Image Evaluation in FDTD by Zernike Aberrations of A MTM Triplet

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***Abstract*—This paper presents a method for calculating the Zernike aberrations in a lens design using transformation optics in FDTD. Currently there is no literature on calculating image aberrations in FDTD. In this paper I divulge the methods to calculate the Zernike aberration using a polynomial fitting for a wavefront. The results are shown to match very well with the Zernike aberrations in an equivalent lens in ZEMAX.**

***Index Terms*—Metamaterials, FDTD, Transformation Optics, Lens Aberrations**

I. Introduction

TO find the aberrations in the MTM Cooke triplet FDTD simulation an alternative approach was taken looking at the wavefront aberration rather than the ray aberration, see Fig. 1. The mathematical functions were described by Frits Zernike in 1934. Frits Zernike went on to win the Nobel prize in 1953 in physics for the development of Phase Contrast Microscopy. This approach is commonly used by opticians for spectacles and contact lenses [1]. Zernike aberration coefficients are the standard way to quantify the aberrations [2] as they provide a method which each aberration coefficient is independent of the others. It is an innovation on the traditional Seidel aberrations so much so that it is widespread in the optical industry for aberration measurements in the eye [3]. Zernike aberrations are used in astronomy, optics, optometry, and ophthalmology. The aberration is classified as a deviation from an ideal Gaussian sphere, see Fig. 1.The wavefront aberration were investigated and provided a better understanding due to the wave nature of the FDTD simulation employed in terms of the wavefront aberration in terms of Seidel aberrations [4]. A perfect lens should perfectly transform a point source placed at the focus into a plane wave. Any deviation from this plane wave can be expressed as aberrations of the lens as a sum of Seidel polynomials [5].

The the wavefront aberration function can be decomposed into a sum of Zernike polynomials

$$W(\rho,\theta) = \sum_{n=0}^{\infty}\sum_{m=0}^{n} z_{nm}\sqrt{(n+1)}V_n^m(\rho)cos(m\theta) \quad (1)$$

The Zernike polynomial is a way to characterise the aberrations of an optical system improving upon Seidel aberrations

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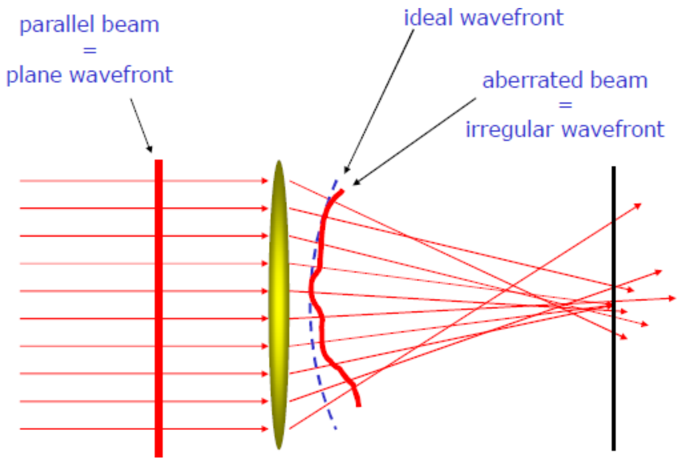

Fig. 1. The wavefront aberration is characterised by the deviation of the aberrated beam / irregular wavefront from an ideal Gaussian wavefront for an incoming parallel beam with a plane wavefront. This Gaussian sphere represents the ideal wavefront from a point source from which any deviation is measured in terms of a wave aberration This is the dual of the ray aberration which is the failure of the individual rays to meet at a stigmatic point. All real lenses and optical systems are subject to some degree of wave or ray aberrations.

due to there property of orthogonality. The radial coefficients for the Zernike polynomial are given by the formulae. The wavefront aberration can be written as a sum of weighted Zernike polynomials,

$$W(x,y) = \sum_{j=0}^{jmax} W_j Z_j(x,y) \quad (2)$$

These have been place in a rectangular co-ordinate system for ease of use and indexed according to a single indexing scheme, j, where

$$j = \frac{n(n+2)+m}{2} \quad (3)$$

Zernike polynomials are an advantage because they are orthogonal. This property allows the Zernike coefficients to be distinct from one another unlike the Seidel aberration calculation.

The spatial frequency along the bottom axis represents how many lines or cycles can fit in one mm therefore the higher the spatial frequency the more difficult it getting to resolve between individual illuminations or slits until a cut of frequency is reached. The y axis is the modulation or contrast between these solid 100 black or white lines. It is one of the most widely used and scientific measure of lensing

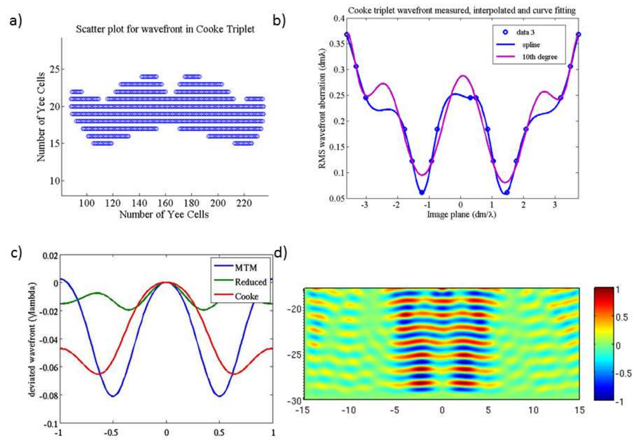

Fig. 2. This figure shows a) the scatter plot containing all the Yee grid cells for the final wavefront with E_z greater than 0. In panel b the figure shows this wavefront plotted and a polynomial fitted to give an analytical solution to the wavefront aberration. In panel c) the wavefront aberration is shown for the MTM lens, the Cooke triplet and the reduced map and in panel d) the original wavefront in FDTD is given.

performance in the optics industry.

The point spread function is the Fourier transform of the wavefront [6]. The Modulation Transfer Function (MTF) is related to the pupil function and the point spread function (PSF) via Fourier transform. It is used to analyse spatial resolving power in industrial application. Any real system, as we have already seen, can be described by its aberrations.

$$PSF_{(x,y)} = \frac{1}{(\lambda^2 d^2 A_p)} FT(p_{(x,y)})(e^{-i2\pi(W(x,y))\lambda})^2 \tag{4}$$

where

$$p(x,y) = P(x,y)exp[jkW(x,y)] \tag{5}$$

is the pupil function and is the aberration function given in Eq.1. The MTF can be calculated from the optical transfer function of which it is the absolute value.

$$OTF(sx,sy) = \frac{FT(PSF)}{FTPSF_{s_x=0,s_y=0}} \tag{6}$$

$$MTF(sx,sy) = ||OTF_{(s_x,s_y)}|| \tag{7}$$

II. The Method for Calculating The Wavefront Aberration Function

The source was placed at the focus in the FDTD simulation described in the chapter on Seidel aberrations. The wavefront results from this when the wave has passed through the lens is seen in figure 2, panel d. The lenses were then simulated again. The aim was to calculate all aberrations from the distorted wavefront function. A program was written in MATLAB in order to find the point where Ez was greater than zero for the region one wavelength beyond the MTM triplet. This program sampled at every grid cell in the transverse axis. The result of this program was a line which represented the actual distorted wavefront in the FDTD simulation. SCATTERPLOT was used to plot all the points where the Efield was above zero. This gave an outline of a single wave contour.

To find the analytical form for the wavefront a polynomial was fitted to the sampled FDTD data. A tenth order polynomial was the highest order in order to give a better fit. The residual was lowest for this order. The fitted polynomial is shown in panel 3, Fig. 2 for the MTM, reduced and Cooke triplet. Low order polynomial fitting is less successful at fitting the correct distribution. Fitting a cubic spline gives the best result for interpolation of the sampled data points for reconstructing the wavefront. The three polynomials are plotted all together in panel c, see Fig. 2. It was more accurately modelled at higher FDTD grid resolution therefore originally the simulation was done at a cell size of 1/10th of the wavelength whereas the graphs shown here are for 1/35 of the wavelength to improve the sampled data which the polynomial was fitted to.

A perfect lens should perfectly transform a point source placed at the focus into a plane wave.

$$a(Q) = C_{40}r^4 + C_{31}h'r^3cos\theta + C_{22}h'^2r^2cos^2\theta + \tag{8}$$

$$C_{20}h'^2r^2 + C_{11}h'^3rcos\theta \tag{9}$$

where C_{40} is spherical, C_{31} is coma, C_{22} and C_{22} are the astigmatism and field curvature and C_{11} is the distortion Seidel coefficient, see Fig. 3. These Seidel Coefficients can be used to describe the aberrations in a field. The wavefront aberration were investigated and provided a better understanding due to the wave nature of the FDTD simulation employed in terms of the wavefront aberration in terms of Seidel aberrations.

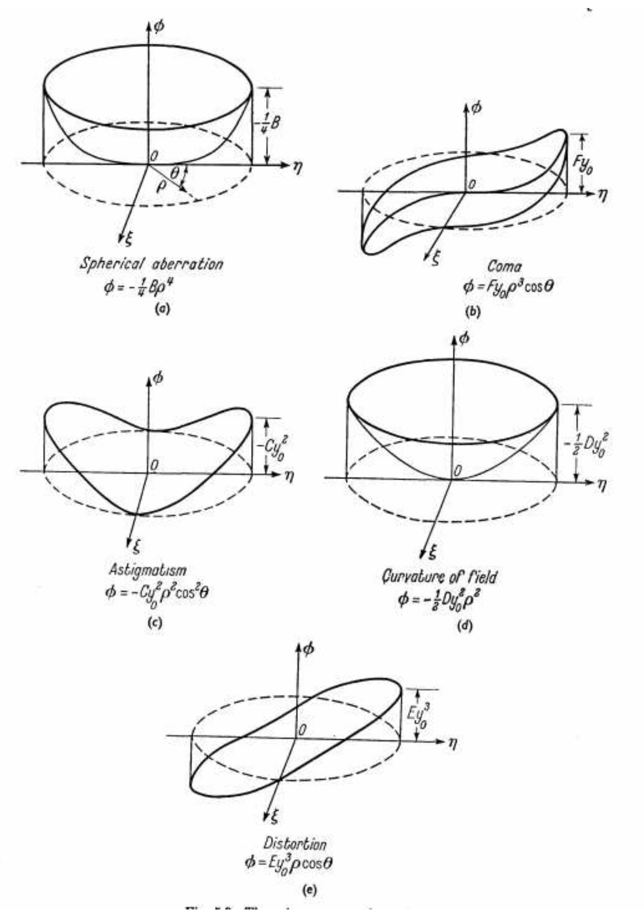

Fig. 3. This figure shows the Seidel wavefront aberrations for A) spherical aberration, B) Coma, C) Astigmatism, D) Field Curvature and E) Distortion. They are accompanied by there function denoting the shape of that particular aberration. The coefficient is specified as the magnitude from peak to valley and is given along with the spherical coordinate system.

III. Results

A. RMS Error

	MTM lens	Reduced map	Cooke triplet	Singlet
RMS Error (/λ)	0.043	0.59	0.15	0.693
Strehl Ratio	0.95	0.48	0.82	0.271
P-V Ratio	0.1333	0.3429	1.4000	0.3506

TABLE I
This table shows the RMS of the aberration function sampled from the FDTD simulation for the four lenses as a function of wavelength. The equivalent Strehl ratio is also given. The MTM lens and Cooke triplet are within typical optical design tolerances.

The average deviation of the wavefront yields the root mean squared optical path difference, see Tab. I which is related to the Strehl ratio using the following equation

$$Strehl = (1 - 2\pi^2\omega^2)^2 \tag{10}$$

This is a common measure for optical systems as is the Peak to Valley (PV) amount which is the difference between the height of the peak of the wavefront at the maximum minus the depth of the valley.

B. Zernike Coefficients Calculation from Q

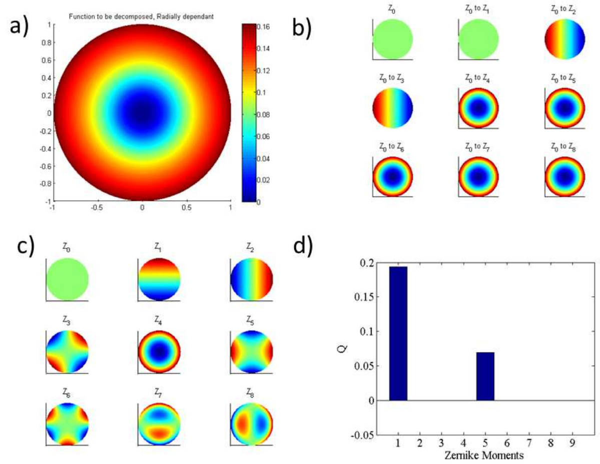

Fig. 4. This figure shows wavefront on a unit disk, the set of Zernike polynomials, the amount of each Zernike polynomial which matches that function. The residual is the left over after all Zernike aberrations have been optimised to best match the input polynomial F. The original function in only 2D therefore results which are not 2D must are not valid.

From the curve fitting of the distorted wavefront a polynomial for each lens is derived, see Fig. 4 and this is the input to a MATLAB script called deco.m which decovolves, as in de-convolution, the polynomial into its constituent Zernike coefficients for each aberration, see Fig. 4. The coefficients for each order of the polynomial are used to calculate the coefficients for the radial Zernike polynomials and because these form an orthogonal set of basis functions will combine and add up to the total RMS wavefront error. The 'closeness' to the original polynomial and the example Zernike aberration is quantified in terms of an integration. The area under the polynomial is calculated in MATLAB using dblquad. It is integrated over the unit cell of a Zernike disk from 0 to 2π. The bar chart shows the Q value which is plotted in the bar chat is the difference between the Zernike polynomial and the wavefront aberration.

C. Spherical Aberration and Secondary Spherical aberration

Zernike aberrations given a values for aberrations which can be separated. This means that there are no overlaps such as occurs in Seidel aberrations. This is achieved by using orthogonal basis functions. The relationship between the aberrations which have integer numbers n and m are given here with there classical aberration relatives. The third order spherical aberration can be calculated from decomposition of the wavefront and the full list of calculated aberrations.

Name	MTM lens	Reduced map	Cooke Triplet	Single Lens
Back Focal Length (dm)	28.7985	27.2625	27.0375	29.1132
RMS Error (waves)	0.043	0.59	0.15	0.693
Z1 (Piston)	0.015	-0.08	-0.0059	-0.0021
Z4 (Defocus)	0.0054	0.0049	-0.0016	0.072
Z10 (Spherical Aberration)	-0.0248	0.1800	0.0290	0.0090
Z22 (Secondary SA)	-3.9E005	-0.0045	0.01	-0.015

TABLE II

THIS TABLE CONTAINS THE FULL WAVE ABERRATIONS FOR ALL PRIMARY ABERRATIONS FOR THE MTM LENS, REDUCED MAP AND THE COOKE TRIPLET. THE ZEROS ARE ABERRATIONS WHICH ARE PRESENT IN THE THIRD DIMENSION. THESE ABERRATIONS COULD NOT BE CALCULATED FROM A 2D SIMULATION ALONE.

D. Defocus

The defocus was used as a measure of how successful the Zernike aberration polynomial was. The polynomial fitted to the wavefront was determined using MATLAB. Then the first order coefficient for x, in the case of the MTM Cooke Triplet this being 0.172. The location of the point source, which was previously placed at the focal length, was placed at the true focal length for this lens in FDTD. This corrected the defocus and improved the calculation of the focal length in the MTM triplet. This method was also applied to the reduced map, conventional and singlet lenses. The method of calculating Zernike aberrations was validated by adjusting the lens according to correct defocus. Defocus or Z_4 in the previous section was used to adjust the original position of the focal point and so recursively improve the aberration correction algorithm.

E. Modulation Transfer Function (MTF)

The MTF is calculated from the Zernike aberration polynomials and is closely related to the pupil function and the point spread function via Fourier transforms and is used widely in optical systems software to give a value to the resolution of the optical system. An aberrated system, as we have already seen, can be described by its deviations. These can be included in the pupil function to give a general pupil function which gives a description of an aperture where the wavefront aberrations are described by a theoretical phase retarding plate

$$\mathbf{P}(x,y) = P(x,y)exp[jkW(x,y)] \tag{11}$$

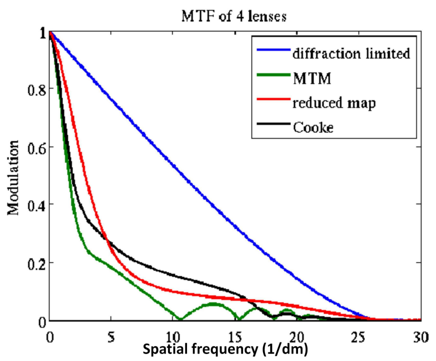

Fig. 5. This figure shows the modulated transfer function (MTF) calculated from the results of the Zernike aberration coefficients for the Cooke triplet, the MTM triplet and the reduced map and a diffraction-limited aberration free case.

The Modulation Transfer function results verify those derived in the section on spherical aberration and resolution however give a more subtle account. The designed lens remains a better performance lens, for spatial resolutions across the range from k= 1 to 25 radians. The reduced map is worse than the Cooke triplet up to k = 5 radians at which point it becomes better than the conventional design up till around 17 radians and the Cooke modestly outperforms the MTM lens at 19 radians. None of the lenses approach the diffraction limited case which would be an image free of aberrations. The MTF can be calculated from the optical transfer function of which it is the absolute value. The optical transfer function is the Fourier transform of the point spread function. The point spread function is a Fourier transform of the pupil function and as we have already shown an aberrated lens can be described by a generalised pupil function. The MTF of the three lens compared with an un-aberrated case are shown in Fig. 5. The results show reasonable agreement with the MTF of a Cooke triplet simulated in OSLO.

IV. CONCLUSION

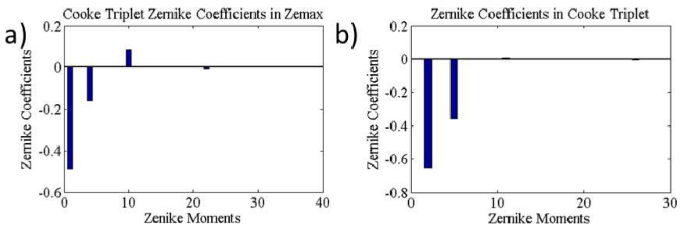

Fig. 6. The figure on the left shows the Zernike coefficients for the Cooke triplet in ZEMAX. In panel b) shows the figure for the Cooke triplet in the FDTD simulation calculation of the Zernike coefficients.

The model can be verified using ZEMAX. ZEMAX give data for the Zernike aberration coefficients in a Cooke triplet. Fig. 6. The results show good agreement validating the design procedure. The MTM lens shows a better imaging performance in terms of RMS wavefront error, aberrations and MTF. The reduced or map which does not require metamaterials shows a similar or worse performance overall when compared to the conventional device. The optical path difference is better for the MTM lens and worse for the reduced lens. The Zernike aberrations shows that the MTM lens has a larger Zernike aberration than the reduced lens and the method was validated using the measure of defocus. This validates our original proposal to design an optical system, the Cooke triplet, using transformation optics and metamaterials.

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Dr Saul J. Wiggin I am completing a PhD at Queen Mary university, school of electronic engineering, antennas and electromagnetics group on FDTD simulations of RF Metamaterials. My supervisor was Professor Yang Hao, Professor Clive Parini FRS and Professor Ian Youngs (DSTL). I was funded partly by the EPRSC and partly by DSTL. I have designed applications for transformation optics to engineering electromagnetic waves and optics. I worked with the QUEST platform grant. Previous to this I completed my Masters at the University of Manchester on Physics where my final year project involved astrophysical simulations of Masers around Late type stars.

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