Visualising Flood Frequency, Flood Volume and Mitigation of Extreme Events

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Two novel visualisation tools on extreme flooding are considered and their appeal discussed. First, an overview of the *Wetropolis Flood Demonstrator* is given. This is a portable set-up that showcases what a return period of an extreme flood event is: it has been received well by the general public. Second, a novel graphical cost-effectiveness diagnostic aimed at the public and decision-makers is explained: it has already led to increasing stakeholder participation in flood-mitigation planning.

1 Introduction

It is often difficult for the general public to understand the frequency and irregularity of extreme flooding events. In particular, what does it mean when a flooding event is denoted as a 1 : 100-year event or, equivalently, as an event with a 1% Annual Exceedance Probability (AEP)? Similarly, it is often difficult for them to grasp how much water is involved in the flooding of a city, as well as the efficacy and variety of flood-mitigation measures proposed to deal with this amount of water. For example, how can one compare the effects of steel-sheet-piling flood-defence walls to those of enhancement of upstream flood-plain storage? Vice versa, it is challenging for flood practitioners to explain to the public such uncertainties, as well as complementary grey and green engineering in flood-defence works, and their designs.

In attempting to bridge this gap in understanding and to minimise misunderstandings, we have devised new visualisation tools; one inspired by the other and both triggered by conversations with professional flood analysts. Neither of these tools replaces engineering calculations; rather they assist in both establishing scenario-based

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designs and communicating more finalised designs of flood-mitigation plans. The first tool is the *Wetropolis Flood Demonstrator*: a physical and portable miniature river set-up with a city intermittently flooded by simulated weather generated in a probabilistic manner, thus showing the meaning of return periods or AEPs of extreme rainfall and flooding events. The second tool is a graphical cost-effectiveness diagnostic of flood-mitigation measures that facilitates scenario-thinking for decision-makers and public planners. Research published in reports and articles [3, 2] evidences how these tools arose, and their societal impacts are summarised in [4]. Following overviews of the two tools, we focus on recently accrued highlights.

2 Wetropolis: from flood to climate and drought demonstrator

Visualisation of extreme flood-event return periods. The goal of Wetropolis is the visualisation of a return period or an AEP for an extreme flooding event. Its creation, propelled by requests from the Environment agency and JBA Trust, was to allow the public to see that a return period of 100 years, say, can occur at random intervals. Wetropolis reaches that goal in a portable model in which probablistically generated weather leads to spatio-temporal rainfall onto an upland moor and/or reservoir intermittently entering the river on top of the constant upstream river inflow.

"Wetropolis days" (WDs) occur in 10s of real time, thus rendering possible the visualisation of rare (e.g. AEP = 1%) events without the audience having to wait over timescales of years. Rainfall durations are toggled by a ball first falling through a skewed Galton board that sets a duration of $(1, 2, 4, 9)$s with respective probabilities $(p_1, p_2, p_3, p_4) = (3, 7, 5, 1)/16$. Rainfall locations occur via a second board assigning rain in: (reservoir, moor and reservoir, moor, nowhere) with probabilities $(q_1, q_2, q_3, q_4) = (3, 7, 5, 1)/16$. The product of these discrete probabilities determines a probability matrix $P_{ij} = p_i q_j$ with $i, j = 1, 2, 3, 4$. The rare event concerns rainfall with the longest duration of 9s, which has probability of $p_4 = 1/16$; the moor-and-reservoir location, with $q_2 = 7/16$, yields $P_{42} = 7/256 \approx 2.7\%$. Note that, for a dry day, rain-duration outcomes of the first Galton board are not used, such that no rain occurs with probability $\sum_{i=1}^{4} P_{i,4} = q_4 \sum_{i=1}^{4} p_i = q_4 = 1/16$. One second of rainfall in either reservoir or moor is driven by aquarium pumps each discharging a volume r_0 of water per second. Scaled by r_0, 1s of WD rainfall has a chance of $p_1(q_1 + q_3)$, probabilities of rainfall flowrates $(0, 1, 2, 4, 8, 9, 18)r_0$ being:

$$\begin{aligned} &0 : q_4, \quad 1 : p_1(q_1 + q_3), \quad 2 : p_2(q_1 + q_3) + p_1 q_2, \\ &4 : p_3(q_1 + q_3) + p_2 q_2, \quad 8 : p_3 q_2, \quad 9 : p_4(q_1 + q_3), \quad 18 : p_4 q_2. \end{aligned} \tag{1}$$

The river channel is circa 4m long, and there is upstream inflow of water of $10r_0$ with the moor halfway downstream of the river, a city at the downstream end of the river and the reservoir between moor and city (Fig. 1). Based on a mathematical and numerical design model of a system of coupled partial and ordinary differential equations, the length of the day and the unit pump strength r_0 were determined such that major flooding occurs when $18r_0$ enters the set-up with an event probability of

$p_e = P_{2,4} = 7/256$. Galton-board outcomes are coupled to matching rainfall using Arduino technology. Detailed modelling and design specifications are found in [2].

The return period T_r of a one-day flooding event uses the Wetropolis Event Probability $p_e = 2.7\%$ and $T_d = 10$s. When a one-day extreme event has happened on day zero, the chance p_n that it reoccurs on day $n > 0$ is given by a geometric distribution $p_n = (1-p_e)^{n-1} p_e$. The mean (the expectation of $t_n = nT_d$) and standard deviation σ_r follow via manipulation of the geometric series and are respectively

$$T_r = \mathbb{E}(t_n) = \frac{T_d}{p_e} = 6\!:\!06\text{min};\quad \sigma_r = \mathbb{E}\left((t_n - \mathbb{E}(t_n))^2\right) = \sqrt{(1-p_e)}T_r = 6\!:\!00\text{min}.$$

Wetropolis was exhibited to over 1200 participants at the Mathematics of Planet Earth (MPE) exhibitions of 2020 and 2022, and a 7m14s educational video of it has been posted at `https://youtu.be/rNgEqWdafKk`, thereby facilitating more reach.

Probability of super- and mega floods, climate and drought. Superfloods occur in Wetropolis when extreme rainfall of 9s/WD in moor and reservoir happens on two consecutive days. Intuitively, this has a return period of $T_r^{(2)} = T_d/p_e^2$. However, return periods and standard deviations for such rainfall on k consecutive days have been calculated exactly using a geometric distribution of order k, yielding:

$$T_r^{(k)}/T_d = \frac{(1-p_e^k)}{(1-p_e)p_e^k},\quad \sigma_r^{(k)}/T_d = \frac{\sqrt{1-(2k+1)(1-p_e)p_e^k - p_e^{2k+1}}}{(1-p_e)p_e^k}. \qquad (2)$$

For small p_e, at leading order $T_r^{(k)} \approx T_d/p_e^k$, which is exact for $k = 1$, and $\sigma_r^{(k)} \approx T_r$.

For the extreme event with $p_e = 7/256$, the return periods for $k = 2, 3$ thus become $T_r^{(2)} = 3.8$hr and $T_r^{(3)} = 140$hr. The two-day superflood makes quite an impression on the audience and has been seen and filmed a few times. The three-day megaflood has not been seen hitherto. The one-day flood event is weaker and sometimes absent if the set-up has not been balanced properly. By changing the event probability to $p_e = 49/256$, so $(7/16)^2$ from both Galton boards, the stronger super- and megafloods with return periods $T_r = 52$s, $T_r^{(2)} = 5{:}25$min and $T_r^{(3)} = 29.11$min become feasible with the disadvantage that 9s rainfall becomes most frequent. Alternatively, one could consider a WD of duration 20s such that rainfall can vary per day.

Climate change has been included and showcased in Wetropolis by adding an extra lake upstream of the moor, with rainfall in this lake and moor triggered by the moor's Galton-board outcome, with a weaker pump for the lake than for the moor. This leads to an increase of $\sim 20\%$ in average rainfall under climate change, and the one-day flooding of the city thus becomes more severe and noticeable. Droughts can be enhanced by allowing a subset of the no-rain days to last longer, say four days, with modified probabilities. A drought can be visualised in the set-up by a drinkwater pipe to the city drying up because, in the absence of rainfall, the moor's water table will sink below an inlet point in the moor. Taken collectively, the above considerations clearly begin to meet requests to link Wetropolis' rainfall to actual weather data, and to illustrate climate change and drought.

Fig. 1 Photograph of the Wetropolis set-up (with Galton boards in the right-hand background) at the 2022 MPE exhibition in London. The reservoir is seen in the foreground as well as the square representing the city on the right. The porous moor is seen as the glass tank in the middle on the left: rainfall (from the pipe above the tank) and the water table can be discerned therein.

3 Graphical cost-effectiveness diagnostic tool on flooding

Though the aftermath of the 2015 Boxing Day flood in Leeds, UK—and the resulting evacuation of a business in the River Aire's flooded plain—led to the inception of Wetropolis, it was inspection of the subsequent public 2017 Flood Alleviation Scheme II (FASII) plans of Leeds City Council (LCC) that triggered a novel flood-mitigation cost-effectiveness diagnostic. This new tool allowed succinct visualisation and summary of the FASII plan and concomitantly identified some inconsistencies and missing information that were reported to LCC. Further public archiving of the general tool allowed its rapid and wider use within the EU, usage later corroborated in the REF2021 impact case study [4].

The tool is summarised schematically in Fig. 2(a), (b) and (c). In sub-figure (a) is a three-panel graph in whose third quadrant directly measured water levels $h(t)$ (axis to left) are displayed as function of time t (axis downwards). Discharge Q (vertical axis shared with quadrant one) is obtained via a (blue) rating curve, alternatively $h(Q)$ or $Q(h)$, in quadrant two. In quadrant one, the discharge $Q(t)$ follows as a function of time (new horizontal axis to right). Given a water-level threshold (right vertical line in quadrants two and three), above which intermediate-to-major flooding occurs around the measurement site, a discharge threshold (lower horizontal line in quadrants two and one) follows. It demarcates the blue area, labelled FEV to represent flood-excess volume; FEV is the fraction of the flood volume that causes the flood damage. Using this approach, the FEV for Leeds' 2015 Boxing Day flood emerged as $\mathrm{FEV}_l = 9.34\mathrm{Mm}^3$: in order to contextualise such a volume of water, FEV is herein expressed as a square lake with human-size (shallow) depth $D = 2$m and side-length $L = \sqrt{\mathrm{FEV}/D}$. Such a lake is shown in sub-figure (b) and, since $L \gg D$ for intermediate-to-large floods (here $L = 2161$m), the vertical scale is exaggerated in the figure. Flood mitigation of the entire FEV amounts to partitioning the square lake into rectangular or quadrilateral shapes for each mitigation measure. For illustration, the three measures chosen are higher flood-defense walls (HW), giving-room-to-the-river (GRR – effectively lowering and widening the river borders so that they convey

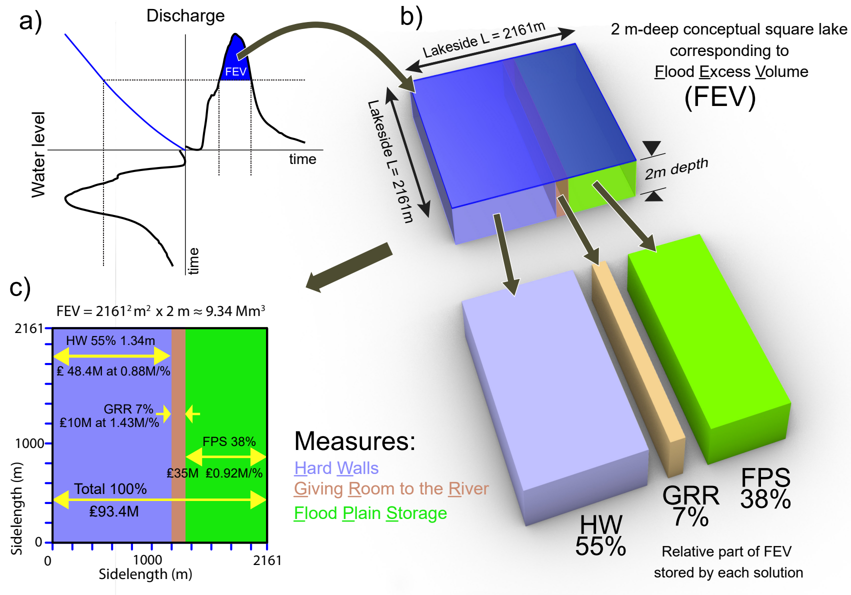

Fig. 2 Schematic of graphical cost-effectiveness diagnostic tool. Abbreviations in figure or text.

more water before flooding the site) and flood-plain storage (FPS – enhancing the storage volume of the flood plain by operating a dynamic weir to further stow up flood waters). In sub-figure (c) the square lake is plotted from above with, for each mitigation measure, its costs, costs per percentage (here under linear scaling) as well as total costs. Refinements can be made by including uncertainties and errors in rating curves, water-level measurements and mitigation measures; it is accordingly accepted that FEV will generally incur potentially large errors. Such uncertainties can readily be shown in the square-lake graphs by using quadrilateral shapes with worst-case values at the bottom and best-case ones at the top; similarly spatially varying rainfall scenarios and mitigation measures can be included (e.g. [1, 4] and references therein). The diagnostic tool is based on a calculated or measured flood hydrograph at one location and therefore appears to be spatially zero-dimensional, but in practice it represents a single stretch of river. Accordingly, the diagnostic can be extended to involve multiple hydrographs from a series of linked river stretches.

With evidence to corroborate its usage [4], the diagnostic tool can and has been applied in three types of investigation: (i) as a consistency check of flood-mitigation plans with proven (as yet uncorroborated) ability to uncover inconsistencies and missing information [1, 4]; (ii) as an *a priori* investigation to define mitigation scenarios before any detailed engineering calculations are done or when these are too costly; as such allowing communication about the physical capacity of measures, stakeholder participation and dismissal of mitigation measures with undersized FEV capacity (as performed by an NGO for the River Glinščica in the Ljubljana, Slovenia [1, 4]);

and, (iii) as an *a posteriori* executive summary of detailed engineering calculations of flood-mitigation plans, as done for a French municipality after devastating floods in 2015 of the River Brague. Consultants picked up on the River Brague case report [5] to perform a comprehensive study for detailing an appropriate GRR-mitigation measure that was first raised by our FEV-cost-effectiveness diagnostic.

Finally, we have introduced the concept of *available flood-storage volume* (see references in [4]) which aids in assessing the efficacy of FPS. It has both highlighted unclear aspects of FPS within Leeds' FASII plan and demonstrated that the efficacy of FPS within (much-publicised) beaver ponds is small, as now quantified. Consider an idealised river with a one-sided flat flood plain of width $W = 100$m and length $L_f = 1$km with an adjacent rectangular river channel of depth d. For a 1:100yr flood the flood-plain depth is $d_1 = 1$m and for a 1:300yr one the depth $d_4 = 4$m, say. A new weir can dynamically raise the flood-plain level to $d_w = 4$m. The available flood-plain storage volume for protection against the 1:100yr flood is then $V_{afps} \leq (d_w - d_1)L_f W = 3\times10^5 m^3$ and, similarly, $V_{afps} \leq (d_w - d_4)L_f W = 0\text{m}^3$ against the 1:300yr flood. Estimation of V_{afps} for a series of dams of a beaver colony in Devon is illustrative. During a rainfall event, the water levels of the beaver ponds can be seen to fluctuate in a limited manner with $V_{afps} \in [200, 1100]\text{m}^3$ for the beaver colony (the 200m^3 estimate arises in [5]). For perfect dams, upscaling to reach 10% protection for Leeds' FEV$_l$ = 9.34Mm^3 would then require FEV$_l/V_{afps} \in [849, 4245]$ beaver colonies (with the other 90% of FEV covered otherwise), which is unrealistic [3, 5]. Hence, overstated promotion of beaver colonies as a flood-mitigation measure draws attention away from their actual and dominant benefit: wildlife enhancement.

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