Lesson answers: Convolution

1 1D discrete convolution

a) The convolution result is the following:

$f(\lambda)$:

1	-1	-2	**0**	-1	1	2	-1

$h(x-\lambda)$:

-2	**2**	1

$\rightarrow$

$g(x)$:

1	1	-6	-2	**3**	-1	6	1	-6	2

b)

$$\begin{pmatrix} 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \\ -2 \\ 0 \\ -1 \\ 1 \\ 2 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -6 \\ -2 \\ 3 \\ -1 \\ 6 \\ 1 \\ -6 \\ 2 \end{pmatrix}$$

Note that the matrix if formed as a Toeplitz matrix.

2 2D discrete convolution

$g(x, y) =$

1	5	**9**	9
3	8	10	3
2	2	2	0

3 2D convolution and correlation applied on images

		8	8	8	8		
		8	8	8	8		
		8	8	8	8		
		8	8	8	8		

f(x,y)

	4	4			−4	−4	
	4	4			−4	−4	
	4	4			−4	−4	
	4	4			−4	−4	

gx(x,y)

		−4	−4	−4	−4		
		−4	−4	−4	−4		
		4	4	4	4		
		4	4	4	4		

gy(x,y)

a) See the figure above.

b) Where the edge goes from 0 to 8, when going from left to right and when going from bottom to top in the image (matrix), the derivative is **positive**. Therefore, along such edges in the original image, we get **positive** values in the resulting images gx and gy after the convolution.

Where the edge goes from 8 to 0, when going from left to right and when going from bottom to top in the image, the derivative is **negative**. Therefore, along such edges in the original image, we get **negative** values in the resulting image gx and gy after the convolution.

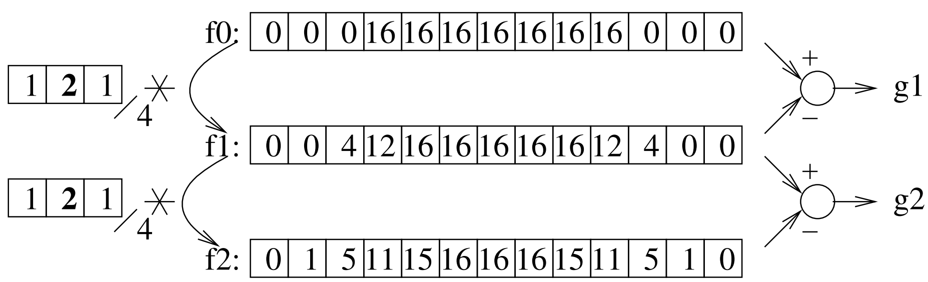
c) We get the same results as in problem a) - see the figure above.

d) $vert$ provides a match for **positive** vertical edges, because we get large **positive** values along these edges in the corresponding positions/pixels in the resulting image. Along **negative** vertical edges, we get corresponding large **negative** values.

$horis$ provides a match for **positive** horisontal edges, because we get large **positive** values along such an edge in the corresponding positions/pixels in the resulting image. Along **negative** horisontal edges, we get corresponding large **negative** values.

(*Note:* Compare this correlation reasoning with the corresponding convolution/derivation reasoning in problem b).)

4 1D lowpass and bandpass filtering



g1:													
	0	0	–4	4	0	0	0	0	0	4	–4	0	0

g2:													
	0	–1	–1	1	1	0	0	0	1	1	–1	–1	0

a) The edges are blurred (smoothed) more and more after each lowpass filtering.

b)
- Smooth surfaces, i.e. with the same pixel values, are set to zero by both filters.
- The edge response from the highpass filter is a negative value next to a positive value (-4, 4) or (4, -4). The edge response from the bandpass filter is similar to the highpass response, but wider and weaker.

c) $\boxed{1 \mid 3 \,\|\, 3 \mid 1}/8$ and $\boxed{1 \mid 4 \mid \mathbf{6} \mid 4 \mid 1}/16$.

d) $$\begin{array}{|c|c|c|}\hline 1 & \mathbf{2} & 1 \\ \hline \end{array}/4 * \begin{array}{|c|}\hline 1 \\ \hline \mathbf{2} \\ \hline 1 \\ \hline \end{array}/4 = \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & \mathbf{4} & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}/16$$

5 The effect of convolution kernels on 2D images

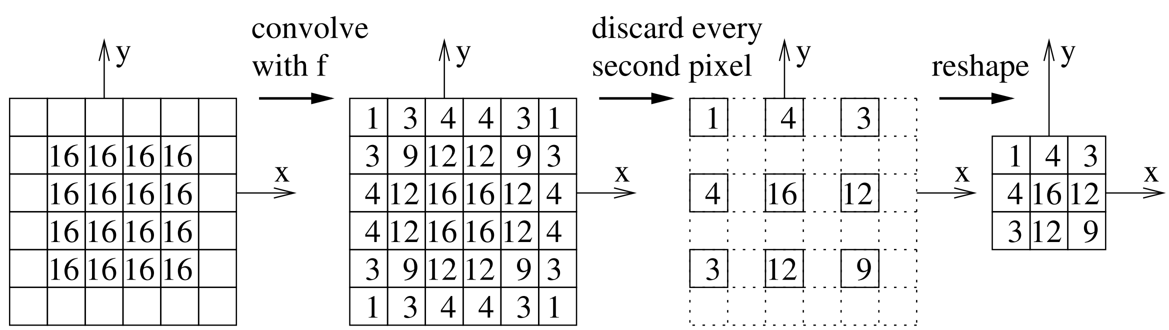
- a and E: E lowpass filters a lot (three times more than A) and results in a very blurry image.
- b and A: A lowpass filters a little and results in a slightly blurred image.
- c and D: Convolving with D only results in a scaling of all pixel values by a factor 0.5.

- d and C: Convolving with C results in a derivation in the vertical direction, which can be seen as large positive or negative pixel values along horisontal edges in the image.

- e and B: Convolving with B results in a derivation in the horisontal direction, which can be seen as large positive or negative pixel values along vertical edges in the image.

- f and F (unknown): In the image F, the convolution has resulted in large positive or negative values along diagonal edges.

6 Downsampling



a) See the figure above.

b) In traditional convolution, the convolution kernel is moved **one** step at a time in each horisontal/forward direction. In strided convolution, the kernel is instead moved **two or more** steps in each direction, which results in an image with lower resolution than the original image.
Hence, traditional convolution followed by a downsampling with a factor 2, as in problem a), can alternatively be performed by a stride 2 convolution.