967
                             3467.995
                                        -3.644 0.000369 ***
## Channel4
                  8489.323
                             4637.993
                                         1.830 0.069164 .
  Channel5
                -10408.967
                             4771.350
                                        -2.182 0.030689 *
## Channel7
                -5376.018
                                        -1.396 0.164847
                             3851.782
## Channel8
                  7215.595
                             4246.489
                                         1.699 0.091342
## Channel11
                -9505.520
                             5721.115
                                        -1.661 0.098692 .
## Channel12
                37240.918
                            12290.648
                                         3.030 0.002878 **
## Channel13
                -41564.547
                            15892.375
                                        -2.615 0.009817 **
## Channel14
                34938.179
                            13290.454
                                         2.629 0.009454 **
## Channel15
                -23761.451
                             6584.006
                                        -3.609 0.000417 ***
## Channel17
                  4296.572
                             3189.730
                                         1.347 0.179998
## Channel19
                14279.808
                             5017.407
                                         2.846 0.005042 **
## Channel20
                -23855.616
                             5153.161
                                        -4.629 7.85e-06 ***
                                         5.454 1.97e-07 ***
## Channel22
                18444.906
                             3381.683
## Channel24
                -20138.426
                             4946.417
                                        -4.071 7.52e-05 ***
## Channel25
                18137.432
                             5374.094
                                         3.375 0.000938 ***
## Channel26
                -7670.318
                             3859.006
                                        -1.988 0.048660 *
## Channel28
                20079.898
                             4991.631
                                         4.023 9.06e-05 ***
## Channel29
                -36351.014
                             7655.223
                                        -4.749 4.72e-06 ***
## Channel30
                18071.276
                             5863.802
                                         3.082 0.002446 **
## Channel32
                  3838.013
                             2722.862
                                         1.410 0.160729
## Channel34
                -9242.884
                             2225.926
                                        -4.152 5.48e-05 ***
## Channel36
                 8070.938
                             3317.588
                                         2.433 0.016152 *
## Channel37
                -9045.588
                             3536.621
                                        -2.558 0.011522 *
## Channel39
                             5986.730
                                         3.118 0.002183 **
                18664.454
## Channel40
                            10701.902
                -20069.709
                                        -1.875 0.062677 .
## Channel41
                22257.776
                            11122.533
                                         2.001 0.047169 *
## Channel42
                -21760.853
                             5833.811
                                        -3.730 0.000270 ***
## Channel45
                18145.804
                             2985.416
                                         6.078 9.50e-09 ***
## Channel46
                -8225.696
                             3715.367
                                        -2.214 0.028330 *
## Channel47
                -4986.549
                             2558.694
                                        -1.949 0.053165 .
## Channel48
                  2876.075
                             2014.985
                                         1.427 0.155546
## Channel50
                -13009.410
                             4535.797
                                        -2.868 0.004720 **
## Channel51
                29251.161
                             6554.297
                                         4.463 1.57e-05 ***
## Channel52
               -26833.976
                             4389.473
                                        -6.113 7.97e-09 ***
## Channel54
                30954.862
                             4392.339
                                         7.047 6.06e-11 ***
## Channel55
                -35183.287
                             5646.314
                                        -6.231 4.39e-09 ***
## Channel56
                14912.986
                             2810.889
                                         5.305 3.93e-07 ***
## Channel59
                -8030.278
                             1887.431
                                        -4.255 3.66e-05 ***
## Channel60
                13071.416
                             2629.374
                                         4.971 1.79e-06 ***
## Channel61
                -7850.189
                             2246.864
                                       -3.494 0.000625 ***
```

```
## Channel63
                 15059.275
                             3231.692
                                         4.660 6.90e-06 ***
                                        -4.211 4.35e-05 ***
## Channel64
               -19909.466
                             4727.696
## Channel65
                             3486.766
                  4190.184
                                         1.202 0.231346
## Channel67
                             3909.121
                                         3.543 0.000526 ***
                 13850.508
## Channel68
                -25873.365
                             5304.223
                                        -4.878 2.69e-06 ***
  Channel69
                             3331.483
                                         5.512 1.50e-07 ***
##
                 18362.385
   Channel71
                 -9223.910
                             1558.752
                                        -5.917 2.11e-08 ***
## Channel73
                 12456.498
                             2386.255
                                         5.220 5.82e-07 ***
   Channel74
                 -5624.411
                             1933.590
                                        -2.909 0.004177 **
  Channel78
                 -7927.105
                             2176.860
                                        -3.642 0.000372 ***
   Channel79
                 15473.188
                             3812.200
                                         4.059 7.89e-05 ***
## Channel80
                -22391.895
                             4490.714
                                        -4.986 1.67e-06 ***
   Channel81
                 13852.453
                             3105.934
                                         4.460 1.59e-05 ***
   Channel84
               -11442.630
                             3457.064
                                        -3.310 0.001167 **
   Channel85
                 20228.671
                             4081.863
                                         4.956 1.91e-06 ***
   Channel87
                -15938.315
                             4102.273
                                        -3.885 0.000153 ***
   Channel88
                  5647.072
                             3236.286
                                         1.745 0.083033 .
   Channel92
                  6595.995
                             1864.595
                                         3.537 0.000537 ***
  Channel94
                 -5497.846
                             1847.113
                                        -2.976 0.003397 **
## Channel98
                 -8728.596
                             2489.314
                                        -3.506 0.000598 ***
## Channel99
                 8554.587
                             1898.010
                                         4.507 1.31e-05 ***
##
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.107 on 151 degrees of freedom
## Multiple R-squared: 0.9947, Adjusted R-squared: 0.9925
## F-statistic: 447.9 on 63 and 151 DF, p-value: < 2.2e-16
##
     (Intercept)
                       Channel1
                                      Channel 2
                                                    Channel4
                                                                   Channel 5
##
        7.093133
                   10559.893784 -12636.966607
                                                 8489.323117 -10408.966948
##
        Channel7
                       Channel8
                                     Channel11
                                                    Channel12
                                                                  Channel13
##
    -5376.017738
                    7215.595409
                                 -9505.520235
                                                37240.918374
                                                              -41564.546571
##
                                                    Channel19
                                                                  Channel 20
       Channel 14
                      Channel 15
                                     Channel17
##
    34938.179314 -23761.450875
                                   4296.572462
                                                14279.808102
                                                              -23855.616123
##
                      Channel24
                                                    Channel26
       Channel22
                                     Channel25
                                                                  Channel28
##
    18444.905722 -20138.426065
                                  18137.431996
                                                -7670.318234
                                                               20079.898191
##
       Channel29
                      Channel30
                                     Channel32
                                                    Channel34
                                                                  Channel36
   -36351.013717
                   18071.275531
                                   3838.013358
                                                                8070.938452
##
                                                -9242.884498
##
       Channel37
                      Channel39
                                     Channel40
                                                    Channel41
                                                                  Channel42
##
    -9045.587624
                   18664.454171
                                 -20069.708579
                                                22257.776227
                                                              -21760.853228
##
       Channel45
                      Channel46
                                     Channel47
                                                    Channel48
                                                                  Channel 50
##
    18145.803786
                   -8225.696060
                                  -4986.549169
                                                 2876.074542
                                                              -13009.409717
##
       Channel51
                      Channel52
                                     Channel54
                                                    Channel55
                                                                  Channel56
##
    29251.160946 -26833.976402
                                 30954.861519 -35183.287363
                                                               14912.986496
##
       Channel59
                      Channel60
                                     Channel61
                                                    Channel63
                                                                  Channel64
##
    -8030.277501
                   13071.415506
                                  -7850.189324
                                                15059.274961 -19909.466348
##
       Channel65
                      Channel67
                                     Channel68
                                                    Channel69
                                                                  Channel71
##
     4190.183533
                   13850.508143
                                -25873.365427
                                                18362.384676
                                                               -9223.909939
##
       Channel73
                      Channel74
                                     Channel78
                                                    Channel 79
                                                                  Channel80
##
    12456.497755
                   -5624.411385
                                  -7927.104791
                                                15473.187794 -22391.894812
##
       Channel81
                      Channel84
                                     Channel85
                                                    Channel87
                                                                   Channel88
##
                                 20228.671387 -15938.315283
    13852.452651 -11442.629734
                                                                5647.072201
##
                      Channel94
                                     Channel98
       Channel92
                                                    Channel99
##
     6595.995241
                  -5497.846381
                                 -8728.596111
                                                 8554.587048
```

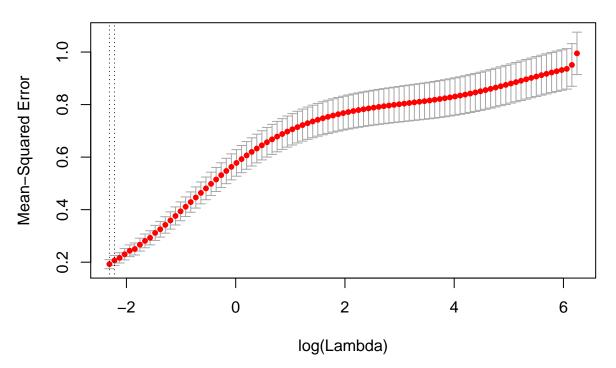
[1] 64

63 Variables were selected and a model of the adjusted R squared of 99.25% was produced.

 $\mathbf{Q5}$



[1] 0.09910954



```
## 101 x 1 sparse Matrix of class "dgCMatrix"
##
##
  (Intercept) -5.984460e-17
               -1.031894e-01
   Channel1
  Channel2
               -1.126216e-01
##
## Channel3
               -1.214028e-01
## Channel4
               -1.296478e-01
## Channel5
               -1.372993e-01
## Channel6
               -1.442254e-01
## Channel7
               -1.502639e-01
## Channel8
               -1.552611e-01
## Channel9
               -1.591851e-01
## Channel10
               -1.618359e-01
## Channel11
               -1.633326e-01
## Channel12
               -1.635877e-01
## Channel13
               -1.624216e-01
## Channel14
               -1.596083e-01
## Channel15
               -1.550072e-01
## Channel16
               -1.485948e-01
## Channel17
               -1.404129e-01
## Channel18
               -1.302440e-01
## Channel19
               -1.180202e-01
## Channel20
               -1.039948e-01
## Channel21
               -8.880577e-02
## Channel22
               -7.361606e-02
## Channel23
               -5.957436e-02
```

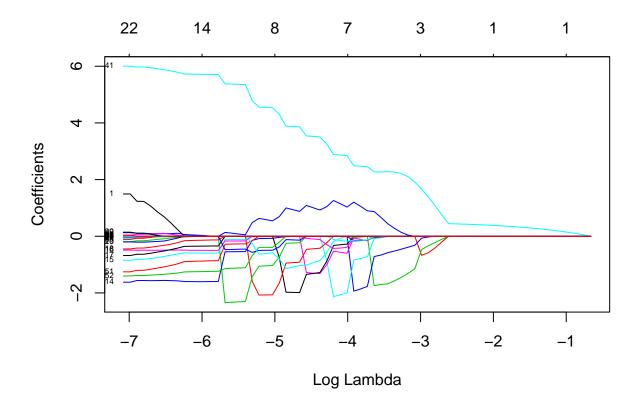
```
## Channel24
               -4.721711e-02
## Channel25
               -3.619736e-02
  Channel26
               -2.534224e-02
  Channel27
               -1.256519e-02
  Channel28
                4.344829e-03
## Channel29
                2.662692e-02
  Channel30
                5.448642e-02
## Channel31
                8.658293e-02
   Channel32
                1.205339e-01
   Channel33
                1.538239e-01
   Channel34
                1.849436e-01
##
   Channel35
                2.140008e-01
##
   Channel36
                2.423180e-01
   Channel37
##
                2.707945e-01
  Channel38
                2.981771e-01
   Channel39
                3.215073e-01
##
  Channel40
                3.370674e-01
   Channel41
                3.409983e-01
##
  Channel42
                3.300942e-01
## Channel43
                3.028880e-01
## Channel44
                2.605030e-01
  Channel45
                2.069027e-01
## Channel46
                1.482506e-01
   Channel47
                9.123195e-02
##
  Channel48
                4.203255e-02
  Channel49
                3.162517e-03
##
  Channel50
                -2.490854e-02
##
   Channel51
               -4.292261e-02
   Channel52
##
               -5.225467e-02
  Channel53
               -5.472501e-02
##
  Channel54
               -5.253630e-02
   Channel55
               -4.809605e-02
   Channel56
               -4.348703e-02
##
  Channel57
               -4.014073e-02
   Channel58
               -3.842608e-02
## Channel59
               -3.841251e-02
## Channel60
               -3.933553e-02
## Channel61
               -4.055333e-02
## Channel62
                -4.184981e-02
## Channel63
               -4.296650e-02
  Channel64
               -4.398656e-02
## Channel65
               -4.493753e-02
##
  Channel66
               -4.583276e-02
##
   Channel67
               -4.675245e-02
  Channel68
               -4.748998e-02
## Channel69
               -4.790305e-02
  Channel70
               -4.780024e-02
   Channel71
               -4.714768e-02
  Channel72
               -4.592782e-02
##
  Channel73
                -4.407423e-02
##
  Channel74
               -4.169460e-02
## Channel75
               -3.889640e-02
## Channel76
               -3.574401e-02
## Channel77
               -3.212053e-02
```

```
-2.774564e-02
## Channel78
## Channel79
               -2.225572e-02
  Channel80
               -1.532490e-02
## Channel81
               -7.268784e-03
##
  Channel82
                1.411225e-03
  Channel83
                1.013919e-02
##
  Channel84
                1.823082e-02
## Channel85
                2.496689e-02
  Channel86
                2.969929e-02
                3.252596e-02
##
  Channel87
## Channel88
                3.435886e-02
## Channel89
                3.649918e-02
  Channel90
                3.989390e-02
##
## Channel91
                4.484971e-02
## Channel92
                5.090499e-02
## Channel93
                5.715961e-02
  Channel94
                6.283132e-02
##
## Channel95
                6.729909e-02
## Channel96
                7.010424e-02
## Channel97
                7.056400e-02
## Channel98
                6.805063e-02
## Channel99
                6.235443e-02
## Channel100
                5.406867e-02
```

[1] 0.09910954

The coefficients converge to 0 as the log lambda increases. They range between -0.1 and 0.3 approximately and almost converge to 0 by reaching the value 4 of log lambda. The least Mean Squared Error is under log lambda of -2.

 $\mathbf{Q6}$



[1] 0.002571916



```
Mean-Squared Error

Wean-Squared Error

-6 -5 -4 -3 -2 -1

log(Lambda)
```

```
## 101 x 1 sparse Matrix of class "dgCMatrix"
##
## (Intercept)
                2.430320e-15
## Channel1
## Channel2
## Channel3
## Channel4
## Channel5
## Channel6
## Channel7
## Channel8
## Channel9
## Channel10
## Channel11
## Channel12
## Channel13
## Channel14
               -1.602254e+00
## Channel15
               -5.920293e-01
## Channel16
               -4.978047e-01
## Channel17
               -3.471019e-01
## Channel18
               -1.377282e-01
## Channel19
## Channel20
## Channel21
## Channel22
## Channel23
```

```
## Channel24
## Channel25
## Channel26
## Channel27
## Channel28
## Channel29
## Channel30
## Channel31
## Channel32
## Channel33
## Channel34
## Channel35
## Channel36
## Channel37
## Channel38
## Channel39
                 5.881678e-03
## Channel40
                 2.533079e-02
## Channel41
                 5.714424e+00
## Channel42
## Channel43
## Channel44
## Channel45
## Channel46
## Channel47
## Channel48
## Channel49
               -2.466071e-05
## Channel50
               -4.082507e-03
## Channel51
               -8.735307e-01
## Channel52
               -1.245545e+00
               -1.069154e-03
## Channel53
## Channel54
## Channel55
## Channel56
## Channel57
## Channel58
## Channel59
## Channel60
## Channel61
## Channel62
## Channel63
## Channel64
## Channel65
## Channel66
## Channel67
## Channel68
## Channel69
## Channel70
## Channel71
## Channel72
## Channel73
## Channel74
## Channel75
## Channel76
## Channel77
```

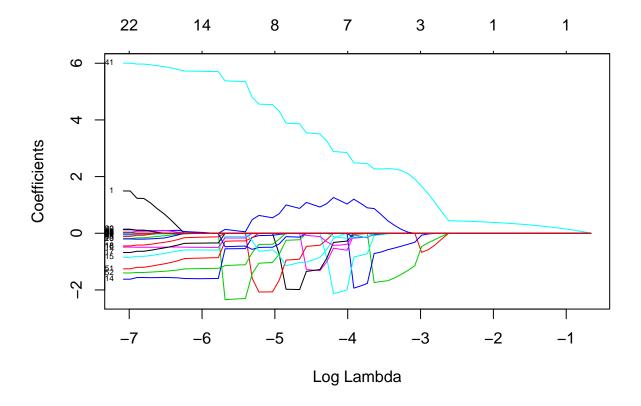
```
## Channel78
## Channel79
## Channel80
## Channel81
## Channel82
##
  Channel83
  Channel84
## Channel85
  Channel86
## Channel87
## Channel88
## Channel89
## Channel90
## Channel91
## Channel92
## Channel93
## Channel94
## Channel95
## Channel96
## Channel97
## Channel98
## Channel99
## Channel100
```

[1] 0.002571916

The coefficient values for the 13 variables selected converge to 0 at the log lambda of -3. the least mse is found between log lambda of -6 to -5.5.

There are 100 variables that have been used in the ridge regression model and only 13 variables used in the lasso regression model. In Ridge regression there is proportional shrinkage that is done while Lasso translates each coefficient by a factor lambda truncating at 0. In the Ridge regression both shrinkage and

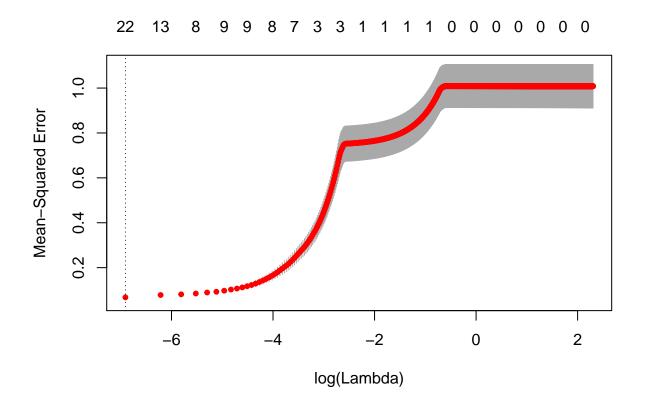
 $\mathbf{Q7}$



```
## 101 x 1 sparse Matrix of class "dgCMatrix"
##
##
   (Intercept)
                3.423729e-15
##
   Channel1
                 2.389874e+00
   Channel2
                 1.988995e-06
##
## Channel3
                 1.193300e-06
## Channel4
               -3.161787e-06
## Channel5
               -5.893738e-06
## Channel6
                3.484921e-01
  Channel7
               -7.283755e-02
## Channel8
               -6.794308e-02
## Channel9
                1.422145e-01
## Channel10
               -7.818637e-02
## Channel11
               -9.791948e-03
## Channel12
               -6.749591e-02
  Channel13
##
                8.343128e-03
   Channel14
               -1.159804e-02
##
##
  Channel15
               -1.105447e+00
## Channel16
               -1.617016e-01
##
  Channel17
               -9.932520e-01
## Channel18
               -7.298657e-01
## Channel19
               -2.581660e+00
## Channel20
               -1.919222e-01
## Channel21
               -1.431399e-01
## Channel22
               -4.840860e-02
## Channel23
               -3.651574e-02
```

```
## Channel24
                -1.864789e-02
## Channel25
                7.570411e-06
   Channel26
                 1.491368e-05
  Channel27
                 1.536979e-05
   Channel28
                 1.409084e-05
  Channel29
##
                 1.370188e-05
  Channel30
                 6.678097e-06
## Channel31
                -6.166308e-06
   Channel32
                -1.319432e-05
   Channel33
                -8.494726e-06
   Channel34
                 1.139730e-05
##
   Channel35
                 3.363313e-05
##
   Channel36
                 5.203122e-05
##
   Channel37
                 6.456039e-05
   Channel38
                 6.255091e-05
   Channel39
                 4.143816e-05
##
  Channel40
                -2.162560e-01
   Channel41
                 6.734801e+00
##
  Channel42
                -1.260505e-05
## Channel43
               -1.435354e-05
## Channel44
               -2.446549e-05
  Channel45
               -3.925484e-05
## Channel46
                -5.191104e-05
   Channel47
                -5.284406e-05
##
  Channel48
               -2.916115e-05
   Channel49
                1.534122e-01
##
               -1.216789e-01
  Channel50
##
   Channel51
               -1.601908e+00
##
   Channel52
               -1.271089e+00
  Channel53
               -7.792935e-02
##
   Channel54
                -1.232931e-01
   Channel55
                -8.068550e-02
   Channel56
                 1.832617e-02
##
  Channel57
                 1.809941e-01
   Channel58
                -5.985316e-03
##
  Channel59
               -4.819876e-02
  Channel60
                 6.648840e-02
## Channel61
                 1.663522e-01
## Channel62
                -1.320815e-01
##
  Channel63
                -9.210347e-02
  Channel64
               -5.989582e-02
## Channel65
               -9.942104e-05
##
   Channel66
               -9.768134e-05
##
   Channel67
               -8.517783e-05
   Channel68
               -7.996782e-05
##
  Channel69
               -8.946121e-05
   Channel70
                -9.530787e-05
   Channel71
               -8.319355e-05
  Channel72
               -6.480279e-05
##
   Channel73
                -6.565899e-05
##
  Channel74
               -7.081182e-05
## Channel75
               -5.436526e-05
## Channel76
               -2.522736e-05
## Channel77
               -1.878956e-05
```

```
## Channel78
               -1.046285e-05
## Channel79
               -4.135561e-06
## Channel80
                7.920261e-06
## Channel81
                1.224882e-05
## Channel82
                1.636592e-05
## Channel83
                1.766812e-05
  Channel84
                1.797741e-05
## Channel85
                2.643758e-05
  Channel86
                3.403708e-05
## Channel87
                4.460767e-05
## Channel88
                5.152972e-05
## Channel89
                5.712904e-05
## Channel90
                5.687547e-05
## Channel91
                5.839429e-05
## Channel92
                6.316667e-05
## Channel93
                6.450139e-05
## Channel94
                6.473916e-05
## Channel95
                6.403011e-05
## Channel96
               -1.350519e-02
## Channel97
                1.275633e-01
## Channel98
                9.754263e-02
## Channel99
                6.085942e-02
## Channel100
                1.261776e-01
```



number of variables chosen = 100

```
## optimal lambda= 0
##
             Length Class Mode
## lambda
              10001 -none- numeric
## cvm
             10001 -none- numeric
## cvsd
             10001 -none- numeric
             10001 -none- numeric
## cvup
## cvlo
             10001 -none- numeric
             10001 -none- numeric
## nzero
## name
                 1 -none- character
## glmnet.fit
                12 elnet list
## lambda.min
                 1 -none- numeric
## lambda.1se
                 1 -none- numeric
## [1] 1.770297
## [1] 48146
```

The value lambda minimum is found to be 0. This would make the MSE 100% which make the model and hence not optimal. The number of variables chosen are 100. The least MSE can be however found at log lambda value -7.

Q8

The linear model made in step 4 selects 63 variables using StepAIC while the lasso model in step 7 no variable selection is done due to lambda value of 0 being considered in the procedure. 'Both' directional selection was used to perform this analysis.

Appendix

```
knitr::opts_chunk$set(echo = TRUE)
library(dplyr)
library(plotly)
library(ggplot2)
library(seriation)
library(glm2)
library(kknn)
library(cvTools)
library(xlsx)
library(MASS)
library(glmnet)
spambase = read.xlsx("spambase.xlsx", sheetName = "spambase_data", header = TRUE)
n=dim(spambase)[1]
set.seed(12345)
id=sample(1:n, floor(n*0.5))
train=spambase[id,]
test=spambase[-id,]
spambase$Spam<-as.factor(spambase$Spam)</pre>
#build model on training dataset
```

```
model <- glm(Spam ~ . , data=train, family = "binomial")</pre>
summary(model)
#predict Y
predictedtest <- predict(model,newdata=test ,type = "response")</pre>
p_class2 <- ifelse(predictedtest > 0.5, "Spam", "Not Spam")
table(p_class2)
tab2 <- table(test$Spam, p_class2)</pre>
tab2
#misClassError
1 - sum(diag(tab2))/sum(tab2) # for test data
#predict Y
predictedtrain<- predict(model,data=train, type = "response")</pre>
p_class1 <- ifelse(predictedtrain > 0.5, "Spam", "Not Spam")
table(p_class1)
\#confusion Matrix
tab1 <- table(train$Spam, p_class1)</pre>
tab1
#misClassError
1 - sum(diag(tab1))/sum(tab1) # for training data
#3.
#predict Y
predictedY3 <- predict(model, newdata=test,type = "response")</pre>
p_class3 <- ifelse(predictedY3 > 0.90, "Spam", "Not Spam")
table(p_class3)
tab3 <- table(test$Spam, p_class3)</pre>
tab3
#misClassError
1 - sum(diag(tab3))/sum(tab3) # for test data
#predict Y
predictedY3.1 <- predict(model,data=train, type = "response")</pre>
p_class3.1 <- ifelse(predictedY3.1 > 0.90, "Spam", "Not Spam")
table(p_class3.1)
tab3.1 <- table(train$Spam, p_class3.1)</pre>
tab3.1
#misClassError
```

```
1 - sum(diag(tab3.1))/sum(tab3.1) # for test data
#Q4 KNN K nearest neighbors
model4.1 <- kknn(Spam ~ .,train,test,k=30)</pre>
summary(model4.1)
predictedY4.1 <- predict(model4.1)</pre>
p_class4.1 <- ifelse(predictedY4.1 > 0.50, "Spam", "Not Spam")
table(p_class4.1)
tab4.1 <- table(test$Spam, p_class4.1)</pre>
tab4.1
1 - sum(diag(tab4.1))/sum(tab4.1)
model4.2 <- kknn(Spam ~ .,train,train,k=30)</pre>
summary(model4.2)
predictedY4.2 <- predict(model4.2)</pre>
p_class4.2 <- ifelse(predictedY4.2 > 0.50, "Spam", "Not Spam")
table(p_class4.2)
tab4.2 <- table(train$Spam, p_class4.2)</pre>
1 - sum(diag(tab4.2))/sum(tab4.2)
model5.1 <- kknn(Spam ~ .,train,test,k=1)</pre>
summary(model5.1)
predictedY5.1 <- predict(model5.1)</pre>
p_class5.1 <- ifelse(predictedY5.1 > 0.50, "Spam", "Not Spam")
table(p_class5.1)
tab5.1 <- table(test$Spam, p_class5.1)</pre>
tab5.1
1 - sum(diag(tab5.1))/sum(tab5.1)
model5.2 <- kknn(Spam ~ .,train,train,k=1)</pre>
summary(model5.2)
predictedY5.2 <- predict(model5.2)</pre>
p_class5.2 <- ifelse(predictedY5.2 > 0.50, "Spam", "Not Spam")
table(p_class5.2)
tab5.2 <- table(train$Spam, p_class5.2)</pre>
tab5.2
1 - sum(diag(tab5.2))/sum(tab5.2)
data <- swiss
y <- as.vector(data[,1])</pre>
x <- as.matrix(data[,c(2:6)])</pre>
```

```
Nfolds <- 5
weighter<- function(X,Y)</pre>
  W \leftarrow ginv(t(X)%*%X)%*%t(X)%*%Y
n <- dim(data)[1]</pre>
set.seed(12345)
sample_n <- sample(1:n)</pre>
ids <- list()</pre>
cvscore <- c()
cv_func <- function(X,Y,Nfolds)</pre>
  start <- 1
  for(i in 1:Nfolds)
    {
       if(i<Nfolds)</pre>
         end <- start+(as.integer(n/Nfolds)-1)</pre>
         ids[[i]] <- sample_n[start:end]</pre>
         start <- end+1
       }
       else if(i==Nfolds)
         end \leftarrow n
         ids[[i]] <- sample_n[start:end]</pre>
       X_test <- X[as.vector(ids[[i]]),]</pre>
       X_train <- X[-as.vector(ids[[i]]),]</pre>
       Y_test <- Y[as.vector(ids[[i]])]</pre>
       Y_train <- Y[-as.vector(ids[[i]])]</pre>
       weights <- as.matrix(weighter(X=X_train,Y=Y_train))</pre>
       y_cap <- X_test%*%weights</pre>
       loss <- y_cap-Y_test</pre>
       cv <- sum(loss*loss)/length(Y_test)</pre>
       cvscore[i] <- cv</pre>
  average_cv <- sum(cvscore)/Nfolds</pre>
cvseq<- matrix(0, nrow = 0, ncol = 3)</pre>
for(k in 1:ncol(x))
  combs <- combn(1:ncol(x),k)</pre>
  for(j in 1:ncol(combs))
    x_new <- as.matrix(x[,combs[,j]])</pre>
    x_new <- cbind(x_new, 1) #adding the incercept</pre>
    seq <- paste(combs[,j], collapse = ",") #converting the sequence into string</pre>
```

```
avg_cvscore <- cv_func(X = x_new,Y = y,Nfolds)</pre>
    cvseq <- rbind(cvseq, c(seq,avg_cvscore, k))</pre>
 }
}
cvseq = as.data.frame(cvseq)
colnames(cvseq) = c("Seq", "Loss", "Xnum")
cvseq$Loss = as.numeric(as.character(cvseq$Loss))
cvseq$Seq = as.character(cvseq$Seq)
cv_func(X= x,Y= y,Nfolds)
cat("Optimal subset of features: ",cvseq$Seq[which.min(cvseq$Loss)])
cat("Cross validation score: ",min(cvseq$Loss))
save1<-ggplot()+geom_point(data= cvseq,aes(x= cvseq$Xnum,y= cvseq$Loss))</pre>
save1
\#spambase = read.xlsx("spambase.xlsx", sheetName = "spambase_data", header = TRUE)
tecator = read.xlsx("tecator.xlsx", sheetName = "data", header = TRUE,row.names = 1)
#tecator=read.csv("tecator.csv", sep=',')
tecator1=as.data.frame(tecator)
n=dim(tecator)[1]
set.seed(12345)
id=sample(1:n, floor(n*0.5))
train=tecator[id,]
test=tecator[-id,]
fit= lm(Moisture~Protein,data=tecator)
plot(Moisture~Protein,data=tecator)
abline(fit,col="red")
coeff=coefficients(fit)
eq = paste0("y = ", round(coeff[2],1), "*x +", round(coeff[1],1))
eq
sm<-summary(fit)</pre>
mse <-mean(sm$residuals^2)</pre>
tecator$Protein2<-(tecator$Protein)^2</pre>
tecator$Protein3<-(tecator$Protein)^3</pre>
tecator$Protein4<-(tecator$Protein)^4
tecator$Protein5<-(tecator$Protein)^5
tecator$Protein6<-(tecator$Protein)^6</pre>
n=dim(tecator)[1]
set.seed(12345)
id=sample(1:n, floor(n*0.5))
train=tecator[id,]
validation=tecator[-id,]
```

```
fit1.1= lm(Moisture~Protein,data=train,x=TRUE,y=TRUE)
coeff1.1=coefficients(fit1.1)
coeff1.1
eq1.1 = paste0("y = ", round(coeff[2],1), "*x +", round(coeff[1],1))
eq1.1
sm1.1<-summary(fit1.1)</pre>
sm1.1
mse1.1 <-mean((fit1.1$fitted.values-fit1.1$y)^2)</pre>
mse1.1
predict.validation1<-predict(fit1.1,validation)</pre>
mse1.1.1<-mean((predict.validation1-validation$Moisture)^2)</pre>
fit2.1= lm(Moisture~Protein+Protein2,data=train,x=TRUE,y=TRUE)
coeff2.1=coefficients(fit2.1)
coeff2.1
eq2.1 = paste0("y = ", round(coeff2.1[2],1), "*x +", round(coeff2.1[3],1),
                "*x^2 +", round(coeff2.1[1],1))
eq2.1
sm2.1<-summary(fit2.1)</pre>
mse2.1 <-mean((fit2.1$fitted.values-fit2.1$y)^2)</pre>
predict.validation2<-predict(fit2.1,validation)</pre>
mse2.1.1<-mean((predict.validation2-validation$Moisture)^2)
fit3.1= lm(Moisture~Protein+Protein2+Protein3,data=train,x=TRUE,y=TRUE)
coeff3.1=coefficients(fit3.1)
coeff3.1
eq3.1 = paste0("y = ", round(coeff3.1[2],1), "*x +", round(coeff3.1[3],1),
                "*x^2 +",round(coeff3.1[4],1), "*x^2 +",round(coeff3.1[1],1))
eq3.1
sm3.1<-summary(fit3.1)</pre>
mse3.1 <-mean((fit3.1$fitted.values-fit3.1$y)^2)</pre>
mse3.1
predict.validation3<-predict(fit3.1,validation)</pre>
mse3.1.1<-mean((predict.validation3-validation$Moisture)^2)</pre>
fit4.1= lm(Moisture~Protein+Protein2+Protein3+Protein4,data=train,x=TRUE,y=TRUE)
coeff4.1=coefficients(fit4.1)
coeff4.1
sm4.1<-summary(fit4.1)</pre>
mse4.1 <-mean((fit4.1$fitted.values-fit4.1$y)^2)</pre>
predict.validation4<-predict(fit4.1,validation)</pre>
mse4.1.1<-mean((predict.validation4-validation$Moisture)^2)</pre>
fit5.1= lm(Moisture~Protein+Protein2+Protein3+Protein4+Protein5,data=train
```

```
,x=TRUE,y=TRUE)
coeff5.1=coefficients(fit5.1)
coeff5.1
sm5.1<-summary(fit5.1)</pre>
mse5.1 <-mean((fit5.1$fitted.values-fit5.1$y)^2)</pre>
mse5.1
predict.validation5<-predict(fit5.1,validation)</pre>
mse5.1.1<-mean((predict.validation5-validation$Moisture)^2)</pre>
fit6.1= lm(Moisture~Protein+Protein2+Protein3+Protein4+Protein5+Protein6,data=train
            , x=TRUE, y=TRUE)
coeff6.1=coefficients(fit6.1)
coeff6.1
sm6.1<-summary(fit6.1)</pre>
mse6.1 <-mean((fit6.1$fitted.values-fit6.1$y)^2)</pre>
predict.validation6<-predict(fit6.1,validation)</pre>
mse6.1.1<-mean((predict.validation6-validation$Moisture)^2)</pre>
msetrain<-c(mse1.1,mse2.1,mse3.1,mse4.1,mse5.1,mse6.1)</pre>
msevalidation <- c (mse1.1.1, mse2.1.1, mse3.1.1, mse4.1.1, mse5.1.1, mse6.1.1)
msevalues<-c(msetrain,msevalidation)</pre>
mseplotdata<-data.frame(id=1:6,msevalues,datasetfrom=c("train","train","train","train","train","train",</pre>
scattermse<-ggplot(mseplotdata, aes(y=mseplotdata$msevalues, x=mseplotdata$id, color=mseplotdata$datasetfr
scattermse
knitr::include_graphics("biasvar.png")
library(MASS)
attach(tecator1)
fit <- lm(Fat~.-Moisture-Protein,data=data.frame(tecator1))</pre>
step <- stepAIC(fit, direction="both",trace = FALSE)</pre>
step$anova
summary(step)
step$coefficients
length(step$coefficients)
covariates1=scale(tecator1[,1:100])
response1=scale(tecator1[, 101])
model1=glmnet(as.matrix(covariates1),response1, alpha=0,family="gaussian")
plot(model1, xvar="lambda", label=TRUE)
model=cv.glmnet(as.matrix(covariates1),response1, alpha=0,family="gaussian")
model $ lambda.min
plot(model)
coef(model, s="lambda.min")
model$lambda.min
covariates2=scale(tecator1[,1:100])
response2=scale(tecator1[, 101])
```

```
model2=glmnet(as.matrix(covariates2),response2, alpha=1,family="gaussian")
plot(model2, xvar="lambda", label=TRUE)
model=cv.glmnet(as.matrix(covariates2),response2, alpha=1,family="gaussian")
model $lambda.min
plot(model)
coef(model, s="lambda.min")
model$lambda.min
\#sum((ynew-mean(y))^2)/sum((y-mean(y))^2)
#sum((ynew-y)^2)
plot(model2, xvar="lambda", label=TRUE)
tecator2<-scale(tecator1)</pre>
covariates3=(tecator2[,1:100])
response3=(tecator2[, 101])
model9=cv.glmnet(as.matrix(covariates3), response3, alpha=1,family="gaussian",lambda=seq(0,10,0.001))
y=validation[,101]
ynew=predict(model9, newx=as.matrix(validation[, 1:100]), type="response")
coef(model9, s="lambda.min")
plot(model9)
cat(paste("number of variables chosen =",length(coef(model9,s="lambda.min"))-1))
optimal<-model9$lambda.min
cat(paste("optimal lambda=",optimal))
summary(model9)
#Coefficient of determination
sum((ynew-mean(y))^2)/sum((y-mean(y))^2)
sum((ynew-y)^2)
```