UE STEOP: Introduction to Mathematics in Data Science
Problem Set 4

Problem 1. Let x and y be nonnegative numbers. Prove that if $x \leq y$, then $\sqrt{x} \leq \sqrt{y}$.

Problem 2. For $n \in \mathbb{N}$ prove that $4 \mid (1 + (-1)^n(2n-1))$.

Problem 3. Prove that $\log_2 3$ is irrational.

Problem 4. Five children found nine mushrooms. Prove that at least two of them found an equal number of mushrooms.

Problem 5. Calculate (a) $0.\overline{12} + 0.\overline{122}$; (b) $0.\overline{3} \cdot 0.\overline{4}$

Problem 6. Find digits a and b such that $\sqrt{0.\overline{a}} = 0.\overline{b}$.

Problem 7. Find a mistake in the proof of the last statement from today's lecture (Oct 13) by induction. (*Hint*: To find it, simulate the proof by considering every $i = 1, 2, 3 \ldots$ manually.)

Problem 8. Find (a) $\gcd(10223, 33341)$; (b) $\gcd(2n+1, 3n+2)$.

Problem 9. Convert $(110100101011011)_2$ to decimal representation. Without converting, can you know for sure whether this number is greater than 2^{15}?