7.2.1 Let $x = x' \cos\theta - y' \sin\theta$ and $y = x' \sin\theta + y' \cos\theta$.

$$ax^2 = ax'^2 \cos^2\theta - ax'y' \sin 2\theta + ay'^2 \sin^2\theta$$
$$bxy = bx'^2 \sin\theta\cos\theta + bx'y' \cos 2\theta - by'^2 \sin\theta\cos\theta$$
$$cy^2 = cx'^2 \sin^2\theta + cx'y' \sin 2\theta + cy'^2 \cos^2\theta$$

Collecting like terms

$$(a\cos^2\theta + b\sin\theta\cos\theta + c\sin^2\theta)x'^2 = (a + \frac{b}{2}\sin 2\theta + (c-a)\frac{1-\cos 2\theta}{2})x'^2$$
$$((c-a)'\sin 2\theta + b\cos 2\theta)x'y' = x'^2 y'^2$$
$$(a\sin^2\theta - b\sin\theta\cos\theta + c\cos^2\theta)y'^2 = (a - \frac{b}{2}\sin 2\theta + (c-a)\frac{1+\cos 2\theta}{2})y'^2$$

We want to choose a suitable θ s.t $x'y' = 0$. Let $\theta^* = \frac{\arctan(\frac{a-c}{b})}{2}$
Let $a' = a + \frac{b}{2}\sin 2\theta^* + (c-a)\frac{1-\cos 2\theta^*}{2}$ and $b' = a - \frac{b}{2}\sin 2\theta^* + (c - a)\frac{1+\cos 2\theta^*}{2}$.
This gives us the desired form $a'x'^2 + b'y'^2$

7.2.2 Let $f(x, y) = ax^2 + bxy + cy^2 + dx + ey + f$ be some quadratic function. **7.2.1** allows us to simplify $ax^2 + bxy + cy^2 \to a'x'^2 + b'y'^2$. Applying the substitution from **7.2.1** $dx + ey + f = d(x'\cos\theta - y'\sin\theta) + e(x'\sin\theta + y'\cos\theta) + f = (d\cos\theta + e\sin\theta)x' + (e\cos\theta - d\sin\theta)y' + f$
Thus, any quadratic can be expressed as $a'x'^2 + b'y'^2 + c'x' + dx' + e'$

7.3.1 Substituting $x = \frac{3at}{1+t^3}$ and $y = \frac{3at^2}{1+t^3}$ into $x^3 + y^3 = 3axy$

$$x^3 + y^3$$
$$(\frac{3at}{1+t^3})^3 + (\frac{3at^2}{1+t^3})^3$$
$$= \frac{27a^3t^3}{(1+t^3)^3} + \frac{27a^3t^6}{(1+t^3)^3}$$
$$= \frac{27a^3t^3(1+t^3)}{(1+t^3)^3} = 3a\frac{9a^2t^3}{(1+t^3)^2}$$
$$= 3a\frac{3at}{1+t^3}\frac{3at^2}{1+t^3}$$
$$= 3axy$$

$x = \frac{3at}{1+t^3}$ and $y = \frac{3at^2}{1+t^3}$ parameterize the folium. At $t = 0$ $x = \frac{3a\cdot 0}{1+0^3} = 0$ and $y = \frac{3a\cdot 0^2}{1+0^3} = 0$, so the folium lies on the x and y axis at 0.

7.3.2 $x^3 + y^3$ can be factored as $(x+y)(x^2 - xy + y^2)$. It follows

$$\begin{aligned}
&x^3 + y^3 = 3axy \\
&\Rightarrow x + y = \frac{3axy}{x^2 - xy + y^2} \text{ where } (x,y) \neq (0,0) \\
&\Rightarrow x + y = \frac{3a}{\frac{x}{y} + \frac{y}{x} - 1} \frac{xy}{xy} \text{ where } x, y \neq= 0 \\
&\Rightarrow x + y = \frac{3a}{\frac{x}{y} + \frac{y}{x} - 1}
\end{aligned}$$

7.3.3 $t \to -1^- \Rightarrow x \to +\infty$ and $t \to -1^+ \Rightarrow x \to -\infty$.
$\frac{x}{y} = \frac{1}{t}$ and $\frac{y}{x} = \frac{t}{1}$, so $\frac{x}{y} \to \frac{1}{-1} = -1$ and $\frac{y}{x} \to \frac{-1}{1} = -1$

7.4.1 $x = t^2$ and y^3 is a simple parameterization by inspection. Finding the second intersection point of the line $y = tx$ we obtain:

$$\begin{aligned}
&y^2 = x^3 \\
&(xt)^2 = x^3 \\
&x^2t^2 = x^3 \\
&t^2x^2 - x^3 = 0 \\
&x^2(t^2 - x)
\end{aligned}$$

, so our points of intersection are $x = 0$ and $x = t^2$. Thus, we again choose the parameterization $x = t^2$ and $y = t^3$.