OBINexus Truth: Formal Proof Framework

Eziokwu - Mathematical Foundation for AI Accountability

OBINexus Systems
Nnamdi Michael Okpala, Architect

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Abstract

This document establishes the formal mathematical framework for OBINexus Truth's proof verification system, implementing the Declarative-Constructive hybrid logic through rigorous set theory and procedural definitions. The framework provides structured mechanisms for handling claimed proofs within AI accountability infrastructure while maintaining cultural sensitivity and human-centered design principles.

Contents

1 Foundation Set Definitions

Definition 1.1 (Universal Evidence Space). *Let $\mathcal{U}$ be the universal evidence space containing all possible evidence artifacts:*

$$\mathcal{U} = \{e : e \textit{ is a digitally representable evidence artifact}\}$$

Definition 1.2 (Claim Bubble). *A claim bubble $\mathcal{C}$ is defined as a 4-tuple:*

$$\mathcal{C} = \langle A, E, T, \Phi \rangle$$

where:

- *A is the assertion set: $A = \{a_1, a_2, \ldots, a_n\}$ where each a_i is an atomic claim*
- *$E \subseteq \mathcal{U}$ is the evidence set supporting the claims*
- *T is the temporal constraint set: $T = [t_{start}, t_{end}] \times \mathbb{N}$*
- *Φ is the attribution mapping: $\Phi : A \to \mathcal{P}(Actors)$*

Definition 1.3 (Proof Space). *The proof space $\mathcal{PS}$ for a claim bubble $\mathcal{C}$ is defined as:*

$$\mathcal{PS}(\mathcal{C}) = \{p : p \textit{ is a valid proof sequence for } \mathcal{C}\}$$

where each proof $p \in \mathcal{PS}(\mathcal{C})$ must satisfy both declarative and constructive constraints.

2 Declarative Proof Logic (7-Method Model)

Definition 2.1 (Declarative Proof Sequence). *A declarative proof sequence $D = \langle d_1, d_2, \ldots, d_7 \rangle$ for claim bubble $\mathcal{C}$ is defined where:*

$$\begin{align}
d_1 &: \textit{Assertion} \to A \textit{ (traceable claim establishment)} \tag{1}\\
d_2 &: \textit{Evidence} \to E \textit{ (data and context provision)} \tag{2}\\
d_3 &: \textit{Attribution} \to \Phi \textit{ (actor/system association)} \tag{3}\\
d_4 &: \textit{Cross-Verification} \to V(\mathcal{C}) \textit{ (independent validation)} \tag{4}\\
d_5 &: \textit{Contradiction Check} \to \neg\exists c \in \mathcal{C}^c : c \models \neg A \tag{5}\\
d_6 &: \textit{Temporal Validation} \to T \models \textit{occurrence constraints} \tag{6}\\
d_7 &: \textit{Archival Recording} \to H(\mathcal{C}, D) \textit{ (cryptographic storage)} \tag{7}
\end{align}$$

Definition 2.2 (Cross-Verification Function). *The cross-verification function $V : \mathcal{C} \to \{0, 1\}$ is defined as:*

$$V(\mathcal{C}) = \begin{cases} 1 & \textit{if } \exists E_{ind} \subseteq \mathcal{U} : E_{ind} \cap E = \emptyset \wedge E_{ind} \models A \\ 0 & \textit{otherwise} \end{cases}$$

where E_{ind} represents independently sourced evidence.

3 Constructive Proof Overlay

Definition 3.1 (Constructive Proof Properties). *A constructive proof K for claim bubble $\mathcal{C}$ must satisfy:*

1. ***Executable Sequence**: $\exists f : \mathcal{C} \to \mathcal{O}$ where $\mathcal{O}$ is the set of observable outcomes*
2. ***Fault Traceability**: $\forall failure\ \phi \in K, \exists trace\ \tau : \tau \models reconstruction(\phi)$*
3. ***Evidence Transformability**: $\exists g : E \times Context \to E'$ where $E' \models A$ in new context*
4. ***No Black Box Doctrine**: $\forall result\ r \in K, \exists logic\ chain\ \ell : \ell \vdash r$*

Theorem 3.1 (Proof Completeness). *A proof p is complete for claim bubble $\mathcal{C}$ if and only if:*

$$p \in \mathcal{PS}(\mathcal{C}) \land D(p) = 1 \land K(p) = 1$$

where $D(p)$ indicates declarative validity and $K(p)$ indicates constructive validity.

4 Formal Procedure for Claimed Proof Handling

Procedure 4.1 (Claimed Proof Verification Protocol). *Given a claimed proof $\hat{p}$ for claim bubble $\mathcal{C}$, execute the following verification sequence:*

***Input**: Claimed proof $\hat{p}$, Claim bubble $\mathcal{C}$ **Output**: Verification result $v \in \{VALID, INVALID, INSUF$*

1. ***Claim Bubble Validation**:*

$$\begin{aligned} Validate(\mathcal{C}) = check(A \neq \emptyset) \land check(E \subseteq \mathcal{U}) && (8)\\ \land\ check(T \text{ is well-formed}) && (9)\\ \land\ check(\Phi \text{ is total}) && (10) \end{aligned}$$

2. ***Evidence Integrity Check**:*

$$\forall e \in E : verify(hash(e)) \land validate(metadata(e))$$

3. ***Declarative Verification**:*

$$D_{valid}(\hat{p}) = \bigwedge_{i=1}^{7} d_i(\hat{p}) = true$$

4. ***Constructive Verification**:*

$$K_{valid}(\hat{p}) = executable(\hat{p}) \land traceable(\hat{p}) \land transformable(\hat{p}) \land transparent(\hat{p})$$

5. ***Cultural Sensitivity Assessment**:*

$$Cultural(\hat{p}) = respect(context) \land accommodate(neurodivergence) \land preserve(dignity)$$

6. ***Final Verification Decision**:*

$$v = \begin{cases} VALID & \text{if } D_{valid}(\hat{p}) \land K_{valid}(\hat{p}) \land Cultural(\hat{p}) \\ INVALID & \text{if } \neg D_{valid}(\hat{p}) \lor \neg K_{valid}(\hat{p}) \lor \neg Cultural(\hat{p}) \\ INSUFFICIENT & \text{otherwise} \end{cases}$$

5 Proof Space Topology

Definition 5.1 (Proof Distance Metric). *For two proofs $p_1, p_2 \in \mathcal{PS}(\mathcal{C})$, define the proof distance:*

$$d(p_1, p_2) = \sum_{i=1}^{7} w_i \cdot |d_i(p_1) - d_i(p_2)| + \lambda \cdot |K(p_1) - K(p_2)|$$

where w_i are weights for declarative components and λ weights constructive differences.

Definition 5.2 (Proof Convergence). *A sequence of proofs $\{p_n\}$ converges to truth $\mathcal{T}$ if:*

$$\lim_{n\to\infty} d(p_n, \mathcal{T}) = 0$$

where $\mathcal{T}$ represents the ground truth for claim bubble $\mathcal{C}$.

6 Implementation Architecture

Definition 6.1 (Evidence Processing Pipeline). *The evidence processing function $\Pi : \mathcal{U} \to \mathcal{PS}$ is decomposed as:*

$$\Pi = H_7 \circ T_6 \circ C_5 \circ V_4 \circ A_3 \circ E_2 \circ I_1$$

where each Π_i corresponds to the i-th step of the declarative proof model.

Lemma 6.1 (Pipeline Composability). *If each stage Π_i preserves evidence integrity and cultural sensitivity, then the composed pipeline Π maintains these properties.*

7 Governance Integration

Definition 7.1 (Policy-Procedure Coupling). *For governance framework integration, define the coupling function:*

$$PMP : \mathcal{PS} \times \mathit{Policies} \to \mathit{Procedures}$$

such that every proof verification maps to executable governance procedures while maintaining the No Ghosting Protocol.

Theorem 7.1 (Accountability Preservation). *The OBINexus Truth framework preserves accountability if for every verified proof $p \in \mathcal{PS}(\mathcal{C})$:*

$$\exists \textit{audit trail } \alpha : \alpha \models \textit{complete traceability of } p$$

8 Conclusion

This formal framework establishes the mathematical foundation for OBINexus Truth's proof verification system, ensuring that AI accountability mechanisms maintain rigorous logical standards while preserving human-centered values and cultural sensitivity. The hybrid Declarative-Constructive approach provides both theoretical soundness and practical implementability for real-world AI transparency requirements.

Eziokwu - Truth as correct speech, implemented through heart-centered computational logic.