

$E[F|J] = -1 > E[C|J] = -2$, so J never calls
king will always call in response to a bet.

P₁ K: Bet Call +2
K: check Bet Call +2

Queen will always pass when passed $E[P_2|Q] = 0 > E[B_2|Q] = -1$

WTS Queen should always pass on first turn ✓

$$\begin{aligned} E[P_1|Q] &= \frac{1}{2} P(P_2|J) + \frac{1}{2} (1 - P(P_2|J)) (3P(C_1|Q) - 1) + \frac{1}{2} (P(C_1|Q) + 1) \\ &= 1 + \frac{3}{2} P(C_1|Q) + \frac{1}{2} - \frac{3}{2} P(P_2|J) P(C_1|Q) + \frac{1}{2} P(P_2|J) - \frac{1}{2} P(C_1|Q) - \frac{1}{2} \\ &= -1 + P(C_1|Q) + P(P_2|J) - \frac{3}{2} P(P_2|J) P(C_1|Q) \end{aligned}$$

$$E[B_1|Q] = -\frac{1}{2} \quad \text{if } P(C_1|Q) \geq \frac{1 - 2P(P_2|J)}{2 - 3P(P_2|J)} \Rightarrow P(C_1|Q) > -\infty$$

king will always bet in response to a pass. $E[B_2|K] > E[P_2|K] = -1$

$E[P_1|J] = -1$ because Queen will pass and king will raise \Rightarrow Jack will fold.

$$\begin{aligned} E[B_1|J] &= -\frac{1}{2} + \frac{1}{2} P(C_2|Q) + 1 + \frac{1}{2} (1 - P(C_2|Q)) \\ &\quad - \frac{1}{2} - \frac{3}{2} P(C_2|Q) \Rightarrow P(C_2|Q) = \frac{1}{3} \end{aligned}$$

$$E[P_1|K] = \frac{3}{2} + \frac{1}{2} P(P_2|J) \Rightarrow P(P_2|J) = \frac{2}{3}$$

$$E[B_1|K] = \frac{1}{2} + \frac{1}{2} \cdot 2 \cdot P(C_2|Q) + \frac{1}{2} (1 - P(C_2|Q)) = \frac{7}{6}$$

Next find $P(B_1|J)$ and $P(B_1|K)$

$$E[\text{PI Raises} | \text{PI folds}] = 1$$

$$E[\text{PI Raises} | \text{PI calls}] = -2P(B_1|J) + 2P(B_1|K) = -2\alpha + 2\beta$$

$$\frac{-2\alpha}{\alpha + \beta} + \frac{2\beta}{\alpha + \beta} = 1 \Rightarrow \beta = 3\alpha$$

What we have so far

Jack always folds in response to call $P(B_2|J) = \frac{1}{3}$ $P(B_1|J) = \alpha$
 $E[J] = -1$

Queen always passes and passes when PI passes $P(C_1|Q) = \frac{1}{3}$

King always bets in response to pass, will always call in response to bet $P(C_2|Q) = \frac{1}{3}$

$P(B_1|K) = 3\alpha$

$E[Q] = -\frac{1}{3}$ $E[K] = \frac{7}{6}$