

Math 170E: Homework 4

Owen Jones

7/26/2023

Problem 1. (1) $M(t) = E[e^{xt}] = \sum_{x \in X} \frac{e^{xt}}{2^{|x|+1}} = \frac{1}{2} \left(\sum_{x \in X: x > 0} \left(\frac{e^t}{2}\right)^x + \sum_{x \in X: x < 0} (2e^t)^x \right) = \frac{1}{2} \left(\frac{\frac{e^t}{2}}{1 - \frac{e^t}{2}} + \frac{\frac{1}{2e^t}}{1 - \frac{1}{2e^t}} \right) =$
 $\frac{e^t + e^{-t} - 1}{5 - 2e^t - 2e^{-t}}$ for $|t| < \ln(2)$

(2) $M(t) = E[e^{xt}] = \frac{1}{2} + \frac{1}{4} \sum_{x=1}^{\infty} \left(\frac{e^t}{2}\right)^x + \frac{1}{4} \sum_{x=1}^{\infty} \left(\frac{1}{2e^t}\right)^x = \frac{1}{2} + \frac{1}{4} \left(\frac{\frac{e^t}{2}}{1 - \frac{e^t}{2}} + \frac{\frac{1}{2e^t}}{1 - \frac{1}{2e^t}} \right) = \frac{4 - e^t - e^{-t}}{10 - 4e^t - 4e^{-t}}$
 for $|t| < \ln(2)$

Problem 2. (1) $\binom{6}{0} (0.25)^0 (0.75)^6 = \frac{729}{4096}$

(2) $(0.25)^2 (0.75)^4 = \frac{81}{4096}$

(3) $\binom{6}{2} (0.25)^2 (0.75)^4 = \frac{1215}{4096}$

Problem 3. (1) $\binom{10}{5} \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^5 = \frac{8064}{59049}$

(2) $\binom{10}{6} \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^4 = \frac{3360}{59049}$

(3) 0 because the particle can only end at even distances away from the start given an even number of moves.

Problem 4. (1) $\frac{\binom{47}{9} \binom{3}{1}}{\binom{50}{10}} = 0.39795 \dots$

(2) $\frac{\binom{47}{9} \binom{3}{1}}{\binom{50}{10}} + \frac{\binom{47}{10} \binom{3}{0}}{\binom{50}{10}} = 0.90204 \dots$

Problem 5. (1) $\sigma^2 = \lambda = 0.5 \Rightarrow P(X = 2) = \frac{\lambda^2 e^{-\lambda}}{2!} = \frac{0.5^2 e^{-0.5}}{2!} = 0.07581 \dots$

(2) $E[X!] = \sum_{x=0}^{\infty} \frac{0.5^x e^{-0.5}}{x!} \cdot x! = e^{-0.5} \sum_{x=0}^{\infty} \frac{1}{2^x} = 2e^{-0.5} = 1.21306 \dots$

Problem 6. $\sum_{x=0}^1 \frac{\left(\frac{225}{150}\right)^x e^{-\frac{225}{150}}}{x!} = e^{-1.5} + 1.5e^{-1.5} = 2.5e^{-1.5} = 0.55782 \dots$