# Pescoid Modeling

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## 1 Force-Balance, Growth, and Cell Patterning

Force Balance:

$$\eta \partial_x^2 u + \partial_x \sigma_s = \gamma u \tag{1}$$

$$\sigma_s = \sigma_a - \Pi \tag{2}$$

$$\sigma_a = \alpha \rho^2, \Pi = B\rho \tag{3}$$

Growth:

$$\partial_t \rho + \partial_x (u\rho) = g\rho(1-\rho) + D\partial_x^2 \rho \tag{4}$$

Cell Patterning:

$$\tau_m(\partial_t m + \partial_x(um) - D\partial_x^2 m) = m(1 - m/m_c)(1 + m/m_c) + r(\sigma - \sigma_c) \tag{5}$$

$$\partial_x \sigma = \gamma u \tag{6}$$

## 2 Nondimensionalization

All variables here are there nondimensional counterparts, but notation has been left the same for simplicity.
Force Balance:

$$\partial_x^2 u + \frac{L_0 \Pi_c}{\eta_0 u_0} \partial_x(\rho(A\rho(1 + \kappa(tanh(m/m_s) + 1)/2) - 1)) = \frac{\gamma}{\eta} L_0^2 u \tag{7}$$

Where we have a screening length $L_n$ such that flows on scales greater than the screening length vanish.

$$L_n = \sqrt{\frac{\eta}{\gamma}} \tag{8}$$

If we choose

$$u_0 = \frac{L_0 \Pi_c}{\eta} \tag{9}$$

and

$$\frac{\gamma}{\eta} L_0^2 = \Gamma, \tag{10}$$

we have the following:

$$f(\rho)\partial_x^2 u + \partial_x(\rho(A\rho(1+\beta tanh(m/m_s)) - 1)) = f(\rho)\Gamma u \tag{11}$$

Growth:

$$\partial_t\rho + \frac{u_o\tau_g}{L_0}\partial_x(u\rho) = \rho(1-\rho) + \frac{D\tau_g}{L_0^2}\partial_x^2\rho \tag{12}$$

$$\tfrac{u_o\tau_g}{L_0} \propto \tau_g/\tau_v,\ F = \tfrac{u_o\tau_g}{L_0},\ \delta = \tfrac{D\tau_g}{L_0^2}$$

$$\partial_t\rho + F\partial_x(u\rho) = \rho(1-\rho) + \delta\partial_x^2\rho \tag{13}$$

Mesoderm Patterning:

$$\frac{\tau_g}{\tau_m}[\partial_t m + F\partial_x(um) + \delta\partial_x^2 m] = m(1-m)(1+m) + r\Pi_c(\sigma-\sigma_c) \tag{14}$$

$$T_m = \tfrac{\tau_g}{\tau_m}\ ,\ R = r\Pi_c$$

$$T_m[\partial_t m + Fu\partial_x(m) + \delta_m\partial_x^2 m] = m(1-m)(1+m) + R(\sigma-\sigma_c) \tag{15}$$

Nondimensionalized parameters are $A, F, T_m, \Gamma, \kappa, R, \delta$ $\Gamma$ is likely a very important variable to consider as it sets the ratio of the length scale of our system to the screening length. we have the $\lim_{\Gamma\to 0}\sigma = 0$. However, we know this is not the case physically, and we are likely looking for a regime that preserves tissue-scale characteristics while still haveing stresses that are significant on the length scale and growth timescale of the system.

$$\frac{1}{2}(tanh(\rho/\rho_s)+1)\partial_x^2 u + \partial_x(\rho(A(\frac{\rho}{1+k\rho^2})(1+\beta tanh(m/m_s))-1)) = \frac{\Gamma}{2}(tanh(\rho/\rho_s)+1)u \tag{16}$$

$$\gamma = \gamma_0 f(\rho) \tag{17}$$

$$\eta = \eta_0 f(\rho) \tag{18}$$

$$f(\rho) = \frac{1}{2}(tanh(\rho/\rho_s)+1) \tag{19}$$

$$\partial_t\rho + F\partial_x(u\rho) = \rho + \delta\partial_x^2\rho \tag{20}$$