UE STEOP: Introduction to Mathematics in Data Science
Problem Set 8

Problem 1. Is this a function from $\mathbb{R}$ to $\mathbb{R}$?

1. $f = \{(x^2, x) \colon x \in \mathbb{R}\}$.
2. $f = \{(x^3, x) \colon x \in \mathbb{R}\}$.

Problem 2. A function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is defined as $f(m,n) = 3n - 4m$. Verify whether this function is injective and whether it is surjective.

Problem 3. Prove that the function $f : \mathbb{R} \setminus \{2\} \to \mathbb{R} \setminus \{5\}$ defined by $f(x) = \frac{5x+1}{x-2}$ is bijective.

Problem 4. Consider all functions from $\{A, B, C, D, E, F, G\}$ to $\{1, 2, 3, 4, 5, 6, 7\}$. How many different functions are there? How many of these functions are injective? How many are surjective? How many are bijective?

Problem 5. Prove that $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ defined as $f(m,n) = 2^{m-1}(2n-1)$ is bijective.

Problem 6. (**Julia.**) Given a list of numbers $1, \ldots, n$, we want to find all possible permutations such that none of the numbers occupies its original place. Let's call them *interesting*. For example, for $n = 3$, we have two interesting permutations: 312 and 231.

1. Find the number of interesting permutations for $n = 4, 5, 6, 7, 8$.
2. Study how the ratio of the total number of permutations to the number of interesting permutations behaves.

Problem 7. (**Julia.**) How many numbers between 1111111 and 9999999 are divisible by 3 or 5 but not 11?

Problem 8. (**Julia.**) Let us call a natural number *charismatic* if the sum of its digits is a perfect square. Find the sum of all charismatic numbers between 111111 and 999999.