Proceedings of the American Control Conference
Arlington, VA June 25-27, 2001

Particle filter based detection for tracking

Y. Boers and J.N. Driessen

THALES NEDERLAND B.V.
P.O. BOX 42
7550 GD Hengelo
The Netherlands
email: y.boers@signaal.nl

Abstract

In this article we will present a new method to perform detection and tracking of a possible target in noise. The classical way in which tracking, e.g. tracking on the basis of radar measurements, is performed, is based on a number of assumptions and a certain format of the a priori information, the measurements. Detection normally is a stage which is performed in the processing chain before measurements are obtained/constructed. Typical radar measurements are range, doppler, bearing and elevation. In this paper we will perform tracking not on the basis of these standard measurements but on the raw radar video data. Detection then is based upon the a posteriori information, i.e. the probability density of the state given these past measurements (in this case video data). This way of data processing/tracking is also referred to as Track Before Detect (TBD) for obvious reasons. An advantage of this method over classical tracking is that in this TBD approach the decision whether a target is present or not is based on integrated and kinematically correlated energy. This method is better suited for tracking weak targets in noise than the classical method. As this problem statement leads to a non linear non Gaussian filtering problem classical filtering methods, e.g. Kalman filtering will result in poor performance. A particle filter is used to deal with the nonlinearities and the non Gaussian nature of the noise. The same particle filter output is also used to perform detection based on a likelihood ratio test.

1 Introduction

Classical tracking methods take as an input so called plots that typically consist of range measurements, bearing measurements, elevation measurements and range rate (doppler) measurements. See [6] and [7]. In this classical radar tracking setting, the tracking proces consists of estimating kinematic state properties, e.g. position, velocity and acceleration on the basis of these measurements.

In the classical setup the measurements (plots) are the output of the extraction, see figure 1. In the detection block thresholding on the basis of radar vidoe has already taken place.

The method, we propose here, will use as measurements the raw video data, see figure 1. If we look at figure

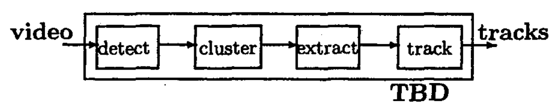

Figure 1: Classical data and signal processing (seperate boxes) and TBD (large box)

1 we see that thresholding in classical tracking is done directly on the basis of the radar video. This means that already at the beginning of the processing chain a hard decision is made w.r.t. the presence of a possible target. Note that this decision is made instantaneously, i.e without using information from the (near) past.

In the newly proposed method this decision is made at the end of the processing chain, i.e. when all information has been used and integrated over time. This method is especially suitable for tracking weak targets in noise, i.e. targets that in the classical setting often will not lead to a detection.

A particle filter will be used to perform the TBD. The *main result* is that the detection is based on the ouput of this filter. At least from a theoretical viewpoint this is the best one can do. This means that the *maximum* of information obtainable by means of the a posteriori probability density of the state given all video data upto the current time instant is obtained.

Furthermore note that although the new results in this article are applied to a TBD application for a radar tracking and detection problem they are certainly not restricted to this particular application.

2 Particle Filter

Here we will give a description of a version of a particle filter, see also [3] and [10].

Given the general nonlinear discrete time system

$$s(k+1) = f(s(k), t(k)) + g(s(k), t(k))w(k), \quad k \in \mathbb{N} \tag{1}$$

$$z(k) = h(s(k), t(k)) + v(k), \quad k \in \mathbb{N}$$

Where $s(k) \in \mathbb{R}^n$ is the state of the system. The process noise and the measurement noise densities are denoted by $p_{w(k)}(w)$ and $p_{v(k)}(v)$. $z(k) \in \mathbb{R}^p$ are the measurements.

We furthermore define the set of measurements upto the current time by

$$Z(k) = \{z(0), \ldots, z(k)\}$$

We can now formulate the optimal filtering problem.

Problem 2.1 (Filtering Problem)
Given a realization of $Z(k)$ associated with system (1) compute $p(s(k) \mid Z(k))$, i.e. the conditional probability density of the state, $s(k)$, given the set of measurements $Z(k)$.

Thus the filtering problem consists of finding the a posteriori probability distribution of the state conditional on all past measurements.

Various state estimators are easily obtained from the a posteriori probability density $p(s(k) \mid Z(k))$.

e.g the minimum variance estimator is obtained as

$$\hat{s}^{MV}(k) = \int_{\mathbb{R}^n} s(k)p(s(k) \mid Z(k))ds(k) \tag{2}$$

or the maximum a posteriori estimator as

$$\hat{s}^{MAP} = \arg \max_{s(k)\in\mathbb{R}^n} p(s(k) \mid Z(k)) \tag{3}$$

Explicit recursive solutions to problem 2.1 are only known in very special cases, e.g. the Kalman filter equations for linear Gaussian systems. A method that has been used to solve problem 2.1 approximately for nonlinear non Gaussian systems is the so called particle filter approach.

The core of the particle filter algorithm is that

$$p(s(k) \mid Z(k))$$

is approximated on the basis of an N point grid (N particles) in the state space.

We will illustrate the particle filter graphically. Suppose that at time $t(k)$ the true a posteriori probability density function

$$p(s(k) \mid Z(k))$$

is given by the probability distribution that is graphically displayed in figure 2, i.e. a tri-modal probability distribution. A particle filter then describes this distribution by a set of particles

$$\{s^i(k)\}_{i=1\ldots N} \tag{4}$$

This set of particles for the number of particles being equal to $N = 500$ is plotted in figure 4.

The number of particles in a certain region of the state space reflects the probability mass of that region. Asymptotically the particle filter will reconstruct the true probability density in the sense that is formulated in theorem 2.3.

We have also graphically displayed how a Kalman filter would approximate the true pdf, see figure 3. Note that the information on the multi modality of the true pdf is completely lost.

We furthermore emphasize that given the particle cloud emprical versions of the estimators (2) and (3) are easily obtained, e.g. for $\hat{s}^{MV}(k)$ we have

$$\hat{s}^{MV}(k) \approx \frac{1}{N}\sum_{i=1}^{N} s^i(k)$$

Below we give an algorithm for a particle filter.

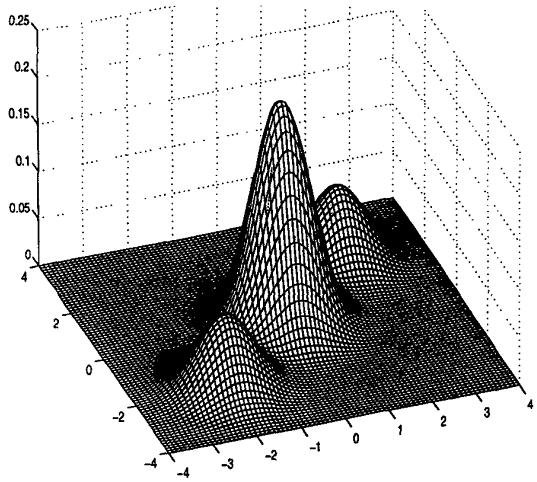

Figure 2: Tri-modal pdf (Gaussian sum)

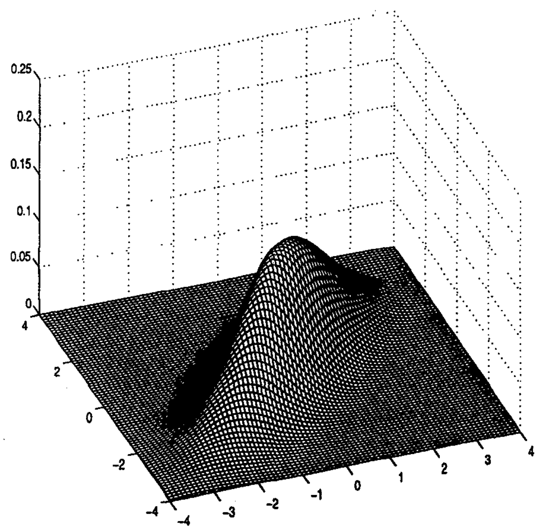

Figure 3: Approximation by single Gaussian (Kalman Filter)

Algorithm 2.2

1. draw $\{w^i(k-1)\}_{i=1,\dots,N}$ according to

$$p_w(k-1)(w)$$

   and compute $\{s^i(k)\}_{i=1,\dots,N}$ using

$$s^i(k) = f(\bar{s}^i(k-1), t(k-1)) +$$

$$+ g(\bar{s}^i(k-1), t(k-1))w^i(k-1)$$

2. Given $z(k)$ define

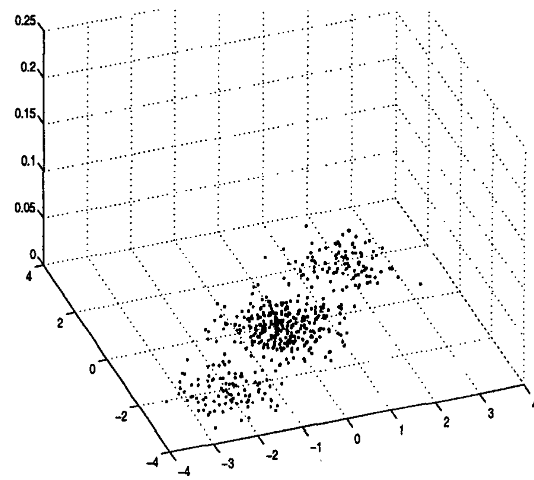

Figure 4: Particle cloud for the tri-modal pdf

$$\tilde{q}^i(k) = p(z(k) \mid s^i(k)), \quad i = 1, \dots, N \ ,$$

$$q^i(k) := \frac{\tilde{q}^i(k)}{\sum_{i=1}^{N} \tilde{q}^i(k)}, \quad i = 1, \dots, N$$

3. Resample N times from

$$\hat{p}(s) := \sum_{j=1}^{N} q^i(k)\delta(s - s^i(k))$$

   and obtain $\{\bar{s}^i(k)\}_{i=1,\dots,N}$ to construct

$$\hat{p}(s(k) \mid Z(k)) := \sum_{j=1}^{N} \frac{1}{N}\delta(s - \bar{s}^i(k))$$

   goto 1.

The following theorem, see [2], can now be formulated

Theorem 2.3
Adopting the notation of algorithm 2.2.

When applying the particle filter to system (1) the following holds

$$\lim_{N\to\infty} \sum_{j=1}^{N} \frac{1}{N}\delta(s - \bar{s}^i(k)) = p(s(k)|Z(k))$$

Remark 2.4
Theorem 2.3 states that in the limit, i.e. for an infinite number of particles , or equivalent, on an infinite dense grid, the particle filter solution equals the true solution to the filtering problem, i.e. problem 2.1.

3 Detection

In this section we will determine how to perform a detection of a possible target on the basis of the information provided by the particle filter. Apart from the problem of tracking a target there is also the problem of deciding whether there is a target present or not. This problem is called the detection problem, see e.g. [8].

Given two hypotheses

- $\mathcal{H}_0$: No target present
$$z(k) = v(k)$$
- $\mathcal{H}_1$: Target present
$$z(k) = h(s(k), t(k)) + v(k),$$
where s(k) evolves according to system (1).

We now define the likelihood ratio.

$$L(z(k)) = \frac{p(z(k) \mid \mathcal{H}_1)}{p(z(k) \mid \mathcal{H}_0)} \tag{5}$$

We now declare a target to be present whenever the likelihood, $L(z(k))$, exceeds a threshold τ, thus when

$$L(z(k)) > \tau \tag{6}$$

This test is known as the likelihood ratio test, see [8].

The choice of a good threshold is a kind of compromise between so called false alarms, i.e. exceeding of the threshold when no target is present and the probability of detection, i.e. the probability that the likelihood exceeds the threshold when indeed there is a target present. We emphasize that the determination of a threshold such that e.g. a certain false alarm rate can be guaranteed is far from trivial, this problem however also arises in classical tracking. Some background can be found in [8]

We now observe that it is easy to obtain $p(z(k) \mid \mathcal{H}_0)$, this is just the pdf of the measurement noise and is assumed to be known. It is however not straightforward to obtain an expression for $p(z(k) \mid \mathcal{H}_1)$, in fact for $p(z(k) \mid \mathcal{H}_1)$ an analytical expression is not available. The following manipulations describe how we obtain an approximation of $p(z(k) \mid \mathcal{H}_1)$ on the basis of the particle filter described in algorithm 2.2.

Under the hypothesis of a target being present, hypothesis $\mathcal{H}_1$, we can write:

$$p(z(k)) = \int p(z(k) \mid s(k \mid k-1)) \cdot \tag{7}$$

$$\cdot p(s(k \mid k-1)) ds(k \mid k-1) =$$

$$= \mathbf{E}_{p(s(k)|Z(k-1))} p(z(k) \mid s(k \mid k-1)) \approx$$

$$\approx \frac{1}{N} \sum_{i=1}^{N} p(z(k) \mid s^i(k)) = \frac{1}{N} \sum_{i=1}^{N} \tilde{q}^i(k)$$

where $\tilde{q}^i(k)$ are the *unnormalized* weights defined in algorithm 2.2.

In fact we have proven the following theorem, i.e. the main result of this paper.

Theorem 3.1 (Main)
Given system (1). The probability

$$p(z(k) \mid \mathcal{H}_1)$$

where the hypothesis $\mathcal{H}_1$ equals

$\mathcal{H}_1$: *Target present,* $\quad z(k) = h(s(k), t(k)) + v(k)$

is approximately equal to

$$\frac{1}{N} \sum_{i=1}^{N} \tilde{q}^i(k)$$

where $\tilde{q}^i(k)$ are the unnormalized *weights defined in algorithm 2.2.*

Thus, using theorem 3.1, we can perform the likelihood ratio test, (6), at least numerically.

Remark 3.2
Note that in fact instead of using the true (analytical) expression for $p(z(k) \mid \mathcal{H}_1)$, which is unknown, we obtain a numerical approximation, see theorem 3.1. We emphasize that for this approximation no additional computational effort is needed. All the information needed to obtain the approximation is already provided by the particle filter, see algorithm 2.2.

Remark 3.3
The approximation will be better if the number of particles, N, is increased. For an infinite number of particles the approximation equals the true probability, see theorem 2.3. The quality of the approximation can be directly related to the number of particles, N. Quantitative results are given in [10].

4 Example

Here we will describe a setup on which the developed theory can be tested.

Consider the following system

For the dynamics we have

$$s(k+1) = F(k)s(k) + G(k)w(k), \quad k \in \mathbb{N} \qquad (8)$$

where $s(k) = [r(k), d(k)]^T$ is the target state, $r(k)$ being the range of the target and $d(k)$ being the doppler speed or range rate of the target,

$$F(k) = \begin{pmatrix} 1 & T \\ 0 & 1 \end{pmatrix},$$

where T is the update time.

$$G(k) = \begin{pmatrix} \sigma_{rp} & 0 \\ 0 & \sigma_{dp} \end{pmatrix},$$

and $w(k)$ is standard white Gaussian Noise.

For the measurement we have

$$z(k) = h(s(k), t(k)) + v(k), \quad k \in \mathbb{N}$$

where $h^{ij}(s(k), t(k)), \quad i = 1, \ldots, N_r, \ j = 1, \ldots, N_d$ is defined as follows

$$h^{ij}(s(k), t(k)) = P - \frac{(r_{ij} - r(k))^2}{R} L_r - \frac{(d_{ij} - d(k))^2}{D} L_d$$

Where P is the target power, R is related to the size of a range gate and D is related to the size of a doppler bin. Furthermore L_r and L_d represent constants for power losses.

Furthermore the probability density of $v(k)$ is a transformation of an exponential type of distribution

Remark 4.1
One measurement consists of $N_r \times N_d$ observations of power, i.e. $z^{ij}(k)$, this is the so called radar video signal. Typical values are; $N_r = 1000$ and $N_d = 10$

Remark 4.2
Note that apart from the fact the measurement equation is nonlinear and the measurement noise is non Gaussian application of e.g. the Kalman filter would also involve computations of and manipulations with the residual covariance matrix which will be of the size $(N_r \times N_d)^2$ and thus result in a very big computational load.

For this system we are currently developing a data generator and a simulation setup to illustrate the theory developed.

5 Conclusions

In this paper we proposed a new method to perform tracking and detection of a target directly on the basis of radar video measurements. The resulting filtering problem is a high dimensional nonlinear Non Gaussian filtering problem and therefore classical techniques, e.g. the Kalman filter, are not suitable. A new detection criterion based on a particle filter has been derived.

References

[1] E. Mazor, A. Averbuch, Y. Bar-Shalom and J. Dayan. Interacting multiple model methods in target tracking: a survey. *IEEE Transactions on Aerospace and Electronic Systems*, vol. 34, pp. 103-123, 1998.

[2] A.F.M. Smith and A.E. Gelfand. Bayesian Statistics without tears: A Sampling-Resampling Perspective. *The American Statistician*, vol. 46, pp. 84-88, 1992.

[3] N.J. Gordon, D.J. Salmond and A.F.M. Smith. Novel approach to nonlinear/non-Gaussian Bayesian state estimation. *IEE Proceedings-F*, vol. 140, no.2, 1993.

[4] A. Doucet, N.J. Gordon and N. de Freitas eds. *Sequential Monte Carlo Methods in Practice*, Springer Verlag, New York, 2001.

[5] D.J. Ballantyne, H. Y. Chan and M.A. Kouritzin. A novel branching particle method for tracking. *SPIE Aerosense 2000 proceedings*, volume 4048, pp. 277-287, Orlando FL, 24-28 April 2000.

[6] S. Blackman and R. Popoli, *Design and Analysis of Modern Tracking Systems*, Artech House, Norwood, MA, 1999.

[7] Y. Bar-Shalom and X.R. Li. *Estimation and Tracking Principles, Techniques and Software*, Artech House, Norwood, MA, 1993.

[8] H.V. Poor. *An introduction to signal detection and estimation*, Springer-Verlag, New York NY, 1994.

[9] Y. Boers and J.N. Driessen. An IMM algorithm based on a hybrid bootstrap filter. *SPIE Aerosense 2000 proceedings*, volume 4048, pp. 300-308, Orlando FL, 24-28 April 2000.

[10] Y. Boers. On the number of samples to be drawn in particle filtering. *proceedings of the IEE Colloquium on Target Tracking: Algorithms and Applications*, pp. 5/1-5/6, London UK, 11-12 November 1999.