# Math 170E: Homework 5

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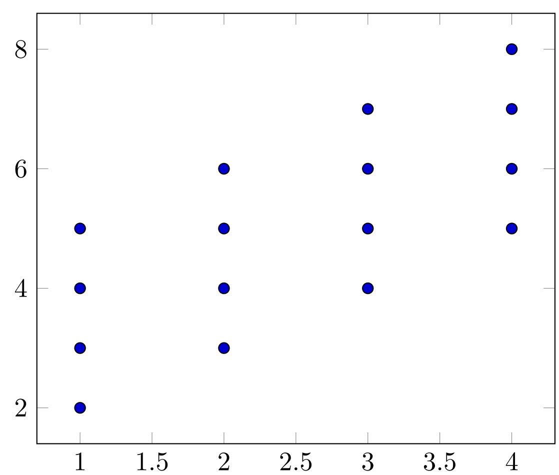
**Problem 1.** (a) $c = \dfrac{1}{\sum_{x=1}^{2}\sum_{y=1}^{3}(x+2y)} = \dfrac{1}{\sum_{x=1}^{2} 3x+12} = \frac{1}{33}$

(b) $c = \dfrac{1}{\sum_{x=1}^{3}\sum_{y=1}^{x}(x+y)} = \dfrac{1}{\sum_{x=1}^{3}(x^2+\dfrac{x(x+1)}{2})} = \frac{1}{24}$

(c) $c = \dfrac{1}{\sum\limits_{\substack{6 \le x+y \le 8 \\ 0 \le y \le 5}} 1} = \frac{1}{3\cdot 6} = \frac{1}{18}$

(d) $c = \dfrac{1}{\sum_{x=1}^{\infty}\sum_{y=1}^{\infty}(\dfrac{1}{4})^x(\dfrac{1}{3})^y} = \dfrac{1}{\sum_{x=1}^{\infty}(\dfrac{1}{4})^x\dfrac{\frac{1}{3}}{1-\frac{1}{3}}} = \frac{1}{6}$

**Problem 2.** (a)




(b) $f(x,y) = \frac{1}{16}$, $x = 1,2,3,4$, $y = x+1, x+2, x+3, x+4$

(c) $f_x(x) = \frac{1}{4}$, $x = 1,2,3,4$

(d) $f_y(y) = \frac{4-|y-5|}{16}$, $y = 2,3,4,5,6,7,8$

(e) $f(1,2) \neq f_x(1)f_y(2)$, so $X$ and $Y$ are dependent.

**Problem 3.** $\mu_x = \sum_{x\in X} x f_x(x) = \sum_{x\in X} x\dfrac{4x+10}{32} = \dfrac{50}{32}$, $\mu_y = \sum_{y\in Y} y f_y(y) = \sum_{y\in y} y\dfrac{2y+3}{32} = \dfrac{90}{32}$, $\sigma_x^2 = E[X^2]-\mu_x^2 = \sum_{x\in X} x^2\dfrac{4x+10}{32} - \mu_x^2 = \dfrac{86}{32} - (\dfrac{50}{32})^2 = \dfrac{63}{256}$, $\sigma_y^2 = E[Y^2]-\mu_y^2 = \sum_{y\in Y} y^2\dfrac{2y+3}{32} - \mu_y^2 = \dfrac{290}{32} - (\dfrac{90}{32})^2 = \dfrac{295}{256}$,

$Cov(X,Y) = E[XY] - \mu_x\mu_y = \sum_{x=1}^{2}\sum_{y=1}^{4} xy\frac{x+y}{32} - \mu_x\mu_y = \sum_{x=1}^{2}\frac{10x^2+30x}{32} - \mu_x\mu_y = \frac{35}{8} - \frac{50\cdot 90}{1024} = -\frac{5}{256}$, $\rho = \frac{Cov(X,Y)}{\sigma_x\sigma_y} = \frac{-5}{\sqrt{295\cdot 63}} = \frac{\sqrt{18585}}{3717}$

**Problem 4.** (a) $\mu_x = \frac{10}{4}$, $\mu_y = \sum_{y=2}^{8} y\frac{4-|y-5|}{16} = 5$, $\sigma_x^2 = E[X^2] - \mu_x^2 = \frac{5}{4}$, $\sigma_y^2 = \sum_{y=2}^{8} y^2\frac{4-|y-5|}{16} - \mu_y^2 = \frac{5}{2}$
$Cov(X,Y) = E[XY] - \mu_x\mu_y = \sum_{x=1}^{x=4}\sum_{y=x+1}^{x+4}\frac{xy}{16} = \sum_{x=1}^{4}\frac{4x^2+10x}{16} = \frac{220}{16} - \frac{200}{16} = \frac{5}{4}$, $\rho = \frac{Cov(X,Y)}{\sigma_x\sigma_y} = \frac{\sqrt{2}}{2}$

(b) $m = \rho\frac{\sigma_y}{\sigma_x} = 1$ $b = \mu_y - m\mu_x = \frac{5}{2}$ $y = x + \frac{5}{2}$