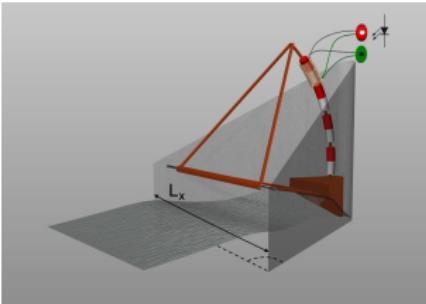


Power optimisation of a heaving buoy in a wave-enhancing contraction

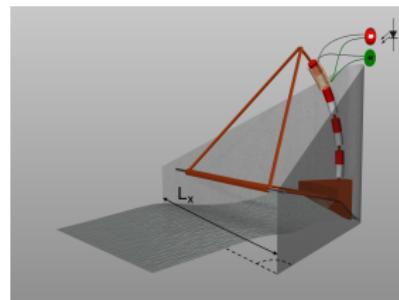
Onno Bokhove, P. Grieve, J. Kim, O. Naar, H. Thompson
£: CDT Fluid Dynamics

Leeds Institute for Fluid Dynamics, UK



Outline: modelling components

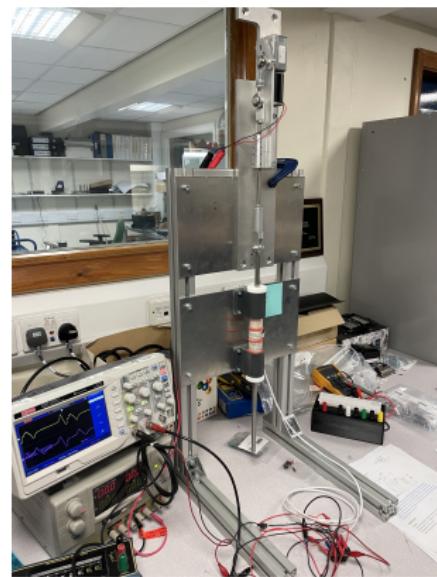
- ▶ Power optimisation of a heaving-buoy wave-energy device considered for a device placed in a wave-amplitude enhancing (V-shaped) contraction.
- ▶ Ideally placed in a breakwater or in an array of contractions moored at sea.
- ▶ Nonlinear wave-to-wire model consisting of 3 components:
 - (i) wave (hydro)dynamics,
 - (ii) buoy motion, and
 - (iii) power generation.



Outline: partitioning & main advances

We have made the following advances for the buoy-generator sub-model (ii-iii):

- ▶ Experimental validation of the model hanging from a driven moving point in its **dry setting**.
- ▶ Same length induction coils placed in parallel and rectified generate most power; three-phase rectifier is best.



Outline: partitioning & main advances

Advances wave-buoy sub-model (i-ii) are:

- ▶ Novel inequality-constraint technique to couple wave and buoy derived and implemented: water surface \leq buoy hull.
- ▶ Nonlinear waves: dispersive, depth-integrated (quadratic in z with 2 dofs) Variational Boussinesq or Green-Naghdi Model (e.g. Gagarina et al. 2013).
- ▶ Spectrally-accurate 2nd- to 5th-order FEM in 2D horizontal numerical wavetank (open source *Firedrake* environment).



Grand variational principle of wave-to-wire model

Equations of motion follow from variational principle (**red**=waves, **blue**=buoy, **green**=EM-generator, coupling, B. et al. 2019):

$$\begin{aligned} 0 = & \delta \int_0^T \int_0^{L_x} \int_{R(t)}^{l_y(x)} \int_0^h -(\partial_t \phi + \frac{1}{2} |\nabla \phi|^2) dz - gh(\frac{1}{2} h - H_0) \\ & - \frac{1}{2\gamma} \left(F_+(\gamma(h - h_b) - \lambda)^2 - \lambda^2 \right) dy dx \\ MW \dot{Z} - \frac{1}{2} MW^2 - MgZ + (L_i I - \underline{K(Z)}) \dot{Q} - \frac{1}{2} L_i I^2 dt \quad (1) \end{aligned}$$

velocity $u = \nabla \phi(x, y, z, t)$, depth $h(x, y, t)$, rest depth H_0 , e.g. buoy $h_b(Z, y) = Z - K_h - \tan \theta (L_y - y)$, piston $R(t)$, coupling function $\gamma_m G(Z) = K'(Z)$, buoy mass M , keel height K_h , buoy coordinate $Z(t)$, buoy velocity $W(t) = \dot{Z}$, charge $Q(t)$, current $I(t) = \dot{Q}$.

Wave-to-wire: PDEs

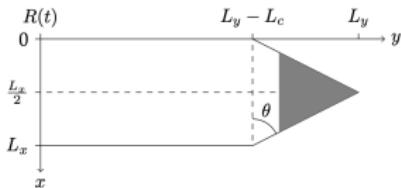
- ▶ Potential-flow water-wave dynamics (Laplace equation in interior, kinematic & Bernoulli equations at free surface):

$$\delta\phi : \nabla^2 \phi = 0 \quad \text{in } \Omega$$

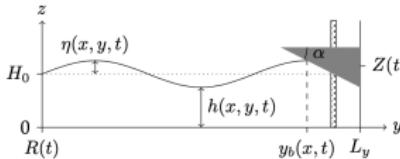
$$(\delta\phi)|_{z=h} : \partial_t h + \nabla \phi \cdot \nabla h = \phi_z \quad \text{at } z = h$$

$$\delta h : \partial_t \phi + \frac{1}{2} |\nabla \phi|^2 + g(z - H_0) - \lambda = 0 \quad \text{at } z = h.$$

- ▶ Coupled **elliptic Laplace equation** to **hyperbolic free-surface equations**, plus a (Lagrange) **multiplier** λ .



(b) Top view of the tank and buoy, outlining the tank's dimensions and how the buoy fits the shape of the contraction.



(c) Side view at time t , with the buoy constrained to move vertically.

Wave-to-wire: inequality constraint & ODEs

- Karush-Kuhn-Tucker inequality conditions satisfied at every space-time x, y, t -position are (Burman et al. 2023):

$$\begin{aligned}\delta\lambda : \lambda &= -[\gamma(h - h_b) - \lambda]_+ = -F_+(\gamma(h - h_b) - \lambda) \\ \implies h(x, y, t) - h_b(Z, y) &\leq 0, \lambda \leq 0, \lambda(h - h_b) = 0.\end{aligned}$$

- Add resistance R_i, R_c & Shockley load $V_s(|I|)$ to submodel:

$$\delta W : \dot{Z} = W,$$

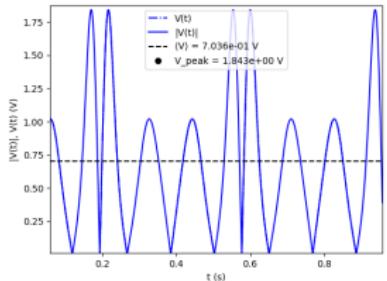
$$\delta Z : M\dot{W} = -Mg \frac{-\gamma_m G(Z)I}{-\gamma_m G(Z)I - \int_0^{L_x} \int_0^{l_y(x)} \lambda dy dx}$$

$$\delta I : \dot{Q} = I,$$

$$\delta Q : L_i \dot{I} = \underline{\gamma_m G(Z) \dot{Z}} - (R_i + R_c)I - \frac{I}{|I|} V_s(|I|).$$

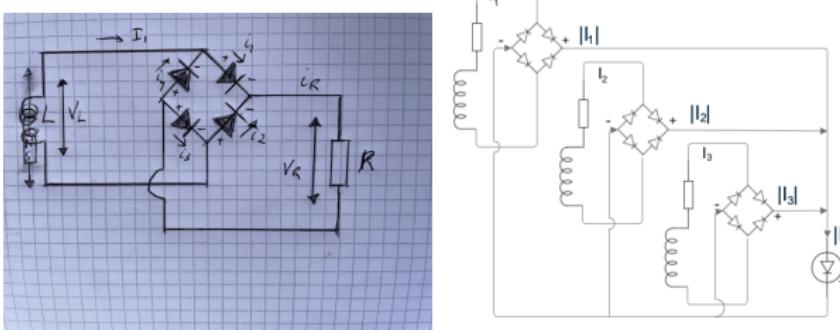
Preliminary experimental validation buoy-generator

- Modeled/observed outputs $V(t)$ from one-coil (or 3 coils in series) with 1 bridge rectifier & 3 magnets.
- **Simulation** of one-coil model with ideal bridge-rectifier:
 $1/T_{period} = 2.6\text{Hz}$, $R = 100\text{Ohm}$, $l_2 = 0.035\text{m}$. Peak/mean values: (1.843, 0.736)V.
- **Laboratory observation:** (1.884, 0.921)V for 0.8s.



Experimental validation buoy-generator

- Same length coils in series or parallel: 3-phase parallel & rectified best:



Experimental validation buoy-generator

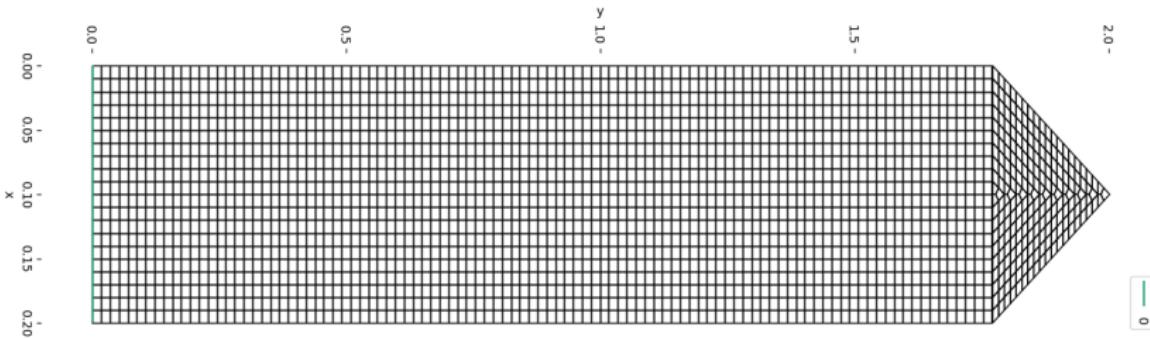
Table: Measurements for resistances/loads $R = 50, 100, 900\text{Ohm}$.

Set-ups: **S series**: 3 coils in series; **3ph**: three-phase rectified of 3 coils;
1ph: 3 coils put in parallel after rectifying each coil; and,
only using *Top, Middle or Bottom* coil.

Name 1 magnet	Max, Mean V	Name 3 magnets	Max, Mean V	Comment
1-100-1ph.csv	1.222,0.348	3-100-1ph.csv	2.341,1.122	2nd best
1-100-3ph.csv	1.750,0.725	3-100-3ph.csv	2.500, 1.528	best
1-100-Bot.csv	nan, nan	3-100-Bot.csv	2.155,1.073	glitch
1-100-Mid.csv	1.046,0.337	3-100-Mid.csv	2.346,0.850	2nd best
1-100-S.csv	0.824,0.306	3-100-S.csv	<u>1.884,0.921</u>	
1-100-Top.csv	0.877,0.300	3-100-Top.csv	1.789,0.733	
1-50-1ph.csv	1.107,0.317	3-50-1ph.csv	2.163,1.079	2nd best
1-50-3ph.csv	1.532,0.482	3-50-3ph.csv	2.500,1.435	
1-50-Bot.csv	0.985,0.258	3-50-Bot.csv	1.896,0.927	
1-50-Mid.csv	0.885,0.299	3-50-Mid.csv	2.045,0.781	
1-50-S.csv	0.635,0.256	3-50-S.csv	1.437,0.703	
1-50-Top.csv	0.767,0.276	3-50-Top.csv	1.584,0.666	

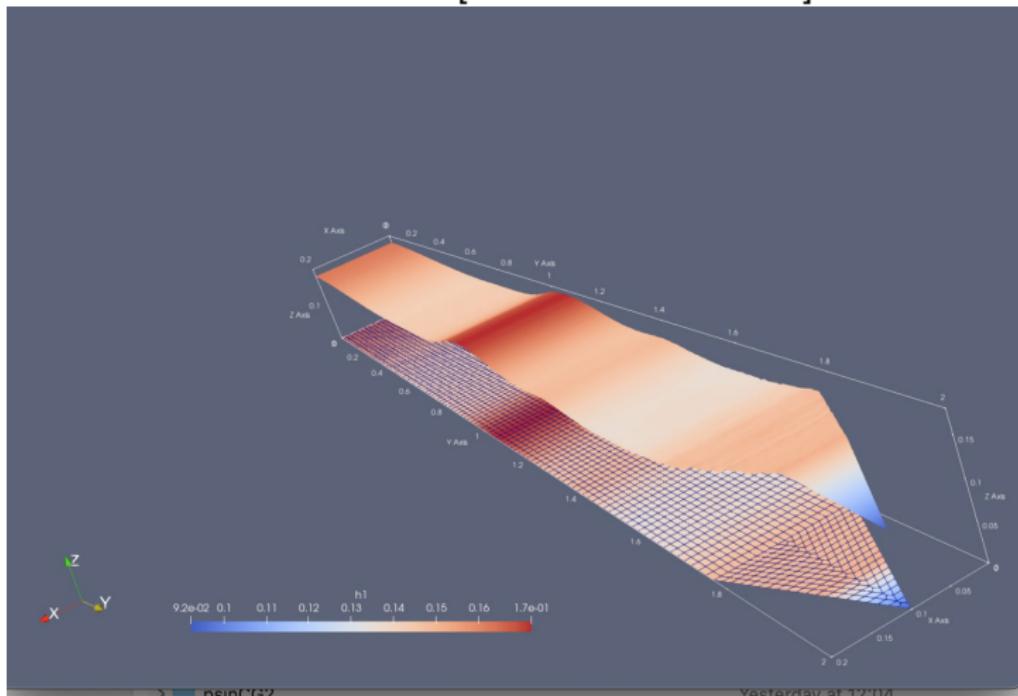
Wave-buoy dynamics: inequality constraints

- ▶ Inequality constraints imposed with *derived, smooth penalty superfunction* as Lagrange multiplier (augmented Lagrangian).
- ▶ FEM mesh, e.g., with 1st, 2nd, 3rd or 4th-order polynomials [dofs: (6,22,50,88)k]; AVF-energy-conservative (-) time integration essential:



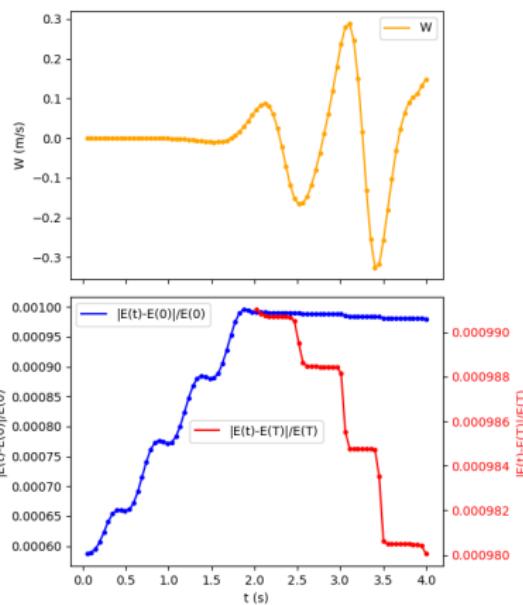
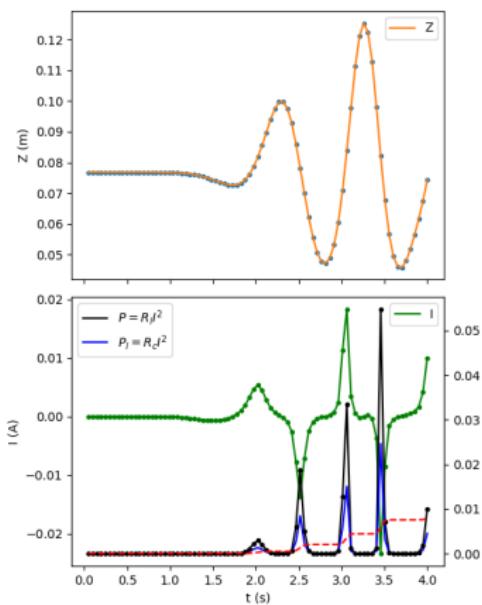
Wave-buoy dynamics: inequality constraints

- Continuous Galerkin CG2 [<https://youtu.be/bBXuj8XBjAk>]:



Wave-buoy dynamics: full model single coil, CG1

- ▶ Preliminary (7am): fully coupled wave-buoy-generator model

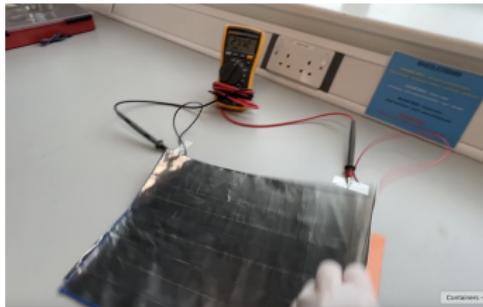


Conclusions

- ▶ Buoy-generator lab/simulation models built: **3-phase rectified parallel circuit best by circa $1.5\times$ to $2\times$.**
- ▶ Nonlinear Variational Boussinesq Model coupled to buoy motion via **novel inequality-constraint technique**: i.e., [Firedrake-GitHub](#) [open source <https://www.firedrakeorg.com> -ask]. Thanks to: Colin Cotter!
- ▶ Full **laboratory validation** with nonlinear model **underway**.
- ▶ Optimisation contraction/buoy geometry (e.g. B. et al. IEEE2024), latching & other control.
- ▶ **Future:** *elastomer sheet* (manufacturing underway, O| 재만, lab and model testing planned using hyperelastic model & VBM).

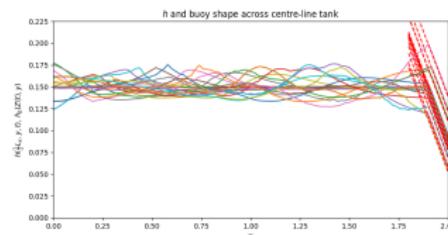
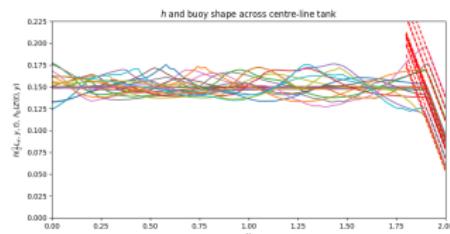
Thank you very much for your attention ...

- ▶ B., Zweers 2013: Proof of principle 2013 https://www.youtube.com/watch?v=SZhe_S0xBWo&t=254s
- ▶ B., Kalogirou, Zweers 2019: From bore-soliton-splash to a new wave-to-wire wave-energy model. *Water Waves* 1 10.1007/s42286-019-00022-9 Bore-soliton-splash:
<https://www.youtube.com/watch?v=YSXsXNX4zW0&list=FL6mc7mUa6M4Bo2VkD970urw>
- ▶ B., Henry, Kalogirou, Thomas 2020: *Int. Marine Energy J.*
- ▶ Choi, Kalogirou, Lu, B., Kelmanson 2024: A study of extreme water waves using a hierarchy of models based on potential-flow theory. *Water Waves* <https://doi.org/10.1007/s42286-024-00084-4>
- ▶ B., Bolton, Thompson, Geometric power optimisation of a rogue-wave energy device in a (breakwater) contraction. 8th IEEE Conference on Control Technology & Applications (CCTA) (2024) 6 pp. Preprint <https://eartharxiv.org/repository/view/7260/>
- ▶ Lu, Gidel, Choi, B., Kelmanson 2025: Subm. *Geoscientific Model Development*.



Wave-buoy dynamics: inequality constraints

- Continuous Galerkin orders CG2, CG3 along centreline:



- Continuous Galerkin orders CG2, CG4:

