Math 116: Worksheet 6

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1. (a)

$$
\begin{array}{r}
x^2 + 2x + 1 \\
x^4 + x^3 + x^2 + x + 1 \,\big)\, \overline{x^6 + 3x^5 + 4x^4 + 5x^3 + 4x^2 + 3x + 1} \\
-\ (x^6 + x^5 + x^4 + x^3 + x^2) \\
\hline
2x^5 + 3x^4 + 4x^3 + 3x^2 + 3x + 1 \\
-\ (2x^5 + 2x^4 + 2x^3 + 2x^2 + 2x) \\
\hline
x^4 + 2x^3 + x^2 + x + 1 \\
-\ (x^4 + x^3 + x^2 + x + 1) \\
\hline
x^3
\end{array}
$$

Thus,

$$x^6 + 3x^5 + 4x^4 + 5x^3 + 4x^2 + 3x + 1 \equiv x^3[\mathbb{Q}]/(P(x))$$

(b)

$$
\begin{aligned}
&x^4 + x^3 + x^2 + x + 1 = (x+1)x^3 + x^2 + x + 1 \\
&x^3 = (x)(x^2 + x + 1) - x - 1 \\
&x^2 + x + 1 = (-x)(-x-1) + 1
\end{aligned}
$$

(c)

$$
\begin{array}{ccc}
 & s(x) & t(x) \\
x^4 + x^3 + x^2 + x + 1 & 1 & 0 \\
x^3 & 0 & 1 \\
x^2 + x + 1 & 1 & -x-1 \\
-x-1 & -x & x^2 + x + 1 \\
1 & 1 - x^2 & x^3 + x^2 - 1
\end{array}
$$

Thus, $x^3 + x^2 - 1$ is x^3 inverse.

2. Suppose for the sake of contradiction $P(x)$ is reducible. Since $\deg(P) = 3$ it must be factorable into one quadratic and one linear factor.
Let $x - a \mid P(x)$.