Math 151A: Problem Set 7

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6/9/2023

Instructions:

- Due on Friday, June 9th by 11:59pm.
- Late HW will not be accepted.
- Write down all of the details and attach your code to the end of the assignment for full credit (as a PDF).
- If you LaTeX your solutions, you will get 5% extra credit.
- (T) are "pencil-and-paper" problems and (C) means that the problem includes a computational/programming component.

Problem 1: (T) Power Method

Use the power method to approximate a dominant eigenvector and eigenvalue of the matrix A:

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 2 & 8 \end{bmatrix},$$

by computing (either by-hand or using a code) $\vec{v}^{(5)}$ and $\lambda^{(5)} = r\left(\vec{v}^{(5)}\right)$ starting with

$$\vec{v}^{(0)} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

Solution:

$$\vec{w}^{(1)} = A \cdot \vec{v}^{(0)} = \begin{bmatrix} 3 \\ 0 \\ 10 \end{bmatrix}, \ \vec{v}^{(1)} = \frac{\vec{w}^{(1)}}{||\vec{w}^{(1)}||} = \begin{bmatrix} \frac{3}{\sqrt{109}} \\ 0 \\ \frac{10}{\sqrt{109}} \end{bmatrix}$$

$$\vec{w}^{(2)} = A \cdot \vec{v}^{(1)} = \begin{bmatrix} \frac{9}{\sqrt{109}} \\ \frac{3}{\sqrt{109}} \\ \frac{80}{\sqrt{109}} \end{bmatrix}, \ \vec{v}^{(2)} = \frac{\vec{w}^{(2)}}{||\vec{w}^{(2)}||} = \begin{bmatrix} \frac{9}{\sqrt{6490}} \\ \frac{3}{\sqrt{6490}} \\ \frac{8\sqrt{10}}{\sqrt{649}} \end{bmatrix}$$

$$\vec{w}^{(3)} = A \cdot \vec{v}^{(2)} = \begin{bmatrix} \frac{27}{\sqrt{6490}} \\ \frac{3\sqrt{2}}{\sqrt{3245}} \\ \frac{323\sqrt{2}}{\sqrt{3245}} \end{bmatrix}, \vec{v}^{(3)} = \frac{\vec{w}^{(3)}}{||\vec{w}^{(3)}||} = \begin{bmatrix} \frac{27}{\sqrt{418081}} \\ \frac{6}{\sqrt{418081}} \\ \frac{38\sqrt{17}}{\sqrt{24593}} \end{bmatrix}$$

$$\vec{w}^{(4)} = A \cdot \vec{v}^{(3)} = \begin{bmatrix} \frac{81}{\sqrt{418081}} \\ \frac{21}{\sqrt{418081}} \\ \frac{5180}{\sqrt{418081}} \end{bmatrix}, \vec{v}^{(4)} = \frac{\vec{w}^{(4)}}{||\vec{w}^{(4)}||} = \begin{bmatrix} \frac{81}{\sqrt{26839402}} \\ \frac{21}{\sqrt{26839402}} \\ \frac{2590\sqrt{2}}{\sqrt{13419701}} \end{bmatrix}$$

$$\vec{w}^{(5)} = A \cdot \vec{v}^{(4)} = \begin{bmatrix} \frac{243}{\sqrt{26839402}} \\ \frac{30\sqrt{2}}{\sqrt{13419701}} \\ \frac{20741\sqrt{2}}{\sqrt{13419701}} \end{bmatrix}, \vec{v}^{(5)} = \frac{\vec{w}^{(5)}}{||\vec{w}^{(5)}||} = \begin{bmatrix} \frac{243}{\sqrt{1720818973}} \\ \frac{60}{\sqrt{1720818973}} \\ \frac{41482}{\sqrt{1720818973}} \end{bmatrix}$$

$$\lambda^{(5)} = r(\vec{v}^{(5)}) = (\vec{v}^{(5)})^T A \vec{v}^{(5)} = \frac{13771216559}{1720818973} \approx 8.0027$$

Problem 2: (T) Power Method

(a) Find the eigenvalues and the corresponding eigenvectors of the matrix A:

$$A = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}.$$

(b) Calculate two iterations of the power method starting with

$$\vec{v}^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

(c) Explain why the power method does not seem to converge to a dominant eigenvector for this problem.

Solution:

(a) $\det(A - \lambda \cdot I) = \lambda^2 - 7\lambda + 14 \Rightarrow \lambda = \frac{7}{2} \pm i\frac{\sqrt{7}}{2}$

$(A - \lambda \cdot I)x = 0 \Rightarrow \begin{bmatrix} (-\frac{1}{2} - i\frac{\sqrt{7}}{2})x_1 - x_2 \\ 2x_1 + (\frac{1}{2} - i\frac{\sqrt{7}}{2})x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} (-\frac{1}{2} + i\frac{\sqrt{7}}{2})x_1 - x_2 \\ 2x_1 + (\frac{1}{2} + i\frac{\sqrt{7}}{2})x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

, so $\vec{v}_1 = \begin{bmatrix} -1 + \sqrt{7}i \\ 4 \end{bmatrix}$ with $\lambda = \frac{7}{2} + i\frac{\sqrt{7}}{2}$ and $\vec{v}_2 = \begin{bmatrix} -1 - \sqrt{7}i \\ 4 \end{bmatrix}$ with $\lambda = \frac{7}{2} - i\frac{\sqrt{7}}{2}$

(b) $\vec{w}^{(1)} = A \cdot \vec{v}^{(0)} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$, $\vec{v}^{(1)} = \frac{\vec{w}^{(1)}}{||\vec{w}^{(1)}||} = \begin{bmatrix} \frac{1}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} \end{bmatrix}$

$\vec{w}^{(2)} = A \cdot \vec{v}^{(1)} = \begin{bmatrix} 0 \frac{7\sqrt{2}}{\sqrt{5}} \end{bmatrix}$, $\vec{v}^{(2)} = \frac{\vec{w}^{(2)}}{||\vec{w}^{(2)}||} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

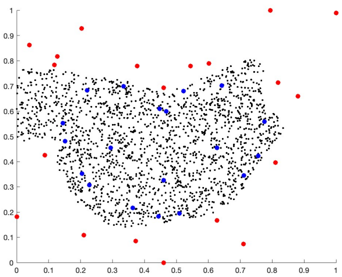
$\lambda^{(2)} = r(\vec{v}^{(2)}) = (\vec{v}^{(2)})^T A \vec{v}^{(2)} = 4$

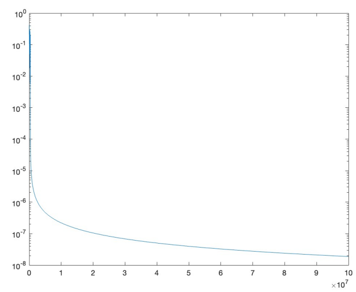
(c) Both of the eigenvalues and the eigenvectors of matrix A are complex, so the power method will not converge to a dominant eigenvalue.

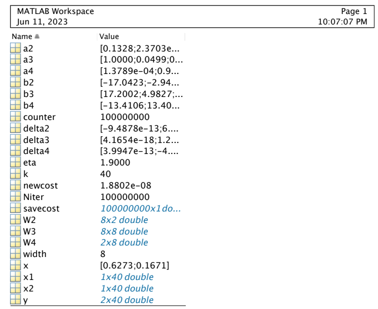
Problem 3: (C) Classification

Complete the template code on linear algebra and deep learning. Apply your completed code to the dataset provided online.

Solution:







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Name	Value
a2	[0.1328;2.3703e...
a3	[1.0000;0.0499;0...
a4	[1.3789e-04;0.9...
b2	[-17.0423;-2.94...
b3	[17.2002;4.9827;...
b4	[-13.4106;13.40...
counter	100000000
delta2	[-9.4878e-13;6....
delta3	[4.1654e-18;1.2...
delta4	[3.9947e-13;-4....
eta	1.9000
k	40
newcost	1.8802e-08
Niter	100000000
savecost	100000000x1do...
W2	8x2 double
W3	8x8 double
W4	2x8 double
width	8
x	[0.6273;0.1671]
x1	1x40 double
x2	1x40 double
y	2x40 double