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Relative illumination calculations

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Abstract

Relative illumination in an optical system is a function of many variables including distortion, vignetting and pupil aberration. It can be obtained by determining the size of the exit pupil in direction cosine space, using the appropriate coordinate system. In general, the calculation is done by tracing bundles of rays through the system. In many cases of interest, it is possible to calculate relative illumination accurately by tracing one on-axis ray and three off-axis rays.

Coordinate system

The key to calculating relative illumination is to define the limiting rays in exit space in terms of direction cosines.[1,2,3] The limiting rays are those which just pass through the edge of the limiting aperture of the system. For a system with no vignetting, the aperture stop is normally the limiting surface. In general, however, different rays will be limited by different surfaces. For systems with obstructions, the rays limited by the obstructions also have to be considered.

The definition of the direction cosines is described in Figure 1. The origin of the coordinate system is at O' with the z-axis along the optical axis. Point Q' is the intersection of the chief ray with the image surface. Point E' is the position of the exit pupil of the system along the optical axis. A reference sphere is constructed whose center is at Q' and which passes through E'. An arbitrary ray intersects the reference sphere at P'. The coordinates of interest are the direction cosines, (l,m,n), of the line segment between P' and Q'. Note that this segment does not coincide with the ray unless there is no aberration.

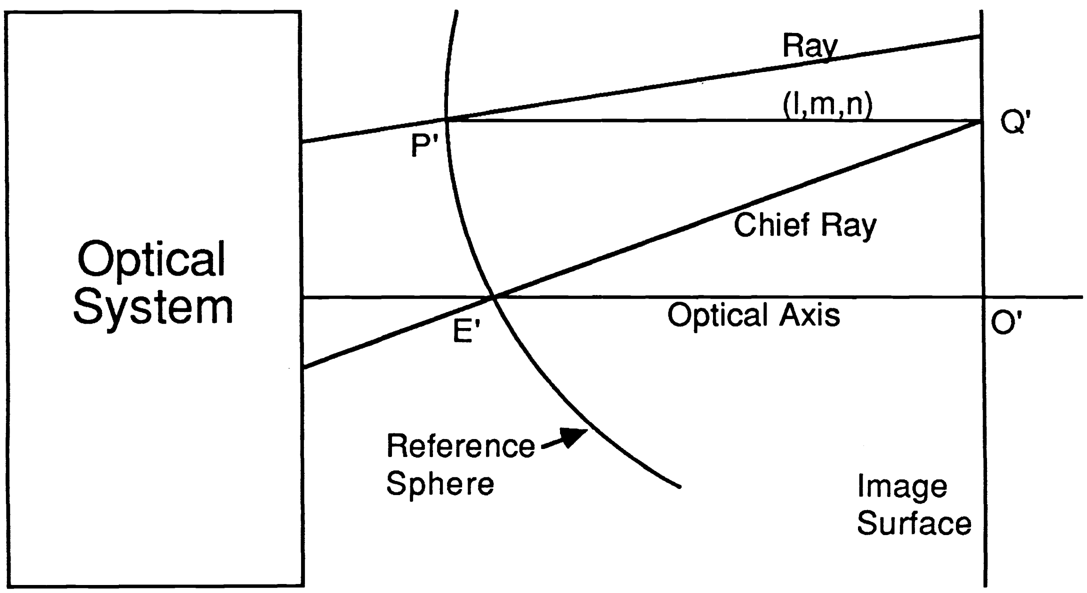

Figure 1. Pupil coordinates

The direction cosines must be defined in a coordinate system whose z-axis is parallel to the surface normal at Q'. For a flat image surface, this is the same as the coordinate system in which the surface is defined, the z-axis being along the optical axis. If the image surface is not flat, (l,m,n) must be rotated into the coordinate system of the surface normal. This is done with the rotation matrix in equation (1), where (L,M,N) is the unit vector along the normal to the surface at point Q'.

$$\begin{bmatrix} \sqrt{1-L^2} & -LM/\sqrt{1-L^2} & -LN/\sqrt{1-L^2} \\ 0.0 & N/\sqrt{1-L^2} & -M/\sqrt{1-L^2} \\ L & M & N \end{bmatrix} \tag{1}$$

Relative illumination

We assume a uniform light intensity over the object and that it behaves like a plane lambertian source. If we plot the exit pupil shape, as defined by the limiting rays, as a function of (l,m) we get a figure whose area is proportional to the illumination on the image surface. This is illustrated in Figure 2. Thus, the relative illumination is obtained by dividing the off-axis pupil area by the on-axis pupil area. It should be noted that this approach automatically takes into account any variation of pupil size as a function of field, including the effects of vignetting and image distortion. It is valid for any general optical system and is not limited to rotationally symmetric systems.

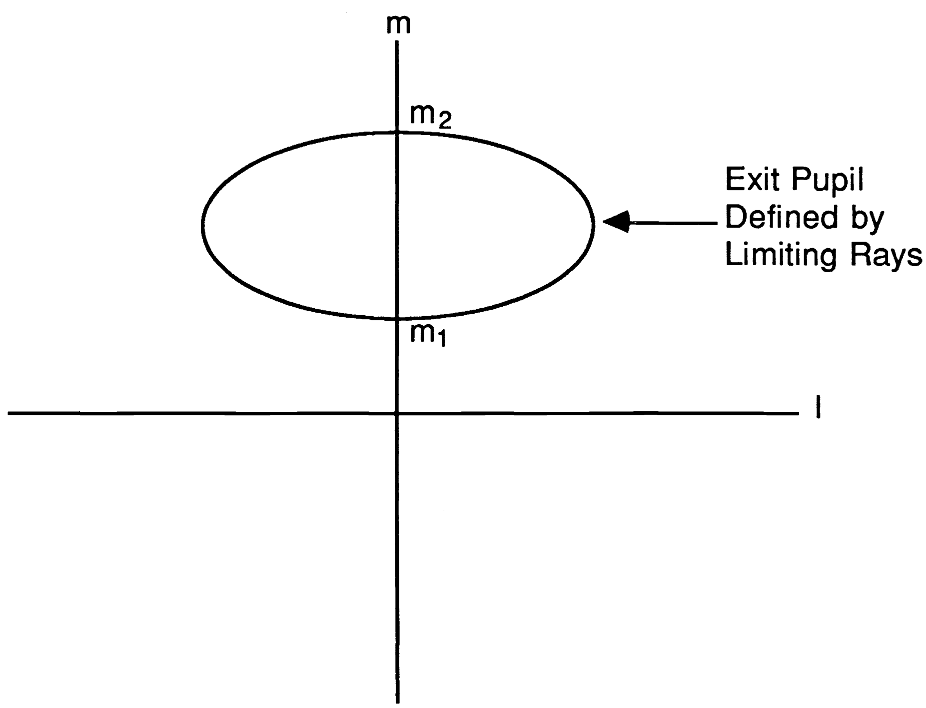

Figure 2. Exit pupil in direction cosine space

The effective F-number can also be obtained from Figure 2. If we denote the points where the exit pupil periphery crosses the m-axis in the figure by m1 and m2, the effective F-number in the m-direction is 1/(m2-m1). A corresponding number can be obtained in the l-direction.

The cosine^4 law

Figure 3 depicts the exit space of a thin lens with stop in contact and shows the limiting rays for the on-axis beam and for an off-axis beam where the chief ray makes an angle, U, with the optical axis. R is the radius of the pupil, D is the distance from the pupil to the image and H is the image height. We can algebraically define the direction cosines of the limiting rays in the meridional and sagittal directions and, assuming the pupil can be represented by an ellipse, generate an expression for the relative illumination. If we now take this expression and let R go to zero (the paraxial limit), we will find that the relative illumination, RI, is

$$RI = \cos^4 U \tag{2}$$

This is the well known cosine^4 law and agrees with the simpler notion that one cosine arises from the obliquity of the beam with the image, one from the obliquity with the pupil, and two from the square of the reciprocal distance. This law more generally applies to a flat-field system whose exit pupil is constant in size and position with respect to field. The angle, U, is measured in image space. If, in addition, the pupils of the system are at the nodal points, the angle in object space is also U.

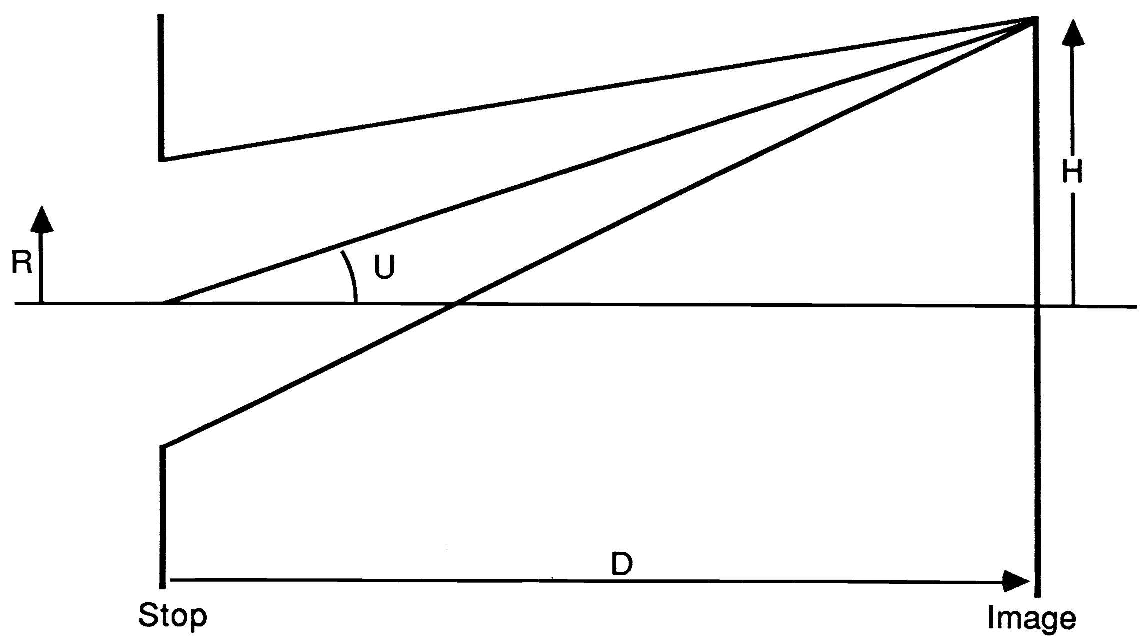

Figure 3. Thin lens with stop in contact

Practical calculations

The straightforward way to calculate the pupil area on a computer is to turn the lens around and trace a grid of rays from the image point equally spaced in the direction cosines (l,m). The area is then proportional to the number of rays which get through the system. It is usually inconvenient to turn the lens around, so an alternate approach is to trace the rays from the object point instead of the image point. This approach, with the following modifications, has proven to be sufficiently accurate for practical purposes.

If the image is aberrated, then the rays will not coincide with the line segment between the image point and the reference sphere. In this case, the raytraced direction cosines in image space can be modified using equation (3), where (l',m') are the raytraced direction cosines, (l,m) are the modified direction cosines, (X,Y) are the transverse aberrations (deviations from the chief ray) on the image surface, and D is the distance from the exit pupil to the image surface. The sign convention used is that a ray which is sloping down in traveling from left to right has a negative direction cosine.

$$
\begin{aligned}
l &= l' - (1-l'^2)X/D + l'm'Y/D \\
m &= m' - (1-m'^2)Y/D + l'm'X/D
\end{aligned}
\tag{3}
$$

It is necessary to space the rays uniformly in exit space direction cosines. To accomplish this, the object-image magnification must be calculated for each field point under consideration. In general, the magnification is different for the meridional and sagittal directions at any one field and linearly relates object space coordinates to image space coordinates. The relationship can be written as shown in equation (4), where (l,m) are the direction cosines in image space, (l_s,m_s) are the direction cosines in object space, $(\bar{l}_s,\bar{m}_s)$ are the direction cosines of the chief ray in object space and $(\bar{l},\bar{m})$ are the direction cosines of the chief ray in image space. When the object is at infinity, entrance pupil coordinates are used instead of object space direction cosines. The matrix elements (A,B,C,D) are obtained by tracing a few close rays to the chief ray and solving a set of simultaneous equations. In the special case of a rotationally symmetric lens, A is the sagittal magnification, D is the meridional magnification and B = C = 0.

$$
\begin{bmatrix} l - \bar{l} \\ m - \bar{m} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} l_s - \bar{l}_s \\ m_s - \bar{m}_s \end{bmatrix} \tag{4}
$$

A simplified calculation

The extensive raytracing described above is not required in many cases. If we assume a rotationally symmetric lens with no obstructions, then in most cases the pupil can be approximated by an ellipse sufficiently accurately. The ellipse, in turn, can be defined by two meridional rays and one sagittal ray. On-axis, of course, only one ray is needed since the pupil is circular. Figure 4 shows on-axis and off-axis exit pupils and gives the directions cosines for the defining rays. The relative illumination is then

$$
RI = \frac{(m2-m1)\ l3}{2\ m0^2} \tag{5}
$$

Thin lens example

Using Figure 3, we can arbitrarily assign the values, R=1, D=10 and H=10. This corresponds to a F/5 lens at a 45 degree obliquity. The values of the limiting direction cosines are m0 = 0.0995, m1 = 0.7399, m2 = 0.6690 and l3 = 0.0705. Evaluating equation (5) gives a relative illumination of 25.2 percent which is very close to the 25.0 percent predicted by the cosine^4 law.

Real lens example

Figure 5 shows a F/6.3 Topogon lens selected from Figure 1 of U.S. Patent 2,031,792. The off-axis field angle is 35 degrees. The relative illumination was calculated in three different ways and compared to the value obtained by R. Kingslake (the average of his two calculations was used). In the first method, approximately 300 on-axis rays were traced through the system. A similar trace was done at the off-axis field using the same exit pupil spacing. Direction cosines were modified as in equation (3). The relative illumination is the ratio of the number of rays traced in the two fields. In the second method, 4 limiting rays were traced and modified using equation (3). The relative illumination was calculated from equation (5). The third method is similar to the second except that the direction cosines were not modified. Table 1 summarizes the results. Even in the case of the 4 limiting rays without correction for image aberration, the results are quite close to the more accurate calculation.

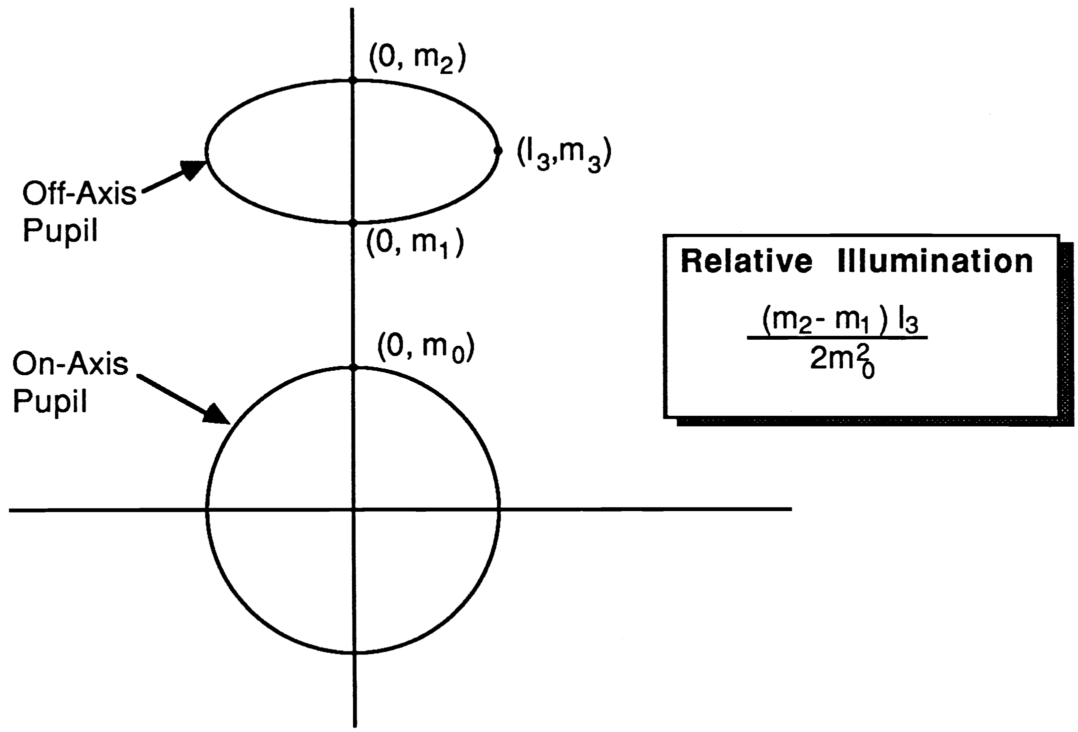

Figure 4. A simplified calculation

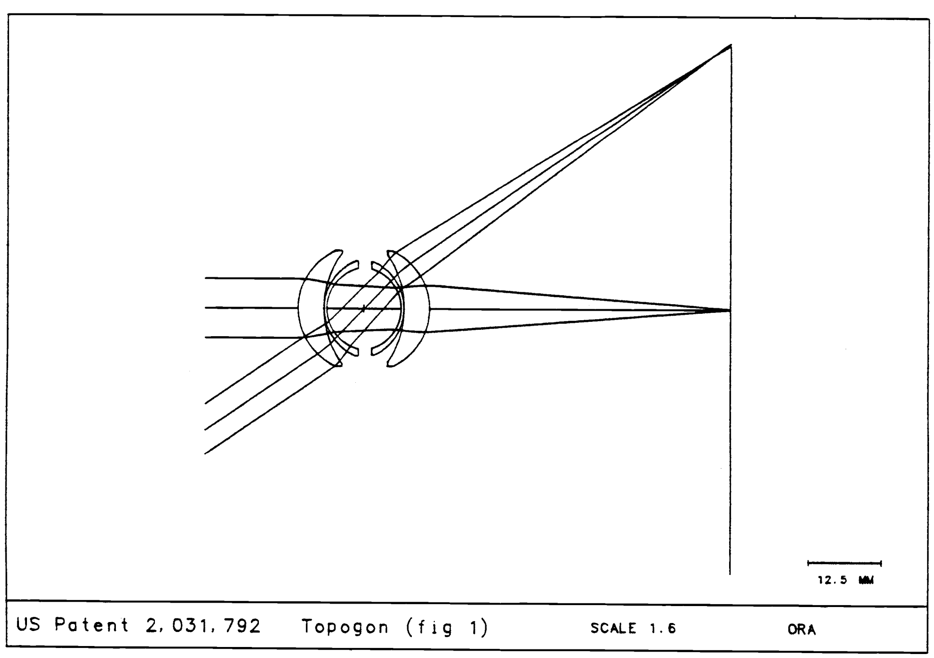

Figure 5. U.S. patent 2,031,792 (Fig. 1)

Table 1. Relative illumination for U.S. Patent 2,031,792 (Fig 1) at 35 degrees

Source of Calculation	Relative Illumination
1 - Grid of rays (corrected for image aberration)	34.7 percent
2 - Equations (5) and (3) (corrected for image aberration)	34.9
3 - Equation (5) (uncorrected for image aberration)	36.7
4 - Kingslake	35.0

Influencing relative illumination

Although a thin lens with stop in contact having a flat image has illumination which follows the $\cos^4$ law, few complex lens systems do. The nature of the $\cos^4$ law suggests ways in which a lens can perform better than predicted by the law. One way is to have in inward curving screen or detector surface. Dome projectors are an example of this. These systems also usually have an F-θ mapping, implying a large amount of negative distortion. This reduces the angle of the chief ray in image space, helping to improve the illumination. For some of these systems, the off-axis illumination can exceed the on-axis illumination.

One can also imagine a system with a central obscuration where the image of the obstruction in the pupil moves toward the edge of the pupil with field. If it moves far enough, it is possible to increase the used pupil area as the field gets bigger and thus obtain higher illumination at the edge of the field than for the axial field. Of course, other factors such as the pupil size and the obliquity of the beam at the image may negate some of this effect.

The Topogon lens shown in Figure 5 is an old, compact, symmetric wide angle lens. It has positive power on the outside elements, causing the chief ray to have a larger angle in the stop space than it does in either the image or object spaces. This has the effect of reducing the pupil size off-axis more than a classical cosine^4 lens resulting in relative illumination which is lower than predicted by the cosine^4 law even without distortion or vignetting. Compare the 35% calculated at 35 degrees, with no vignetting, with the 45% predicted by the cosine^4 law. Most modern wide angle lenses have negative power on the outside. As a result, they are longer but have much better relative illumination than cosine^4 even with low distortion because the angle of the chief ray at the stop is less than the angle in object space.

Conclusions

Relative illumination in a general optical system can be calculated by tracing bundles of rays at the appropriate field points. In many cases of interest it is not necessary to do extensive ray tracing. By tracing only three off-axis rays and one on-axis ray it is possible to get accurate results including the effects of distortion, vignetting, and pupil aberrations.

Acknowledgments

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