Math 170S: Homework 6

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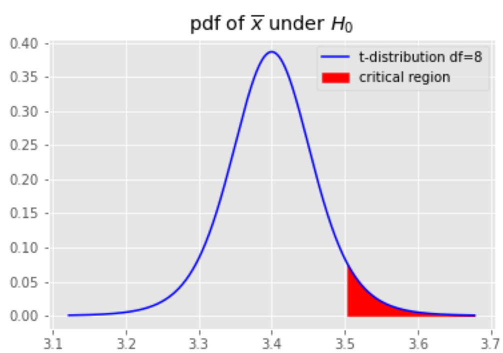
Problem 1.

1. $H_0 : \mu = 3.4$ liters.
2. $H_1 : \mu > 3.4$ liters.
3. Let $T := \frac{\overline{x}-\mu}{\frac{s}{3}} \sim t^{(8)}$
4. Graph below shows t distribution with 8 degrees of freedom. Shaded region descibes $C := \{\overline{x} | \overline{x} > 3.5033\}$



5. $\overline{x} = \frac{1}{9}\sum_{i=1}^{9} x_i = 3.55$, $s_x = \sqrt{\frac{\sum_{i=1}^{9}(x_i - \overline{x})^2}{8}} = 0.167 \Rightarrow T = \frac{\overline{x}-\mu}{\frac{s_x}{3}} = 2.80$
6. We choose to reject the null in favor of the alternative hyppothesis.
7. $p - value = P(\overline{x} > 3.55 | H_0) = 0.0115 < 0.05$

Problem 2. Let $Z := \frac{Y-np}{\sqrt{np(1-p)}} = \frac{Y-0.08\cdot 100}{\sqrt{100\cdot 0.08(1-0.08)}} \approx \mathcal{N}(0,1)$ by CLT.
Let $z_\alpha := |\frac{6-0.08\cdot 100}{\sqrt{100\cdot 0.08(1-0.08)}}|$. Want to find α s.t $P(Z < -z_\alpha) = \alpha$. $z_\alpha = 0.737 \Rightarrow \alpha = 0.230$

Without normal approximation $P(Y \leq 6 | p = 0.08) = \sum_{k=0}^{6}\binom{100}{k} 0.08^k 0.92^{100-k} = 0.3032 \Rightarrow \alpha = 0.3032$

Problem 3.

1. Let $Z := \frac{Y-np}{\sqrt{np(1-p)}} = \frac{Y-0.75\cdot 192}{\sqrt{192\cdot 0.75(1-0.75)}} \approx \mathcal{N}(0,1)$ by CLT.
Let $z_\alpha := \frac{152-0.75\cdot 192}{\sqrt{192\cdot 0.75(1-0.75)}} = 1.33$. $\alpha = P(Y \geq 152 | p = 0.75) \approx P(Z > z_\alpha) = 0.091$
2. Let $Z := \frac{Y-np}{\sqrt{np(1-p)}} = \frac{Y-0.08\cdot 192}{\sqrt{192\cdot 0.08(1-0.08)}} \approx \mathcal{N}(0,1)$ by CLT.
Let $z_\beta := \frac{152-0.08\cdot 192}{\sqrt{192\cdot 0.08(1-0.08)}} = 36.34$. $\beta = P(Y < 152 | p = 0.08) \approx P(Z < z_\beta) = 1$

Problem 4. We choose $H_0 : m = 0$ and $H_1 : m > 0$

Let $W := \sum_{i=1}^{15} [sgn(X_i - Y_i)]R_i = 54$

Let $Z := \frac{W}{\sqrt{\frac{n(n+1)(2n+1)}{6}}} \approx \mathcal{N}(0, 1)$

$P(W \geq 54|H_0) \Leftrightarrow P(Z \geq \frac{54}{\sqrt{\frac{15(16)(31)}{6}}}|H_0) = 0.0625$, so we would fail to reject H_0 at an $\alpha = 0.05$.