

R you ready?

One-Day Workshop 24 July, Staple Inn, London

Claims reserving in R The *ChainLadder* package

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Content

- Introduction
- ChainLadder package
- R and databases
- R and MS Office interfaces

Double click the paperclip symbol in the bottom of this slide to access all the R code of this presentation.



Claims reserving in insurance

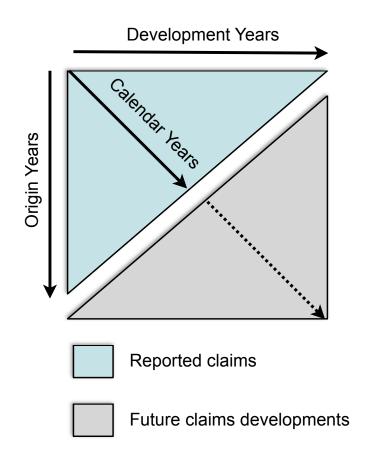
- Insurers sell the promise to pay for future claims occurring over an agreed period for an upfront received premium
- Unlike other industries insurers don't know the production cost of their product
- The estimated future claims have to be held in the reserves, one of the biggest liability items on an insurer's balance sheet

Reserving in insurance

- Reserves cover IBNR (Incurred But Not Reported) claims
- Reserves are usually estimated based on historical claims payment/reporting patterns
- In the past a point estimator for the reserves was sufficient
- New regulatory requirements (→ Solvency II) foster stochastic methods

Typical scenario

- Usually an insurance portfolio is split into "homogeneous" classes of business, e.g. motor, marine, property, etc.
- Policies are aggregated by class and looked at in a triangle view of reported claims to forecast future claims developments



Stochastic reserving

- Over recent years stochastic methods have been developed and published
- Excel is often the standard tool, but is not an ideal environment for implementing those stochastic methods
- Idea: Use R to implement stochastic reserving methods, and CRAN to share them
- Use the RExcel Add-in as a front end for Excel to use R functions

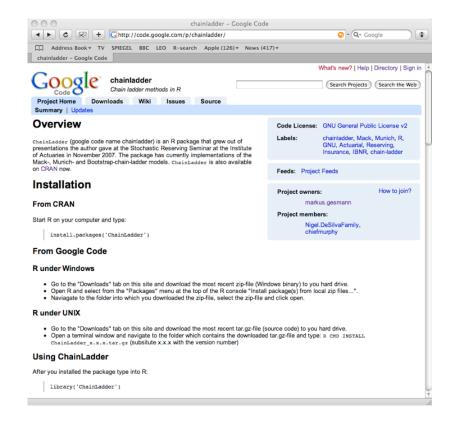
The ChainLadder package

- Started out of presentations given at the Institute of Actuaries on stochastic reserving
- Mack-, Munich- and Bootstrap-chain-ladder implemented
- Example spreadsheet shows how to use the functions with Excel using the RExcel Add-in
- Available from CRAN sources and binaries
- Contribution most welcome!

The ChainLadder package

Agenda:

- Getting started
- ChainLadder package philosophy
- Examples for
 - MackChainLadder
 - MunichChainLadder
 - BootChainLadder



Project web page: http://code.google.com/p/chainladder/

Current version: 0.1.2-11



Getting started

- Start R and type for
 - Installation: install.packages("ChainLadder")
 - Loading the package: library(ChainLadder)
 - Help: ?ChainLadder
 - Examples: example(ChainLadder)

Example data sets

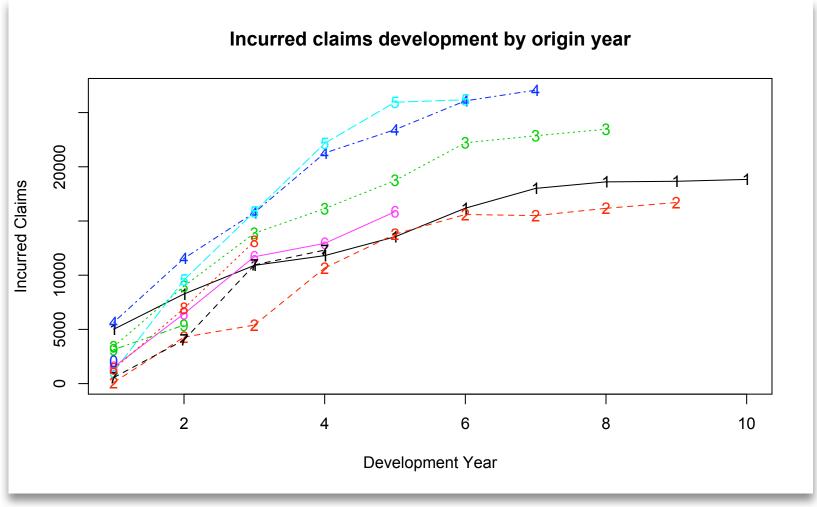
The ChainLadder package comes with some example data sets, e.g.

```
> library(ChainLadder)
```

> RAA

dev origin 2 3 5 8 10 1981 5012 8269 10907 11805 13539 16181 18009 18608 18662 18834 1982 106 5396 10666 13782 15599 15496 16169 16704 4285 NA 16141 18735 22214 22863 23466 1983 3410 8992 13873 NA NA 15766 21266 23425 26083 27067 1984 5655 11555 NA NA NA 1985 1092 9565 15836 22169 25955 26180 NA NA NA NA 1986 1513 6445 11702 12935 15852 NA NA NA NA NA 1987 557 4020 10946 12314 NA NA NA NA NA NA 1988 1351 6947 13112 NA NA NA NA NA NA NA 1989 3133 5395 NA NA NA NA NA NA NA NA 1990 2063 NA NA NA NA NA NA NA NA NA

Triangle plot



> matplot(t(RAA), t="b")

Working with triangles

Transform from cumulative to incremental

```
incRAA <- cbind(RAA[,1], t(apply(RAA,1,diff)))</pre>
```

Transform from incremental to cumulative

```
cumRAA <- t(apply(incRAA,1, cumsum))</pre>
```

Triangles to long format

```
lRAA <- expand.grid(origin=as.numeric(dimnames(RAA)
$origin), dev=as.numeric(dimnames(RAA)$dev))
lRAA$value <- as.vector(RAA)</pre>
```

Long format to triangle (see later for <u>as.ArrayTriangle</u> function, works much better with ChainLadder)

```
reshape(lRAA, timevar="dev", idvar="origin",
v.names="value", direction="wide")
```

ChainLadder package philosophy

- Use the linear regression function "1m" as much as possible and utilise its output
- The chain-ladder model for volume weighted average link ratios is expressed as a formula:

$$y \sim x + 0$$
, weights=1/x

- and can easily be changed
- Provide tests for the model assumptions

Chain-ladder as linear regression

Chain-ladder can be regarded as weighted linear regression through the origin:

```
x <- RAA[,1] # dev. period 1
y <- RAA[,2] # dev. period 2

model <- lm(y ~ x + 0, weights=1/x)

Call:
lm(formula = y ~ x + 0, weights = 1/x)
Coefficients:</pre>
```

X

(2.999) ← chain-ladder link-ratio

Full regression output

```
> summary(model)

    chain-ladder link

Call:
                                             ratio
lm(formula = y \sim x + 0, weights = 1/x)
                                            • std. error of the link
Residuals:
                                             ratio
   Min
           10 Median
                           30
                                  Max

    P-value

-95.54 -71.50 49.03 99.55 385.32

    Residual std. error

Coefficients:
  Estimate Std. Error t value Pr(>|t|
     2.999
                 1.130)
                          2.654
                                  (0.0291
X
                    '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
Signif. codes:
Residual standard error: (167) on 8 degrees of freedom
Multiple R-squared: 0.4682, Adjusted R-squared: 0.4017
F-statistic: 7.043 on 1 and 8 DF, p-value: 0.02908
```

The output shows:

model formula

Chain-ladder using the "1m" function

Idea: Create linear model for each development period

Accessing regression statistics

```
CL <- ChainLadder(RAA)
# Get chain-ladder link-ratios
sapply(CL, coef)
# 2.999 1.62 1.27 1.17 1.11 1.04 1.03 1.016 1.0
# Get residual standard errors
sapply(lapply(CL, summary), "[[", "sigma")
# 166.98 33.29 26.295 7.82 10.9 6.389 1.159 2.8 NaN
# Get R squared values
sapply(lapply(ChainLadder(RAA), summary), "[[",
"r.squared")
# 0.468 0.95 0.97 0.997 0.995 0.998 0.999 0.999 1.00</pre>
```

Mack-chain-ladder

Mack's chain-ladder method calculates the standard error for the reserves estimates.

The method works for a cumulative triangle C_{ik} if the following assumptions are hold:

$$E\left[\frac{C_{i,k+1}}{C_{ik}}|C_{i1},C_{i2},\ldots,C_{ik}\right]=f_k$$

$$\operatorname{Var}\left(\frac{C_{i,k+1}}{C_{ik}}|C_{i1},C_{i2},\ldots,C_{ik}\right) = \frac{\sigma_k^2}{C_{ik}}$$

All accident years are independent

MackChainLadder

Usage:

- Triangle: cumulative claims triangle
- weights: default (1/Triangle) volume weighted CL
- est.sigma: Estimator for sigma_{n-1}
- tail, tail.se, tail.sigma: estimators for the tail

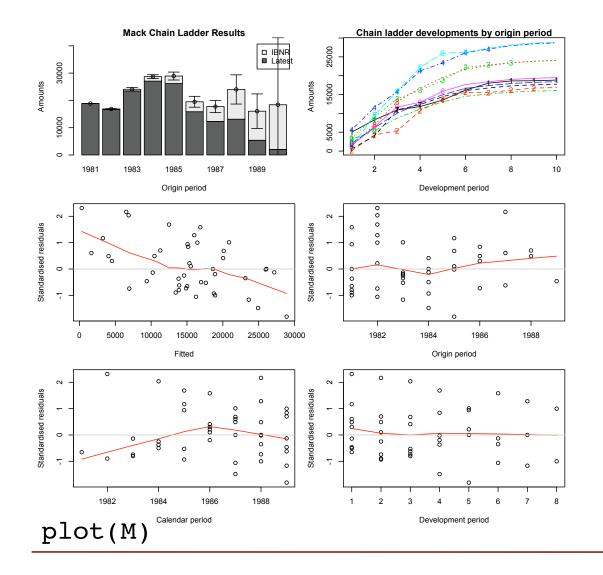
MackChainLadder example

```
library(ChainLadder)
M <- MackChainLadder(Triangle = RAA, est.sigma = "Mack")</pre>
Μ
     Latest Dev.To.Date Ultimate
                                   IBNR Mack.S.E CV(IBNR)
1981 18,834
                  1.000
                          18,834
                                      0
                                               0
                                                      NaN
1982 16,704
                  0.991
                        16,858
                                    154
                                             206
                                                     1.339
1983 23,466
                  0.974
                        24,083
                                    617
                                             623
                                                     1.010
1984 27,067
                  0.943
                          28,703
                                  1,636
                                             747
                                                    0.457
1985 26,180
                  0.905
                          28,927
                                  2,747
                                           1,469
                                                    0.535
1986 15,852
                  0.813
                          19,501
                                 3,649
                                           2,002
                                                    0.549
1987 12,314
                  0.694
                        17,749 5,435
                                           2,209
                                                    0.406
                  0.546 24,019 10,907
                                           5,358
1988 13,112
                                                    0.491
                        16,045 10,650
                                           6,333
1989 5,395
                  0.336
                                                    0.595
                                          24,566
1990 2,063
                  0.112
                          18,402 16,339
                                                     1.503
               Totals
           160,987.00
Latest:
Ultimate:
           213,122.23
IBNR:
       52,135.23
Mack S.E.: 26,909.01
```

CV(IBNR):

0.52

plot.MackChainLadder



The residual plots show the standardised residuals against fitted values, origin period, calendar period and development period.

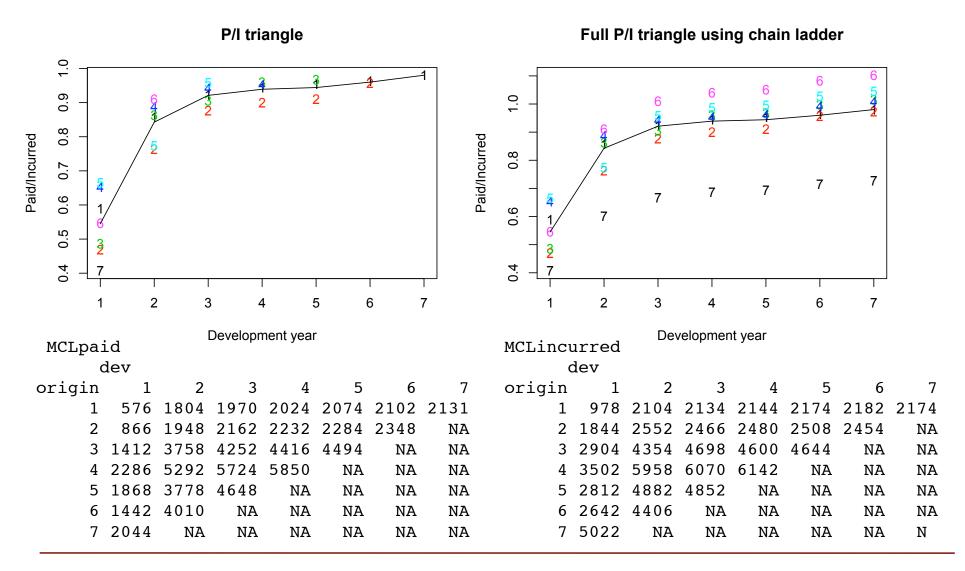
All residual plots should show no pattern or direction for Mack's method to be applicable.

Pattern in any direction can be the result of trends and require further investigations.

Munich-chain-ladder

- Munich-chain-ladder (MCL) is an extension of Mack's method that reduces the gap between IBNR projections based on paid (P) and incurred (I) losses
 - Mack has to be applicable to both triangles
- MCL adjusts the chain-ladder link-ratios depending if the momentary (P/I) ratio is above or below average
- MCL uses the correlation of residuals between P vs. (I/P) and I vs. (P/I) chain-ladder link-ratio to estimate the correction factor

Munich-chain-ladder example



MunichChainLadder

Usage:

- Paid: cumulative paid claims triangle
- Incurred: cumulative incurred claims triangle
- est.sigmaP, est.sigmaI: Estimator for sigma_{n-1}
- tailP, tailI: estimator for the tail

MunichChainLadder example

MCL

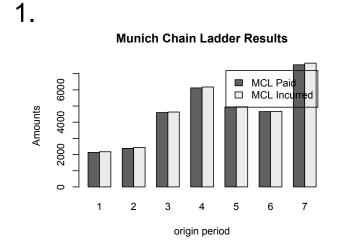
	Latest Paid	Latest Incurred	Latest P/I Ratio	Ult. Paid	Ult. Incurred Ul	lt. P/I Ratio
1	2,131	2,174	0.980	2,131	2,174	0.980
2	2,348	2,454	0.957	2,383	2,444	0.975
3	4,494	4,644	0.968	4 , 597	4,629	0.993
4	5 , 850	6,142	0.952	6,119	6 , 176	0.991
5	4,648	4,852	0.958	4,937	4,950	0.997
6	4,010	4,406	0.910	4,656	4,665	0.998
7	2,044	5,022	0.407	7 , 549	7 , 650	0.987
				(/

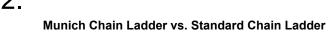
Totals

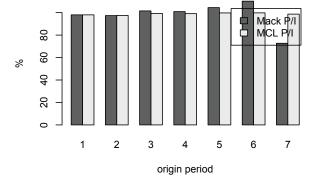
		Paid	Incurred	P/I	Ratio
	Latest:	25,525	29,694		0.86
	Ultimate:	32,371	32,688		0.99

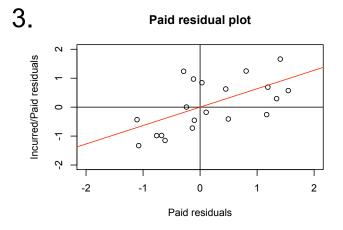
Munich-chain-ladder forecasts based on paid and incurred losses

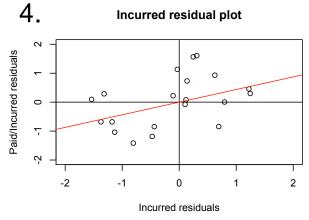
plot.MunichChainLadder











1. MCL forecasts on P and I

- 2. Comparison of Ultimate P/I ratios of MCL and Mack
- 3. I/P link-ratio residuals against P link-ratio residuals
- 4. P/I link-ratio residuals against I link-ratios residuals

plot(MCL)



Bootstrap-chain-ladder

- BootChainLadder uses a two-stage approach.
 - 1. Calculate the scaled Pearson residuals and bootstrap R times to forecast future incremental claims payments via the standard chain-ladder method.
 - 2. Simulate the process error with the bootstrap value as the mean and using an assumed process distribution.
- The set of reserves obtained in this way forms the predictive distribution, from which summary statistics such as mean, prediction error or quantiles can be derived.

BootChainLadder

Usage:

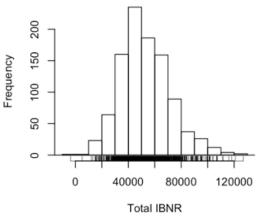
- Triangle: cumulative claims triangle
- R: Number of resampled bootstraps
- process.distr: Assumed process distribution

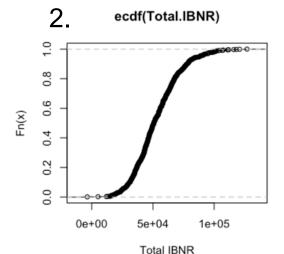
BootChainLadder example

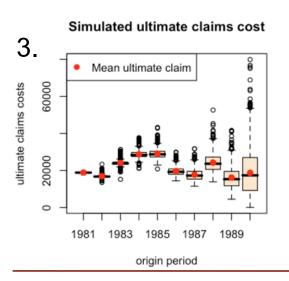
```
set.seed(1)
BootChainLadder(Triangle = RAA, R = 999, process.distr = "od.pois")
     Latest Mean Ultimate Mean IBNR SD IBNR IBNR 75% IBNR 95%
1981 18,834
                  18,834
                                 0
                                         0
                                                  0
                                                           0
1982 16,704
                  16,921
                                       710
                                                253
                                                       1,597
                               217
1983 23,466
                  24,108
                                              1,074
                               642
                                     1,340
                                                       3,205
1984 27,067
                  28,739
                             1,672
                                    1,949
                                              2,679
                                                       4,980
1985 26,180
                  29,077
                             2,897
                                    2,467
                                              4,149 7,298
                                              4,976
1986 15,852
                  19,611
                             3,759
                                    2,447
                                                       8,645
1987 12,314
                             5,410
                                    3,157
                                             7,214
                                                      11,232
                  17,724
                                           14,140
                                                      20,651
1988 13,112
                  24,219
                            11,107
                                    5,072
                  16,119
                            10,724
                                    6,052 14,094
                                                     21,817
1989 5,395
                                             24,459
    2,063
                  18,714
                            16,651
                                    13,426
                                                      42,339
1990
                Totals
               160,987
Latest:
               214,066
Mean Ultimate:
                53,079
Mean IBNR:
                18,884
SD IBNR:
                64,788
Total IBNR 75%:
Total IBNR 95%:
                88,037
```

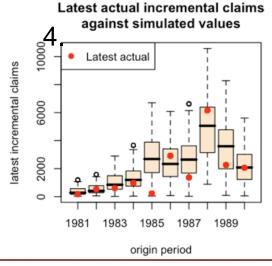
plot.BootChainLadder











- 1.Histogram of simulated total IBNR
- 2.Empirical distribution of total IBNR
- 3.Box-whisker plot of simulated ultimate claims cost by origin period
- 4. Test if latest actual incremental loss could come from simulated distribution of claims cost

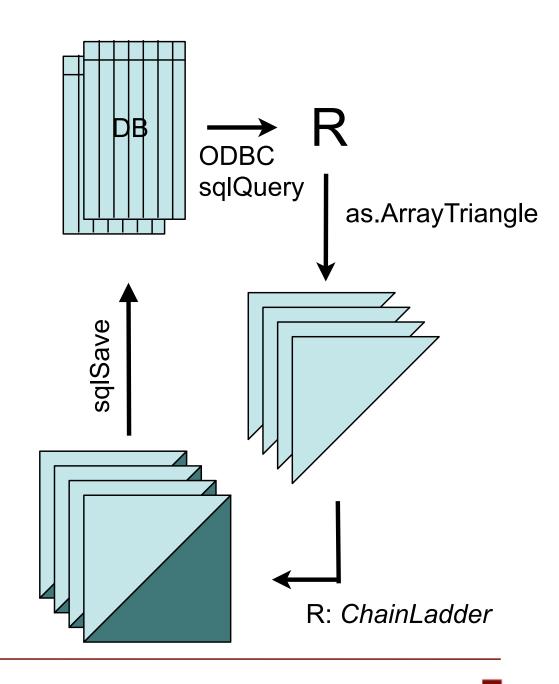
Generic Methods

- Mack-, Munich-, BootChainLadder
 - names: gives the individual elements back
 - summary: summary by origin and totals
 - print: nice formatted output
 - plot: plot overview of the results
- MackChainLadder
 - residuals: chain-ladder residuals
- BootChainLadder
 - mean: mean IBNR by origin and totals
 - quantile: gives quantiles of the simulation back

R and databases

Agenda:

- Create test data
- DDBC SQL
- Query database
- Tables to triangles
- Apply functions
- Write to database



Working with databases

- Triangles are usually stored in databases
 - Triangles are stored in long tables
- Use ODBC to connect to databases
- Use SQL to interact with databases
- Use R to transform tables into triangles
- Apply ChainLadder function across many triangles in one statement
- Write results back into database

Create sample data in a table format

Use example data sets to create a sample data table

Write test data into database

Example with MS Access 2003 See also documentation for RODBC

Read tables from database

Access data via ODBC and SQL-statements

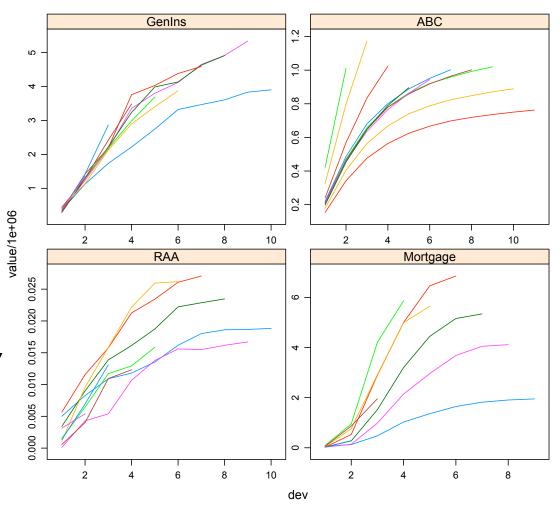
ChainLadderTestData.mdb attached at the bottom of this page (double click on paper clip) and save in C:/Temp



As an aside: Plot tables with lattice

Triangles stored in long tables are much easier to plot than triangles in cross-tab formats

```
# Plot long triangles
library(lattice)
xyplot(
 value/1e6 ~ dev | LOB,
 groups=origin, t="1",
 data=myData,
 scales="free"
)
```



Transform tables into triangles

We use the *array* function rather than *reshape*, as its output is ready to be used by *ChainLadder*

Use by to apply ChainLadder functions

- by function applies functions on sub sets of data
 - convert table for each LOB into a triangle
 - apply MackChainLadder for each triangle
- Output is stored in a list

```
myResults <- by(myData, list(LOB=myData$LOB),
    function(x){
        triangle <- as.ArrayTriangle(x)
        M <- MackChainLadder(triangle, est.sigma="Mack")
        return(M)
    })
myResults</pre>
```

Combine results in tables

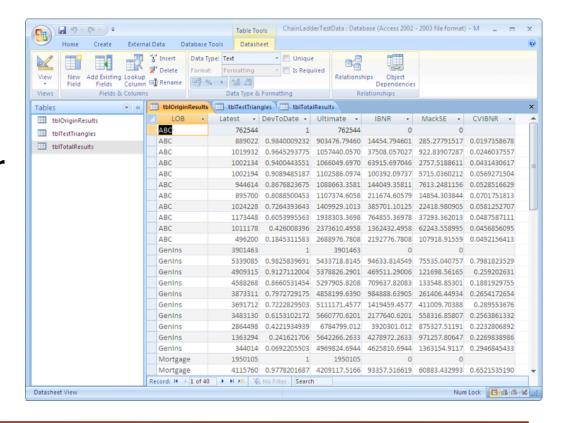
- Use lapply to access MackChainLadder output
 - Access origin year and total results separately

Write results into database

 Write results back into new tables of the database via QDBC and sqlSave

Database summary

- Use R to query DB
- Transform table to triangles
- Apply ChainLadder function across all triangles
- Summaries results
- Save results in DB



R and MS Office interfaces

Agenda:

- win.metafile
- Clipboard
- RExcel
- COM-server
- rcom



Windows meta-file

- Windows meta-file (WMF, or EMF (Enhanced meta-file) is a vector graphic format
- High quality, but editable format for MS Office
- Create WMF-files in R with win.metafile()

```
win.metafile(file="C:/Temp/Testplot.wmf")
plot(sin(seq(0,round(2*pi,2),0.01)))
dev.off()
```

Clipboard to exchange data

Copy and paste from R to and from Excel

R -> Excel

```
mydf=data.frame(x=1:10, y=letters[1:10])
write.table(mydf, file="clipboard",
sep="\t", row.names=FALSE)
```

• Excel -> R
read.table(file= "clipboard", sep="\t")

RExcel - Using R from within Excel

RExcel Add-in allows to use R functions from Excel, see:

http://sunsite.univie.ac.at/rcom/

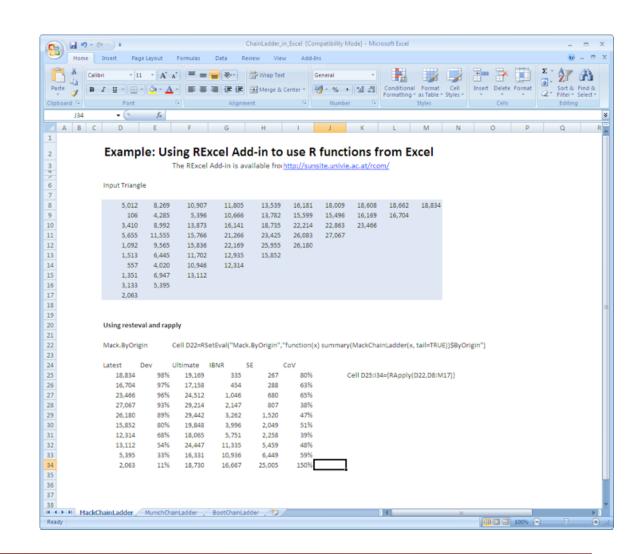
There are at least three different ways of using R from within Excel

- Scratchpad mode
 - Writing R Code directly in an Excel worksheet and transferring scalar, vector, and matrix variables between R and Excel
- Macro mode
 - Writing macros using VBA and the macros supplied by RExcel, attaching the macros to menu items or toolbar items
- Worksheet functions
 - R can be called directly in functions in worksheet cells

Source: http://sunsite.univie.ac.at/rcom/server/doc/RExcel.html

ChainLadder_in_Excel.xls

- RExcel allows to use R functions within Excel
- Package comes with example file
- R function can be embedded and are interactive
- Use R graphics



Using the COM server (VBA Example)

StatConnector allows to use R within MS Office VBA

Add reference to **StatConnectorSrv** 1.1 Type Library

```
Sub FirstR()
Dim nrandom As Integer, x As Double
nrandom = 100
Set StaR = New StatConnector
StaR.Init ("R")
With StaR
   .SetSymbol "n", nrandom
   .EvaluateNoReturn ("x <- rnorm(n)")
   .EvaluateNoReturn ("pdf(file='c:/Temp/Testplot.pdf')")
   .EvaluateNoReturn ("hist(x)")
   .EvaluateNoReturn ("dev.off()")
x = .Evaluate("mean(x)")
End With
Debug.Print x
End Sub</pre>
```

rcom: Control MS Office from R

- Using the rcom R-package you can write output from R into MS Office application
 - Example: Create PowerPoint slide with MackChainLadder output

```
library(ChainLadder)
R <- MackChainLadder(RAA)
myfile=tempfile()
win.metafile(file=myfile)
plot(R)
dev.off()
#
library(rcom)
ppt<-comCreateObject("Powerpoint.Application")
comSetProperty(ppt, "Visible", TRUE)
myPresColl<-comGetProperty(ppt, "Presentations")
myPres<-comInvoke(myPresColl, "Add")
mySlides<-comGetProperty(myPres, "Slides")
mySlide<-comInvoke(mySlides, "Add", 1, 12)
myShapes<-comGetProperty(mySlide, "Shapes")
myPicture<-comInvoke(myShapes, "AddPicture", myfile, 0, 1, 100, 10)</pre>
```

More help ...

- See examples on project web page
- Read documentation on CRAN: http://cran.r-project.org/web/packages/ChainLadder/ChainLadder.pdf
- Read help pages in R:
 - ?MackChainLadder
 - ?MunichChainLadder
 - ?BootChainLadder
- Follow examples in R:
 - example(MackChainLadder)
 - example(MunichChainLadder)
 - example(BootChainLadder)

Conclusions

- R is ideal for reserving
 - Built-in functions for statistical modelling
 - Powerful language for data manipulations
 - Fantastic graphical capabilities for analysis and presentation
 - Easy to set-up connections to databases (ODBC)
 - RExcel add-in allows to share R functions with colleagues without R knowledge
 - rcom allows to control MS Office from R
 - Effective knowledge transfer plain text files

For a laugh - fancy 3d plot

```
5
library(rgl) #provides interactive 3d plotting functions
MCL=MackChainLadder(GenIns/1e6)
                                                                 4
FT <- MCL$FullTriangle
                                                                 3
FTpSE <- FT+MCL$Mack.S.E
                                                                 2
FTpSE[which(MCL$Mack.S.E==0, arr.ind=TRUE)] <- NA</pre>
FTmSE <- FT-MCL$Mack.S.E
FTmSE[which(MCL$Mack.S.E==0, arr.ind=TRUE)] <- NA</pre>
                                                                10
zr <- round(FT/FT[1,10]*100)
zlim <- range(zr, na.rm=TRUE)</pre>
                                                                         dev
zlen \leftarrow zlim[2] - zlim[1] + 1
colorlut <- terrain.colors(zlen) # height color lookup table</pre>
cols <- colorlut[ zr -zlim[1]+1 ] # assign colors to heights for each point
x <- as.numeric(dimnames(FT)$origin)</pre>
y <- as.numeric(dimnames(FT)$dev)</pre>
persp3d(x, y=y,
        z=(FT), col=cols, xlab="origin", ylab="dev", zlab="loss",back="lines")
mSE <- data.frame(as.table(FTmSE))</pre>
points3d(xyz.coords(x=as.numeric(as.character(mSE$origin)),
    y=as.numeric(as.character(mSE$dev)),z=mSE$Freq), size=2)
pSE <- data.frame(as.table(FTpSE))</pre>
points3d(xyz.coords(x=as.numeric(as.character(pSE$origin)),
    y=as.numeric(as.character(pSE$dev)),z=pSE$Freq), size=2)
```

6

loss

origin

References

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- Thomas Mack. The standard error of chain ladder reserve estimates:
 Recursive calculation and inclusion of a tail factor. Astin Bulletin. Vol. 29. No
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- P.D.England and R.J.Verrall, Stochastic Claims Reserving in General Insurance, British Actuarial Journal, Vol. 8, pp.443-544, 2002.
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