Quantum Polar Filter

Let's see how light behaves going through polarising filters!

You can try this yourself, use polarised lenses from sunglasses or a science supply shop (don't use *circular* polarisers that are common on cameras).

Polarization

Light is normally free to vibrate in any direction at right angles to its path.

But **polarized** light vibrates in one plane only:

Light gets polarized when passing through a polarizing filter.

Photon

Before the filter we can write the state of the photon like this (see Unit Circle):

cos(θ)→ + sin(θ)↑

Where

- → means left-right direction, and
- ↑ means up-down direction:

First Encounter

What happens when the photon meets a polarising filter?

Let us say the filter is aligned in the left-right direction.

After passing through the filter the photon is either **blocked** or emerges as

→

with a **probability of** $\mathbf{cos^2(\theta)}$

Note: this follows the same rule as light intensity (Malus' Law): intensity is proportional to cos^2(angle).

Probability

One of the basic rules in quantum mechanics is that the probability equals the amplitude magnitude squared, in other words:

Probability = $|\text{Amplitude}|^2$

The || means magnitude of a vector, not absolute value.

This example may help:

Example θ = 45°

At 45° we have

cos(45°)→ + sin(45°)↑

cos(45°) = 1√2, and sin(45°) = 1√2 (see Unit Circle), so we have:

1√2→ + 1√2↑

So, is the probability of passing 1√2 ?

Not quite, because total probability should equal 1

But with a little help from pythagoras we have:

$(1\sqrt{2})^2 + (1\sqrt{2})^2 = 1^2$

12 + 12 = 1

Each probability is 12. At 45° that makes sense, right?

We can use pythagoras each time, or simply remember:

The probability of each state is the amplitude magnitude squared:

$(1\sqrt{2})^2 = 12$

Let's try another angle just to be sure, how about 30°?

cos(30°)→ + sin(30°)↑

cos(30°) = √32 and sin(30°) = 12, so:

√32→ + 12↑

The probability of each state is the amplitude magnitude squared:

$(\sqrt{32})^2 = 34$ and $(12)^2 = 14$

And 34 + 14 = 1

OK, enough examples, back to our filtering.

We are currently polarised in the left-right direction, like this:

→

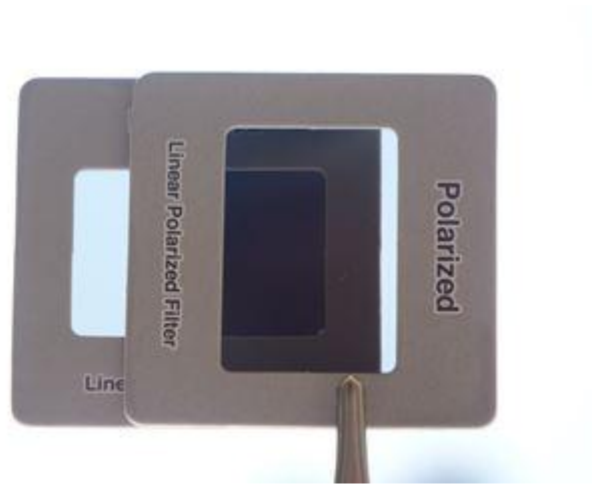
100% probability left-right, 0% probability up-down.

Next Filter!

The next filter we use is **up-down polarised**.

But we currently have 0% probability of up-down.

So too bad. All gone. And the result is **blackness**.



But What If We Add a 45° In Between?

Now we place a third filter in between the other two, and orient it at 45 degrees.

Our "intuition" says adding more filtering should block the light even more, making for a blacker black, right?

Well, let's work through the mathematics!

After the first (left-right) filter we have (as before):

→

Now the photon faces the middle filter at 45°

We have already seen an example of what happens at 45°. Well, the photon doesn't care what orientation our nice graph is at, so this works just as well:

The result is:

1√2↗ + 1√2↘

and faces a 1/2 chance of being blocked, and if it gets through it is now at:

↘

Now the photon faces the final filter at 45°

Sorry? Isn't that 90°? To us maybe, but from the photon's *current* point of view it is another 45°. Like this:

The result is:

1√2↓ + 1√2→

And again there is a 12 chance of being blocked, or getting through at:

↓

The total for the last two filters is 12 × 12 = 14

Meaning that a photon that got through the first filter has a 1-in-4 chance of **getting through** the next two filters. So there is **a modest chance** that a photon can get through all 3 filters!

And it looks like this:

You can see that 0°⇒90° is black (lower centre triangle), but 0°⇒45°⇒90° (upper centre triangle) actually lets some light through. Adding that middle filter at 45° lets more light through.

Wow, mathematics rules!

What We Learned Here