ISE 4623/5023: Deterministic Systems Models / Systems Optimization University of Oklahoma School of Industrial and Systems Engineering Fall 2024

Individual Assignment 1: Linear Algebra Basics and Mathematical Notation

1. (18 points) Let

$$A = \begin{bmatrix} 4 & 1 & 8 \\ 2 & -7 & 5 \end{bmatrix}, B = \begin{bmatrix} -9 & 7 \\ 1 & 5 \\ 8 & 3 \end{bmatrix}$$

Solve:

(a) (3 points) AB

(b) (3 points) BA

(c) (3 points) $-B$

(d) (3 points) $B - A$

(e) (3 points) A^T

(f) (3 points) $-B^T B$

Solution:

(a)

$$AB = \begin{bmatrix} 29 & 57 \\ 15 & -6 \end{bmatrix}$$

(b)

$$BA = \begin{bmatrix} -22 & -58 & -37 \\ 14 & -34 & 33 \\ 38 & -13 & 79 \end{bmatrix}$$

(c)

$$-B = \begin{bmatrix} 9 & -7 \\ -1 & -5 \\ -8 & -3 \end{bmatrix}$$

(d) $B - A$: Not possible, different dimensions

(e)

$$A^T = \begin{bmatrix} 4 & 2 \\ 1 & -7 \\ 8 & 5 \end{bmatrix}$$

(f)

$$-B^T B = \begin{bmatrix} -146 & 34 \\ 34 & -83 \end{bmatrix}$$

2. (21 points) If possible, solve for x and y. If they can't be computed, give a brief explanation of why:

(a) (7 points)

$$3x + y = 5$$
$$8x + 5y = 2$$

Solution:

$$y = 5 - 3$$
$$25 - 15x + 8x = 2$$
$$x = 23/7, y = -34/7$$

(b) (7 points)

$$15x + 5y = 40$$
$$3x - y = 8$$

Solution:

$$y = 3x - 8.$$
$$30x - 40 = 40.$$
$$x = 8/3, y = 0.$$

(c) (7 points)

$$2x - 24y = 42$$
$$-x + 12y = -20$$

Solution: No solutions

3. (20 points) Let:

$$C = \begin{bmatrix} 3 & -2 \\ 12 & -8 \end{bmatrix}, D = \begin{bmatrix} -1 & 4 \\ 6 & -8 \end{bmatrix}, E = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} F = \begin{bmatrix} -3 & 5 & 9 \\ 8 & -2 & 5 \\ 1 & -2 & 14 \end{bmatrix}$$

Solve:

(a) (5 points) C^{-1}

(b) (5 points) D^{-1}

(c) (5 points) E^{-1}

(d) (5 points) F^{-1}

Solution:

(a)

$$C^{-1} : \text{No solution}$$

(b)

$$D^{-1} : \begin{bmatrix} 0.5 & 0.25 \\ 0.375 & 0.0625 \end{bmatrix}$$

(c)

$$E^{-1} : \text{No solution}$$

(d)

$$F^{-1} = \begin{bmatrix} \frac{18}{607} & \frac{88}{607} & -\frac{43}{607} \\ \frac{107}{607} & \frac{51}{607} & -\frac{87}{607} \\ \frac{14}{607} & \frac{1}{607} & \frac{34}{607} \end{bmatrix}$$

4. (15 points) Translate into words:

(a) (5 points) $\sum_{j \in \mathbb{R}} j$ Solution: Summation of all real numbers.

(b) (5 points) $\sum_{j \in \mathbb{Z}^+ | j \bmod 5 = 0} j$ Solution: Summation of all integer possitive numbers multiples of 5.

(c) (5 points) $\sum_{\substack{j=1 \\ j \in \mathbb{Z}}}^{40} j$ Solution: Summation of all integers from 1 to 40.

5. (12 points) Translate into mathematical expression:

(a) (4 points) Summation of every element j over the set J such that the element is odd. Solution: $\sum_{j \in J | j \bmod 2 != 0} j$

(b) (4 points) Summation of every variable x_{ij} over j that belongs to the set J for all i that belongs to the set I. Solution: $\sum_{j \in J} x_{ij} \ \forall \ i \in I$

(c) (4 points) Summation of every variable x_{ij} over j that belongs to the set J and i that belongs to the set I, if the pair (i,j) belongs to the set of pairs A. Solution: $\sum_{j\in J, i\in I:(i,j)\in A} x_{ij}$

6. (14 points) Expand the expressions:

   (a) (7 points) $\sum_{j\in\{0,1,\ldots,12\}:j \bmod 2=0} j$ Solution: $2+4+6+8+10+12$

   (b) (7 points) Let

$$N=\{1,2,3,4\},\ A=\{(1,2),(2,3),(2,4),(3,4),(2,1)\},\ D=\{3\},\ S=\{1,2\},\ T=\{4\}$$

$$b_i=\begin{cases}\alpha_i & i\in S\\ \beta_i & i\in D\\ 0 & i\in T\end{cases} \quad \forall\, i\in N$$

   Expand:

$$\sum_{j\in N:(i,j)\in A} x_{ij} - \sum_{j\in N:(j,i)\in A} x_{ji} = b_i \ \forall\, i\in N$$

   Solution:

$$x_{12}-x_{21}=\alpha_1$$

$$x_{23}+x_{24}+x_{21}-x_{12}=\alpha_2$$

$$x_{34}-x_{23}=\beta_3$$

$$-x_{24}-x_{34}=0$$

The Greeks, just to keep in mind:

Figure 1: Retrieved from `https://www.instagram.com/joshi_physics_classes/`