

On floods
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Wetropolis' weather
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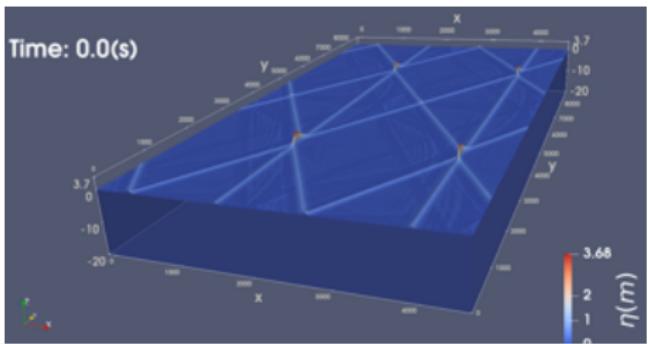
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Extreme events in Wetropolis flood investigator & dynamics of extreme water waves

Onno Bokhove [et al.], KAIST 21-08-2024
£€: EU Eagre GA859983

Leeds Institute for Fluid Dynamics, UK



Outline

I will give an overview of our work on the mathematics and statistics of:

- ▶ Wetropolis flood investigator (B et al. (2020, 2024)),
- ▶ a novel wave-energy device based on extreme and extremely-high water waves (e.g., Kadomtsev & Petviashvili 1970, Benney & Luke 1964, Luke 1967, Kodama (2010, 2018), B. and Kalogirou 2016, Choi et al. (2022,2024), B. et al (2019,IEEE2024)).

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On floods: Wetropolis flood investigator



Inspiration for Wetropolis: **Boxing Day 2015 floods** of the River Aire in Leeds

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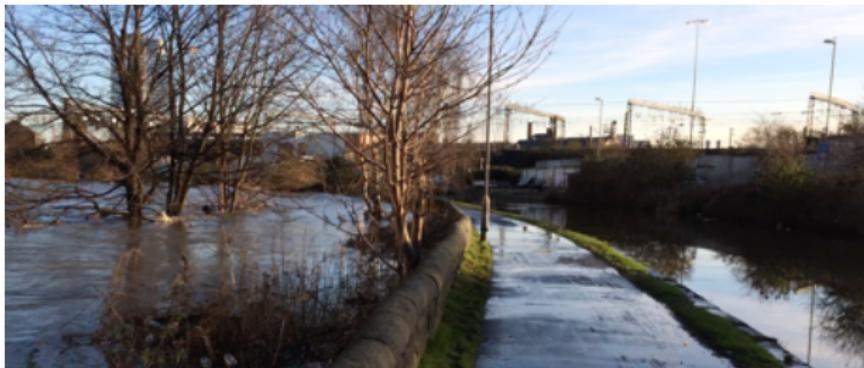
Tour of Wetropolis: visualising extreme events

Goal: visualising return period/Annual Exceedance Probability
(request EA & JBA Trust). <https://www.youtube.com/watch?v=rNgEqWdafKk>



Legacy of our work on flooding

- ▶ **Analysis Leeds' public 2017 Flood-Alleviation Scheme II:** led to graphical flood-mitigation cost-effectiveness tool, laying bare inconsistencies in FASII.
- ▶ **Wetropolis inspired cost-effectiveness tool:** used in flood cases France & Slovenia
- ▶ Tool shows that **efficacy of Natural Flood Management** small to minute, e.g. beaver dams, somewhat contrary to overstated promotion of NFM & beavers by Environment Agency, council & media.
- ▶ See **REF Impact case study ICS 2021:** Wetropolis & flood-mitigation effectiveness tool for decision-makers. <https://results2021.ref.ac.uk/impact/submissions/1eedb5bd-8f92-4737-a6f0-1e61c997e4f0/impact>



How (well) can we mitigate flooding?

Flood-mitigation measures, but which ones to choose?

- ▶ Higher walls (HW)
- ▶ Flood-plain storage (FPS): dynamic using weirs and optimal control (underdeveloped)
- ▶ Giving-room-to-the-river (GRR)
- ▶ Natural _{Flood} Management (NFM): tree planting, peat land, leaky dams
- ▶ Beaver colonies
- ▶ Sustainable urban drainage systems (SUDS)
- ▶ Dredging
- ▶ Resilience?

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How (well) can we mitigate flooding?

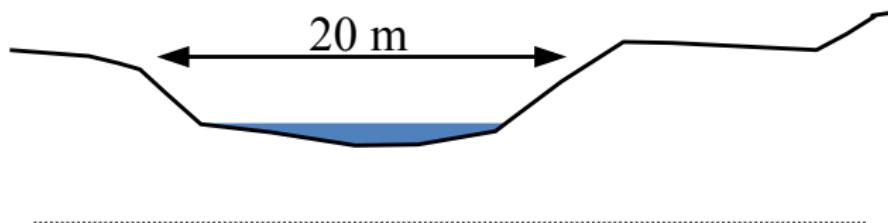
Higher flood defence walls – HW (2m high proposed in Leeds):



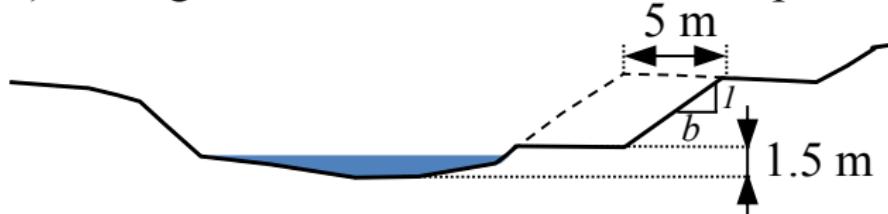
How (well) can we mitigate flooding?

Giving-room-to-the-river – GRR:

a) Current transverse profile



b) Giving-room-to-the-river transverse profile



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How (well) can we mitigate flooding?

Giving-room-to-the-river – GRR, extra channel in River Aire at *Aire River at Kirkstall The Forge* (Leeds):

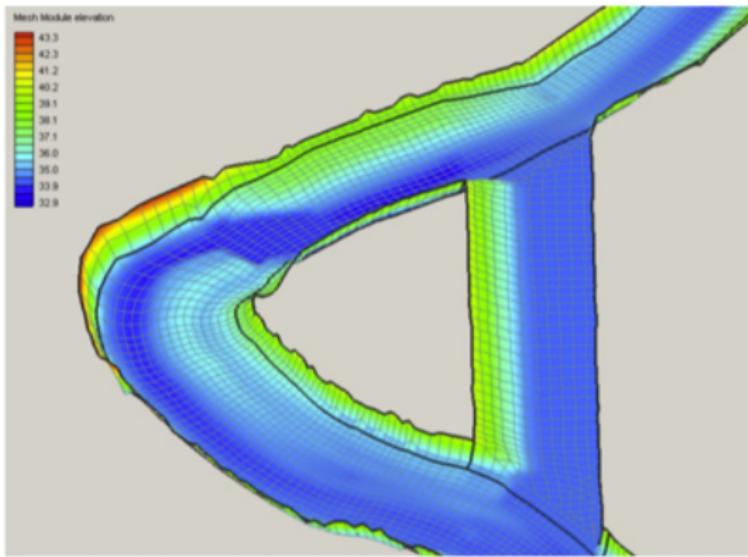


Illustration 1: Meander bend with flood relief channel, TUFLOW FV mesh

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How (well) can we mitigate flooding?

Giving-room-to-the-river – GRR, extra channel in River Waal/Rhine Nijmegen (NL):



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How (well) can we mitigate flooding?

Flood-plain storage –FPS & dynamic weir control:



How (well) can we mitigate flooding?

Extra storage –FPS active flooding of certain areas (Merwede, Storm Ciara, NL, 20Mm³):



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How (well) can we mitigate flooding?

Natural flood management – NFM 1300 leaky dams & trees (public engagement & co-benefits, e.g. carbon sequestration)



Central part of one of the two experimental timber bunds in the River Seven catchment

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How (well) can we mitigate flooding? Beavers nonsense!

Imagine your home is flooded. Lots of [beaver colonies](#) then? Extra water storage behind dams: $\sim 1100\text{m}^3 = 1.1\text{Mlitres}$ (or $1/5^{\text{th}}$).

How beavers can help stop homes from flooding

○ 17 Feb 2020 Last updated at 11:08



[Beavers can play an important role in helping to keep our homes from being flooded.](#)

That's according to scientists at Exeter University, who have carried out a five year study of wild animals living in Devon.

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How (well) can we mitigate flooding?

Dredging –Wainfleet Flood Action Group (flood June 2019, 67 homes & lots of farmland flooded):



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How (well) can we mitigate flooding?

Resilience: raising of new houses now mandatory in Wainfleet:

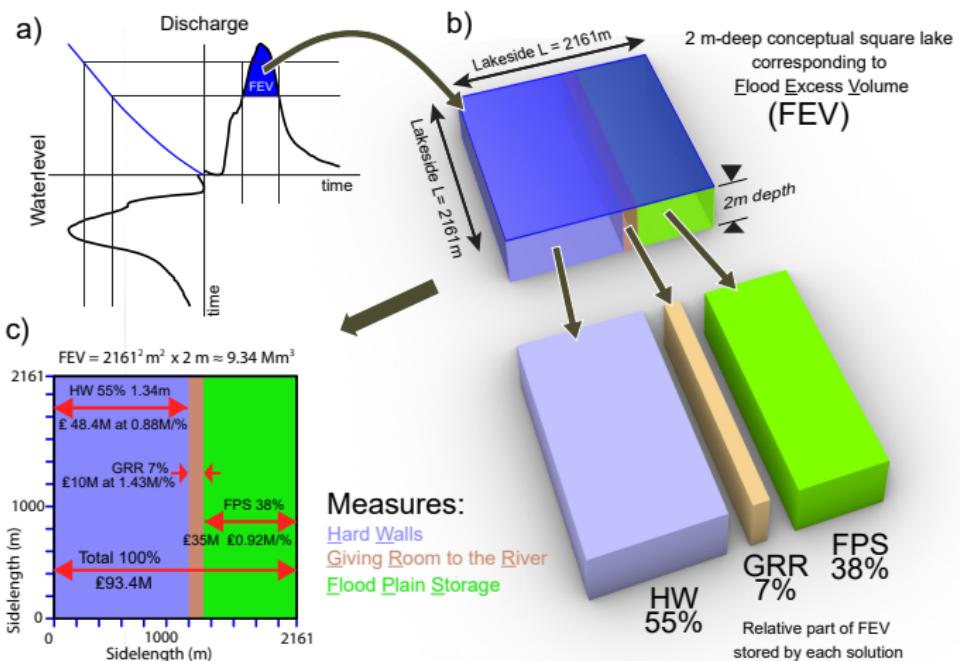


How (well) can we mitigate flooding?

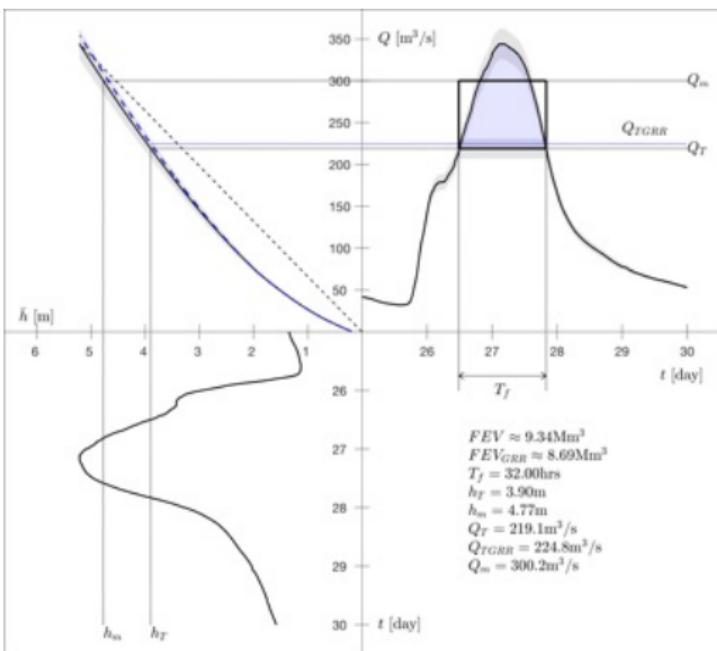
Resilience: responsible flood-plain development (**zero-sum or negative volume rule**), Rhine valley:



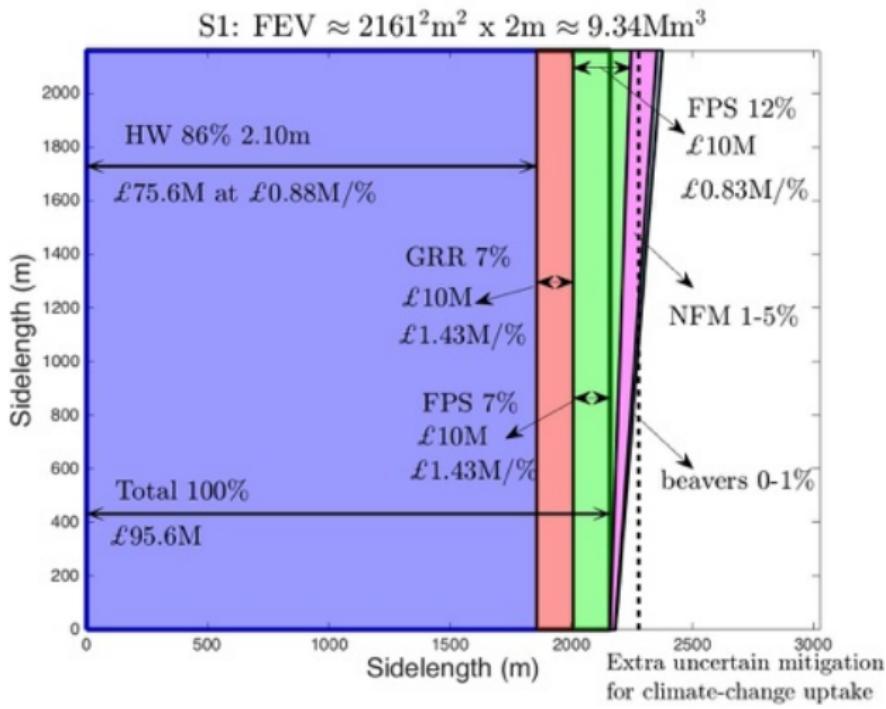
Graphical cost-effectiveness tool for flood mitigation



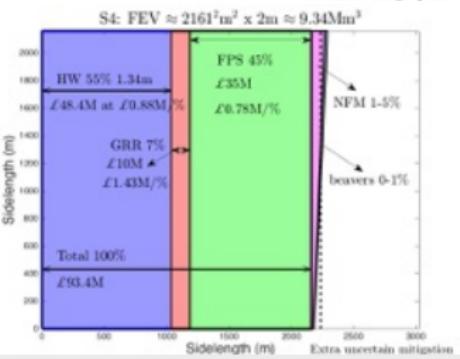
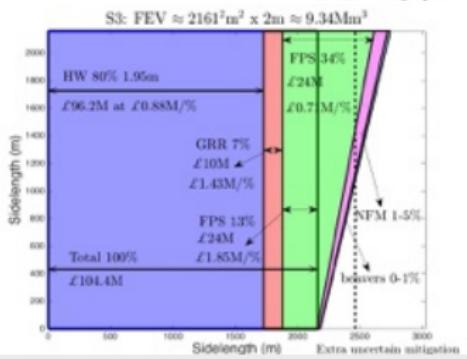
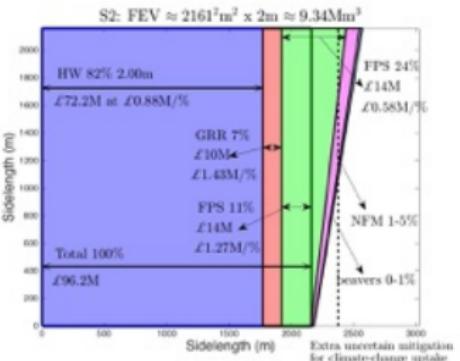
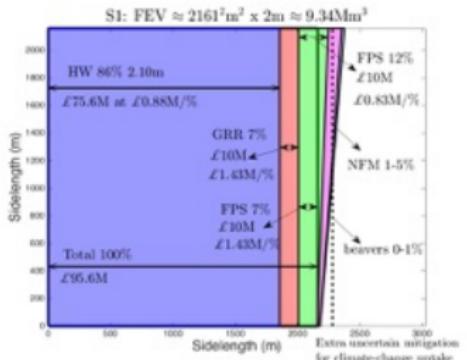
Graphical cost-effectiveness tool: three-panel graphs



Graphical cost-effectiveness tool: square lake (1 : 200yr design flood)



Graphical cost-effectiveness tool: square lake scenarios (1 : 200yr design flood)



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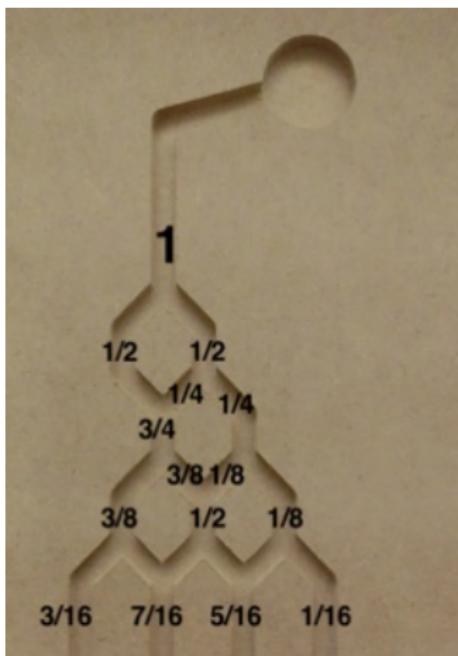
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Weather in Wetropolis: skew Galton boards

Ball falls through, peak chance at $7/16$ & “rare” event at $1/16$:



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Weather in Wetropolis-I: two skew Galton boards

Two Galtonboards, one rain duration & one for rain location:



Wetropolis-I's weather: probability and statistics

- ▶ X, Q : probabilities p_i rainfall duration/wd versus q_j rain location:
- ▶ p_i, q_j with $i, j = 1, 2, 3, 4$ and $\sum p_i = 1, \sum q_j = 1$.
- ▶ For old case, $p_1 = q_1 = 3/16, p_2 = q_2 = 7/16, q_3 = p_3 = 5/16, q_4 = p_4 = 1/16$:

Table: Probability matrix $P_{ij} = p_i q_j$ times 256.

	1s	2s	4s	9s
p_1	p_2	p_3	p_4	
reservoir q_1	9	21	15	3
both q_2	21	49	35	7
moor q_3	15	35	25	5
no rain q_4	3	7	5	1

Return period of floods: geometric distribution

- ▶ Rain amount per $T_d = 10\text{s} = 1\text{wd}$ determined by **design**: no to minor flooding for (1, 2, 4) & (8, 9), **flooding** for 18 units.
- ▶ **Return period** T_r of extreme flooding at $t_n = nT_d$ determined by geometric distribution with here $p_n = (1 - p_e)^{n-1} p_e$ where $p_e = P_{24} = q_2 p_4 = 7/256$, s.t.

$$T_r = \mathbb{E}(t_n) = \sum_{n=1}^{\infty} T_d n (1 - p_e)^{n-1} p_e = \frac{T_d}{p_e} \approx 365.7\text{s} \approx \mathbf{6 : 06\text{min.}}$$

- ▶ Standard deviation σ_r (thanks to Daan C & Jason F):

$$\begin{aligned}\sigma_r^2 &= \mathbb{E}((t_n - \mathbb{E}(t_n))^2) = (1 - p_e) \frac{T_d^2}{p_e^2} \\ &= (1 - p_e) T_r^2 \implies \sigma_r = 36.07\text{wd} = 360.7\text{s} \approx 6\text{min.}\end{aligned}$$

Super- and megafloods: geometric distribution of order k

- ▶ Two consecutive “2015 Boxing Days” extreme rainfall WEP
 $p_e^2 = (7/256)^2$ s.t.

$$T_r^{(2)} \approx \frac{T_d}{p_e^2} = (256/7)^2 \times 10\text{s} \approx 223\text{min} \approx 3 : 43\text{hr.}$$

Movie “Wetropolis Boxing Day flood” on <https://github.com/obokhove/wetropolis20162020>

- ▶ $T_r^{(2)}$ & $\sigma_r^{(2)}$ follow from **geometric distribution of order $k = 2$**
 (Viveros & Balakrishnan 1993, Koutras & Eryilmaz 2017):

$$\frac{T_r^{(k)}}{T_d} = \frac{(1 - p_e^k)}{(1 - p_e)p_e^k}, \quad \frac{\sigma_r^{(k)}}{T_d} = \frac{\sqrt{1 - (2k + 1)(1 - p_e)p_e^k - p_e^{2k+1}}}{(1 - p_e)p_e^k}.$$

Super- and megafloods: Wetropolis-II revisited design

- ▶ For floods on two consecutive days with old $p_e = 7/256$:

$$T_r^{(2)} = T_d \frac{(1 + p_e)}{p_e^2} = 1374 \text{wd} = 13740 \text{s} = 3.8 \text{hr},$$

$$\sigma_r^{(2)} = T_d \frac{\sqrt{1 - 5(1 - p_e)p_e^3 - p_e^5}}{(1 - p_e)p_e^3} = 3.8 \text{hr}.$$

- ▶ Long waiting times suggest *redesign*, e.g. take Galton board outcome $p_e = p_2 q_2 = 49/256 \approx 1/5$ for 9s rainfall in moor & reservoir, yielding **return periods for $k = 2, 3$ -day floodings:**

$$T_r = 5.2 \text{wd} = 52 \text{s}, T_r^{(2)} = 32.5 \text{wd} = 5 : 25 \text{min},$$

$$T_r^{(3)} = 175 \text{wd} = 29 : 11 \text{min}, \sigma_r^{(k)} \approx T_r^{(k)}, k = 1, 2, 3.$$

Wetropolis-II's' weather: revisited

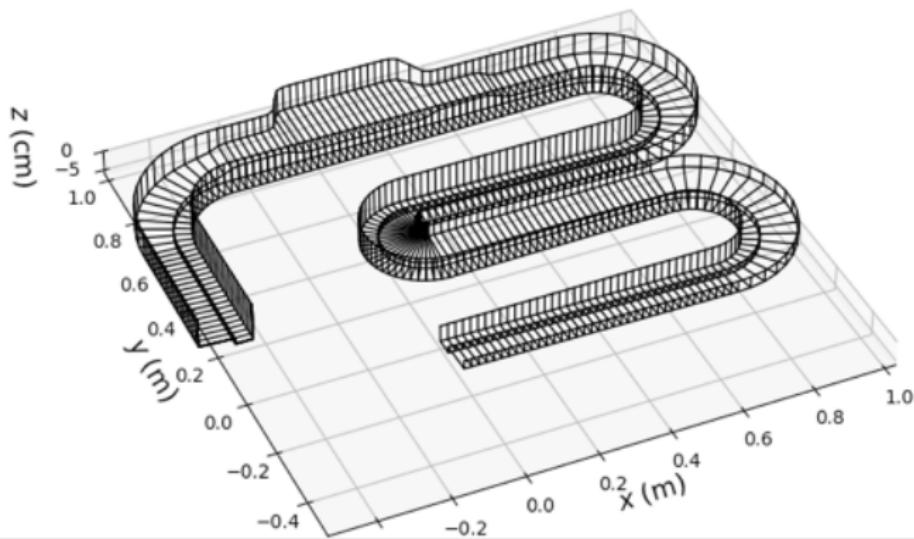
- ▶ X, Q : probabilities p_i rainfall duration/wd versus q_j rain location: <https://www.youtube.com/watch?v=g8znktYpxvY>
- ▶ p_i, q_j with $i, j = 1, 2, 3, 4$ and $\sum p_i = 1, \sum q_j = 1$.
- ▶ For current case, $p_1 = q_1 = 3/16, p_2 = q_2 = 7/16, q_3 = p_3 = 5/16, q_4 = p_4 = 1/16$:

Table: Probability matrix $P_{ij} = p_i q_j$ times 256.

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Wetropolis mathematical models

Kinematic river flow in **design model** fixed wd, r_0 ; 1D **predictive** shallow-water river model with ground-water & reservoir dynamics, in progress (PDEs & ODEs: B. et al, HESS, 2020). Bathymetry:



Wetropolis 1D model: maths $A, u, h_m, h_{res}, h_{1c}, h_{2c}, h_{3c}$

River:

$$\begin{cases} \partial_t A + \partial_s(Au) = S_A \\ \partial_t(Au) + \partial_s(Au^2) + gA\partial_sh = -g \left(A\partial_sb + \frac{C_m^2 Au|u|}{R^{4/3}} \right) + uS_A \end{cases} \quad \text{on } s \in [0, L]$$

with $h = h(A(s, t))$, $h(s, 0) = h_0(s)$, $u(s, 0) = u_0(s)$,

and $S_A(t) = (1 - \gamma)Q_{res}(t)\delta(s - s_{res}) + Q_{moor}(t)\delta(s - s_{moor}) + Q_{1c}(t)\delta(s - s_{1c})$

(37a)

Moor:

$$\partial_t(w_v h_m) - \alpha g \partial_y(w_v h_m \partial_y h_m) = \frac{w_v R_{moor}(t)}{m_{por}\sigma_e} \quad \text{on } y \in [0, L_y]$$

with $\partial_t h_m|_{y=L_y} = 0$, $h_m(0, t) = h_{3c}(t)$, $h_m(y, 0) = h_{m0}(y)$

(37b)

Reservoir:

$$w_{res}L_{res} \frac{dh_{res}}{dt} = w_{res}L_{res}R_{res}(t) - Q_{res}, \text{ with } h_{res}(0) = h_{r0}$$
(37c)

Canal-1:

$$w_c(L_{1c} - L_{2c}) \frac{dh_{1c}}{dt} = Q_{2c} - Q_{1c}, \text{ with } h_{1c}(0) = h_{10}$$
(37d)

Canal-2:

$$w_c(L_{2c} - L_{3c}) \frac{dh_{2c}}{dt} = Q_{3c} - Q_{2c}, \text{ with } h_{2c}(0) = h_{20}$$
(37e)

Canal-3:

$$w_c L_{3c} \frac{dh_{3c}}{dt} = \gamma Q_{res} - Q_{3c}, \text{ with } h_{3c}(0) = h_{30},$$
(37f)

Influxes:

$$Q_{1c} = C_f \sqrt{g} w_c \max(h_{1c} - P_{1w}, 0)^{3/2}$$
(37g)

$$Q_{2c} = C_f \sqrt{g} w_c \max(h_{2c} - P_{2w}, 0)^{3/2}$$
(37h)

$$Q_{3c} = C_f \sqrt{g} w_c \max(h_{3c} - P_{3w}, 0)^{3/2}$$
(37i)

$$Q_{moor} = \frac{1}{2} m_{por} \sigma_e w_v \alpha g (\partial_y h_m)^2|_{y=0}$$
(37j)

$$Q_{res} = C_f \sqrt{g} w_{res} \max(h_{res} - P_{wr}, 0)^{3/2}$$
(37k)

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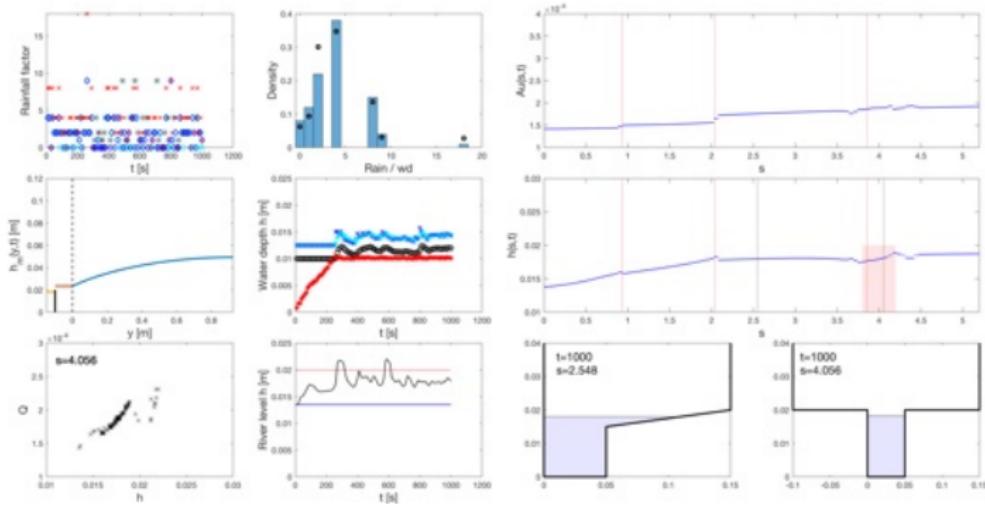
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Wetropolis modelling

Simulations, https://github.com/tkent198/hydraulic_wetro:



Wetropolis flood investigator: future work & proposal

How can a **Wetropolis laboratory set-up** and a “**Numerical Wetropolis Prediction**” model be used to understand:

- ▶ risk, extreme weather & flooding probability statistics –revisit **spatial-temporal rainfall** & change-point analysis;
- ▶ rare-event simulations (for events of “intermediate rarity”);
- ▶ flood control –e.g., reservoirs in Wetropolis;
- ▶ data assimilation & parameter estimation –experiment as “**truth run**”; test (limits of) **machine learning**;
- ▶ Wetropolis World’s **goal**: investigate “classical” PDE & Data Assimilation “**NWP**” model with ML predictions.
- ▶ **Proposal EPSRC-F⁺**: PDE/ML, info-gap theory on decision-making, 1/4 educational-version, board game, workshops.

On waves: Modelling extremely high water waves

- ▶ Origin 2010 *bore-soliton-splash*:
- ▶ **Definition** rogue/extreme waves:
 $AI = H_r/H_s > 2$.
- ▶ To what extent do exact but idealised extreme- or rogue-wave solutions survive in more realistic settings?
- ▶ Will such *extreme waves* fall apart due to dispersion or other mechanisms?
- ▶ Use fourfold and ninefold **KPE-link** amplifications of interacting solitons/cnoidal waves.
- ▶ What do you think: will we be **able to reach the ninefold wave amplification** in more realistic calculations or in reality?



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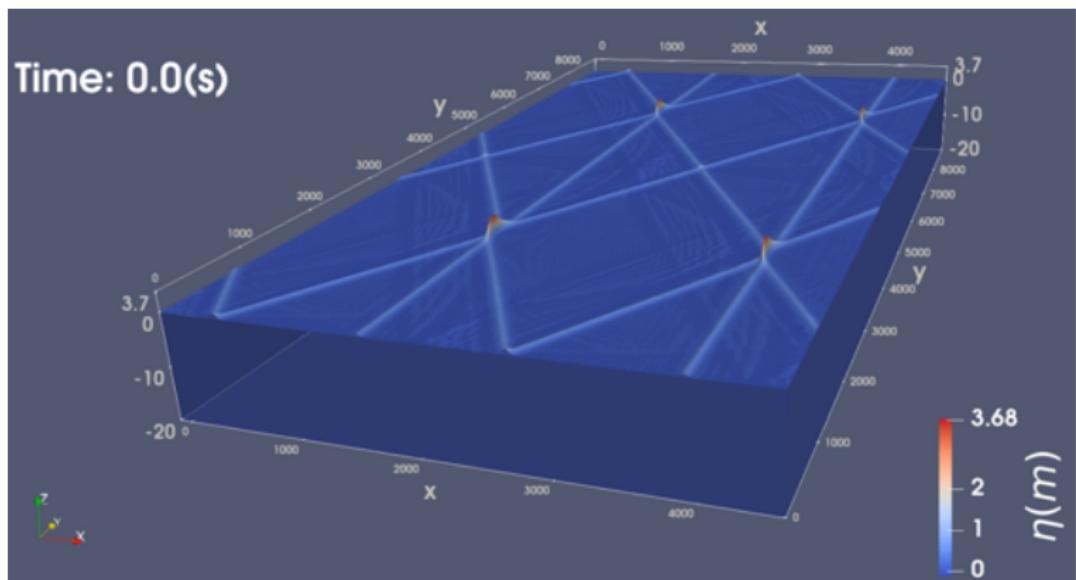
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Results simulation three-soliton interaction (dimensional)

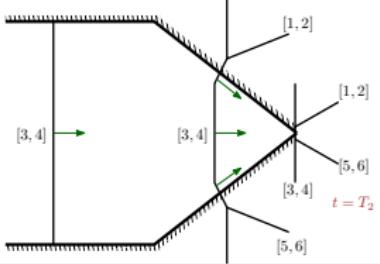
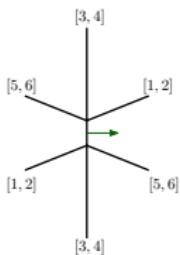
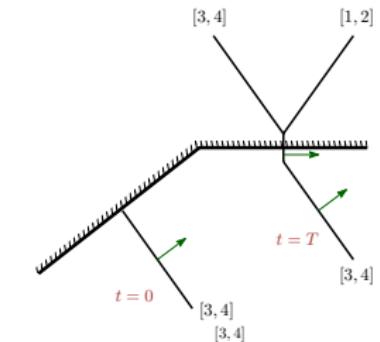
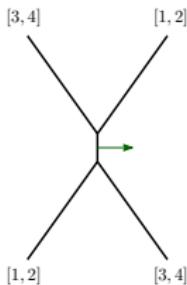
Crossing seas (8 domains combined) <https://www.youtube.com/watch?v=EGhpQ7BM2jA>



Two & three-soliton interactions in plane vs. wave tank

Left: on infinite horizontal plane; **right:** top view of wave tank.

Top/bottom: 2 or 3 solitons. (Exact solutions to KP equation.)



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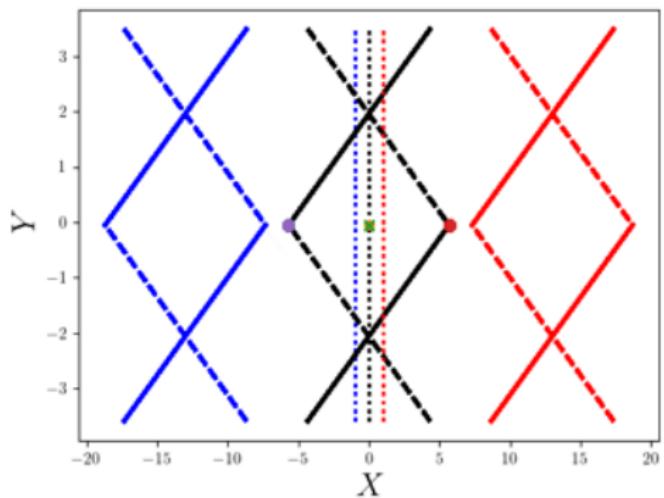
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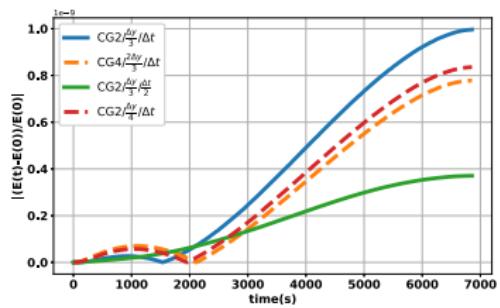
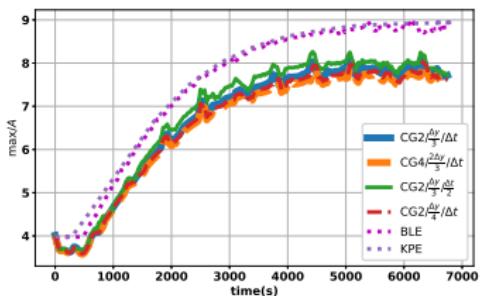
Three-soliton interactions

Sketch far-field line solitons at times prior to (blue), at (black) & after (red) maximum amplification (geometric & analytical proofs):



Results simulations three-soliton interaction (dimensional)

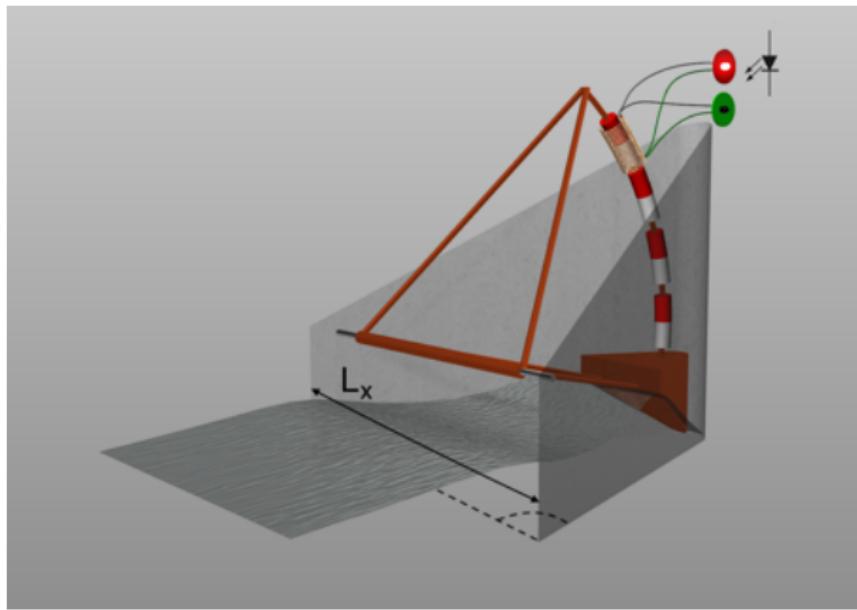
Kadomtsev-Petviashvili equation (KPE, exact), Benney-Luke equations (BLE), potential-flow equations (PFE, CG-FEM):



Novel wave-energy device in a breakwater contraction

Proof of principle: https://www.youtube.com/watch?v=SZhe_S0xBWo&t=254s.

Sketch wave amplification in contraction with angle θ_c :



Grand variational principle of novel wave-energy device

Equations of motion follow from variational principle (**red**=waves, **blue**=buoy, **green**=EM-generator, **coupling**):

$$\begin{aligned}
 0 = & \delta \int_0^T \int_0^{L_x} \int_{R(t)}^{l_y(x)} \int_0^h -(\partial_t \phi + \frac{1}{2} |\nabla \phi|^2) dz - gh(\frac{1}{2} h - H_0) \\
 & - \frac{1}{2\gamma} \left(F_+(\gamma(h - h_b) - \lambda) - \lambda^2 \right) dy dx \\
 MW \dot{Z} - \frac{1}{2} MW^2 - MgZ + (L_i I - K(Z)) \dot{Q} - \frac{1}{2} L_i I^2 dt \quad (1)
 \end{aligned}$$

velocity $u = \nabla \phi(x, y, z, t)$, depth $h(x, y, t)$, rest depth H_0 , buoy $h_b(Z, x) = Z - K - \tan \theta(L_y - x)$, piston $R(t)$, coupling function $\gamma_m G(Z) = K'(Z)$, buoy mass M , keel height K , buoy coordinate $Z(t)$, buoy velocity $W(t) = \dot{Z}$, charge $Q(t)$, current $I(t) = \dot{Q}$.

Mathematical modelling: PDEs

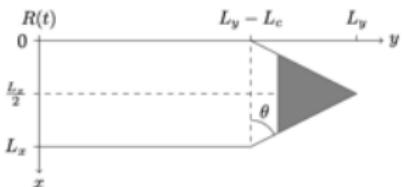
- ▶ Potential-flow water-wave dynamics (Laplace equation in interior, kinematic & Bernoulli equations at free surface):

$$\delta\phi : \nabla^2 \phi = 0 \quad \text{in } \Omega$$

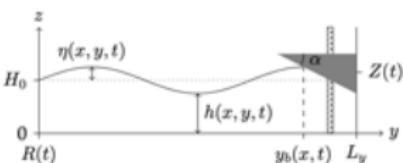
$$(\delta\phi)|_{z=h} : \partial_t h + \nabla \phi \cdot \nabla h = \phi_z \quad \text{at } z = h$$

$$\delta h : \partial_t \phi + \frac{1}{2} |\nabla \phi|^2 + g(z - H_0) - \lambda = 0 \quad \text{at } z = h.$$

- ▶ Coupled **elliptic Laplace equation** to **hyperbolic free-surface equations**, plus a (Lagrange) **multiplier** λ .



(b) Top view of the tank and buoy, outlining the tank's dimensions and how the buoy fits the shape of the constriction.



(c) Side view at time t , with the buoy constrained to move vertically.

Mathematical modelling: inequality constraint & ODEs

- Karush-Kuhn-Tucker inequality conditions satisfied at every space-time x, t -position are:

$$\begin{aligned}\delta\lambda : \lambda &= -[\gamma(h - h_b) - \lambda]_+ = -F_+(\gamma(h - h_b) - \lambda) \\ \implies h(x, t) - h_b(Z, x) &\leq 0, \lambda \leq 0, \lambda(h - h_b) = 0.\end{aligned}$$

- Add resistance R_i, R_c & Shockley load $V_s(|I|)$ to submodel:

$$\delta W : \dot{Z} = W,$$

$$\delta Z : M\dot{W} = -Mg\underline{-\gamma_m G(Z)I} - \int_0^{L_x} \int_0^{l_y(x)} \lambda \, dy \, dx$$

$$\delta I : \dot{Q} = I,$$

$$\delta Q : L_i \dot{I} = \underline{\gamma_m G(Z)\dot{Z}} - (R_i + R_c)I - \frac{I}{|I|} V_s(|I|).$$

On floods

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Wetropolis' weather

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W-investigator

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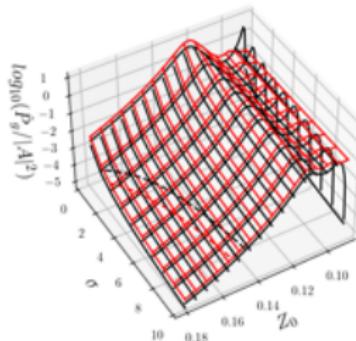
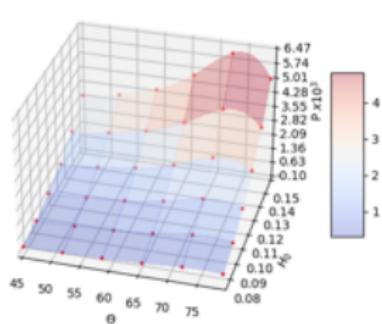
On waves

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Optimisation wave-energy device

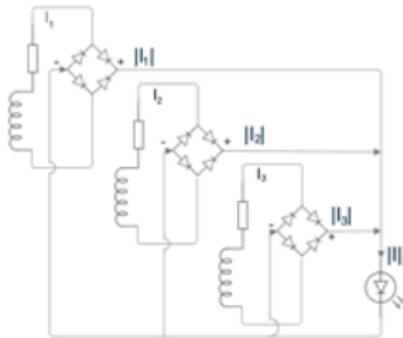
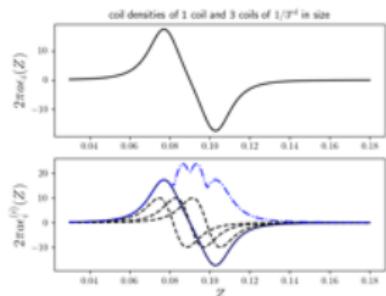
Surrogate modelling geometry angle/rest depth θ_c-H_0 &
1-coil (**black**) vs. 3-coil (**red**) power P_g (B. et al IEEE2024):

RBF approximation of Power Output beta=2.0 and n=36



Novel wave-energy device: future work

- ▶ **Brief overview given** of wave-energy device, based on extreme-wave amplification in a contraction. Showcased modelling with VPs & geometric numerical integrators.
- ▶ **Future work:** optimisation, real-time control using Pontryagin's principle, surrogate modelling & ML.
- ▶ Experiment soliton interactions: <https://www.youtube.com/watch?v=cBhr0DnVclU>
- ▶ **Laboratory realisation under development**, for testing of submodels & wave-to-wire models.



Thanks very much for your attention ...

- ▶ Knotters, B, Lamb, Poortvliet 2024: How to cope with uncertainty monsters in flood risk management? *Water Prisms*. <https://doi.org/10.1017/wat.2024.4> (Nominated paper.)
- ▶ B, Kelmanson, Piton, Tacnet 2024: Visualising Flood Frequency, Flood Volume and Mitigation of Extreme Events. <https://obokhove.github.io/UKsuccessFEVpreprint23102023.pdf>
<https://www.youtube.com/watch?v=g8znktYpxvY>
- ▶ B, Kelmanson, Hicks, Kent: 2022: Flood mitigation: from outreach demonstrator to a graphical cost-effectiveness diagnostic for policy makers. UK Research Excellence Framework Impact Case Study.
<https://results2021.ref.ac.uk/impact/submissions/1eedb5bd-8f92-4737-a6f0-1e61c997e4f0/impact>
- ▶ B, Hicks, Kent, Zweers 2020: Wetropolis extreme rainfall and flood demonstrator: from mathematical design to outreach and research. *Hydrology and Earth System Sciences* **24**.
[/doi.org/10.5194/hess-24-2483-2020](https://doi.org/10.5194/hess-24-2483-2020)
- ▶ B., Kalogirou, Zweers 2019: From bore-soliton-splash to a new wave-to-wire wave-energy model. *Water Waves* **1** [10.1007/s42286-019-00022-9](https://doi.org/10.1007/s42286-019-00022-9) Bore-soliton-splash:
<https://www.youtube.com/watch?v=YSXsXNX4zW0&list=FL6mc7mUa6M4Bo2VkJ970urw>
- ▶ Choi, Kalogirou, Lu, B., Kelmanson 2024: A study of extreme water waves using a hierarchy of models based on potential-flow theory. *Water Waves* <https://doi.org/10.1007/s42286-024-00084-4>
- ▶ B., Bolton, H. Thompson, Geometric power optimisation of a rogue-wave energy device in a (breakwater) contraction. **8th IEEE Conference on Control Technology & Applications (CCTA)** (2024) 6 pp. Preprint
<https://eartharxiv.org/repository/view/7260/>