$$
\begin{aligned}
&\underset{\mathbf{w}}{\operatorname{argmax}}\, p(t|x,\mathbf{w},\beta) = \underset{\mathbf{w}}{\operatorname{argmax}} \prod_{n=1}^{N} \mathcal{N}(t_n|y(x_n,\mathbf{w}),\beta^{-1}) \\
&= \underset{\mathbf{w}}{\operatorname{argmax}} \prod_{n=1}^{N} \sqrt{\frac{\beta}{2\pi}} \exp\{-\frac{\beta}{2}(t_n - \mathbf{w}^\top \phi(x_n))^2\} \\
&= \underset{\mathbf{w}}{\operatorname{argmax}} \frac{N}{2}\log(\beta) - \frac{N}{2}\log(2\pi) - \frac{\beta}{2}\sum_{n=1}^{N} (t_n - \mathbf{w}^\top \phi(x_n))^2 \\
&= \underset{\mathbf{w}}{\operatorname{argmin}} \frac{1}{2}\sum_{n=1}^{N} (t_n - \mathbf{w}^\top \phi(x_n))^2 \\
&\Rightarrow \sum_{n=1}^{N}(t_n - \mathbf{w}^\top \phi(x_n))(-\phi(x_n)) = 0 \\
&\Rightarrow \sum_{n=1}^{N} t_n\phi(x_n) = \sum_{n=1}^{N}(\mathbf{w}^\top \phi(x_n))\phi(x_n) \\
&\Rightarrow \Phi^\top t = \Phi^\top \Phi \mathbf{w} \Rightarrow \mathbf{w} = (\Phi^\top \Phi)^{-1}\Phi^\top t
\end{aligned}
$$