Power optimisation of a heaving buoy in a wave-enhancing contraction

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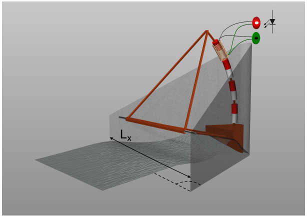
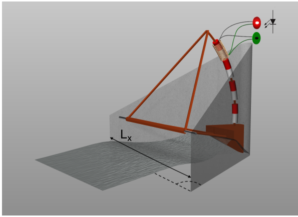


Outline: modelling components

- Power optimisation of a heaving-buoy wave-energy device considered for a device placed in a wave-amplitude enhancing (V-shaped) contraction.
- Ideally placed in a breakwater or in an array of contractions moored at sea.
- Nonlinear wave-to-wire model consisting of 3 components:

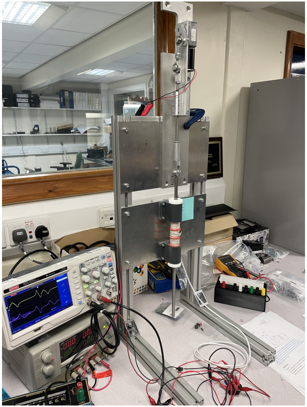
(i) wave (hydro)dynamics,
(ii) buoy motion, and
(iii) power generation.



Outline: partitioning & main advances

We have made the following advances for the buoy-generator sub-model (ii-iii):

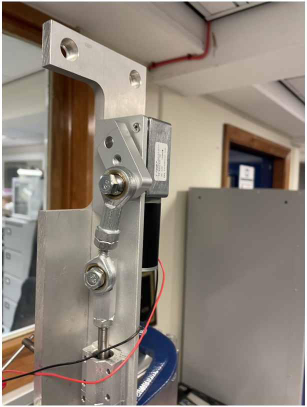
- Experimental validation of the model hanging from a driven moving point in its dry setting.
- Same length induction coils placed in parallel and rectified generate most power; three-phase rectifier is best.



Outline: partitioning & main advances

Advances wave-buoy sub-model (i-ii) are:

- Novel inequality-constraint technique to couple wave and buoy derived and implemented: water surface $\leq$ buoy hull.
- Nonlinear waves: dispersive, depth-integrated (quadratic in z with 2 dofs) Variational Boussinesq or Green-Naghdi Model (e.g. Gagarina et al. 2013).
- Spectrally-accurate 2^{nd}- to 5^{th}-order FEM in 2D horizontal numerical wavetank (open source *Firedrake* environment).



Grand VP of wave-to-wire model

Equations of motion follow from variational principle (VP; red=waves, blue=buoy, green=EM-generator, coupling, B. et al. 2019):

$$
\begin{aligned}
0 =&\delta \int_0^T \int_0^{L_x} \int_{R(t)}^{l_y(x)} \int_0^h -(\partial_t\phi + \frac{1}{2}|\nabla\phi|^2)\mathrm{d}z - gh(\frac{1}{2}h - H_0) \\
&\underline{- \frac{1}{2\gamma}\Big(F_{+}(\gamma(h-h_b)-\lambda)^2 - \lambda^2\Big)}\mathrm{d}y\mathrm{d}x \\
&MW\dot{Z} - \frac{1}{2}MW^2 - MgZ + (L_i I - \underline{K(Z)})\dot{Q} - \frac{1}{2}L_i I^2 \mathrm{d}t \qquad (1)
\end{aligned}
$$

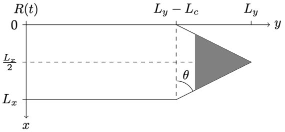
velocity $\mathbf{u} = \nabla\phi(x,y,z,t)$, depth $h(x,y,t)$, rest depth H_0, e.g. buoy $h_b(Z,y) = Z - K_h - \tan\theta(L_y - y)$, piston $R(t)$, coupling function $\gamma_m G(Z) = K'(Z)$, buoy mass M, keel height K_h, buoy coordinate $Z(t)$, buoy velocity $W(t) = \dot{Z}$, charge $Q(t)$, current $I(t) = \dot{Q}$.

Wave-to-wire: PDEs

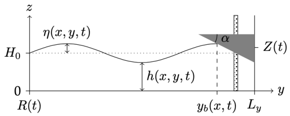
- Potential-flow water-wave dynamics (Laplace equation in interior, kinematic & Bernoulli equations at free surface):

$$
\begin{aligned}
\delta\phi: &\quad \nabla^2\phi = 0 \quad \text{in} \quad \Omega \\
(\delta\phi)|_{z=h}: &\quad \partial_t h + \nabla\phi\cdot\nabla h = \phi_z \quad \text{at} \quad z = h \\
\delta h: &\quad \partial_t\phi + \frac{1}{2}|\nabla\phi|^2 + g(z - H_0) - \lambda = 0 \quad \text{at} \quad z = h.
\end{aligned}
$$

- Coupled elliptic Laplace equation to hyperbolic free-surface equations, plus a (Lagrange) **multiplier** λ.



(b) Top view of the tank and buoy, outlining the tank's dimensions and how the buoy fits the shape of the contraction.



(c) Side view at time t, with the buoy constrained to move vertically.

Wave-to-wire: inequality constraint & ODEs

- Karush-Kuhn-Tucker inequality conditions satisfied at every space-time x, y, t-position are (Burman et al. 2023):

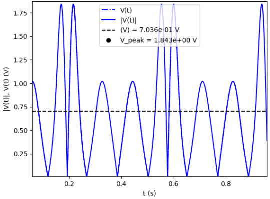
$$\delta\lambda : \lambda = -[\gamma(h-h_b)-\lambda]_+ = -F_+(\gamma(h-h_b)-\lambda)$$
$$\Longrightarrow \underline{h(x,y,t)-h_b(Z,y)\leq 0}, \lambda \leq 0, \lambda(h-h_b)=0.$$

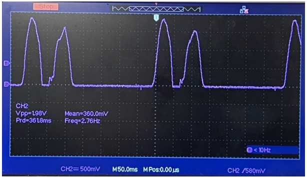
- Add resistance R_i, R_c & Shockley load $V_s(|I|)$ to submodel:

$$\delta W : \ \dot{Z} = W,$$
$$\delta Z : \ M\dot{W} = -Mg \underline{-\gamma_m G(Z) I} - \int_0^{L_x}\int_0^{l_y(x)} \lambda \, \mathrm{d}y\mathrm{d}x$$
$$\delta I : \ \dot{Q} = I,$$
$$\delta Q ; \ L_i \dot{I} = \underline{\gamma_m G(Z)\dot{Z}} - (R_i+R_c)I - \frac{I}{|I|}V_S(|I|).$$

Preliminary experimental validation buoy-generator

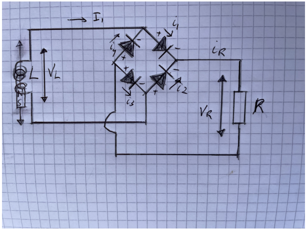
- Modeled/observed outputs $V(t)$ from one-coil (or 3 coils in series) with 1 bridge rectifier & 3 magnets.
- Simulation of one-coil model with ideal bridge-rectifier: $1/T_{period} = 2.6\mathrm{Hz}$, $R = 100\mathrm{Ohm}$, $l_2 = 0.035\mathrm{m}$. Peak/mean values: $(\underline{1.843}, 0.736)\mathrm{V}$.
- Laboratory observation: $(\underline{1.884}, 0.921)\mathrm{V}$ for $0.8\mathrm{s}$.

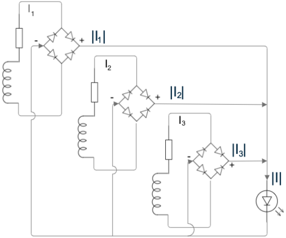




Experimental validation buoy-generator

- Same length coils in series or parallel: 3-phase parallel & rectified best:





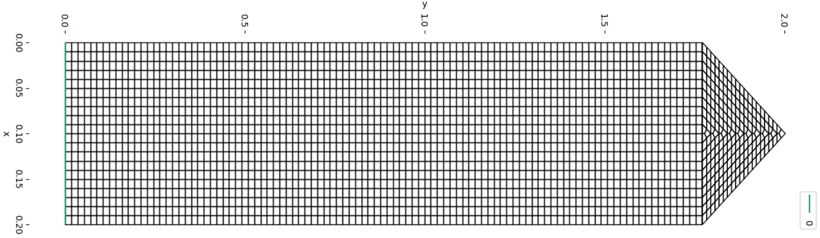
Experimental validation buoy-generator

Table: Measurements for resistances/loads $R = 50, 100, 900\mathrm{Ohm}$.
Set-ups: **S series**: 3 coils in series; 3ph: three-phase rectified of 3 coils; 1ph: 3 coils put in parallel after rectifying each coil; and, only using *Top, Middle or Bottom* coil.

Name 1 magnet	Max, Mean V	Name 3 magnets	Max, Mean V	Comment
1-100-1ph.csv	1.222,0.348	3-100-1ph.csv	2.341,1.122	2nd best
1-100-3ph.csv	1.750,0.725	3-100-3ph.csv	2.500, 1.528	best
1-100-Bot.csv	nan, nan	3-100-Bot.csv	2.155,1.073	glitch
1-100-Mid.csv	1.046,0.337	3-100-Mid.csv	2.346,0.850	2nd best
1-100-S.csv	0.824,0.306	3-100-S.csv	1.884,0.921	
1-100-Top.csv	0.877,0.300	3-100-Top.csv	1.789,0.733	
1-50-1ph.csv	1.107,0.317	3-50-1ph.csv	2.163,1.079	2nd best
1-50-3ph.csv	1.532,0.482	3-50-3ph.csv	2.500,1.435	best
1-50-Bot.csv	0.985,0.258	3-50-Bot.csv	1.896,0.927	
1-50-Mid.csv	0.885,0.299	3-50-Mid.csv	2.045,0.781	
1-50-S.csv	0.635,0.256	3-50-S.csv	1.437,0.703	
1-50-Top.csv	0.767,0.276	3-50-Top.csv	1.584,0.666	

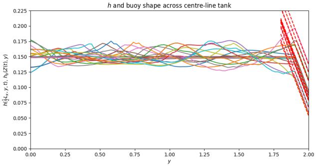
Wave-buoy dynamics: inequality constraints

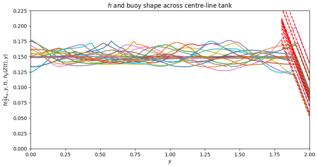
- Inequality constraints imposed with *derived, smooth* penalty superfunction as Lagrange multiplier (augmented Lagrangian).
- FEM mesh, e.g., with $1^{\mathrm{st}}, 2^{\mathrm{nd}}, 3^{\mathrm{rd}}$ or 4^{th}-order polynomials [dofs: (6,22,50,88)k]; AVF-energy-conservative (-) time integration essential:



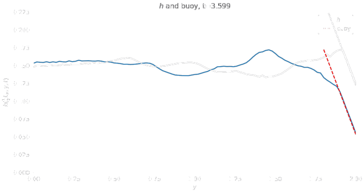
Wave-buoy dynamics: inequality constraints

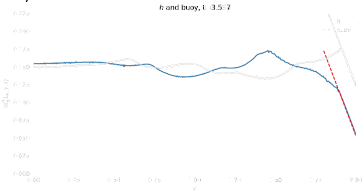
- Continuous Galerkin orders CG2, CG3 along centreline:





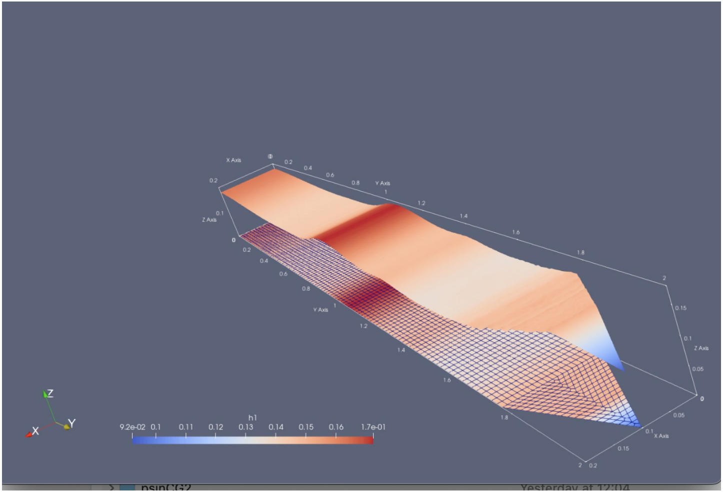
- Continuous Galerkin orders *CG2, CG4*:





Wave-buoy dynamics: inequality constraints

- Continuous Galerkin *CG2* [https://youtu.be/bBXuj8XBjAk]:



Conclusions

- Buoy-generator lab/simulation models built: 3-phase rectified parallel circuit best by circa 1.5× to 2×.
- Nonlinear Variational Boussinesq Model coupled to buoy motion via novel inequality-constraint technique: i.e., Firedrake-GitHub [open source `https://www.firedrakeorg.com` –ask]. Thanks to: Colin Cotter!
- Full laboratory validation with nonlinear model underway.
- Optimisation contraction/buoy geometry (e.g. B. et al. IEEE2024), latching & other control.
- **Future**: *elastomer sheet* (manufacturing underway, 이 재만, lab and model testing planned using hyperelastic model & VBM).

Thank you very much for your attention ...

- B., Zweers 2013: Proof of principle 2013 https://www.youtube.com/watch?v=SZhe_SOxBWo&t=254s
- B., Kalogirou, Zweers 2019: From bore-soliton-splash to a new wave-to-wire wave-energy model. *Water Waves* **1** 10.1007/s42286-019-00022-9 Bore-soliton-splash: https://www.youtube.com/watch?v=YSXsXNX4zW0&list=FL6mc7mUa6M4Bo2VkD970urw
- B., Henry, Kalogirou, Thomas 2020: *Int. Marine Energy J.*
- Choi, Kalogirou, Lu, B., Kelmanson 2024: A study of extreme water waves using a hierarchy of models based on potential-flow theory. *Water Waves* https://doi.org/10.1007/s42286-024-00084-4
- B., Bolton, Thompson, Geometric power optimisation of a rogue-wave energy device in a (breakwater) contraction. 8th IEEE Conference on Control Technology & Applications (CCTA) (2024) 6 pp. Preprint https://eartharxiv.org/repository/view/7260/
- Lu, Gidel, Choi, B., Kelmanson 2025: Subm. *Geoscientific Model Development*.