Math 170S: Homework 5

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Problem 1. $\frac{x}{N} = \frac{150}{1000}$. To obtain a maximum error less than 0.04 choose n to be large enough s.t $z_{0.025}\sqrt{\frac{\frac{x}{N}(1-\frac{x}{N})}{n}} \leq 0.04$. Thus, we want $n \geq \frac{z_{0.025}^2 0.15 \cdot 0.85}{0.0016} = 306.11$, so choose $n = 307$

Problem 2. Want to choose n large enough s.t $\epsilon = z_{0.005}\sqrt{\frac{p(1-p)}{n}} \leq z_{0.005}\sqrt{\frac{\frac{1}{2}\cdot\frac{1}{2}}{n}} \leq 0.02$. Want $n \geq \frac{z_{0.005}^2 \frac{1}{4}}{0.004} = 4146.81038$. Choose $n = 4147$

Problem 3. Want to choose n large enough s.t $\epsilon = z_{0.025}\frac{\sigma}{\sqrt{n}} \leq 5 \Rightarrow n \geq \frac{z_{0.025}^2 33.7^2}{25} = 174.5 \Rightarrow n = 175$.

Problem 4.

1. $P(Y_3 < m < Y_{10}) = \displaystyle\sum_{k=3}^{9} \binom{12}{k} 0.5^{12} = 0.9614 \Rightarrow (5.4, 6.3)$ gives us a 96.14% confidence interval for m.
2. $P(Y_1 < m < Y_7) = \displaystyle\sum_{k=1}^{6} \binom{12}{k} 0.3^k 0.7^{12-k} = 0.9476 \Rightarrow (4.8, 5.8)$ gives us a 94.76% confidence interval for $\pi_{0.3}$.

Problem 5.

1. $X \sim \mathcal{N}(32, 4) \Rightarrow \overline{x} = \dfrac{1}{36}\displaystyle\sum_{i=1}^{36} X_i \sim \mathcal{N}(32, \frac{1}{9}) \Rightarrow Z := \dfrac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \dfrac{\overline{x} - 32}{\frac{1}{3}} \sim \mathcal{N}(0, 1) \Rightarrow P(Z > 9)$ basiscally 0.
2. We should reject H_0.