Dear All,

Here are some common mistakes on Homework 3:

3 I did not penalize points for this, but almost every student made the following mistake in computing the Bayes factor.

Recall the Bayes factor is

$$B = \frac{\mathbb{P}[x|H0]}{\mathbb{P}[x|H1]}. \tag{1}$$

We have

$$\mathbb{P}[x|H0]$$

represents the chance of seeing our data under the null hypothesis

$$H0$$

. In particular, if

$$H0 = \{a \leq \theta \leq b\}$$

then

$$\mathbb{P}[x|H0] = \int_a^b f_X(x|\theta, H_0)\pi(\theta|H0)d\theta \tag{2}$$

. The main mistake is most students assumed for 3 that

$$\pi(\theta|H0)$$

is a uniform distributed over

$$[0, 1]$$

so its density is 1. This is not true! This is the prior conditioned on

$$H0$$

, which in that problem was

$$\{0 \leq \theta \leq \frac{1}{2}\}$$

. When you condition a uniform

$$(0, 1)$$

r.v. to be at most

$$1/2$$

, you will end up with a uniform

$$(0, 1/2)$$

r.v., which has a density of 2. Similar for

$$\pi(\theta|H1)$$

, but most students got the right answer since the 2 divided away in the Bayes factor.

5 A lot of students seemed to be confused about the difference between

$$\mathbb{P}[x|H0]$$

and

$$\mathbb{P}[H0|x]$$

. So this problem gave you the posterior, so you want to compute

$$\mathbb{P}[H0|x]$$

. This quantity is the chance of the null hypothesis being true under the posterior measure. So we have if

$$f(\theta|x)$$

is the posterior density and

$$H_0 = \{a \leq \theta \leq b\}$$

then

$$\mathbb{P}[H0|x] = \int_a^b f(\theta|x)d\theta. \tag{3}$$

In this problem you were given the posterior is

$$\mathcal{N}(\mu_1, \sigma_1^2)$$

and

$$H_0 = \{\theta \leq 175\}$$

, so you just integrate the density of a

$$\mathcal{N}(\mu_1, \sigma_1^2)$$

over the region

$$(-\infty, 175)$$

to compute

$$\mathbb{P}[H0|x]$$

. And for

$$\mathbb{P}[H1|x]$$

just compute the integral of that

$$\mathcal{N}(\mu_1, \sigma_1^2)$$

density over

$$(175, \infty)$$

.

Best,
Raymond