
UE STEOP: Introduction to Mathematics in Data Science

Problem Set 3

Problem 1. Prove that $3 \mid n^3 - n$ for all $n \in \mathbb{N}$.

Problem 2. Prove that $2^n < n!$ for all $n \geq 4$.


Problem 3. Prove that $\sin nx \leq n \sin x$ for all $x \in \mathbb{R}$ and $n \in \mathbb{N}$.

Problem 4. (Bernoulli inequality.) Prove that $(1 + x)^n \geq 1 + nx$ for all real $x \geq -1$ and all $n \in \mathbb{N}$.

Problem 5. Calculate $\sum_{k=1}^n \frac{1}{k(k+1)}$.

Problem 6. Let S be a set containing n elements. Use induction to prove that $|\mathcal{P}(S)| = 2^n$.

Problem 7. The bank has an unlimited supply of 3 and 5 euros bills. Prove that it can dispense any number of euros greater than 8 (without change).

Problem 8. There is a 16×16 chess board with one square removed. Prove that it can be tiled using triminoes  (you can rotate them). We have already proved it in class, and now your goal is to express the proof in words.

Problem 9. Prove that there exists an $n \in \mathbb{N}$ for which $11 \mid (2^n - 1)$.