
UE STEOP: Introduction to Mathematics in Data Science

Problem Set 5

Problem 1. Find $\gcd(\underbrace{11\dots 1}_m, \underbrace{11\dots 1}_n)$.

Problem 2. Find all $n \in \mathbb{N}$ for which the fraction $\frac{n^2-n+1}{n^2+1}$ is irreducible.

Problem 3. Prove that if n is an odd positive integer, then $n^2 \equiv 1 \pmod{8}$.

Problem 4. Find the remainder from the division of 2^{2025} by 11.

Problem 5. Solve the equation $x^2 - 16y = 3$, where $x, y \in \mathbb{Z}$.

Problem 6. Prove that a number whose decimal representation contains three 1 and several 0 cannot be a perfect square.

Problem 7. Find all $p \in \mathbb{N}$ such that $p, p + 10$, and $p + 14$ are prime numbers.

Problem 8. It is October today. What are the chances that, in 250 months' time, people will be swimming in the Danube?

Problem 9. (Julia.) Prove that there exists an $n \in \mathbb{N}$ for which $11 \mid (2^n - 1)$.

Problem 10. (Julia.) Prove that there exists a power of 3 that ends with the digits 001. (In other words, $3^n = \dots 001$ for some $n \in \mathbb{N}$.)

Problem 11. (Julia.) A natural number is called perfect if it is equal to the sum of all its divisors. For example, 28 is perfect, because $28 = 1 + 2 + 4 + 7 + 14$. Find all perfect numbers under 10000.