

```
In [2]: import math as m
import numpy as np
import cmath as cm
from matplotlib import pyplot as plt
from scipy import integrate
```

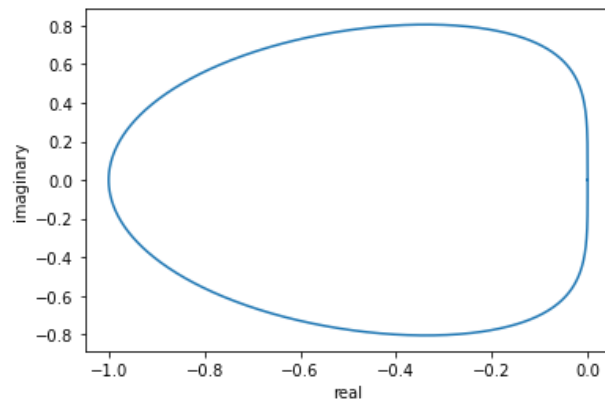
```
In [13]: rho_poly = lambda r: r**2-r
sigma_poly = lambda r: 3/2*r-1/2
thvec=np.linspace(0,2*m.pi,1000)
root_con = lambda z: max(abs((1+3/2*z+cm.sqrt(9/4*z**2+z+1)/2)),abs((1+3/2*z-cm.s
```

```
In [26]: z=np.array([rho_poly(cm.exp(t*1j))/sigma_poly(cm.exp(t*1j)) for t in thvec])
```

```
In [15]: root_con(-0.5)
```

```
Out[15]: 0.7653882032022076
```

```
In [27]: plt.plot(z.real,z.imag)
plt.xlabel('real')
plt.ylabel('imaginary')
plt.savefig('AB_2_boundary')
```



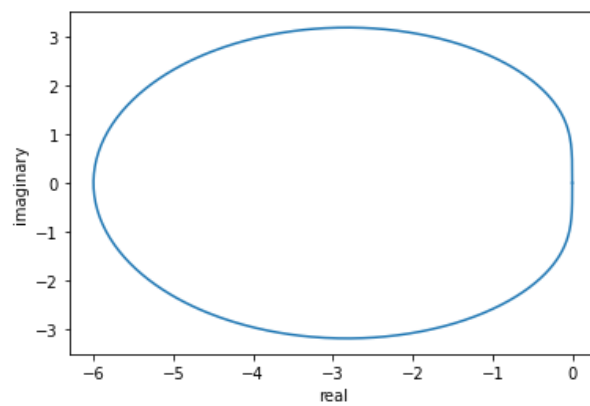
```
In [17]: rho_poly_2 = lambda r: r**2-r
sigma_poly_2 = lambda r: 5/12*r**2+8/12*r-1/12
root_con_2 = lambda z: max(abs((1+2/3*z+cm.sqrt(23/48*z**2+z+1))/(2-5/6*z)),abs(
```

```
In [24]: z_2=np.array([rho_poly_2(cm.exp(t*1j))/sigma_poly_2(cm.exp(t*1j)) for t in thvec])
```

```
In [19]: root_con_2(-3)
```

```
Out[19]: 0.5601534739054566
```

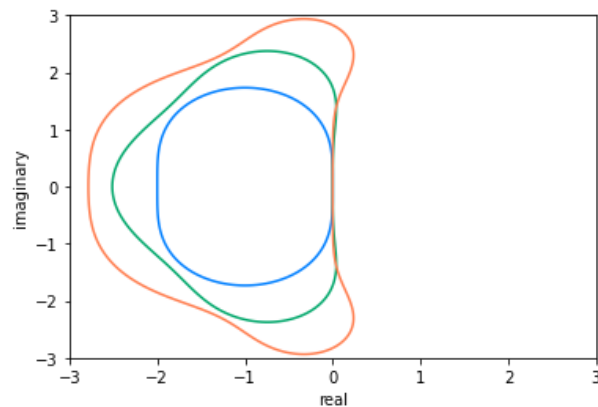
```
In [25]: plt.plot(z_2.real,z_2.imag)
plt.xlabel('real')
plt.ylabel('imaginary')
plt.savefig('AM_2_boundary')
```



```
In [21]: RK_2_T=lambda z:abs(1+z+1/2*z**2)
RK_3_T=lambda z:abs(1+z+1/2*z**2+1/6*z**3)
RK_4_T=lambda z:abs(1+z+1/2*z**2+1/6*z**3+1/24*z**4)
```

```
In [22]: azure = [(0, 128/255, 1.0)]
jade = [(0, 168/255, 107/255)]
coral = [(1.0, 127/255, 80/255)]
```

```
In [23]: xv = np.linspace(-3, 3, 301)
yv = np.linspace(-3, 3, 301)
xx, yy = np.meshgrid(xv, yv)
zz=xx+yy*1j
plt.contour(xx,yy,RK_2_T(zz),[0,1],colors=azure)
plt.contour(xx,yy,RK_3_T(zz),[0,1],colors=jade)
plt.contour(xx,yy,RK_4_T(zz),[0,1],colors=coral)
plt.xlabel('real')
plt.ylabel('imaginary')
plt.savefig('RK_boundaries')
```

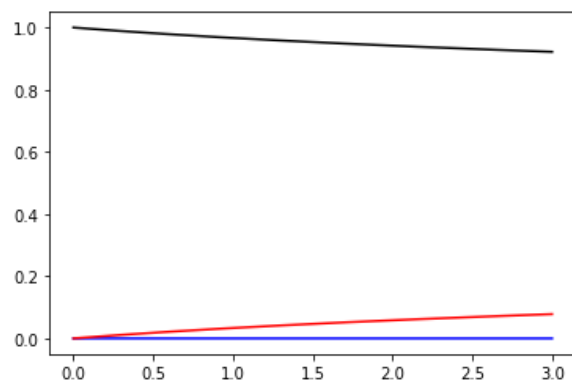


```
In [4]: concentration_f= lambda t,y: np.array([-4e-2*y[0]+1e4*y[1]*y[2],4e-2*y[0]-1e4*y[1]
```

```
In [34]: c_RK45=integrate.solve_ivp(concentration_f,[0,3],np.array([1,0,0]),method='RK45')
```

```
In [35]: plt.plot(c_RK45.t,c_RK45.y[0], 'k-')
plt.plot(c_RK45.t,c_RK45.y[1], 'b-')
plt.plot(c_RK45.t,c_RK45.y[2], 'r-')
```

```
Out[35]: [<matplotlib.lines.Line2D at 0x7f95281fa400>]
```



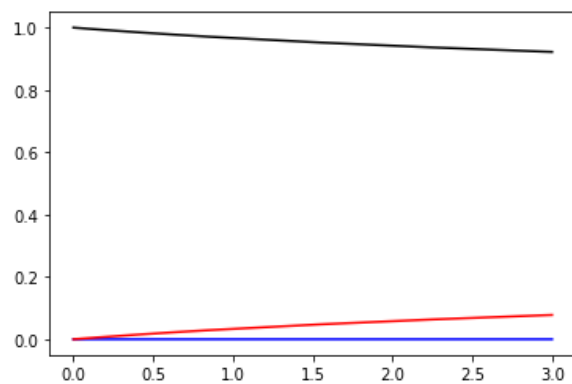
In [36]: `len(c_RK45.t)`

Out[36]: 30001

In [43]: `c_BDF=integrate.solve_ivp(concentration_f,[0,3],np.array([1,0,0]),method='BDF')`

In [53]: `plt.plot(c_BDF.t,c_BDF.y[0],'k-')  
plt.plot(c_BDF.t,c_BDF.y[1],'b-')  
plt.plot(c_BDF.t,c_BDF.y[2],'r-')`

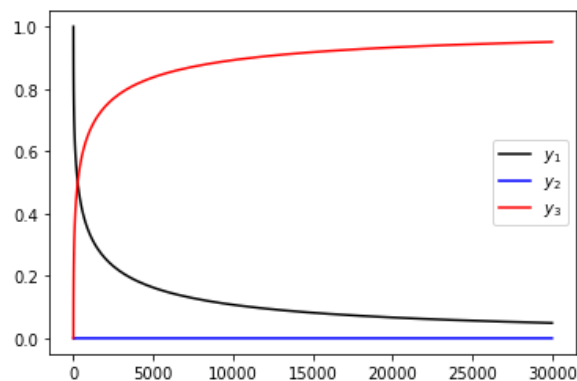
Out[53]: [<matplotlib.lines.Line2D at 0x7f8af9fd4730>]



In [46]: `len(c_BDF.t)`

Out[46]: 27

In [8]: `c_BDF_2=integrate.solve_ivp(concentration_f,[0,3e4],np.array([1,0,0]),method='BDF')  
plt.plot(c_BDF_2.t,c_BDF_2.y[0],'k-',label='$y_1$')  
plt.plot(c_BDF_2.t,c_BDF_2.y[1],'b-',label='$y_2$')  
plt.plot(c_BDF_2.t,c_BDF_2.y[2],'r-',label='$y_3$')  
plt.legend()  
c_BDF_2.y[2][-1]  
plt.savefig('stiff_solver_chemical_reaction_rate')`



```
In [12]: len(c_BDF_2.t)
```

```
Out[12]: 95
```

```
In [11]: c_BDF_2.y[2][-1]
```

```
Out[11]: 0.9509191008000204
```

```
In [ ]:
```