

MATH5453M Numerical Exercise 1.



Given :

$$u_t - a(t)u_x - \epsilon u_{xx} = 0 \quad \text{for } x \in [l_p, L]$$

$$u(x, 0) = u_0(x)$$

$$u(l_p, t) = u(L, t) = 0,$$

(small) constant diffusion ϵ & a given function $a(t)$. The B.C. are classical homogeneous Dirichlet conditions.

Solutions

Problem 1:-

$$u_t - a(t)u_x - \epsilon u_{xx} = 0 \quad \text{for } x \in [l_p, L]$$

- The above given equation is linear because dependent variable $u(x, t)$ and its derivative appears linear in the above equation.

① Linear : if we look into the equation there is no term containing multiplication

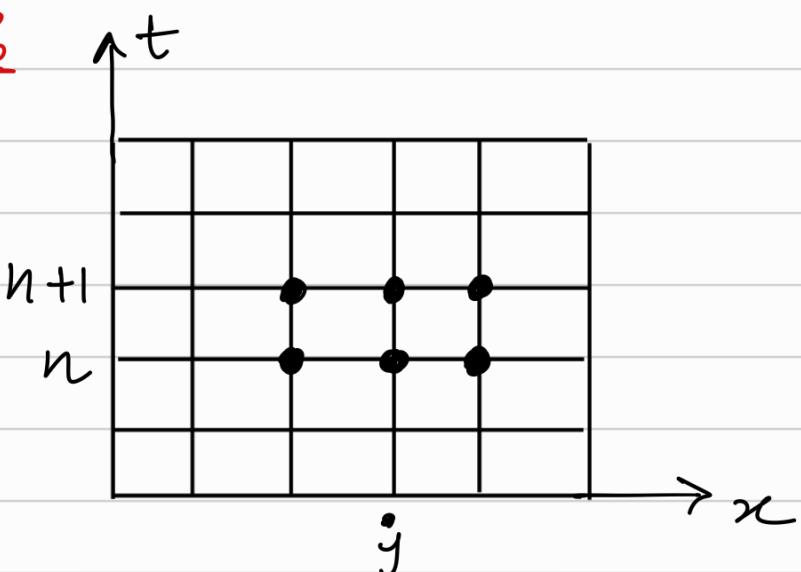
there is no term containing multiplication of $U(x, t)$ & its derivatives or any power of dependent variable $U(x, t)$.

② Advection: ' $-a(t)U_x$ ' represents the advection because it is showing the advection of U_x .

$$-a(t) \frac{\partial U}{\partial x}$$

③ Diffusion: ' ϵU_{xx} '
 \downarrow
 $\epsilon \frac{\partial^2 U}{\partial x^2}$ → represents the diffusion in the equation, with diffusion coefficient of ' ϵ '.

Problem 2:



Taylor Series expansion of U_j^{n+1} & U_j^n about $(\frac{\Delta t}{2})$

$$U_j^{n+1} = \left[U + \frac{1}{2} \Delta t U_t + \frac{1}{2} \left(\frac{1}{2} \Delta t \right)^2 U_{tt} + \frac{1}{6} \left(\frac{1}{2} \Delta t \right)^3 U_{ttt} + \dots \right]_j^{n+1/2}$$

$$\text{& } U_j^n = \left[U - \frac{1}{2} \Delta t U_t + \frac{1}{2} \left(\frac{1}{2} \Delta t \right)^2 U_{tt} - \frac{1}{6} \left(\frac{1}{2} \Delta t \right)^3 U_{ttt} + \dots \right]_j^{n+1/2}$$

Subtracting these two, all odd will cancel

$$U_j^{n+1} - U_j^n = \left[\Delta t U_t + \frac{1}{24} (\Delta t)^3 U_{ttt} + \dots \right]_j^{n+1/2}$$

$$U_j^{n+1} - U_j^n = \delta_t U_j^{n+1/2}$$

that means, (2.80)

$$\delta_t U_j^{n+1/2} = \left[\Delta t U_t + \frac{1}{24} (\Delta t)^3 U_{ttt} + \dots \right]_j^{n+1/2} - (2.80)$$

from eqn (2.30),

$$\sum_x^2 U(x, t) = U_{nn} (\Delta x)^2 + \frac{1}{12} U_{nnnn} (\Delta x)^4 + \dots$$

$$\text{& from } \sum_x^2 U_j^{n+1} = \frac{U_j^{n+1} - 2U_j^n + U_j^{n-1}}{(\Delta x)^2}$$

Expanding

$$U_j^{n+1} = U_j + \Delta n U_{j\bar{x}} + \frac{1}{2} (\Delta n)^2 U_{j\bar{x}\bar{x}} + \frac{1}{6} (\Delta n)^3 U_{j\bar{x}\bar{x}\bar{x}}$$

$$U_{j-1}^{n+1} = U_j - \Delta n U_{j\bar{x}} + \frac{1}{2} (\Delta n)^2 U_{j\bar{x}\bar{x}} - \frac{1}{6} (\Delta n)^3 U_{j\bar{x}\bar{x}\bar{x}}$$

$$\therefore \delta_x^2 U_j^{n+1} = \cancel{\left(U_j + \Delta n U_{j\bar{x}} + \frac{1}{2} (\Delta n)^2 U_{j\bar{x}\bar{x}} + \frac{1}{6} (\Delta n)^3 U_{j\bar{x}\bar{x}\bar{x}} \right)} \\ - \cancel{2U_j} + \cancel{\left(U_j - \Delta n U_{j\bar{x}} + \frac{1}{2} (\Delta n)^2 U_{j\bar{x}\bar{x}} \right)} \\ - \cancel{\frac{1}{6} (\Delta n)^3 U_{j\bar{x}\bar{x}\bar{x}}} \\ \underline{\hspace{10em}} \\ (\Delta n)^2$$

(2.80) ↓

$$\delta_x^2 U_j^{n+1} = U_{j\bar{x}\bar{x}} + \frac{1}{12} (\Delta n)^2 U_{j\bar{x}\bar{x}\bar{x}\bar{x}} + \dots$$

— (2.81)

- Now considering weighted Average of spatial derivatives

$$\theta \delta_x^2 U_j^{n+1} + (1-\theta) \delta_x^2 U_j^n$$

Using expansion of $\delta_x^2 U_j^{n+1}$ & $\delta_x^2 U_j^n$

$$\delta_x^2 U_j^{n+1} = \left[(\Delta n)^2 U_{j\bar{x}\bar{x}} + \frac{1}{12} (\Delta n)^4 U_{j\bar{x}\bar{x}\bar{x}\bar{x}} + \frac{1}{12} (\Delta n)^6 U_{j\bar{x}\bar{x}\bar{x}\bar{x}\bar{x}} + \dots \right]^{n+1} \\ + \frac{1}{2} \theta t \left[(\Delta n)^2 U_{j\bar{x}\bar{x}t} + \frac{1}{12} (\Delta n)^4 U_{j\bar{x}\bar{x}\bar{x}x} + \dots \right]^{\frac{n+1}{2}}$$

$$+\frac{1}{2} \left(\frac{1}{2} \Delta t \right)^2 \int (\Delta t)^2 u_{xxx} dt + \dots \sim_j^{\frac{n+1}{2}}$$

Similarly :-

$$\begin{aligned} \delta_x^2 u_j^n &= \left[(\Delta n)^2 u_{xx} + \frac{1}{12} (\Delta n)^4 u_{xxxx} + \dots \right]_j^{\frac{n+1}{2}} \\ &\quad - \frac{1}{2} \Delta t \left[(\Delta n)^2 u_{xxt} + \frac{1}{12} (\Delta n)^4 u_{xxxxt} + \dots \right]_j^{\frac{n+1}{2}} \\ &\quad + \frac{1}{2} \left(\frac{1}{2} \Delta t \right)^2 + \int (\Delta t)^2 u_{xxtt} dt + \dots \end{aligned}$$

using these two in

$$\begin{aligned} \theta \delta_x^2 u_j^{n+1} + (1-\theta) \delta_x^2 u_j^n \\ = \left[(\Delta n)^2 u_{xx} + \frac{1}{12} (\Delta n)^4 u_{xxxx} + \dots \right] \\ + \left(\theta - \frac{1}{2} \right) \Delta t \left[(\Delta n)^2 u_{xxt} + \frac{1}{12} (\Delta n)^4 u_{xxxxt} \right. \\ \left. + \dots \right] \\ + \frac{1}{8} (\Delta t)^2 (\Delta n)^2 \int u_{xxtt} dt + \dots \\ \quad \text{--- (2.82)} \end{aligned}$$

Truncation Error :

$$\begin{aligned} T_j^{\frac{n+1}{2}} &= \frac{\delta_t u_j^{\frac{n+1}{2}}}{\Delta t} - \frac{\theta \delta_x^2 u_j^{n+1} + (1-\theta) \delta_x^2 u_j^n}{(\Delta n)^2} \\ &\quad \text{--- (2.83)} \end{aligned}$$

$$\text{Using } \frac{\partial_t u_j}{\Delta t} = u_t + \frac{1}{2\Delta t} (\Delta t)^2 u_{ttt} + \dots$$

$$\text{L} \frac{\theta \delta_x^2 u_j^{n+1} + (1-\theta) \delta_x^2 u_j^n}{(\Delta x)^2} = u_{xx} + \frac{1}{12} (\Delta x)^2 u_{xxxx} + \dots$$

$$\begin{aligned} \therefore u_j^{n+\frac{1}{2}} &= [u_t - u_{xx}] + \left[\left(\frac{1}{2} - \theta \right) \Delta t u_{xt} - \frac{1}{12} (\Delta x)^2 u_{xxxx} \right] \\ &\quad + \left[\frac{1}{12} (\Delta t)^2 u_{ttt} - \frac{1}{8} (\Delta t)^2 \right] \\ &\quad + \left[\frac{1}{12} \left(\frac{1}{2} - \theta \right) \Delta t (\Delta x)^2 u_{xxxx} - \frac{2}{6!} (\Delta x)^4 u_{xxxxxx} \right] \\ &\quad - (2.84) \end{aligned}$$

Problem 3:

The given advection-diffusion eqns

$$u_t - a(t) u_x - \epsilon u_{xx} = 0$$

$$u_x = \frac{u_{i+1}^n - u_i^n}{\Delta x}$$



$$-a(t) u_x = -a(t) \frac{u_{i+1}^n - u_i^n}{\Delta x}$$

for negative $a(t)$ & $a(t) = 1$: consider the forward difference

$$U_{\Delta t n} = \frac{U_{i+1}^n - 2U_i^n + U_{i-1}^n}{\Delta t^2}$$

θ -discretisation :-

$$\frac{U_i^{n+1} - U_i^n}{\Delta t} = \theta \left(-a(t) \frac{U_{i+1}^n - U_i^n}{\Delta t} + \epsilon \frac{U_{i+1}^{n+1} - 2U_i^{n+1} + U_{i-1}^n}{(\Delta t)^2} \right) + (1-\theta) \left(-a(t) \frac{U_{i+1}^n - U_i^n}{\Delta t} + \epsilon \frac{U_{i+1}^n - 2U_i^n + U_{i-1}^n}{\Delta t^2} \right)$$

taking unknown to LHS:

$$U_i^{n+1} = U_i^n + \Delta t \left[\theta \left(-\frac{a(t)}{\Delta t} (U_{i+1}^{n+1} - U_i^{n+1}) \right) + \frac{\epsilon}{\Delta t^2} (U_{i+1}^{n+1} - 2U_i^{n+1} + U_{i-1}^{n+1}) \right] + (1-\theta) \left(-\frac{a(t)}{\Delta t} (U_{i+1}^n - U_i^n) + \frac{\epsilon}{\Delta t^2} (U_{i+1}^n - 2U_i^n + U_{i-1}^n) \right)$$

$$\left. \begin{aligned} \text{assuming } \frac{\Delta t}{\Delta x} = \lambda \\ \text{& } \frac{\Delta t}{(\Delta x)^2} = \mu \end{aligned} \right\}$$

$$\begin{aligned}
 & \underline{u}_i^{n+1} + \theta \lambda a(t) \underline{u}_{i+1}^{n+1} - \theta \lambda a(t) \underline{u}_i^{n+1} \\
 & - \epsilon M \theta \underline{u}_{i+1}^{n+1} + \theta \epsilon M 2 \underline{u}_i^{n+1} - \epsilon \theta M \underline{u}_{i-1}^{n+1} \\
 & = (1-\theta) \left(\frac{-a(t)}{\Delta n} (u_{i+1}^n - u_i^n) + \frac{\epsilon}{\Delta n^2} (u_{i+1}^n - 2u_i^n + u_{i-1}^n) \right)
 \end{aligned}$$

$$\Rightarrow [1 + \theta \epsilon M 2 - \theta \lambda a(t)] \underline{u}_i^{n+1} + [\theta \lambda a(t) - \epsilon \theta] \underline{u}_{i+1}^{n+1} \\
 - \epsilon \theta M \underline{u}_{i-1}^{n+1} = (1-\theta) \left[-a(t) \lambda (u_{i+1}^n - u_i^n) + \epsilon M (u_{i+1}^n - 2u_i^n + u_{i-1}^n) \right]$$

$$\Rightarrow [1 + \theta \epsilon M 2 - \theta \lambda a(t)] \underline{u}_i^{n+1} + [\theta \lambda a(t) - \epsilon M \theta] \underline{u}_{i+1}^{n+1} \\
 - \epsilon \theta M \underline{u}_{i-1}^{n+1} = -a(t) \lambda \underline{u}_{i+1}^n + a(t) \lambda \underline{u}_i^n + \epsilon M \underline{u}_{i+1}^n \\
 - \epsilon M 2 \underline{u}_i^n + \epsilon M \underline{u}_{i-1}^n + a(t) \theta \lambda \underline{u}_{i+1}^n \\
 - \theta a(t) \lambda \underline{u}_i^n - \epsilon \theta M \underline{u}_{i+1}^n + 2 \epsilon M \theta \underline{u}_i^n \\
 - \epsilon M \theta \underline{u}_{i-1}^n$$

$$\Rightarrow [1 + \theta \epsilon M 2 - \theta \lambda a(t)] \underline{u}_i^{n+1} + [\theta \lambda a(t) - \epsilon M \theta] \underline{u}_{i+1}^{n+1} \\
 - \epsilon \theta M \underline{u}_{i-1}^{n+1} = [a(t) \theta \lambda - a(t) \lambda - \epsilon M \theta] \underline{u}_{i+1}^n + [a(t) \lambda - 2 \epsilon M \\
 + 2 \epsilon M \theta] \underline{u}_i^n + [\epsilon M - \epsilon M \theta] \underline{u}_{i-1}^n$$

Now: $x_i = l_p + i \Delta n$

$$x_i = l_p + i \Delta n \quad i = 0, 1, 2, \dots, J$$

for $i=0$, $x_0 = l_p$ (left boundary)
 $i=J$, $x_J = L$ (right boundary)

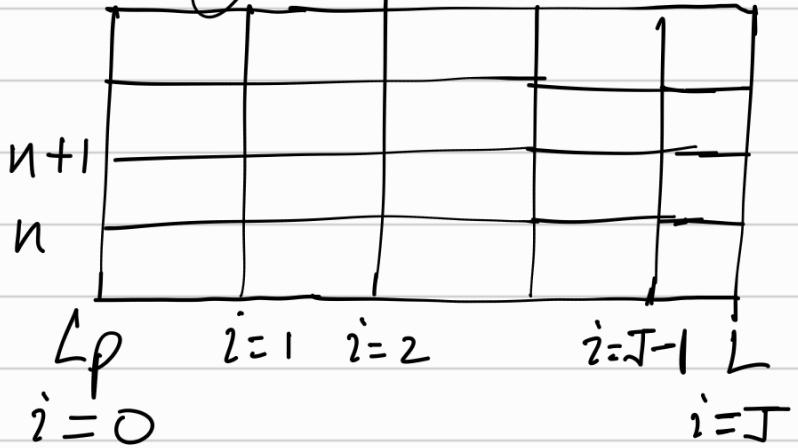
$$\text{let } t^n = (n \Delta t), \quad n = 0, 1, \dots, N$$

So Near Left Boundary: $i=1$
 Near Right Boundary: $i=J-1$

and as given

$$U_0^n = U_J^n = 0$$

for $n = 0, 1, 2, \dots$



Problem: 4

Given :

$$U_t = U_{xx} \quad \text{for } t > 0, \quad 0 < x < 1$$

by

$$U(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq \frac{1}{2} \\ 2 - 2x & \text{if } \frac{1}{2} \leq x \leq 1 \end{cases}$$

use $J = 20 \rightarrow$ Total grid points

$\Delta x = 0.05 \rightarrow$ Spatial step size

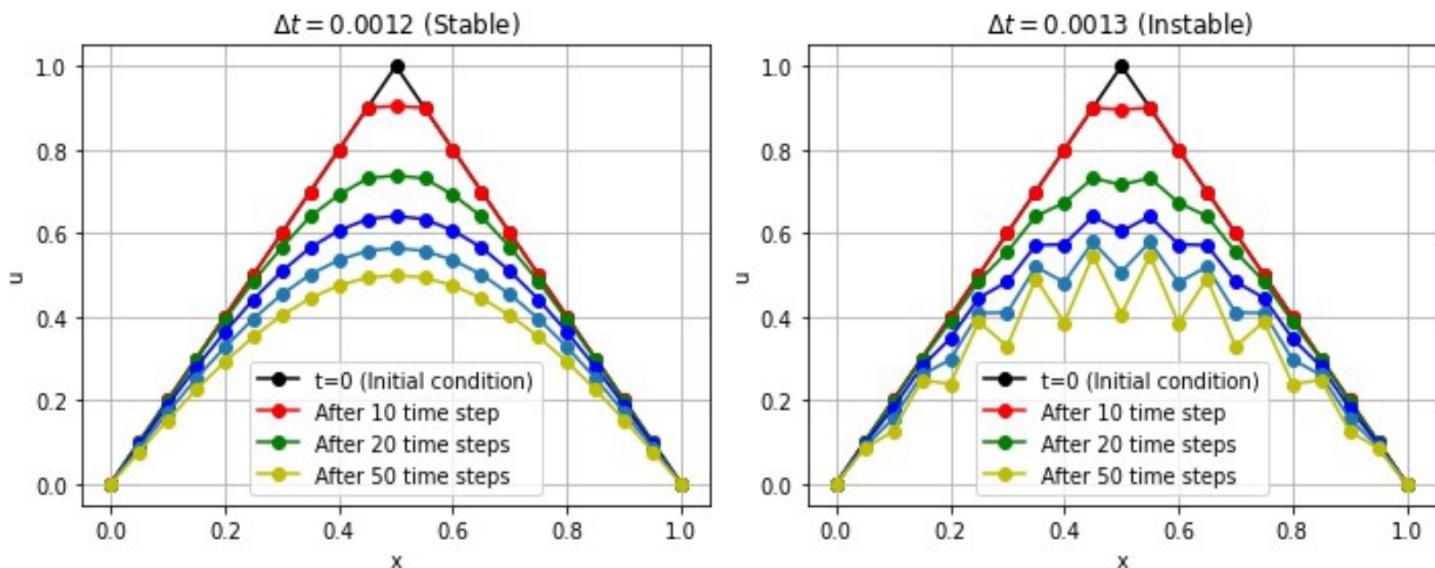
$\Delta t_1 = 0.0012 \rightarrow$ First time step size

$\Delta t_2 = 0.0013 \rightarrow$ 2nd time step size

using explicit scheme, a python code written in 'problem4.py' file & results has been matched & compared with fig 2.2 (MGM)

- for two different time steps the output

'profile is' given below!



- The above output image clearly shows the effect of time step size & as we can see for time step size 0.0013, there is instability can be observed.
- Now we will check the stability of this scheme, with both Fourier analysis and max. principle

① Fourier Analysis :-

$$(a) \quad U_j^{n+1} = \lambda^{n+1} e^{i k_j \Delta x} = \lambda \underbrace{[e^{i k_j \Delta x}]}_{\rightarrow U_j^n}$$

$$U_j^{n+1} = \lambda U_j^n$$

$$(b) \quad U_j^{n+1} = \lambda^n e^{i k_j (j+1) \Delta x}$$

$$= \lambda^n e^{i k \Delta x} e^{ik \Delta x}$$

$$U_{j+1}^n = U_j^n e^{i k \Delta x}$$

Similarly

$$U_{j-1}^n = U_j^n e^{-i k \Delta x}$$

Now use all these terms in explicit scheme difference eqn

$$U_j^{n+1} = U_j^n + \mu \left[U_{j+1}^n - 2U_j^n + U_{j-1}^n \right]$$

$$\lambda U_j^n = U_j^n + \mu \left[U_j^n e^{i k \Delta x} - 2U_j^n + U_j^n e^{-i k \Delta x} \right]$$

divided by U_j^n

$$\Rightarrow \lambda = 1 + \mu \left[e^{i k \Delta x} - 2 + e^{-i k \Delta x} \right]$$

$$\lambda = 1 + 2\mu \left[\frac{e^{i k \Delta x} + e^{-i k \Delta x}}{2} - 1 \right]$$

$$= 1 + 2\mu \left[\cos(k \Delta x) - 1 \right]$$

$$\lambda = 1 - 4M \sin^2 \frac{K_0 n}{2}$$

L $\rightarrow \lambda = f(K)$

for $K = m\pi$, we get

$$U_j^n = \sum_{m=-\infty}^{\infty} A_m e^{imn\pi} (\text{from } f(A_m))$$

$\therefore \lambda \rightarrow$ complex no.

L $\rightarrow \lambda = Re(\lambda) + i Im(\lambda)$

for stability requires that,

$$|\lambda| \leq 1$$



$\therefore \sin^2 \frac{K_0 n}{2} \leq 1$ is the
most

$$\Rightarrow |\lambda| \leq 1 \Rightarrow |1 - \lambda| \leq 1$$

$\mu \leq \frac{1}{2}$

is the stability criterion

&

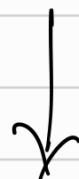
for $\lambda = 1 - 4\mu < -1$

$\mu > \frac{1}{2}$

\rightarrow unstable

Now, $\because \mu \leq \frac{1}{2}$

$$\frac{\Delta t}{\Delta n^2} \leq \frac{1}{2}$$



$\Delta t \leq \frac{1}{2} \Delta n^2$

\therefore (1) for $\Delta t_1 = 0.0012$ } Stable
 $\Delta n = 0.05$

$$\mu = \frac{0.0012}{(0.05)^2} \Rightarrow 0.48 \leq \underline{\underline{\frac{1}{2}}}$$

② for $Dt_2 = 0.0013$ } unstable
 $Dn = 0.05$

$$\lambda = \frac{0.0013}{(0.05)^2} = 0.52 > \frac{1}{2}$$

② Max. principle :-

According to Max. principle, a difference approximation to $U_t = U_{xx}$ should possess beyond consider as

$$Dn, Dt \rightarrow 0$$

$$\therefore \text{If } (1-\theta) \leq \frac{1}{2}$$

& for explicit scheme, $\theta = 0$

$$\lambda \leq \frac{1}{2}$$

\rightarrow which will give

$\Delta t_1 = 0.0012$ stable |
 $\Delta t_2 = 0.0013$ unstable |

\Rightarrow we extend this explicit scheme
to ①

$$U_t - a(t)U_{xx} - \varepsilon U_{xxx} = 0$$

$$U(x, 0) = U_0(x)$$

$$U(L_p, t) = U(L, t) = 0$$

$$\begin{aligned} & [1 + \theta \varepsilon U_x - \theta \lambda a(t)] U_i^{n+1} + [\theta \lambda a(t) - \varepsilon U \theta] U_{i+1}^{n+1} \\ & - \varepsilon \theta U_{i-1}^{n+1} = [a(t) \theta \lambda - a(t) \lambda - \varepsilon U \theta] U_i^n + [a(t) \lambda - 2\varepsilon \\ & + 2\varepsilon U \theta] U_i^n + [\varepsilon U - \varepsilon U \theta] U_{i-1}^n \end{aligned}$$

for $\theta = 0$: Explicit Scheme

$$U_i^{n+1} = -a(t)\lambda U_{i+1}^n + (a(t)\lambda - 2\varepsilon) U_i^n + \varepsilon U U_{i-1}^n$$

$$M = \frac{\Delta t}{\Delta x^2} \quad \lambda = \frac{\Delta t}{\Delta x}$$

Problem 5 :-

Implemented Q-Method and used the linear Algebra process in Problem Soly

$$\therefore u_t - \alpha(t)u_{xx} - \varepsilon u_{xxx} = 0$$

$$u(x, 0) = u_0(x)$$

$$u(L_p, t) = u(L, t) = 0$$

used $\zeta = 0$ & $L = 1$ (B.Cs)

& G.C $\rightarrow u_0(x) = \begin{cases} 2x & \text{if } 0 < x \leq 0.5 \\ 2 - 2x & \text{if } 0.5 \leq x \leq 1 \end{cases}$

$$\therefore u_x = \frac{u_{i+1} - u_{i-1}}{\Delta x}$$

& $u_{xx} = \frac{2u_i}{(\Delta x)^2}$

$$u_t = \frac{u_i^{i+1} - u_i^i}{\Delta t}$$

Q-Method implemented:-

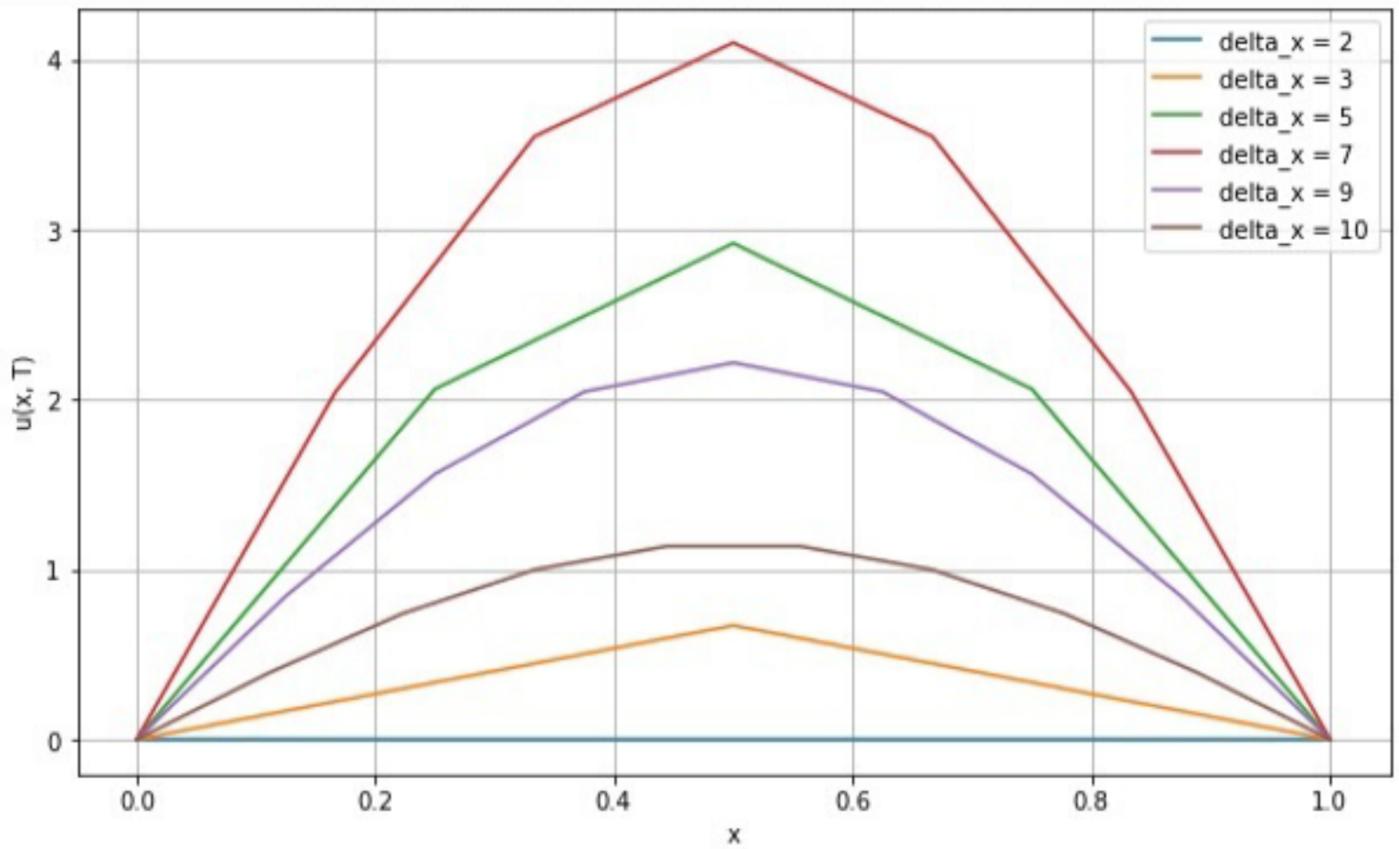
$$\begin{aligned} & \left[1 + \theta \varepsilon u_{xx} - \theta \lambda \alpha(t) \right] u_i^{i+1} + \left[\theta \lambda \alpha(t) - \varepsilon u \theta \right] u_{i+1}^{i+1} \\ & - \varepsilon \theta u_{i-1}^{i+1} = \left[\alpha(t) \theta \lambda - \dot{\alpha}(t) \lambda - \varepsilon u \theta \right] u_i^i + \left[\alpha(t) \lambda - 2\varepsilon u \right. \\ & \quad \left. + 2\varepsilon u \theta \right] u_i^i + \left[\varepsilon u - \varepsilon u \theta \right] u_{i-1}^i \end{aligned}$$

$\rightarrow \theta = 0 \rightarrow$ Explicit Scheme
 $\theta = 1 \rightarrow$ Implicit Scheme
 $\theta = 0.5 \rightarrow$ Crank-Nicolson

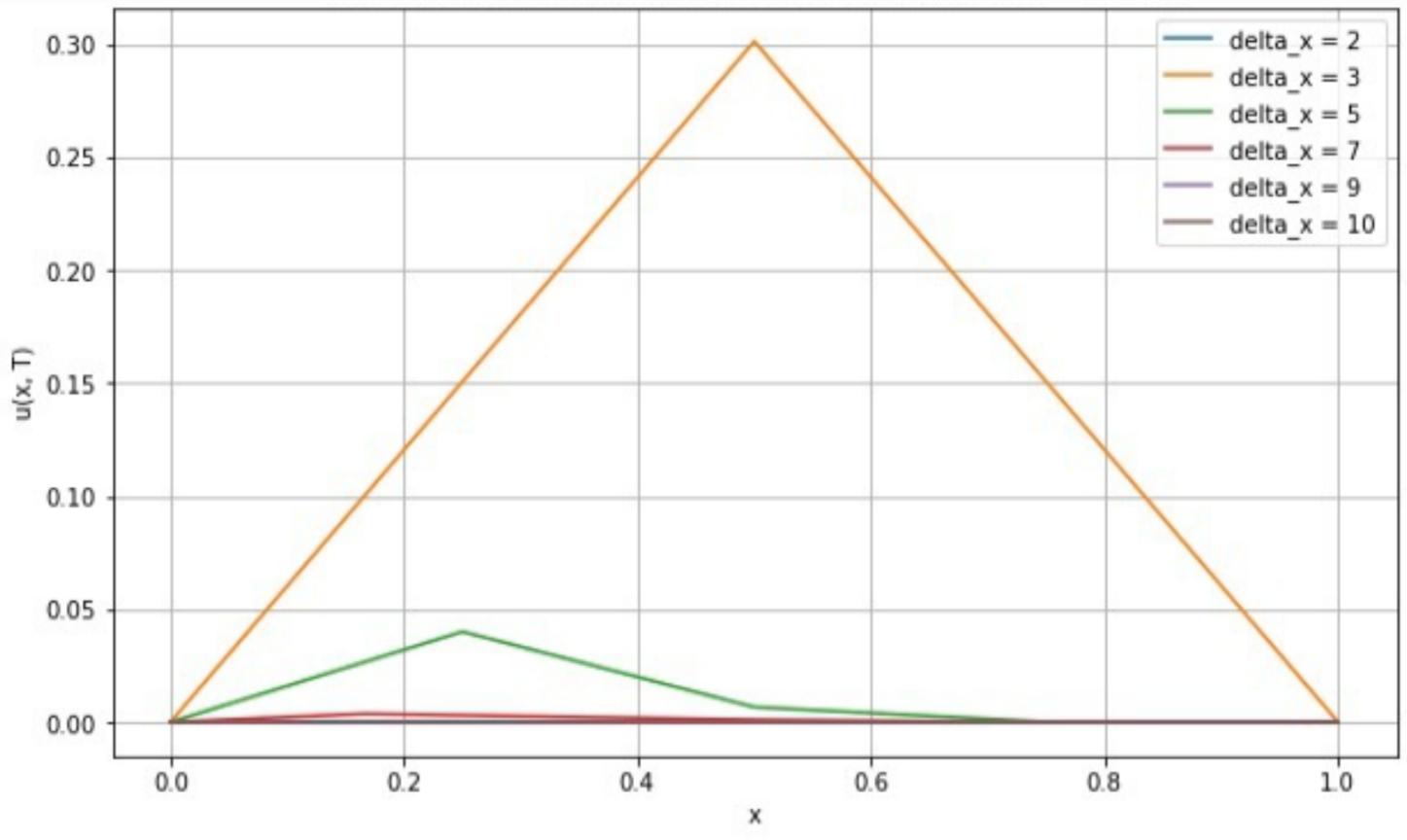
by varying θ value in Problem Soly

python file to 0, 1, 0.5, following
plot were found

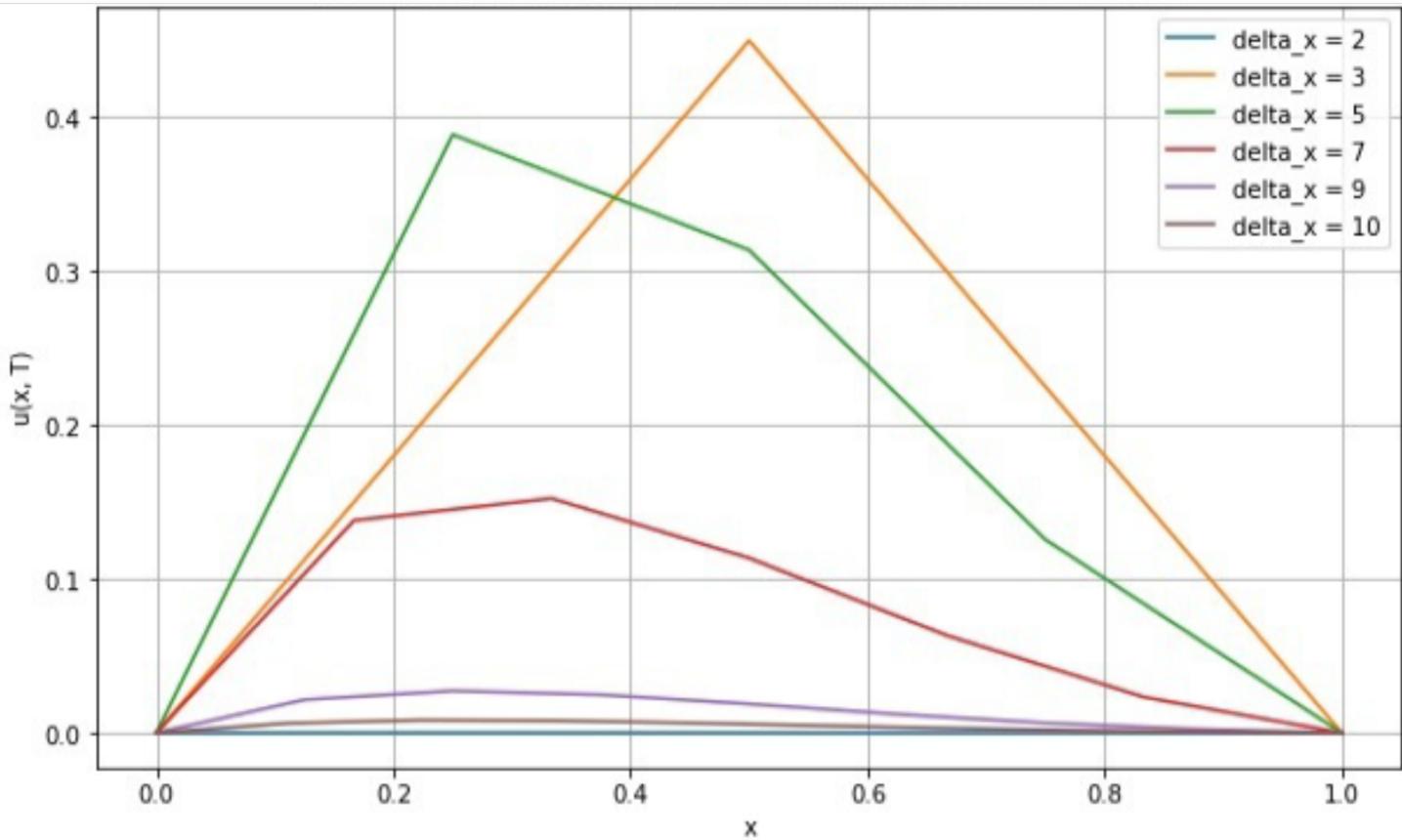
① → at $\theta = 1, \epsilon = 0.1, \Delta t = 0.0012$



② at $\theta = 0, \epsilon = 0.1, \Delta t = 0.0012$



③ at $\theta = 0.5$, $\epsilon = 0.1$, $\Delta t = 0.0012$



Now for Stability :-

$$U_j^{n+1} = U_j^n + \theta M \epsilon (U_{j+1}^{n+1} - 2U_j^{n+1} + U_{j-1}^{n+1}) + (1-\theta) M \epsilon (U_{j+1}^n - 2U_j^n + U_{j-1}^n)$$

$$\text{let } U_j^n = \lambda^n e^{ijk\pi n}$$

$$\lambda^{n+1} e^{ijk\pi n} - \lambda^n e^{ijk\pi n} = \theta M \epsilon \lambda^{n+1} (e^{i(j+1)\pi n} -$$

$$-2e^{ijk\pi n} + e^{i(j-1)\pi n}) + (1-\theta) M \epsilon (\lambda^n (e^{i(j+1)\pi n} - 2e^{ijk\pi n} + e^{i(j-1)\pi n}))$$

÷ by $\lambda^n e^{ikon}$

$$\lambda - 1 = \theta M \epsilon \lambda \left(e^{ikon} - \frac{2 + e^{-ikon}}{2} \right) + (1-\theta) M \epsilon \left(e^{ikon} - \frac{2 + e^{-ikon}}{2} \right)$$

using

$$\frac{e^{ikon} + e^{-ikon}}{2} = \cos kon$$

↓

$$\lambda - 1 = \theta M \epsilon \lambda (1 - \cos kon) + (1-\theta) M \epsilon 2 (1 - \cos kon)$$

↓

$$\lambda - 1 = -\theta M \epsilon \left(4 \sin^2 \frac{1}{2} kon \right) + (1-\theta) M \epsilon \left(-4 \sin^2 \frac{1}{2} kon \right)$$

$$\lambda = \frac{1 - 4(1-\theta) M \epsilon \sin^2 \left(\frac{1}{2} kon \right)}{1 + 4 \theta M \epsilon \sin^2 \left(\frac{1}{2} kon \right)}$$

for Stability, $|\lambda| \leq 1$

↓

$$(1-2\theta) M \epsilon \sin^2 \left(\frac{1}{2} kon \right) \leq \frac{1}{2}$$

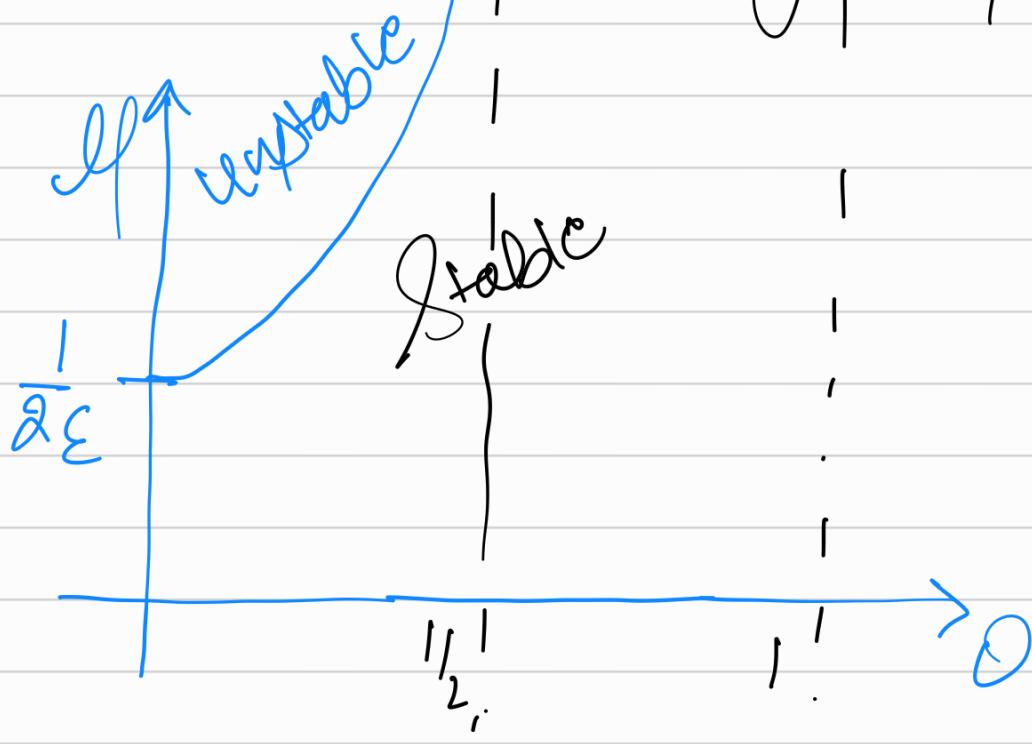
worst will be

$$\sin^2\left(\frac{\theta \Delta n}{2}\right) = 1$$

$$\text{So } (1-2\theta)\mu \epsilon \leq \frac{1}{2}$$

$$\boxed{\mu \leq \frac{1}{2\epsilon(1-2\theta)}}$$

$\mu = \frac{1}{2\epsilon(1-2\theta)}$ \rightarrow Stability criterion
for diffusion part



Now for advection part :-

■ $\lambda = \frac{1 + (1-\theta) \mu q^n D_n (c_i^{n+1} - c_i^n)}{1 - \theta q^n D_n (c_i^n - c_i^{n-1})}$

for worst value of (θ) , $|\lambda|$ to be higher.

$$\max |e^{ik\theta n} - 1|$$

$$e^{ik\theta n} = \cos \theta n + i \sin \theta n$$

∴ when $\cos \theta n = -1$

$$\downarrow \quad \max |e^{ik\theta n} - 1| = \sqrt{4} = 2$$

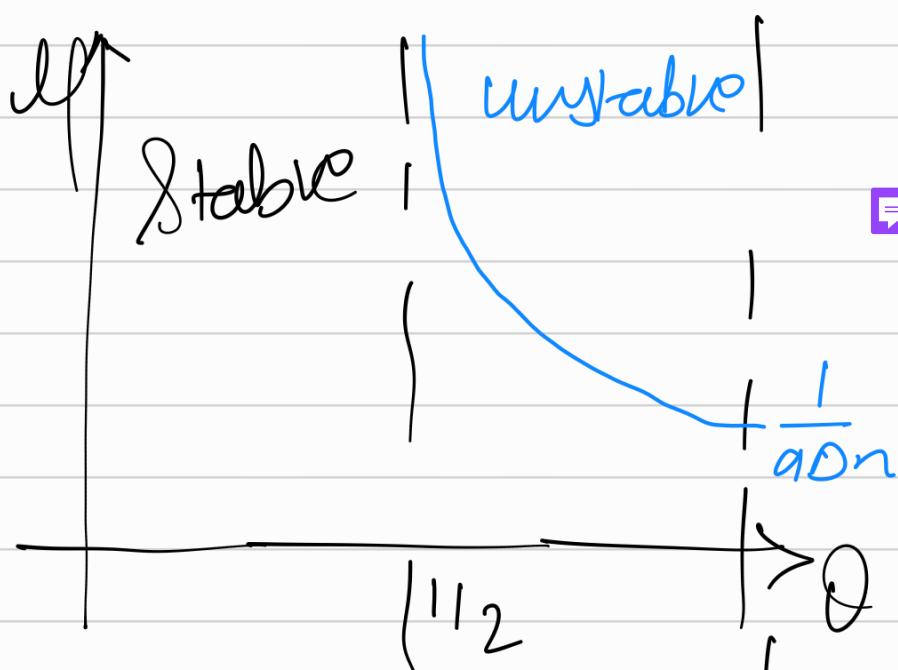
$$\downarrow$$

$$\lambda \leq 1$$

$$\therefore -1 \leq \frac{1+2(1-\theta) \frac{1}{a} a^n \theta n}{1-2\theta \frac{1}{a} a^n + \frac{1}{a} \theta n}$$

for $a = \cos \theta +$

$$\boxed{\left| \frac{1}{a \theta n (2\theta - 1)} \right|}$$



Problem 6 :-

In python code problem 6.py.

Case 1 :-

$$u(x, 0) = (1-x)^4 (1+x)$$

Case 2 :-

$u(x, 0)$ with random value of \textcircled{R}

we have

$$\left\{ \begin{array}{l} u_t - \alpha l f) u - \varepsilon u_{xx} = 0 \\ u(x, 0) = u_0 x \\ u(L_f, t) = u(L, t) = 0 \end{array} \right.$$

$$L_f = -1, L = 1, \varepsilon = 10^{-3}, T = 1$$

