Math 167: Homework 1

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Exercise 1.9 We know one of the two players has a winning strategy because we are playing a progressively bounded game with no ties. We assume for the sake of contradiction that Player II has a winning strategy which we will call S. Player I places their first hexagon in an arbitrary space. After Player II places their first hexagon, Player I ignores their initial placement and plays strategy S rotated 120° which we will call S^*. If Player I's first placement is not included in S^* or the center hexagon is included in strategy S, Player I chooses a different arbitrary hexagon or the center space respectively for their first placement. Thus, because the first placement can only help Player I, they must also have a winning strategy which is a contradiction because only one player can have a winning strategy. Hence, Player I must have a winning strategy.

Exercise 2.1 Let (i^*, j^*) be a saddle point for matrix $A_{m\times n}$. Suppose for the sake of contradiction there exists another saddle point (i^+, j^+) such that $a_{i^*j^*} \neq a_{i^+j^+}$. If (i^+, j^+) is a saddle point, then for each i and j $a_{i^+j} \geq a_{i^+j^+} \geq a_{ij^+}$. It follows $a_{i^+j^*} \geq a_{i^+j^+} \geq a_{i^*j^+}$. Since (i^*, j^*) is also a saddle point, then for each i and j $a_{i^*j} \geq a_{i^*j^*} \geq a_{ij^*}$. It follows $a_{i^*j^+} \geq a_{i^*j^*} \geq a_{i^+j^*}$. However, this implies $a_{i^*j^*} \geq a_{i^+j^+} \geq a_{i^*j^*}$ which is a contradiction because $a_{i^*j^*} \neq a_{i^+j^+}$. Hence, all saddle points must have the same payoff for each player.

Exercise 2.9 $\frac{3}{4}x_1 + 0x_2 + 0x_3 = 0x_1 + \frac{1}{4}x_2 + 0x_3 = 0x_1 + 0x_2 + \frac{1}{2}x_3$
$x_1 + x_2 + x_3 = 1$
$x_1 = \frac{2}{11}, x_2 = \frac{6}{11}, x_3 = \frac{3}{11}$
Symmetric, so payoff probabilities same for y

	1	2	3
1	3/4	0	0
2	0	1/4	0
3	0	0	1/2

Exercise 2.11 $-4x_1 + 6(1 - x_1) = 6x_1 - 9(1 - x_1)$
$x_1 = \frac{3}{5}, x_2 = \frac{2}{5}$
Symmetric, so same for y

		P2	
		2	3
P	2	-4	6
1	3	6	-9

Exercise 2.12 The below matrix has a saddle point at (C, RL), so the game has a value of $\frac{1}{4}$

Exercise 2.13 $\begin{pmatrix} p & 1-p \end{pmatrix} \begin{pmatrix} 5000 & 1000 \\ 1000 & 6000 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} p & 1-p \end{pmatrix} \begin{pmatrix} 3000 \\ 3500 \end{pmatrix} = 3500 - 500p \Rightarrow p = 0$

P2

		RR	RC	RL	LL	LC	CC
P 1	R	1/3	3/16	1/4	1/2	3/8	3/8
	L	1/2	3/8	1/4	1/3	3/16	3/8
	C	5/8	5/16	1/4	5/8	5/16	1/3