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% HW Problem 5 template; implement 2nd order Taylor Series (TS) method for IVP:
% y' = f(t,y) with y(t_0) = y0

%% Define the ODE

% RHS function of ODE
f = @(t,y) -(5*t)*y.^2 + 5/t - 1/t^2 ;

% Partial derivatives of RHS function (needed for TS)

%---------------------- Your edits (uncomment below) -----------------------

f_y = @(t,y) -10*y*t ;
f_t = @(t,y) -5*y.^2-5/t^2+2/t^3 ;

%---------------------- End of your edits for this part ----------------------


%% Time-stepping

% Time step
h = 0.2 ;

% Initial and final time
t0 = 1 ;
tf = 2 ;

% Discretization of time
tset = (t0 : h : tf)' ;

% Number of time steps
numsteps = length(tset) ;

% Initial value
y0 = 1 ;

% Initialize time
t = t0  ;

%% Initialization of numerical solution

% Initialize numerical solutions with the initial value
y_FE = y0 ; % Forward Euler method
y_TS = y0 ; % Taylor Series method

% Store the numerical solution
y_FE_set = zeros(numsteps, 1) ; y_FE_set(1) = y0 ;
y_TS_set = zeros(numsteps, 1) ; y_TS_set(1) = y0 ;


%% Actual time stepping

for i = 2:numsteps

    % Update and store FE numerical solution
    y_FE = y_FE + h*f(t,y_FE) ; % update
    y_FE_set(i) = y_FE ; % store
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    % Update TS numerical solution

    %------------------- Your edits (uncomment below) ---------------------
    y_TS = y_TS + h*f(t,y_TS)+h^2/2*(f_t(t,y_TS)+f(t,y_TS)*f_y(t,y_TS)) ;
    y_TS_set(i) = y_TS ;
    %------------------- End of your edits for this part ------------------

    % Update time
    t = t + h ;


end


%% Plot

figure(1) ; clf ;

% Plot the exact solution on [1,2]
tset_fine = linspace(t0, tf, 1000) ;
plot(tset_fine, 1./tset_fine, 'k-', 'LineWidth', 2) ; hold on ;

% Plot the Forward Euler numerical solution
plot(tset, y_FE_set, 'b:o', 'LineWidth', 1.8, 'MarkerSize', 10, ...
     'MarkerFaceColor', 'k')

% Plot the Taylor Series method numerical solution (uncomment below once your TS solution↙
is ready)
 plot(tset, y_TS_set, 'r:o', 'LineWidth', 1.8, 'MarkerSize', 10, ...
     'MarkerFaceColor', 'r')

% Plot settings for making a nice figure
grid on ;
set(gca, 'FontSize', 20)
set(gcf, 'defaultTexTInterpreter', 'Latex')
set(gcf, 'Position', [223   215   641   449])
leg = legend('Exact sol. $y(t) = 1/t$', 'FE sol.', 'TS sol.') ;
set(leg, 'Interpreter', 'Latex')
xlabel('Time $t$')
```