7.5/20 (no numerics)

3) flux $F(A,s)|_{s_{k+\frac{1}{2}}} = F_{k+\frac{1}{2}}(\bar{A}_k^n, \bar{A}_k^{n+1}) \approx \frac{1}{\Delta t}\int_{t_n}^{t_{n+1}} F(A, s_{k+\frac{1}{2}})\,dt$

So $\bar{A}_k^{n+1} = \bar{A}_k^n - \frac{1}{h_k}\int_{t_n}^{t_{n+1}} \cancel{F_{k+\frac{1}{2}}(\bar{A}_k^n, \bar{A}_k^{n+1}) - F_{k-\frac{1}{2}}(\bar{A}_k^n, \bar{A}_k^{n+1})\,dt}$ $F(A,s)|_{s_{k+\frac{1}{2}}} - F(A,s)|_{s_{k-\frac{1}{2}}}\,dt$

$\Rightarrow \bar{A}_k^{n+1} = \bar{A}_k^n - \frac{\Delta t}{h_k}\left(F_{k+\frac{1}{2}}(\bar{A}_k^n, \bar{A}_k^{n+1}) - F_{k-\frac{1}{2}}(\bar{A}_k^n, \bar{A}_k^{n+1})\right)$

4) From question 2, we saw that the characteristic 'speed' $F'(A) > 0$, so the characteristics move to the right. So, the flux taken at a boundary $k+\frac{1}{2}$ is determined by the volume to the left of the boundary. Thus, the Godunov flux $F_{k+\frac{1}{2}} = F(\bar{A}_k^n)$ which is equal to ~~Ak from the Riemann problem solution~~ $F(A_L)$, using A_L from the Riemann Problem solution.

5) The CFL stability condition for the Godunov scheme above defines a timestep, Δt, that must satisfy:

$$\Delta t < CFL \cdot \min_k \frac{h_k}{|\lambda_k|}$$

For $CFL \in (0,1)$, and $\lambda_k = F'(\bar{A}_k^n) > 0$ from question 2.

So $\Delta t < \min_k \frac{h_k}{F'(\bar{A}_k^n)}$ is our time-step restriction.

6) At $s=0$, define our boundary condition, $\bar{A}_{-1}^n$, as our inflow area. Since $F'(A) > 0$, our characteristics move from left to right in the domain $s = [0, 1, \ldots, L_e$.

Fix the flux at $s=0$ $F_{-\frac{1}{2}}(\bar{A}_{-1}^n) = Q_0$

Since $\lambda(A) > 0$, giving us an upwind scheme, the characteristics simply leave the domain at $s = L_e$, so fixing the flux at the end of the domain would not have an effect on the rest of the domain. Thus, no boundary condition is needed at the downstream boundary.