

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

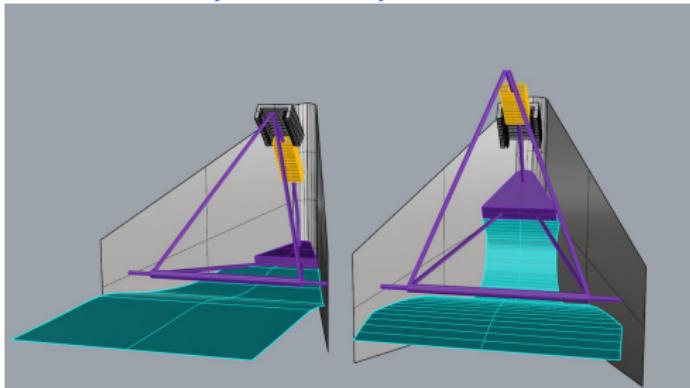
Introduction
Modelling
Linearise
wave-2-wire
model
Discretise
wave-2-wire
model
Preliminary
results
Conclusions

On a novel wave-to-wire energy device in a breakwater contraction

Onno Bokhove (Jonathan Bolton, Anna Kalogirou, Duncan Borman, Harvey Thompson, Wout Zweers)

Leeds Institute for Fluid Dynamics (LIFD –EPSRC funding)

COER, Maynooth University, Ireland, 11-03-2021



Onno
Bokhove

(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction

Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

1 Introduction

2 Modelling

3 Linearise wave-2-wire model

4 Discretise wave-2-wire model

5 Preliminary results

6 Conclusions

Wave-to-wire energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction

Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions



Introduction: maths wave-2-wire device

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction

Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

Goals & challenges:

- introduce novel wave-energy device;
- provide **nonlinear model** for new wave-2-wire device;
- provide **linearised shallow-water model** of device;
- **discretise** in space, in time & in space-time;
- show **preliminary simulations**; and,
- **open questions**: validation, optimisation,

Motivation: rogue waves

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction

Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

Rogue waves are anomalously high waves defined relative to a significant wave height H_s .

- Index (Kharif et al. '09, Dysthe et al. '08):

$$AI = H_{rw}/H_s > 2 \quad \text{or} \quad AI = \eta_{rw}/H_s > 1.25$$

- Relevance in maritime & coastal engineering
- Pyramidal rogue wave (Faulkner 2001):



Fig.1. Pyramidal wave off south Japan

Motivation: rogue waves

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zwaanswijk)

Introduction

Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

There are many causes of rogue waves, see Kharif et al. (2009):

- E.g., crossing seas, nearly standing/pyramidal waves
- We made a *bore-soliton splash* rogue wave:
$$AI = \frac{H_{rw}}{H_s} = \frac{3.5}{0.35} \approx 10$$
- It inspired our wave-2-wire wave-energy device.



Introduction: working wave-2-wire device

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction

Modelling

Linearise
wave-2-wire
model

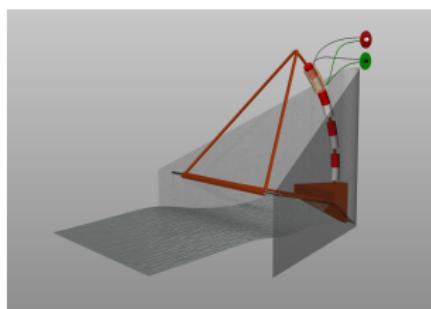
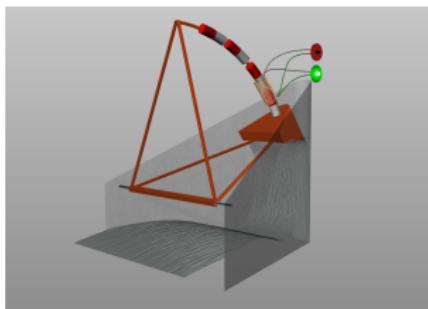
Discretise
wave-2-wire
model

Preliminary
results

Conclusions

Design by B. & Zweers:

- Sketch with **water waves**, wave-activated **buoy motion** & electro-magnetic generator & **LED**-loads:



- ***Proof-of-principle*** of laboratory device (2013)

https://www.youtube.com/watch?v=SZhe_S0xBWo

Modelling

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction

Modelling

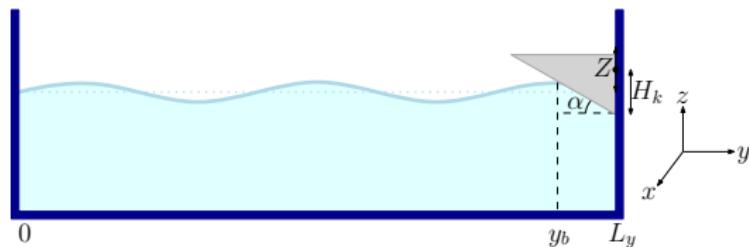
Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

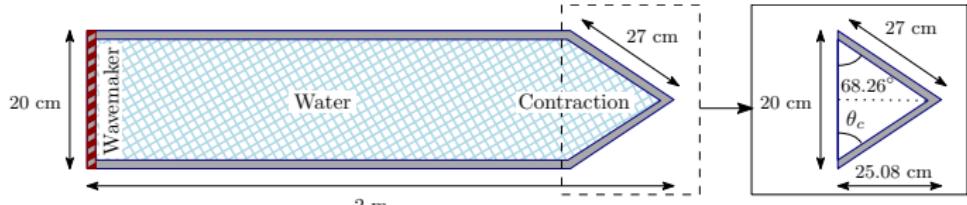
Preliminary
results

Conclusions

- Wave-2-wire model in a **wavetank**, side view along center-line:



- Wave-2-wire model in a **wavetank**, top view:



Nonlinear wave-2-wire model

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction
Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

Dynamics of water waves $\{\phi, h\}$, wave-activated buoy motion $\{Z, W\}$ & electro-magnetic generator $\{Q, P_Q\}$ with current I , $I = \dot{Q}$ and $P_Q = L_i I - K(Z)$ & LED-loads:

$$\nabla^2 \phi = 0$$

$$\partial_t h + \nabla_H \phi \cdot \nabla h - \partial_z \phi = 0 \text{ on } z = h$$

$$\partial_t \phi + \frac{1}{2} |\nabla \phi|^2 + g(h - H_0) + \underline{\lambda \Theta(y - y_b)} = 0 \quad \text{on } z = h$$

$$\underline{h - h_b} = 0 \quad \text{for } y \geq y_b(x, t)$$

$$\dot{Z} = W$$

$$M \dot{W} = -Mg - \gamma(P_Q + K(Z)) \frac{G(Z)}{L_i} + \rho_0 \int_0^{L_x} \int_0^{l_y(x)} \underline{\lambda \Theta(y - y_b)} \, dx \, dy$$

$$\dot{Q} = (P_Q + K(Z))/L_i \equiv I$$

$$\dot{P}_Q = -(R_c + R_i)I - \underline{\text{sign}(I)n_q V_T \ln(|I|/I_{sat} + 1)}.$$

Nonlinear wave-2-wire model: hydrodynamics

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction

Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

Variables & parameters **water waves**:

- potential flow $\mathbf{u} = \nabla\phi(x, y, z, t)$
- (single-valued) water depth $h = h(x, y, t)$, surface potential $\phi_s(x, y, t) = \phi(x, y, h(x, y, t), t)$

$$\nabla^2\phi = 0$$

$$\partial_t h + \nabla_H \phi \cdot \nabla h - \partial_z \phi = 0 \text{ on } z = h$$

$$\partial_t \phi + \frac{1}{2} |\nabla \phi|^2 + g(h - H_0) + \underline{\lambda \Theta(y - y_b)} = 0 \quad \text{on } z = h$$

$$\underline{h - h_b} = 0 \quad \text{for } y \geq y_b(x, t)$$

- acceleration of gravity g and vertical (free-surface) rest level H_0
- (single-valued) **waterline** $y = y_b(x, t)$.

Nonlinear wave-2-wire model: buoy motion

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction

Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

Variables & parameters:

- **wave-activated buoy**: position $Z(t)$ & velocity $W(t)$

$$\underline{h - h_b} = 0 \quad \text{for } y \geq y_b(x, t)$$

$$\dot{Z} = W$$

$$M\dot{W} = -Mg - \gamma(P_Q + K(Z)) \frac{G(Z)}{L_i} + \rho_0 \int_0^{L_x} \int_0^{l_y(x)} \underline{\lambda \Theta(y - y_b)} \, dx \, dy$$

- *Lagrange multiplier* $\lambda(x, y, t)$ imposes that water depth h equals wetted buoy shape h_b :

$$\underline{h(x, y, t) - h_b(x, y; Z(t))} = 0$$

Nonlinear wave-2-wire model: EM motor

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction
Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

Variables & parameters:

- electromagnetic generator: charge $Q(t)$ & conjugate $P_Q(t) = L_i \dot{Q} - K(Z); \dot{P}_Q(t) = \underline{L_i \dot{I}} - \gamma G(Z) \dot{Z}$
- EM-theory $K(Z) = \int^Z \gamma G(\tilde{Z}) d\tilde{Z}$ thin-wire & symmetric
- induction L_i , combined buoy-magnet mass M
- **two LEDs in parallel:** adapted Shockley equation.

$$\dot{Q} = (P_Q + K(Z))/L_i \equiv I$$

$$\dot{P}_Q = -(R_c + R_i)I - \underline{\text{sign}(I)n_q V_T \ln \left(|I|/I_{sat} + 1 \right)}.$$

Wave-2-wire model: EM motor

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction

Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

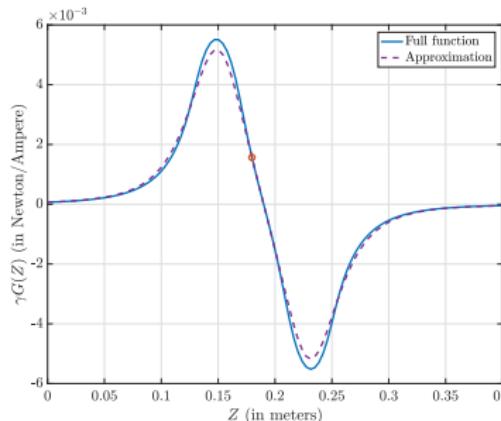
Conclusions

- By solving the Maxwell's equations, the function

$$G(Z) = \frac{\pm 1}{\pi A_m^2 L_m a} \int_{-L/2}^{L/2} \int_0^{2\pi} \int_0^{A_m} F(z) dr d\theta dq$$

- arises with $F(z) = f(-z) - f(z)$, $z = q + \bar{Z} + \alpha_h H_m - Z$,

$$f(z) = r(a - r \cos \theta) / (r^2 + (\frac{L_m}{2} + z)^2 + a^2 - 2ra \cos \theta)^{3/2}.$$



Wave-2-wire model

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction
Modelling
Linearise
wave-2-wire
model
Discretise
wave-2-wire
model
Preliminary
results
Conclusions

- When the change of current I in time & loads small, s.t.
 $\dot{I} \approx 0$:

$$(R_c + R_i + \underline{R_l})I \approx \gamma G(Z)\dot{Z},$$

- then the vertical momentum equation for buoy, mast and magnet becomes approximately

$$M\dot{W} = -Mg - \frac{\gamma^2 G^2(Z)}{L_i(R_c + R_i + \underline{R_l})} W + \rho_0 \int_0^{L_x} \int_0^{l_y(x)} \lambda \Theta(y - y_b) dx dy.$$

- Resistance and loads combined act as drag* on the buoy-mast-magnet unit in this partial linear limit.

Wave-2-wire model: circuits

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction

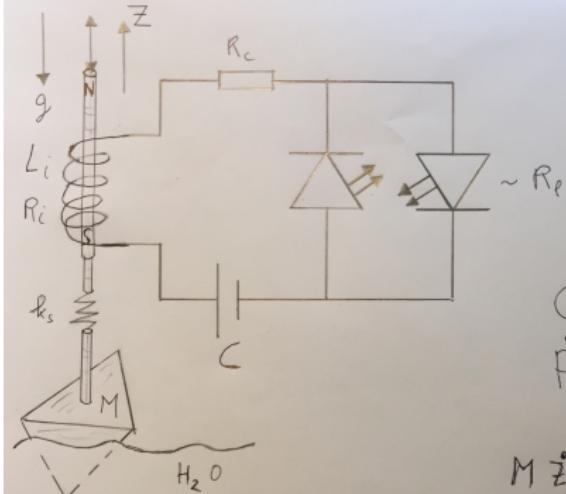
Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions



$$\dot{Q} = (P_Q + K(z)) / L_i \equiv I$$

$$\begin{aligned}\dot{P}_Q &= L_i \dot{I} - \gamma G(z) \dot{z} \\ &= -(R_c + R_i) I - \frac{Q}{C} + V_{\text{LEO}}(Q, I)\end{aligned">\text{BATTERY}}$$

$$M \ddot{z} = M W$$

$$M W = -Mg - k_s z - \gamma G(z) I$$

$$\begin{aligned}K(z) &= \int^z \gamma G(\tilde{z}) d\tilde{z} \\ &= 3D \text{ symmetric EN THEORY}\end{aligned}$$

Wave-2-wire model: queries

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction
Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

- Charge equation not needed (Bolton), unless a capacitor is added and/or a battery depending on Q, I .
- Mathematical model for **battery** (Li & Ke, 2011):
 - Shepherd and modified Shepherd models $V_{battery}(Q, I)$.
- Addition of spring with adjustable spring constant $k_s(t)$ in motor & model for **active control of resonance?**
- Originally, we used schematic of Wellstead (2000), but that **model does not linearise?**

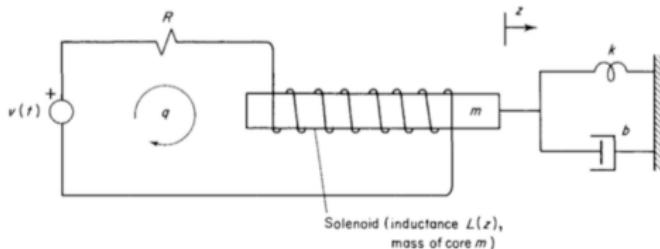


Figure 7.25

Wave-2-wire variational principle

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction
Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

Wave-to-wire model posited using 3 combined VPs:

- Luke (1967), Cotter & B. (2010, JEM), $1 + 1 + 1 = 3$:

$$\begin{aligned} 0 &= \delta \int_0^T \mathcal{L}[D, \phi, h, \phi_s, Z, W, Q, P_Q, p, \lambda] dt \\ &\equiv \delta \int_0^T \rho_0 \int_0^{L_x} \int_{R(t)}^{l_y(x)} \int_0^{h(x,y,t)} D \partial_t \phi \, dz \, dy \, dx - MW \dot{Z} - P_Q \dot{Q} + \mathcal{H} \, dt \\ &\equiv \delta \int_0^T \rho_0 \int_0^{L_x} \int_{R(t)}^{l_y(x)} \int_0^{h(x,y,t)} D \partial_t \phi + \frac{1}{2} D |\nabla \phi|^2 + g D (z - H_0) \\ &\quad + p(D - 1) \, dz \, dy \, dx \\ &\quad + \rho_0 \int_0^{L_x} \int_{R(t)}^{l_y(x)} \lambda(h - h_b) \Theta(y - y_b(x, t)) \, dx \, dy \\ &\underline{\qquad\qquad\qquad} \\ &- MW \dot{Z} - P_Q \dot{Q} + \frac{1}{2} MW^2 + MgZ + \frac{1}{2} \frac{(P_Q + K(Z))^2}{L_i} \, dt \end{aligned}$$

Linearise wave-2-wire equations

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction

Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

Linearise & simplify to shallow water with only (x, y) :

$$\eta - \tilde{Z} = 0 \quad \text{for } y \geq L_b, \quad \dot{R} = \partial_y \tilde{\phi} \quad \text{at } y = 0$$

$$\partial_t \eta + \nabla \cdot (H \nabla \tilde{\phi}) = 0, \partial_t \tilde{\phi} + g\eta + \tilde{\lambda} \Theta(y - L_b) = 0$$

$$\dot{\tilde{Z}} = \tilde{W}$$

$$M \dot{\tilde{W}} + \gamma G(\bar{Z}) \tilde{I} - \rho_0 \int_0^{L_x} \int_0^{l_y(x)} \tilde{\lambda} \Theta(y - L_b) \, dy \, dx = 0$$

$$\dot{\tilde{Q}} = \tilde{I}, \quad \dot{\tilde{P}}_Q = -\left(R_c + R_i + \frac{n_q V_T}{I_{sat}}\right) \tilde{I}, \quad \tilde{I} = \frac{(\tilde{P}_Q + \gamma G(\bar{Z}) \tilde{Z})}{L_i}$$

$$\implies \nabla \cdot (H \nabla \tilde{\lambda}) - \frac{\rho_0}{M} \int_0^{L_x} \int_0^{l_y(x)} \tilde{\lambda} \Theta(y - L_b) \, dy \, dx =$$

$$- \nabla \cdot (g H \nabla \eta) - \frac{\gamma}{M} G(\bar{Z}) \tilde{I} \quad \text{for } y \geq L_b.$$

Discrete wave-2-wire equations

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zwaanswijk)

Introduction

Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

- Finite elements in space CG1, symplectic Euler in time.
- Consistent discretisation.

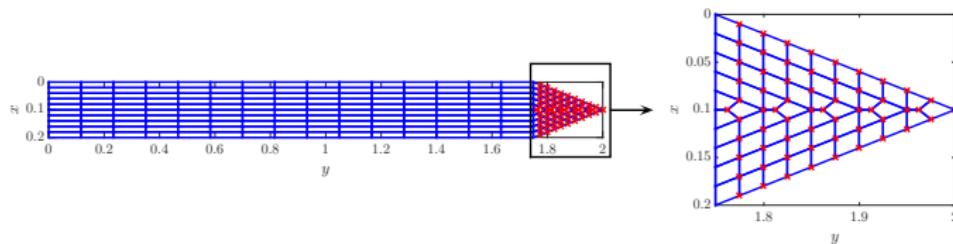


Figure: Computational mesh for $N_x = 10$, $N_y = 15$.

- For Δx & $\Delta x/2$: $O(\sqrt{\Delta t})$ & $O(\Delta t^{3/4})$ & $O(\Delta x^{1.7})$.
- Energy partitioning over subsystems consistent.

Convergence in time

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction

Modelling

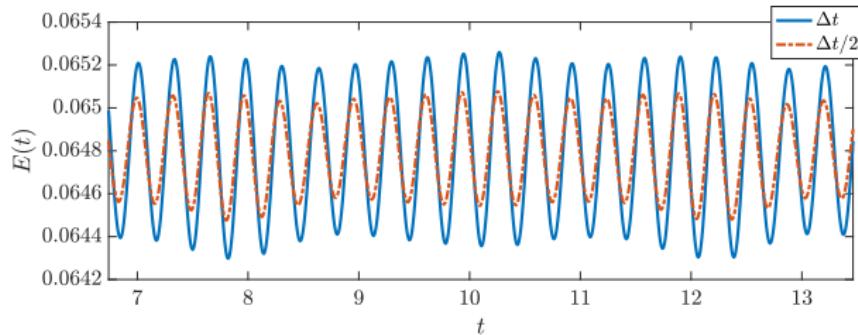
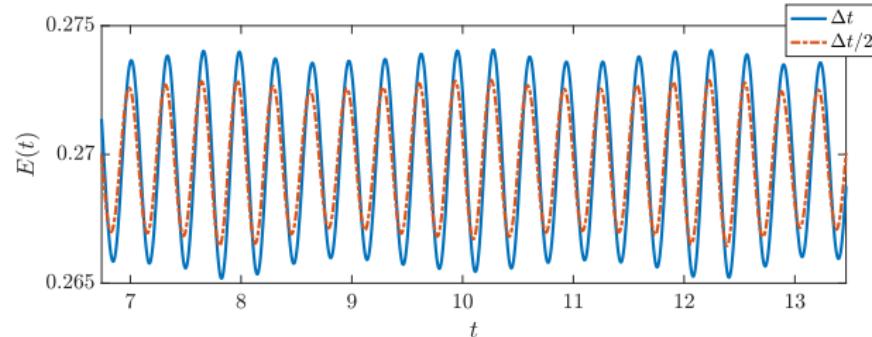
Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

For Δx & $\Delta x/2$, we find $O(\sqrt{\Delta t})$ & $O(\Delta t^{3/4})$:



Convergence in space

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction

Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

Table: Convergence rates n using three different norms (\mathcal{L}_1 , \mathcal{L}_2 , \mathcal{L}_∞) evaluated using the value of the velocity potential $\tilde{\phi}$ at the final time of the simulation, i.e. at $t = T$.

	x-el	y-el	Total el	Nodes	Rate
Symbol	N_x	N_y	N_k	N_n	n
Mesh 1	6	30	201	241	\mathcal{L}_1 : 1.711293
Mesh 2	12	60	798	877	\mathcal{L}_2 : 1.696554
Mesh 3	24	120	3180	3337	\mathcal{L}_∞ : 1.765833

Preliminary results full model

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction

Modelling

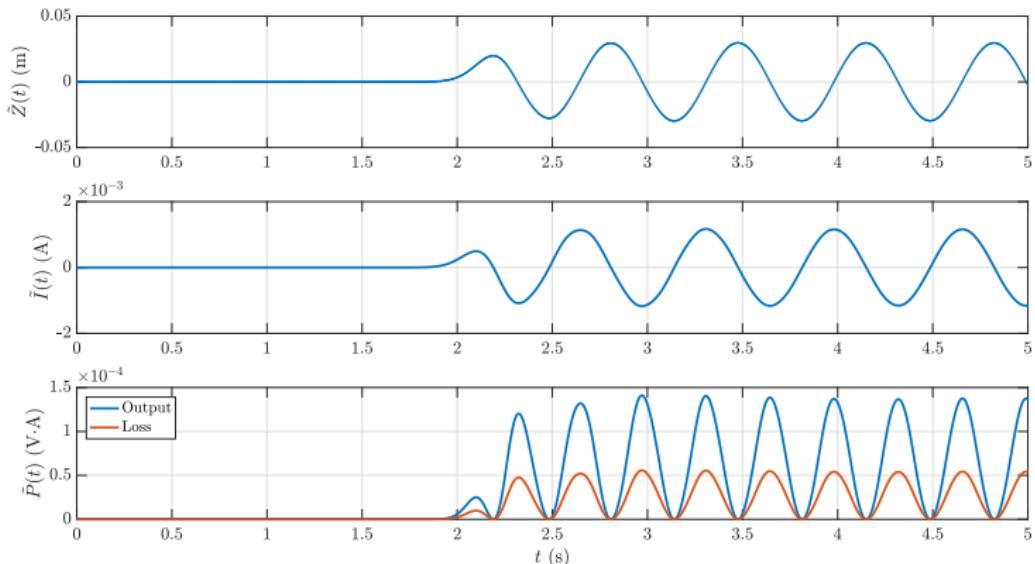
Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

Vertical displacement, current & total power
 $(R_c + R_i + \underline{n_q V_T / I_{sat}}) \tilde{I}^2$:



Preliminary results

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction

Modelling

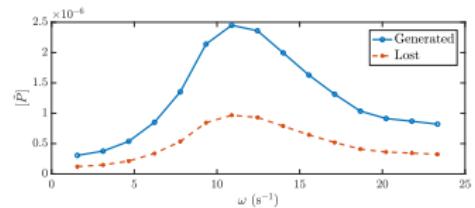
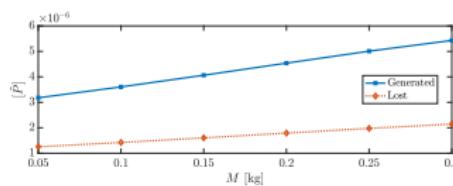
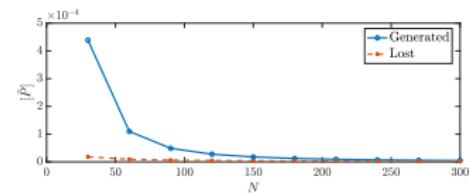
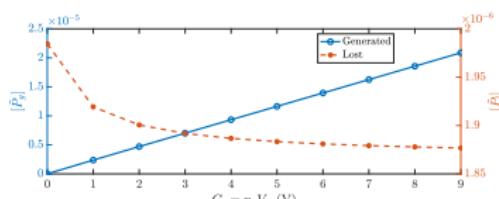
Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

Optimal harvest in linear case at resonance, maximum loads and minimum windings:



Conclusions & future work

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction
Modelling

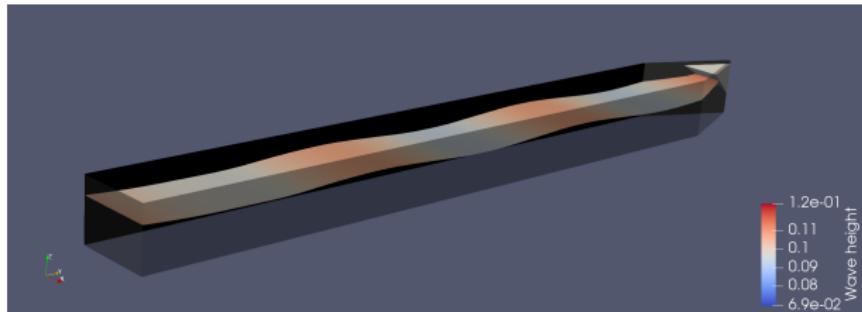
Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

- Full wave-2-wire maths model from *1st principles*.
- Established space-time compatible discretisation of linearised model.
- Explored parameter space in preliminary fashion.
- Leeds' Fluid Dynamics CDT project with *J. Bolton*:
 - nonlinear Benney-Luke hydrodynamic model (2DH)
 - faster than 3D potential-flow for optimization of parameter space (geometries, mass, spring constant, loads)



Conclusions & future work

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction
Modelling

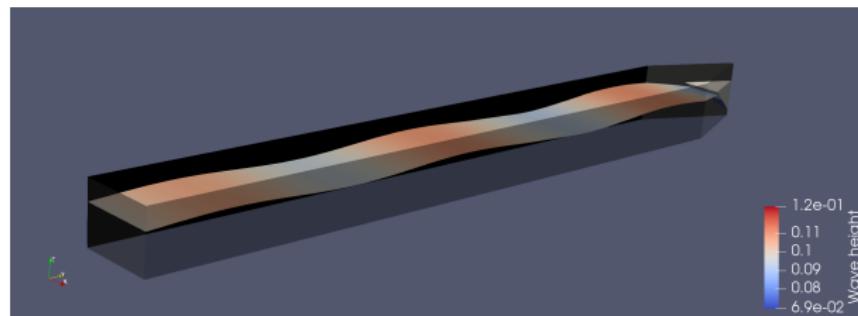
Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

- **Validation!** Preferred location: **breakwater**.
- Discretise to **higher-order** in space & time.
- Numerically implement **nonlinear models**.
- **Optimise** wave-2-wire device wrt energy-output efficiency.
- B., Kalogirou & Zweers 2019: On wave-2-wire model. *Water Waves* **1**.
<https://link.springer.com/article/10.1007/s42286-019-00022-9>
- B., Kalogirou, Henry & Thomas 2020: *Int. Marine Energy J.* **30** (EWTEC2019, Napoli)
- Bolton et al. 2021: EWTEC2021, Plymouth, pending.



Thank you!

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction

Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

$$\partial_t \phi - g \eta = 0$$
$$\sigma = \int \left(\frac{1}{2} (\partial_t \eta)^2 - \frac{1}{2} (\nabla \eta)^2 + (\eta - c) \right) dx dt$$
$$\lambda = \eta - \partial_x^2 \eta - \partial_x^2 \eta = 0 \text{ in } [0, L] \times [0, T]$$

$\eta = \text{const}$

$\boxed{\eta - c \leq 0}$

- variational inequ. $\eta(x, t) = c \quad \forall x \in I \subset [0, L]$
- complementarity problem $\forall t$
- active set methods $\forall \eta \in H^1([0, L]) : \int_I (\eta - c) \eta' dx = 0 \quad \forall \eta' \in L^2(I)$
- contact problem

$$= \langle \eta \rangle^T Z - \langle \eta \rangle^T Z W^T \langle Z W \rangle^{-1} \langle Z \rangle^T$$

AVE

Appendix Wave-2-wire VP

Wave-to-wire
energy device

Onno

Bokhove

(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction

Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

Vary & get eqns of motion (conj. pairs + multipliers):

$$\delta D : \partial_t \phi + \frac{1}{2} |\nabla \phi|^2 + g(z - H_0) + p = 0 \quad (13a)$$

$$\delta \phi : \partial_t D + \nabla \cdot (D \nabla \phi) = 0 \quad (13b)$$

$$\delta p : D = 1 \quad (13c)$$

$$\delta \lambda : h - h_b = 0 \quad \text{for } y \geq y_b(x, t) \quad (13d)$$

$$\delta \phi_s : \partial_t h + \nabla \phi \cdot \nabla h = \partial_z \phi \quad \text{at } z = h(x, y, t) \quad (13e)$$

$$\delta h : \partial_t \phi + \frac{1}{2} |\nabla \phi|^2 + g(h - H_0) + \lambda \Theta(y - y_b(x, y, t)) = 0 \quad \text{at } z = h(x, y, t) \quad (13f)$$

$$\delta \phi_R : \dot{R} = \partial_y \phi \quad \text{at } y = R(t) \quad (13g)$$

$$\delta W : \dot{Z} = W \quad (13h)$$

$$\begin{aligned} \delta Z : M \dot{W} + Mg + \frac{\gamma G(Z)}{L_i} (P_Q + K(Z)) \\ - \rho_0 \int_0^{L_x} \int_{R(t)}^{l_y(x)} \lambda \Theta(y - y_b(x, t)) dx dy = 0 \end{aligned} \quad (13i)$$

$$\delta P_Q : \dot{Q} = \frac{(P_Q + K(Z))}{L_i} \equiv I \quad (13j)$$

$$\delta Q : \dot{P}_Q = 0, \quad (13k)$$

Wave-2-wire VP

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction

Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

- Free surface: $z = h(x, y, t)$ for $y < y_b(x, t)$
- Constrained surface: $z = h(x, y, t)$ for $y \geq y_b(x, t)$
- **Waterline motion** $y = y_b(x, t)$; evaluate Bernoulli in interior at surface, compare with free-surface Bernoulli:

$$p(x, y, h(x, y, t), t) = 0 \quad y < y_b(x, t)$$

$$p(x, y, h(x, y, t), t) = \lambda(x, y, t) \quad y \geq y_b(x, t),$$

- so $\lambda(x, y_b(x, t), t) = 0$ at $y = y_b$ is derived
- when $D = 1$ imposed strongly, like in Luke (1967 –WW only), then need to impose $\lambda(x, y_b, t) = 0$ a priori.

Wave-2-wire VP

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction
Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

- Add electrical resistance & LED loads (Shockley eqns in parallel):

$$\dot{P}_Q = - (R_c + R_i)I - \text{sign}(I)n_q V_T \ln \left(\frac{|I|}{I_{sat}} + 1 \right)$$

$$\text{with } I = \frac{(P_Q + K(Z))}{L_i}.$$

- Simplex buoy shape:

$$z = h_b(x, y; Z(t)) = Z(t) - H_k - \tan \alpha (y - L_y).$$

Appendix Discrete wave-2-wire equations

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction

Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

Space CG (second-order, tent functions) FEM needs to be consistent; trick: local extension around waterline $y = L_b$

$$N_{\bar{k}l}^T (\eta_l - 1_l \tilde{Z}) = 0 \quad (28a)$$

$$M_{kl} \dot{\phi}_l = -g M_{kl} \eta_l - N_{k\bar{l}} \lambda_{\bar{l}} - N_{k\bar{b}} \lambda_{\bar{b}} \quad (28b)$$

$$M_{kl} \dot{\eta}_l = S_{kl} \phi_l + T_k \dot{R} \quad (28c)$$

$$\dot{\tilde{Z}} = \tilde{W} \quad (28d)$$

$$\dot{\tilde{W}} = C \tilde{Q}_{\bar{l}} \lambda_{\bar{l}} + C \tilde{Q}_{\bar{b}} \lambda_{\bar{b}} - C_1 G(\bar{Z})(\tilde{P}_Q + \gamma G(\bar{Z}) \tilde{Z}) \quad (28e)$$

$$\dot{\tilde{Q}} = \frac{(\tilde{P}_Q + \gamma G(\bar{Z}) \tilde{Z})}{L_i} \quad (28f)$$

$$\dot{\tilde{P}}_Q = -C_2 (\tilde{P}_Q + \gamma G(\bar{Z}) \tilde{Z}) \quad (28g)$$

$$\begin{aligned} (\tilde{S}_{\bar{k}\bar{l}} + C \tilde{Q}_{\bar{k}} \tilde{Q}_{\bar{l}}) \lambda_{\bar{l}} &= -g S_{\bar{k}l} \eta_l - C \tilde{Q}_{\bar{k}} \tilde{Q}_{\bar{b}} \lambda_{\bar{b}} - \tilde{S}_{\bar{k}\bar{b}} \lambda_{\bar{b}} \\ &\quad + C_1 \tilde{Q}_{\bar{k}} G(\bar{Z})(\tilde{P}_Q + \gamma G(\bar{Z}) \tilde{Z}) \end{aligned} \quad (28h)$$

Discrete wave-2-wire equations

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction

Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

- Indexing of nodes: N_n nodes $k, l = 1, \dots, N_n$;
 $N_n - N_p + 1$ nodes under buoy $\tilde{k}, \tilde{l} = N_p, \dots, N_n$;
 N_b waterline nodes $\tilde{b} = N_p, \dots, N_p + N_b - 1$
- Mass & Laplace matrices:

$$M_{kl} = \int_0^{L_x} \int_0^{l_y(x)} \varphi_k(x, y) \varphi_l(x, y) dx dy, \quad (29a)$$

$$S_{kl} = \int_0^{L_x} \int_0^{l_y(x)} H(y) \nabla \varphi_k(x, y) \cdot \nabla \varphi_l(x, y) dx dy, \quad (29b)$$

$$\tilde{S}_{\tilde{k}\tilde{l}} = \int_0^{L_x} \int_0^{l_y(x)} H(y) \nabla \varphi_{\tilde{k}}(x, y) \cdot \nabla \varphi_{\tilde{l}}(x, y) dx dy, \quad (29c)$$

$$\tilde{Q}_{\tilde{k}} = \int_0^{L_x} \int_0^{l_y(x)} \varphi_{\tilde{k}}(x, y) dx dy, \quad (29d)$$

$$N_{k\tilde{l}} = \int_0^{L_x} \int_0^{l_y(x)} \varphi_k(x, y) \varphi_{\tilde{l}}(x, y) dx dy, \quad (29e)$$

$$T_k = \int_0^{L_x} H(0) \varphi_k(0, y) dy. \quad (29f)$$

Discrete wave-2-wire equations

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction

Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

Consistency on discrete level (but only with **trick**):

The key consistency check is to ensure that the first seven equations in (28) are consistent with the last seven equations in (28). Consider the first seven equations. Take the time derivative of the primary constraint and eliminate the time derivatives by using two of the other seven equations, to obtain the secondary constraint

$$N_{kl}^T \left(M_{lk}^{-1} (S_{km}\phi_m + T_k \dot{R}) - 1_l \tilde{W} \right) = 0. \quad (30)$$

Now take the time derivative of this secondary constraint above and again eliminate the time derivatives by using two different equations of these seven equations, to obtain the consistency equation

$$\begin{aligned} & \left(N_{kl}^T M_{lk}^{-1} S_{km} M_{mn}^{-1} N_{nl} + C N_{kl}^T 1_l \tilde{Q}_l \right) \lambda_{\bar{l}} = -g N_{kl}^T M_{lk}^{-1} S_{km} \eta_m \\ & - \left(N_{kl}^T M_{lk}^{-1} S_{km} M_{mn}^{-1} N_{n\bar{b}} + C N_{kl}^T 1_l \tilde{Q}_{\bar{b}} \right) \lambda_{\bar{b}} \\ & + N_{kl}^T M_{lk}^{-1} T_k \ddot{R} + C_1 N_{kl}^T 1_l G(\bar{Z}) \left(\tilde{P}_Q + \gamma G(\bar{Z}) \tilde{Z} \right). \end{aligned} \quad (31a)$$

This consistency equation matches the last equation (28h) if and only if the following relations hold

$$\tilde{S}_{\bar{k}\bar{l}} = N_{kl}^T M_{lk}^{-1} S_{km} M_{mn}^{-1} N_{n\bar{l}} \quad (31b)$$

$$\tilde{S}_{\bar{k}m} = N_{kl}^T M_{lk}^{-1} S_{km} \quad (31c)$$

$$\tilde{S}_{\bar{k}\bar{b}} = N_{kl}^T M_{lk}^{-1} S_{km} M_{mn}^{-1} N_{n\bar{b}} \quad (31d)$$

$$\tilde{Q}_{\bar{k}} = N_{kl}^T 1_l \quad (31e)$$

$$N_{kl}^T M_{lk}^{-1} T_k = 0. \quad (31f)$$

Discrete wave-2-wire equations

Wave-to-wire
energy device

Onno
Bokhove
(Jonathan
Bolton, Anna
Kalogirou,
Duncan
Borman,
Harvey
Thompson,
Wout Zweers)

Introduction

Modelling

Linearise
wave-2-wire
model

Discretise
wave-2-wire
model

Preliminary
results

Conclusions

■ Time only: symplectic Euler first order

$$\eta^{n+1} - \bar{Z}^{n+1} = 0 \quad \text{for } y \geq L_b \quad (25a)$$

$$\frac{(\tilde{\phi}^{n+1} - \tilde{\phi}^n)}{\Delta t} + g\eta^n + \bar{\lambda}^n \Theta(y - L_b) = 0 \quad (25b)$$

$$\begin{aligned} \frac{(\tilde{W}^{n+1} - \tilde{W}^n)}{\Delta t} + C_1 G(\bar{Z})(\tilde{P}_Q^n + \gamma G(\bar{Z})\bar{Z}^n) \\ - C \int_0^{L_x} \int_0^{l_y(x)} \bar{\lambda}^n \Theta(y - L_b) \, dy \, dx = 0 \end{aligned} \quad (25c)$$

$$\frac{(\tilde{P}_Q^{n+1} - \tilde{P}_Q^n)}{\Delta t} = -C_2 (\tilde{P}_Q^{n+1} + \gamma G(\bar{Z})\bar{Z}^n) \quad (25d)$$

$$\frac{(\tilde{Z}^{n+1} - \tilde{Z}^n)}{\Delta t} = \tilde{W}^{n+1} \quad (25e)$$

$$\frac{(\tilde{Q}^{n+1} - \tilde{Q}^n)}{\Delta t} = \frac{(\tilde{P}_Q^{n+1} + \gamma G(\bar{Z})\bar{Z}^n)}{L_i}, \quad (25f)$$

$$\frac{(\eta^{n+1} - \eta^n)}{\Delta t} + \nabla \cdot (H \nabla \tilde{\phi}^{n+1}) = 0$$

with $\partial_y \tilde{\phi}^{n+1}|_{y=l_y(x)} = 0, \quad \partial_y \tilde{\phi}^{n+1}|_{y=0} = \dot{R}^{n+1}$

$$\nabla \cdot (H \nabla \tilde{\lambda}^n) - C \int_0^{L_x} \int_0^{l_y(x)} \bar{\lambda}^n \Theta(y - L_b) \, dy \, dx = -g \nabla \cdot (H \nabla \eta^n)$$

$$- C_1 G(\bar{Z})(\tilde{P}_Q^n + \gamma G(\bar{Z})\bar{Z}^n) \quad y \geq L_b$$

$$\text{with } \tilde{\lambda}^n(x, L_b, t) = g \left(\eta^n(x, L_b^-, t) - \bar{Z}^n(t) \right) \text{ and } \hat{n} \cdot \nabla \tilde{\lambda}^n|_{\partial \Omega_h, y > L_b} = 0, \quad (25h)$$