Math 170S Final Exam Cheet Sheet

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1 Formulas

Sample Mean: $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, **Variance:** $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2$, **Sample Variance:** $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2$

Linearity rules: $\sum_{i=1}^{n} E[X_i] = E[\sum_{i=1}^{n} X_i], \sum_{i=1}^{n} Var[X_i] = Var[\sum_{i=1}^{n} X_i], E[nX] = nE[X], Var[nX] = n^2 Var[X] \Rightarrow$ if $X \sim \mathcal{N}(\mu, \sigma^2)$, $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} X_i \sim \mathcal{N}(\mu, \frac{\sigma^2}{n})$

Sample Percent.: $\tilde{\pi_p} = y_r$ if there exists y_r s.t $y_r = (n+1)p$, $\tilde{\pi_p} = y_r + \frac{a}{b}(y_{r+1} - y_r)$ for $(n+1)p = r + \frac{a}{b}$

Freq. Histogram: $h(x) = \frac{f_i}{n(c_i - c_{i-1})} \quad c_{i-1} < x < c_i, i = 1, 2, \ldots k$

PDF Order Stat.: $P(Y_r \leq y) = G_r(y) = \binom{n}{k}\sum_{k=r}^{n}[F(y)]^k[1-F(y)]^{n-k}$

$100(1-\alpha)\%$ **z:** $(\overline{x} - z_{\frac{\alpha}{2}}(\frac{\sigma}{\sqrt{n}}), \overline{x} + z_{\frac{\alpha}{2}}(\frac{\sigma}{\sqrt{n}}))$ $100(1-\alpha)\%$ **t:** $(\overline{x} - t_{\frac{\alpha}{2}}^{n-1}(\frac{s}{\sqrt{n}}), \overline{x} + t_{\frac{\alpha}{2}}^{n-1}(\frac{s}{\sqrt{n}}))$

Diff. of Means: $(\overline{x} - \overline{y} - z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}}, \overline{x} - \overline{y} + z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}})$

Unknown Var.: $(\overline{x} - \overline{y} - t_{\frac{\alpha}{2}}^{n+m-2}\sqrt{\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}}\sqrt{\frac{(n-1)s_x^2}{\sigma_x^2(n+m-2)} + \frac{(m-1)s_y^2}{\sigma_y^2(n+m-2)}}, \overline{x} - \overline{y} + t_{\frac{\alpha}{2}}^{n+m-2}\sqrt{\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}}\sqrt{\frac{(n-1)s_x^2}{\sigma_x^2(n+m-2)} + \frac{(m-1)s_y^2}{\sigma_y^2(n+m-2)}})$

Proportions: $\frac{\hat{p} + \frac{z_{\frac{\alpha}{2}}^2}{2n} \pm z_{\frac{\alpha}{2}}\sqrt{\hat{p}(1-\hat{p})/n + z_{\frac{\alpha}{2}}^2/(4n^2)}}{1 + z_{\frac{\alpha}{2}}^2/n} \Rightarrow \hat{p} \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ if n is large.

Sample Size: For C. interval $(\overline{x} - \epsilon, \overline{x} + \epsilon)$ we want $\frac{z_{\frac{\alpha}{2}}^2 \sigma^2}{\epsilon^2} \leq n$, $\frac{(t_{\frac{\alpha}{2}}^{n-1})^2 s^2}{\epsilon^2} \leq n$, or $\frac{z_{\frac{\alpha}{2}}^2 \hat{p}(1-\hat{p})}{\epsilon^2} \leq \frac{z_{\frac{\alpha}{2}}^2}{4\epsilon^2} \leq n$

C.I for Percentile: $P(Y_i < \pi_p < Y_j) = \sum_{k=i}^{j-1}\binom{n}{k}p^k(1-p)^{n-k}$

2 Maximum Likelihood Estimator

$L(\theta) = L(\theta; x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} f(x_i; \theta)$ Goal: Find $\arg\max_\theta L(\theta) \Leftrightarrow \arg\max_\theta \log(L(\theta))$.

3 Regression

$Y = y(x; w) + \epsilon$ where $\epsilon \sim \mathcal{N}(0, \sigma^2)$. Use MLE to solve for σ^2 and w.
$\hat{\alpha} = \overline{y} - \beta\overline{x}, \hat{\beta} = \frac{\sum(y_i - \overline{y})(x_i - \overline{x})}{\sum(x_i - \overline{x})^2}, \sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - (\hat{\alpha} + \beta x_i))^2$

4 Hypothesis Tests

$\alpha := P(\text{reject } H_0|H_0 \text{ is true}), \beta := P(\text{accept } H_0|H_0 \text{ is false})$, $\phi(z) :=$ cdf standard normal variable., $z_\alpha :=$ a value s.t $P(Z \geq z_\alpha) = \alpha$ where $Z \sim \mathcal{N}(0,1)$

$H_1 : \mu_1 > \mu_0$ $p := P(z \geq \frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n}}}|H_0) = 1 - \phi(\frac{\overline{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}})$ reject if $p < \alpha$.

$H_1 : \mu_1 < \mu_0$ $p := P(z \leq \frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n}}}|H_0) = \phi(\frac{\overline{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}})$ reject if $p < \alpha$.

$H_1 : \mu_1 \neq \mu_0$ $p := P(|z| \geq |\frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n}}}||H_0) = 1 - \phi(|\frac{\overline{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}|) + \phi(-|\frac{\overline{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}|)$ reject if $p < \alpha$.

For unknown variance and proportions, use $T := \frac{\overline{x}-\mu}{\frac{s}{\sqrt{n}}} \sim t^{n-1}$ and $Z := \frac{Y-np}{\sqrt{p(1-p)}} \sim \mathcal{N}(0,1)$. Difference of means and proporrtions use linearity rules.

Willcoxon Test : $W = \sum_{i=1}^{n} sign(x_i - m)R_i$. $Z := \frac{W}{\sqrt{\frac{n(n+1)(2n+1)}{6}}} \sim \mathcal{N}(0,1)$. reject if $p < \alpha$.

Statistical power: $\beta = \beta(\theta)$. Define $K(\mu) = P(\text{reject } H_0|\mu) = 1 - \beta(\mu) \Leftrightarrow 1 - \phi(\frac{c-\mu}{\frac{\sigma}{\sqrt{n}}})$ where $c := \mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}}$ for $H_1 : \mu > \mu_0$. Similar for different alternative hypotheses.

5 Best Critical Region

Neyman-Pearson: Let $X_1, \ldots, X_n$ be i.i.d r.v and let $\alpha \in (0,1)$. Assume $\mathcal{C}$ s.t

(1.) $P((x_1, \ldots, x_n) \in \mathcal{C}|H_0) = \alpha$

(2.) $\frac{L(\theta_0)}{L(\theta_1)} \leq k$ for each $(x_1, \ldots, x_n) \in \mathcal{C}$

(3.) $\frac{L(\theta_0)}{L(\theta_1)} \geq k$ for each $(x_1, \ldots, x_n) \in \mathcal{C}^c$

Then $\mathcal{C}$ is a best critical region.

6 Likelihood Ratio

Goal is to develop a test for when H_0 and H_1 are composite.
$H_0 : \theta \in \omega \quad H_1 : \theta \in \omega^c$ where $\Omega := \omega \cup \omega^c$
Let $\lambda := \frac{L(\hat{\omega})}{L(\hat{\Omega})}$ where $\hat{\omega} := \arg\max_{\theta \in \omega} L(x_1, \ldots, x_n; \theta)$ and $\hat{\Omega} := \arg\max_{\theta \in \Omega} L(x_1, \ldots, x_n; \theta)$.
By Neyman-Pearson $\lambda \leq k$ for some $0 < k < 1$, so that a desired significance α is attained.

7 χ^2 test

$H_0 : p_i = p_{i0}$ for all $i = 1 \ldots k$ $Q_{k-1} := \sum_{i=1}^{k} \frac{(Y_i - np_{i0})^2}{np_{i0}}$ where Y_i is the observed number of outcomes in A_i, $p_i = P(A_i)$, and $Y_k = n - y_1 - y_2 - \ldots - y_{k-1}$.
$Q_{k-1} \sim \chi(k-1)$
Contingency tables: $p_{ij} := P(A_i) \quad j = 1 \ldots h$, $H_0 : p_{i1} = p_{i2} = \ldots = p_{ih}$, $\hat{p_i} := \frac{\sum_{j=1}^{h} Y_{ij}}{\sum_{j=1}^{h} n_j}$, $\hat{p_k} = 1 - \hat{p_1} - \ldots - \hat{p_{k-1}}$, $Q := \sum_{i=1}^{k} \sum_{j=1}^{h} \frac{(Y_{ij} - n_j\hat{p_i})^2}{n_j\hat{p_i}} \sim \chi((h-1)(k-1))$.

8 ANOVA

Statistical test for all means are the same
$\overline{X} := \frac{1}{n}\sum_{i=1}^{m}\sum_{j=1}^{n_i} X_{ij} \quad n : \sum_{i=1}^{m} n_i$, $\overline{X_i} := \frac{1}{n_i}\sum_{j=1}^{n_i} X_{ij} \quad i = 1, \ldots, m$
$(n-1)S_{grand}^2 := \sum_{i=1}^{m}\sum_{j=1}^{n_i}\left(X_{ij} - \overline{X}\right)^2$, $(n_i - 1)S_i^2 := \sum_{j=1}^{n_i}\left(X_{ij} - \overline{X_i}\right)^2$ $V := \sum_{i=1}^{m} n_i(\overline{X_i} - \overline{X})^2$
$(n-1)S_{grand}^2 = \sum_{i=1}^{m}(n_i - 1)S_i^2 + V$
$\frac{\frac{V}{m-1}}{\frac{\sum_{i=1}^{m}(n_i-1)S_i^2}{n-m}} \sim F(m-1, n-m)$

9 Common PDFs

Normal $f(z, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z-\mu)^2}{2\sigma^2}}$

Gamma $f(x; \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, x > 0$

Beta $f(x; \alpha, \beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$

Binomial $f(x; m, p) = \binom{m}{x} p^x (1-p)^{m-x}$

Bernoulli $f(x; p) = \begin{cases} p & \text{for } x = 1 \\ 1-p & \text{for } x = 0 \end{cases}$

STATISTICAL TABLES

Cumulative normal distribution

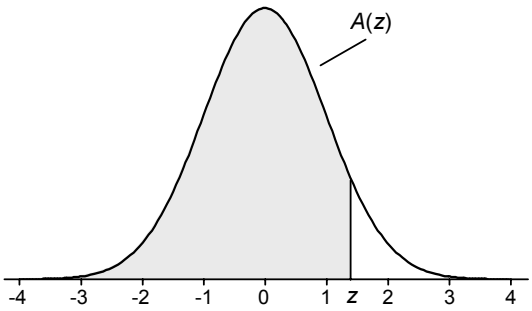
Critical values of the *t* distribution

Critical values of the *F* distribution

Critical values of the chi-squared distribution

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

TABLE A.2

t Distribution: Critical Values of *t*

		Significance level					
Degrees of freedom	*Two-tailed test*: *One-tailed test*:	10% 5%	5% 2.5%	2% 1%	1% 0.5%	0.2% 0.1%	0.1% 0.05%
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15		1.753	2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725
26		1.706	2.056	2.479	2.779	3.435	3.707
27		1.703	2.052	2.473	2.771	3.421	3.690
28		1.701	2.048	2.467	2.763	3.408	3.674
29		1.699	2.045	2.462	2.756	3.396	3.659
30		1.697	2.042	2.457	2.750	3.385	3.646
32		1.694	2.037	2.449	2.738	3.365	3.622
34		1.691	2.032	2.441	2.728	3.348	3.601
36		1.688	2.028	2.434	2.719	3.333	3.582
38		1.686	2.024	2.429	2.712	3.319	3.566
40		1.684	2.021	2.423	2.704	3.307	3.551
42		1.682	2.018	2.418	2.698	3.296	3.538
44		1.680	2.015	2.414	2.692	3.286	3.526
46		1.679	2.013	2.410	2.687	3.277	3.515
48		1.677	2.011	2.407	2.682	3.269	3.505
50		1.676	2.009	2.403	2.678	3.261	3.496
60		1.671	2.000	2.390	2.660	3.232	3.460
70		1.667	1.994	2.381	2.648	3.211	3.435
80		1.664	1.990	2.374	2.639	3.195	3.416
90		1.662	1.987	2.368	2.632	3.183	3.402
100		1.660	1.984	2.364	2.626	3.174	3.390
120		1.658	1.980	2.358	2.617	3.160	3.373
150		1.655	1.976	2.351	2.609	3.145	3.357
200		1.653	1.972	2.345	2.601	3.131	3.340
300		1.650	1.968	2.339	2.592	3.118	3.323
400		1.649	1.966	2.336	2.588	3.111	3.315
500		1.648	1.965	2.334	2.586	3.107	3.310
600		1.647	1.964	2.333	2.584	3.104	3.307
∞		1.645	1.960	2.326	2.576	3.090	3.291

TABLE A.3

F Distribution: Critical Values of *F* (5% significance level)

v_2 \ v_1	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.36	246.46	247.32	248.01
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.42	19.43	19.44	19.45
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.71	8.69	8.67	8.66
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.87	5.84	5.82	5.80
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.64	4.60	4.58	4.56
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.96	3.92	3.90	3.87
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.53	3.49	3.47	3.44
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.24	3.20	3.17	3.15
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.03	2.99	2.96	2.94
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.86	2.83	2.80	2.77
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.74	2.70	2.67	2.65
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.64	2.60	2.57	2.54
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.55	2.51	2.48	2.46
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.48	2.44	2.41	2.39
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.42	2.38	2.35	2.33
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.37	2.33	2.30	2.28
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.33	2.29	2.26	2.23
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.29	2.25	2.22	2.19
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.26	2.21	2.18	2.16
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.22	2.18	2.15	2.12
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.20	2.16	2.12	2.10
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.17	2.13	2.10	2.07
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.15	2.11	2.08	2.05
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.13	2.09	2.05	2.03
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.11	2.07	2.04	2.01
26	4.22	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.09	2.05	2.02	1.99
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.08	2.04	2.00	1.97
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.06	2.02	1.99	1.96
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.05	2.01	1.97	1.94
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.04	1.99	1.96	1.93
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	2.04	1.99	1.94	1.91	1.88
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.95	1.90	1.87	1.84
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.95	1.89	1.85	1.81	1.78
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.86	1.82	1.78	1.75
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97	1.89	1.84	1.79	1.75	1.72
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95	1.88	1.82	1.77	1.73	1.70
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94	1.86	1.80	1.76	1.72	1.69
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.85	1.79	1.75	1.71	1.68
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.78	1.73	1.69	1.66
150	3.90	3.06	2.66	2.43	2.27	2.16	2.07	2.00	1.94	1.89	1.82	1.76	1.71	1.67	1.64
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.80	1.74	1.69	1.66	1.62
250	3.88	3.03	2.64	2.41	2.25	2.13	2.05	1.98	1.92	1.87	1.79	1.73	1.68	1.65	1.61
300	3.87	3.03	2.63	2.40	2.24	2.13	2.04	1.97	1.91	1.86	1.78	1.72	1.68	1.64	1.61
400	3.86	3.02	2.63	2.39	2.24	2.12	2.03	1.96	1.90	1.85	1.78	1.72	1.67	1.63	1.60
500	3.86	3.01	2.62	2.39	2.23	2.12	2.03	1.96	1.90	1.85	1.77	1.71	1.66	1.62	1.59
600	3.86	3.01	2.62	2.39	2.23	2.11	2.02	1.95	1.90	1.85	1.77	1.71	1.66	1.62	1.59
750	3.85	3.01	2.62	2.38	2.23	2.11	2.02	1.95	1.89	1.84	1.77	1.70	1.66	1.62	1.58
1000	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89	1.84	1.76	1.70	1.65	1.61	1.58

TABLE A.3 (continued)

F Distribution: Critical Values of *F* (5% significance level)

ν_2 \ ν_1	25	30	35	40	50	60	75	100	150	200
1	249.26	250.10	250.69	251.14	251.77	252.20	252.62	253.04	253.46	253.68
2	19.46	19.46	19.47	19.47	19.48	19.48	19.48	19.49	19.49	19.49
3	8.63	8.62	8.60	8.59	8.58	8.57	8.56	8.55	8.54	8.54
4	5.77	5.75	5.73	5.72	5.70	5.69	5.68	5.66	5.65	5.65
5	4.52	4.50	4.48	4.46	4.44	4.43	4.42	4.41	4.39	4.39
6	3.83	3.81	3.79	3.77	3.75	3.74	3.73	3.71	3.70	3.69
7	3.40	3.38	3.36	3.34	3.32	3.30	3.29	3.27	3.26	3.25
8	3.11	3.08	3.06	3.04	3.02	3.01	2.99	2.97	2.96	2.95
9	2.89	2.86	2.84	2.83	2.80	2.79	2.77	2.76	2.74	2.73
10	2.73	2.70	2.68	2.66	2.64	2.62	2.60	2.59	2.57	2.56
11	2.60	2.57	2.55	2.53	2.51	2.49	2.47	2.46	2.44	2.43
12	2.50	2.47	2.44	2.43	2.40	2.38	2.37	2.35	2.33	2.32
13	2.41	2.38	2.36	2.34	2.31	2.30	2.28	2.26	2.24	2.23
14	2.34	2.31	2.28	2.27	2.24	2.22	2.21	2.19	2.17	2.16
15	2.28	2.25	2.22	2.20	2.18	2.16	2.14	2.12	2.10	2.10
16	2.23	2.19	2.17	2.15	2.12	2.11	2.09	2.07	2.05	2.04
17	2.18	2.15	2.12	2.10	2.08	2.06	2.04	2.02	2.00	1.99
18	2.14	2.11	2.08	2.06	2.04	2.02	2.00	1.98	1.96	1.95
19	2.11	2.07	2.05	2.03	2.00	1.98	1.96	1.94	1.92	1.91
20	2.07	2.04	2.01	1.99	1.97	1.95	1.93	1.91	1.89	1.88
21	2.05	2.01	1.98	1.96	1.94	1.92	1.90	1.88	1.86	1.84
22	2.02	1.98	1.96	1.94	1.91	1.89	1.87	1.85	1.83	1.82
23	2.00	1.96	1.93	1.91	1.88	1.86	1.84	1.82	1.80	1.79
24	1.97	1.94	1.91	1.89	1.86	1.84	1.82	1.80	1.78	1.77
25	1.96	1.92	1.89	1.87	1.84	1.82	1.80	1.78	1.76	1.75
26	1.94	1.90	1.87	1.85	1.82	1.80	1.78	1.76	1.74	1.73
27	1.92	1.88	1.86	1.84	1.81	1.79	1.76	1.74	1.72	1.71
28	1.91	1.87	1.84	1.82	1.79	1.77	1.75	1.73	1.70	1.69
29	1.89	1.85	1.83	1.81	1.77	1.75	1.73	1.71	1.69	1.67
30	1.88	1.84	1.81	1.79	1.76	1.74	1.72	1.70	1.67	1.66
35	1.82	1.79	1.76	1.74	1.70	1.68	1.66	1.63	1.61	1.60
40	1.78	1.74	1.72	1.69	1.66	1.64	1.61	1.59	1.56	1.55
50	1.73	1.69	1.66	1.63	1.60	1.58	1.55	1.52	1.50	1.48
60	1.69	1.65	1.62	1.59	1.56	1.53	1.51	1.48	1.45	1.44
70	1.66	1.62	1.59	1.57	1.53	1.50	1.48	1.45	1.42	1.40
80	1.64	1.60	1.57	1.54	1.51	1.48	1.45	1.43	1.39	1.38
90	1.63	1.59	1.55	1.53	1.49	1.46	1.44	1.41	1.38	1.36
100	1.62	1.57	1.54	1.52	1.48	1.45	1.42	1.39	1.36	1.34
120	1.60	1.55	1.52	1.50	1.46	1.43	1.40	1.37	1.33	1.32
150	1.58	1.54	1.50	1.48	1.44	1.41	1.38	1.34	1.31	1.29
200	1.56	1.52	1.48	1.46	1.41	1.39	1.35	1.32	1.28	1.26
250	1.55	1.50	1.47	1.44	1.40	1.37	1.34	1.31	1.27	1.25
300	1.54	1.50	1.46	1.43	1.39	1.36	1.33	1.30	1.26	1.23
400	1.53	1.49	1.45	1.42	1.38	1.35	1.32	1.28	1.24	1.22
500	1.53	1.48	1.45	1.42	1.38	1.35	1.31	1.28	1.23	1.21
600	1.52	1.48	1.44	1.41	1.37	1.34	1.31	1.27	1.23	1.20
750	1.52	1.47	1.44	1.41	1.37	1.34	1.30	1.26	1.22	1.20
1000	1.52	1.47	1.43	1.41	1.36	1.33	1.30	1.26	1.22	1.19

TABLE A.3 (continued)

F Distribution: Critical Values of *F* (1% significance level)

v_2 \ v_1	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20
1	4052.18	4999.50	5403.35	5624.58	5763.65	5858.99	5928.36	5981.07	6022.47	6055.85	6106.32	6142.67	6170.10	6191.53	6208.73
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.44	99.44	99.45
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.92	26.83	26.75	26.69
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.25	14.15	14.08	14.02
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.77	9.68	9.61	9.55
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.60	7.52	7.45	7.40
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.36	6.28	6.21	6.16
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.56	5.48	5.41	5.36
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	5.01	4.92	4.86	4.81
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.60	4.52	4.46	4.41
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.29	4.21	4.15	4.10
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.05	3.97	3.91	3.86
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.86	3.78	3.72	3.66
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.70	3.62	3.56	3.51
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.56	3.49	3.42	3.37
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.45	3.37	3.31	3.26
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.35	3.27	3.21	3.16
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.27	3.19	3.13	3.08
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.19	3.12	3.05	3.00
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.13	3.05	2.99	2.94
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.07	2.99	2.93	2.88
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	3.02	2.94	2.88	2.83
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.97	2.89	2.83	2.78
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.93	2.85	2.79	2.74
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.89	2.81	2.75	2.70
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.86	2.78	2.72	2.66
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.82	2.75	2.68	2.63
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.79	2.72	2.65	2.60
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.77	2.69	2.63	2.57
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.74	2.66	2.60	2.55
35	7.42	5.27	4.40	3.91	3.59	3.37	3.20	3.07	2.96	2.88	2.74	2.64	2.56	2.50	2.44
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.56	2.48	2.42	2.37
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70	2.56	2.46	2.38	2.32	2.27
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.39	2.31	2.25	2.20
70	7.01	4.92	4.07	3.60	3.29	3.07	2.91	2.78	2.67	2.59	2.45	2.35	2.27	2.20	2.15
80	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.64	2.55	2.42	2.31	2.23	2.17	2.12
90	6.93	4.85	4.01	3.53	3.23	3.01	2.84	2.72	2.61	2.52	2.39	2.29	2.21	2.14	2.09
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50	2.37	2.27	2.19	2.12	2.07
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.23	2.15	2.09	2.03
150	6.81	4.75	3.91	3.45	3.14	2.92	2.76	2.63	2.53	2.44	2.31	2.20	2.12	2.06	2.00
200	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.27	2.17	2.09	2.03	1.97
250	6.74	4.69	3.86	3.40	3.09	2.87	2.71	2.58	2.48	2.39	2.26	2.15	2.07	2.01	1.95
300	6.72	4.68	3.85	3.38	3.08	2.86	2.70	2.57	2.47	2.38	2.24	2.14	2.06	1.99	1.94
400	6.70	4.66	3.83	3.37	3.06	2.85	2.68	2.56	2.45	2.37	2.23	2.13	2.05	1.98	1.92
500	6.69	4.65	3.82	3.36	3.05	2.84	2.68	2.55	2.44	2.36	2.22	2.12	2.04	1.97	1.92
600	6.68	4.64	3.81	3.35	3.05	2.83	2.67	2.54	2.44	2.35	2.21	2.11	2.03	1.96	1.91
750	6.67	4.63	3.81	3.34	3.04	2.83	2.66	2.53	2.43	2.34	2.21	2.11	2.02	1.96	1.90
1000	6.66	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43	2.34	2.20	2.10	2.02	1.95	1.90

TABLE A.3 (continued)

F Distribution: Critical Values of *F* (1% significance level)

ν_1 / ν_2	25	30	35	40	50	60	75	100	150	200
1	6239.83	6260.65	6275.57	6286.78	6302.52	6313.03	6323.56	6334.11	6344.68	6349.97
2	99.46	99.47	99.47	99.47	99.48	99.48	99.49	99.49	99.49	99.49
3	26.58	26.50	26.45	26.41	26.35	26.32	26.28	26.24	26.20	26.18
4	13.91	13.84	13.79	13.75	13.69	13.65	13.61	13.58	13.54	13.52
5	9.45	9.38	9.33	9.29	9.24	9.20	9.17	9.13	9.09	9.08
6	7.30	7.23	7.18	7.14	7.09	7.06	7.02	6.99	6.95	6.93
7	6.06	5.99	5.94	5.91	5.86	5.82	5.79	5.75	5.72	5.70
8	5.26	5.20	5.15	5.12	5.07	5.03	5.00	4.96	4.93	4.91
9	4.71	4.65	4.60	4.57	4.52	4.48	4.45	4.41	4.38	4.36
10	4.31	4.25	4.20	4.17	4.12	4.08	4.05	4.01	3.98	3.96
11	4.01	3.94	3.89	3.86	3.81	3.78	3.74	3.71	3.67	3.66
12	3.76	3.70	3.65	3.62	3.57	3.54	3.50	3.47	3.43	3.41
13	3.57	3.51	3.46	3.43	3.38	3.34	3.31	3.27	3.24	3.22
14	3.41	3.35	3.30	3.27	3.22	3.18	3.15	3.11	3.08	3.06
15	3.28	3.21	3.17	3.13	3.08	3.05	3.01	2.98	2.94	2.92
16	3.16	3.10	3.05	3.02	2.97	2.93	2.90	2.86	2.83	2.81
17	3.07	3.00	2.96	2.92	2.87	2.83	2.80	2.76	2.73	2.71
18	2.98	2.92	2.87	2.84	2.78	2.75	2.71	2.68	2.64	2.62
19	2.91	2.84	2.80	2.76	2.71	2.67	2.64	2.60	2.57	2.55
20	2.84	2.78	2.73	2.69	2.64	2.61	2.57	2.54	2.50	2.48
21	2.79	2.72	2.67	2.64	2.58	2.55	2.51	2.48	2.44	2.42
22	2.73	2.67	2.62	2.58	2.53	2.50	2.46	2.42	2.38	2.36
23	2.69	2.62	2.57	2.54	2.48	2.45	2.41	2.37	2.34	2.32
24	2.64	2.58	2.53	2.49	2.44	2.40	2.37	2.33	2.29	2.27
25	2.60	2.54	2.49	2.45	2.40	2.36	2.33	2.29	2.25	2.23
26	2.57	2.50	2.45	2.42	2.36	2.33	2.29	2.25	2.21	2.19
27	2.54	2.47	2.42	2.38	2.33	2.29	2.26	2.22	2.18	2.16
28	2.51	2.44	2.39	2.35	2.30	2.26	2.23	2.19	2.15	2.13
29	2.48	2.41	2.36	2.33	2.27	2.23	2.20	2.16	2.12	2.10
30	2.45	2.39	2.34	2.30	2.25	2.21	2.17	2.13	2.09	2.07
35	2.35	2.28	2.23	2.19	2.14	2.10	2.06	2.02	1.98	1.96
40	2.27	2.20	2.15	2.11	2.06	2.02	1.98	1.94	1.90	1.87
50	2.17	2.10	2.05	2.01	1.95	1.91	1.87	1.82	1.78	1.76
60	2.10	2.03	1.98	1.94	1.88	1.84	1.79	1.75	1.70	1.68
70	2.05	1.98	1.93	1.89	1.83	1.78	1.74	1.70	1.65	1.62
80	2.01	1.94	1.89	1.85	1.79	1.75	1.70	1.65	1.61	1.58
90	1.99	1.92	1.86	1.82	1.76	1.72	1.67	1.62	1.57	1.55
100	1.97	1.89	1.84	1.80	1.74	1.69	1.65	1.60	1.55	1.52
120	1.93	1.86	1.81	1.76	1.70	1.66	1.61	1.56	1.51	1.48
150	1.90	1.83	1.77	1.73	1.66	1.62	1.57	1.52	1.46	1.43
200	1.87	1.79	1.74	1.69	1.63	1.58	1.53	1.48	1.42	1.39
250	1.85	1.77	1.72	1.67	1.61	1.56	1.51	1.46	1.40	1.36
300	1.84	1.76	1.70	1.66	1.59	1.55	1.50	1.44	1.38	1.35
400	1.82	1.75	1.69	1.64	1.58	1.53	1.48	1.42	1.36	1.32
500	1.81	1.74	1.68	1.63	1.57	1.52	1.47	1.41	1.34	1.31
600	1.80	1.73	1.67	1.63	1.56	1.51	1.46	1.40	1.34	1.30
750	1.80	1.72	1.66	1.62	1.55	1.50	1.45	1.39	1.33	1.29
1000	1.79	1.72	1.66	1.61	1.54	1.50	1.44	1.38	1.32	1.28

TABLE A.3 (continued)

F Distribution: Critical Values of *F* (0.1% significance level)

v_2 \ v_1	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20
1	4.05e05	5.00e05	5.40e05	5.62e05	5.76e05	5.86e05	5.93e05	5.98e05	6.02e05	6.06e05	6.11e05	6.14e05	6.17e05	6.19e05	6.21e05
2	998.50	999.00	999.17	999.25	999.30	999.33	999.36	999.37	999.39	999.40	999.42	999.43	999.44	999.44	999.45
3	167.03	148.50	141.11	137.10	134.58	132.85	131.58	130.62	129.86	129.25	128.32	127.64	127.14	126.74	126.42
4	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.47	48.05	47.41	46.95	46.60	46.32	46.10
5	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	27.24	26.92	26.42	26.06	25.78	25.57	25.39
6	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.69	18.41	17.99	17.68	17.45	17.27	17.12
7	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.33	14.08	13.71	13.43	13.23	13.06	12.93
8	25.41	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.77	11.54	11.19	10.94	10.75	10.60	10.48
9	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37	10.11	9.89	9.57	9.33	9.15	9.01	8.90
10	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.96	8.75	8.45	8.22	8.05	7.91	7.80
11	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	8.12	7.92	7.63	7.41	7.24	7.11	7.01
12	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.48	7.29	7.00	6.79	6.63	6.51	6.40
13	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.98	6.80	6.52	6.31	6.16	6.03	5.93
14	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.58	6.40	6.13	5.93	5.78	5.66	5.56
15	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.26	6.08	5.81	5.62	5.46	5.35	5.25
16	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.98	5.81	5.55	5.35	5.20	5.09	4.99
17	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.75	5.58	5.32	5.13	4.99	4.87	4.78
18	15.38	10.39	8.49	7.46	6.81	6.35	6.02	5.76	5.56	5.39	5.13	4.94	4.80	4.68	4.59
19	15.08	10.16	8.28	7.27	6.62	6.18	5.85	5.59	5.39	5.22	4.97	4.78	4.64	4.52	4.43
20	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24	5.08	4.82	4.64	4.49	4.38	4.29
21	14.59	9.77	7.94	6.95	6.32	5.88	5.56	5.31	5.11	4.95	4.70	4.51	4.37	4.26	4.17
22	14.38	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.99	4.83	4.58	4.40	4.26	4.15	4.06
23	14.20	9.47	7.67	6.70	6.08	5.65	5.33	5.09	4.89	4.73	4.48	4.30	4.16	4.05	3.96
24	14.03	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80	4.64	4.39	4.21	4.07	3.96	3.87
25	13.88	9.22	7.45	6.49	5.89	5.46	5.15	4.91	4.71	4.56	4.31	4.13	3.99	3.88	3.79
26	13.74	9.12	7.36	6.41	5.80	5.38	5.07	4.83	4.64	4.48	4.24	4.06	3.92	3.81	3.72
27	13.61	9.02	7.27	6.33	5.73	5.31	5.00	4.76	4.57	4.41	4.17	3.99	3.86	3.75	3.66
28	13.50	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.50	4.35	4.11	3.93	3.80	3.69	3.60
29	13.39	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.45	4.29	4.05	3.88	3.74	3.63	3.54
30	13.29	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39	4.24	4.00	3.82	3.69	3.58	3.49
35	12.90	8.47	6.79	5.88	5.30	4.89	4.59	4.36	4.18	4.03	3.79	3.62	3.48	3.38	3.29
40	12.61	8.25	6.59	5.70	5.13	4.73	4.44	4.21	4.02	3.87	3.64	3.47	3.34	3.23	3.14
50	12.22	7.96	6.34	5.46	4.90	4.51	4.22	4.00	3.82	3.67	3.44	3.27	3.41	3.04	2.95
60	11.97	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69	3.54	3.32	3.15	3.02	2.91	2.83
70	11.80	7.64	6.06	5.20	4.66	4.28	3.99	3.77	3.60	3.45	3.23	3.06	2.93	2.83	2.74
80	11.67	7.54	5.97	5.12	4.58	4.20	3.92	3.70	3.53	3.39	3.16	3.00	2.87	2.76	2.68
90	11.57	7.47	5.91	5.06	4.53	4.15	3.87	3.65	3.48	3.34	3.11	2.95	2.82	2.71	2.63
100	11.50	7.41	5.86	5.02	4.48	4.11	3.83	3.61	3.44	3.30	3.07	2.91	2.78	2.68	2.59
120	11.38	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.38	3.24	3.02	2.85	2.72	2.62	2.53
150	11.27	7.24	5.71	4.88	4.35	3.98	3.71	3.49	3.32	3.18	2.96	2.80	2.67	2.56	2.48
200	11.15	7.15	5.63	4.81	4.29	3.92	3.65	3.43	3.26	3.12	2.90	2.74	2.61	2.51	2.42
250	11.09	7.10	5.59	4.77	4.25	3.88	3.61	3.40	3.23	3.09	2.87	2.71	2.58	2.48	2.39
300	11.04	7.07	5.56	4.75	4.22	3.86	3.59	3.38	3.21	3.07	2.85	2.69	2.56	2.46	2.37
400	10.99	7.03	5.53	4.71	4.19	3.83	3.56	3.35	3.18	3.04	2.82	2.66	2.53	2.43	2.34
500	10.96	7.00	5.51	4.69	4.18	3.81	3.54	3.33	3.16	3.02	2.81	2.64	2.52	2.41	2.33
600	10.94	6.99	5.49	4.68	4.16	3.80	3.53	3.32	3.15	3.01	2.80	2.63	2.51	2.40	2.32
750	10.91	6.97	5.48	4.67	4.15	3.79	3.52	3.31	3.14	3.00	2.78	2.62	2.49	2.39	2.31
1000	10.89	6.96	5.46	4.65	4.14	3.78	3.51	3.30	3.13	2.99	2.77	2.61	2.48	2.38	2.30

TABLE A.3 (continued)

F Distribution: Critical Values of *F* (0.1% significance level)

ν_1 / ν_2	25	30	35	40	50	60	75	100	150	200
1	6.24e05	6.26e05	6.28e05	6.29e05	6.30e05	6.31e05	6.32e05	6.33e05	6.35e05	6.35e05
2	999.46	999.47	999.47	999.47	999.48	999.48	999.49	999.49	999.49	999.49
3	125.84	125.45	125.17	124.96	124.66	124.47	124.27	124.07	123.87	123.77
4	45.70	45.43	45.23	45.09	44.88	44.75	44.61	44.47	44.33	44.26
5	25.08	24.87	24.72	24.60	24.44	24.33	24.22	24.12	24.01	23.95
6	16.85	16.67	16.54	16.44	16.31	16.21	16.12	16.03	15.93	15.89
7	12.69	12.53	12.41	12.33	12.20	12.12	12.04	11.95	11.87	11.82
8	10.26	10.11	10.00	9.92	9.80	9.73	9.65	9.57	9.49	9.45
9	8.69	8.55	8.46	8.37	8.26	8.19	8.11	8.04	7.96	7.93
10	7.60	7.47	7.37	7.30	7.19	7.12	7.05	6.98	6.91	6.87
11	6.81	6.68	6.59	6.52	6.42	6.35	6.28	6.21	6.14	6.10
12	6.22	6.09	6.00	5.93	5.83	5.76	5.70	5.63	5.56	5.52
13	5.75	5.63	5.54	5.47	5.37	5.30	5.24	5.17	5.10	5.07
14	5.38	5.25	5.17	5.10	5.00	4.94	4.87	4.81	4.74	4.71
15	5.07	4.95	4.86	4.80	4.70	4.64	4.57	4.51	4.44	4.41
16	4.82	4.70	4.61	4.54	4.45	4.39	4.32	4.26	4.19	4.16
17	4.60	4.48	4.40	4.33	4.24	4.18	4.11	4.05	3.98	3.95
18	4.42	4.30	4.22	4.15	4.06	4.00	3.93	3.87	3.80	3.77
19	4.26	4.14	4.06	3.99	3.90	3.84	3.78	3.71	3.65	3.61
20	4.12	4.00	3.92	3.86	3.77	3.70	3.64	3.58	3.51	3.48
21	4.00	3.88	3.80	3.74	3.64	3.58	3.52	3.46	3.39	3.36
22	3.89	3.78	3.70	3.63	3.54	3.48	3.41	3.35	3.28	3.25
23	3.79	3.68	3.60	3.53	3.44	3.38	3.32	3.25	3.19	3.16
24	3.71	3.59	3.51	3.45	3.36	3.29	3.23	3.17	3.10	3.07
25	3.63	3.52	3.43	3.37	3.28	3.22	3.15	3.09	3.03	2.99
26	3.56	3.44	3.36	3.30	3.21	3.15	3.08	3.02	2.95	2.92
27	3.49	3.38	3.30	3.23	3.14	3.08	3.02	2.96	2.89	2.86
28	3.43	3.32	3.24	3.18	3.09	3.02	2.96	2.90	2.83	2.80
29	3.38	3.27	3.18	3.12	3.03	2.97	2.91	2.84	2.78	2.74
30	3.33	3.22	3.13	3.07	2.98	2.92	2.86	2.79	2.73	2.69
35	3.13	3.02	2.93	2.87	2.78	2.72	2.66	2.59	2.52	2.49
40	2.98	2.87	2.79	2.73	2.64	2.57	2.51	2.44	2.38	2.34
50	2.79	2.68	2.60	2.53	2.44	2.38	2.31	2.25	2.18	2.14
60	2.67	2.55	2.47	2.41	2.32	2.25	2.19	2.12	2.05	2.01
70	2.58	2.47	2.39	2.32	2.23	2.16	2.10	2.03	1.95	1.92
80	2.52	2.41	2.32	2.26	2.16	2.10	2.03	1.96	1.89	1.85
90	2.47	2.36	2.27	2.21	2.11	2.05	1.98	1.91	1.83	1.79
100	2.43	2.32	2.24	2.17	2.08	2.01	1.94	1.87	1.79	1.75
120	2.37	2.26	2.18	2.11	2.02	1.95	1.88	1.81	1.73	1.68
150	2.32	2.21	2.12	2.06	1.96	1.89	1.82	1.74	1.66	1.62
200	2.26	2.15	2.07	2.00	1.90	1.83	1.76	1.68	1.60	1.55
250	2.23	2.12	2.03	1.97	1.87	1.80	1.72	1.65	1.56	1.51
300	2.21	2.10	2.01	1.94	1.85	1.78	1.70	1.62	1.53	1.48
400	2.18	2.07	1.98	1.92	1.82	1.75	1.67	1.59	1.50	1.45
500	2.17	2.05	1.97	1.90	1.80	1.73	1.65	1.57	1.48	1.43
600	2.16	2.04	1.96	1.89	1.79	1.72	1.64	1.56	1.46	1.41
750	2.15	2.03	1.95	1.88	1.78	1.71	1.63	1.55	1.45	1.40
1000	2.14	2.02	1.94	1.87	1.77	1.69	1.62	1.53	1.44	1.38

TABLE A.4

χ^2 (Chi-Squared) Distribution: Critical Values of χ^2

	Significance level		
Degrees of freedom	5%	1%	0.1%
1	3.841	6.635	10.828
2	5.991	9.210	13.816
3	7.815	11.345	16.266
4	9.488	13.277	18.467
5	11.070	15.086	20.515
6	12.592	16.812	22.458
7	14.067	18.475	24.322
8	15.507	20.090	26.124
9	16.919	21.666	27.877
10	18.307	23.209	29.588