$$F = \frac{A^{5/3}\sqrt{-\frac{\partial b}{\partial s}}}{C_m\left(w_0+\frac{2A}{w_0}\right)^{2/3}} = \frac{1}{C_m} A^{5/3}\left(w_0+\frac{2A}{w_0}\right)^{-\frac{2}{3}}\sqrt{-\frac{\partial b}{\partial s}}$$

$$\frac{\partial F}{\partial A} = \frac{1}{C_m}\sqrt{-\frac{\partial b}{\partial s}}\left[\left(\frac{5}{3}A^{2/3}\right)\left(w_0+\frac{2A}{w_0}\right)^{-\frac{2}{3}} + A^{5/3}\left(-\frac{4}{3w_0}\left(w_0+\frac{2A}{w_0}\right)^{-\frac{5}{3}}\right)\right]$$ by product rule

$$= \frac{1}{3C_m}\sqrt{-\frac{\partial b}{\partial s}}\left[\underbrace{5A^{2/3}\left(w_0+\frac{2A}{w_0}\right)^{-\frac{2}{3}}}_{\text{multiply this term by } \left(w_0+\frac{2A}{w_0}\right) \text{ to get equal powers of } \left(w_0+\frac{2A}{w_0}\right)^{-5/3}} - \frac{4}{w_0}A^{5/3}\left(w_0+\frac{2A}{w_0}\right)^{-5/3}\right]$$

$$= \frac{1}{3C_m}\sqrt{-\frac{\partial b}{\partial s}}\left[5A^{2/3}w_0\left(w_0+\frac{2A}{w_0}\right)^{-5/3} + \frac{10A^{5/3}}{w_0}\left(w_0+\frac{2A}{w_0}\right)^{-5/3} - \frac{4}{w_0}A^{5/3}\left(w_0+\frac{2A}{w_0}\right)^{-5/3}\right]$$

$$= \frac{1}{3C_m}\sqrt{-\frac{\partial b}{\partial s}}\left(5w_0A^{2/3} + \frac{6}{w_0}A^{5/3}\right)\left(w_0+\frac{2A}{w_0}\right)^{-5/3} > 0$$

In F, the only s-dependent variable is w_0, so use the chain rule on w_0 to find $\frac{\partial F}{\partial s}$

$$\frac{\partial F}{\partial s} = \frac{1}{C_m}\sqrt{-\frac{\partial b}{\partial s}}\,A^{5/3}\left[-\frac{2}{3}\left(w_0+\frac{2A}{w_0}\right)^{-\frac{5}{3}}\cdot\left(1-\frac{2A}{w_0^2}\right)\right]\frac{dw_0}{ds}$$

$$= -\frac{2}{3C_m}\sqrt{-\frac{\partial b}{\partial s}}\,A^{5/3}\left(1-\frac{2A}{w_0^2}\right)\left(w_0+\frac{2A}{w_0}\right)^{-\frac{5}{3}}\frac{dw_0}{ds}$$

2) Say that w_0 is independent of s now, so the width is uniform in space.
So $F(A,s) \equiv F(A)$ since the only s dependence in F was through w_0.
So the kinematic river equation is now a ~~scalar~~ conservation law:

$$\frac{\partial A}{\partial t} + \frac{\partial F(A)}{\partial s} = 0$$

The 'eigenvalue' of a scalar conservation law $\lambda(A) = F'(A)$ is ~~the~~ ~~characteristic~~ ~~speed~~ $F'(A) = \frac{\partial F}{\partial A} > 0$ from question (1).
So waves always propagate downstream, since our 'eigenvalue' $\lambda(A)$ is the characteristic speed

Now, set up the Riemann problem
use initial conditions $A(s,0) = \begin{cases} A_l, & s<s' \\ A_r, & s>s' \end{cases}$

If $A_l > A_r$, a shock wave occurs with shock speed $d = \frac{F(A_l)-F(A_r)}{A_l-A_r}$
If $A_l < A_r$, a rarefaction wave occurs creating an expanding fan with left-edge $F'(A_l)$ and right edge $F'(A_r)$