Math 170S: Homework 3

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10/27/2023

Problem 1. Want to find $a(X), b(X)$ with $C(X) = (a(X), b(X))$ s.t $P(\mu \in C(X)) = 0.99$. $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = 14.01429$. Let $z_{\frac{\alpha}{2}} = 2.57583$ for $\alpha = 0.01$. $a(X) = \overline{x} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} = 11.26061$, $b(X) = \overline{x} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} = 16.76796$. Thus, $C(X) = (11.26061, 16.76796)$

Problem 2. Want to find $a(X), b(X)$ with $C(X) = (a(X), b(X))$ s.t $P(\mu \in C(X)) = 0.95$. $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = 89.5$. Let $t_{\frac{\alpha}{2}}^{(n-1)} = 2.26216$ for $n = 10$ and $\alpha = 0.05$. $s_x = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2} = 16.58145$. $a(X) = \overline{x} - t_{\frac{\alpha}{2}}^{(n-1)}\frac{s_x}{\sqrt{n}} = 77.63835$ $b(X) = \overline{x} + t_{\frac{\alpha}{2}}^{(n-1)}\frac{\sigma}{\sqrt{n}} = 101.36165$. Thus, $C(X) = (77.63835, 101.36165)$.

Problem 3.

1. $\pi(\lambda|x) \propto \pi(x|\lambda)\pi(\lambda)$. Let $\pi(\lambda) = \Gamma(a,b) = \frac{b^a\lambda^{a-1}e^{-b\lambda}}{\Gamma(a)}$. Let $\pi(x|\lambda) = \prod_{i=1}^{n} \lambda e^{-x_i\lambda}$. Thus, $\pi(\lambda|x) \propto \frac{b^a\lambda^{a+n-1}e^{-\lambda(b+\sum_{i=1}^{n} x_i)}}{\Gamma(a)} \Rightarrow \pi(\lambda|x) = Gamma(a+n, b+\sum_{i=1}^{n} x_i)$.
2. Want to find a, b s.t $\int_a^b \pi(\lambda|x)d\lambda = \int_a^b Gamma(15, 153) = 0.8$. Let $C = (0.06732, 0.13156)$

Problem 4.

1. $P(h_0) = 0.5, P(h_1) = 0.5$ Prior odds $1:1$
2. $\frac{P(h_0|x)}{P(h_1|x)} = \frac{P(x|h_0)}{P(x|h_1)}\frac{P(h_0)}{P(h_1)} = \frac{\int_0^{\frac{1}{2}} \pi(x|p)\pi(p|h_0)dp}{\int_{\frac{1}{2}}^1 \pi(x|p)\pi(p|h_1)dp} = \frac{\int_0^{\frac{1}{2}} p^x(1-p)^{n-x}dp}{\int_{\frac{1}{2}}^1 p^x(1-p)^{n-x}dp}\frac{P(h_0)}{P(h_1)}$
3. $\frac{\int_0^{\frac{1}{2}} p^x(1-p)^{n-x}dp}{\int_{\frac{1}{2}}^1 p^x(1-p)^{n-x}dp} = \frac{\int_0^{\frac{1}{2}} p^n dp}{\int_{\frac{1}{2}}^1 p^n dp} = \frac{\frac{1}{n+1}(\frac{1}{2})^{n+1}}{\frac{1}{n+1}(1-(\frac{1}{2})^{n+1})} = \frac{(\frac{1}{2})^{n+1}}{1-(\frac{1}{2})^{n+1}}$

Problem 5.

1. $P(H_0) = \int_{-\infty}^{175} \frac{1}{\sqrt{50\pi}}e^{-\frac{(x-170)^2}{50}}dx = 0.84134, P(H_1) = 1 - P(H_0) = 0.15865$ Prior odds $\approx 5.30297 : 1$
2. Substituting 10, 176, and 9 for n, $\overline{x}$, and σ^2 respectively, $\mu_1 = \frac{\frac{175}{25}+\frac{176\cdot 10}{9}}{\frac{1}{25}+\frac{10}{9}} = 175.96525$ It follows $P(H_0|x) = \int_{-\infty}^{175} \frac{1}{\sqrt{50\pi}}e^{-\frac{(x-175.96525)^2}{50}}dx = 0.15019$, so the posterior odds $P(H_0|x) : P(H_1|x) \Leftrightarrow P(H_0|x) : 1 - P(H_0|x)$ are $0.17673 : 1$. $\frac{0.17673}{5.30297} << 1$, so we choose to reject the H_0.

In [1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
import scipy.stats as st
import math
```

In [2]:
```
question_1=np.array([12.3, 14.2, 13.8, 14.5, 15.1, 12.7, 13.9, 15.2, 13.6, 14.0, 14.8, 14.4, 13.0, 14.7
])
```

In [3]:
```
sigma_1=4
```

In [4]:
```
n_1=len(question_1)
```

In [5]:
```
mean_1=np.mean(question_1)
mean_1
```

Out[5]: 14.014285714285714

In [6]:
```
z_99=st.norm.ppf(0.995)
z_99
```

Out[6]: 2.5758293035489004

In [7]:
```
b_1=mean_1+z_99*sigma_1/math.sqrt(n_1)
b_1
```

Out[7]: 16.76796306876929

In [8]:
```
a_1=mean_1-z_99*sigma_1/math.sqrt(n_1)
a_1
```

Out[8]: 11.260608359802138

In [9]:
```
question_2=np.array([72, 116, 79, 97, 90, 67, 115, 82, 95, 82])
```

In [10]:
```
sigma_2=np.std(question_2,ddof=1)
sigma_2
```

Out[10]: 16.58144880414388

In [11]:
```
n_2=len(question_2)
```

In [12]:
```
mean_2=np.mean(question_2)
mean_2
```

Out[12]: 89.5

In [13]:
```
t_95=st.t.ppf(0.975,n_2-1)
t_95
```

Out[13]: 2.2621571627409915

In [14]:
```
a_2=mean_2-t_95*sigma_2/math.sqrt(n_2)
a_2
```

Out[14]: 77.63834608725665

In [15]:
```
b_2=mean_2+t_95*sigma_2/math.sqrt(n_2)
b_2
```

Out[15]: 101.36165391274335

In [16]:
```
st.gamma.ppf(0.1,a=15,scale=1/(153))
```

Out[16]: 0.06731776017838348

In [17]:
```
st.gamma.ppf(0.9,a=15,scale=1/(153))
```

Out[17]: 0.13155563313304508

In [18]:
```
st.norm.cdf(1)
```

Out[18]: 0.8413447460685429

In [19]:
```
1-st.norm.cdf(1)
```

Out[19]: 0.15865525393145707

In [20]:
```
st.norm.cdf(1)/(1-st.norm.cdf(1))
```

Out[20]: 5.302974375068753

In [26]:
```
mu=(175/25+176*10/9)/(1/25+10/9)
```

In [27]:
```
s=1/(1/25+10/9)
```

In [28]:
```
st.norm.cdf((175-mu)/math.sqrt(s))
```

Out[28]: 0.15019059769993914

In [29]:
```
st.norm.cdf((175-mu)/math.sqrt(s))/(1-st.norm.cdf((175-mu)/math.sqrt(s)))
```

Out[29]: 0.17673445044669917

In [ ]:
```

```