A Short Explanation of Aberration Theory.

Propagation of light through an optical system is governed by Snell law (see Fig. 1).

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1} \qquad (1)$$

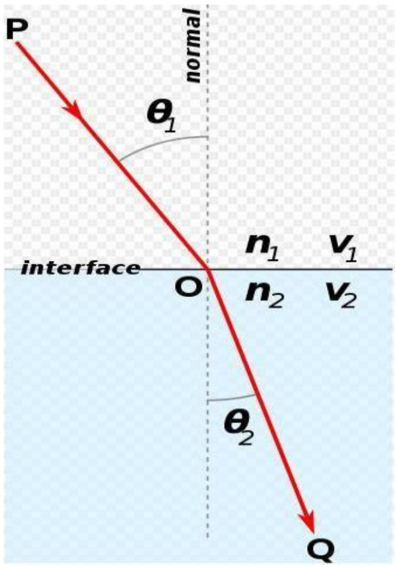

Figure 1: Snell law.

Sin is a nonlinear function:

$$\sin\theta \approx \theta - \frac{\theta^3}{3!} + \dots$$

So, rays that start from the same point**,** and pass through the optical system at different heights, do not meet at a single point (see Fig. 2).

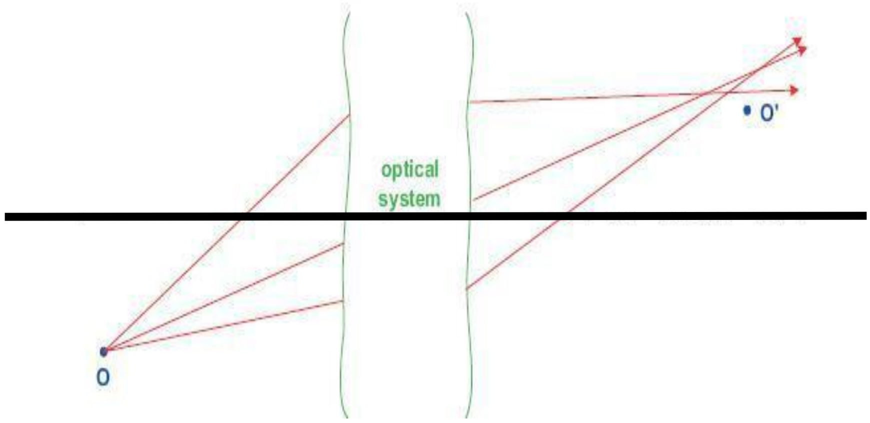

Figure 2: Real optical system.

The perfect image is created when the rays meet at a single point. This happens if the propagation of light is governed by the following law:

$$\frac{\theta_1}{\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1} \qquad (2)$$

Where $\theta_{1,2}$ are angles represented in Fig. 1 and measured in radians. This is Gaussian or paraxial optics law.

Deviation of real ray coordinates from coordinates of perfect ray in an image plane is represented by the following formulas[1]. See Fig. 3.

$$x' = -\frac{R}{n'}\left\{2b_1x_p + 4c_1x_p(x_p^2 + y_p^2) + 2c_2yx_py_p + 2c_4y^2x_p + \ldots\right\}$$

$$y' = -\frac{R}{n'}\left\{2b_1y_p + b_2y + 4c_1y_p(x_p^2 + y_p^2) + c_2y(x_p^2 + 3y_p^2) + 2(c_3 + c_4)y^2y_p + c_5y^3 + \ldots\right\} \qquad (3)$$

Where *R*, *n'* are coefficients. The deviation is the function of the ray coordinates at object and pupil planes. The formulas were obtained in the case *x=0* for simplicity.

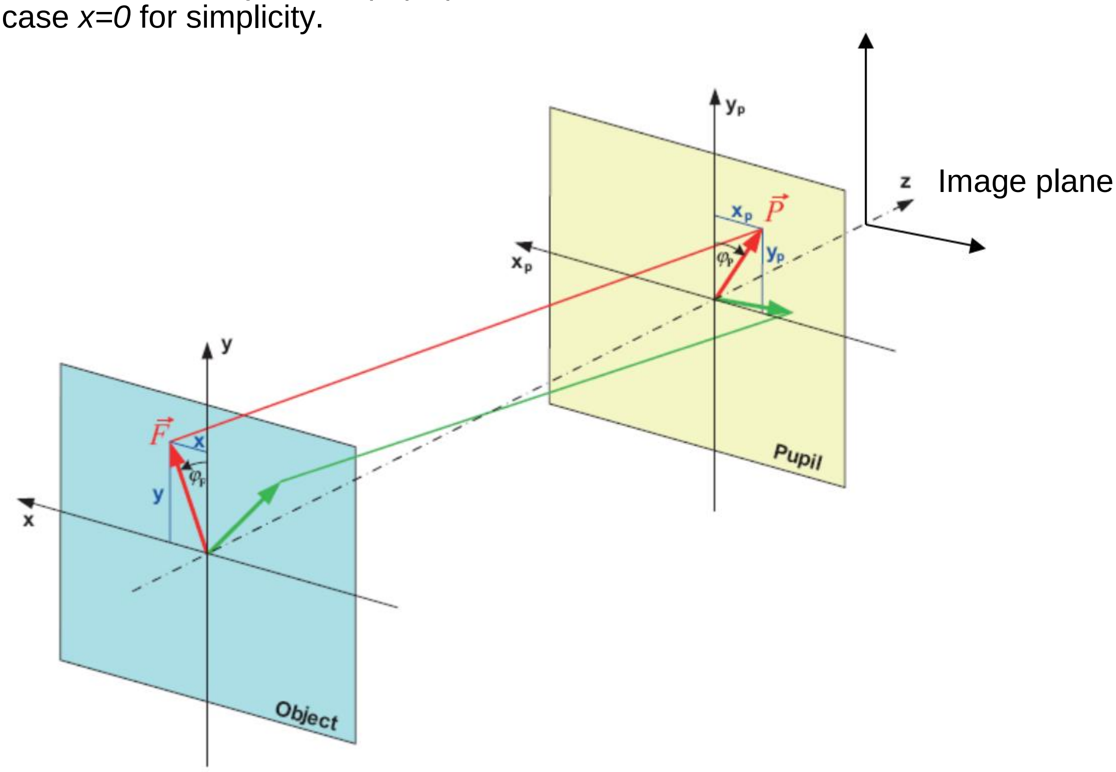

Figure 3: Propagation of light through real system.

Other coefficients in the formulas represent seven types of optical aberrations (see Fig. 4). The coefficients are proportional to Seidel sums that are usually used in the description of third-order aberrations. Next coefficients in the formulas are fifth-order coefficients that considerably smaller. So, they are not written.

Aberration	Coeffi-cient	Seidel sum
Spherical aberr.	c_1	S_{I}
Coma	c_2	S_{II}
Astigmatism	c_3	S_{III}
Field curvatures	(sagittal) c_4	(Petzval) S_{IV}
Distortion	c_5	S_{V}
Axial color	$\tilde{b}_1$	C_{I}
Lateral color	$\tilde{b}_2$	C_{II}

Figure 4: Coefficients and aberrations.

The terms Spherical aberration, Coma, Astigmatism, Field curvature, Distortion, Axial and Lateral Colors are explained in reference 2. An optical system is represented as a set of surfaces in an optical design program. Every surface is a boundary between two media with different refraction indexes. For example, a single lens can be represented by two surfaces (see Fig. 5)

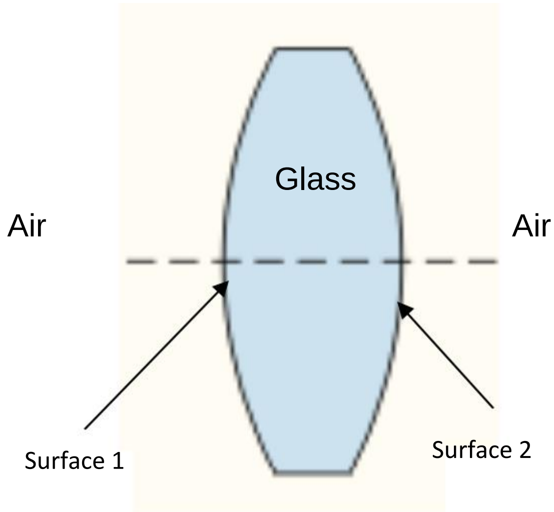

Figure 5: Representation of lens as a set of two surfaces.

Every surface introduces aberration coefficients with "+" or "-" signs. Each kind of the aberration coefficients is balanced in image plane. See Fig. 6. To obtain a high image resolution, the aberrations should be well balanced. Therefore, incident ray angles on every surface should be as small as possible. This requirement reduces nonlinear terms in **sin** expansion. The requirement causes to smooth refraction of rays. It allows avoiding high values of the aberration coefficients that does not create the balance. To perform the smooth refraction diameter of some lens should be larger than the diameter of other lenses in the imaging system (see the front lens in Fig. 6). Lenses that introduce small values of aberration coefficients are less sensitive to tolerances than lenses that introduce larger values of the coefficients.

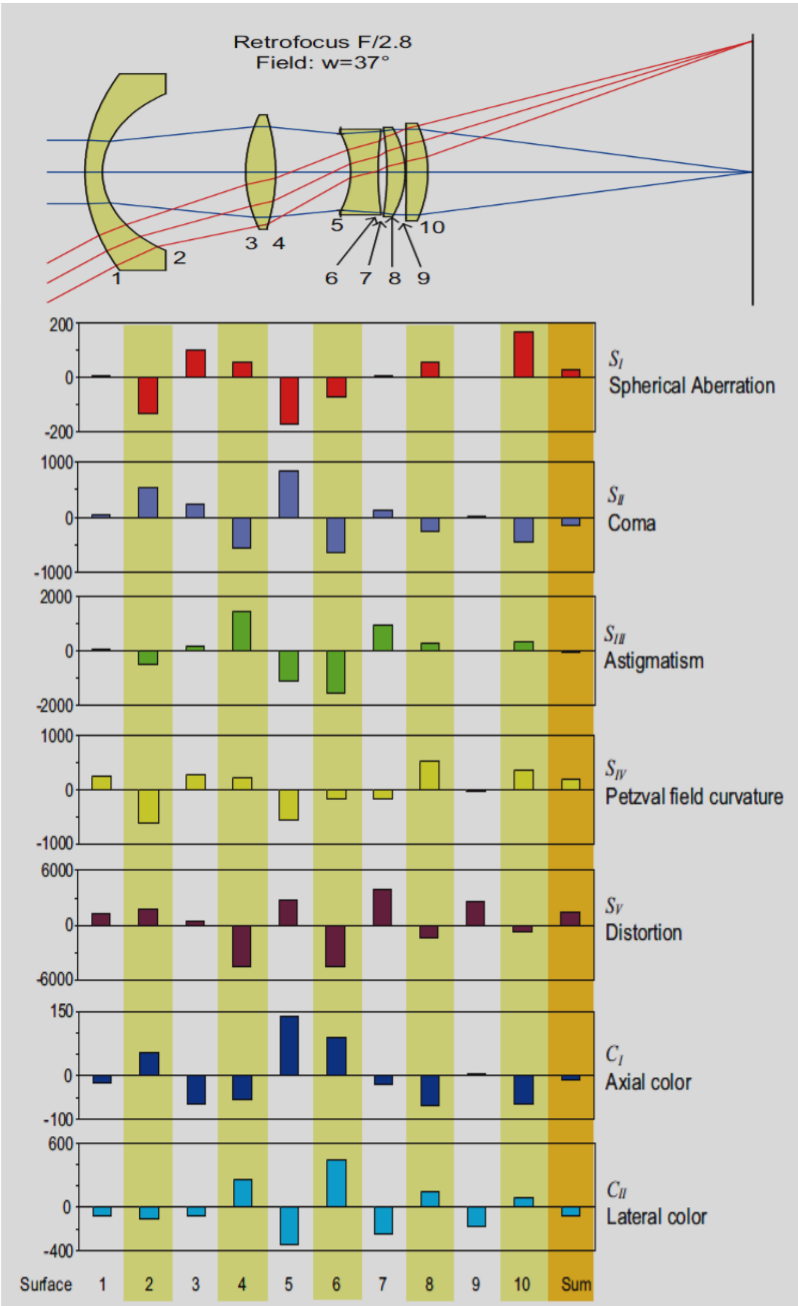

Figure 6: Balance of aberration coefficients[1].

To improve the resolution of specific design, the diffraction-limited PSF (Point Spread Function) should be made narrower. It can be done by enlarging the diameter of the pupil. It causes to decreasing of F/#. But, in this case the deviation of the real ray from perfect ray increases according to Eq.(3). Hence, the diameters of ray spots increase, while optical performance deteriorates. Consequently, more spherical lenses are necessary for improving the balance of aberrations.

If FOV (Field of View) of the camera increases, the deviation of the real ray from perfect ray also increases according to Eq. (3). For the same reason, the larger FOV demands a larger number of spherical lenses for improving the balance of aberrations.

Therefore, an experienced optical designer chooses a start design with F/# and FOV close to client's requirements for achieving the balance of aberrations. After that, he optimizes the design using merit function until required optical performance is obtained.

References.

1. Handbook of Optical Systems edited by Herbert Gross, Vol. 3: Aberration Theory and Correction of Optical Systems. Eq. (29-11) and Eq. (29-12).
2. Photonics Handbook, Lens aberrations, Bruce H. Walker, Walker Associates.

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