Time Series Analysis — Teaching assignment 1

Assignment 12

Let $\{x_t\}$ be a zero mean unit variance stationary process with ACF $= \rho_n$

$$y_t = \mu_t + \sigma_t x_t$$

a) Find mean and covariance function of $\{y_t\}$ process

$$E(y_t) = E(\mu_t + \sigma_t x_t)$$

μ_t and σ_t are non-stochastic

$$\mu_t = \mu_t + \sigma_t \underbrace{E(x_t)}_{0}$$

$$\operatorname{cov}(y_s, y_t) = E[(y_s - E(y_s))(y_t - E(y_t))]$$

$$y_t - E(y_t) = \cancel{\mu_t} + \sigma_t x_t - \cancel{\mu_t} = \sigma_t x_t$$

$$y_s - E(y_s) = \mu_s - \sigma_s x_s - \mu_s = \sigma_s x_s$$

$$E(\sigma_s \sigma_t x_s x_t) = \sigma_s \sigma_t E(x_s, x_t) = \sigma_s \sigma_t E[(x_s - E(x_s))(x_t - E(x_t))]$$

$$= \sigma_s \sigma_t \gamma_x(s,t) = \sigma_s \sigma_t \operatorname{cov}(x_s, x_t)$$

b) Show that the ACF for the $\{y_t\}$ process depends only on time lag. Is $\{y_t\}$ stationary?

$$\rho(s,t) = \frac{\gamma(s,t)}{\sqrt{\gamma(s,s)\,\gamma(t,t)}}$$

$$\rho_y(t, t+h) = \frac{\gamma_y(t, t+h)}{\sqrt{\gamma_y(t,t)\,\gamma_y(t+h, t+h)}} = \frac{\gamma_y(t, t+h)}{\sqrt{\operatorname{var}(y_t^2)^2 \operatorname{var}(y_{t+n}^2)}}$$

$$= \frac{\cancel{\sigma_{y_t}}\ \cancel{\sigma_{y_{t+h}}}\ \operatorname{cov}(y_t, y_{t+n})}{\sigma_{y_t}^2\ \sigma_{y_{t+h}}^2} = \frac{\gamma_y(t, t+h)}{\sigma_{y_t}\ \sigma_{y_{t+h}}}$$

y_t is stationary

$$\gamma_y(t, t+h) = \gamma_y(|t+h-t|) = \gamma_y(h)$$

$\Rightarrow$ Autocovarinance depends only on lag.

Autocovariance for stationary process $\gamma_y(h) = cov(y_t, y_{t+h})$

c) Yes. It is possible where $var(y_t) = \infty$ ($\mu_t = const$, $\gamma(s,t) = \gamma(s-t)$, $var(y_t) < \infty$)

Assignment 18:

For each of the ARMA models, find the roots of the AR and MA polynomials, identify the values of p, q for which they are ARMA (p, q) (be careful of parameter redundancy), determine whether they are causal and determine whether they are invertible. In each case $W_t \sim Wn(0,1)$

c) $x_t - 3x_{t-1} = w_t + 2w_{t-1} - 8w_{t-2}$

$1 - 3B = 0$

$3B = 1$

$B = 1/3$

$B < 1$, $\therefore$ not causal

$1 + 2B - 8B^2 = 0$

$8B^2 - 2B - 1 = 0$

$B = \left(\frac{2+6}{16}, \frac{2-6}{16}\right) = \left(\frac{1}{2}, -\frac{1}{4}\right)$

$B < 1$, $\therefore$ not invertible

d) $x_t - 2x_{t-1} + 2x_{t-2} = w_t - \frac{8}{9}w_{t-1}$

$1 - 2B + 2B^2 = 0$

$2B^2 - 2B + 1 = 0$

$B = (0.5 + 0.5i, 0.5 - 0.5i)$

$\therefore$ not causal

$1 - \frac{8}{9}B = 0$

$\frac{8}{9}B = 1 \Rightarrow B = \frac{9}{8} > 1$

$\therefore$ invertible.

e) $x_t - 4x_{t-2} = w_t - w_{t-1} + 0.5w_{t-2}$

$1 - 4B^2 = 0$

$B = (1/2, -1/2)$

$\therefore$ not causal.

$1 - B + 0.5B^2 = 0$

$0.5B^2 - B + 1 = 0$

$B = ~~(1+i, 1-i)~~$ $(-2.73, 0.73)$

$\therefore$ non-invertible. $B < 1$

Scanned by CamScanner