MARIN

European Commission

UNIVERSITY OF LEEDS

HORIZON 2020

Eagre/Aegir: High-Seas Wave-Impact Modelling

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Yang Lu, Wajiha Rehman, Onno Bokhove
mmyl@leeds.ac.uk, mmwr@leeds.ac.uk, o.bokhove@leeds.ac.uk

MARIN supervisors: Dr Ir Tim Bunnik (with, e.g., Ir Sanne van Essen and Dr Bulent Duz)
PI and Leeds advisor: Prof Onno Bokhove

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Chapter 1

Introduction

The Marie-Curie European Industry Doctorate (EID) project was established to support collaboration between academic and industrial centres by educating new researchers in specific disciplines of vital importance to strategic sectors of the economy. The “Eagre/Aegir: High-Seas Wave-Impact Modelling” project, as part of EID, is a collaboration between the University of Leeds and the Maritime Research Institute Netherlands (MARIN). Its remit is to develop and implement novel mathematical methods to problems currently faced by the maritime industry. The research project consists of two related subprojects:

- WP1: “Extreme Waves” with Early-Stage-Researcher (George) Yang Lu, supervised by Prof. Onno Bokhove from the University of Leeds;
- WP2: “Wave Turbine Impact” with Early-Stage-Researcher (ESR) Wajiha Rehman, supervised by Prof. Onno Bokhove from the University of Leeds.

Dr. Tim Bunnik with Sanne van Essen and Bulent Duz are the supervisors from MARIN. Onno Bokhove is the principal investigator. This report is the mid-term report for the mid-term meeting in March 2021.

The two ESRs, Yang Lu and Wajiha Rehman started in November 2020 (excluding two weeks of self-isolation due to the Covid-19 pandemic). Their progress up to date is described in this report.

The first 18 months of the project the ESRs will be based at the University of Leeds, where the predominant activity is the development of mathematical and numerical models and their implementation. The next 18 months will be based at MARIN where, in addition to further model development, validation of simulation results against experimental data in the maritime engineering environment at MARIN will be undertaken.

1.1 WP1: Extreme Waves project

The Marie Curie actions offer the opportunity to combine academic research with industrial work, and support multi-field and international research. Within this framework, a collaboration between the University of Leeds and the Maritime Research Institute Netherlands (MARIN) has been set up to model and test variational water waves through an European Industry Doctorate project. MARIN has wave basins available for experiments, set with wave makers on two sides, as well as bottom topography and a beach. The key idea is to derive a three-dimensional model of the water waves in such a domain, including coupling between deep and shallow water, and compare numerical modelling results to experiments.

In summary, work package WP1 “Extreme Waves”, offers the following innovations:

- To create a complete numerical finite-element wavetank for high-amplitude potential-flow water waves. The state-of- the-art concerns direct numerical solvers of potential-flow dynamics in 2D and 3D simulations, based on compatible discretizations, with a piston wave-maker [16, 19] or wave-breaking parameterizations [7] or wave-beach interactions in 2D [20]. Building upon these results, the innovation consists of combining all these elements in a 3D *Firedrake* solver[1]. Furthermore, exploration of coordinate transformations and dynamic mesh motion, first in 2D, will be original

[1]Firedrake is a finite-element modelling environment, see `https://www.firedrakeproject.org/`

and will greatly enhance the performance robustness and scope of the methodology available for investigating water-wave problems born of a variety of applications.to be investigated.

- To develop and deliver a series of (novel) benchmark cases. Benchmarking using the two- and three three-soliton splashes will be original. The innovation in exploring irregular waves, random waves and short-crested waves lies in its use for testing scale models and the robustness of the potential potential-flow solvers. This development step can be done with existing solvers, as well as improved ones; it is a crucial step to facilitate widespread use of our methodology/ tools.
- To derive the mathematical and variational/Hamiltonian formulation of wave-current interactions. Extending classical work on geometric structures for water-wave equations [25, 26, 9, 18], to wave-current interactions will be a valid doctoral- training step and introduction to the topic for the ESR, yet it will also contain innovative new elements, i.e. the extension to the wave-current flows.
- To deliver an open-access, fast and easy-to-use water-wave simulation and scientific-computation tool. This delivery will focus on innovative computer-science elements and make the tool more robust and operational, e.g. in Leeds and at MARIN Academy BV. Testing and improving the tool's robustness is an important and practical innovation, because its subsequent usage feeds directly into the design and testing of maritime-engineering hardware.
- To validate 3D numerical potential-flow water-wave-tank against existing and/or new measurements at MARIN Academy BV will be a challenging and novel endeavor. New measurements will tentatively include measurements those that can be used to assess the damping/dynamics of the waves at the a beach.
- Similarly, to validate of 2D numerical potential-flow water-wave-current tank against existing and/or new measurements at MARIN will provide novel insights into testing maritime structures in waves and currents.
- To explore compatible/variational numerical formulations of wave-current interactions. These space-time finite-element discretizations based on the geometric wave-current models will be entirely novel (optional).

1.2 WP2 Wave Turbine Impact project

The search for alternative and effective energy sources that support balanced growth has led to an increased focus on offshore wind energy. Both visibility issues and wind supply play major roles in the development of this particular branch of wind energy. There are two main directions of active research in this field, namely offshore floating platforms with wind turbines, and fixed-bottom monopile wind farms in shallow water: a review of this research is given in [1].

In summary, work package WP2 "Extreme Waves", offers the following:

- Theory of potential-flow water waves coupled to a nonlinear hyperelastic beam. Extending Salwa's preliminary results [32, 31, 22] with a full and concise derivation of the nonlinear equations of motion, as well as incorporating our new asymptotic two-way coupling based on one monolithic variational principle, constitutes be a novel and innovative step forwards, one that allows our approach to solve the problem in not only a mathematically consistent and justified manner, but also one in which the new asymptotics offer a means of incorporating implicitly prescribed boundary conditions to a controllable (and high) degree of accuracy, thus enhancing computational efficiency and speed.
- While piston wave-makers have now been successfully included in theoretical and numerical variational principles for water waves, the mathematical and computational counterpart inclusion of a (more realistic and relevant to deep-water maritime engineering) waveflap into the mathematical and computational counterparts is a new challenge. MARIN's most prominent wave basins have waveflaps on two basin sides to create focussed waves. Both the coordinate transforms and the mesh motion integrated in the VP will be explored and developed (cf. [2]).

- Development of a compatible finite-element discretization of the coupled wave-structure system using a monolithic VP is an innovation with immediate application for testing wave impact on wind-turbine masts. The iterative asymptotic approach will be integrated within the numerical approach to obtain faster numerical computations. Integrating mesh motion within the overall VP using distinctive equations for mesh motion will be completely entirely novel.

- The inclusion of wave breaking parameterizations into our numerical wave-structure modeling will complete our numerical tool for elaborate and novel testing against experimental measurements of wave impact on wind-turbine masts.

- Open-access, fast and easy-to-use water-wave-structure simulation and scientific-computation tool. Establishing and completing the numerical tool with benchmark tests will complete our main innovative approach on wave impact modeling using compatible numerical techniques.

- The validation of our new monolithic geometric water-wave-beam model is novel since such a model has to date never been validated.

Both ESRs have successfully installed *Firedrake* on their laptops and they have commenced to work through the tutorials [30].

1.3 Report overview

The report furthermore consists of a presentation of the following topics:

- Scientific Results and Research Training, divided into two parts, one per workpackage;

- Networking and Transfer of Knowledge;

- Outreach Activities; and,

- Management.

Acknowledgements: Parts of this report will also be used for the PhD transfer reports of the ESRs George Yang Lu and Wajiha Rehman. The PhD transfer is the formal evaluation of the PhD research quality in the School of Mathematics at the University of Leeds. These transfers are planned for April/May 2021. We thank the entire "Eagre" team for their input.

Chapter 2

Scientific Results and Research Training

2.1 WP1- ExtremeWaves: Extreme water-waves computational modelling using advanced geometric methods with wave generation, breaking, and currents (by (George) Yang Lu and Onno Bokhove)

The overall goal of the extreme-wave modelling workpackage is to create and deliver computational/-mathematical modelling tools for solving problems in maritime engineering, based on advanced mathematical/numerical analysis and efficient implementation and testing in a general finite-element simulation environment provided by Firedrake.

2.1.1 Training: Taught Courses, i.e. Deliverable D1.14

On deliverable D1.14, the MSc courses 1 and 2, the following updates:

- D1.14 MSc course 1: numerical assignment N3 is still due to date; N3 is worth circa 10% of the first MSc course and is relevant to the ESRs' research; one can simply give zero marks for N3 and formally complete the first MSc course but that serves no purpose since the work still has to be completed and the delayed late start is not the ESRs fault.
- D1.14 MSc course 2: the first assignment has been submitted to date and the next two assignments are due late March and late April and then the assignments get marked. This submission schedule is later than last year due to (pandemic) changes this academic 2020/2021 year. Also note that the ESRs started two to three months late both due to delays at the University of Leeds and the pandemic combined.
- Both MSc courses are taught as part of the Centre for Doctoral Training (CDT) in Fluid Dynamics at the University of Leeds and have been taught to circa 15 participants, PhD students in their MSc phase, including the two ESRs. Due to the delayed started, caused a.o. by the pandemic, the numerical component of the first MSc course has been taught twice by Prof Bokhove (at the expense of his other work), for the ESRs benefit.

Yang Lu works as an early-stage researcher to participate in the project "ExtremeWaves", which has as objective to create a numerical wavetank concerning modelling of extreme or rogue waves in wave basins [29, 20, 13, 38, 11]. This research direction requires development of new methods to analyse extreme or rogue waves in wave basins. According to the requirements, the following two taught courses have been taken in order to acquire a solid fundamental acknowledge for the project during the first year:

- **MATH5453M Foundations of Fluid Dynamics** (Semester 1, 30 credits)

  This module consists of two parts, theoretical and numerical. It provides fundamental theoretical concepts of fluid dynamics and their application on solving engineering and scientific problems.

In addition, three principle numerical methods, namely Finite Difference, Finite Volume and Finite Element Methods, have been introduced and used to numerically solve practical problems. I have finished the assignments except the third numerical assignment N3, which requires teamwork between the two ESRs and is in progress, and I have successfully undertaken the final examination.

There are respectively three numerical assignments each corresponding to one of the three numerical methods for the numerical part of the course. All of them require us to use Python programming language for the coding. So after finishing N1 and N2, we also have acquired basic Python-programming skills. Furthermore, these assignments are directly relevant to my research project, which focuses on better modelling of water waves. For the assignment N2, we are asked to predict surf height at beaches based on linearised shallow-water system of equations and they are numerically solved by Finite Volume Method. In this assignment, our numerical model was first validated by an exact standing-wave solution. Then it was extended to simulate the free surface profile and used to investigate the effect of an in depth decreasing seabed profile on the wave amplitude at beaches. The numerical and the corresponding asymptotic results are shown in Fig.2.1. It can be seen that as the seabed becomes more shallow (and steeper), the wave length decreases while its amplitude increases along the propagation direction, which is a reflection of mass and energy conservation. However, for a flat bottom in deep water, the wave length and the wave speed are all constant, and it takes less time for the wave to propagate from the open left to the right boundaries. In the ongoing assignment N3, we are going to numerically solve linear potential flow shallow water equations by the Ritz-Galerkin Finite Element Method. The finite element discretization is derived through two related approaches and the numerical implementation will be verified against an exact solution in a rectangular domain with solid walls.

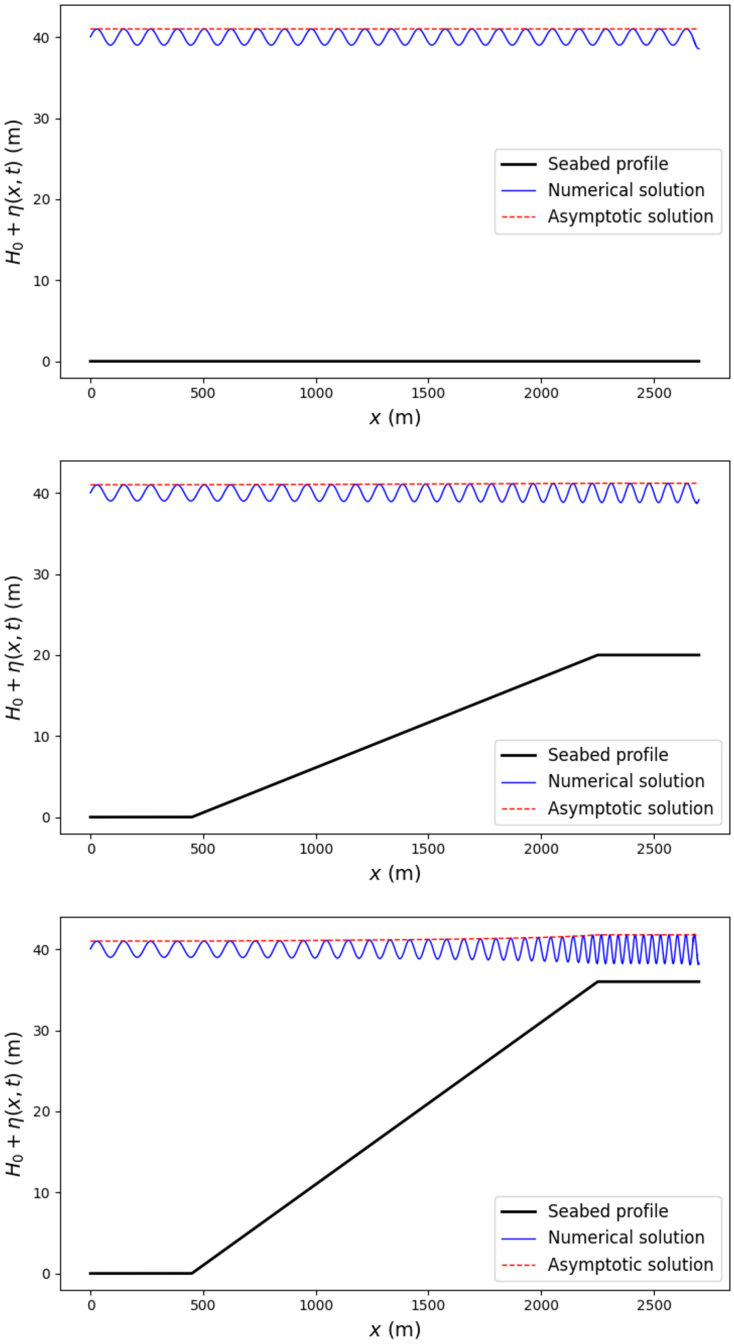

Figure 2.1: Effects of bottom topography on wave amplitude at beaches. It can be seen that as the seabed becomes more shallow (and steeper), the wave length decreases while its amplitude increases along the propagation direction, which is a reflection of mass and energy conservation.

- **COMP5454M Fluid-Structure Interactions** (Semester 2, 15 credits)

  This module is concerned with the coupled interaction between the flow of the fluid and the displacement of the solid in fluid flow problems where the movement and the deformation of the solid cannot be neglected. The module is comprised of theories of linear and nonlinear elastic and viscoelastic solids, techniques for coupling fluid flow and structure equations, as well as the development and application of appropriate numerical methods. All the lectures and tutorials have been delivered and we are doing the course assignments, which consist of derivation of equations, software-aided problems and fluid-structure interactions (FSI) literature review. The assignments relating to aeroelasticity, and water-wave and wind-turbine interactions have been finished. The topic chosen for the literature review concerns how different types of fluid models can used when they are coupled to a solid body.

2.1.2 Training: Short-Term Training Sessions

Short-term training programmes concerning paper writing, graphing, software and coding etc. are intended to attend. The workshops in which I have participated are listed below:

- PGR Core Language Skills: Becoming a Doctoral Researcher workshop (17 February 2021);
- PGR Core Language Skills: Reading Critically to Write Critically Workshop (24 February 2021);
- PGR Core Language Skills: What is proofreading and how do I go about doing it? (3 March 2021);
- PGR Core Language Skills: Writing Purposefully Workshop (9 March 2021);
- PGR Core Language Skills: Reading to Improve Writing Workshop (10 March 2021);
- PGR Core Language Skills: Grammar A Workshop (23 March 2021); and,
- COMP5992M: Outreach Training (by Dr Katie Chicot, CEO of MathsWorldUK).

2.1.3 Scientific Results: Relevance to planned research

During the training courses, I have done some basic work that is intrinsically related to my planned research project "ExtremeWaves". It can be summarised into three parts, as follows:

- The first part concerns a comparison between Godunov's and alternating fluxes in a Finite Volume Method applied to the linearised shallow-water equations. This is related to my research project, because the first objective is to create a complete numerical finite-element wave tank for high-amplitude potential flow water waves, while the linearised shallow-water equations describe the limit of small-amplitude shallow-water wave dynamics. Therefore, it serves as a foundation for understanding wave dynamics towards the final goal. In addition, the results obtained from the numerical model were compared against an exact standing-wave solution that we derived first, which means the one-dimensional numerical tank has been validated against a benchmark case. These procedures not only help me to learn how to create an numerical model for water waves properly, but also improve my programming skills. The results and analysis are shown in detail in the following section.
- The second part focuses on the derivation of the equations of motion for water-wave dynamics using a variational principle. According to the third objective of my research project, we need to derive the mathematical and variational formulation of wave-current interactions, and the starting point lies in the classical work on variational principles for water-wave dynamics by Luke [25], which I learned from FSI course. In the water-wave and wind-turbine interactions part of the FSI course, the nonlinear potential flow water-wave equations are derived by using variational principle and then they are linearised around a hydro-static state of rest. In this report, the derivation of linearised equations of motion for water-wave dynamics is shown in detail.
- The third part is the ongoing literature review assignment for the FSI course. Since the project "ExtremeWaves" focuses on better modelling of nonlinear water waves, I chose the topic "fluid models used in fluid-solid interaction problems" as literature review assignment in the course. We have considered a variational principle governing surface gravity waves [25, 26], and its advantage lies in its simplicity, accuracy and fast speed in numerical calculations [15, 16, 19, 14, 17]. However, it involves an irrational, incompressible, potential-flow fluid approximation that put limitations on the modelling of wave-breaking. In reality, impact events with steep waves usually involve wave breaking, which cannot be simulated with the incompressible potential flow due to lack of rotational degrees of freedom. Therefore, several attempts has been made to extend the model so that wave-breaking can be simulated. For example, some improved models that have the capability to simulate wave breaking are developed by using an air-water mixture model [3, 28, 33, 32, 12]. I am writing a short literature review on improved water-wave models and at the same time, thus learning more relevant background and existing research for this planned project.

Comparison between Godunov's and alternating fluxes in a Finite Volume Method for linearised shallow-water equations

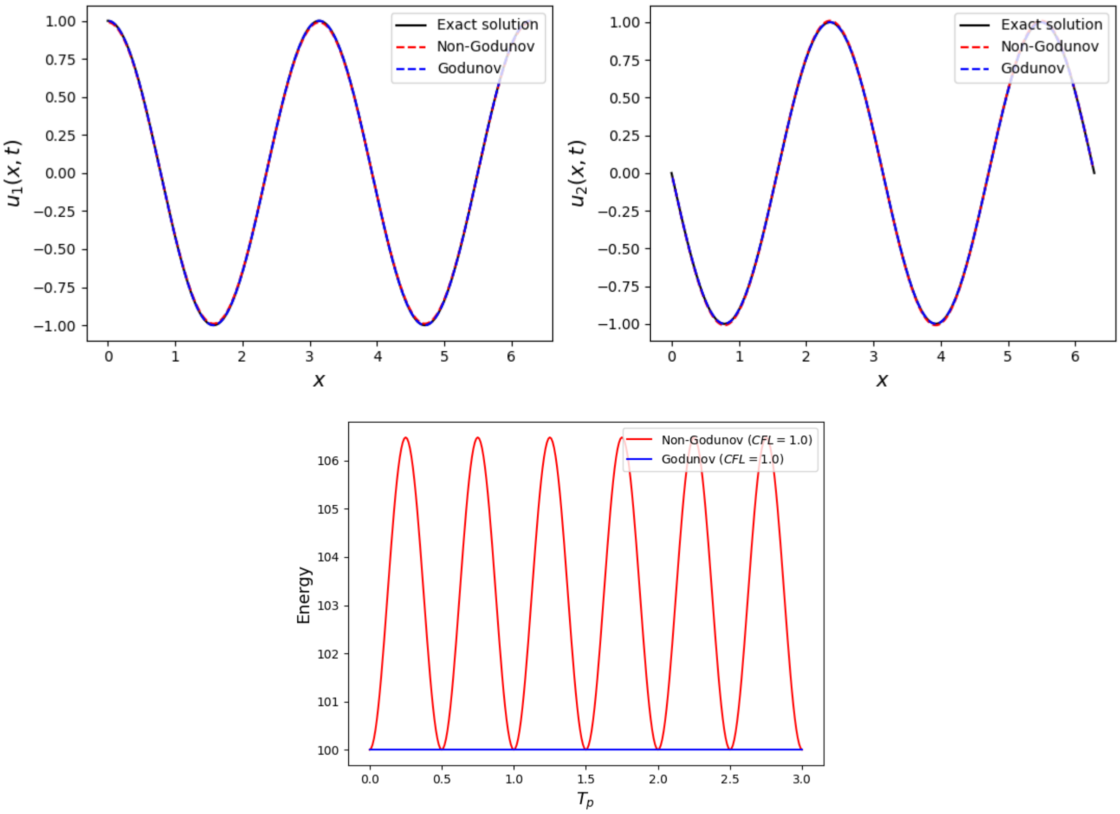

Figure 2.2: Comparison between two flux schemes when CFL=1.0. It can be seen that numerical results obtained from both flux schemes agree well with the exact standing-wave solution. Also, the energy for the system using Godunov's flux scheme could remain at its initial value, while for the system using the alternative flux scheme and geometric time integrator [23], its energy oscillates with the amplitude being $\sim 6.5\%$. Top figures: snapshots of profiles versus space x. Bottom figure: energy as function of time in terms of wave period T_p.

In the second numerical assignment for the Foundations of Fluid Dynamics course, we focused on numerically solving the linearised shallow-water systems of equations, which read

$$\frac{\partial \eta}{\partial t} + \frac{\partial (Hu)}{\partial x} = 0 \tag{2.1a}$$

$$\frac{\partial u}{\partial t} + \frac{\partial (g\eta)}{\partial x} = 0, \tag{2.1b}$$

where $u = u(x,t)$ is the velocity and $\eta = \eta(x,t)$ is the free-surface deviation, $H(x)$ is the rest depth and g represents the acceleration of gravity. After the numerical and computational modelling using Godunov's method, we compare the numerical results for standing wave solutions between Godunov's flux and an alternative flux to enhance our understanding, which space-discrete energy is defined as follows

$$F_{\eta,j+1/2} = \theta H(x_{j+1/2})U_{j+1}^n + (1-\theta)H(x_{j+1/2})U_j^n \tag{2.2a}$$

$$F_{u,j+1/2} = (1-\theta)g\bar{\eta}_{j+1}^{n+1} + \theta g\bar{\eta}_j^{n+1} \tag{2.2b}$$

with $\theta \in [0,1]$ and while assuming that $H(x)$ is continuous. The flux is alternating because the position of the weights θ and $(1-\theta)$ is reversed in the respective fluxes. In addition, the discrete energy $E(t)$ for the system was monitored for each time step, which takes the form

$$E(t) = \frac{1}{2}\sum_{j=1}^{J} H_0 U_j^2 + g\bar{\eta}_j^2, \tag{2.3}$$

where J is the number of finite volumes in the domain.

Next we compare the results after three time period $3T_p$ with two different time steps using a $CFL = 1$ and $CFL = 0.1$ for both flux schemes. In Fig. 2.2, where $CFL = 1$, it can be seen that numerical results obtained by both flux schemes agree well with the exact standing wave solution. In addition, the energy for the system using Godunov's flux scheme could remains at its initial value, while for the system using the alternative flux scheme (2.2), its energy oscillates with its minimum being the initial energy E_0 and its amplitude around $\sim 6.5\%$. However, when $CFL = 0.1$, which is shown in Fig. 2.3, the numerical results obtained from Godunov's method deviate from the exact solution while its counterpart agrees well with the exact solution. From the perspective of energy, the energy of the system using Godunov's flux scheme decreases with time, but the energy of the counterpart system is nearly conserved. It still oscillates with its minimum value being E_0 while its amplitude is now around 0.65, which is ten times smaller than that of the last case. Therefore, the order of accuracy for this alternative scheme should be first-order in time. It can be deduced that the energy is conserved for the system using the alternative flux scheme when the time step is infinitesimal.

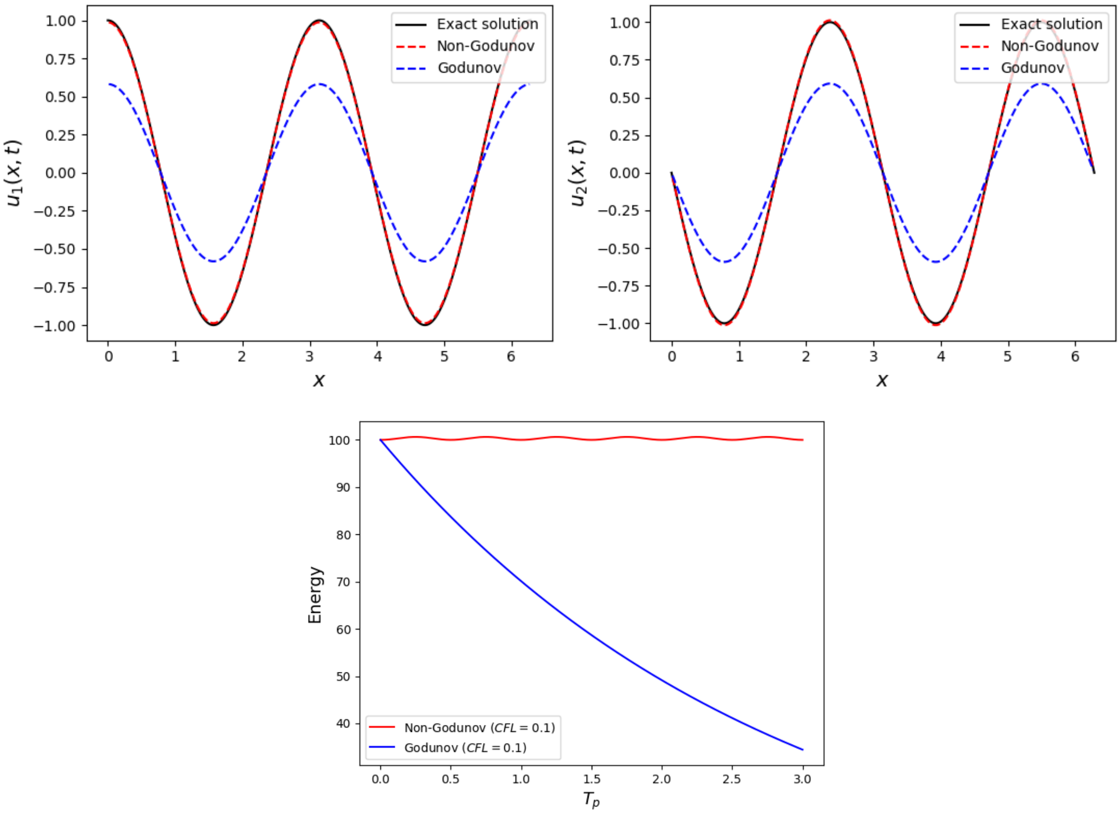

Figure 2.3: Comparison between two flux schemes when CFL=0.1. It can be seen that the numerical results obtained from Godunov's method deviate from the exact solution while its counterpart agrees well with the exact solution. Also, the energy of the system using Godunov's flux scheme decreases with time, but the energy of the counterpart system is nearly conserved with respect to time.

Derivation of equations of motion for water-wave dynamics using a variational principle

Consider the hydrodynamics of water as an incompressible fluid with a sharp air-water interface moving under the influence of the Earth's gravity. Gravity acts downward in the z-direction and the 3D velocity is approximated as $\mathbf{u} = \nabla\phi$ using a velocity potential $\phi = \phi(x, y, z, t)$, with horizontal coordinates x and y, vertical coordinate z and time t. The water-wave dynamics follows from Luke's variational principle

[37, 25, 35], i.e.,

$$0 = \delta \int_0^T \mathcal{L}[\phi, h] \,\mathrm{d}t \tag{2.4a}$$

$$= \lim_{\epsilon\to 0} \int_0^T \frac{\mathcal{L}[\phi + \epsilon\delta\phi, h + \epsilon\delta h] - \mathcal{L}[\phi, h]}{\epsilon} \,\mathrm{d}t \tag{2.4b}$$

with Lagrangian functional

$$\mathcal{L}[\phi, h] = \iint_{\Omega_H} \int_0^h \partial_t\phi + \frac{1}{2}|\nabla\phi|^2 + g(z - H_0) \,\mathrm{d}z \,\mathrm{d}x\mathrm{d}y \tag{2.4c}$$

with horizontal extent Ω_H of the domain and rest height H_0. Let $\eta(x, y, t)$ be the free-surface perturbation from the rest state, such that we have $z = H_0 + \eta(x, y, t) = h(x, y, t)$. Based on (2.4), we can show that the linearised equations of motion follow from the simplified variational principle

$$0 = \delta \int_0^T \left(\iint_{\Omega_H} \phi\partial_t\eta - \frac{1}{2}g\eta^2 \mathrm{d}x\mathrm{d}y - \iint_{\Omega_H} \int_0^{H_0} \frac{1}{2} |\nabla\phi|^2 \,\mathrm{d}z \,\mathrm{d}x \,\mathrm{d}y \right) \mathrm{d}t. \tag{2.5}$$

Considering the end-point conditions $\delta\eta|_{t=0} = \delta\eta|_{t=T} = 0$, the first two terms in (2.5) become

$$\begin{aligned}
&\delta \int_0^T \left(\iint_{\Omega_H} \phi\partial_t\eta - \frac{1}{2}g\eta^2 \mathrm{d}x\mathrm{d}y \right) \mathrm{d}t \\
&= \lim_{\epsilon\to 0} \frac{1}{\epsilon} \int_0^T \iint_{\Omega_H} \left[(\phi + \epsilon\delta\phi)\, \partial_t\,(\eta + \epsilon\delta\eta) - \frac{1}{2}g\,(\eta + \epsilon\delta\eta)^2 \right] - \left(\phi\partial_t\eta - \frac{1}{2}g\eta^2 \right) \mathrm{d}x\mathrm{d}y\mathrm{d}t \\
&= \int_0^T \iint_{\Omega_H} \phi\partial_t(\delta\eta) + \delta\phi\partial_t\eta - g\eta\delta\eta \,\mathrm{d}x\mathrm{d}y\mathrm{d}t \\
&= \iint_{\Omega_H} (\phi\delta\eta)|_{t=0}^{t=T} \,\mathrm{d}x\mathrm{d}y + \int_0^T \iint_{\Omega_H} \delta\phi\partial_t\eta - \delta\eta\partial_t\phi - g\eta\delta\eta \,\mathrm{d}x\mathrm{d}y\mathrm{d}t \\
&= \int_0^T \iint_{\Omega_H} \partial_t\eta\delta\phi - (\partial_t\phi + g\eta)\,\delta\eta \,\mathrm{d}x\mathrm{d}y\mathrm{d}t.
\end{aligned} \tag{2.6}$$

After using Gauss's theorem and considering that the outward normal at the upper surface $z = H_0$ is $\hat{\mathbf{n}} = (0, 0, 1)^T$ and solid wall boundary conditions $\hat{\mathbf{n}} \cdot \nabla\phi = 0$ at domain boundaries $\partial\Omega_w$, the last term in (2.5) becomes

$$\begin{aligned}
&\delta \int_0^T \iint_{\Omega_H} \int_0^{H_0} -\frac{1}{2} |\nabla\phi|^2 \,\mathrm{d}z\mathrm{d}x\mathrm{d}y\mathrm{d}t \\
&= \lim_{\epsilon\to 0} -\frac{1}{2\epsilon} \int_0^T \iint_{\Omega_H} \int_0^{H_0} |\nabla(\phi + \epsilon\delta\phi)|^2 - |\nabla\phi|^2 \,\mathrm{d}z\mathrm{d}x\mathrm{d}y\mathrm{d}t \\
&= \int_0^T \iint_{\Omega_H} \int_0^{H_0} -\nabla\phi \cdot \nabla(\delta\phi) \,\mathrm{d}z\mathrm{d}x\mathrm{d}y\mathrm{d}t \\
&= \int_0^T \iint_{\Omega_H} \int_0^{H_0} \nabla^2\phi\delta\phi - \nabla \cdot (\delta\phi\nabla\phi) \,\mathrm{d}z\mathrm{d}x\mathrm{d}y\mathrm{d}t \\
&= \int_0^T \iint_{\Omega_H} \int_0^{H_0} \nabla^2\phi\delta\phi \,\mathrm{d}z\mathrm{d}x\mathrm{d}y\mathrm{d}t - \int_0^T \iint_{\Omega_H} \partial_z\phi\delta\phi \,\mathrm{d}x\mathrm{d}y\mathrm{d}t.
\end{aligned} \tag{2.7}$$

Combining (2.6) and (2.7), the equation (2.5) becomes

$$0 = \int_0^T \iint_{\Omega_H} (\partial_t\eta - \partial_z\phi)\delta\phi - (\partial_t\phi + g\eta)\,\delta\eta \,\mathrm{d}x\mathrm{d}y\mathrm{d}t + \int_0^T \iint_{\Omega_H} \int_0^{H_0} \nabla^2\phi\delta\phi \,\mathrm{d}z\mathrm{d}x\mathrm{d}y\mathrm{d}t. \tag{2.8}$$

Finally, using the arbitrariness of variations $\delta\phi$ and $\delta\eta$, we can derive the following linearised equations of motion:

$$\delta\phi: \quad \nabla^2\phi = 0 \quad \text{on } \Omega_0 \tag{2.9a}$$

$$\delta\phi|_{z=H_0}: \quad \partial_t\eta - \partial_z\phi = 0 \quad \text{at } z = H_0 \tag{2.9b}$$

$$\delta\eta|_{z=H_0}: \quad \partial_t\phi = -g\eta \quad \text{at } z = H_0. \tag{2.9c}$$

2.2 WP2: WaveTurbineImpact: Water-wave impact on dynamic and flexible (wind-turbine) structures (by Wajiha Rehman and Onno Bokhove)

The objective of the project is to create a numerical wavetank concerning the wave-structure interactions, especially wave-impact, on a dynamic wind-turbine mast. The modelling of water waves and wave-structure interactions is planned to be undertaken with (dis)continuous Galerkin finite-element methods, cf. [32]. To complete the project it is essential to have a background in fluid mechanics, fluid-structure interactions, and numerical modelling. Therefore, based on the requirements, following modules taken for the postgraduate (PG) training:

2.2.1 Training: Taught courses, i.e. Deliverable D1.14

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- D1.14 MSc course 1: numerical assignment N3 is still due to date; N3 is worth circa 10% of the first MSc course and is relevant to the ESRs' research; one can simply give zero marks for N3 and formally complete the first MSc course but that serves no purpose since the work still has to be completed and the delayed late start is not the ESRs fault.
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- Both MSc courses are taught as part of the Centre for Doctoral Training in Fluid Dynamics at the University of Leeds and have been taught to circa 15 participants, PhD students in their MSc phase, including the two ESRs. Due to the delayed started, caused a.o. by the pandemic, the numerical component of the first MSc course has been taught twice by Prof Bokhove (at the expense of his other work), for the ESRs benefit.

MATH5453M Foundations of Fluid Dynamics (Semester 1, 30 credits)

This module includes lectures, seminars, practical training sessions and assignments related to fluid dynamics and numerical methods. The assessment is done via a final exam and assignments. The theoretical part of the module helped to build a solid theoretical foundation which is required to complete the project while the numerical part of the module includes three assignments related to finite difference method (FDM), finite volume method (FVM) and finite element method (FEM). The concepts learned during each assignment are explained as follows:

- The finite difference method (**FDM**) assignment is related to the numerical modelling of heat transfer (advection-diffusion equation) between two materials by using an Explicit Euler scheme, an Implicit Euler scheme and the Crank-Nicolson method. The results obtained from these methods are compared and their pros and cons are observed and analysed. This analysis helped to build the basis of numerical methods by using the Python programming language, usage of different stencils, treatment of different boundary conditions like Dirichlet, Neumann and Robin boundary conditions, and the effect of the CFL-condition on the numerical stability of the results.
- The finite volume method (**FVM**) assignment is related to the numerical modelling of water waves by using linearised shallow water equations in 1D. The objective is to use this model to predict surf height at beaches. The modelling is done by comparing two ways of analysis: numerical analysis and simulations versus the given results of an asymptotic analysis. The numerical scheme is using the Godunov's scheme and the second scheme is similar but employs an energy-preserving numerical flux. This assignment developed the understanding related to the Riemann problem, flux calculations at the boundary of two cells, the usage of an alternating flux to conserve discrete energy, and the implementation of different boundaries conditions like solid-wall and open domain conditions. As a part of this assignment, the shallow water equations (SWE) are solved to model the behaviour of water waves when they reach the sea shore. It is observed that the wavelength of waves get shorter

when the water depth decreases, while the amplitude increases potentially to a point where the wave can break (not modelled in the assignment). This combined effect confers to mass and energy conservation. This wave-steeping and -shortening effect is verified by using the numerical code of the linear shallow water wave equations for the varying topography. The results obtained from the modelling are inline with the simulations, as shown in the Fig. 2.4.

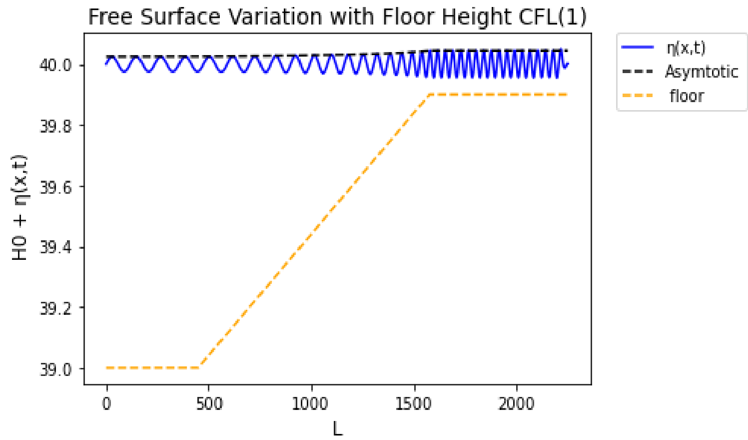

Figure 2.4: Variation of wave height and wavelength as the topography varies.

- The finite element method (**FEM**) assignment concerns the modelling of wave dynamics by using Ritz-Galerkin finite element method. The equations of motion are derived from the variational principle. The equations to be solved numerically are in turn: the linear potential-flow shallow water equations, Benney-Luke type equations and mlinear potential flow equations. The numerical scheme is implemented by using Störmer-Verlet time stepping routine. In the final part of this assignment, the developed approach is (possibly) extended by using *Firedrake* finite-element environment. The *Firedrake* numerical modelling environment will be used in the PhD project to undertake the numerical simulations and, therefore, the assignment covers the training required for *Firedrake*. The assignment is is still in progress to date.

COMP5454M Fluid-Structure Interactions (Semester 2, 15 credits)

This module comprises of lectures, seminars and projects to develop a deeper understanding for the modelling of fluid-structure interactions (FSI) analysis by building solid mathematical foundation. Besides learning the mathematical foundations (Conservation laws; deformation gradients; Piola-Kirchhoff stress; linear and nonlinear elasticity; hyperelasticity; compressible and incompressible models; fluid-solid interface conditions; and thin-structure limits), developing simplified models (FSI cases that may be reduced to analytic solutions; elastohydrodynamic lubrication (EHL); poro-elastic materials; aeroelasticity in wings) and different numerical methods (meshing; single and multiple mesh methods; fitted and non-fitted approaches for the fluid-solid interface; explicit versus implicit coupling; common numerical techniques (interface tracking, immersed finite element and fictitious domain methods)). The training gives hands-on-experience on various in-house and commercial software (use of commercial software such as ANSYS or COMSOL as segregated solvers, based upon coupling separate fluid and solid solvers, or COMSOL as a monolithic solver; use and modification of in-house software; applications based upon open source tools such as OpenFOAM). Some of the results produced during this module are shared in the next section of this report.

2.2.2 Training: Short-Term Training Sessions

In addition to the modules, following training courses are taken for the sake of professional development and research skills improvement. The training courses are as follow:

- PGR Core Language Skills: Reading Critically to Write Critically Workshop (1.5h), 25/11/2020;
- Increasing the Visibility of Your Research (1.5h), 15/12/2020; and,

- COMP5992M: Outreach Training (by Dr Katie Chicot, CEO of MathsWorldUK).

2.2.3 Scientific Results: Relevance to planned research

The wave-impact project is related to the fluid-structure interaction (FSI) analysis of water waves with a flexible and dynamic wind turbine mast. At this stage, a literature review is undertaken including learning different techniques to carry out FSI analysis. Some outcomes of the training are briefly discussed in this section, including:

- The topic selected for the literature study is *'Different Numerical Methods for the FSI Analysis of Offshore Wind Turbines'*. In the maritime industry, the significance of numerical modelling cannot be denied because scaled-model testing in wave basins is an expensive and time consuming process. In this literature study, the numerical models developed or used by different researchers, for the testing of offshore wind turbines, are studied and compared. Various computational models for FSI analysis use the Navier-Stokes equations, this include Reynolds-Averaged Navier Stokes (RANS), Large-Eddy Simulations (LES), [8], Smoothed Particle Hydrodynamics (SPH) [10] and Immersed boundary Method (IBM) [36]. Although, these models can accurately predict the complex non-linear wave phenomena when they are combined with turbulence models, they are computationally expensive because ocean wave modelling requires a disparate range of scales. Therefore, the researchers are developing alternative numerical techniques which can accurately predict the floating offshore turbine (FOWT) performance without being computationally intensive. These solvers are normally based on potential-flow theory while some of the solvers also couple Morison's equation [27] with potential theory to capture the viscous effects [24]. The hydroelastic behaviour of the structure can be modelled by using multi-body dynamics method (MBD). The coupling of the fluid and structure solvers can either be monolithic or partitioned.

- In this PhD project, we are combining the variational potential-flow approach and the variational hyperelastic-beam formulation in one nonlinear, monolithic variational principle [21]. The derivation of equations of motion of an hyper-elastic beam by using the variational principle is explained in this section.

- The FSI analysis, shown later as part of the FSI-course, of linear and non-linear hyperelastic beam, is done by using *'foam-extend'*, which is an extension of OpenFOAM. The fluid and solid equations and the coupling of the two solvers are explained in the upcoming section.

Derivation of equations of motions for the hyperelastic beam

The derivation of the equations of motion of an hyper-elastic beam by using the variational principle is done by Salwa [32]. As a part of the literature study, the derivation is studied and each step is elaborated. Consider a wind-turbine mast in the Lagrangian framework which is modelled by using the positions $\mathbf{X}$ = $\mathbf{X}(a,b,c,t) = (X,Y,Z)^T = (X_1,X_2,X_3)^T$ as function of Lagrangian label coordinates $\mathbf{a} = (a,b,c)^T = (a_1,a_2,a_3)^T$ and time t. These label coordinates $\mathbf{a}$ are defined in a reference domain Ω_O with a boundary $\partial\Omega_O$. A sketch of the beam is shown in Fig. 2.5.

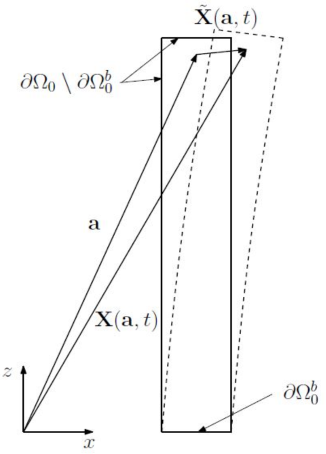

Figure 2.5: Cross section of a beam in the *x*–*z* plane [32].

In the sketch, **a**= **X**(**a**,0) is the Lagrangian coordinate in the reference state (boundary denoted by solid line); **X**(**a**,t) is the position of a point in the deformed beam (boundary denoted by dashed line) and $\tilde{\mathbf{X}}(\mathbf{a},t)$ its deflection; $\partial\Omega_O = \partial\Omega_0$ denotes the structure boundary and $\partial\Omega_O^b = \partial\Omega_0^b$ denotes the bottom which is fixed. The variational formulation of the hyperelastic material [34] closely follows that of a linear, elastic solid obeying Hooke's law, but the movement of parcels of solid material in the Lagrangian framework adds additional non-linearity. The Lagrangian framework deals with these finite rather than infinitesimal displacements. The relevant variational principle is classical, consisting of a kinetic energy $\frac{1}{2}\rho_0(\boldsymbol{a})|\boldsymbol{U}|^2)$ depending only on the velocity vector $\boldsymbol{U} \equiv \partial \boldsymbol{X}/\partial t$ (and label $\boldsymbol{a}$), plus the internal energy $W(\underline{\mathbf{E}})$, the latter depending only on the positions **X** (and its derivatives with respect to a) through the tensor $\boldsymbol{E}$ (and $\boldsymbol{a}$), and potential energy gZ. The density of the material is $\rho_0(\boldsymbol{a})$ and can vary in label space; it then depends on $\boldsymbol{a}$ as indicated. The variational principle for a hyperelastic material [34]is given by

$$0 = \delta \int_0^T \iiint_{\Omega_O} \rho_0 \mathbf{U}\cdot\partial_t\mathbf{X} - \frac{1}{2}\rho_0|\mathbf{U}|^2 - \rho_0 g Z - W(\underline{\mathbf{E}})\,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c\,\mathrm{d}t \tag{2.10}$$

with internal energy, and Lamé parameters λ and μ,

$$W(\underline{\mathbf{E}}) = \frac{1}{2}\lambda\,|tr(\underline{\mathbf{E}})|^2 + \mu\, tr(\underline{\mathbf{E}}^2). \tag{2.11}$$

After applying the variations and using end-point conditions $\delta\mathbf{X}(\mathbf{a},0) = \delta\mathbf{X}(\mathbf{a},T) = 0$, the first term in (2.10), it becomes:

$$\begin{aligned}
&\delta \int_0^T \iiint_{\Omega_O} \rho_0 \mathbf{U}\cdot\partial_t\mathbf{X}\,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c\,\mathrm{d}t\\
&= \lim_{\epsilon\to 0}\frac{1}{\epsilon}\int_0^T \iiint_{\Omega_O} \rho_0(\mathbf{U}+\epsilon\delta\mathbf{U})\cdot\partial_t(\mathbf{X}+\epsilon\delta\mathbf{X}) - \rho_0\mathbf{U}\cdot\partial_t\mathbf{X}\,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c\,\mathrm{d}t\\
&= \int_0^T \iiint_{\Omega_O} \rho_0\delta\mathbf{U}\cdot\partial_t\mathbf{X} + \rho_0\mathbf{U}\cdot\partial_t(\delta\mathbf{X})\,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c\,\mathrm{d}t\\
&= \int_0^T \iiint_{\Omega_O} \rho_0\delta\mathbf{U}\cdot\partial_t\mathbf{X} - \rho_0\delta\mathbf{X}\cdot\partial_t\mathbf{U}\,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c\,\mathrm{d}t + \iiint_{\Omega_O} \rho_0\mathbf{U}\cdot\delta\mathbf{X}|_{t=0}^{t=T}\;\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c\\
&= \int_0^T \iiint_{\Omega_O} \rho_0\delta\mathbf{U}\cdot\partial_t\mathbf{X} - \rho_0\delta\mathbf{X}\cdot\partial_t\mathbf{U}\,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c\,\mathrm{d}t.
\end{aligned} \tag{2.12}$$

Similarly, the second term in (2.10) becomes:

$$\begin{aligned}
\delta \int_0^T \iiint_{\Omega_O} -\frac{1}{2}\rho_0|\mathbf{U}|^2\,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c\,\mathrm{d}t &= \lim_{\epsilon\to 0}\frac{1}{\epsilon}\int_0^T \iiint_{\Omega_O} -\frac{1}{2}\rho_0|\mathbf{U}+\epsilon\delta\mathbf{U}|^2 + \frac{1}{2}\rho_0|\mathbf{U}|^2\,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c\,\mathrm{d}t\\
&= \int_0^T \iiint_{\Omega_O} -\rho_0\mathbf{U}\cdot\delta\mathbf{U}\,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c\,\mathrm{d}t.
\end{aligned} \tag{2.13}$$

Furthremore, the third term in (2.10) becomes:

$$\begin{aligned}\delta \int_0^T \iiint_{\Omega_O} -\rho_0 g Z \,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c\,\mathrm{d}t &= \lim_{\epsilon\to 0}\frac{1}{\epsilon}\int_0^T \iiint_{\Omega_O} -\rho_0 g\delta_{3l}(X_l+\epsilon\delta X_l)+\rho_0 g\delta_{3l}X_l \,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c\,\mathrm{d}t \\ &= \int_0^T \iiint_{\Omega_O} -\rho_0 g\delta_{3l}\delta X_l \,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c\,\mathrm{d}t.\end{aligned} \tag{2.14}$$

Considering the Lagrangian-Green stress tensor $E_{ij} = \frac{1}{2}(F_{ki}F_{kj} - \delta_{ij}) = E_{ji}$, we have $E_{ij}(F_{ki}\delta F_{kj} + F_{kj}\delta F_{ki}) = 2E_{ij}F_{ki}\delta F_{kj}$ with deformation gradient $F_{kj} = \partial X_k/\partial a_j$. The fourth term in (2.10) becomes

$$\begin{aligned}&\delta \int_0^T \iiint_{\Omega_O} -W(\underline{\mathbf{E}}) \,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c\,\mathrm{d}t \\ &= \lim_{\epsilon\to 0}\frac{1}{\epsilon}\int_0^T \iiint_{\Omega_O} -\frac{1}{2}\lambda(E_{ii}+\epsilon\delta E_{ii})(E_{jj}+\epsilon\delta E_{jj}) - \mu(E_{ij}+\epsilon\delta E_{ij})^2 + \frac{1}{2}\lambda E_{ii}E_{jj} + \mu E_{ij}^2 \,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c\,\mathrm{d}t \\ &= -\int_0^T \iiint_{\Omega_O} \frac{1}{2}\lambda(E_{ii}\delta E_{jj} + E_{jj}\delta E_{ii}) + 2\mu E_{ij}\delta E_{ij} \,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c\,\mathrm{d}t \\ &= -\int_0^T \iiint_{\Omega_O} \lambda E_{ii}\delta E_{jj} + \mu E_{ij}(F_{ki}\delta F_{kj} + F_{kj}\delta F_{ki}) \,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c\,\mathrm{d}t \\ &= -\int_0^T \iiint_{\Omega_O} \lambda \mathrm{tr}(\underline{\mathbf{E}})F_{kj}\delta F_{kj} + 2\mu E_{ij}F_{ki}\delta F_{kj} \,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c\,\mathrm{d}t \\ &= -\int_0^T \iiint_{\Omega_O} [\lambda \mathrm{tr}(\underline{\mathbf{E}})F_{kj} + 2\mu E_{ij}F_{ki}]\frac{\partial(\delta X_k)}{\partial a_j} \,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c\,\mathrm{d}t.\end{aligned} \tag{2.15a}$$

Next we change subscripts in (2.15a) and denote the stress tensor as $T_{li} = \lambda\,\mathrm{tr}(\underline{\mathbf{E}})F_{li} + 2\mu\,E_{ki}F_{lk}$. After using Gauss's theorem and considering $\delta X_l = 0$ at the bottom of the beam $\partial\Omega_O^b$, (2.15a) becomes

$$\begin{aligned}&\delta \int_0^T \iiint_{\Omega_O} -W(\underline{\mathbf{E}}) \,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c\,\mathrm{d}t \\ &= -\int_0^T \iiint_{\Omega_O} [\lambda \mathrm{tr}(\underline{\mathbf{E}})F_{li} + 2\mu E_{ki}F_{lk}]\frac{\partial(\delta X_l)}{\partial a_i} \,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c\,\mathrm{d}t \\ &= \int_0^T \left(\iiint_{\Omega_O} \frac{\partial T_{li}}{\partial a_i}\delta X_l \,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c - \iint_{\partial\Omega_O/\partial\Omega_O^b} n_i T_{li}\delta X_l \,\mathrm{d}S \right)\mathrm{d}t,\end{aligned} \tag{2.15b}$$

where $\mathrm{d}S$ denotes a surface element on the free boundaries of the beam. After substituting (2.12), (2.13), (2.14) and (2.15b) into (2.10), it becomes

$$\begin{aligned}0 = &\int_0^T \iiint_{\Omega_O} \rho_0(\partial_t\mathbf{X} - \mathbf{U})\cdot\delta\mathbf{U} - \rho_0\partial_t\mathbf{U}\cdot\delta\mathbf{X} - \rho_0 g\delta_{3l}\delta X_l + \frac{\partial T_{li}}{\partial a_i}\delta X_l \,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c\,\mathrm{d}t \\ &- \int_0^T \iint_{\partial\Omega_O/\partial\Omega_O^b} n_i T_{li}\delta X_l \,\mathrm{d}S\,\mathrm{d}t.\end{aligned} \tag{2.16}$$

After collecting the various analysed expressions, the final variations can be written as

$$\begin{aligned}0 = &\int_0^T \iiint_{\Omega_O} \rho_0(\partial_t\mathbf{X} - \mathbf{U})\cdot\delta\mathbf{U} - \left(\rho_0\partial_t U_l + \rho_0 g\delta_{3l} - \frac{\partial T_{li}}{\partial a_i}\right)\delta X_l \,\mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c\,\mathrm{d}t \\ &- \int_0^T \iint_{\partial\Omega_O/\partial\Omega_O^b} n_i T_{li}\delta X_l \,\mathrm{d}S\,\mathrm{d}t.\end{aligned} \tag{2.17}$$

By using the arbitrariness of variations on (2.17), we finally obtain the following equations of motion:

$$\delta\mathbf{U}: \quad \partial_t\mathbf{X} = \mathbf{U} \quad \text{on } \Omega_O \tag{2.18a}$$

$$\delta X_l: \quad \rho_0\partial_t U_l = -\rho_0 g\delta_{3l} + \frac{\partial T_{li}}{\partial a_i} \quad \text{on } \Omega_O \tag{2.18b}$$

$$\delta X_l|_{\partial\Omega_O/\partial\Omega_O^b}: \quad 0 = n_i T_{li} \quad \text{on } \partial\Omega_O/\partial\Omega_O^b, \tag{2.18c}$$

where stress tensor $T_{li} \equiv \lambda \text{tr}(\underline{\mathbf{E}})F_{li} + 2\mu E_{ki}F_{lk}$ and n_i is the outward normal component.

FSI analysis of the hyperelastic beam by using "foam-extend"

In another assignment of the fluid-structure interactions module, the modelling of a linear and non-linear hyperelastic beam is done by using "foam-extend", which is part of the *OpenFoam* software environment. The problem is solved by using boundary-fitted approaches known as Arbitrary Lagrangian-Eulerian (ALE) method. In this method, the fluid problem is solved on a mesh that deforms around a Lagrangian structure mesh as the structure deforms. At the shared interface both fluid and structure meshes match with each other.

i. Fluid governing equations:

Isothermal incompressible Newtonian fluid flow is governed by mass and linear momentum conservation laws

$$\oint_s \mathbf{n} \cdot \boldsymbol{\nu} \, dS = 0 \tag{2.19}$$

$$\frac{d}{dt} \int \boldsymbol{\nu} \, dV + \oint_s \boldsymbol{n} \cdot (\boldsymbol{\nu} - \boldsymbol{\nu_s}) \nu \, dS = \oint_s \boldsymbol{n} \cdot (\nu \boldsymbol{\nabla} \boldsymbol{\nu}) \, dS - \frac{1}{\rho} \int_V \nabla p \, dV. \tag{2.20}$$

The arbitrary Lagrangian-Eulerian (ALE) formulation is defined by the geometric (space) conservation law:

$$\frac{d}{dt} \int_V dV + \oint_S \boldsymbol{n} \cdot \boldsymbol{\nu_s} \, dS = 0. \tag{2.21}$$

ii. Solid governing equations:

The deformation of the solid is assumed to be elastic and compressible. It can be described by the linear momentum conservation law in the total Lagrangian form:

$$\int_V \rho_o \frac{\partial}{\partial t} (\frac{\partial \boldsymbol{u}}{\partial t}) \, dV = \oint_S \boldsymbol{n} \cdot (2\mu \boldsymbol{E} \boldsymbol{F^T} + \lambda tr(\boldsymbol{E}) \boldsymbol{I} \boldsymbol{F^T}) \, dS + \int_{V_0} \rho_0 \boldsymbol{b} \, dV, \tag{2.22}$$

where $\boldsymbol{E}$ is Green-Lagrange strain tensor. It is given as follows:

$$\boldsymbol{E} = \frac{1}{2} [\nabla \boldsymbol{\mu} + (\nabla \boldsymbol{\mu})^T + \nabla \boldsymbol{\mu} \cdot (\nabla \boldsymbol{\mu})^T]$$

in which the $\boldsymbol{F}$ is the deformation gradient tensor:

$$\boldsymbol{F} = \boldsymbol{I} + (\boldsymbol{\nabla} \boldsymbol{u}^T).$$

After putting the equations of $\boldsymbol{E}$ and $\boldsymbol{F}$ in (2.22) and simplifying, we get:

$$\rho_o \int_V \frac{\partial}{\partial t} (\frac{\partial \boldsymbol{u}}{\partial t}) \, dV - \oint_S \boldsymbol{n} \cdot (2\mu + \lambda) \boldsymbol{\nabla} \boldsymbol{u} \, dS = \oint \boldsymbol{n} \cdot \boldsymbol{q} \, dS + \rho_o \int_{V_o} \boldsymbol{b} \, dV, \tag{2.23}$$

where $\boldsymbol{q}$ is defined as:

$$\boldsymbol{q} = \mu (\boldsymbol{\nabla} \boldsymbol{u})^T + \lambda tr(\boldsymbol{\nabla} \boldsymbol{u}) \boldsymbol{I} - (\mu + \lambda) \boldsymbol{\nabla} \boldsymbol{u} + \mu \boldsymbol{\nabla} \boldsymbol{u} \cdot (\boldsymbol{\nabla} \boldsymbol{u})^T + \frac{1}{2} \lambda tr[\boldsymbol{\nabla} \boldsymbol{u} \cdot (\boldsymbol{\nabla} \boldsymbol{u})^T] \boldsymbol{I} + ((2\mu \boldsymbol{E} + \lambda tr(\boldsymbol{E}) \boldsymbol{I}) \cdot \boldsymbol{\nabla} \boldsymbol{u}$$

iii. Boundary conditions at the fluid-solid interface:

The fluid and solid models are coupled by kinematic and dynamic boundary conditions which must be satisfied at the fluid-solid interface.

- The kinematic conditions are given as:

$$\boldsymbol{\nu}_{F,i} = \boldsymbol{\nu}_{S,i}$$

$$\boldsymbol{u}_{F,i} = \boldsymbol{u}_{S,i},$$

where subscripts F denotes the fluid, S denotes solid and i stands for interface.

- The dynamic conditions are:

$$\boldsymbol{n_i} \cdot \boldsymbol{\sigma_{F,i}} = \boldsymbol{n_i} \cdot \boldsymbol{\sigma_{S,i}}$$

$$\boldsymbol{\sigma_{F,i}} = -p\boldsymbol{I} + \mu[\boldsymbol{\nabla\nu} + \boldsymbol{\nabla\nu}^T].$$

The results obtained after running the simulations on “foam-extend” are given in Fig. 2.6 and Fig. 2.7. The fluid inlet is at the left side of the domain and the hyperelastic beam is fixed at the bottom. The contours in blue and red show the variations of fluid velocity as the fluid interacts with the structure.

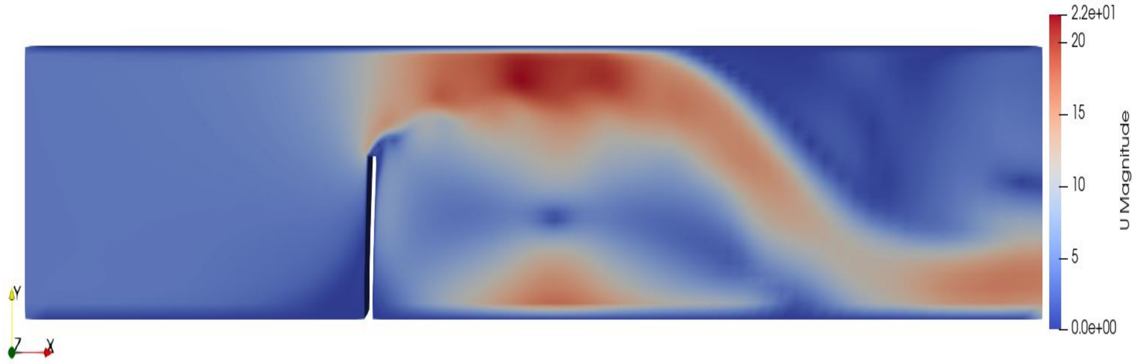

Figure 2.6: Snapshot of FSI analysis for a linear elastic beam.

In the linear elastic beam case, it can be noticed that the deflection of the beam is small and the flow is also changing because of the structure deformation.

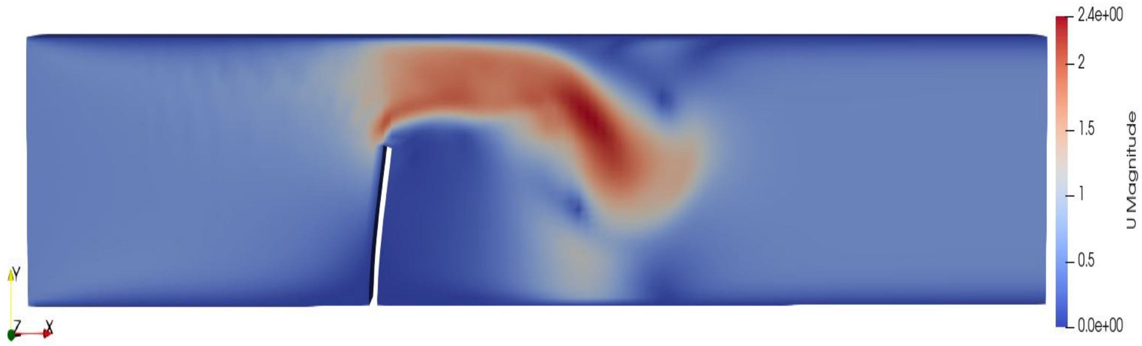

Figure 2.7: Snapshot of FSI analysis for a non-linear elastic beam.

In non-linear elastic beam case, it is noticed that the deflection in the beam is significant. The non-linearity taken into account is this case is due to geometry (large deformation in the structure) while the material non-linearity is not considered.

Chapter 3

Networking and Transfer of Knowledge

3.1 Function of EID, partners, in practice

Hitherto, one online Teams' meeting on the progress of the ESRs was held with MARIN and Leeds' supervisors present:

- 18-01-2021 – collective kick-off meeting with minutes found on the Eagre Github site.

3.2 Interaction with private sector hitherto

Besides the main project partner MARIN, none. Potentially *HydroTec Ltd.* can be involved regarding the design of a wave tank for public demonstrations, since we have excellent working contacts. Existing wave-tanks in the mathematics fluid dynamics laboratory can be used [6, 5]. However, due to the Covid-19 pandemic, access to the School of Mathematics and laboratories has been forbidden to date (of this report). To date the ESRs have been working in Leeds for circa five months.

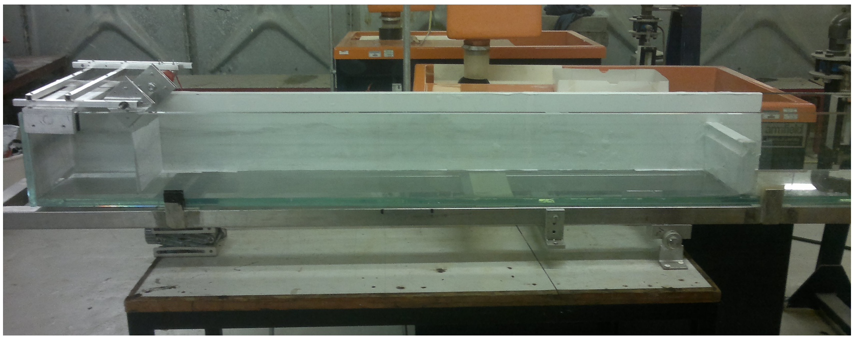

Figure 3.1: In the mathematics fluid dynamics laboratory, a glass wave tank with a wave maker (left) is available for training and testing. In addition, we have also acquired a portable perspex wave tank fitting in a car booth for demonstrations to the general public.

3.3 Internet Presence, i.e. part of Deliverable D4.1

Finally, we have a project webpage, a GitHub page, and the ESRs maintain webpages, etc. on their developing research and outreach activities (click on links):

- **project webpage**;
- **webpage George Yang Lu**;
- **webpage Wajiha Rehman**; and,
- **GitHub page**.

Chapter 4

Outreach Activities: Public outreach plan, i.e. part of Deliverable D4.1

Both ESRs have attended an online outreach training workshop related to public outreach for mathematics and fluid dynamics, a workshop that is of the CDT Fluid Dynamics courses. The training consists of three sessions which are summarised as follows:

- The first session is related to the generation of ideas and content for the public/audience engagement. In this session we looked at what makes a good workshop activity. We also tried out activities and then developed materials for masterclasses and school-aged children.
- The second part is focused on creation of STEM-related content like videos, recorded interviews, animations and virtual games to engage the audience virtually. These ideas can be used in the current COVID19 situation as the physical activities cannot be carried out. This approach can address a wide range of audiences from across the world. Therefore in this session, we first watched and compared our favourite videos and then looked at what makes a good science/maths video.
- The third part is related to the physical demonstration of science concepts in public spaces. In this session, we first looked at what maths exhibits already exist, and then the ideas and activities were discussed that can engage the audience physically. Especially, we exchanged ideas for exhibits for a fluid-dynamics exhibition. These ideas are applicable once the government restrictions are over. Alternatively, live-recorded demonstrations with direct interactions are still possible, also during the pandemic.

4.1 Virtual outreach

- Deliverable D4.1 (social media pages) is completed as reported in sections 3 and 4 of this report.

4.1.1 Social media platforms

- **Twitter account: EAGRE-H2020**
  A Twitter account is created and the updates of the project will be posted on it. This account is created to target the audience from all over the world and the material will be posted in English.
- **Weibo account: EAGRE-Horizon2020**
  A Sina Weibo account is created to post micro-blogs concerning news and progress of the project both in Chinese and in English. Since it is one of the biggest social media platforms in China, we intend to use this platform to reach audiences in China.

4.1.2 Personal web-pages

Both ESRs have developed their personal web-pages where they will post the updates, results and codes related to the project. The links of the pages are as follows:

- **Wajiha Rehman**
- **Yang Lu**

4.2 Physical outreach

Both ESRs have signed up for a **In2scienceUK mentoring scheme** organised by In2science with whom the University of Leeds is working in partnership. During this activity, both ESRs will mentor a group of students from low-income and under-represented backgrounds to promote social mobility and diversity within STEM. The programme runs throughout August, 2021. Mentors will meet with their students online for two 45-minute sessions, before hosting a one-day workplace visit (Covid-19 restrictions permitting). In addition, we are intended to join the following activities for public outreach in the future.

- Headingley Café Scientifique in Leeds (September, 2021);
- School of Mathematics Open days (April, 2022); and,
- Maritime Research Institute Netherlands Open Days (Need to be discussed with MARIN).

Chapter 5

Management

5.1 Recruitment report

We had numerous (circa 30) applicants applying to the two ESR–posts in two calls for applicants, in which we extended and changed the advertisement and posted it also on *ResearchGate* in the second call, besides the usual UK sites the School of Mathematics tends to use (Find-a-PhD, EuraAxess, et cetera). We online-interviewed six candidates in the summer of 2020, with a committee of five men and women, including Dr Tim Bunnik and Prof Onno Bokhove, resulting in the appointment of the ESRs Wajiha Rehman and (George) Yan Lu.

5.2 Management meetings

We held the following online meetings with all partners:

- 01-03-2020 Kick-off meeting with Dr Tim Bunnik and Prof Onno Bokhove; and
- 18-01-2021 Kick-off meeting with the ESRs and several industrial and acdemics supervisors.
- Minutes of these meetings are found at `https://github.com/obokhove/EagreEUEID20202023`.

Formal PhD transfer meetings, as part of the School of Mathematics and university requirements, will be held to assess the progress of the ESRs in April/May 2021.

5.3 Discussion of possible minor re-orientations

Except for the potential WP2.7 adaptation (see below) at a later stage, no re-orientations are planned given that we are in the training phase of the project. Due to the delayed start of the ESRs as well as the Covid-19 pandemic, deadlines for deliverables and milestones will face a two-to-three month delay.

- Deliverable D1.14 on the two MSc courses will face some delays.
- WP1.1 Create a complete numerical finite-element wavetank for high-amplitude potential-flow water waves with a breaking-wave parameterization, optimized for parallel computing, wave generation and wave damping at beaches, in both two and three dimensions (2D and 3D). Explore coordinate transformations as well as dynamic mesh motion. *Deadline corresponding deliverable will be delayed by starting and pandemic delays.*
- WP1.2 Develop and deliver a (new) series of benchmark cases (soliton splashes [4], Stokes, Rienecker-Fenton, (ir)regular, short-crested waves, random waves, etc.) for the wavetank of WP1.1. *Deadline corresponding deliverable will be delayed by starting and pandemic delays.*
- WP2.1 Formulate the nonlinear mathematical theory of potential-flow water waves coupled to a non-linear hyperelastic beam (wind-turbine mast) in 2D and 3D, also using the applicants' new asymptotic analysis of the two-way feedback mechanism (cf. [22]) *Deadline corresponding deliverable will be delayed by starting and pandemic delays.*

- WP2.2 Derive a compatible numerical discretization of potential-flow water-wave motion and a prescribed beam (or waveflap) motion in 2D. *Deadline corresponding deliverable will be delayed by starting and pandemic delays.*

- WP4 Launching and maintenance of active Wordpress blog, Facebook page, webpages and Twitter account throughout the projects (launching completed); items for MARIN's/MARIN BV's website and news items, announcement of presentations, new results, activity summaries etc., augmented by the presentation of movies and photo impressions; and, proactive external stimulation to seed invitations invited to give public presentations.

- WP2.7 Provide and explore the variational formulation of a mixture-theory water-wave model in the Eulerian framework, using Euler-Poincaré theory and its Euler-Boussinesq-equation limit. Couple the resulting water-wave model variationally to the nonlinear beam (wind-turbine mast). Consider and explore numerical water-wave motion in a compressible Van-der-Waals-fluid model, in its potential-flow limit, and compare this computational model with a classic finite-volume formulation using a continuous equation of state. Explore the imposition of incompressibility (optional explorations).

  The proposal mentions to consider replacing WP2.7 at the mid-term review by a particular applied and end-user topic of interest to MARIN Academy BV, to be defined by the MARIN Academy BV supervisors depending on the progress at the time in discussion with ESR Wajiha Rehman and the academic advisors.

5.4 Financial aspects

Financial expenses to date have included salaries and the acquisition of computer equipment to ensure that the finite-element environment Firedrake, as planned for the research, can be used and used efficiently. A few extra minor expenses regarding the more extensive homeworking during the pandemic have taken place or are expected (screens, data storage, etc). Travel expenses have been virtually absent during the pandemic, except for the initial arrival travel. These cost saving by far surpass the extra costs of extra small electronic equipment

In addition, we have budgeted expenses for dedicated experiments at MARIN, as well as participation in international conferences. We should now also start to plan our participation in several (online) international conferences shortly (e.g., International Conference on Ocean, Maritime and Offshore/Artic Engineering (OMAE) and European Geophysical Union (EGU), their annual General Assembly; Firedrake conference).

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