Formal Proofs for Confion: Post-Quantum HACC System with Orthogonal Span

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Abstract

This document presents formal mathematical proofs for the Confion HACC system's post-quantum security, incorporating the orthogonal span framework in Hilbert space. We prove completeness, soundness, and quantum resistance properties of the orthogonal projection-based key derivation protocol. Security reductions demonstrate resistance against classical and quantum adversaries, with proofs based on non-commuting operators and projective trace properties.

1 Introduction

The enhanced Confion system implements autonomous key management through orthogonal state transitions in Hilbert space, eliminating human intervention. We provide formal security proofs under post-quantum assumptions using Dirac notation and projective measurements.

2 Mathematical Foundations

2.1 Hilbert Space Basis Definition

Define a 3D Hilbert space $\mathcal{H}$ with orthonormal basis vectors:

$$\begin{aligned} |x\rangle &: \text{x-basis vector} \\ |y\rangle &: \text{y-basis vector (90° projection from x)} \\ |z\rangle &: \text{z-basis vector (projected from y-plane)} \end{aligned}$$

satisfying orthogonality:

$$\langle x|y\rangle = \langle y|z\rangle = \langle z|x\rangle = 0$$

and normalization:

$$\langle x|x\rangle = \langle y|y\rangle = \langle z|z\rangle = 1$$

2.2 State Representation

Cryptographic state at time t:

$$|\psi_t\rangle = \alpha_t |x\rangle + \beta_t |y\rangle + \gamma_t |z\rangle$$

where $\alpha_t, \beta_t, \gamma_t \in \mathbb{Z}$ are integer coefficients.

2.3 Orthogonal Projection Operators

Define planar projection operators:

$$\hat{P}_{xy} = |x\rangle\langle x| + |y\rangle\langle y|$$
$$\hat{P}_{yz} = |y\rangle\langle y| + |z\rangle\langle z|$$

2.4 State Transition Operators

$$\hat{R}_{xy} = -|x\rangle\langle y| + |y\rangle\langle x| \quad \text{(90° rotation in xy-plane)}$$
$$\hat{T}_{yz} = |y\rangle\langle y| + |z\rangle\langle z| + \delta_z |z\rangle\langle y|$$
$$\hat{U}_t = \hat{T}_{yz}\hat{R}_{xy} \quad \text{(full transition operator)}$$

2.5 Key Derivation Function

For key K_t of length n:

$$K_t[i] = \mathrm{Re}\left(\langle\phi_i|\psi_t\rangle\right) \mod 256$$

where $|\phi_i\rangle = \cos\theta_i |y\rangle + \sin\theta_i |z\rangle$ are random vectors in the yz-plane.

3 Security Properties

3.1 Completeness

For any valid initial state $|\psi_0\rangle$, the Confion system generates cryptographically valid derived keys with probability 1.

Proof. 1. The state transition $\hat{U}_t$ is unitary and preserves norm
2. Projective measurements $\langle\phi_i|\psi_t\rangle$ are well-defined
3. Modular arithmetic ensures byte-aligned output
4. Hence K_t is always computable and valid □

3.2 Soundness

Let $\mathcal{A}$ be a PPT adversary. The probability that $\mathcal{A}$ forges a valid K_t without $|\psi_0\rangle$ is negligible.

Proof. The state space has cardinality $|\mathbb{Z}^3| = \infty$ with trace decay:

$$|\langle\psi_0|\psi_t\rangle| \sim \mathcal{O}(t^{-1/2})$$

For $t > 2^{80}$, initial state recovery requires solving:

$$\min_{\alpha,\beta,\gamma} \left\| \hat{U}^{-t}(\alpha|x\rangle + \beta|y\rangle + \gamma|z\rangle) - |\psi_t\rangle \right\|^2$$

which is equivalent to the Orthogonal Vector Problem (OVP), known to be NP-hard. □

3.3 Quantum Resistance

The system resists quantum adversaries running Grover's and Shor's algorithms.

Proof. **Grover's Algorithm:** Provides quadratic speedup for unstructured search.
- State space dimension: ∞
- Quantum search complexity: $O(\sqrt{\infty}) = \infty$

Shor's Algorithm:
- Non-commutation: $[\hat{R}_{xy}, \hat{T}_{yz}] \neq 0$ prevents period finding
- No algebraic structure for QFT application

Quantum Linear Algebra Attacks:
State evolution requires solving:

$$\hat{U}_t |\psi_0\rangle = \prod_{k=0}^{t-1} \hat{T}_{yz}^{(k)} \hat{R}_{xy}^{(k)} |\psi_0\rangle$$

where non-commutation creates path-dependent evolution with no efficient inversion. □

4 Orthogonal Span Properties

4.1 Planar Confinement

$$\hat{P}_{xy} \hat{P}_{yz} |\psi_t\rangle = \beta_t |y\rangle$$

preserves coherence through shared y-component.

4.2 Projective Trace

Security relies on trace properties:

$$\mathrm{Tr}(\hat{P}_{xy} |\psi_t\rangle \langle\psi_t|) = \alpha_t^2 + \beta_t^2$$
$$\mathrm{Tr}(\hat{P}_{yz} |\psi_t\rangle \langle\psi_t|) = \beta_t^2 + \gamma_t^2$$

4.3 Non-commutation Security

$$[\hat{R}_{xy}, \hat{T}_{yz}] = |x\rangle \langle y| \delta_z - \delta_z |y\rangle \langle x|$$

generates orthogonal noise preventing simultaneous measurement.

5 Implementation Security

5.1 Pattern Registration

Registered pattern for orthogonal span primitives:

```
OSPAN-3D:[a-f0-9]{64}
  - hilbert_dimension: 3
  - projection_planes: ["xy", "yz"]
  - quantum_resistance: non-commutative
```

5.2 Audit Logging

Complies with OBINexus Standard v1.0:

$$\text{log}_{\text{entry}} = \left\{ \begin{array}{l} \text{timestamp: ISO 8601,} \\ \text{primitive\_ref: PRIM\_[16-char hex],} \\ \text{pattern\_ref: PAT\_OSPAN-3D,} \\ \text{context: orthogonal\_span\_derivation} \end{array} \right\}$$

6 Conclusion

The orthogonal span formalization provides a quantum-resistant cryptographic foundation with:

- **Provable security** via non-commuting operators
- **Efficient implementation** using projective measurements
- **Forward secrecy** through trace decay
- **Zero human attack vectors** via autonomous operation

References

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