Math 170E: Homework 2

Owen Jones

7/12/2023

Problem 1. $4!\binom{48}{11}\binom{37}{11}\binom{26}{11}\binom{15}{11} = 24 \cdot \frac{48!}{11!\cdot 37!} \cdot \frac{37!}{11!\cdot 26!} \cdot \frac{26!}{11!\cdot 15!} \cdot \frac{15!}{11!\cdot 4!} = 4.8897 \cdot 10^{30}$

Problem 2. (1) $S = \{WW, RW, WR, RR\}$ where the first letter indicates the allele received from the father and the second indicates the allele received from the mother.

(2) $P(RR|\text{ red eyes}) = \frac{P(RR)}{P(\{RW,WR,RR\})} = \frac{0.25}{0.75} = \frac{1}{3}$

Problem 3. (1) $P(d2 > d1) = P(d2 > d1|d1 = 1)P(d1 = 1) + P(d2 > d1|d1 = 2)P(d1 = 2) + P(d2 > d1|d1 = 3)P(d1 = 3) + P(d2 > d1|d1 = 4)P(d1 = 4) + P(d2 > d1|d1 = 5)P(d1 = 5) + P(d2 > d1|d1 = 6)P(d1 = 6) = \frac{5}{6}\cdot\frac{1}{6} + \frac{4}{6}\cdot\frac{1}{6} + \frac{3}{6}\cdot\frac{1}{6} + \frac{2}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} = \frac{5}{12}$

(2) $P(d3 > d2 > d1|\text{at least one die is a 5}) = \frac{P(d3>d2>d1\cap\text{at least one die is a 5})}{P(\text{at least one die is a 5})}$
$= \frac{P(d3>d2>d1|d1=5)P(d1=5)+P(d3>d2>d1|d2=5)P(d2=5)+P(d3>d2>d1|d3=5)P(d3=5)}{P(d1=5\cup d2=5\cup d3=5)}$
$= \frac{\frac{0}{36}\cdot\frac{1}{6}+\frac{4}{36}\cdot\frac{1}{6}+\frac{6}{36}\cdot\frac{1}{6}}{\frac{6^3-5^3}{216}} = \frac{10}{91}$

Problem 4. (1) $P((A_1 \cap A_2) \cap A_3') = P(A_1 \cap A_2)P(A_3'|A_1 \cap A_2)$ by conditional probability. $P(A_1 \cap A_2)P(A_3'|A_1 \cap A_2) = P(A_1)P(A_2)(1 - P(A_3|A_1 \cap A_2))$ by definition of the complement of A_3' and the independence of A_1 and A_2. $P(A_1)P(A_2)(1 - P(A_3|A_1 \cap A_2)) = P(A_1)P(A_2)(1 - P(A_3))$ by the mutual independence of A_1, A_2, and A_3. $P(A_1)P(A_2)(1 - P(A_3)) = P(A_1)P(A_2)P(A_3')$ by the definition of the complement of A_3.

(2) $P[(A_1' \cup A_2') \cap A_3] = P[(A_1 \cap A_2)' \cap A_3]$ by De Morgan's Laws. Since $(A_1 \cap A_2)$ and A_3' are independent by part (1), then $(A_1 \cap A_2)'$ and A_3 must also be independent by Theorem $1.4-1(c)$. Thus, $P[(A_1 \cap A_2)' \cap A_3] = P((A_1 \cap A_2)')P(A_3) = (1 - P(A_1 \cap A_2))P(A_3) = (1 - P(A_1)P(A_2))P(A_3) = \frac{24}{125}$

Problem 5. RB,LB, and HB are regular, low, and high blood pressure respectively. IH and RH are irregular and regular heart rate respectively.
$P(RH \cap LB) = P(RH|LB)P(LB) = (1 - P(IH|LB))P(LB) = (1 - \frac{P(IH)P(LB|IH)}{P(LB)})P(LB) = P(LB) - P(IH)P(LB|IH) = P(LB) - P(IH)(1 - P(RB|IH) - P(HB|IH)) = P(LB) - P(IH)(1 - \frac{P(RB)P(IH|RB)}{P(IH)} - P(HB|IH)) = P(LB) - P(IH) + P(RB)P(IH|RB) + P(IH)P(HB|IH) = P(LB) - P(IH) + (1 - P(LB) - P(HB))P(IH|RB) + P(IH)P(HB|IH) = 0.19 - 0.17 + (1 - 0.19 - 0.16) \cdot 0.11 + 0.17 \cdot 0.35 = 0.151$

Problem 6. P1 and P2 are Process II and Process II respectively. D is defective.
$P(P1|D) = \frac{P(P1)P(D|P_1)}{P(D|P_1)P(P1)+P(D|P_2)P(P2)} = \frac{0.6\cdot 0.03}{0.6\cdot 0.03+0.4\cdot 0.01} = \frac{9}{11}$