MATHS453 NUMERICS EX2 (Written work) ID: 201116379

$\boxed{Q1}$

Scaling the system -

$$\partial\eta/\partial t + \partial/\partial x(Hu) \sim \frac{1}{(L_s/U_0)} \cdot H_{0s}\,\partial\eta'/\partial t' + \frac{U_0 \cdot H_{0s}}{L_s}\left(u'\frac{\partial H'}{\partial x'} + H'\frac{\partial u'}{\partial x'}\right)$$

Then -

$$0 \sim \frac{U_0 H_{0s}}{L_s}\left(\partial\eta'/\partial t' + \partial/\partial x'(H'u')\right)$$

Dropping the primes $\frac{U_0H_{0s}}{L_s}$ gives -

$$0 \sim \partial\eta'/\partial t + \partial/\partial x'(H'u')$$

$$\partial u/\partial t + \partial(g\eta)/\partial x \propto \frac{U_0}{(L_s/U_0)}\partial u'/\partial t' + g\frac{H_{0s}}{L_s}\partial\eta'/\partial x'$$

$$0 \sim U_0^2/L_s\,\partial u'/\partial t' + \frac{gH_{0s}}{L_s}\partial\eta'/\partial x'$$

$$\sim 1/L_s\,\partial u'/\partial t' + 1/L_s\,\frac{gH_{0s}}{U_0^2}\partial\eta'/\partial x'$$

Setting dimensionless $g' = \frac{gH_{0s}}{U_0^2}$:

$$0 \sim 1/L_s\left(\partial u'/\partial t' + g'\partial\eta'/\partial x'\right)$$

Dropping prime $1/L_s$ gives -

$$0 \sim \partial u'/\partial t' + g'\,\partial\eta'/\partial x'$$

<u>Defining the Reimann Problem</u>

We want $q = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$ st $q_t + Aq_x = 0$ for some matrix A.

Then we will have equations:

$$\begin{pmatrix} q_{1t} + a_{11}q_{1x} + a_{12}q_{2x} \\ q_{2t} + a_{21}q_{1x} + a_{22}q_{2x} \end{pmatrix} = \begin{pmatrix} \eta_t + H_0u_x \\ u_t + g\eta_x \end{pmatrix}$$

For $H(x) = H_0$ const, $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

clearly $q_1 = \eta$, $a_{11} = 0$, $a_{22} = 0$. We have the choice of:

$q_2 = u$, $a_{12} = H_0$, $a_{21} = g$. or
$q_2 = H_0 u$, $a_{12} = 1$, $a_{21} = gH_0$

To simplify the matrix, choose $q_2 = H_0 u$, st we have -

$$\partial_t \begin{pmatrix} \eta \\ H_0 u \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ gH_0 & 0 \end{pmatrix} \partial_x \begin{pmatrix} \eta \\ H_0 u \end{pmatrix} = 0$$

When $H_0 u_t + gH_0 \eta_x = H_0(u_t + g\eta_x) = 0$ so $u_t + g\eta_x = 0$
as desired.
Let $c_0 = \sqrt{gH_0}$ where c_0 is phase speed.

Then we have the Riemann problem -

$$\partial_t q + \begin{pmatrix} 0 & 1 \\ c_0^2 & 0 \end{pmatrix} \partial_x q = 0$$

to be coupled with constant initial conditions. This is a system of coupled PDEs, so we can now decouple the equations.

Eigenvalues and eigenvectors of A

$$\mathrm{Det}(A - \lambda I) = \left| \begin{pmatrix} -\lambda & 1 \\ c_0^2 & -\lambda \end{pmatrix} \right| = (-\lambda)^2 - c_0^2 = 0$$

gives eigenvalues $\lambda = \pm c_0$, that is, $\lambda_1 = c_0 = \sqrt{(gH_0)}$, $\lambda_2 = -c_0 = -\sqrt{(gH_0)}$.
Then we have eigenvectors -

$\lambda_1 = c_0$;

$$\begin{pmatrix} -c_0 & 1 \\ c_0^2 & -c_0 \end{pmatrix} \begin{pmatrix} r_{11} \\ r_{12} \end{pmatrix} = \begin{pmatrix} -c_0 r_{11} + r_{12} \\ c_0^2 r_{11} - c_0 r_{12} \end{pmatrix} = \underline{0} \quad \text{then} \quad r_{12} = c_0 r_{11}$$

Let $r_{11} = 1$, then $r_{12} = c_0$ for eigenvector $\underline{r}_1 = \begin{pmatrix} 1 \\ c_0 \end{pmatrix}$

$\lambda_2 = -c_0$;

$$\begin{pmatrix} c_0 & 1 \\ c_0^2 & c_0 \end{pmatrix} \begin{pmatrix} r_{21} \\ r_{22} \end{pmatrix} = \begin{pmatrix} c_0 r_{21} + r_{22} \\ c_0^2 r_{21} + c_0 r_{22} \end{pmatrix} = \underline{0} \quad \text{then} \quad r_{22} = -c_0 r_{21}$$

Let $r_{21} = 1$, then $r_{22} = -c_0$. for eigenvector $\underline{r}_2 = \begin{pmatrix} 1 \\ -c_0 \end{pmatrix}$.

[These are both clearly scalar multiples of given eigenvectors-

$\frac{1}{2c_0}\begin{pmatrix}1\\c_0\end{pmatrix}$ and $-\frac{1}{2c_0}\begin{pmatrix}1\\-c_0\end{pmatrix}$.]

Our matrix of eigenvectors is: $B=\begin{pmatrix}1 & 1\\ c_0 & -c_0\end{pmatrix}$ with $B^{-1}=-\frac{1}{2c_0}\begin{pmatrix}-c_0 & -1\\ -c_0 & 1\end{pmatrix}$.

Then

$$B^{-1}B=\begin{pmatrix}-c_0 & -1\\ -c_0 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\ c_0 & -c_0\end{pmatrix}\cdot -\frac{1}{2c_0}=\begin{pmatrix}-2c_0 & 0\\ 0 & -2c_0\end{pmatrix}\cdot -\frac{1}{2c_0}=I$$

and

$$B^{-1}AB=-\frac{1}{2c_0}\begin{pmatrix}-c_0 & -1\\ -c_0 & 1\end{pmatrix}\begin{pmatrix}0 & 1\\ c_0^2 & 0\end{pmatrix}\begin{pmatrix}1 & 1\\ c_0 & -c_0\end{pmatrix}=-\frac{1}{2c_0}\begin{pmatrix}-c_0 & -1\\ -c_0 & 1\end{pmatrix}\begin{pmatrix}c_0 & -c_0\\ c_0^2 & c_0^2\end{pmatrix}$$

$$=-\frac{1}{2c_0}\begin{pmatrix}-2c_0^2 & 0\\ 0 & 2c_0^2\end{pmatrix}=\begin{pmatrix}c_0 & 0\\ 0 & -c_0\end{pmatrix}=\begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{pmatrix}I.$$

Let

$$\underline{v}=B^{-1}\underline{q} \quad \text{ie} \quad \underline{v}=-\frac{1}{2c_0}\begin{pmatrix}-c_0 & -1\\ -c_0 & 1\end{pmatrix}\begin{pmatrix}\eta\\ H_0u\end{pmatrix}=-\frac{1}{2c_0}\begin{pmatrix}-c_0\eta-H_0u\\ -c_0\eta+H_0u\end{pmatrix}$$

$$\begin{pmatrix}c_0 & 0\\ 0 & -c_0\end{pmatrix} \qquad =\frac{1}{2c_0}\begin{pmatrix}c_0\eta+H_0u\\ c_0\eta-H_0u\end{pmatrix}$$

} slightly different to given Σ due to using different (but correct) eigenvectors.

Then $\underline{q}=B\underline{v}$ and we have:

$$B\partial_t\underline{v}+AB\partial_x\underline{v}=0$$

as our equations. Then multiplying by B^{-1}, we have:

$$B^{-1}B\partial_t\underline{v}+B^{-1}AB\partial_x\underline{v}=0, \quad \text{then}$$

$$\partial_t\underline{v}+\begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{pmatrix}\partial_x\underline{v}=\underline{0} \quad \text{ie} \quad \frac{1}{2c_0}\left[\begin{pmatrix}c_0\partial_t\eta+H_0\partial_t u\\ c_0\partial_t\eta-H_0\partial_t u\end{pmatrix}+\begin{pmatrix}c_0^2\partial_x\eta+c_0H_0\partial_x u\\ -c_0^2\eta+c_0H_0\partial_x u\end{pmatrix}\right]=\underline{0}$$

Giving decoupled equations:

(Eq 1) $\quad c_0\partial_t\eta+H_0\partial_t u+c_0^2\partial_x\eta+c_0H_0\partial_x u=0$

(Eq 2) $\quad c_0\partial_t\eta-H_0\partial_t u-c_0^2\partial_x\eta+c_0H_0\partial_x u=0$

Rearranging original equations

From the original equations $\partial_t \eta + \partial_x (H_0 u) = 0$, $\partial_t u + \partial_x (g\eta) = 0$ (with $H(x) = H_0$), we have -

$$c_0 (\partial_t \eta + H_0 \partial_x u) = 0 \ , \quad H_0 (\partial_t u + g \partial_x \eta) = 0$$

Then: $$0 + 0 = c_0 (\partial_t \eta + H_0 \partial_x u) + H_0 (\partial_t u + g \partial_x \eta),$$

$$0 = c_0 \partial_t \eta + c_0 H_0 \partial_x u + H_0 \partial_t u + c_0^2 \partial_x \eta$$

That is, it equals Eq 1 as required.

And
$$0 - 0 = c_0 (\partial_t \eta + H_0 \partial_x u) - H_0 (\partial_t u + g \partial_x \eta)$$

$$0 = c_0 \partial_t \eta + c_0 H_0 \partial_x u - H_0 \partial_t u - c_0^2 \partial_x \eta$$

That is, we have Eq 2 as required.

Q2 Using the given $\underline{r}$, $\partial_t \underline{r} + \begin{pmatrix} c_0 & 0 \\ 0 & -c_0 \end{pmatrix} \partial_x \underline{r} = 0$ gives us the two linear advection equations:

Eq 1 : $\partial_t (r_1) + c_0 \partial_x (r_1) = 0$

Eq 2 : $\partial_t (r_2) - c_0 \partial_x (r_2) = 0$ where $\underline{r} = \begin{pmatrix} r_1 \\ r_2 \end{pmatrix} = \begin{pmatrix} H_0 u + c_0 \eta \\ H_0 u - c_0 \eta \end{pmatrix}$

For Eq 1:

As a linear advection eq, evidently the characteristics are defined by:

$$dx/dt = c_0 \ \left[= \sqrt{g H_0}\right]$$

and $r_1(x,t)$ is constant along these characteristics as:

$$dr_1/dt = \partial r_1/\partial x \ dx/dt + \partial r_1/\partial t = \partial r_1/\partial x \cdot dx/dt + \left(-c_0 \partial r_1/\partial x\right)$$

$$= \partial r_1/\partial x \left(dx/dt - c_0\right) = \partial r_1/\partial x \cdot 0 = 0$$

For $t=0$, $x(0) = \xi$, then characteristic curves are given by

$x(t) = c_0 t + [\text{const}]$

$x(0) = [\text{const}] = \xi$

then $x(t) = c_0 t + \xi$. (and $\xi = x - c_0 t$).

These are straight lines with x-intercept ξ, slope $\lambda_1 = c_0$. Note r_1 is constant along these lines, then

$$r_1(x,t) = r_1(x - c_0 t, 0) = r(\xi, 0) = \begin{cases} r_{1l}, & \xi < 0 \\ r_{1r}, & \xi \geq 0 \end{cases}$$

on the characteristics.

Then as $\xi = x - c_0 t$, then we can adjust the conditions:

$$\xi < 0 \Rightarrow x - c_0 t < 0 \Rightarrow x < c_0 t$$
$$\xi \geq 0 \Rightarrow x - c_0 t \geq 0 \Rightarrow x \geq c_0 t$$

giving solution:

$$r_1(x,t) = \begin{cases} r_{1l}, & x < c_0 t \\ r_{1r}, & x \geq c_0 t \end{cases}$$

as required.

For Eq 2:

We can find the solution for Eq 2 in a similar way, with characteristics defined by

$$dx/dt = -c_0 \quad [= -\sqrt{gH_0}]$$

where we have

$$dr_2/dt = \partial r_2/\partial t + \partial r_2/\partial x \frac{dx}{dt} = c_0 \partial r_2/\partial x + \frac{\partial r_2}{\partial x} dx/dt = \partial r_2/\partial x \left(c_0 + dx/dt\right)$$
$$= \partial r_2/\partial x \cdot 0 = 0$$

that is, r_2 is constant on the characteristics.

Integrating and using $x(0) = \xi$ for some ξ, we have characteristic curves:

$$x(t) = -c_0 t + \xi, \quad \xi = x + c_0 t$$

As r_2 is constant on these characteristic curves –

$$r_2(x,t) = r_2(x + c_0 t, 0) = r_2(\xi, 0) = \begin{cases} r_{2l}, & \xi < 0 \\ r_{2r}, & \xi \geq 0 \end{cases}$$

Adjusting conditions:

$$\xi < 0 \Rightarrow x + c_0 t < 0 \Rightarrow x < -c_0 t$$
$$\xi \geq 0 \Rightarrow x + c_0 t \geq 0 \Rightarrow x \geq -c_0 t$$

Then we have:
$$r_2(x,t) = \begin{cases} r_{2l}, & x < -c_0t \\ r_{2r}, & x \geq -c_0t \end{cases}$$

on the characteristics as required.

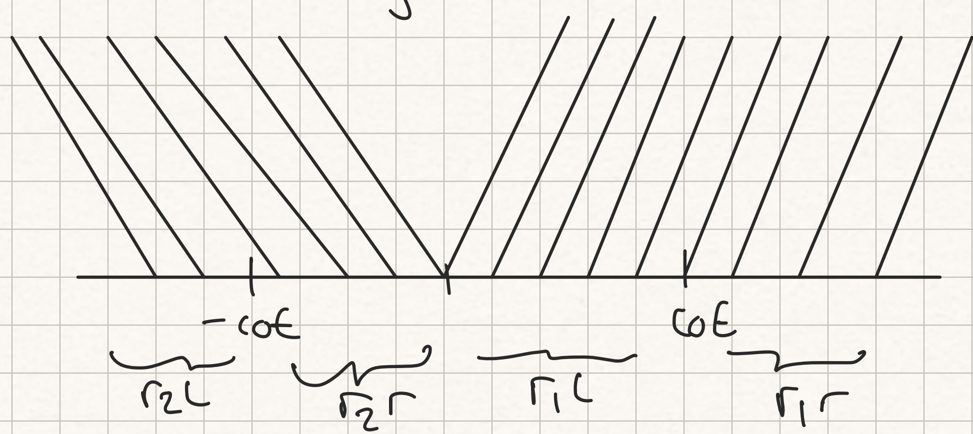
Solving for $u(x,t)$, $\eta(x,t)$:

Evidently, we have that:

$$H_0u = r_1 - c_0\eta = r_1 + r_2 - H_0u \Rightarrow 2H_0u = r_1 + r_2, \; H_0u = \tfrac{1}{2}(r_1+r_2)$$
and
$$\eta = \frac{r_1 - H_0u}{c_0} = \frac{r_1 - (r_2 + c_0\eta)}{c_0} = \frac{r_1 - r_2 - c_0\eta}{c_0} \Rightarrow 2c_0\eta = r_1 - r_2$$

ie $$\eta = \tfrac{1}{2c_0}(r_1 - r_2).$$

Sketching characteristics for $\lambda_1 = c_0$ (corresponding to r_1) and $\lambda_2 = -c_0$ (corresponding to r_2), so far –



Noting that λ_1, λ_2 are constant and do not depend on u or η.

So we are seeking solutions for (H_0u, η) in the intervals:

$x < -c_0t$: corresponds to (H_0u_l, η_l)
$-c_0t \leq x < c_0t$: corresponds to (H_0u_f, η_f)
$x \geq c_0t$: corresponds to (H_0u_r, η_r)

where for left and right states, we can use initial data of r_1, r_2, that is,
$$r_{1l} = H_0u_l + c_0\eta_l, \quad r_{2l} = H_0u_l - c_0\eta_l$$
$$r_{1r} = H_0u_r + c_0\eta_r, \quad r_{2r} = H_0u_r - c_0\eta_r$$
clearly define
$$(H_0u_l, \eta_l) = \left(\tfrac{1}{2}(r_{1l}+r_{2l}), \tfrac{1}{2c_0}(r_{1l} - r_{2l})\right)$$

$$(H_0u_r, \eta_r) = \left(\tfrac{1}{2}(r_{1r} + r_{2r}), \tfrac{1}{2c_0}(r_{1r} - r_{2r})\right).$$

(H_0u_f, η_f) defines the solution in the middle region, which we need to find. Clearly, these characteristics must be in a fan.

Looking at characteristics $\lambda_1 = c_0$ in the fan, they come from the origin at slope x/t, ie –

$$\lambda_1 = c_0 = x/t, \quad \lambda_2 = -c_0 = x/t$$

Then on $-c_0t < x < c_0t$

$$r_{2r} = H_0u_r + x/t\,\eta_r \qquad \text{const on } -c_0 = \lambda_2.$$
$$= H_0u + x/t\,\eta$$

$$r_{1L} = H_0u + x/t\,\eta \qquad \text{const on } c_0 = \lambda_1.$$
$$H_0u_L + x/t\,\eta_L$$

Then:

$$H_0u_f = \tfrac{1}{2}(r_{1L} + r_{2r}) = \tfrac{1}{2}\left(H_0u_r + x/t\,\eta_r + H_0u_L + x/t\,\eta_L\right)$$
$$= \frac{H_0}{2}(u_r + u_L) + \frac{x}{2t}(\eta_r + \eta_L)$$

$$\eta_f = \tfrac{1}{2}(r_{1L} - r_{2r}) = \tfrac{1}{2}\left(H_0u_L - x/t\,\eta_L - (H_0u_r - x/t\,\eta_r)\right)$$
$$= \frac{H_0}{2}(u_L - u_r) + \frac{x}{2t}(\eta_r - \eta_L)$$

giving:

$$H_0u(x,t) = \begin{cases} H_0u_L, & x < -c_0t \\ \frac{H_0}{2}(u_r + u_L) + \frac{x}{2t}(\eta_r + \eta_L), & -c_0t < x < c_0t \\ H_0u_r, & x > c_0t \end{cases}$$

$$\eta(x,t) = \begin{cases} \eta_L, & x < -c_0t \\ \frac{H_0}{2}(u_L - u_r) + \frac{x}{2t}(\eta_r - \eta_L), & -c_0t < x < c_0t \\ \eta_r, & x > c_0t \end{cases}$$

* Maybe some sign errors.

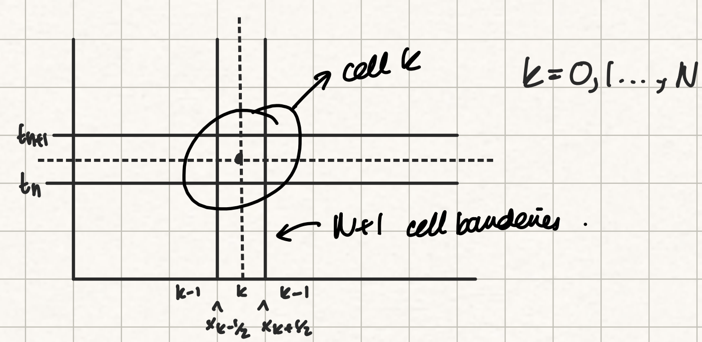
$\boxed{Q3}$

We're using: $\qquad$ (For $H(x) = \text{const.} = H_0$)

$$q_t + f(q)_x = 0$$

where
$$q = \begin{pmatrix} \eta \\ H_0 u \end{pmatrix}, \quad f(q) = \begin{pmatrix} 0 & 1 \\ gH_0 & 0 \end{pmatrix} \begin{pmatrix} \eta \\ H_0 u \end{pmatrix} = B \begin{pmatrix} \eta \\ H_0 u \end{pmatrix}$$

Diagram of cell arrangement in this method:



Deriving Godunov Scheme

For cell $\qquad x_{k-1/2} < x_{k+1/2}$:

$$Q_k(t) = \frac{1}{h_k} \int_{x_{k-1/2}}^{x_{k+1/2}} q(x,t)\, dx$$

where $h_k = x_{k+1/2} - x_{k-1/2}$ (not assuming uniform cell size)

where $\quad Q_k(t)$ is a vector st

$$Q_k(t) = \frac{1}{h_k} \begin{pmatrix} \int_{x_{k-1/2}}^{x_{k+1/2}} \eta(x,t)\, dx \\ H_0 \int_{x_{k-1/2}}^{x_{k+1/2}} u(x,t)\, dx \end{pmatrix}$$

As
$$0 = q_t + f(q)_x$$

Then
$$\int_{t_n}^{t_{n+1}} \int_{x_{k-1/2}}^{x_{k+1/2}} \partial q/\partial t + B\, \partial q/\partial x \; dx\,dt =$$

$$\int_{t_n}^{t_{n+1}} \int_{x_{k-1/2}}^{x_{k+1/2}} 0 \; dx\,dt = 0$$

Then:

$$0 = \int_{x_{k-1/2}}^{x_{k+1/2}} \int_{t_n}^{t_{n+1}} \frac{\partial}{\partial t} q \, dt\, dx + B \int_{t_n}^{t_{n+1}} \int_{x_{k-1/2}}^{x_{k+1/2}} \frac{\partial}{\partial x} q \, dx\, dt$$

$$= \int_{x_{k-1/2}}^{x_{k+1/2}} q(x, t_{n+1})\, dx - \int_{x_{k-1/2}}^{x_{k+1/2}} q(x, t_n)\, dx + B \int_{t_n}^{t_{n+1}} q(x_{k+1/2}, t)\, dt$$

$$- B \int_{t_n}^{t_{n+1}} q(x_{k-1/2}, t)\, dt.$$

Dividing by h_k:

$$0 = \frac{1}{h_k} \int_{x_{k-1/2}}^{x_{k+1/2}} q(x, t_{n+1})\, dx - \frac{1}{h_k} \int_{x_{k-1/2}}^{x_{k+1/2}} q(x, t_n)\, dx + \frac{B}{h_k} \int_{t_n}^{t_{n+1}} q(x_{k+1/2}, t)\, dt$$

$$- \frac{B}{h_k} \int_{t_n}^{t_{n+1}} q(x_{k-1/2}, t)\, dt.$$

$$= Q_k(t_{n+1}) - Q_k(t_n) + \frac{B}{h_k} \int_{t_n}^{t_{n+1}} q(x_{k+1/2}, t)\, dt - \frac{B}{h_k} \int_{t_n}^{t_{n+1}} q(x_{k-1/2}, t)\, dt.$$

Giving: where Q_k^n is constant.

$$Q_k^n - Q_k^{n+1} = \frac{B}{h_k} \int_{t_n}^{t_{n+1}} q(x_{k+1/2}, t)\, dt - \frac{B}{h_k} \int_{t_n}^{t_{n+1}} q(x_{k-1/2}, t)\, dt$$

We define approximate numerical flux. For uniform time step Δt.

$$F(Q_k^n, Q_{k+1}^n) = \frac{B}{\Delta t} \int_{t_n}^{t_{n+1}} q(x_{k+1/2}, t)\, dx = \frac{1}{\Delta t} \int_{t_n}^{t_{n+1}} f(q(x_{k+1/2}, t))\, dx$$

Hence:

$$Q_k^n - Q_k^{n+1} = \frac{\Delta t}{h_k} \left[F(Q_k^n, Q_{k+1}^n) - F(Q_{k-1}^n, Q_k^n) \right]$$

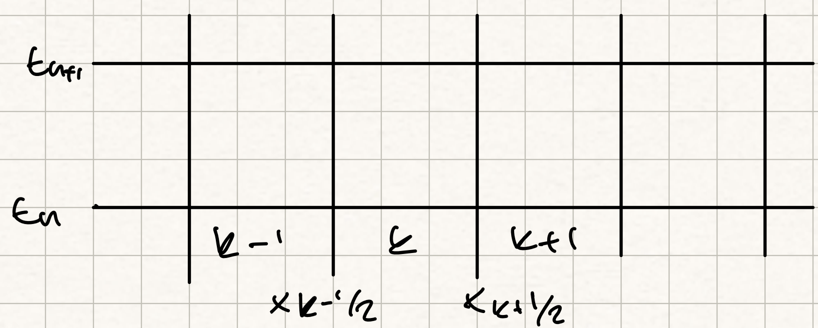
Our Godunov Scheme.

Note these are vector values!

eg $$F(Q_k^n, Q_{k+1}^n) = \frac{1}{\Delta t}\begin{pmatrix} 0 & 1 \\ gH_0 & 0 \end{pmatrix}\begin{pmatrix} \int_{t_n}^{t_{n+1}} \eta(x_{k+1/2},t)\,dt \\ H_0\int_{t_n}^{t_{n+1}} u(x_{k+1/2},t)\,dt \end{pmatrix}$$

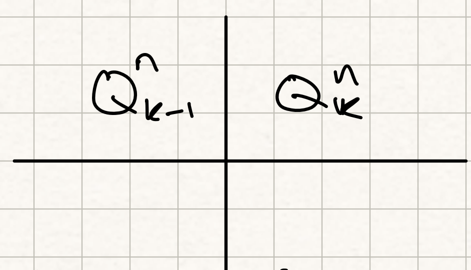
$$= \frac{1}{\Delta t}\begin{pmatrix} H_0\int_{t_n}^{t_{n+1}} u(x_{k+1/2},t)\,dt \\ gH_0\int_{t_n}^{t_{n+1}} \eta(x_{k+1/2},t)\,dt \end{pmatrix}$$

Mapping out solution domain:

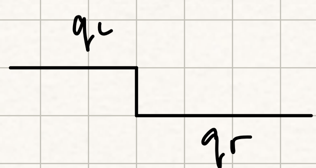


where

Q_k^n is constant in its cell – each cell has their own Riemann problem and at faces

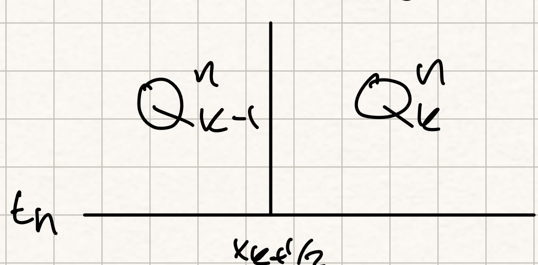


we can model it like a Riemann problem.



where $q_l = Q_{k-1}^n$, $q_r = Q_k^n$

Let us look at cell edge $x_{k+1/2}$



Returning to our original Riemann problem:

* Maybe did not complete Qu2! Possibly incorrect values here.

We have co-ordinate shift $(0,0) \mapsto (x_{j+1/2}, t_n)$
so $(x,t) \mapsto (x+x_{j+1/2}, t+t_n)$

* possibly wrong.

Note

$$Q_k^n = \begin{pmatrix} E_k^n \\ U_k^n \end{pmatrix}$$ for E_k^n corresponding to η, U_k^n corresponding to u.

Then

$$Q_k^n = \begin{pmatrix} E_k^n \\ U_k^n \end{pmatrix} \text{ corresponds to } \begin{pmatrix} \eta_L \\ u_L \end{pmatrix}$$

$$Q_{k+1}^n = \begin{pmatrix} E_{k+1}^n \\ U_{k+1}^n \end{pmatrix} \text{ corresponds to } \begin{pmatrix} \eta_r \\ u_r \end{pmatrix}$$

for $\underline{Q_k^n \mid Q_{k+1}^n}$

So we have: Riemann problems -

$$H_0 u = \begin{cases} H_0 U_k^n, & x < c_0 t_n \\ \frac{H_0}{2}(U_{k+1}^n + U_k^n) + \frac{x_{k+1/2}}{2t_n}(E_{k+1}^n + E_k^n), & -c_0 t_n < x < c_0 t_n \\ H_0 U_{k+1}^n, & x > c_0 t_n \end{cases}$$

$$\eta = \begin{cases} E_k^n, & x < -c_0 t_n \\ \frac{H_0}{2}(U_k^n - U_{k+1}^n) + \frac{x_{k+1/2}}{2t_n}(E_{k+1}^n - E_k^n), & -c_0 t_n < x < c_0 t_n \\ E_{k+1}^n, & x > c_0 t_n \end{cases}$$

!! Coords transformation on (x,t) is a bit wrong in the above !!

Note: Think there may be shocks in each cell - Riemann problems overlap?

Time step

CFL < 1 Then

$$\Delta t = \min_k \frac{h_k}{c_0}$$

where CFL ≈ 1.

Boundary conditions

At E_1^n, set $E_1^n = E_0^n = \eta_L$?

At U_1^n, set $U_1^n = -U_0^n = -u_L$ } Extrapolated BCs.

for $\forall n$.

And

$$\left.\begin{aligned} E_{J-1}^n &= E_J^n = \eta_r \\ U_{J-1}^n &= U_J^n = -u_r \end{aligned}\right\} \text{At right boundary?}$$

?

J: Right edge of domain.