Teaching session 2: Hand In assignment.

Assignment 1:

$$\text{ARIMA}(p, d, q) \times (P, D, Q)_S$$

using the backshift operator:

$$\phi^P(B^S)\,\phi^p(B)\,(1-B^S)^D\,(1-B)^d\,x_t = \theta^Q(B^S)\,\theta^q(B)\,w_t$$

you may use the following

$$p=3,\ d=2,\ q=1,\ P=2,\ D=1,\ Q=1,\ S=5$$

$$z_t = A z_{t-1} + e_t,$$

$$x_t = C z_t + v_t$$

e_t and v_t are noise that we want to get rid of.

x_t is the measure.

z_t is the latent variable that we want to find without the noise (i.e we want to remove the noise)

ARIMA models in state space form: $\phi^p(B)\,x_t = \theta^q(B)\,w_t$

let $r = \max(p, q+1)$, $= \max(3, 1+1) = 3$

$$\phi^r(B) = 1 - \phi_1 B - \dots - \phi_r B^r$$

$$\phi^3(B) = 1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3$$

$$\theta^r(B) = 1 + \theta_1 B - \dots - \theta_{r-1} B^{r-1}$$

$$\theta^3(B) = 1 + \theta_1 B - \theta_2 B^2$$

$$\phi^r(B)\,(\theta^r(B))^{-1}\,x_t = w_t$$

Hence, for $z_t = (\theta^r(B))^{-1} x_t$, we can have $\phi^r(B)\,z_t = w_t$

$$\phi^3(B)\,(\theta^3(B))^{-1}\,x_t = w_t$$

$$(\theta^3(B))^{-1}\,x_t = z_t, \quad \phi^3(B)\,z_t = w_t$$

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$$z_t = \begin{bmatrix} z_t \\ z_{t-1} \\ z_{t-2} \\ z_{t-3} \\ z_{t-4} \end{bmatrix} = \begin{bmatrix} z_t \\ z_{t-1} \\ \vdots \\ z_{t-r+1} \end{bmatrix}$$

and

$$z_t = \begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_r \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} z_{t-1} + \begin{bmatrix} w_t \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\boxed{\begin{aligned} z_t &= \begin{bmatrix} \phi_1 & \phi_2 & \phi_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} z_{t-1} + \begin{bmatrix} w_t \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ x_t &= \begin{bmatrix} 1 & \theta_1 & \theta_2 & \cdots & \theta_r \end{bmatrix} z_t \\ &= \begin{bmatrix} 1 & \theta_1 & \theta_2 & \theta_3 \end{bmatrix} z_t \end{aligned}}$$

$$\phi^p(B)\, x_t = \theta^q(B)\, w_t$$

$\cancel{z_t} =$

Time Series Analysis - Teaching assignment 2:

The state space model is given by

$$z_t = A z_{t-1} + e_t \quad ; \quad e_t \sim N(0, R)$$

$$x_t = C z_t + v_t \quad ; \quad v_t \sim N(0, Q)$$

Find the state space representation for ARIMA $(p, d, q) \times (P, D, Q)_S$

$$\phi^P(B^S)\,\phi^p(B)(1-B^S)^D(1-B)^d x_t = \theta^Q(B^S)\,\theta^q(B)\,w_t$$

$$\phi^P(B^S)\,\phi^p(B)(1-B^S)^D(1-B)^d \underbrace{\left[\theta^Q(B^S)\,\theta^q(B)\right]^{-1} x_t}_{z_t} = w_t$$

$$\phi^P(B^S)\,\phi^p(B)(1-B^S)^D(1-B)^d z_t = w_t$$

Matrix A will be,

$$A = \begin{bmatrix} \phi_1 & \phi_2 & \phi_3 & \cdots & \phi_r \\ 1 & 0 & \cdots & \cdots & 0 \\ \vdots & 1 & & & \\ \vdots & & 1 & & \\ & & & \ddots & \\ 0 & & & & 1 \end{bmatrix}$$

where $r = \max(P + p + sD + d,\ Q + sq + 1)$

$\therefore$ The AR equation can be written as:

$$\begin{bmatrix} z_t \\ z_{t-1} \\ \vdots \\ z_{t-r+1} \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_r \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \\ 0 & & & 1 \end{bmatrix} \begin{bmatrix} z_{t-1} \\ \vdots \\ z_{t-r} \end{bmatrix} + \begin{bmatrix} w_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Matrix C;

$$C = \begin{bmatrix} 1 & \theta_1 & \theta_2 & \cdots & \theta_{r-1} \end{bmatrix}$$

The MA equation can be written as:

$$x_t = \begin{bmatrix} 1 & \theta_1 & \theta_2 & \cdots & \theta_{r-1} \end{bmatrix} \begin{bmatrix} z_t \\ z_{t-1} \\ \vdots \\ z_{t-r+1} \end{bmatrix}$$

Substituting the given values;

$p = 3$, $d = 2$, $q = 1$, $P = 2$, $D = 1$, $Q = 1$, $s = 5$

$$r_{max} = \max(3+2+2\cdot1+5,\ 5+2\cdot1+1)$$

$$= \max(12, 8) = 12$$

$$\begin{bmatrix} z_t \\ z_{t-1} \\ \vdots \\ z_{t-13} \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_{12} \\ 1 & - & - & 0 \\ \vdots & 1 & - & 0 \\ \vdots & & \ddots & \\ 0 & & & 1 \end{bmatrix} \begin{bmatrix} z_{t-1} \\ z_{t-2} \\ \vdots \\ z_{t-12} \end{bmatrix} + \begin{bmatrix} w_t \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

And;

$$x_t = \begin{bmatrix} 1 & \theta_1 & \theta_2 & \cdots & \theta_{11} \end{bmatrix} \begin{bmatrix} z_t \\ z_{t-1} \\ \vdots \\ z_{t-13} \end{bmatrix}$$