Math 170E: Homework 3

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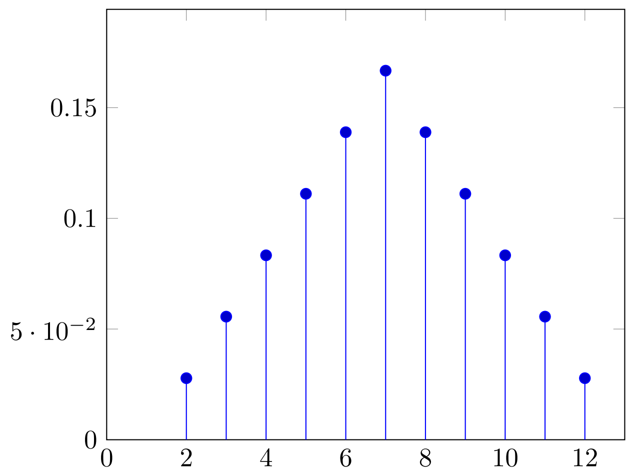
Problem 1. (1) pmf $f(x) = \frac{6-|7-x|}{36}$ for $x \in X$

probability histogram for pmf $f(x)$



(2)

Problem 2. (1) $E[X] = \frac{1+2+3+4}{4} = \frac{5}{2}$

(2) $E[X^2] = \frac{1^2+2^2+3^2+4^2}{4} = \frac{15}{2}$

(3) $E[(2X+1)^2] = E[4X^2+4X+1] = 4E[X^2] + 4E[X] + 1 = 41$

(4) $E[\frac{1}{X}] = \frac{\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{4} = \frac{25}{48}$

Problem 3. $E[(X-bY)^2] = E[X^2] - 2bE[XY] + b^2E[Y^2] = 1 - 2b + 2b^2$. Let $g(b) = 2b^2 - 2b + 1$ which is minimized when $g'(b) = 4b - 2 = 0 \Rightarrow b = \frac{1}{2}$. $g(\frac{1}{2})$ is a minimum because $g''(\frac{1}{2}) = 4 > 0$ i.e $g(b)$ is convave up. Thus, $E[(X-bY)^2]$ assumes a minimum for $b = \frac{1}{2}$.

Problem 4. (1) $E[\frac{X-\mu}{\sigma}] = \frac{1}{\sigma}E[X] - \frac{\mu}{\sigma} = \frac{\mu}{\sigma} - \frac{\mu}{\sigma} = 0$

(2) $E[(\frac{X-\mu}{\sigma})^2] = \frac{1}{\sigma^2}E[(X-\mu)^2] = \frac{Var(X)}{\sigma^2} = \frac{\sigma^2}{\sigma^2} = 1$

Problem 5. $\mu = \frac{1}{16} \cdot 1 + \frac{3}{16} \cdot 2 + \frac{5}{16} \cdot 3 + \frac{7}{16} \cdot 4 = \frac{1+6+15+28}{16} = \frac{25}{8}$, $E[X^2] = \frac{1}{16} \cdot 1 + \frac{3}{16} \cdot 2^2 + \frac{5}{16} \cdot 3^2 + \frac{7}{16} \cdot 4^2 = \frac{85}{8}$, $Var(X) = E[X^2] - \mu^2 = \frac{55}{64}$, $\sigma = \sqrt{Var(X)} = \frac{\sqrt{55}}{8}$

Problem 6. $\mu = M'(0) = \frac{2}{5}e^0 + \frac{2}{5}e^{2\cdot 0} + \frac{6}{5}e^{3\cdot 0} = 2$, $E[X^2] = M''(0) = \frac{2}{5}e^0 + \frac{4}{5}e^{2\cdot 0} + \frac{18}{5}e^{3\cdot 0} = \frac{24}{5}$, $Var(X) = E[X^2] - \mu^2 = \frac{4}{5}$, $\sigma = \sqrt{Var(X)} = \frac{2\sqrt{5}}{5}$