UE STEOP: Introduction to Mathematics in Data Science
Problem Set 6

Problem 1. In a school, every boy knows 32 girls, and every girl knows 29 boys. Who are there more of in the school: boys or girls, and by how many times?

Problem 2. How many 10-digit numbers are there in which there are at least two identical digits?

Problem 3. How many permutations of the letters ABCDEFGH contain the string 'ABC'?

Problem 4. Are there more seven-digit numbers that contain the digit 1, or more that don't?

Problem 5. Give a proof of the Binomial Theorem by induction on n.

Problem 6. Find the coefficient of x^4 in $(2x^3 - \frac{1}{x^2})^8$.

Problem 7. Let p be prime and $0 \leq k \leq p$. Prove that $p \mid \binom{p}{k}$.

Problem 8. Prove if n and m are integers with $1 \leq m \leq n$, then $m\binom{n}{m} = n\binom{n-1}{m-1}$. (Try to provide a combinatorial proof as well as an algebraic one.)

Problem 9. How many permutations of the 26 letters of the English alphabet do not contain any of the strings 'math', 'love', and 'quiz'?

Problem 10. How many 6-digit numbers have strictly decreasing digits from left to right? (For example, as 987620).

Problem 11. Let $m, n \in \mathbb{N}$. How many solutions does the equation

$$x_1 + x_2 + \cdots + x_m = n$$

have if 1) all x_i are nonnegative integer numbers; 2) all x_i are natural numbers? (*Hint.* If you're not confident in your solution, try verifying particular cases for small values of m and n.)