

can be written as

$$b_j^{n+1} = b_j^n - C_1 [(b_j^n)^3 - (b_{j-1}^n)^3 + (b_{j+1}^n)^3 - (b_{j+1}^n)^3] \\ + C_2 [(b_{j+1}^n + b_{j-1}^n)^3 (b_{j+1}^n - b_j^n) - (b_j^n + b_{j-1}^n)^3 (b_j^n - b_{j-1}^n)] \\ + C_2 [(b_{j+1}^n - b_{j-1}^n)^3 (b_{j+1}^n - b_j^n) - (b_j^n + b_{j-1}^n)^3 (b_j^n - b_{j-1}^n)]$$

Where  $C_1 = \alpha \frac{\Delta t}{2\Delta x^2}$  and  $C_2 = \beta \frac{\Delta t}{16(\Delta x)^2}$

$$\bar{J}_{ij} = \frac{\partial R_{ij}}{\partial b_{ij}} = \frac{R_{ij, \text{next}} - R_{ij}}{\epsilon}$$

Solve for  $J \delta b = -R$

$$\Rightarrow b^{n+1} = b^n + \delta b$$

- Crank-nicholson takes longer to compute given more calculations at each step of the calculation.
- Slightly more accurate solution achieved.