Midterm Redo

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(Q-1) (a) $N_0^{k+1} = \frac{1}{2}N_0^k + \frac{1}{10}N_1^k + \frac{1}{10}N_2^k$
$N_1^{k+1} = (1-\frac{1}{2})N_0^k + 0N_1^k + 0N_2^k$
$N_2^{k+1} = 0N_0^k + (1-\frac{1}{5}+\frac{1}{10})N_1^k + (1-\frac{1}{10})N_2^k$

$$\begin{bmatrix} N_0^{k+1} \\ N_1^{k+1} \\ N_2^{k+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{10} & \frac{1}{10} \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{9}{10} & \frac{9}{10} \end{bmatrix}$$

(b) $N^{(k)} = L^k N^{(0)}$

(Q-2) $\lim_{k\to\infty} N^{(k)} = \lambda_2^k v_2 = \infty$ because there exists a dominant eigenvalue greater than 1, so we obtain exponential growth.
$\lim_{k\to\infty} \frac{v_{21}}{sum(v_{2i})} = \frac{0.904455}{0.904455+0.408457+0.122975}$

(Q-3) $a_{k+1} = a_k + 100mg/L - 0.2a_k = 0.8a_k + 100$ $a_0 = 0$

(Q-4) $\rho_t - D\rho_{xx} = b\rho(x,t)$ with B.C $q(-1,t) = q(1,t) = 0$. Using boundary conditions $\rho(t) = \frac{N_0}{2}e^{bt}$ for $-1 < x < 1$ and $\rho(t) = 0$ elsewhere.