

Wave-to-wire
energy device

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(Jonathan
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Kalogirou,
Duncan
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Harvey
Thompson)

Introduction

Modelling

Linearise
wave-2-wire
model

Discretise
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model

Preliminary
results

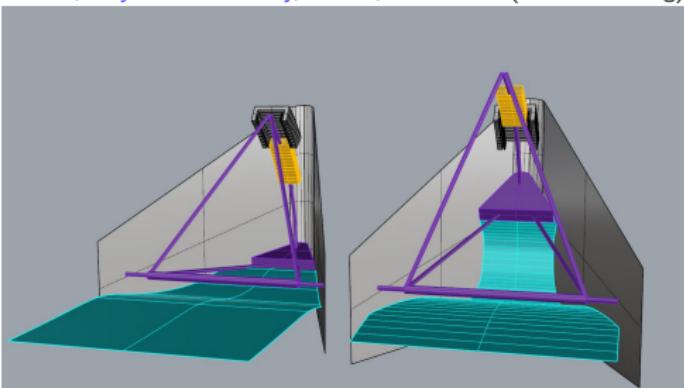
Conclusions

On a novel wave-to-wire energy device in a breakwater contraction

Onno Bokhove (Jonathan Bolton, Anna Kalogirou, Duncan Borman, Harvey Thompson)

Leeds Institute for Fluid Dynamics (LIFD)

COER, Maynooth University, Ireland, 11-03-2021 (EPSRC funding)



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Introduction: maths wave-2-wire device

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Goals & challenges:

- introduce novel wave-energy device;
- provide **nonlinear model** for new wave-2-wire device;
- provide **linearised shallow-water model** of device;
- **discretise** in space, in time & in space-time;
- show **preliminary simulations**; and,
- **open questions**: validation, optimisation,

Motivation: rogue waves

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Rogue waves are anomalously high waves defined relative to a significant wave height H_s .

- Index (Kharif et al. '09, Dysthe et al. '08):

$$AI = H_{rw}/H_s > 2 \quad \text{or} \quad AI = \eta_{rw}/H_s > 1.25$$

- Relevance in maritime & coastal engineering
- Pyramidal rogue wave (Faulkner 2001):



Fig.1. Pyramidal wave off south Japan

Motivation: rogue waves

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There are many causes of rogue waves, see Kharif et al. (2009):

- E.g., crossing seas, nearly standing/pyramidal waves
- We made a *bore-soliton splash* rogue wave:
$$AI = \frac{H_{rw}}{H_s} = \frac{3.5}{0.35} \approx 10$$
- It inspired our wave-2-wire wave-energy device.



Introduction: working wave-2-wire device

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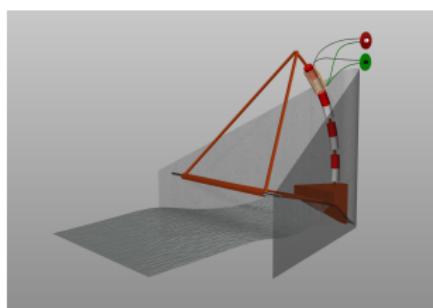
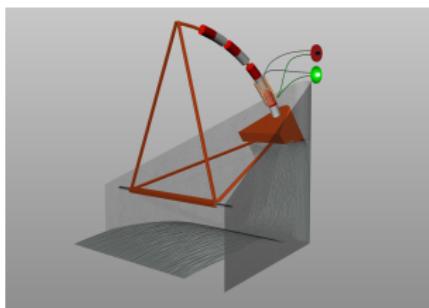
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Design by B. & Zweers:

- Sketch with **water waves**, wave-activated **buoy motion** & electro-magnetic generator & **LED**-loads:



- ***Proof-of-principle*** of laboratory device (2013)

https://www.youtube.com/watch?v=SZhe_S0xBWo

Modelling

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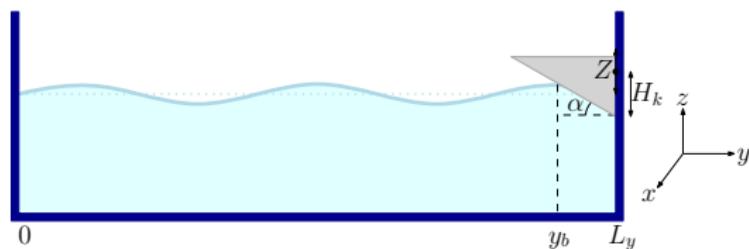
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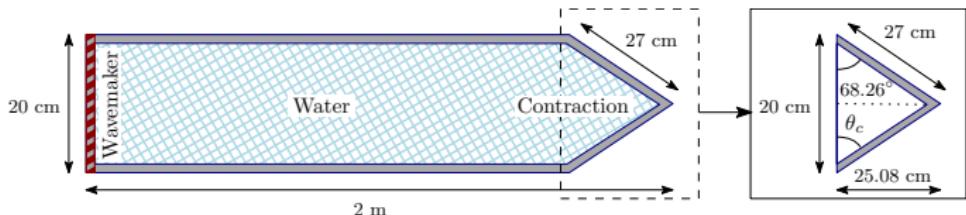
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- Wave-2-wire model in a **wavetank**, side view along center-line:



- Wave-2-wire model in a **wavetank**, top view:



Nonlinear wave-2-wire model

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Dynamics of water waves $\{\phi, h\}$, wave-activated buoy motion $\{Z, W\}$ & electro-magnetic generator $\{Q, P_Q\}$ with current I , $I = \dot{Q}$ and $P_Q = L_i I - K(Z)$ & LED-loads:

$$\nabla^2 \phi = 0$$

$$\partial_t h + \nabla_H \phi \cdot \nabla h - \partial_z \phi = 0 \text{ on } z = h$$

$$\partial_t \phi + \frac{1}{2} |\nabla \phi|^2 + g(h - H_0) + \underline{\lambda \Theta(y - y_b)} = 0 \quad \text{on } z = h$$

$$h - h_b = 0 \quad \text{for } y \geq y_b(x, t)$$

$$\dot{Z} = W$$

$$M \dot{W} = -Mg - \gamma(P_Q + K(Z)) \frac{G(Z)}{L_i} + \rho_0 \int_0^{L_x} \int_0^{l_y(x)} \underline{\lambda \Theta(y - y_b)} \, dx \, dy$$

$$\dot{Q} = (P_Q + K(Z))/L_i \equiv I$$

$$\dot{P}_Q = -(R_c + R_i)I - \underline{\text{sign}(I)n_q V_T \ln(|I|/I_{sat} + 1)}.$$

Nonlinear wave-2-wire model: hydrodynamics

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Variables & parameters:

- **water waves:** potential $\mathbf{u} = \nabla\phi(x, y, z, t)$
- **water waves:** water depth $h = h(x, y, t)$, surface potential $\phi_s(x, y, t) = \phi(x, y, h(x, y, t), t)$

$$\nabla^2\phi = 0$$

$$\partial_t h + \nabla_H \phi \cdot \nabla h - \partial_z \phi = 0 \text{ on } z = h$$

$$\partial_t \phi + \frac{1}{2} |\nabla \phi|^2 + g(h - H_0) + \underline{\lambda \Theta(y - y_b)} = 0 \quad \text{on } z = h$$

$$\underline{h - h_b} = 0 \quad \text{for } y \geq y_b(x, t)$$

- acceleration of gravity g and rest position H_0 .

Nonlinear wave-2-wire model: buoy motion

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Variables & parameters:

- **wave-activated buoy**: position $Z(t)$ & velocity $W(t)$

$$\underline{h - h_b} = 0 \quad \text{for } y \geq y_b(x, t)$$

$$\dot{Z} = W$$

$$M\dot{W} = -Mg - \gamma(P_Q + K(Z)) \frac{G(Z)}{L_i} + \rho_0 \int_0^{L_x} \int_0^{l_y(x)} \underline{\lambda \Theta(y - y_b)} \, dx \, dy$$

- *Lagrange multiplier* $\lambda(x, y, t)$ imposes that water depth h equals buoy shape h_b :

$$\underline{h(x, y, t) - h_b(x, y; Z(t))} = 0$$

Nonlinear wave-2-wire model: EM motor

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Variables & parameters:

- electromagnetic generator: charge $Q(t)$ & conjugate $\dot{P}_Q(t) = L_i \dot{Q} - K(Z)$; $\dot{P}_Q(t) = \underline{L}_i \dot{\underline{I}} - \gamma G(Z) \dot{\underline{Z}}$
- EM-theory $K(Z) = \int^Z \gamma G(\tilde{Z}) d\tilde{Z}$ thin-wire & symmetric
- induction L_i , buoy/magnet mass M , reference water level H_0 , etc.
- **two LEDs in parallel:** adapted Shockley equation.

$$\dot{Q} = (P_Q + K(Z))/L_i \equiv I$$

$$\dot{P}_Q = -(R_c + R_i)I - \underline{\text{sign}(I)n_q V_T \ln \left(|I|/I_{sat} + 1 \right)}.$$

Wave-2-wire model: EM motor

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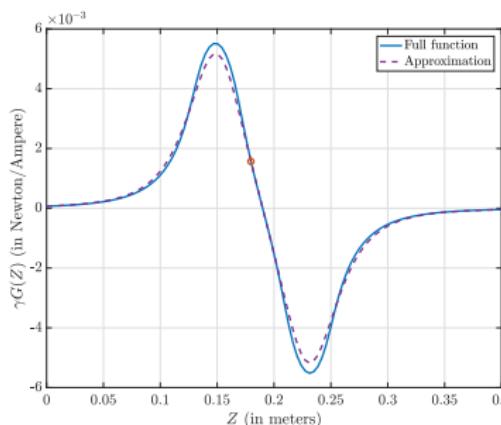
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- By solving the Maxwell's equations, the function

$$G(Z) = \frac{\pm 1}{\pi A_m^2 L_m a} \int_{-L/2}^{L/2} \int_0^{2\pi} \int_0^{A_m} F(z) dr d\theta dq$$

- arises with $F(z) = f(-z) - f(z)$, $z = q + \bar{Z} + \alpha_h H_m - Z$,

$$f(z) = r(a - r \cos \theta) / (r^2 + (\frac{L_m}{2} + z)^2 + a^2 - 2ra \cos \theta)^{3/2}.$$



Wave-2-wire model

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- When the change of current I in time & loads small, s.t.
 $\dot{I} \approx 0$:

$$(R_c + R_i + \underline{R_l})I \approx \gamma G(Z)\dot{Z},$$

- then the vertical momentum equation for buoy, mast and magnet becomes approximately

$$\begin{aligned} M\dot{W} = -Mg - \frac{\gamma^2 G^2(Z)}{L_i(R_c + R_i + \underline{R_l})} W \\ + \rho_0 \int_0^{L_x} \int_0^{l_y(x)} \lambda \Theta(y - y_b) dx dy. \end{aligned}$$

- Resistance and loads combined act as drag* on the buoy-mast-magnet unit in this partial linear limit.

Wave-2-wire variational principle

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Wave-to-wire model posited using 3 combined VPs:

- Luke (1967), Cotter & B. (2010, JEM), $1 + 1 + 1 = 3$:

$$\begin{aligned} 0 &= \delta \int_0^T \mathcal{L}[D, \phi, h, \phi_s, Z, W, Q, P_Q, p, \lambda] dt \\ &\equiv \delta \int_0^T \rho_0 \int_0^{L_x} \int_{R(t)}^{l_y(x)} \int_0^{h(x,y,t)} D \partial_t \phi \, dz \, dy \, dx - MW \dot{Z} - P_Q \dot{Q} + \mathcal{H} \, dt \\ &\equiv \delta \int_0^T \rho_0 \int_0^{L_x} \int_{R(t)}^{l_y(x)} \int_0^{h(x,y,t)} D \partial_t \phi + \frac{1}{2} D |\nabla \phi|^2 + g D (z - H_0) \\ &\quad + p(D - 1) \, dz \, dy \, dx \\ &+ \rho_0 \int_0^{L_x} \int_{R(t)}^{l_y(x)} \lambda(h - h_b) \Theta(y - y_b(x, t)) \, dx \, dy \\ &\underline{- MW \dot{Z} - P_Q \dot{Q} + \frac{1}{2} MW^2 + MgZ + \frac{1}{2} \frac{(P_Q + K(Z))^2}{L_i} \, dt} \end{aligned}$$

Wave-2-wire model: queries

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Conclusions

- Charge equation not needed (Bolton), unless a capacitor is added and/or a battery depending on Q ?
- Instead of LEDs: (mathematical) model for battery?
- Addition of spring with adjustable spring constant in motor & model for active control of resonance?

Linearise wave-2-wire equations

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Linearise & simplify to shallow water with only (x, y) :

$$\eta - \tilde{Z} = 0 \quad \text{for } y \geq L_b, \quad \dot{R} = \partial_y \tilde{\phi} \quad \text{at } y = 0$$

$$\partial_t \eta + \nabla \cdot (H \nabla \tilde{\phi}) = 0, \partial_t \tilde{\phi} + g\eta + \tilde{\lambda} \Theta(y - L_b) = 0$$

$$\dot{\tilde{Z}} = \tilde{W}$$

$$M \dot{\tilde{W}} + \gamma G(\tilde{Z}) \tilde{I} - \rho_0 \int_0^{L_x} \int_0^{l_y(x)} \tilde{\lambda} \Theta(y - L_b) \, dy \, dx = 0$$

$$\dot{\tilde{Q}} = \tilde{I}, \quad \dot{\tilde{P}}_Q = -(R_c + R_i + \frac{n_q V_T}{I_{sat}}) \tilde{I}, \quad \tilde{I} = \frac{(\tilde{P}_Q + \gamma G(\tilde{Z}) \tilde{Z})}{L_i}$$

$$\implies \nabla \cdot (H \nabla \tilde{\lambda}) - \frac{\rho_0}{M} \int_0^{L_x} \int_0^{l_y(x)} \tilde{\lambda} \Theta(y - L_b) \, dy \, dx =$$

$$- \nabla \cdot (g H \nabla \eta) - \frac{\gamma}{M} G(\tilde{Z}) \tilde{I} \quad \text{for } y \geq L_b.$$

Discrete wave-2-wire equations

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- Finite elements in space CG1, symplectic Euler in time.
- Consistent discretisation.

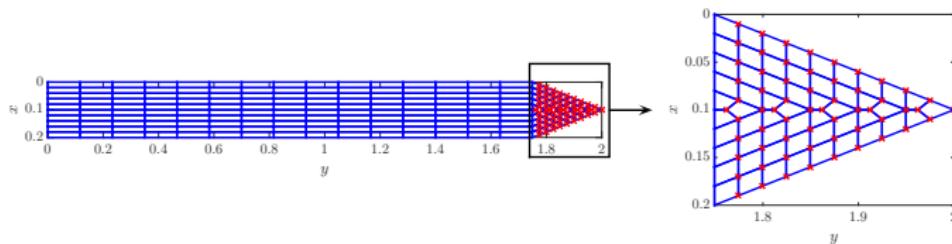


Figure: Computational mesh for $N_x = 10$, $N_y = 15$.

- For Δx & $\Delta x/2$: $O(\sqrt{\Delta t})$ & $O(\Delta t^{3/4})$ & $O(\Delta x^{1.7})$.
- Energy partitioning over subsystems consistent.

Convergence in time

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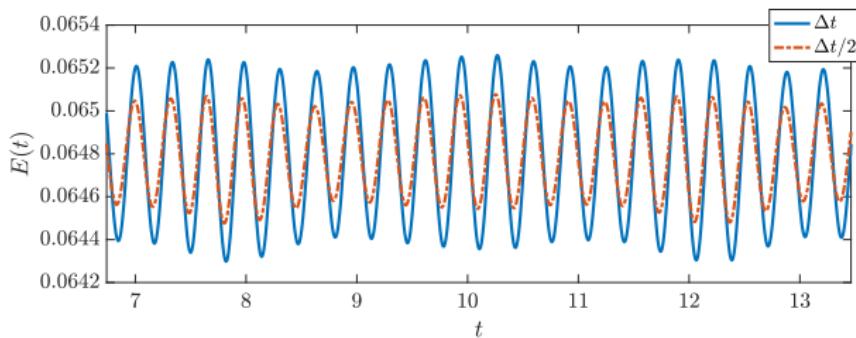
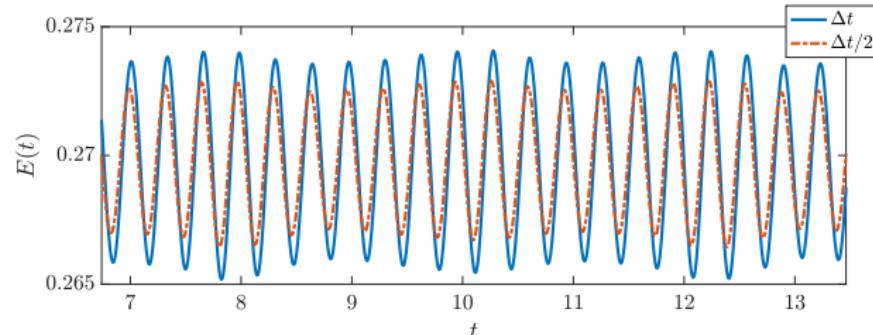
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For Δx & $\Delta x/2$, we find $O(\sqrt{\Delta t})$ & $O(\Delta t^{3/4})$:



Convergence in space

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Table: Convergence rates n using three different norms (\mathcal{L}_1 , \mathcal{L}_2 , \mathcal{L}_∞) evaluated using the value of the velocity potential $\tilde{\phi}$ at the final time of the simulation, i.e. at $t = T$.

| | x-el | y-el | Total el | Nodes | Rate |
|--------|-------|-------|----------|-------|---------------------------------|
| Symbol | N_x | N_y | N_k | N_n | n |
| Mesh 1 | 6 | 30 | 201 | 241 | \mathcal{L}_1 : 1.711293 |
| Mesh 2 | 12 | 60 | 798 | 877 | \mathcal{L}_2 : 1.696554 |
| Mesh 3 | 24 | 120 | 3180 | 3337 | \mathcal{L}_∞ : 1.765833 |

Preliminary results full model

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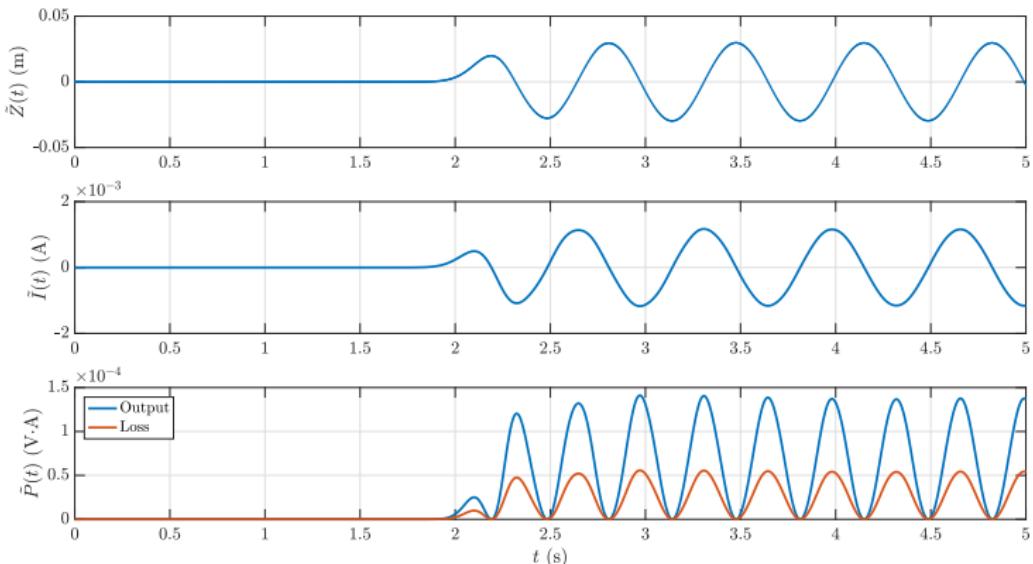
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Vertical displacement, current & total power
 $(R_c + R_i + \underline{n_q V_T / I_{sat}}) \tilde{I}^2$:



Preliminary results

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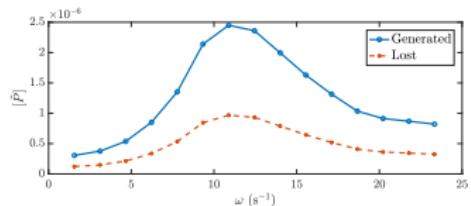
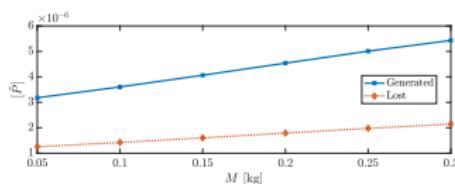
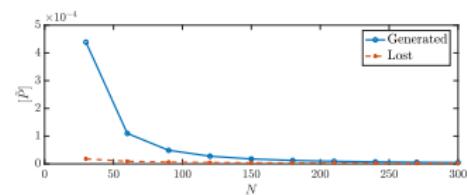
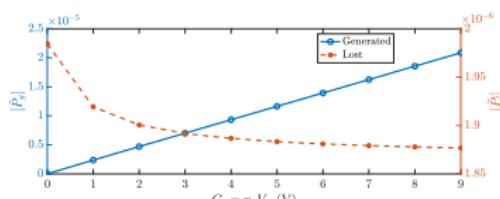
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Optimal harvest in linear case at resonance, maximum loads and minimum windings:



Conclusions & Future work

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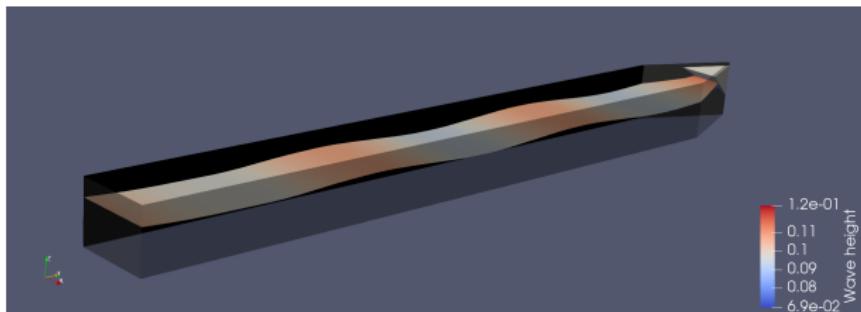
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- Full wave-2-wire maths model from *1st principles*.
- Established space-time compatible discretisation of linearised model.
- Explored parameter space in preliminary fashion.
- Leeds' Fluid Dynamics CDT project with *J. Bolton*:
 - nonlinear Benney-Luke hydrodynamic model (2DH)
 - faster than 3D potential-flow for optimization of parameter space (geometries, mass, spring constant, loads)



Conclusions & Future work

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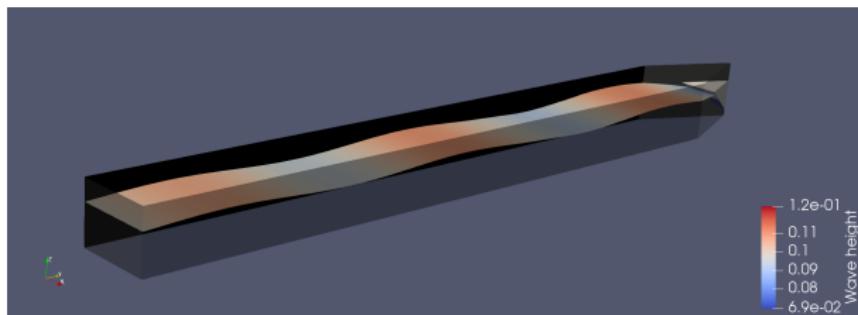
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- **Validation!** Preferred location: **breakwater**.
- Discretise to **higher-order** in space & time.
- Numerically implement **nonlinear models**.
- **Optimise** wave-2-wire device wrt energy-output efficiency.
- B., Kalogirou & Zweers 2019: On wave-2-wire model. *Water Waves* **1**.
<https://link.springer.com/article/10.1007/s42286-019-00022-9>
- B., Kalogirou, Henry & Thomas 2020: *Int. Marin Energy J.* **30** (EWTEC2019, Napoli)
- Bolton et al. 2022: EWTEC2021, Southampton, pending.



Thank you!

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$$\partial_t \phi - g \eta = 0$$
$$\sigma = \frac{1}{2} (\partial_t \eta)^2 - \frac{1}{2} (\nabla \eta)^2 + (h - c) \partial_x \eta$$
$$\lambda = \eta - \partial_x^2 \eta - \partial_x^2 \eta = 0 \text{ in } [0, L] \times [0, T]$$

$\eta = \text{const}$

$\boxed{\eta - c \leq 0}$

- variational inequ. $\eta(x, t) = C$ $\forall x \in I \subset [0, L]$
- complementarity problem $\forall t$
- active set methods $\forall \eta \in H^1([0, L]) : \int_I (\eta'(x)) \eta(x) dx = 0$ $\forall \eta \in L^2(I)$
- contact problem
- $= UU^T Z - UU^T Z W^T (I - ZW^T)^{-1} \leq C$

Appendix Wave-2-wire VP

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Vary & get eqns of motion (conj. pairs + multipliers):

$$\delta D : \partial_t \phi + \frac{1}{2} |\nabla \phi|^2 + g(z - H_0) + p = 0 \quad (13a)$$

$$\delta \phi : \partial_t D + \nabla \cdot (D \nabla \phi) = 0 \quad (13b)$$

$$\delta p : D = 1 \quad (13c)$$

$$\delta \lambda : h - h_b = 0 \quad \text{for } y \geq y_b(x, t) \quad (13d)$$

$$\delta \phi_s : \partial_t h + \nabla \phi \cdot \nabla h = \partial_z \phi \quad \text{at } z = h(x, y, t) \quad (13e)$$

$$\delta h : \partial_t \phi + \frac{1}{2} |\nabla \phi|^2 + g(h - H_0) + \lambda \Theta(y - y_b(x, y, t)) = 0 \quad \text{at } z = h(x, y, t) \quad (13f)$$

$$\delta \phi_R : \dot{R} = \partial_y \phi \quad \text{at } y = R(t) \quad (13g)$$

$$\delta W : \dot{Z} = W \quad (13h)$$

$$\begin{aligned} \delta Z : M \dot{W} + Mg + \frac{\gamma G(Z)}{L_i} (P_Q + K(Z)) \\ - \rho_0 \int_0^{L_x} \int_{R(t)}^{l_y(x)} \lambda \Theta(y - y_b(x, t)) \, dx \, dy = 0 \end{aligned} \quad (13i)$$

$$\delta P_Q : \dot{Q} = \frac{(P_Q + K(Z))}{L_i} \equiv I \quad (13j)$$

$$\delta Q : \dot{P}_Q = 0, \quad (13k)$$

Wave-2-wire VP

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- Free surface: $z = h(x, y, t)$ for $y < y_b(x, t)$
- Constrained surface: $z = h(x, y, t)$ for $y \geq y_b(x, t)$
- **Waterline motion** $y = y_b(x, t)$; evaluate Bernoulli in interior at surface, compare with free-surface Bernoulli:

$$p(x, y, h(x, y, t), t) = 0 \quad y < y_b(x, t)$$

$$p(x, y, h(x, y, t), t) = \lambda(x, y, t) \quad y \geq y_b(x, t),$$

- so $\lambda(x, y_b(x, t), t) = 0$ at $y = y_b$ is derived
- when $D = 1$ imposed strongly, like in Luke (1967 –WW only), then need to impose $\lambda(x, y_b, t) = 0$ a priori.

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- Add electrical resistance & LED loads (Shockley eqns in parallel):

$$\dot{P}_Q = - (R_c + R_i)I - \text{sign}(I)n_q V_T \ln \left(\frac{|I|}{I_{sat}} + 1 \right)$$

$$\text{with } I = \frac{(P_Q + K(Z))}{L_i}.$$

- Simplex buoy shape:

$$z = h_b(x, y; Z(t)) = Z(t) - H_k - \tan \alpha (y - L_y).$$

Appendix Discrete wave-2-wire equations

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Space CG (second-order, tent functions) FEM needs to be consistent; trick: local extension around waterline $y = L_b$

$$N_{\bar{k}l}^T (\eta_l - 1_l \tilde{Z}) = 0 \quad (28a)$$

$$M_{kl} \dot{\phi}_l = -g M_{kl} \eta_l - N_{k\bar{l}} \lambda_{\bar{l}} - N_{k\bar{b}} \lambda_{\bar{b}} \quad (28b)$$

$$M_{kl} \dot{\eta}_l = S_{kl} \phi_l + T_k \dot{R} \quad (28c)$$

$$\dot{\tilde{Z}} = \tilde{W} \quad (28d)$$

$$\dot{\tilde{W}} = C \tilde{Q}_{\bar{l}} \lambda_{\bar{l}} + C \tilde{Q}_{\bar{b}} \lambda_{\bar{b}} - C_1 G(\bar{Z})(\tilde{P}_Q + \gamma G(\bar{Z})\tilde{Z}) \quad (28e)$$

$$\dot{\tilde{Q}} = \frac{(\tilde{P}_Q + \gamma G(\bar{Z})\tilde{Z})}{L_i} \quad (28f)$$

$$\dot{\tilde{P}}_Q = -C_2 (\tilde{P}_Q + \gamma G(\bar{Z})\tilde{Z}) \quad (28g)$$

$$\begin{aligned} (\tilde{S}_{\bar{k}\bar{l}} + C \tilde{Q}_{\bar{k}} \tilde{Q}_{\bar{l}}) \lambda_{\bar{l}} &= -g S_{\bar{k}l} \eta_l - C \tilde{Q}_{\bar{k}} \tilde{Q}_{\bar{b}} \lambda_{\bar{b}} - \tilde{S}_{\bar{k}\bar{b}} \lambda_{\bar{b}} \\ &\quad + C_1 \tilde{Q}_{\bar{k}} G(\bar{Z})(\tilde{P}_Q + \gamma G(\bar{Z})\tilde{Z}) \end{aligned} \quad (28h)$$

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- Indexing of nodes: N_n nodes $k, l = 1, \dots, N_n$;
 $N_n - N_p + 1$ nodes under buoy $\tilde{k}, \tilde{l} = N_p, \dots, N_n$;
 N_b waterline nodes $\tilde{b} = N_p, \dots, N_p + N_b - 1$
- Mass & Laplace matrices:

$$M_{kl} = \int_0^{L_x} \int_0^{l_y(x)} \varphi_k(x, y) \varphi_l(x, y) dx dy, \quad (29a)$$

$$S_{kl} = \int_0^{L_x} \int_0^{l_y(x)} H(y) \nabla \varphi_k(x, y) \cdot \nabla \varphi_l(x, y) dx dy, \quad (29b)$$

$$\tilde{S}_{\tilde{k}\tilde{l}} = \int_0^{L_x} \int_0^{l_y(x)} H(y) \nabla \varphi_{\tilde{k}}(x, y) \cdot \nabla \varphi_{\tilde{l}}(x, y) dx dy, \quad (29c)$$

$$\tilde{Q}_{\tilde{k}} = \int_0^{L_x} \int_0^{l_y(x)} \varphi_{\tilde{k}}(x, y) dx dy, \quad (29d)$$

$$N_{k\tilde{l}} = \int_0^{L_x} \int_0^{l_y(x)} \varphi_k(x, y) \varphi_{\tilde{l}}(x, y) dx dy, \quad (29e)$$

$$T_k = \int_0^{L_x} H(0) \varphi_k(0, y) dy. \quad (29f)$$

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Consistency on discrete level (but only with **trick**):

The key consistency check is to ensure that the first seven equations in (28) are consistent with the last seven equations in (28). Consider the first seven equations. Take the time derivative of the primary constraint and eliminate the time derivatives by using two of the other seven equations, to obtain the secondary constraint

$$N_{kl}^T \left(M_{lk}^{-1} (S_{km}\phi_m + T_k \dot{R}) - 1_l \tilde{W} \right) = 0. \quad (30)$$

Now take the time derivative of this secondary constraint above and again eliminate the time derivatives by using two different equations of these seven equations, to obtain the consistency equation

$$\begin{aligned} \left(N_{kl}^T M_{lk}^{-1} S_{km} M_{mn}^{-1} N_{nl} + C N_{kl}^T 1_l \tilde{Q}_{\bar{l}} \right) \lambda_{\bar{l}} &= -g N_{kl}^T M_{lk}^{-1} S_{km} \eta_m \\ - \left(N_{kl}^T M_{lk}^{-1} S_{km} M_{mn}^{-1} N_{n\bar{b}} + C N_{kl}^T 1_l \tilde{Q}_{\bar{b}} \right) \lambda_{\bar{b}} \\ + N_{kl}^T M_{lk}^{-1} T_k \ddot{R} + C_1 N_{kl}^T 1_l G(\bar{Z}) \left(\tilde{P}_Q + \gamma G(\bar{Z}) \tilde{Z} \right). \end{aligned} \quad (31a)$$

This consistency equation matches the last equation (28h) if and only if the following relations hold

$$\tilde{S}_{\bar{k}\bar{l}} = N_{kl}^T M_{lk}^{-1} S_{km} M_{mn}^{-1} N_{n\bar{l}} \quad (31b)$$

$$\tilde{S}_{\bar{k}m} = N_{kl}^T M_{lk}^{-1} S_{km} \quad (31c)$$

$$\tilde{S}_{\bar{k}\bar{b}} = N_{kl}^T M_{lk}^{-1} S_{km} M_{mn}^{-1} N_{n\bar{b}} \quad (31d)$$

$$\tilde{Q}_{\bar{k}} = N_{kl}^T 1_l \quad (31e)$$

$$N_{kl}^T M_{lk}^{-1} T_k = 0. \quad (31f)$$

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■ Time only: symplectic Euler first order

$$\eta^{n+1} - \bar{Z}^{n+1} = 0 \quad \text{for } y \geq L_b \quad (25a)$$

$$\frac{(\tilde{\phi}^{n+1} - \tilde{\phi}^n)}{\Delta t} + g\eta^n + \bar{\lambda}^n \Theta(y - L_b) = 0 \quad (25b)$$

$$\begin{aligned} \frac{(\tilde{W}^{n+1} - \tilde{W}^n)}{\Delta t} + C_1 G(\bar{Z})(\tilde{P}_Q^n + \gamma G(\bar{Z})\bar{Z}^n) \\ - C \int_0^{L_x} \int_0^{l_y(x)} \bar{\lambda}^n \Theta(y - L_b) \, dy \, dx = 0 \end{aligned} \quad (25c)$$

$$\frac{(\tilde{P}_Q^{n+1} - \tilde{P}_Q^n)}{\Delta t} = -C_2 (\tilde{P}_Q^{n+1} + \gamma G(\bar{Z})\bar{Z}^n) \quad (25d)$$

$$\frac{(\tilde{Z}^{n+1} - \tilde{Z}^n)}{\Delta t} = \tilde{W}^{n+1} \quad (25e)$$

$$\frac{(\tilde{Q}^{n+1} - \tilde{Q}^n)}{\Delta t} = \frac{(\tilde{P}_Q^{n+1} + \gamma G(\bar{Z})\bar{Z}^n)}{L_i}, \quad (25f)$$

$$\frac{(\eta^{n+1} - \eta^n)}{\Delta t} + \nabla \cdot (H \nabla \tilde{\phi}^{n+1}) = 0$$

with $\partial_y \tilde{\phi}^{n+1}|_{y=l_y(x)} = 0, \quad \partial_y \tilde{\phi}^{n+1}|_{y=0} = \dot{R}^{n+1}$

$$\nabla \cdot (H \nabla \tilde{\lambda}^n) - C \int_0^{L_x} \int_0^{l_y(x)} \bar{\lambda}^n \Theta(y - L_b) \, dy \, dx = -g \nabla \cdot (H \nabla \eta^n)$$

$$- C_1 G(\bar{Z})(\tilde{P}_Q^n + \gamma G(\bar{Z})\bar{Z}^n) \quad y \geq L_b$$

$$\text{with } \tilde{\lambda}^n(x, L_b, t) = g \left(\eta^n(x, L_b^-, t) - \bar{Z}^n(t) \right) \text{ and } \hat{n} \cdot \nabla \tilde{\lambda}^n|_{\partial \Omega_h, y > L_b} = 0, \quad (25h)$$