Exercise sheet 2

$$\frac{\partial A}{\partial t} + \frac{\partial (AU)}{\partial s} = 0 \,, \qquad \frac{\partial U}{\partial t} + U\frac{\partial U}{\partial s} = -g\frac{\partial (h+b)}{\partial s} - g\,C_m^2\, U|U|/R^{4/3} \qquad (1)$$

velocity $U = U(s,t)$, coordinate s, time t

Cross-sectional wetted area $A = A(s,t)$

water depth $h = h(s,t) \equiv h(A(s,t),s)$

Hydraulic radius $R = R(A,s) = A/P$

for wetted Perimeter $P = P(A,s)$

Manning relation for U and discharge $Q = AU$

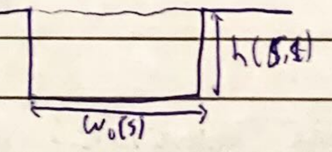
$$U = \frac{R^{2/3}\sqrt{-\frac{\partial b}{\partial s}}}{C_m} \qquad Q = AU = \frac{A^{5/3}\sqrt{-\frac{\partial b}{\partial s}}}{C_m P^{2/3}} \qquad (2)$$

Substituting into (1) gives

$$\frac{\partial A}{\partial t} + \frac{\partial}{\partial s}\left(\frac{AR^{2/3}\sqrt{-\frac{\partial b}{\partial s}}}{C_m}\right) = S \qquad \text{'volume' source } S(s,t) \qquad (3)$$

$$\Rightarrow \frac{\partial A}{\partial t} + \frac{\partial}{\partial s}F(A,s) = S \qquad \text{with } F(A,s) = \frac{A^{5/3}\sqrt{-\frac{\partial b}{\partial s}}}{C_m P^{2/3}} = Q \qquad (4)$$

i) Rectangular river cross-section, varying river width $w_0(s)$



The cross-sectional area $A(s,t) = w_0(s)\cdot h(s,t)$

$$\Rightarrow h(s,t) = \frac{A}{w_0}$$

The wetted perimeter is the perimeter of the rivers walls and bed.

so $P(A,s) = w_0(s) + 2h(s,t)$

$$= w_0(s) + \frac{2A}{w_0}$$

$$F(A,s) = \frac{A^{5/3}\sqrt{-\frac{\partial b}{\partial s}}}{C_m\left(w_0 + \frac{2A}{w_0}\right)^{2/3}}$$

Let $S = 0$, so $\frac{\partial A}{\partial t} + \frac{\partial F}{\partial s} = 0$ for $F = F(A,s)$

$\frac{\partial F}{\partial s} = \frac{\partial F}{\partial A}\frac{\partial A}{\partial s} + \frac{\partial F}{\partial s}$ by the chain rule

so now $\frac{\partial A}{\partial t} + \frac{\partial F}{\partial A}\frac{\partial A}{\partial s} + \frac{\partial F}{\partial s} = 0$