UE STEOP: Introduction to Mathematics in Data Science
Problem Set 1

Problem 1. Write each of the following sets by listing their elements between braces

1. $\{x \in \mathbb{Z} \colon |2x| < 5\}$
2. $\{x \in \mathbb{R} \colon x^3 + 5x^2 = -6x\}$
3. $\{x \in \mathbb{R} \colon \sin \pi x = 0\}$
4. $\{5x \colon x \in \mathbb{Z}, |2x| \leq 8\}$

Problem 2. Write each of the following sets in set-builder notation (the opposite to the previous problem)

1. $\{2, 4, 8, 16, 32, 64, \dots\}$
2. $\{-5, -4, -3, -2, -1, , 0, 1\}$
3. $\left\{\dots, \frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27\right\}$
4. $\{3, 6, 11, 18, 27, 38, \dots\}$

Problem 3. Write out the indicated sets by listing their elements between braces.

1. $\{x \in \mathbb{R} \colon x^2 = 2\} \times \{a, b, c\}$
2. $\{0, 1\}^4$
3. $\{\pi, 1, 0\} \times \{-\pi, -1, -0\}$
4. $\{\{\mathbb{R}\}\}$

Problem 4. List all the subsets of the following sets.

1. $\{1, 2, 3, 4\}$
2. $\{\mathbb{R}, \mathbb{Q}, \mathbb{N}\}$
3. $\{\{0, 1\}, \{0, 1, \{2\}\}, \{0\}\}$

Problem 5. Write the following sets by listing their elements between braces.

1. $\mathscr{P}(\{\{\varnothing\}, 4\})$
2. $\mathscr{P}(\mathscr{P}(\{3\}))$

Problem 6. Check in Julia whether the following is true. Afterwards, make sure you understand why the statements are true or false.

1. $\{1\} \in \{1, \{1\}\}$
2. $\{1\} \subset \{1, \{1\}\}$

3. $\varnothing \notin \mathbb{N}$

4. $\varnothing \subset \mathbb{N}$

Problem 7. Write a function in Julia that takes two sets A and B and returns the Cartesian product $A \times B$ as a set of tuples.

Problem 8. Let $A = \{b, c, d\}$ and $B = \{a, b\}$. Find a) $\mathscr{P}(A) \cap \mathscr{P}(B)$; b) $(A \times B) \setminus (B \times B)$.

Problem 9. Draw an Euler-Venn diagram for a) $(A \setminus B) \cap C$ and b) $A \setminus (B \cap C)$.

Problem 10. Sketch the set $A = [1, 3] \times [1, 2]$ on the plane $\mathbb{R}^2$. On separate drawing, shade in the sets $A^{\complement}$ and $A^{\complement} \cap ([0, 2] \times [0, 3])$. The set $A^{\complement}$ denotes the complement of A (in this context, the universal set is $\mathbb{R}^2$).

Problem 11. Simplify

1. $\bigcup_{i \in \mathbb{N}} [i, i+1] =$

2. $\bigcap_{i \in \mathbb{N}} [i, i+1] =$

3. $\bigcap_{i \in \mathbb{N}} \mathbb{R} \times [i, i+1] =$

Problem 12*.[1] Write in Julia the function that outputs the power set of a given set (without using external libraries).

[1] The star denotes a more difficult problem than usual. It's alright if you cannot solve it (at least for now)