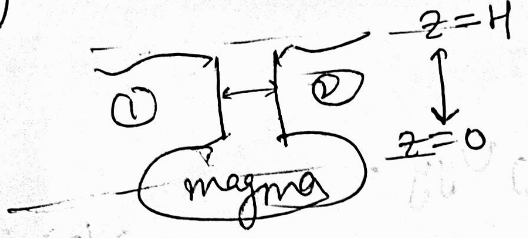
1



width: $b(z,t)$

velocity: $u(z,t)$

$\frac{\partial b}{\partial z}$

$$\frac{\partial b}{\partial t} + \frac{\partial (ub)}{\partial z} = 0 \rightarrow (1) \qquad u = \alpha b^2 - \beta b^2 \frac{\partial b}{\partial z} \rightarrow (2)$$

BC [dirichlet]:

$$b(0,T) = b_B - (3), \quad b(H,T) = b_T \rightarrow (H)$$

$$b(z,0) = b_i(z) \rightarrow (IT) \rightarrow \text{initial condition.}$$

$\alpha(z) = \frac{g(K\rho_r(z) - \rho_m)}{\eta \mu_m}$ and $\beta = \frac{1}{\eta \mu_m \lambda} \geq 0$

(a) $$\partial_t b + \partial_z(\alpha b^3 - \beta b^3 \partial_z b) = 0 \longrightarrow (B)$$

using eq (1) and (2) eq (B) is derived.

$$\frac{\partial b}{\partial t} + \frac{\partial}{\partial z}\left(\alpha b^3 - \beta b^3 \frac{\partial b}{\partial z}\right) = 0$$

$$\frac{\partial b}{\partial t} + 3\alpha b^2 \frac{\partial b}{\partial z} - 3\beta b^2 \left(\frac{\partial b}{\partial z}\right)^2 - \beta b^3 \frac{\partial^2 b}{\partial z^2} = 0 \rightarrow (4)$$

$b = D_0 + b'$ [D_0 = constant, b' = perturbation]

$$\therefore \frac{\partial b'}{\partial t} + 3\alpha (D_0 + b')^2 \frac{\partial b'}{\partial z} - 3\beta (D_0 + b')^2 \left(\frac{\partial b'}{\partial z}\right)^2 - \beta (D_0 + b')^3 \frac{\partial^2 b'}{\partial z^2} = 0$$

neglecting the higher order smaller terms.

$$(D_0 + b')^2 = D_0^2 + (b')^2 + 2D_0 b'$$

$$(D_0 + b')^3 = D_0^3 + (b')^3 + 3D_0^2 b' + 3D_0 (b')^2$$

∴ After neglecting the higher order terms (in b') (having ε^2 or less contributions)

$$\boxed{\frac{\partial b'}{\partial t} + 3\alpha D_0^2 \frac{\partial b'}{\partial z} - \beta D_0^3 \frac{\partial^2 b'}{\partial z^2} = 0} \rightarrow (5)$$

Eq (3) and (5) are nonlinear convection-diffusion eq^n.

As both eq have $\frac{\partial}{\partial z}$ → convection term and $\frac{\partial^2}{\partial z^2}$ → diffusion term and are non-linear before linearization (or in it's original form) as shown by eq (4)

(b) Forward euler time discretization.

upwind scheme ⟹ Convective term

2nd order central difference ⟹ diffusive term

time [forward Euler] →

$$\boxed{\frac{\partial b}{\partial t} + \frac{\partial}{\partial z}\left(\alpha b^3 - \beta b^3 \frac{\partial b}{\partial z}\right) = 0}$$

$$\frac{\partial b}{\partial t} \simeq \frac{b_j^{n+1} - b_j^n}{\Delta t}$$

Convective term :- $\frac{\partial}{\partial z}(\alpha b^3)$ → upwind scheme

$$\cancel{3\alpha b^2 \frac{\partial b}{\partial z} \simeq 3\alpha b_j^2 \left(\frac{b_j^n - b_{j-1}^n}{\Delta z}\right)}$$

$$3\alpha b^2 \frac{\partial b}{\partial z} \simeq 3\alpha (b_j^n)^2 \left(\frac{b_j^n - b_{j-1}^n}{\Delta z}\right)$$

diffusive term:- 2nd order central difference

$$-\beta \frac{\partial}{\partial z}\left(b^3 \frac{\partial b}{\partial z}\right) \longrightarrow \text{diffusive term}$$

$$\left(b^3 \frac{\partial b}{\partial z}\right)_{j+1/2}^{n} \simeq \left(b_{j+1/2}^{n}\right)^3 \left(\frac{b_{j+1}^{n} - b_j^{n}}{\Delta z}\right)$$

$$\left(b^3 \frac{\partial b}{\partial z}\right)_{j-1/2}^{n} \simeq \left(b_{j-1/2}^{n}\right)^3 \left(\frac{b_j^{n} - b_{j-1}^{n}}{\Delta z}\right)$$

$$\frac{\partial}{\partial z}\left(b^3 \frac{\partial b}{\partial z}\right) \simeq \frac{\left(b^3 \frac{\partial b}{\partial z}\right)_{j+1/2} - \left(b^3 \frac{\partial b}{\partial z}\right)_{j-1/2}}{\Delta z}$$

$$\therefore -\beta \frac{\partial}{\partial z}\left(b^3 \frac{\partial b}{\partial z}\right) \simeq \left(b_{j+1/2}^{n}\right)^3 \left[\frac{b_{j+1}^{n} - b_j^{n}}{(\Delta z)^2}\right] - \left(b_{j-1/2}^{n}\right)^3 \left[\frac{b_j^{n} - b_{j-1}^{n}}{(\Delta z)^2}\right]$$

$\therefore$ final eqn :-

$$\frac{b_j^{n+1} - b_j^{n}}{\Delta t} + 3\alpha \left(b_j^{n}\right)^2 \left(\frac{b_j^{n} - b_{j-1}^{n}}{\Delta z}\right)$$

$$-\frac{\beta}{(\Delta z)^2}\left[\underbrace{\left(b_{j+1/2}^{n}\right)^3 \left(b_{j+1}^{n} - b_j^{n}\right) - \left(b_{j-1/2}^{n}\right)^3 \left(b_j^{n} - b_{j-1}^{n}\right)}_{\Downarrow}\right] = 0$$

$$b_{j+1}^{n}\left(b_{j+1/2}^{n}\right)^3 - b_j^{n}\left[\left(b_{j+1/2}^{n}\right)^3 + \left(b_{j-1/2}^{n}\right)^3\right] + b_{j-1}^{n}\left(b_{j-1/2}^{n}\right)^3$$

$$\left(b_{j+1/2}^{n}\right)^3 \simeq \frac{\left(b_{j+1}^{n}\right)^3 + \left(b_j^{n}\right)^3}{2} \longrightarrow (6)$$

$$\left(b_{j-1/2}^{n}\right)^3 \simeq \frac{\left(b_j^{n}\right)^3 + \left(b_{j-1}^{n}\right)^3}{2}$$

discretization of linearized eqn

$$\frac{\partial b'}{\partial t} + 3\alpha D_0^2 \frac{\partial b'}{\partial z} - \beta D_0^3 \frac{\partial^2 b'}{\partial z^2} = 0$$

$$\frac{b_j'^{n+1} - b_j'^{n}}{\Delta t} + 3\alpha D_0^2 \left(\frac{b_j'^{n} - b_{j-1}'^{n}}{\Delta z} \right) - \beta D_0^3 \left(\frac{b_{j+1}'^{n} - 2b_j'^{n} + b_{j-1}'^{n}}{(\Delta z)^2} \right) = 0 \quad \to (7)$$

check (7) is the linearized version of (6)

<u>Convective terms</u>

$$3\alpha (b_j^n)^2 \left(\frac{b_j^n - b_{j-1}^n}{\Delta z} \right) \qquad \boxed{b_j^n = D_0 + b_j'^n}$$

neglecting the non-linear terms

$$3\alpha (D_0 + b_j'^n)^2 \left[\frac{D_0 + b_j'^n - D_0 - b_{j-1}'^n}{\Delta z} \right] = \boxed{3\alpha D_0^2 \left(\frac{b_j'^n - b_{j-1}'^n}{\Delta z} \right)}$$

same as in eq (7)

<u>diffusive term</u>:

$$\frac{-\beta}{(\Delta z)^2} \left\{ b_{j+1}^n \left[\frac{(b_{j+1}^n)^3 + (b_j^n)^3}{2} \right] - b_j^n \left[\frac{(b_{j+1}^n)^3 + (b_{j-1}^n)^3}{2} \right] + b_{j-1}^n \left[\frac{(b_j^n)^3 + (b_{j-1}^n)^3}{2} \right] \right\}$$

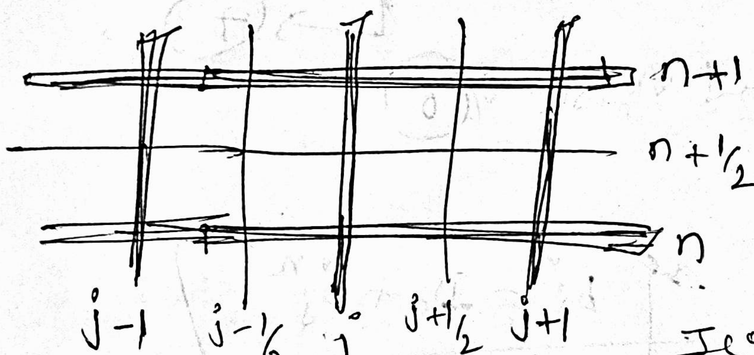
$$\Rightarrow \frac{-\beta}{2(\Delta z)^2} \left[(b_j^n)^3 \left[b_{j+1}^n - 2b_j^n + b_{j-1}^n \right] + (b_{j+1}^n)^3 \left[b_{j+1}^n - b_j^n \right] - (b_{j-1}^n)^3 \left[b_j^n - b_{j-1}^n \right] \right]$$

Substituting $b_j^n = D_0 + b_j'^n$ and neglecting non-linear terms $\therefore (b_j^n)^3 = D_0^3 = (b_{j+1}^n)^3 = (b_{j-1}^n)^3$

$$\Rightarrow \frac{-\beta D_0^3}{2(\Delta z)^2}\left[b'^n_{j+1} - 2b'^n_j + b'^n_{j-1} + b'^n_{j+1} - b'^n_j - b'^n_j + b'^n_{j-1}\right]$$

$$\Rightarrow \boxed{\frac{-\beta D_0^3}{(\Delta z)^2}\left[b'^n_{j+1} - 2b'^n_j + b'^n_{j-1}\right]}$$

↳ same as Eq (7)



BC:
$b(0,t) \simeq b(0,t_n) = b_B \;\forall n$
$b(H,t) \simeq b(H,t_n) = b_T \;\forall n$

IC: $b(z,0) \simeq b(z_j,0) = b_i(z) \simeq b_i(z_j) \;\forall j$

(c) Fourier Analysis

(1) $\alpha = 0,\ \beta \neq 0$ → Pure diffusion.

$$\frac{b'^{n+1}_j - b'^n_j}{\Delta t} - \frac{\beta D_0^3}{(\Delta z)^2}\left(b'^n_{j+1} - 2b'^n_j + b'^n_{j-1}\right) = 0$$

$$b'^n_j = \lambda_k^n \exp(ikj\Delta x) \qquad \boxed{\frac{\beta D_0^3 \Delta t}{(\Delta z)^2} = \mu}$$

$$b'^{n+1}_j = b'^n_j + \mu\left(b'^n_{j+1} - 2b'^n_j + b'^n_{j-1}\right)$$

$$\lambda_k^{n+1}\exp(ikj\Delta x) = \lambda_k^n \exp(ikj\Delta x) + \mu\left[\begin{array}{l}\lambda_k^n \exp[ik(j+1)\Delta x] \\ -2\lambda_k^n \exp[ikj\Delta x] \\ +\lambda_k^n \exp[ik(j-1)\Delta x]\end{array}\right]$$

$$\lambda_k = 1 + \mu\left[\exp(ik\Delta x) - 2 + \exp(-ik\Delta x)\right]$$

$$\lambda_k = 1 + \mu\left[2\cos(k\Delta x) - 2\right] \qquad \cos 2x = 1 - 2\sin^2 x$$

$$2\cos 2x - 2 = -4\sin^2 x$$

$$\boxed{\lambda_k = 1 + \mu\left\{-4\sin^2\left(\frac{k\Delta x}{2}\right)\right\}}$$

$d_k = \cancel{1-4\mu\sin} \; 1 - 4\mu \sin^2\left(\frac{k\Delta x}{2}\right)$

$\sin^2\theta \rightarrow$ always $+$ve

$\boxed{d_{k\,min} = 1 - 4\mu}$

$\boxed{d_{k\,max} = 1}$

~~stability~~ stability :- $-1 \le d \le 1$

$\therefore 1 - 4\mu \geq -1 \rightarrow 4\mu \leq 2 \Rightarrow \boxed{\mu \leq \frac{1}{2}}$

$$\mu = \frac{\beta D_0^3 \Delta t}{(\Delta z)^2} \leq \frac{1}{2} \Rightarrow \boxed{\Delta t \leq \frac{\Delta z^2}{2\beta D_0^3}}$$

(b) $\alpha \neq 0, \beta = 0 \Rightarrow$ Pure ~~conduction~~ convection

$$\frac{b_j'^{\,n+1} - b_j'^{\,n}}{\Delta t} + 3\alpha D_0^2 \left(\frac{b_j'^{\,n} - b_{j-1}'^{\,n}}{\Delta z}\right) = 0$$

$\boxed{b_j'^{\,n} = d_k^n \exp(ikj\Delta x)}$

$\boxed{\frac{3\alpha D_0^2 \Delta t}{\Delta z} = \mu_2}$

$$b_j'^{\,n+1} - b_j'^{\,n} + \left(\frac{3\alpha D_0^2 \Delta t}{\Delta z}\right)\left(b_j'^{\,n} - b_{j-1}'^{\,n}\right) = 0$$

$$b_j'^{\,n+1} - b_j'^{\,n} + \mu_2\left(b_j'^{\,n} - b_{j-1}'^{\,n}\right) = 0$$

$$d_k^{n+1}\exp(ikj\Delta x) = d_k^n \exp(ikj\Delta x) - \mu_2\left(d_k^n \exp(ikj\Delta x) - d_k^n \exp(ik\Delta x(j-1))\right)$$

$$d_k = 1 - \mu_2\left(1 - \exp(-ik\Delta x)\right)$$

$$d_k = 1 - \mu_2\left(1 - \cos(k\Delta x) + i\sin(k\Delta x)\right)$$

magnitude of $d_k \Rightarrow |d_k|^2 = \left[1 - \mu_2(1-\cos(k\Delta x))\right]^2 + \left[\mu_2 \sin(k\Delta x)\right]^2$

$$\Rightarrow 1 - 2\mu_2(1-\cos(k\Delta x)) + \underbrace{\mu_2^2(1-\cos(k\Delta x))^2}_{} + \mu_2^2\sin^2(k\Delta x)$$

$$1 - 2\mu_2 + 2\mu_2\cos(k\Delta x) + \mu_2^2 - 2\mu_2^2\cos(k\Delta x) + \mu_2^2$$

$$\Rightarrow 1 - 2\mu_2 + 2\mu_2\cos(k\Delta x) + 2\mu_2^2 - 2\mu_2^2\cos(k\Delta x)$$

$$\Rightarrow |\lambda_k|^2 = 1 - 2\mu_2(1-\cos(k\Delta x)) + 2\mu_2^2(1-\cos(k\Delta x))$$

ⓐ $\cos(k\Delta x) = -1 \longrightarrow |\lambda_k|^2_{max} \rightarrow$ maximum

$$|\lambda_k|^2_{max} = 1 - 4\mu_2 - 4\mu_2^2 = (1-2\mu_2)^2$$

Stability

$$|1-2\mu_2| \leq 1$$

$$\Updownarrow$$

$$0 \leq \mu_2 \leq 1 \qquad \mu_2 = \frac{3\alpha D_0^2\Delta t}{\Delta z}$$

$$\frac{3\alpha D_0^2\Delta t}{\Delta z} \leq 1 \longrightarrow \boxed{\Delta t \leq \frac{\Delta z}{3\alpha D_0^2}}$$

ⓓ Maximum principle $\longrightarrow$ stable time step for discretization

$$\frac{b_j'^{n+1} - b_j'^{n}}{\Delta t} + 3\alpha D_0^2\frac{b_j'^{n} - b_{j-1}'^{n}}{\Delta z} - \frac{\beta D_0^3}{(\Delta z)^2}\left(b_{j+1}'^{n} - 2b_j'^{n} + b_{j-1}'^{n}\right) = 0$$

$$b_j'^{n+1} = \left(1 - 3\alpha D_0^2\frac{\Delta t}{\Delta z} - 2\beta D_0^3\frac{\Delta t}{\Delta z^2}\right)b_j'^{n} + \underbrace{\beta D_0^3\frac{\Delta t}{\Delta z^2}}_{a}\left(b_{j+1}'^{n}\right)$$

$$+ \underbrace{\left(3\alpha D_0^2\frac{\Delta t}{\Delta z} + \beta D_0^3\frac{\Delta t}{\Delta z^2}\right)}_{b} b_{j-1}'^{n}$$

if $\alpha, \beta, \Delta t$ and $\Delta z > 0$ $\therefore$ a & b are +ve

$$\therefore 1 - 3\alpha D_0^2\frac{\Delta t}{\Delta z} - 2\beta D_0^3\frac{\Delta t}{\Delta z^2} > 0$$

$$\Delta t\frac{(3\alpha D_0^2\Delta z + 2\beta D_0^3)}{(\Delta z)^2} < 1 \longrightarrow \boxed{\Delta t < \frac{(\Delta z)^2}{2\beta D_0^3 + 3\alpha D_0^2\Delta z}}$$

for Maximum principle

(e)

$$\frac{b_j^{n+1} - b_j^n}{\Delta t} + \frac{3\alpha (b_j^n)^2}{\Delta z}\left(b_j^n - b_{j-1}^n\right)$$

$$- \frac{\beta}{2(\Delta z)^2}\begin{bmatrix} \left[(b_j^n)^3 + (b_{j+1}^n)^3\right]\left[b_{j+1}^n - b_j^n\right] \\ - \left[(b_{j-1}^n)^3 + (b_j^n)^3\right]\left[b_j^n - b_{j-1}^n\right]\end{bmatrix} = 0$$

$$b_j^{n+1} = b_j^n - \frac{3\alpha\Delta t}{\Delta z}(b_j^n)^3 + \frac{3\alpha\Delta t}{\Delta z}(b_j^n)^2(b_{j-1}^n)$$

$$+ \frac{\beta\Delta t}{2(\Delta z)^2}\begin{bmatrix} (b_j^n)^3 b_{j+1}^n - 2(b_j^n)^4 + (b_{j+1}^n)^4 \\ - b_j^n (b_{j+1}^n)^3 - (b_{j-1}^n)^3 b_j^n + (b_{j-1}^n)^4 \\ + (b_j^n)^3 b_{j-1}^n \end{bmatrix}$$

$$b_j^{n+1} = \left[1 - \frac{3\alpha\Delta t}{\Delta z}(b_j^n)^2 - \frac{\beta\Delta t\, 2}{2(\Delta z)^2}(b_j^n)^3\right] b_j^n +$$

$$b_{j-1}^n\left[\frac{3\alpha\Delta t}{\Delta z}(b_j^n)^2 + \frac{\beta\Delta t}{(\Delta z)^2 2}\left((b_{j-1}^n)^3 + (b_j^n)^3 - b_j (b_{j-1}^n)^2\right)\right]$$

$$+ b_{j+1}^n\left[\frac{\beta\Delta t}{2(\Delta z)^2}\left((b_j^n)^3 + (b_{j+1}^n)^3 - b_j^n (b_{j+1}^n)^2\right)\right]$$

$\alpha > 0$, $\beta > 0$, $b_j > 0$ for maximum priciple.
we are taking ~~terms~~ that might ~~give~~ negative influ

$$\therefore\ 1 - \frac{3\alpha\Delta t}{\Delta z}(b_j^n)^2 - \frac{\beta\Delta t}{2(\Delta z)^2}\left[2(b_j^n)^3 + (b_{j-1}^n)^3 + (b_{j+1}^n)^3\right] > 0$$

$$\Delta t\left[3\alpha(\Delta z)(b_j^n)^2 - \frac{\beta}{2}\left[2(b_j^n)^3 + (b_{j-1}^n)^3 + (b_{j+1}^n)^3\right]\right] < (\Delta z)^2$$

$$\boxed{\Delta t < \frac{(\Delta z)^2}{3\alpha(\Delta z)(b_j^n)^2 + \beta/2\left(2 (b_j^n)^3 + (b_{j-1}^n)^3 + (b_{j+1}^n)^3\right)}}$$

(f) 2nd order spatial discretization of convective term

$$b(t_n, z_j + \Delta z) = b_j + \Delta z \frac{\partial}{\partial z} b_j^n + \frac{(\Delta z)^2}{2} \frac{\partial^2}{\partial z^2} b_j^n + O(z^3)$$

$$b(t_n, z - \Delta z) = b_j - \Delta z \frac{\partial}{\partial z} b_j^n + \frac{(\Delta z)^2}{2} \frac{\partial^2 b_j^n}{\partial z^2} - O(z^3)$$

$$b_{j+1}^n - b_{j-1}^n = 2 \Delta z \frac{\partial b_j}{\partial z}$$

$$\therefore \boxed{\frac{\partial b_j}{\partial z} = \frac{b_{j+1}^n - b_{j-1}^n}{2(\Delta z)}}$$

$\therefore$ eqn becomes :-

$$\frac{b_j^{n+1} - b_j^n}{\Delta t} + \frac{3\alpha (b_j^n)^2}{2\Delta z}\left(b_{j+1}^n - b_{j-1}^n\right)$$

$$-\frac{\beta}{2(\Delta z)^2}\begin{bmatrix} b_{j+1}^n\left[(b_{j+1}^n)^3 + (b_j^n)^3\right] - b_j^n\left[(b_{j+1}^n)^3 + (b_{j-1}^n)^3\right] \\ + b_{j-1}^n\left[(b_j^n)^3 + (b_{j-1}^n)^3\right] \end{bmatrix} = 0$$

$$\Rightarrow b_j^{n+1} = b_j^n - \frac{3\alpha \Delta t (b_j^n)^2 (b_{j+1}^n)}{2\Delta z} + \frac{3\alpha (b_j^n)^2}{2\Delta z} b_{j-1}^n$$

$$+ \frac{\beta \Delta t}{2(\Delta z)^2}\begin{bmatrix} b_{j+1}^n\left[(b_{j+1}^n)^3 + (b_j^n)^3\right] \\ - b_j^n\left[(b_{j+1}^n)^3 + (b_{j-1}^n)^3\right] \\ + b_{j-1}^n\left[(b_j^n)^3 + (b_{j-1}^n)^3\right] \end{bmatrix}$$

for $b_j > 0$ and for maximum principle, considering -ve terms

$$1 - \frac{3\alpha \Delta t}{2(\Delta z)}(b_j^n)(b_{j+1}^n) - \frac{\beta \Delta t}{2(\Delta z)^2}\left[(b_{j+1}^n)^3 + (b_{j-1}^n)^3\right] > 0$$

< 1

end

$$\boxed{\Delta t < \frac{2(\Delta z)^2}{3\alpha \Delta z (b_j^n)(b_{j+1}^n) + \beta\left[(b_{j+1}^n)^3 + (b_{j-1}^n)^3\right]}}$$

② steady state solution of

ⓐ $\cancel{\frac{\partial b}{\partial t}} + \frac{\partial}{\partial z}\left(\alpha b^3 - \beta b^3 \frac{\partial b}{\partial z}\right) = 0$

$0 \quad\quad$ ∵

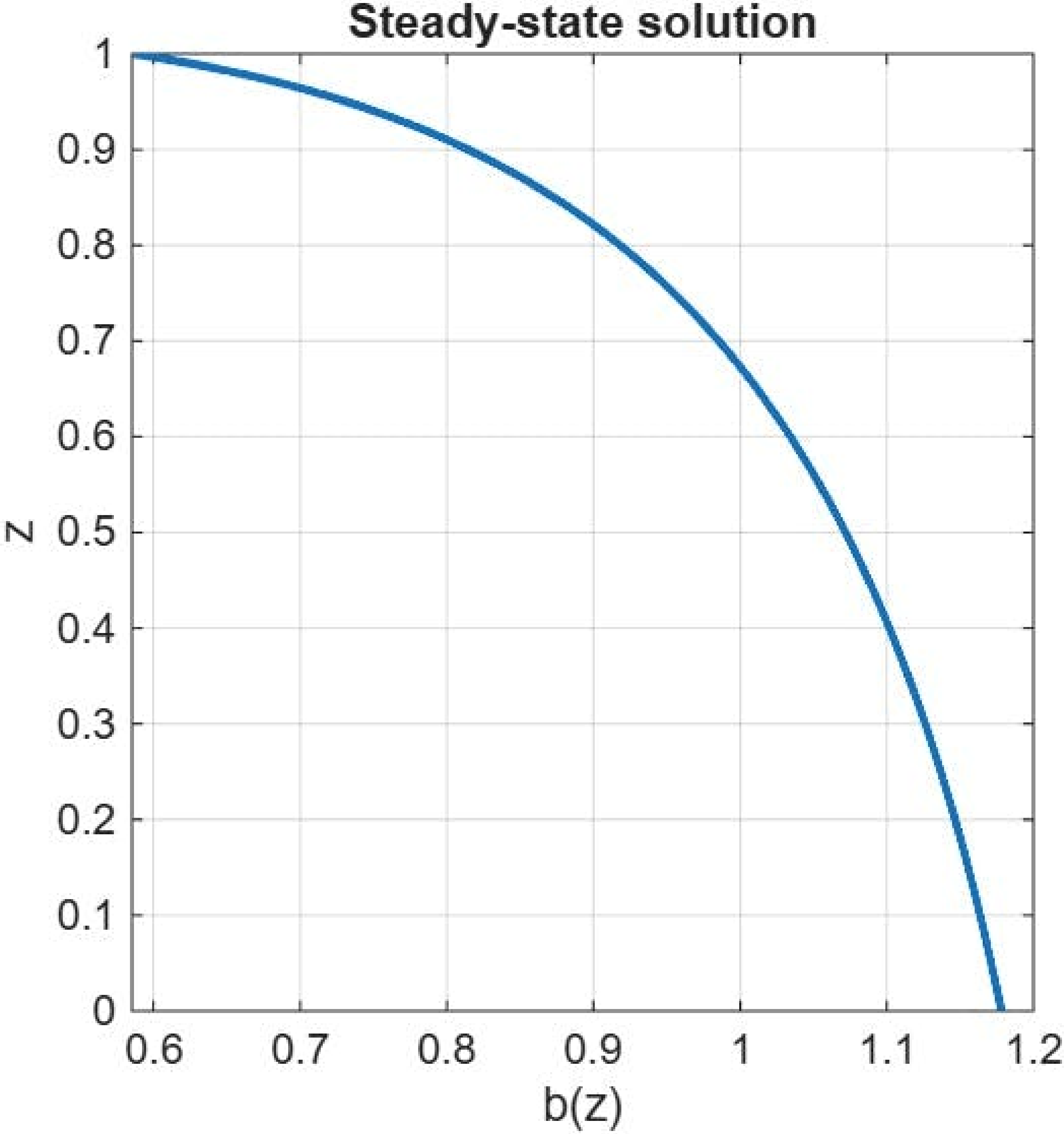
steady state $\quad \therefore \frac{\partial}{\partial z}\left(\alpha b^3 - \beta b^3 \frac{\partial b}{\partial z}\right) = 0$

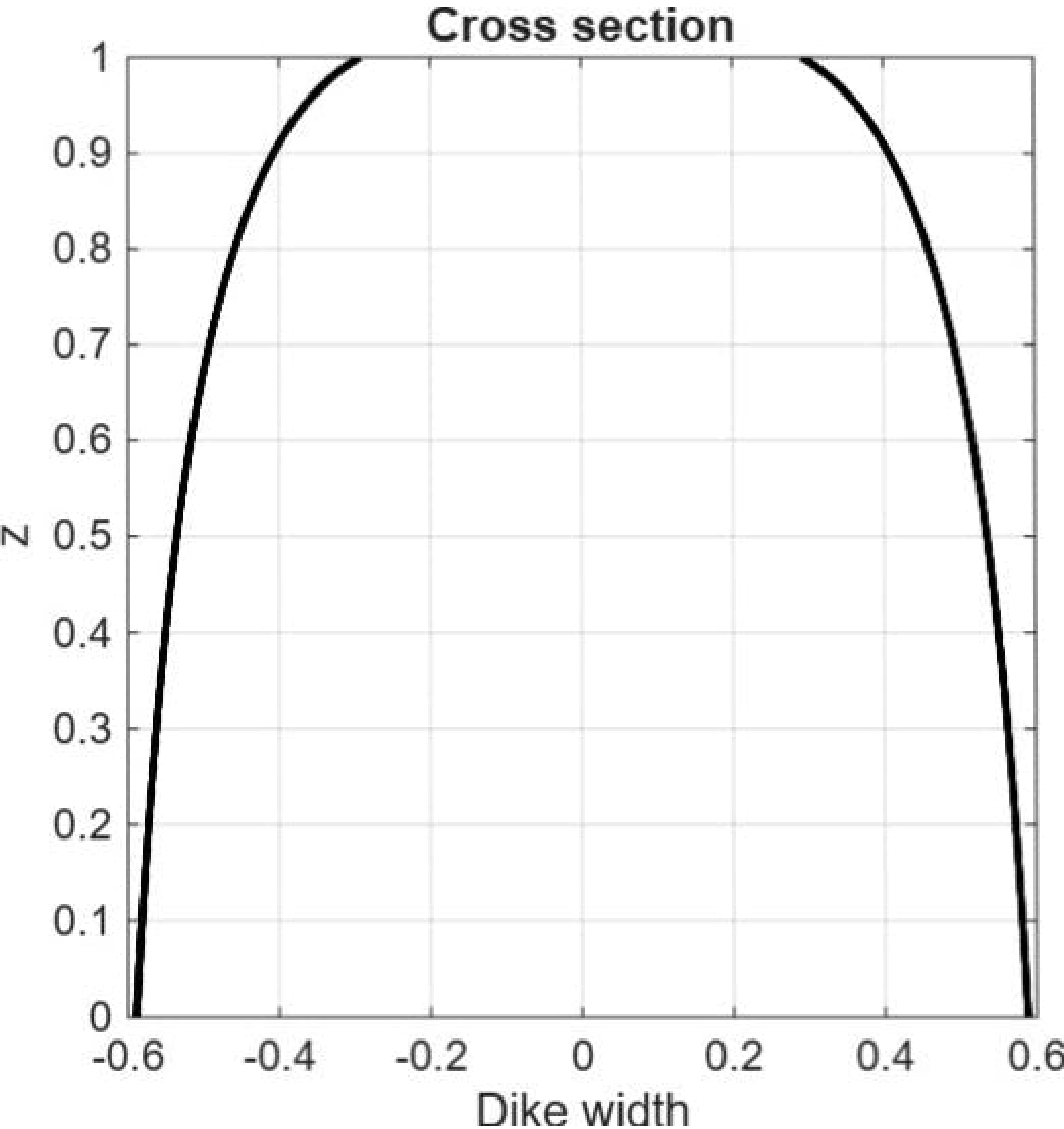
$$\alpha b^3 - \beta b^3 \frac{\partial b}{\partial z} = \varphi$$

$$\therefore \beta b^3 \frac{\partial b}{\partial z} = \alpha b^3 - \varphi \quad , \quad \frac{\partial b}{\partial z} = \frac{b_{j+1}^n - b_j^n}{\Delta z}$$

$$\frac{b_{j+1}^n - b_j^n}{\Delta z} = \frac{\alpha b^3}{\beta b^3} - \frac{\varphi}{\beta b^3}$$

$$\boxed{b_{j+1}^n = \frac{\alpha}{\beta}\Delta z - \frac{\varphi \Delta z}{\beta b^3} + b_j^n}$$


Steady-state solution
z
b(z)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.6
0.7
0.8
0.9
1
1.1
1.2



Cross section
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
z
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
Dike width

(b) Nonlinear convection - diffusion Eqn:-

$t = 0.05, 0.1, 0.2, 0.5, 1, 2$

grid points 11, 21, 41

$$\frac{\partial b}{\partial t} + \frac{\partial}{\partial z}\left(\alpha b^3 - \beta b^3 \frac{\partial b}{\partial z}\right) = 0$$

$b_j^0 = b_T \cong 0.58537373798$

I'm using $dt = 1e-5$

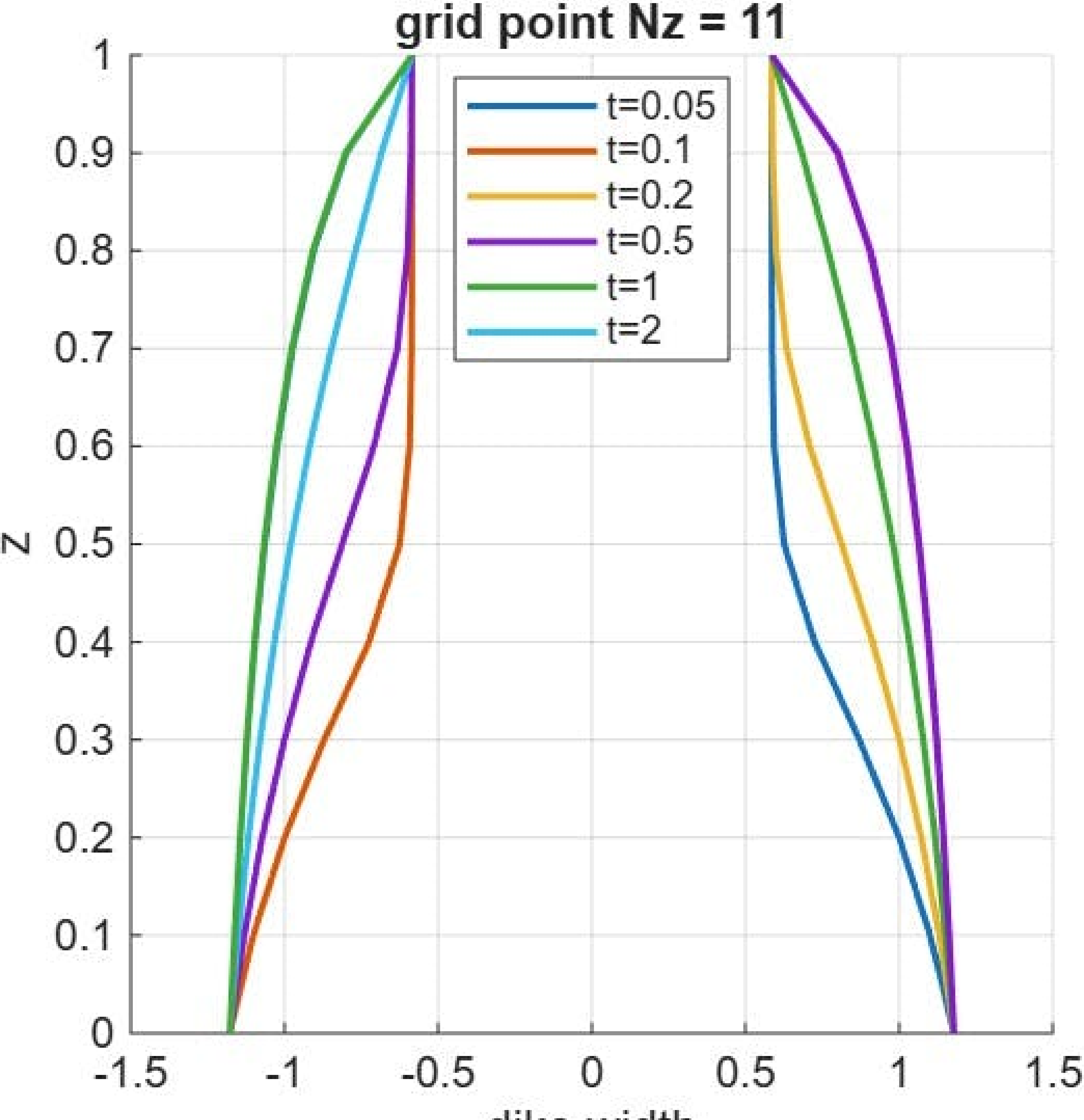
* Solution goes to ∞ for 1e-3 for grid point 41
stable time step ~~for~~ discretization should be

$$\Delta t < \frac{(\Delta z)^2}{3\alpha(\Delta z)(b_j^n)^2 - \frac{\beta}{2}\left(2(b_j^n)^3 + (b_{j-1}^n)^3 + (b_{j+1}^n)^3\right)}$$

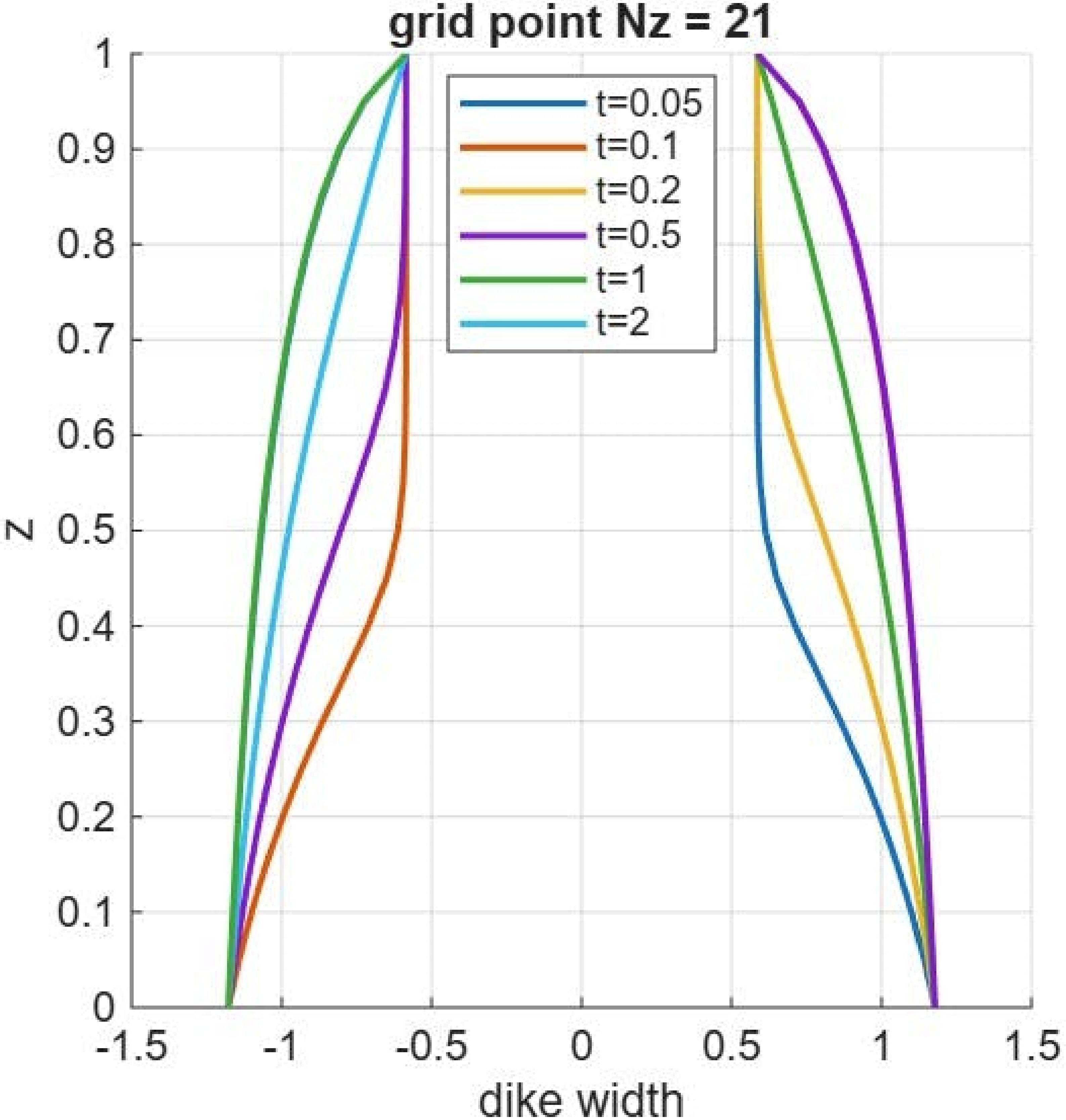
or

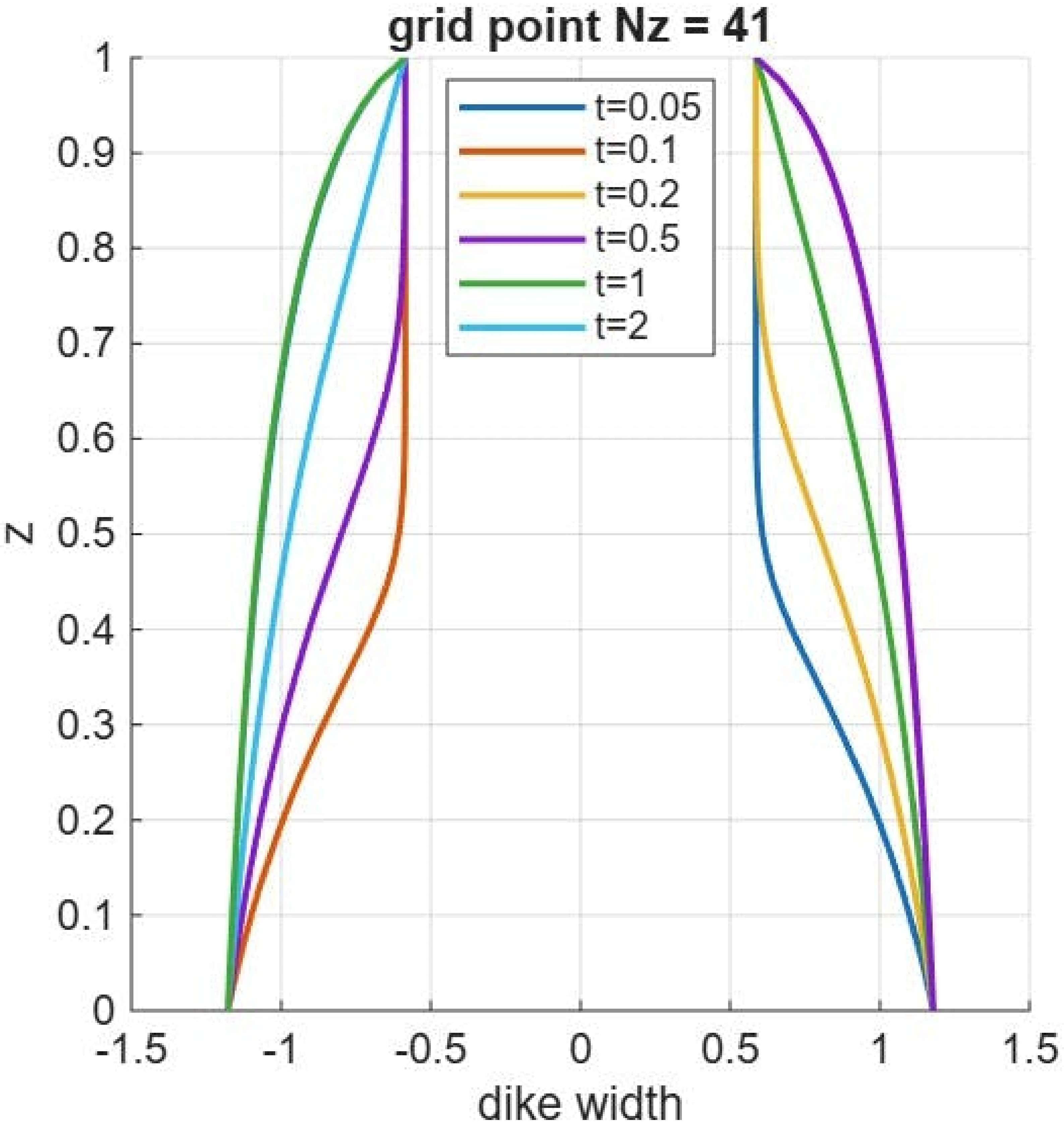
$$\Delta t < \frac{(\Delta z)^2}{2\beta D_0^2 + 3\alpha D_0^2 \Delta z}$$

∴ to avoid this I'm going with 1e-5



grid point Nz = 11
t=0.05
t=0.1
t=0.2
t=0.5
t=1
t=2
z
dike width
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
-1.5
-1
-0.5
0
0.5
1
1.5


grid point Nz = 21
t=0.05
t=0.1
t=0.2
t=0.5
t=1
t=2
z
dike width
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-1.5
-1
-0.5
0
0.5
1
1.5



grid point Nz = 41
t=0.05
t=0.1
t=0.2
t=0.5
t=1
t=2
z
dike width