The Physics of Subcritical Multiplying Systems

Augusto GANDINI & Massimo SALVATORES

TECHNICAL REPORT

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Augusto GANDINI[1,*] and Massimo SALVATORES[2]

[1]*University of Rome, Nuclear Engineering Department (DINCE), Corso Vittorio Emanuele II, 244, 00186 Rome, Italy*
[2]*Commissariat Energie Atomique DEN/DDIN-CEA Cadarache, 131088 Saint Paul Lez Durance CEDEX, France*

In recent years an increasing interest is observed with respect to subcritical, accelerator driven systems (ADS), for their possible role in future nuclear energy scenarios, as actinide (Pu and MA) incinerators, and/or as claimed energy plants with potential enhanced safety characteristics. Important research programs are devoted to the various related fields of research. Extensive studies on the ADS behavior under incidental conditions are in particular made, for verifying their claimed advantage, under the safety point of view, with respect to the corresponding critical reactors. Related medium and long range scenarios are being considered to cope with a number of concerns associated with safety (power excursions, residual heat risk), as well as with fuel flow (criticality accidents, fuel diversion, radiological risk, proliferation). In the present work we shall comment on some issues relevant to these new reactor concepts, with the intent of giving a general view of the present state-of-the-art in the reactor physics domain. Specific calculation methods to be adopted for intercomparison analysis of these systems and experimental data interpretation are also discussed, as well as ongoing and perspective relevant experimental campaigns on experimental facilities.

I. Introduction

The role of accelerator-driven systems has been recognized worldwide[1–3] as a potential highly relevant tool in order to transmute very large amounts of radioactive wastes and, consequently, to lower the burden on a deep geological storage.

Both the two major variants of the strategies of transmutation which make use of accelerator driven systems (ADS)[4] call for subcritical cores with a fast neutron spectrum and a fuel dominated by mixtures of Pu and minor actinides (MA), essentially fertile-free.

These cores are characterized by a very low fraction of delayed neutrons and by a low (even near zero) Doppler reactivity coefficient. In principle, the subcriticality will help to reduce (or to eliminate) the negative consequences of these characteristics on the safety of the multiplying medium. This is one of the points that will be discussed in the present article.

In fact, the physics of the ADS and of its subcritical core is well understood, and there are several publications, which deal extensively with the subject (see, for example, Refs. 5) and 6), among many others). However, several concepts are new and their understanding requires experimental validation.

In the present article, despite the obvious interest of a global description of on ADS and of the coupling phenomena among the different basic components (*i.e.* high intensity proton accelerator, spallation target, subcritical core), we will give a description of the basic physics phenomena in the subcritical multiplying core, with reference to the coupling phenomena and their impact on the subcritical core (SC), when needed.

We shall treat successively:
 a) the integral parameters which characterize the SC,
 b) the kinetics of such systems,

 c) the control of the reactivity,
 d) some safety features.

We shall also indicate the areas which need particular care for experimental validation and we will quote some ongoing experimental programs and preliminary results.

Finally, the inspection of some "visual" images of SC as they are presently studied in several laboratories, will be the occasion to point out some relevant design-oriented problems of subcritical cores and their integration in an ADS.

II. The Subcritical Multiplying Core in Stationary Regime

1. The Flux Distribution

In a critical system, the condition of balance of neutron production and neutron disparition at each point of the phase space (energy E, space r, angle Ω) is expressed by the Boltzmann equation:

$$
\begin{aligned}
&-\Omega\nabla\phi_o(r,\ E) - \Sigma_o(r,\ E)\phi_o(r,\ E) \\
&+ \iint \Sigma_o(r,\ E')f_o(r;\ \Omega',\ E' \to \Omega,\ E) \\
&\times \phi_o(r,\ \Omega',\ E')d\Omega'dE' + \frac{\chi(E)}{4\pi} \\
&\times \iint \nu\Sigma_{o,f}(E')\phi_o(r,\ \Omega',\ E')d\Omega'dE' = 0, \qquad (1)
\end{aligned}
$$

where ϕ_o is the neutron flux, Σ_o and $\Sigma_{o,f}$ the total and fission macroscopic cross-sections, while f_o is the probability of neutron scattering transfer from $\Omega',\ E'$ to $\Omega,\ E$.

Assuming a neutron energy group representation, Eq. (1) can be expressed in matrix form:

$$
A_o\boldsymbol{\phi}_o + P_o\boldsymbol{\phi}_o = 0, \qquad (2)
$$

where $\boldsymbol{\phi}_o$ represents the multigroup neutron flux vector, A_o is

*Corresponding author, Tel. +39066868095, Fax. +39066868489,
 E-mail: augusto.gandini@uniroma1.it

a matrix operator accounting for neutron leakage, absorption and scattering transfer, while P_o is a matrix operator accounting for fission neutron production.

In the same system, made subcritical by the alteration of some system parameter (so that A_o and P_o become A and P, respectively), the condition to reach a stationary condition is to have an external source $s(E, \, r, \, \Omega)$ such that the governing equation becomes:

$$A\phi_{in} + P\phi_{in} + s = 0, \tag{3}$$

ϕ_{in} representing the solution of the multigroup inhomogeneous equation. The continuous distribution in space and angle of ϕ_{in}, as well as the discrete one relevant to the multigroup energy representation, are obviously generally different from those of ϕ_o. Of course, ϕ_{in} is close to ϕ_o if the system parameter alterations are small.

For an ADS, once defined the material properties, the geometry of the system, the relevant cross-sections and the source intensity (in units of neutrons/s), the distribution of the inhomogeneous flux is fully determined by Eq. (3).

Relevant integral parameters, characterizing the subcritical core (SC), such as reaction rates (*i.e.* integral quantities of the type $R = \iiint \phi_{in} \Sigma \, dr \, dE \, d\Omega$) can be easily calculated. This would allow to evaluate the power deposited in different points of the system, the damage rate, the breeding ratio *etc.*

There are integral parameters, such as the subcriticality level, the reactivity coefficients or even the effective fraction of delayed neutrons, which deserve a special attention.

2. The Reactivity of the Subcritical Core

For the critical core, it is possible to define a meaningful adjoint function $\phi^{*7)}$ such that, with the same matrix notation adopted above,

$$\langle \phi^*, \, (A + P)\phi \rangle = \langle (A^* + P^*)\phi^*, \, \phi \rangle, \tag{4}$$

where $\langle \cdot \rangle$ represents angle-space integration over the whole system, ϕ is the flux of the corresponding critical system, defined as governed by the homogeneous equation

$$A\phi + \frac{1}{K_{eff}}P\phi = 0, \tag{5}$$

K_{eff} being a parameter introduced to "restore" the balance equation. Function ϕ^* is then given by equation:

$$A^*\phi^* + \frac{1}{K_{eff}}P^*\phi^* = 0. \tag{6}$$

For systems not too far from criticality conditions ϕ may be assumed as an approximation of ϕ_{in}. Multiplying Eq. (5) by ϕ^* and space integrating, recalling Eq. (3), we may then write:

$$K_{eff} = -\frac{\langle \phi^*, \, P\phi \rangle}{\langle \phi^*, \, A\phi \rangle} \simeq \frac{\langle \phi^*, \, P\phi \rangle}{\langle \phi^*, \, s \rangle + \langle \phi^*, \, P\phi \rangle}. \tag{7}$$

To improve (7), by taking into account the change in shape of the flux, it has been proposed a different definition of the subcriticality, introducing a "K-source" coefficient (K_s). The procedure consists in considering an integral balance condition obtained by integrating Eq. (5), with ϕ replaced by ϕ_{in}

and K_{eff} by coefficient K_s, such that

$$\langle u, \, A\phi_{in} \rangle = \frac{\langle u, \, P\phi_{in} \rangle}{K_s}, \tag{8}$$

u being a unit vector. Recalling that $A\phi_{in} = -(P\phi_{in} + s)$, we obtain:

$$K_s = \frac{\langle u, \, P\phi_{in} \rangle}{\langle u, \, P\phi_{in} \rangle + \langle u, \, s \rangle}. \tag{9}$$

This new definition does not account for the difference of "importance" between the neutrons generated by fission and the "importance" of the source neutrons.

A more satisfactory definition of the subcriticality has been proposed (Ref. 8)), introducing an importance function n_o^* associated with the relative power level in the subcritical system. This importance function is obtained from the following equation:

$$A^*n_o^* + P^*n_o^* + \frac{\gamma}{W_o}\Sigma_f = 0, \tag{10}$$

where γ is the number if energy units per fission, W_o the reactor power, Σ_f the vector of the macroscopic fission cross-sections.

The following definition is obtained:

$$K_{sub} = \frac{\langle n_o^*, \, P\phi_{in} \rangle}{\langle n_o^*, \, s \rangle + \langle n_o^*, \, P\phi_{in} \rangle} \tag{11}$$

which takes into account both the inhomogeneous flux distribution and the importance of neutrons with respect to the relevant "observable" of the system (*i.e.* its power level).

In Ref. 8), it is shown that when approaching criticality, Eq. (11) becomes equal to Eq. (7), as required on physical grounds.

Another elaborated definition of the subcriticality, which distinguishes between "multiplication of source neutrons" and "multiplication of fission neutrons," has been proposed in Ref. 9).

These different definitions of the multiplication coefficient imply an ambiguity in the definition of reactivity, *i.e.*, of its relative change due to a reactor parameter perturbation. Along with its current definition, the reactivity is in fact associated with the coefficient K_{eff} and then applicable only to extraneous source-free systems. In Ref. 8), the definition of generalized reactivity is given, associated with the coefficient K_{sub} defined by Eq. (11). This generalized concept may be applied to critical, as well as subcritical systems.

3. Reactivity Coefficients

Reactivity coefficients in a subcritical system, associated with alterations of neutron absorption and/or leakage, like Doppler effect, coolant void affect *etc.*, have been so far generally defined in terms of the standard perturbation theory:[10]

$$\Delta\rho = \frac{\langle \phi^*, \, \Delta A\phi'_{in} \rangle}{\langle \phi^*, \, P\phi'_{in} \rangle}, \tag{12}$$

where ϕ'_{in} the perturbed neutron flux, solution of equation

$$A'\phi'_{in} + P\phi'_{in} + s = 0.$$

An ambiguity similar to that previously indicated (relevant to the choice of the appropriate importance function), is again

found here. Its effect is of course smaller, the smaller the distance from criticality, the contribution of the importance of the source neutrons to the weighting process implied by Eq. (12) becoming negligible.

The concept of generalized reactivity mentioned above, implying the use of the importance function n_o^* defined by Eq. (10), appears in many circumstances a more appropriate, unambiguous choice (see Chap. III).

4. The φ^* Parameter

We have seen from the previous discussion that to the understanding of the behavior of a source-driven subcritical core it is of relevance the evaluation of the relative importance of the source neutrons with respect to the fission neutrons generated in the SC.

Starting from Eq. (7), we can write:

$$\frac{1}{K_{eff}} - 1 = \frac{\langle \phi^*, s \rangle}{\langle \phi^*, P\phi \rangle} = \frac{\langle \phi^*, s \rangle}{\langle \phi^*, P\phi \rangle} \cdot \frac{\langle u, s \rangle}{\langle u, P\phi \rangle} \cdot \frac{\langle u, P\phi \rangle}{\langle u, s \rangle}. \tag{7'}$$

We can now rewrite Eq. (7') in terms also of K_S:

$$\frac{1}{K_{eff}} - 1 = \frac{\langle \phi^*, s \rangle}{\langle \phi^*, P\phi \rangle} \left(\frac{1}{K_s} - 1 \right) \frac{\langle u, P\phi \rangle}{\langle u, s \rangle}. \tag{13}$$

The parameter φ^*:

$$\varphi^* = \frac{\langle \phi^*, s \rangle}{\langle \phi^*, P\phi \rangle} \bigg/ \frac{\langle u, s \rangle}{\langle u, P\phi \rangle} \tag{14}$$

is the required ratio of the average importance of the source neutrons to the average importance of fission neutrons. In other terms:

$$\varphi^* = \left(\frac{1}{K_{eff}} - 1 \right) \bigg/ \left(\frac{1}{K_s} - 1 \right). \tag{15}$$

If we introduce the average number of prompt neutrons per fission $\bar{\nu}$, and the average source neutrons per fission Γ:

$$\bar{\nu} = \frac{\langle u, P\phi \rangle}{\langle \Sigma_f, \phi \rangle}, \quad \Gamma = \frac{\langle u, s \rangle}{\langle \Sigma_f, \phi \rangle}, \tag{16}$$

Σ_f being a vector representing the macroscopic fission cross-section in a multigroup form, we obtain the relation:

$$\frac{\Gamma}{\bar{\nu}} \varphi^* = \frac{1}{K_{eff}} - 1 \tag{17}$$

given in Ref. 11), where the experimental determination of φ^* is discussed.

The φ^* parameter plays an important role in the ADS performance parameters assessment. In fact in Ref. 13), it is shown that the relation among the proton beam current i_p, the power in the SC and its subcriticality is given by:

$$i_p (\text{Ampere}) = \frac{\bar{\nu} \left(\frac{1}{K_{eff}} - 1 \right)}{\varphi^* Z} \cdot \frac{W}{\varepsilon_f}, \tag{18}$$

where W is the power of the SC in magawatts, ε_f the energy per fission (in MeV) and Z is the number of neutrons per incident proton.

It can be seen from Eq. (18) that a value of φ^* higher than 1, can reduce proportionally the proton beam current requirement, for a given subcriticality level.

Measurements of φ^* are made in the CEA facility MASURCA in Cadarache, in the frame of the MUSE program,[11] which will be described shortly in a successive paragraph.

III. The Kinetics of a Subcritical System

1. Basic Equation and Point Kinetics

We shall describe in the following the point kinetics equations to be solved for investigating the kinetic behavior of a SC.

Let us now consider the equations governing the neutron flux ϕ of components ϕ_g ($g = 1, \cdots, G$) and the precursor densities c_i ($i = 1, 2, \cdots, I$) relevant to a SC:

$$\begin{cases} V^{-1} \dfrac{d\phi}{dt} = A\phi + (1 - \beta)\chi_P F\phi + \chi_D u \sum_{i=1}^{I} \lambda_i c_i + s \\[2ex] \dfrac{dc_i}{dt} = \beta_i \nu \Sigma_f^T \phi - \lambda_i c_i \qquad \left(\beta = \sum_{i=1}^{I} \beta_i \right) \end{cases} \tag{19}$$

where V is the neutron speed diagonal matrix, u a G-component unit vector, β_i the i'th species delayed neutron fraction, λ_i the associated time delay constant, and where

$$F = \begin{vmatrix} \nu \Sigma_{f,1} & \cdots & \nu \Sigma_{f,G} \\ \cdots & \cdots & \cdots \\ \nu \Sigma_{f,1} & \cdots & \nu \Sigma_{f,G} \end{vmatrix},$$

$$\begin{cases} \Sigma_f^T = |\Sigma_{f,1} \cdot\cdot \Sigma_{f,G}| \\ \chi_z = \text{diag}|\chi_{z,1} \cdot\cdot \chi_{z,G}| \quad (z = P, D) \end{cases}$$

$\chi_{P,g}$ and $\chi_{D,g}$ representing the prompt and delayed fission neutrons spectra, respectively.

Equation (10) may be rewritten in the form:

$$A^* n_o^* + F[(1 - \beta)\chi_P + \beta \chi_D] n_o^* + \frac{\gamma}{W_o} \Sigma_f = 0. \tag{20}$$

Following the derivation described in Refs. 14), 15), using Eq. (20), and assuming as unperturbed the neutron flux density appearing in ratios, Eq. (19) can be reformulated in terms of normalized reactor power P_N as follows:

$$\begin{cases} l_{eff} \dfrac{dP_N}{dt} = (\rho_{gen} - \alpha\beta)P_N + \alpha \sum_{i=1}^{I} \lambda_i \xi_i \\[1ex] \qquad\qquad + \zeta(1 - P_N) + \rho_{\text{source}} \\[2ex] \dfrac{d\xi_i}{dt} = \beta_i P_N - \lambda_i \xi_i \end{cases} \tag{21}$$

with initial conditions (at $t = 0$): $P_N(0) \equiv P_{N,o} = 1$ and $\xi_i(0) \equiv \xi_{i,o} = \beta_i / \lambda_i$. These are the "point kinetics" equations for a subcritical system, in which the coefficients represent physically significant quantities, with the following expressions and definitions:

Normalized power:

$$P_N(t) = \frac{W(t)}{W_o(1 + q)} \left(q = \frac{\langle \delta\Sigma_f, \phi_o \rangle}{\langle \Sigma_{f,o}, \phi_o \rangle} \right) \tag{22}$$

i'th effective precursor density:

$$\xi_i = \frac{\langle c_i u, \chi_D n_o^* \rangle}{I} \tag{23}$$

Effective prompt neutron lifetime:

$$l_{eff} = \frac{\langle \boldsymbol{n}_o^*, \, V^{-1}\boldsymbol{\phi}_o \rangle}{I} \tag{24}$$

Generalized reactivity relevant to a perturbation at $t=0$:
$A \to A + \delta A, \; F \to F + \delta F$

$$\rho_{gen} = \frac{\langle \boldsymbol{n}_{s,o}^*, \, (\delta A + \bar{\chi}\delta F)\boldsymbol{\phi}_o \rangle}{\langle \boldsymbol{n}_{s,o}^*, \, \bar{\chi} F_o \boldsymbol{\phi}_o \rangle} \tag{25}$$

Source reactivity related to a source perturbation: $s \to s + \delta s$
at $t=0$

$$\rho_{\text{source}} = \frac{\langle \boldsymbol{n}_o^*, \, \delta s \rangle}{I} \tag{26}$$

Coefficient accounting for the delayed neutron distribution:

$$\alpha = \frac{\langle \boldsymbol{n}_o^*, \, \chi_D F \boldsymbol{\phi} \rangle}{I} \tag{27}$$

Subcriticality index:

$$\zeta = \frac{1}{I} \equiv \frac{1 - K_{sub}}{K_{sub}} \tag{28}$$

Effective delayed neutron fractions:

$$\beta_{i,eff} = \frac{\displaystyle\sum_{g=1}^{G}\sum_{j=1}^{J} \langle n_{s,o,g}^* c_j \chi_{D,g}^j \beta_{i,g}^j \nu \sigma_{f,g}^j \phi_g \rangle}{\displaystyle\sum_{g=1}^{G}\sum_{j=1}^{J}\sum_{i=1}^{I} \langle n_{s,o,g}^* c_j \chi_{D,g}^j \nu \sigma_{f,g}^j \phi_g \rangle} \tag{29}$$

$$\beta_{eff} = \sum_{i=1}^{I} \beta_{i,eff} \tag{30}$$

Normalization integral:

$$I = \langle \boldsymbol{n}_o^*, \, \bar{\chi} F \boldsymbol{\phi}_o \rangle, \tag{31}$$

having defined $\bar{\chi} = (1 - \beta)\chi_P + \beta\chi_D$.

In Ref. 16), the concept of generalized reactivity ρ_{gen} is commented in relation to other definitions of the multiplication factor.

For the system approaching criticality conditions (and correspondingly vanishing extraneous source), Eq. (21) would reduce to the classical point kinetics equations as given, for example, in Ref. 17). In this case the solution would reduce to a series of exponential functions, with time constants solution of the well known inhour equation.

2. The Asymptotic Behavior

The asymptotic power following the insertion of a perturbation which maintains the system subcritical may be obtained from Eq. (21). Setting the time derivatives equal to zero and substituting $\xi_i = \beta_{i,eff}/\lambda_i$, it is:

$$\rho_{gen} P_N + \zeta(1 - P_N) + \rho_{\text{source}} = 0 \tag{32}$$

and then, recalling Eq. (28),

$$P_N = \frac{1 - K_{sub} + K_{sub}\rho_{\text{source}}}{1 - K_{sub} - K_{sub}\rho_{gen}}. \tag{33}$$

As expected, P_N increases with ρ_{source} (*i.e.*, with increasing source), and with ρ_{gen} (*i.e.*, with positive reactivity). At the limit, for $\rho_{gen} \to (1 - K_{sub})/K_{sub}$, *i.e.*, with the system ap-

proaching criticality, P_N diverges.

To note that an increase by a factor h of the source (so that $s' = hs$, with $h > 1$) would induce a quasi-instantaneous increase of the power by this same factor. The same power increase would occur following a decrease by the same factor h of the subcriticality, corresponding to a reactivity insertion

$$\rho_{gen}' = \frac{1 - h}{h} \cdot \frac{1 - K_{sub}}{K_{sub}}. \tag{34}$$

If, *e.g.*, the system is subcritical corresponding to -10β, a reactivity insertion of $+5\beta$, would produce a doubling of the power (see **Fig. 1**). This of course is totally different from the behavior of a critical system (which would become prompt critical).

In more general terms, the kinetic behavior of a critical system is generally characterized by the delayed neutrons and their time constants (about 10 s), whereas the kinetic behavior of a SC is characterized by the time constants related to the external source, in the sense that an instantaneous variation of source has an effect on the time scale of the prompt neutron lifetime (typically of the order of microseconds).

The evolution of the power with time and then the consequent temperature changes induce reactivity feedbacks due to the Doppler effect, fuel expansion, structural materials and coolant concentration changes. These reactivity effects are essential for the safety of a critical reactor. In a subcritical core, they have a different relevance according to the level of subcriticality. In fact, for a core strongly subcritical, the dynamic behavior is dominated by the external source and its variation with time. With the system closer to criticality conditions, these feedback effects became more important, the behavior of the core approaching that of the corresponding critical core.

3. Reactivity and Loss-of-Flow Accidents

Fast external reactivity insertions give rise to different consequences in critical or subcritical cores. Examples have been given in Refs. 18), 19). In Ref. 18), a $0.55\beta/s$ reactivity insertion in a PHENIX fast reactor type core, critical, or subcritical at $k_{eff}=0.95$, gives rise (at constant external source level) to the following power and average temperature values, as given in **Table 1**.

In Ref. 19), a reactivity ramp of $170\beta/s$ is inserted in a critical core ($W_o = 1$ GW), or in the same core made subcritical at $-1\beta, -2\beta, -3\beta$. The results show that prompt criticality is reached in the critical core after 6 ms with a first power peak of 700 GW at 8.5 ms and a second peak of 500 GW at 13.2 ms. In the subcritical mode, the peaks are respectively of 530 GW at -1β, 6 GW at -2β and 2.2 GW at -3β ($t=16$ ms) (see **Fig. 2**).

The increase in power is considerably slower in a sub-

Table 1 Power average

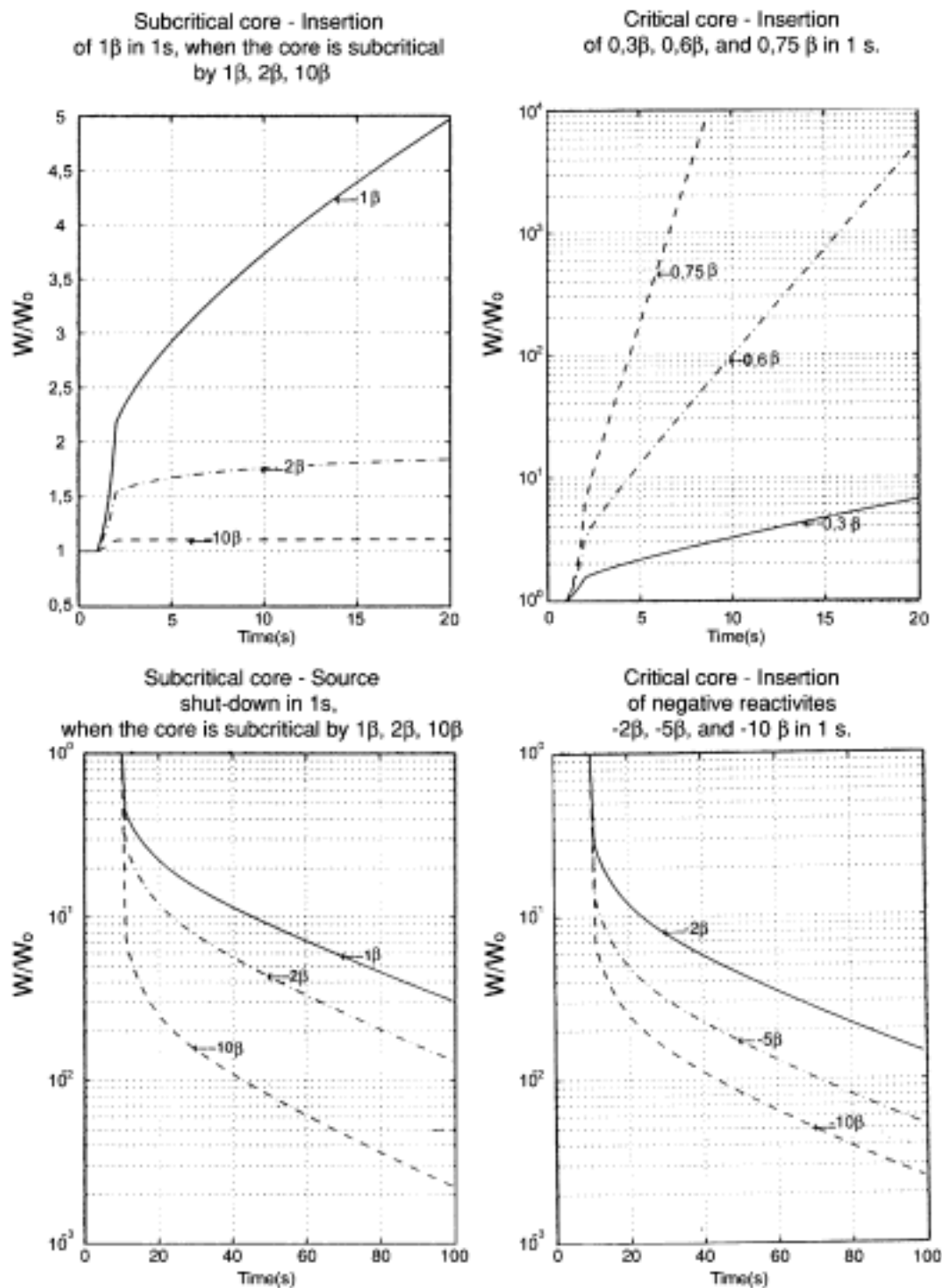|	Critical core	Subcritical core ($k=0.95$)
Delay before fuel fusion	2 s	12 s
Inserted reactivity	1.1β	6.6β
Power increase W'/W	2.2	1.5

Fig. 1 Relative power *vs.* time following reactivity insertions in a subcritical system (Vanier: private communication)

critical system, and then the total energy deployed is much smaller.

In the case of loss-of-coolant-flow accidents, Refs. 18) and 19) give again simple examples, which show that, in the case of no shut-down of the source, the behavior of a -10β subcritical system is less favorable, since in a critical system the increase of the coolant temperature is slower and lower, due to the more effective reactivity feedbacks. It may be seen, again, how the choice of the correct subcriticality level is crucial for fully exploiting the potentiality of a subcritical reactor. This

choice has of course to take into account the characteristics of the system (core size, fuel type, coolant, *etc.*).

4. Cores with Low Doppler Effect

In an ADS dedicated to transmutation, and then in absence of ^{238}U, the fuel will be dominated by MA. As well known, in this case a small Doppler effect will result.

The dynamic behavior of the core will be differently affected by such characteristics, according to the level of subcriticality (see **Fig. 3** and Ref. 18)).
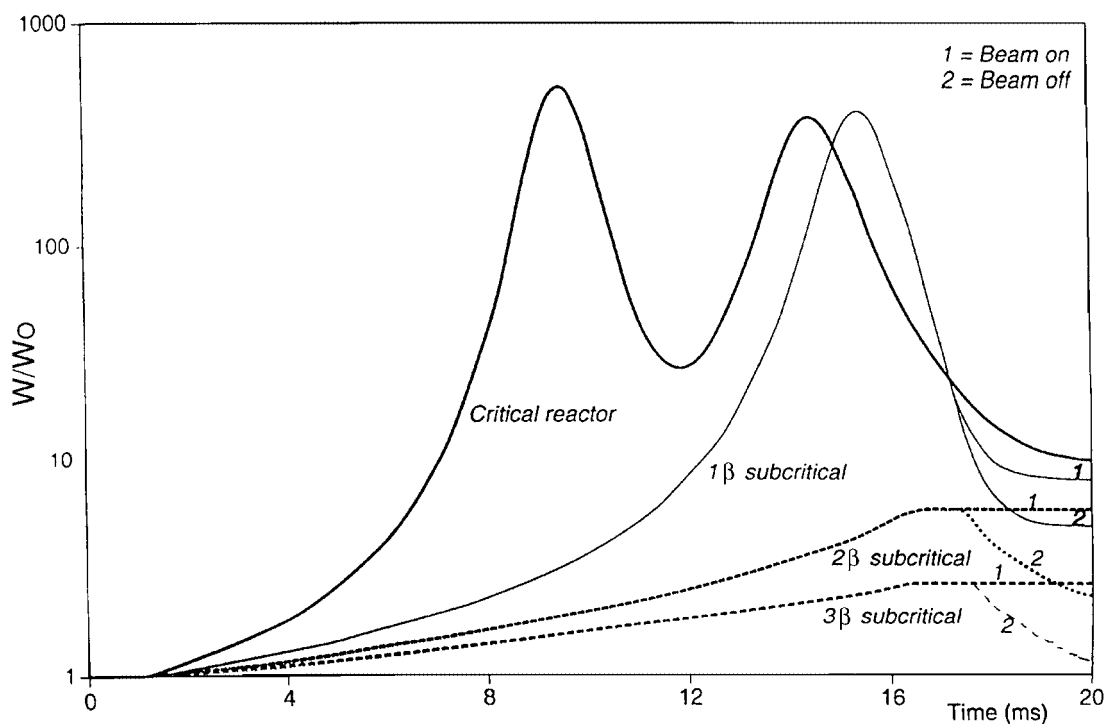
Fig. 2 Relative power *vs.* time following reactivity insertions in critical and subcritical systems (Rief and Takahashi[19])
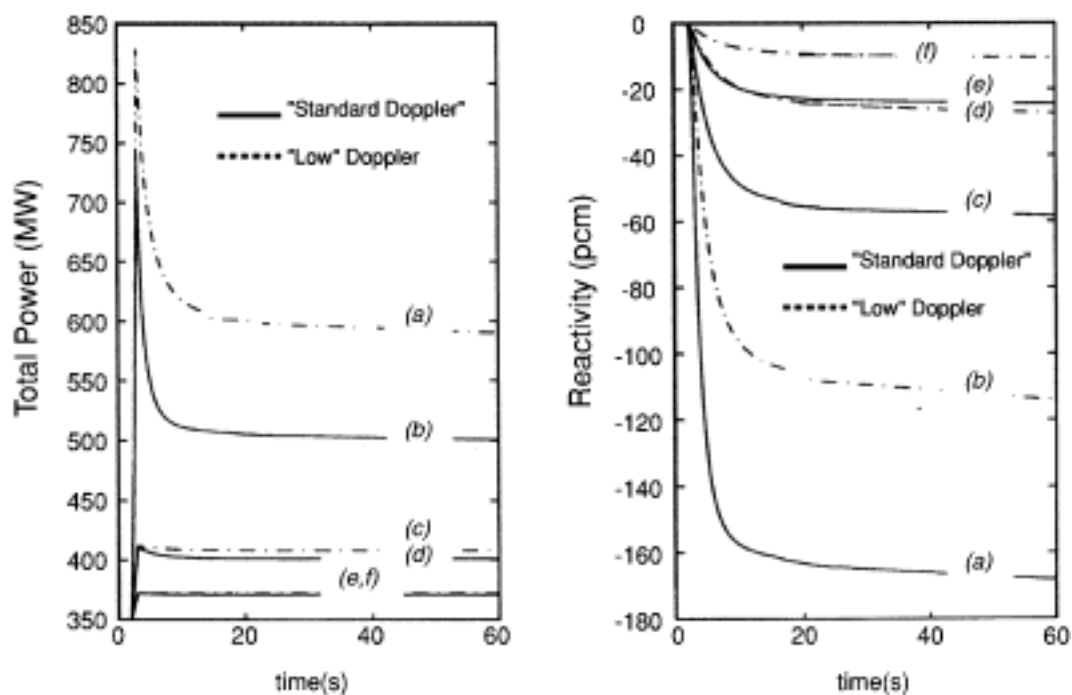

Fig. 3 Total power and temperature feedback reactivity *vs.* time, following $\Delta K_{eff}/K_{eff}=1\beta$ insertion, for two values of the Doppler coefficient and three levels of subcriticality [K_{eff}=0.995 (*a*, *b*), 0.98 (*c*, *d*), 0.95 (*e*, *f*)] (Vanier: Private Communication)

At deep subcriticality, the calculations of the effect of reactivity insertion performed with a "standard" Doppler coefficient K_D, and with a "low" Doppler (K'_D=0.1K_D), indicate no significant difference in the power evolution. On the contrary, close to criticality the effect becomes significant.

5. Choice of the Subcritical Level

No final criteria have been established up to now for defining an "optimal" level of subcriticality. However the above considerations indicate the importance of finding a compromise between the "source-dominated" and the "feedback dominated" regimes.

More quantitatively, (see for example Ref. 16)), in the case that no control rods are foreseen in the SC, the level of subcriticality should be such that the core stays subcritical when going from a "hot" state (*i.e.* normal operation) to a "cold" state (*i.e.* reactor shut-down). Since thermal feedbacks induce generally (*e.g.* in standard fast reactors) a positive reactivity effect (ΔK_{FB}) going from "hot" to "cold" state, one can require that the "cold" core stay subcritical even in the case of an accidental reactivity insertion (ΔK_{AC}), due for example to coolant voiding. In that case the required "K_{eff}" should respect the following relation:

$$K_{eff} + \Delta K_{FB} + \Delta K_{AC} < 1. \qquad (35)$$

During operation, the maximum reactivity insertion (ΔK_{AC}^M) can be higher than ΔK_{AC}. In that case one has the requirement that:

$$K_{eff} + \Delta K_{AC}^M < 1. \qquad (36)$$

Moreover, during the reactor operation, the reactivity varies due to the irradiation (burn-up) of the fuel and its isotopic evolution. In general, this reactivity variation ΔK_{BU} is negative, but in some case (*e.g.* a fuel made essentially by minor actinides, which act as "fertile" materials, since they are transmuted in more "reactive" elements, as it is the case for example of ^{241}Am), ΔK_{BU} can be positive. In that case, one should have:

$$K_{eff} + \Delta K_{AC}^M + \Delta K_{BU} < 1. \qquad (37)$$

Looking for a compromise among the different criteria indicated above, one has also to consider that a very deep subcriticality cannot be necessarily the optimal solution. In fact, besides obvious considerations on the "cost" of a strong external source, and the parallel reduction of the plant efficiency, a deep subcritical core has a peaked power distribution, dominated by the source distribution and, therefore very far from the flat behavior in space, required to optimise the fuel irradiation (and, consequently, the fuel "transmutation").

6. Reactivity Control and Monitoring

The control of reactivity and of the power level in a critical reactor is made essentially using control rods. In an ADS, in principle, the control can be made using only the external source. As an example, the variation of reactivity with the fuel burn-up can be compensated with an appropriate change of the beam current intensity. It can also be conceived a similar system to control the reactivity change between "hot" and "cold" states. However, major variations of the current would be necessary. For example in a SC without control rods, which has K_{eff}=0.99 in the "cold" state, K_{eff}=0.98 at "hot" state at the beginning of irradiation cycle and K_{eff}=0.95 at the end of irradiation cycle, the source intensity should change by a factor of approximately 5, to account for both the trip towards nominal power and the operation cycle. In this context, it is clear that the use of control rods to insure at least some of the functions of reactivity control, should be carefully examined.

Moreover, if it is true that in a SC, in particular in a "source dominated" mode, the shut-down of the source has an instantaneous effect to reduce power, the inverse effect, *e.g.* an

"overshoot" due to a sudden increase of the external source, has as a consequence an instantaneous power increase. Although more limited than the potential power increase in a critical reactor following an inadvertent control rod extraction, such accidental situation should be examined.

Also, when the reactor is shut down, the consequences of the insertion of the full "reserve" of beam current should be analyzed. In fact, if the insertion of the full "reserve" of beam current cannot be excluded, this accidental event could lead to a power relative increase (see Ref. 20)),

$$\frac{W'}{W} = 1 - \frac{\bar{\rho}}{(A+B) - \bar{\rho}} \cdot \frac{\Delta i_p}{i_p}, \qquad (38)$$

where $\bar{\rho}=1-K_{eff}$, and A and B are reactivity (normalized) power coefficients.

If W_{Max} is the maximum allowable power in a short time interval, one can deduce the maximum allowable subcriticality such that $W \le W_{\text{Max}}$.

Finally, we should mention that, in principle, long term variations of the reactivity can be achieved by an appropriate variation of the φ^* parameter, as suggested by Eq. (18).

This can obtained with changes, *e.g.*, of the geometrical arrangement of the buffer (or of the buffer material) surrounding the spallation source.

As for the monitoring of the level of subcriticality, different methods can be envisaged and experimentally validated. Some examples are as follows:

- Use of a pulse mode of the source. The recording of the time evolution of the counting rates of incore neutron detectors can allow to extract the reactivity. In fact, the point kinetics predicts the prompt decay of the neutron population after a pulse, to be of the type $\exp(-\alpha t)$ with $\alpha=(\rho-\beta)/l$. For β and l known one can extract ρ from the decay of the neutron population obtained experimentally.

- If control rods are foreseen, the modified source multiplication method (MSM, see Ref. 21)) can be used, but the calibration of the control rod reactivity should be performed at near-critical level. In fact in a subcritical configuration, a reactor will multiply the source neutrons according to $S/(1-K_{eff})$. A detector will record C counts per second according to the expression:

$$C = \frac{\varepsilon S}{1 - K_{eff}}, \qquad (39)$$

where ε is the detector efficiency. Between two states, $K_{eff,1}$ and $K_{eff,2}$, we can write the ratio of count rates as seen by the detector:

$$\frac{C_1}{C_2} = \frac{\varepsilon_1 S_1 (1 - K_{eff,2})}{\varepsilon_2 S_2 (1 - K_{eff,1})}. \qquad (40)$$

The MSM method allows to correct for the fact that the ratios of the detector efficiencies and of the effective source strength are not equal to unity with a change in state. If one takes state $K_{eff,1}$ as a state where an absolute measurement of $(1-K_{eff})$ can be made (*e.g.*, by a rod-drop method), all other reactivities for subcritical states can be obtained using the above method.

- A different method could be used, considering changes

of control rod insertion giving rise to $(\delta K_{eff}/K_{eff})_{B,exp}$ and changes of external source intensity $\delta S_{exp}/S$, such that the power level remains, unaffected. In that case and not too far from criticality one obtains:[22]

$$\left(\frac{\delta K_{eff}}{K_{eff}}\right)_{B,exp} \frac{1}{1-K_{eff}} + \frac{\delta S_{exp}}{S} = 0. \qquad (41)$$

This relation allows then to easily evaluate the subcriticality $(1-K_{eff})$.

Finally, it can be mentioned that an original concept to control the source feed to the SC has been proposed in Ref. 23), the so-called "Accelerator Coupled System" concept. In that concept, the accelerator current is provided by coupling directly the power produced in the SC with the accelerator power supply. In such way, an intrinsic control of the source feed is obtained and the system works as a "critical" system, where the fraction of "delayed" neutrons is increased by a quantity equal to the subcriticality. Applications of that concept have also been illustrated.

7. Beam Trips

As far as the coupling of the accelerator with the subcritical core is concerned, one significant point that has been raised,[24] is the effect on the SC by frequent beam trips. Since, as we have mentioned before, the time scale for power variation due source changes is very short, and the heat removal time from fuel to coolant is of the order of $0.1 \div 1$ s, the heat so generated remains stored in the fuel for ~ 1 s and, consequently, an high thermal conductivity fuels may be required. In a similar way, thermal stresses in the core structures may be expected, due to the difference of time constants relevant to power increase and consequent temperatures changes in the structures. In the case of frequent beam trips, fatigue failures of the structures could then occur and cause safety concerns.

IV. Experimental Validation

The physics characteristics and the predicted behavior of a SC, as outlined in previous paragraphs, need an experimental validation, in order to calibrate the calculation tools and to gain confidence in the prediction of the basic safety features of an eventual future ADS, which will be fuelled with very innovative fuels.

The main fields that need experimental validation are:
- the effects of the relative contributions of the source neutrons and of the neutrons generated by fission. In particular, φ^* (see Sec. II-4) measurements should allow to achieve that objective in stationary conditions. Kinetic experiments performed at different subcritical levels, with or without feedback effects, may be essential to understand the transition between a "source-dominated" and a "feedback dominated" regime,
- space and energy distributions of neutrons and their variations close to the external source,
- subcriticality level assessment and monitoring,
- relation between the external source and the power in the core,
- *etc.*

1. The MUSE Experimental Program—The Physical Principle

A first experiment related to the verification of the physical principles of an ADS, was performed by C. Rubbia at CERN (FEAT experiment, Ref. 25)). A proton beam did hit directly a natural uranium block, and the "energy amplification" was experimentally verified.

Since 1995, at the MASURCA facility of CEA in Cadarache, a series of experiments called "MUSE" (MUltiplication avec Source Externe) has been performed. The principle of these experiments (Ref. 11)) lies in the hypothesis of the separability of the effects of the source and of the multiplication in the SC.

In a subcritical reactor, approaching criticality conditions, the number of fission generations associated, on average, with an external source neutron, increases asymptotically, so that the overall flux generated by an arbitrary external source would gradually approach the fundamental space-energy distribution. In case of a moderate subcriticality (*e.g.* with $K_{eff} \geq 0.95$, it is then possible to study the neutronics of the source-driven SC, using, instead of a true spallation source, a different, well-known, external source. The first MUSE experiments were performed using a ^{252}Cf spontaneous fission source, placed at the center of a SC (Ref. 11)). The present MUSE experiments (MUSE-4) use a neutron source obtained from (d, t) or (d, d) reactions.

In fact, a deuteron accelerator, GENEPI built at ISN-Grenoble has been coupled to the MASURCA facility, and a deuterium or a tritium target is placed at the center of the SC (see **Figs. 4** and **5**). These targets are surrounded by a lead buffer, to simulate the neutron diffusion of an actual lead (or lead-bismuth) target. Numerical simulations have shown the validity of the basic hypothesis of the experiments, namely that using a spallation neutron source or the neutrons issued from the (d, t) or (d, t) reactions, the neutron spectrum in the core close to the buffer region is very much the same, whatever the neutron source energy distribution.

This result is shown in **Figs. 6** and **7**, where the neutron spectra calculated at CEA-Cadarache by P. Seltborg,[12] are shown at the interface buffer/core and at 10 cm from that interface. Only at the buffer/core interface some differences are observed.

2. The MUSE Experimental Results and Techniques

Experimental results of relevance have already been obtained. For example, in **Figs. 8** and **9** (from Ref. 11)), the flux distribution inside the SC of the MUSE-1 configuration is shown in terms of the measured ^{235}U fission rate, in presence of the ^{252}Cf source.

Figure 8 shows the radial flux distribution in the core with and without external source. The presence of the source gives a more "peaked" distribution, as expected.

Figure 9 shows the axial distributions when the source is at the core/upper reflector interface ($+25$ cm from core midplane). Three axial distributions of the fission rates are shown. Far away from the source, the axial profile becomes less sensitive to the asymmetrical position of the source itself.

Moreover, φ^* measurements have been performed and **Table 2** gives the comparison of calculated and experimental
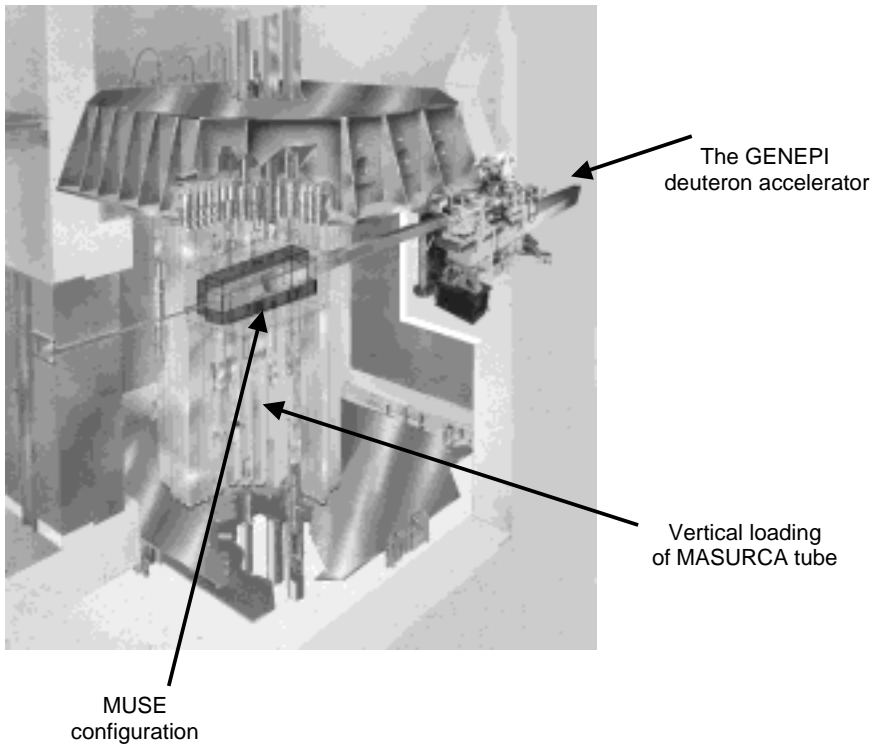
Fig. 4 The MASURCA installation for the MUSE program (general view)
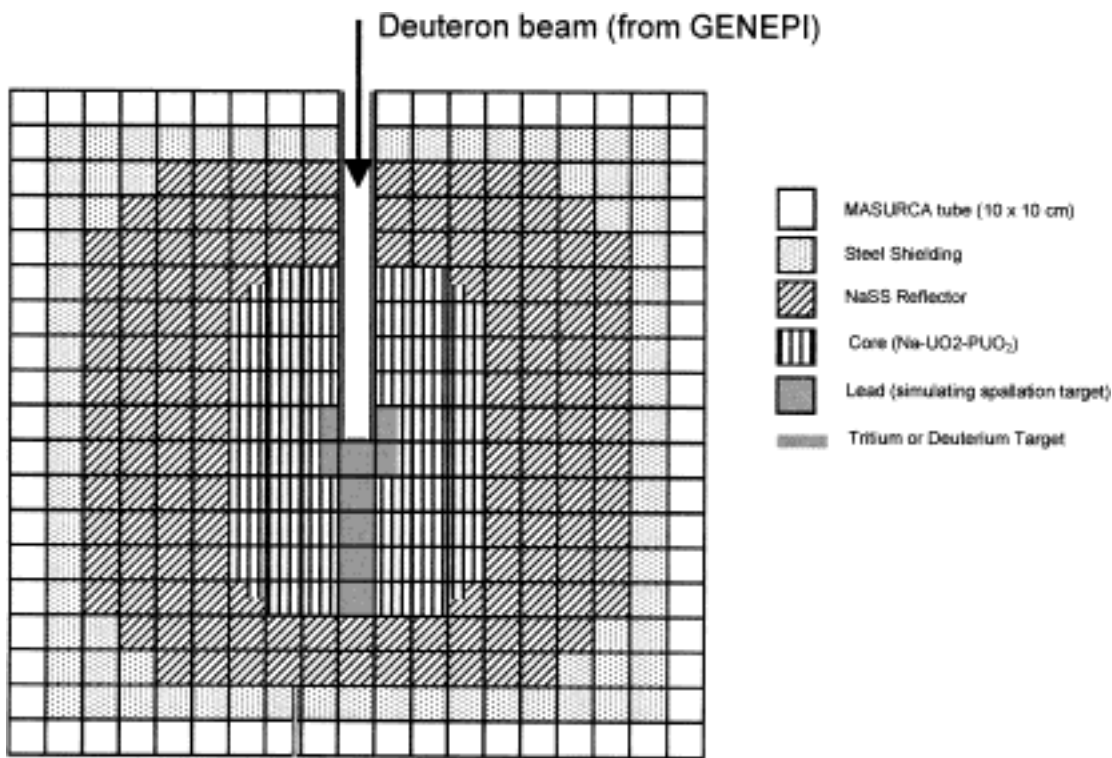

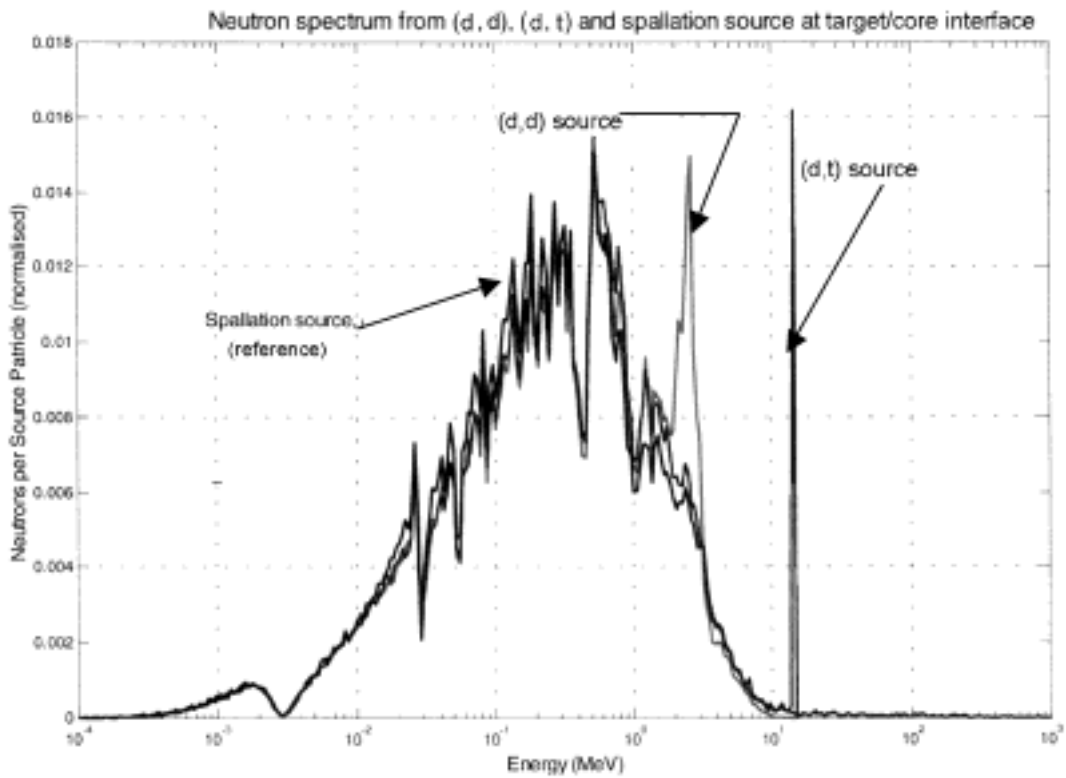**Fig. 5** The MASURCA installation for the MUSE program (vertical layout)

Fig. 6   Neutron spectrum from (d, d), (d, t) and spallation source at target/core interface (Seltborg[12])
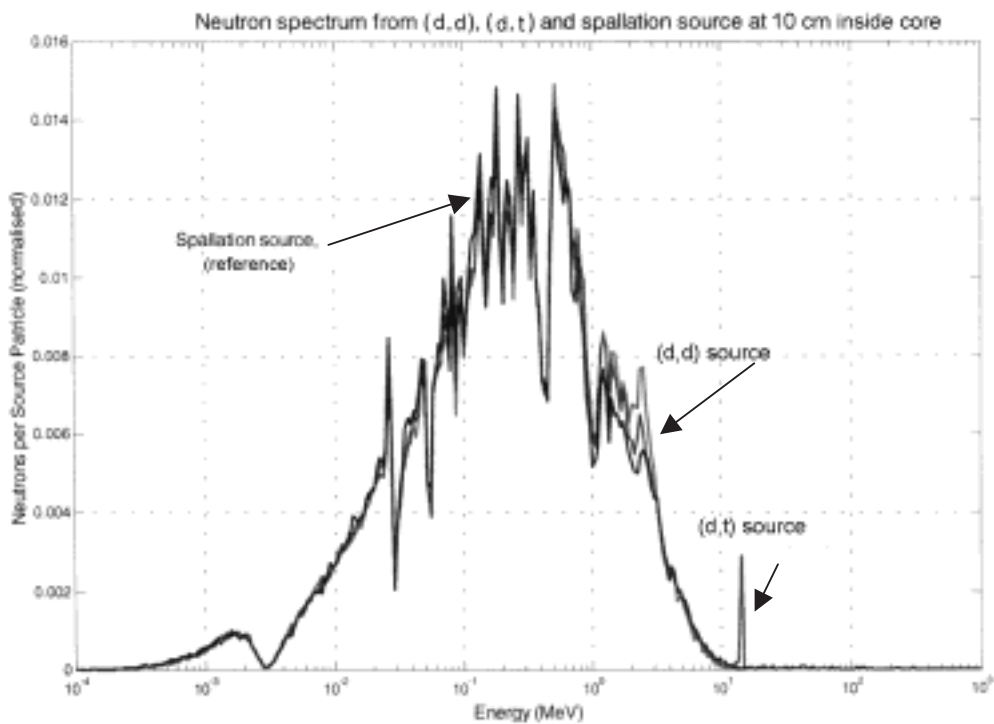

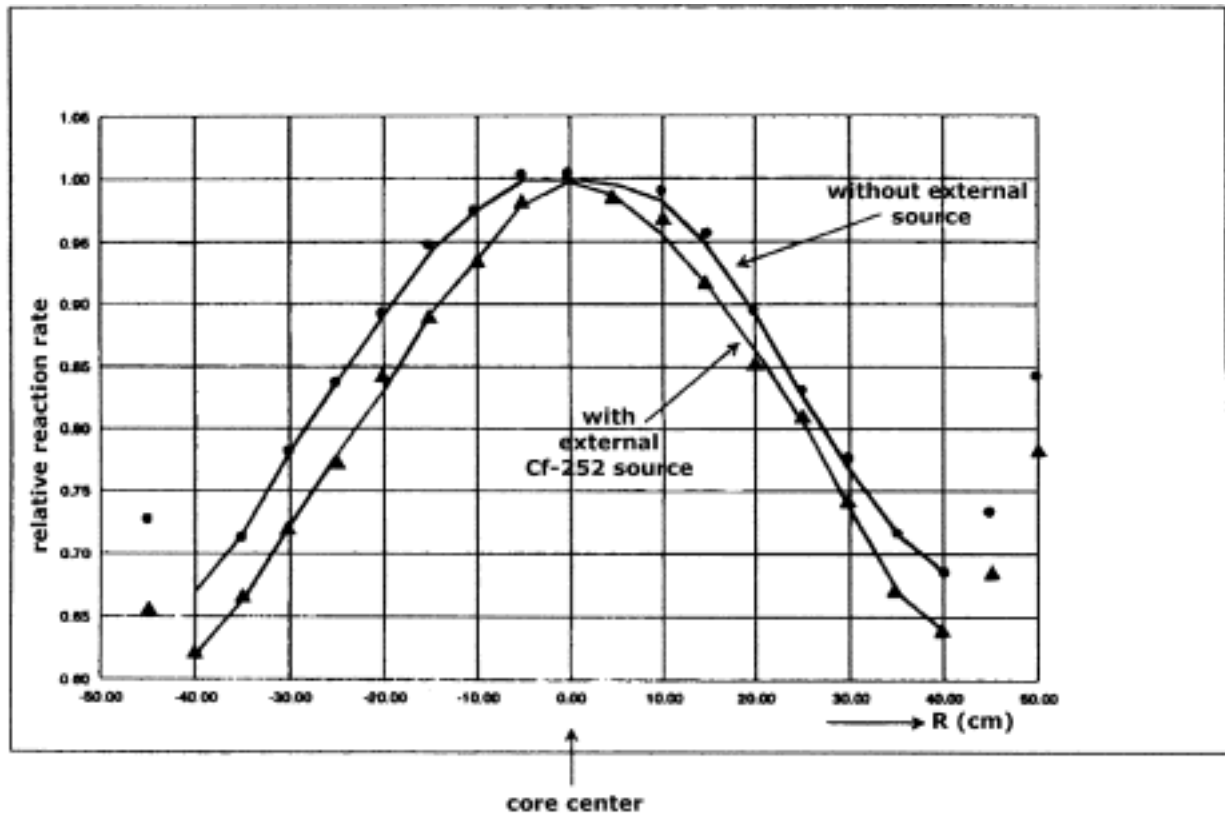**Fig. 7**   Neutron spectrum from (d, d), (d, t) and spallation source at 10 cm inside core (Seltborg[12])

Fig. 8 Radial ^{238}U fission rate in MUSE-1 (Salvatores, *et al.*[11])

Table 2 φ^* measurements in MUSE-1

Configuration	Calculated φ^*	Measured φ^*
^{252}Cf source at core center	1.25	1.19±4%
^{252}Cf source at core/blanket axial interface	0.91	0.90±4%

results.

Finally, in the configuration MUSE-3, the SC was driven by a (d, t) neutron source and the counting rate evolution in time of in-core detectors after a neutron pulse at different subcritical levels is shown in **Fig. 10**. From the decay rate, the subcriticality level can be deduced, as indicated in Sec. III-6.

More experiments are planned in the new configuration with the GENEPI accelerator (MUSE-4, see Figs. 4 and 5), and different experimental techniques (transfer function, MSM method, Rossi-α and Feynman-α, Ref. 26)) will be used in order to measure the subcriticality level, but also l_{eff} and β_{eff} and control rod worths in the SC.

3. Further Experimental Validation at Power

The MUSE experiments are essential to understand and validate the physics characteristics of a SC at near-zero power. They are also essential to establish and validate the experimental techniques needed to assess and monitor the subcriticality level in an actual ADS.

However, these experiments lack feedback effects, which can only be observed with a power SC.

In this respect, an interesting experiment has been proposed reactor by Rubbia, in order to use an existing low power reactor (like a TRIGA) and to run it subcritical, driven by an external source, which could be obtained inserting in the central channel (generally used in those reactors for irradiation purposes) a spallation target (*e.g.* in tungsten), fed by a proton beam coming from an accelerator to be coupled with the core.

V. Some ADS "IMAGES" and Technological Problems

Several countries and leading research laboratories are actively working on the development of ADS, in particular in the frame of radioactive waste minimization strategies.

Conceptual designs have been developed. A typical example is the Energy Amplifier proposed by Rubbia,[27] see **Fig. 11**.

Conceptual designs have also been developed for experimental ADS, in the power range 80–100 MWt. **Figures 12** and **13** indicate two of these configurations: one, lead-cooled, developed at Ansaldo-Italy, and one, gas-cooled, developed at Framatome/Novatome-France.[1]

All these conceptual layouts are of a preliminary nature and some relevant technological problems are still to be accounted for in a satisfactory way.

This is the case, for example, of the shielding configurations in the upper part of the systems. The shielding in fact should account for the potential deep penetration of high energy neutrons ($E_n \geq 100$ MeV) issued from the spallation of
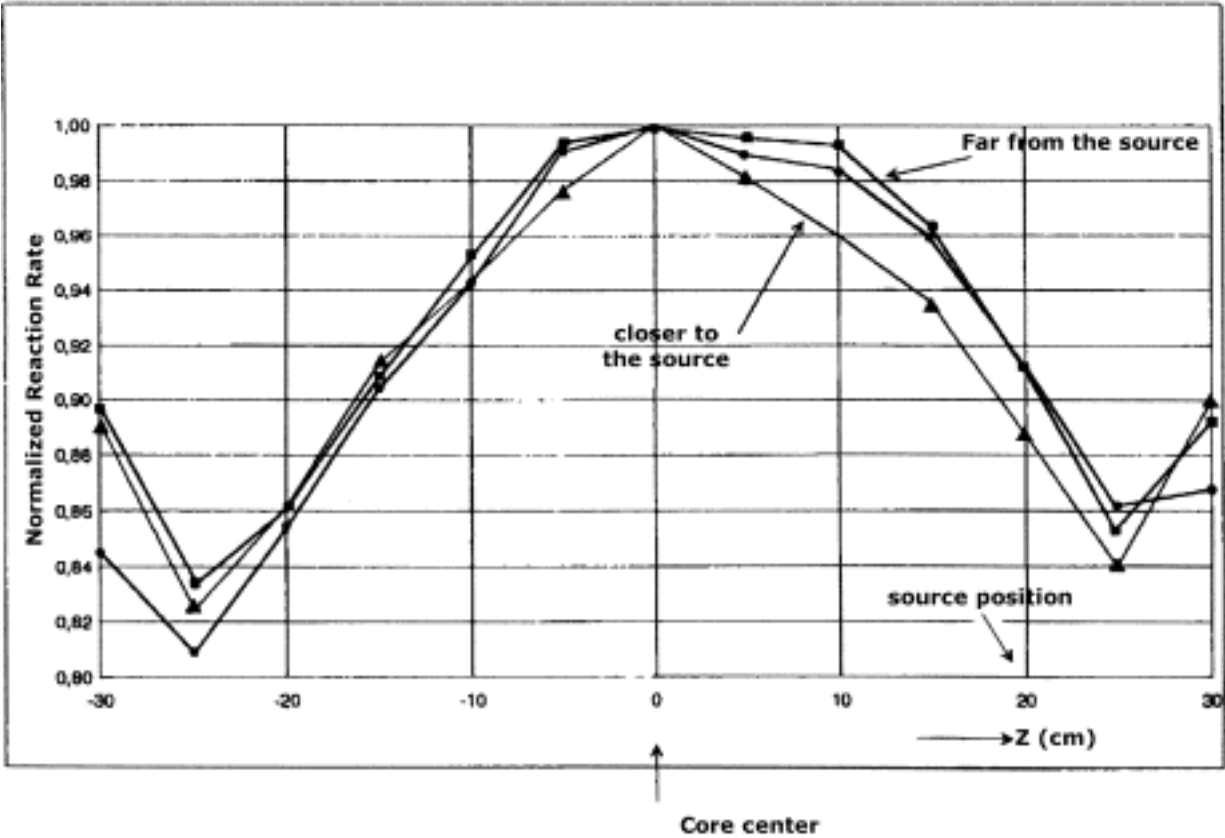
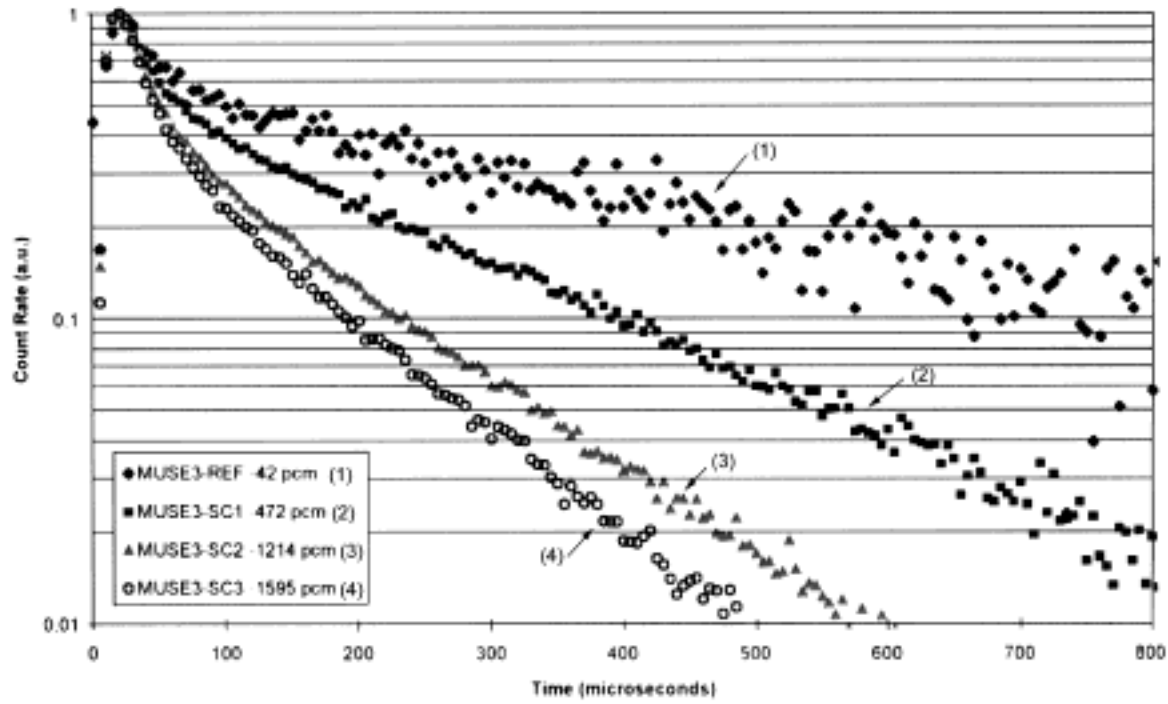**Fig. 9** Axial ^{238}U fission rate in MUSE-1 at three radial positions (Salvatores, *et al.*[11])

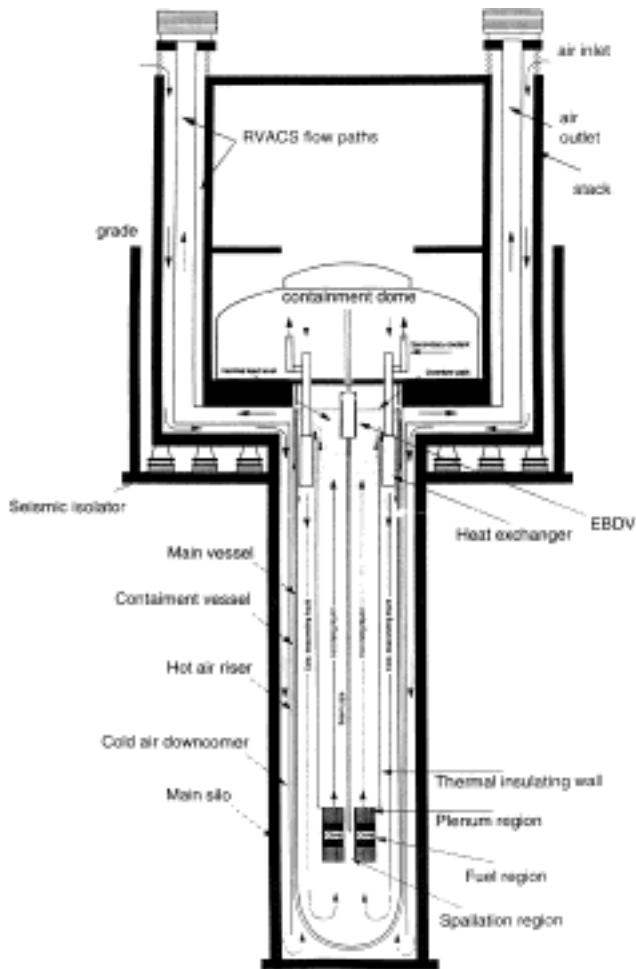**Fig. 10** Dynamic measurements MUSE3-REF, MUSE3-SC1, MUSE3-SC2, MUSE3-SC3

Fig. 11 C. Rubbia Energy Amplifier

Fig. 12 Sketch of ADS, liquid metal cooled (not to scale) Representative of the European XADS proposals (Ansaldo).

Fig. 13 Sketch of ADS, gas cooled (not to scale) Representative of the European XADS proposals (Framatome).

protons (typically E_p=0.6–1.5 GeV).

High energy neutron penetration experimental studies performed in Japan confirm the very large thickness of material (like concrete or stainless steel) needed in order to reduce to acceptable level the doses around the structures.

The beam entrance configuration is also a matter of concern. In fact, a simple vertical entrance of the beam can imply a very complicated system for the fuel loading-unloading system and can also be not optimal with respect to the need to guarantee the beam tube void from the intrusion of backscattered neutrons.

These are just a few examples of technological problems that may have an impact on the coupling of the different components of an ADS, and which might need substantial efforts in order to develop a robust ADS design.

VI. Conclusions

In previous sections we have commented on some issues relevant to subcritical reactors driven by an external neutron source. We think the at present the core physics of these systems is generally well understood. However, some specific features have never been fully experimentally demonstrated.

Several steps have been undertaken in that direction and planned experiments (like the MUSE experiments), or po-
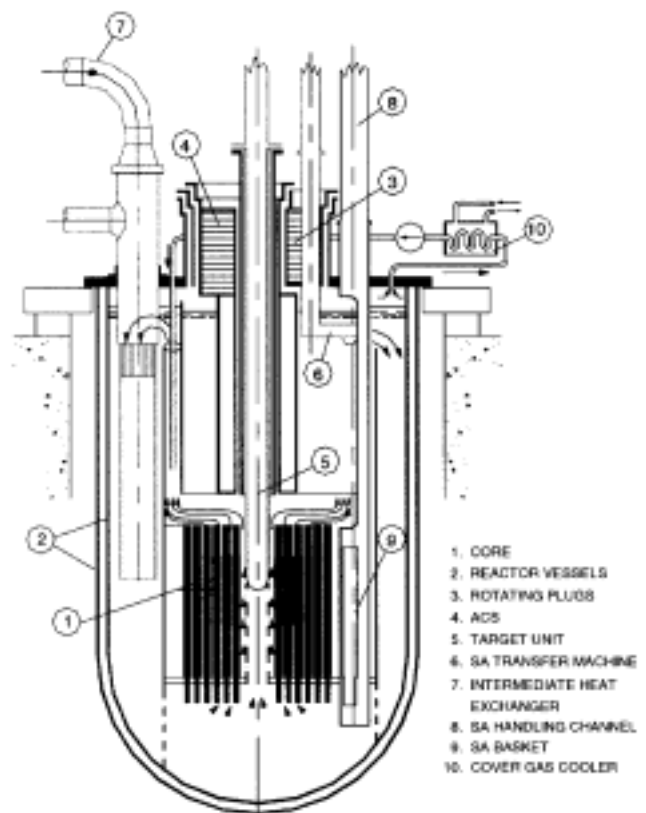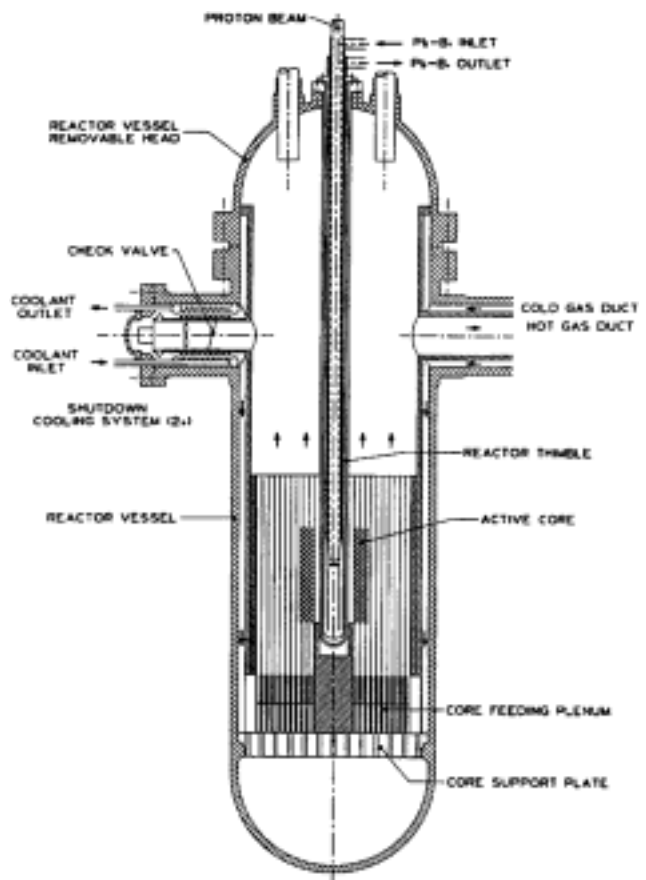
tential experiments being investigated in terms of feasibility (like the "TRIGA" experiments mentioned above), should give most of the demonstrations needed in order to proceed to a sound design of an experimental accelerator driven systems (ADS), as proposed, *e.g.*, in the European Roadmap towards and ADS demonstration.[1]

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