

## Chapter 2

### How do humans and beavers view river floods?

**Abstract** Advanced river-level and (intermittent) flow/discharge measurements are analysed for four river floods in particular. Where possible rating curves are introduced as a fit of water-level measurements versus discharge measurements. Flood-excess volume (FEV) is defined as the water volume causing the flood damage, based on introducing both water-level and discharge thresholds. Simplifications as well as complications in determining FEV are discussed. Various storage-based flood-mitigation measures are introduced, and exemplified visually by showing photographs, including higher defence walls, giving-room-to-the-river options, natural flood management, flood-plain storage and nature based solutions (to flooding). Herein the concept of available flood-storage volume is needed and thus defined. The potential of beaver dams for flood protection, or rather their lack of potential, serves as a nice first example of using FEV and available flood-storage volume. FEVs of a large number of river floods are presented and give a sense of the sizes of floods, relative to the size of the relevant river-valley landscape, for which the introduction of (dynamic) square lakes is and will become insightful. A more advanced mathematical example of available flood storage volume is crafted in an active or a dynamic control problem, involving a dam with adjustable porosity or gates permitting the control, in which both humans and beavers are seen to have different optimisation goals and cost functions.

#### 2.1 Introduction

The do-it-yourself example described in the Chapter 1 introduced how one can measure water-level and discharge data in a case when there are no officially maintained river-gauge data of water levels and discharge available. In many countries, however, there is a network of river gauges set up across the country and either associated data are publicly available online, available on request, or both, with online data generally

given over a limited interval backwards in time<sup>1</sup>. Data from these networks tend to be used in the forecasts of river floods. They are also used to monitor and maintain levels in navigation channels for shipping. In addition, while water-level data can be available online, discharge data tend to be measured less often, either directly or indirectly, which means that one needs additional, intermittently-measured data to convert water-level data, e.g., measured in metres (with unit m), into discharge data, e.g., measured in cubic metres per second (unit m<sup>3</sup>/s or cumecs), at a given river cross-section. In England, the Environment Agency (EA) maintains such a network of river gauges with some of these data available online, while further information can (readily) be made available upon request under the Freedom-of-Information act.

In this chapter, such data measured by government agencies will be displayed, analysed and discussed for a variety of river floods, including the River Don in Rotherham Tesco (2019), River Ouse in York (2015) and the Tamar River in Gunnislake (2012), all in the UK, as well as the River Ciliwung near Djakarta (2020), Indonesia<sup>2</sup>. These four rivers have been chosen because their data formats are different, regarding the directly measured river-level data, their rating curves and, hence, the way approximate discharge data are calculated. Furthermore, for each location it will be argued what threshold flood level  $h_T$  is chosen and why, whereafter the flood-excess volume (FEV) can be determined. A formal definition of FEV is provided, recalling its importance as the flooding volume that caused the flood damage, which when reduced to zero would in principle lead to flood protection for a flood volume recurrence of that magnitude. To get a sense of the size of floods, FEVs for a large range of floods will be summarised in a table for intercomparison.

Before the concept of available flood-storage volume is defined and discussed, an overview is given of a series of storage-based flood-protection measures. Beavers have been hailed as natural engineers who can (potentially) play a role in flood mitigation, both in some scientific literature and the (UK) media. The analysis of three beaver-dam complexes [40, 26, 48, 35] in three different rivers and their floods, in combination with the FEVs involved in a series of floods, shows that the potential of beaver colonies mitigating floods is rather limited. Nonetheless, such an analysis yields valuable insights in some (straightforward) river-flood hydraulics. It also triggers a brief investigation of mathematical flood control such as to either maximise or minimise the available flood-storage volume behind a porous dam with controls on its throughflow, operated by humans or beavers, respectively. Naturally, the reintroduction of beavers in the UK seems a good idea from a wildlife perspective even without the need to elude to the (minute) flood-mitigation potential by a multitude of beaver-dam complexes.

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<sup>1</sup> E.g., Rijkswaterstaat The Netherlands:

<https://waterdata.wrij.nl/index.php?wat=standardgraph&deeplink=1>;

Environment Agency: <https://flood-warning-information.service.gov.uk/station/8184>; France.

<sup>2</sup> The initial data gathering and displays were acquired in student projects by Zheming Zhang (River Don), Antonia Feilden (River Ouse) and Nico Septianus (Ciliwung river), and by me (River Tamar). I have built on these projects and compiled material into one *Python* code.

## 2.2 River-level measurements

Water-level measurements are presented for four river floods in Fig. 2.2, concerning the November 18<sup>th</sup> 2019 of the River Don in Rotherham, UK, the January 1<sup>st</sup> 2020 flood of the Ciliwung river near Djakarta, Indonesia, the December 25<sup>th</sup> 2015 flood of the River Ouse in York, UK; and, the December 24<sup>th</sup> 2013 flood of the River Tamar, at Gunnislake, UK<sup>3</sup>. At the Tesco gauge, Depok Floodgate, Skelton and Gunnislake gauges the in-situ water depths  $h(t)$  are measured at discrete time intervals  $t_k$  of time  $t$  with (regular) time intervals  $\Delta t_k = 15\text{min}, 30\text{min}$  and  $15\text{min}$ , respectively, for the first three gauges. The time interval at the Gunnislake gauge station is not given but daily mean, minimum and maximum water levels are reported by the EA. Note that the water-level is commonly measured relative to the deepest datum in the river's cross-section, which may change due to bed erosion and accretion, while the water-level marker or scale will stay fixed for a long time<sup>4</sup>.

The station information generally does not go into the details of what this reference datum is but given that the level-scale is fixed, cf. Fig. 2.1, small changes in its position relative to deepest riverbed point at a given cross section are irrelevant.

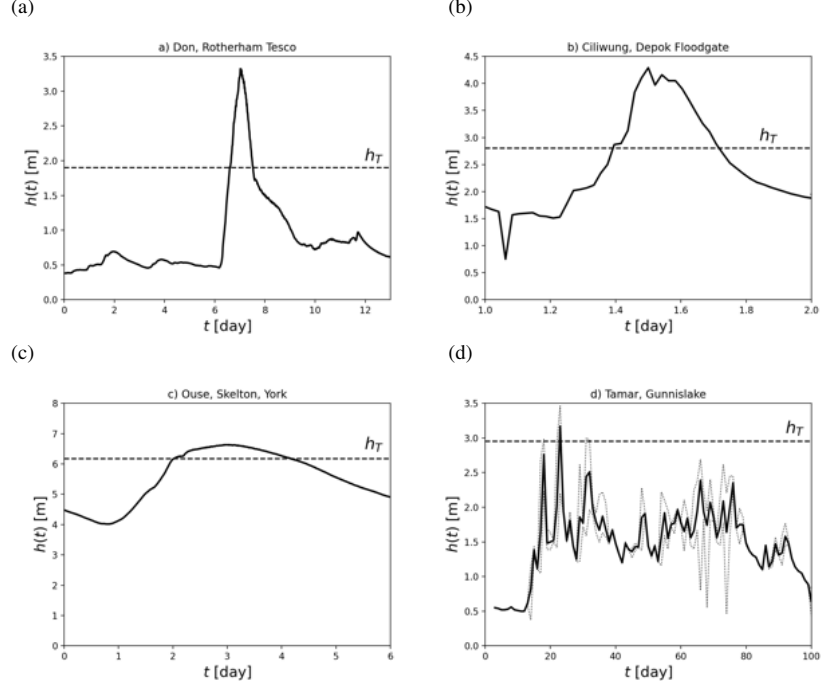


**Fig. 2.1** Shown is the scale of the Ciliwung river in flood at the Depok Flood gate. Photo courtesy: Inews/Rizky <https://www.inews.id/news/megapolitan/pintu-air-depok-siaga-ii-warga-di-bantaran-kali-ciliwung-diminta-waspada>, see also Fig. 4.2 in [44].

The choice of the threshold  $h_T$  is open to some debate. Often the respective environment agencies provide thresholds for minor, intermediate and major flooding associated with a specific river gauge. It is important to have a clear rational for

<sup>3</sup> <https://flood-warning-information.service.gov.uk/station/8175>, e.g., <http://poskobanjirdsda.jakarta.go.id/Pages/grafikDataTinggiMukaAir.aspx>, <https://flood-warning-information.service.gov.uk/station/8184>, <https://flood-warning-information.service.gov.uk/station/3201>

<sup>4</sup> For the Yangtze River, however, the river level is given relative to Chinese mean sea level at Wusong station near the city of Shanghai.



**Fig. 2.2** Water-level measurements of water depth  $h(t)$  versus time  $t$  for: (a) River Don 08-11-2019 flood at Tesco gauge station, Rotherham, UK, made every 15min; (b) Ciliwung river 01-01-2020 flood at Depok Floodgate gauge station near Djakarta, Indonesia, made every 30min; (c) River Ouse 25-12-2015 flood at Skelton gauge station, York, UK, made every 15min; and, (d) River Tamar 24-12-2013 flood at Gunnislake, UK, (minimum, mean and maximum) data collected per day. For the River Tamar the (daily) minimum and maximum (dashed, thin lines) as well as mean (solid, fat lines) water levels have been displayed.

the choice of  $h_T$  at each relevant location and to explore the effects of varying  $h_T$ , e.g., [4]. Bespoke local information can be added to the agencies' information based on eyewitness and/or photographic information. For the River Don case, the minor flooding threshold of 1.7m was slightly increased, yielding a choice of  $h_T = 1.9$ m. For the Ciliwung river, the above-mentioned Jakarta-flood website states specific river heights for the top two warning stages, as follows: a first, worst stage of overflowing and a second stage of initial flooding for  $h > 2.8$ m and, hence,  $h_T = 2.8$ m was taken. For the River Ouse, minor flooding near Skelton is possible for  $h > 5.8$ m. At the Viking-recorder-5 upstream, the EA reported flooding for  $h > 4.55$ m, which corresponded to a river level at 27-12-2015 of  $h_T = 6.17$ m at Skelton (email of EA official to Antonia Feilden 19-03-2019). Finally, for the River Tamar the EA employs the following warnings: a flood alert at full banks is issued for a river level of 2.65m; a flood warning level is issued when  $h > 2.95$ m; and, a severe

flood warning is issued for levels over 3.45m, see [2]. Hence, a threshold value of  $h_T = 2.95\text{m}$  was taken. Each threshold  $h_T$  is indicated by a horizontal dashed line in each panel of Fig. 2.2.

### 2.3 The role of rating curves

Rating curves convert water-level measurements to a discharge. In the simplest or ideal case, one water level  $h(t)$  (expressed in m) corresponds to one discharge rate  $Q(t)$  (expressed in  $\text{m}^3/\text{s}$ ) across a river's cross-section, yielding a rating curve  $Q(t) = Q(h(t)) = Q(h)$  at every time  $t$ . River dynamics is, of course, spatially three-dimensional and turbulent, especially during floods. However, for rivers with steeper bed slopes, one rating curve as one-to-one function between  $Q$  and  $h$  tends to be a good approximation. Rating curves are obtained by relating water-level measurements at a certain time with flow measurements at that time or over a sufficiently short time interval. By combining knowledge of the bed topography in that cross-section and these flow measurements (e.g. via usage of Acoustic Doppler Current Profilers –ADCPs, e.g., [32, 37]), one can approximate the discharge via integration of the velocity over the river cross-section, as follows

$$Q(t) \approx \int_{y_l(t)}^{y_r(t)} \int_{b(x,y)}^{b(x,y)+h(x,y,t)} V(x,y,z,t) dz dy, \quad (2.1)$$

with the  $y$ -coordinate aligned across the river, the  $x$ -coordinate in the local downstream direction and the locally vertical upward  $z$ -direction; the water depth  $h(x,y,t)$  (assumed single-valued under no wave breaking or over a brief time interval); bed topography  $b(x,y)$  (assumed steady over the time interval considered);  $U(x,y,z,t)$  the local velocity component in the  $x$ -direction at a fixed location; and, with extent  $y_l(t) < y < y_r(t)$ . The time interval for velocity measurements must be sufficiently short such that the time dependence of  $Q(t)$  is negligible. Velocity measurements are either taken over much longer time intervals (than the water-level measurements) or, occasionally, water level and discharge measurements are taken over the same time interval. In the former case, the results of such measurement campaigns generally accrue over days to months or even years such that one obtains a scatter plot of discharges for different water levels. Where possible, the scatter plots of  $Q$  versus  $h$  resulted in a curve fitting  $Q(h)$ . For the rivers Don, Ciliwung and Tamar, discharge measurements are infrequent while for the river Ouse, discharge and water level are measured every 15min. The fitted  $Q(h)$  are shown in Fig. 2.3a,b,c) and the data in Fig. 2.3d). For the rivers Don, Ciliwung and Tamar, the following type of power-law fit is used for certain river-level stages or intervals

$$Q(h) = c_j(h - a_j)^{b_j} \quad (2.2)$$

with coefficients  $c_j, a_j, b_j$  for the  $j^{\text{th}}$ -stage  $h_{j-1} < h < h_j$  with  $h_{j-1} > a_j$  and  $j = 1, \dots, n_s$  and beyond the last stage/interval, for  $h > h_{n_s}$  extrapolation is used. While the unit of  $a_j$  is in metres, such that the unit (denoted using the square brackets)  $[a_j] = \text{m}$ , and  $b_j$  is dimensionless, the unit of  $c_j$  is  $[c_j] = \text{m}^{3-b_j}/\text{s}$ . It would make more sense to redefine (2.2) as follows

$$Q(h) = \bar{c}_j (h/a_j - 1)^{b_j} \quad (2.3)$$

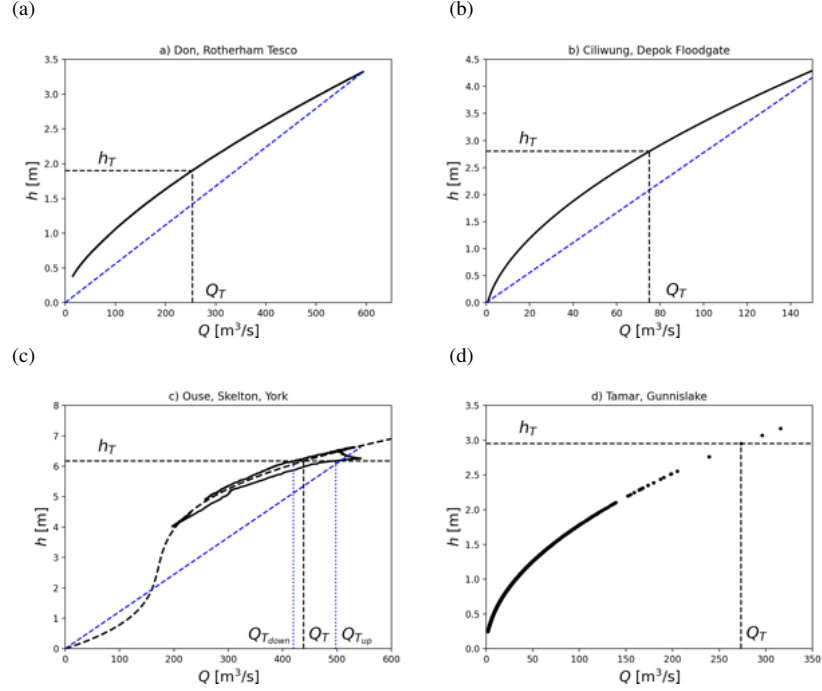
with coefficients  $\bar{c}_j, a_j, b_j$  such that  $\bar{c}_j$  has units of a discharge typical to the  $j^{\text{th}}$ -stage.

**Table 2.1** The 2018 coefficients  $c_j, a_j, b_j$  and limb thresholds  $h_0 = 0.1890$  and  $h_j$  for  $j = 1, 2, 3$  for the rating curve of the River Tamar at the river-level gauge station Gunnislake, UK. Courtesy: EA.

$j$	$h_j$	$c_j$	$a_j$	$b_j$
	m	$\text{m}^{3-b_j}/\text{s}$	m	-
1	0.365	30.45	-0.238	3.89
2	3.9840	31.44	-0.00174	2.00

Beyond the heighest water and discharge measurements there are no data and the river is in flood so extrapolation tends to be a poor approximation. Sometimes, hydraulic flood simulations are used to create additional rating-curve data. For the River Don at Rotherham Tesco, the EA provided water-level and discharge data and Fig. 2.3a) shows that a smooth rating curve has been used to create smooth discharge data. For the river Ciliwung, there is only one stage such that  $Q(h) = c(h - a)^b$  with  $c = 11.403 \text{m}^{3-b_j}/\text{s}$ ,  $b = 1, 715$ ,  $a = -0.2\text{m}$ , see [44], displayed in Fig. 2.3b). Linear approximations to these two rating curves based on the maximum flow rate recorded are shown by the blue dashed slanted lines. For the River Tamar at Gunnislake, coefficients for (2.2) are given in Table 2.1 for a case with two stages/intervals, with rating curve data displayed in Fig. 2.3d). Given that daily data are rather coarse, rating curve data are displayed as individual points  $\{Q(h), h\}$ . For these three river floods (a,b,d), the intersection of the threshold level  $h_T$  with the single-valued rating curve yields one threshold discharge value  $Q_T$  as indicated by the dashed thin horizontal and vertical lines. For the River Ouse at Skelton, water level and discharge data are measured in unison every 15 minutes and the situation is seen to be different in Fig. 2.3c). The solid thick data points form a lower branch in the  $h(Q)$ -plot when the river is rising and a different upper branch when the river level is sinking. A fourth-order polynomial  $Q(h) = a_1 h + a_2 h^2 + a_3 h^3 + a_4 h^4$ , forced through the origin, is fit through the data and displayed by the dashed curved line. This fit is likely poor and should be ignored outside the region with data for  $Q < 180^3/\text{s}$  and  $Q > 550^3/\text{s}$ . An average threshold value  $Q_T = Q(h_T)$  emerges from this fit but the intersections of the horizontal line at  $h = h_T$  with the data shows that there are two bounds  $Q_{T_{\text{down}}}$  and  $Q_{T_{\text{up}}}$  emerging. In the Ouse case it is therefore not clear whether there is a threshold value of  $Q$ , above which there is flooding, in contrast to the other three river-flood cases. Hysteresis, as observed in the Ouse data  $h-Q$  plot, often

emerges for river with smaller river slopes, see [33]. The river slopes at the other three flood locations are all larger than the river slope at Skelton in York. Finally, it is a question whether the powerlaw behaviour (2.2) can be based on mathematical analysis of (simplified) hydrodynamic equations, such as the St. Venant equations used in Chapter 4.



**Fig. 2.3** Rating curves  $h = h(Q)$  or measurements of water depth  $h$  versus discharge  $Q$  for: (a) River Don November 08-11-2019 flood at Tesco gauge station, Rotherham, UK; (b) Ciliwung river 01-01-2020 flood at Depok Floodgate gauge station near Djakarta, Indonesia; (c) River Ouse 25-12-2015 flood at Skelton gauge station, York, UK; and, (d) River Tamar 24-12-2013 flood at Gunnislake, UK. Note the nested time dependence  $h(t) = h(Q(t))$  or  $Q(t) = Q(h(t))$  for the singled-valued rating curves, with a linear approximation indicated by the blue dashed line going through the origin. For the River Ouse both water level and discharge are determined via flow measurements. There are two main rating curves for the River Ouse, one when the river level rises and one when the river level recedes.

## 2.4 Hydrographs and flood-excess volume

For the four river floods, the hydrographs of discharge  $Q(t)$  versus time  $t$  are displayed in Fig. 2.4. The discharge data in panels (a,b,d) follow from the water-levels measurements by using rating curves of the form (2.2) while the discharge for the Ouse flood is directly measured. In the latter case, the corresponding  $h$ - $Q$  plot was seen to be a scatter plot of the combined water-level and discharge measurements at several times  $t$ . Consequently, when there is a single-valued rating curve, each threshold  $h_T$  leads to a corresponding and unique discharge threshold  $Q_T$ , allowing a straightforward definition of the flood-excess volume.

The flood-excess volume (FEV) is the volume  $V_e = V_e(T_f)$  that caused the flooding and, given  $Q(t)$  and  $Q_T$ , is defined as the volume of the integrated difference  $Q(t) - Q_T$  over the duration  $T_f$  of the flood, for which this difference is positive. That is, for one identified flood event,

$$V_e = \int_{Q(t) > Q_T} (Q(t) - Q_T) dt, \quad (2.4)$$

which is one interval  $T_f$  when there is one peak with  $Q(t) > Q_T$  in a flood event or concerns multiple intervals such that  $T_f = T_{f1} + \dots + T_{fn_p}$  when there are several,  $n_p$ , local peaks with  $Q(t) > Q_T$  in one flood event. When there is one discharge peak  $Q_{max}$  with maximum water level height  $h_{max}$ , one observes that the FEV goes to zero, such that  $V_e \rightarrow 0$ , when the maximum discharge and water levels are reached, with  $Q_T \rightarrow Q_{max}$  or  $h \rightarrow h_{max}$ . A straightforward flood defence concerns the building or raising of flood walls or dikes along the river stretch relevant to the chosen river-gauge, thus raising the threshold level  $h_T > h_{max}$  for a flood of that magnitude. However, when one raises  $h_T$  by introducing flood walls, the discharge patterns and rating curve will also change and one will need to check the extent to which this is the case, potentially via detailed hydraulic simulations. As a first approximation, one can raise  $h_T$  while keeping the original rating curve and hydrograph to redefine the FEV.

From the definitions (2.4) of flood-excess volume and flood duration  $T_f$ , one can define a mean discharge

$$Q_m = Q_T + V_e/T_f \quad (2.5)$$

and, graphically, an FEV equivalent can also be depicted by a rectangle between  $Q_m$  and  $Q_T$  over the flooding time  $T_f$ .

When one only has the water-level measurements available as well as the maxima  $h_{max}$  and  $Q_{max}$ , the linear approximation of the rating curve indicated by the straight slanted dashed lines in Fig. 2.3a,b,d) can be employed. Such a linear approximation yields the mean discharge or  $Q_m$ -approximations

$$Q_m \approx (h_m/h_{max})Q_{max} \quad \text{and} \quad Q_T \approx (h_T/h_{max})Q_{max}, \quad (2.6)$$



provided one knows the mean water depth  $h_m$  during a flood. For idealised rectangular (with peak duration  $T_f$ ), triangular (with one peak point) or trapezoid (with peak duration  $T_f/2$ ) shape hydrographs (note that for a linear rating curve the shapes of  $h(t)$  and  $Q(t)$  are the same), the mean water depth can be calculated to be

$$h_m = h_{max}, h_m = \frac{1}{2}(h_{max} + h_T), h_m = \frac{(3h_{max} + h_T)}{4}, \quad \text{respectively.} \quad (2.7)$$

With (2.6), these lead to the following FEV approximations, see, e.g., [4],

$$V_e \equiv T_f (Q_m - Q_T) \approx V_{e1} \approx T_f (Q_{max} - Q_T), \quad (2.8)$$

$$V_e \approx V_{e2} = \frac{1}{2} T_f \frac{Q_{max}}{h_{max}} (h_{max} - h_T), \quad (2.9)$$

$$V_e \approx V_{e3} = \frac{3}{4} T_f \frac{Q_{max}}{h_{max}} (h_{max} - h_T), \quad \text{respectively.} \quad (2.10)$$

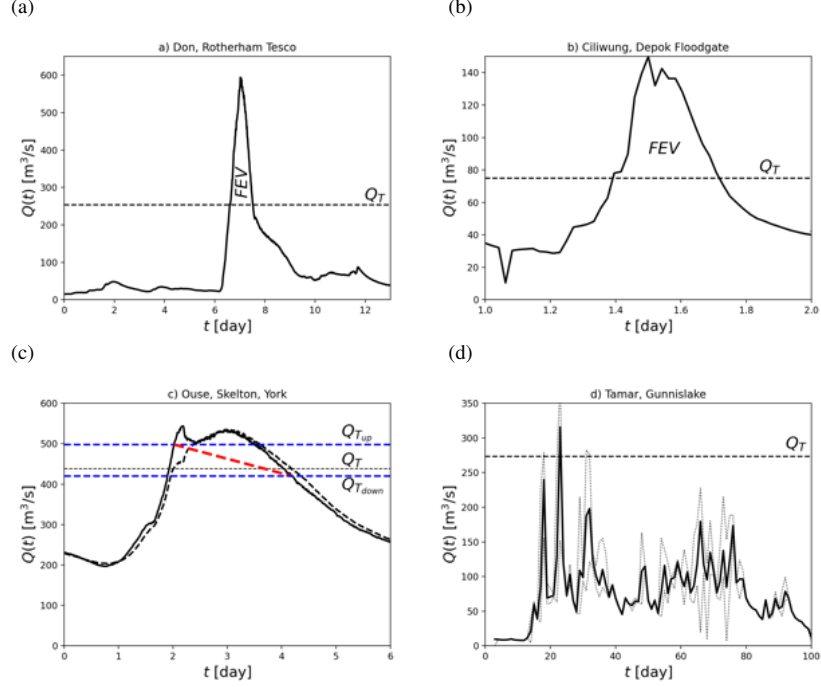
Such approximations are useful when only  $Q_{max}$  is given or when a quick verification is needed.

For the Ouse case, the definition of FEV is ambiguous because there are not only three choices  $Q_{Tup}$ ,  $Q_T$ ,  $Q_{Tdown}$  of threshold discharges but one may also decide to make  $Q_T = Q_T(t)$ , a function of time  $t$ . The three thresholds are indicated in Fig. 2.4 by the horizontal dashed (thicker) blue and (thin) black lines, each line defining a different FEV after integrating the (positive part of the) difference  $(Q(t) - Q_{Tup})$ ,  $(Q(t) - Q_T)$ ,  $(Q(t) - Q_{Tdown})$ , respectively, for one flood event. Indicated as the red dashed and slanted line is another time-dependent threshold  $Q_T(t)$  (notationally distinguished by highlighting this time dependence explicitly), linking the thresholds  $Q_{Tup}$ ,  $Q_{Tdown}$  in a linear fashion and yielding another definition of the FEV, as follows

$$V_e = \int_{Q(t) > Q_T(t)} (Q(t) - Q_T(t)) dt. \quad (2.11)$$

One way to resolve these issues is by pursuing detailed hydrodynamic simulations and comparing hydrographs with and without flood-mitigation measures. Another way may be to derive analytical approximations of the river flow that capture some elements of the hysteresis observed, including detailed approximate expressions linking flow and water level.

The FEV is a fraction of the water volume flowing through a river cross-section over the duration of a flood event, during its ramp-up towards flooding and easing to more normal water levels. In addition, during the flood defined as the time  $T_f$  for which  $h(t) > h_T$  most of the flood waters go through the main channel. FEV is simply defined as the integrated time difference  $Q(t) - Q_T$  or, equivalently, the difference between the flooded hydrograph and the same hydrograph with a chopped, flat peak at  $Q_T$  over the duration  $T_f$  of the flood. To obtain a sense of the sizes of rivers in flood, Table 2.2 provides a summary of various floods with FEVs spanning several orders of magnitude from the 2014 Finchingfield flood [20] with  $V_e = 0.053 \text{ Mm}^3$  to



**Fig. 2.4** Discharge  $Q(t)$  versus time  $t$  for: (a) River Don November 08-11-2019 flood at Tesco gauge station, Rotherham, UK; (b) Ciliwung river 01-01-2020 flood at Depok Floodgate gauge station near Djakarta, Indonesia; (c) River Ouse 25-12-2015 flood at Skelton gauge station, York, UK, made every 15min; and, (d) River Tamar 24-12-2013 flood at Gunnislake, UK. The River Ouse has a multi-valued rating curve, essentially one when the river level rises (lower branch in  $h-Q$  data) and one when the river levels sinks (upper branch in  $h-Q$ ). For the River Tamar the (daily) minimum and maximum (dashed, thin lines) as well as mean (solid, fat lines) discharges have been displayed.

several floods with  $V_e > 3\text{Mm}^3$  and the 2020 Yangtze River flood at Cuntan, China, with an FEV of nearly  $V_e \approx 100\text{Mm}^3$ , with in particular the 2007/2019 River Don floods in Sheffield and Rotherham as well as the 2015 Rivers Aire and Calder floods causing a lot of damage. Some of the FEVs in Table 2.2 contain error bars, which tend to be rather large, from 16% to over 50%. Calculation of such error bars will be considered in the next chapter. It may, however, still be difficult to attach meaning to the size of floods with FEVs of a million cumecs or more. Consider therefore a flood with  $V_e = 2\text{Mm}^3$ , divide this FEV by a depth  $D = 2\text{m}$  and subsequently take the square root to obtain a side length

$$L_D = \sqrt{V_e/D} = 1000\text{m} \quad (2.12)$$

river -	location -	flood date(s) -	FEV $V_e$ Mm <sup>3</sup>	$h_T$ m	$L_D$ m
Aire	Armley/Leeds, UK	26-12-2015	$9.34 \pm 1.50$	3.9	2161
Calder	Mytholmroyd, UK	26-12-2015	$1.65 \pm 0.60$	4.5	908
Don	Sheffield, UK	25/26-06-2007	$3.00 \pm 0.71$	2.9	1225
Don	Rotherham Tesco, UK	08-11-2019	14.2	1.9	2665
Ciliwung	Depok Floodgate, Indonesia	01-01-200	1.05	2.8	725
Ouse	Skelton/York, UK	25-12-2015	$\sim 10.34$ (?)	6.17	2274
Brague	Biot, France	03-10-2015	$0.488 \pm 0.311$	3.06	494
Tamar	Gunnislake, UK	23-12-2012	1.96	2.95	990
Tamar	Gunnislake, UK	24-12-2013	3.65	2.95	1351
Finchingfield Brooke	Finchingfield, UK	07-02-2014	0.053	(?)	163
beaver dam (1 <sup>st</sup> estimate)	Devon, UK	2015	0.0002	-	10
beaver dam (2 <sup>nd</sup> estimate)	Devon, UK	2015	0.001	-	22
Yangtze	Cuntan, Chongqing, China	2020	$96.25 \pm 5$	180.5	6973 <sup>†</sup>

**Table 2.2** FEVs are given of several river floods with river name, river-gauge location, flood date, FEV (plus error), threshold  $h_T$ , and square-lake side length  $L_D$  stated. For some floods, error estimates of the FEV are provided as well. FEV estimates for a Devon beaver colony have been added as well, for which  $h_T$  is not relevant. For Finchingfield, [20] does not provide a threshold  $h_T$ . <sup>†</sup> Provisional data by Shunyu Yao: the error bar is a symmetrised approximate and  $h$ ,  $h_T$  are height above the mean sea level at Wusong near Sjanghai with the river bottom at the Cuntan gauge station lying at circa 174m above sea level.

of a (dynamic) square lake with a two-metres deep human-size depth. The lake is denoted to be dynamic because the FEV generally concerns flowing rather than static flood waters. It can therefore be dubbed a hypothetical or effective lake. If one could find space in the river valley upstream of the flooded region of interest, to store extra flood waters in a lake of that size or in an accumulation of smaller lakes with that total size, then the flood could likely be annulled or mitigated. Given the size of such a square lake of a flood, it can be compared with the dimensions of potential storage sites in the river valley. If the river valley is narrow and/or build-up, then finding such storage sites may be a task that is difficult to achieve. The (dynamic) lake side-lengths  $L_D$  for each flood are found in Table 2.2 as well and are seen to range from 163m to 2665m and even  $\sim 7000$ m for the 2020 Yangtze flood at Cuntan.

## 2.5 Storage-based protection measures

Both the (critical) water levels  $h_T$  and  $h_{max}$  as well as the FEV,  $V_e$ , for a particular flood classify its magnitude. To protect against flood damage of floods with a similar magnitude, people may wish or decide to reduce the FEV to zero and possibly also partially raise  $h_T$  by introducing a variety of flood-mitigation measures. When FEV is used, the focus will lie on effective storage-based measures, or their equivalent, reducing the in-situ  $h_{max}$ , while when  $h_T \rightarrow h_{max}$  focus lies on reaching the peak

water-level value with flood defences. Both approaches are valuable. Focus here will be placed on storage-based methods since the use of the chosen threshold  $h_T$  includes the limiting of  $h_{max} \leq h_T$ . Also recall that the discussion on building or raising flood-defence walls is seen to translate into  $Q_T(t) \rightarrow Q_{max}$  such that the FEV reduces to zero,  $V_e \rightarrow 0$ , given that  $h_T \rightarrow h_{max}$ . One caveat in using the FEV (or threshold  $h_T$ ) of a particular flood, using either measured or simulate data is that the FEV generally changes a (little) bit when one introduces flood-mitigation measures because these measures will affect the river-bed and flood-plain characteristics. When the FEV is kept unchanged in discussing and quantifying flood-mitigation measures, one has to keep in mind this caveat, which can be overcome by detailed simulations or approximated calculations that include changes in those characteristics. A brief, pictorial overview of storage-based flood-mitigation measures will be provided next.

*Flood-defence walls:* Heightening flood-defence walls or berms/dikes (HW) leads to a reduction of the FEV by increasing  $h_T$  and, hence,  $Q_T(t)$ , see Fig. 2.5a,b). More simply put: (dynamic) water volume is “stored” between the walls when these are raised.

*GRR:* Giving-room-to-the-river (GRR) concerns a series of interventions which increase the conveyance of the river bed and flood plains. GRR can include lowering and widening river banks, as well as opening up old or creating new river branches to allow more flow and storage in floods, see Figs. 2.5c and 2.6d).

*Flood-plain storage:* Flood-plain storage (FPS) can be achieved in various ways: by lowering summer dikes such that the river overflows onto the adjacent flood plains more easily (e.g., also called washlands) or by constricting the river banks with dikes or berms on the flood plain such that more flood waters are stowed onto the flood plain upstream of such a constriction. Such constrictions can be made via static berms upstream of fixed weirs or dynamically by using berms and dikes on the flood plain with a moveable, i.e. controllable, weir in the river. Dynamic or active control of the weir can then be used to optimize flood-water storage on the adjacent flood plains. In both these passive and active cases, extra water volume is stored onto the flood plains and in the river relative to the flood situation without these interventions, see Fig. 2.5e).

*Natural Flood Management:* Natural flood management (NFM) is a broad category involving various options, such as a multitude of leaky dams built from natural materials, or tree planting to retain more precipitation/water, generally in the natural upstream portions of a river catchment, see Fig. 2.5f) and Fig. 2.7g). Each leaky dam by itself is an example of (the use of a passive weir constructed by using natural materials in) FPS but given that the storage volume per leaky dam tends to be small relative to the required FEV, the accumulation of FPS behind an accumulation of such leaky dams in an area, placed in series on a tributary or in parallel over several tributaries connected to a main channel, is categorised as NFM [18].

*Beaver dams:* Beaver colonies and dams (BD), essentially a form of NFM, are thought by some people to create and deliver flood protection since their dams are thought to hold a sufficient amount of flood waters [40, 35, 26], see Fig. 2.7h). Such beaver dams are a particularisation of the leaky dams mentioned under NFM.



**Fig. 2.5** Storage-based flood-mitigation measures (a) flood-defence walls (HW) along the Holbeck near the River Aire in the centre of Leeds have been raised after the 2015 Boxing Day Floods; (b) a heightened and repaired berm or dike (HW) along the relief canal north of Wainfleet, Lincolnshire, UK, after the berm breached in June 2019, flooding parts of Wainfleet and the surrounding area; (c) river-bank lowering and widening (GRR) in the form of the multi-usage Porter Brooke Pocket Park along the River Sheaf in Sheffield, UK, established in response to the 2007 floods in Sheffield; Courtesy: photo of Porter Park by Sheffield Society of Architects. **Placeholders photo for c); alternative River Dinkel in NL available or travel to Sheffield to take photo.**

(d)



**Fig. 2.6** Storage-based flood-mitigation measures (d) a new alleviation channel or cut (GRR) for the River De Waal (the largest tributary of the River Rhine) at Nijmegen, The Netherlands; the main river is seen on the left and an old river branch on the right has been reopened and deepened as alleviation channel during high water levels and as recreational space during normal and low water levels. A similar alleviation channel is found at Kirkstall The Forge in the River Aire upstream part of Leeds. It is a new cut made after the 2015 Boxing Day floods, which is dry during normal conditions but tempers the water-level increase in part caused by the sharp bend in the river during flood conditions, thus keeping the railway tracks dry longer. Courtesy: 2015 photo of River De Waal by Wout Zweers.

*Nature-based Solutions:* Nature-based Solutions (NBS) are a broad category concerning flood-mitigation measures inspired by natural aspects. NFM and beaver dams are consequently included but NBS is a broader category and can include green and hybrid grey-green engineering solutions. Some GRR measures can sometimes also be dubbed as NBS.

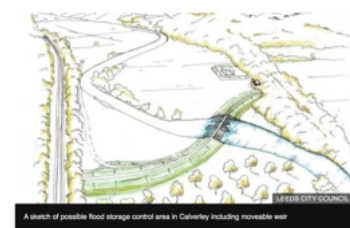
## 2.6 Available flood-storage volume

The storage-based flood-mitigation measures discussed above can be quantified by using the concept of available flood-storage volume [4]. Available flood-storage volume is the volume available above and beyond the storage capacity of the river and its floodplains already occupied by a flood of a certain magnitude. It is important to realise that it is an effective and generally dynamic storage volume, since the river water will in most cases be flowing through but possibly at a lower speed. Regarding the quantification of new measures, available flood-storage volume can alternatively be defined as the difference between the total storage volume for a certain flood-

(e)



(f)



(g)



(h)



**Fig. 2.7** Continued. (e) flood-plain storage (FPS) on washlands of the River Aire with its summer dikes visible during Storm Ciara 09-02-2020 floods; (f) sketch of dynamic FPS with a moveable weir; (g) leaky dam (NFM) made of logs over the Sutherland Beck in North Yorkshire (guided tour to a beaver colony in North Yorkshire on 12-03-2020); (h) beaver dam (BD) across the the Sutherland Beck in North Yorkshire. Courtesy images: e) Robin Attrill (aerial) and f) sketch by Leeds City Council. Placeholder photos for e,f). Alternatives under consideration.

mitigation measure and the existing total storage volume in a certain area prior to implementation of that measure for an event of the same magnitude. For example, if the flood-plain providing storage volume  $A_p d_p$  upstream of a contraction in the river, natural or otherwise, involved a flood depth  $d_p = 3\text{m}$  on average over an area  $A_p$ , and if extra flood storage would be achieved by further (partial) blocking of the contraction leading to an average water depth of  $d_e = 4.5\text{m}$  over an area  $A_e \geq A_p$ , with a total storage volume of  $A_e d_e$ , then the available flood-storage volume would be the difference  $A_e d_e - A_p d_p > A_p 1.5\text{m}$ . For a flood of lower magnitude the average depth  $d_p$  and area  $A_p$  will be smaller and there would potentially be more available flood-storage volume achievable if the maximum  $d_e = 4.5\text{m}$  can still be reached, for example by further reduction of the contraction area in a controllable fashion.

The manner in which the contraction is controlled or controls itself via its fluid dynamics does matter, as the following example illustrates. If prior to the arrival of the water-level increase during a flood event or prior to the flood peak the flood-storage area is already (partially) filled above  $d_p > 3\text{m}$  and possibly even filled fully to level of  $d_e = 4.5\text{m}$  then there is less or no flood-storage capacity left and the available flood-storage volume is suboptimal or even zero. Using the maximum available flood-storage volume therefore becomes a control problem when active control of the contraction area is available, for example by opening or closing the dynamic weir in a prescribed manner [10, 11, 47]. The concept of available flood-storage volume will be explored in two examples in the next two sections, one involving beavers as natural engineers creating storage ponds behind their beaver dams and one concerning control, either optimised by humans or beavers, each with their own cost function.

## 2.7 Upscaling flood-water storage behind beaver dams unrealistic?

In both scientific literature [40, 26, 35] and the (British) media, water storage via colonisation of a river valley by beavers has been proposed as a (potential) flood-mitigation measure to reduce floods. To assess that potential it is necessary to look at available measurements of the hydrology and hydrodynamics of beavers colonies and the particular effects of their beaver ponds and dams. The change in time of the storage volume  $V_b$  of a beaver pond or a series of beaver ponds depends on the influx  $Q_{in}$ , outflux  $Q_{out}$  and evaporation  $E_v$  (all expressed as rates). Cf. the analysis in [26] we find

$$\frac{dV_b}{dt} = Q_{in} - E_v - Q_{out}, \quad (2.13)$$

in which  $Q_{out}$  can be divided further into outflow from the dam  $Q_{dam}$ , groundwater outflow  $Q_{gw}$  and return flow  $Q_{rf}$  from the floodplain downstream of the dam such that  $Q_{out} = Q_{dam} + Q_{gw} + Q_{rf}$ . Measurements of the inflow  $Q_{in}$  and (part of)



the  $Q_{out}$  will be shown for beaver-dam complexes in the River Tamar catchment in Devon, UK [40], along the Chevril River in the Ardennes, Belgium [35], and in streams crossing coastal wetland in Northern Ontario, Canada [48].

Two hydrographs are shown in Fig. 2.8 at a weir of the river flow upstream of a series of 13 beaver dams and ponds over a length of circa 200m in Devon and at a weir downstream of this beaver colony for a flood peak around 12-12-2014. Puttock et al. [40] investigated a series of such flood peaks, the in- and outflows, and report a storage volume of circa  $V_b = 1100\text{m}^3$  (here taking a convenient limit above the mean but within the reported error range). In addition, they conclude that “*beavers are likely to have a significant flow attenuation impact*” and “*to develop and understanding how beavers may form a ‘nature based solution’ to . . . flooding problems faced by society*”. Given this storage volume  $V_b$  and the FEVs of several river floods, as found Table 2.2, it follows how many beaver colonies of the Devon-type are required to mitigate 1%, 10% or 100% of the FEV, where in the first two cases 99% or 90% of the relevant FEV preferably ought to be covered by other mitigation measures to provide full protection against floods of the chosen magnitude. The number of beaver colonies of the Devon-type, in the optimal case of the full storage being used and without any dam collapse, is then

$$(0.01, 0.1, 1)V_e/V_b, \quad (2.14)$$

respectively, for (1, 10, 100)% coverage. For the various river floods, that number of required beaver colonies lies between 48 and 9400 (and 5 to 940 colonies for 10% coverage). E.g. 48 beaver colonies would be required to reach 100% coverage of the FEV for the smallest flood in the table, the Finchingfield flood. For a flood with  $V_e = 1.1\text{Mm}^3$ , one would need (10, 100, 1000) beaver colonies to achieve (1, 10, 100)% coverage, involving (130, 1300, 13000) beaver dams (13 beaver dams per colony) and (40, 400, 4000) beavers (using four beavers per colony) over a length of (2, 20, 200)km distributed over the relevant river catchment (using 200m per colony). Table 2.3 provides a summary of some of these numbers. Hence, such large numbers of beaver colonies required reveal that there seems to be no potential for serious flood mitigation by using beavers, even for the smaller fractions of FEV-coverage by the available flood-storage volume of beaver ponds.

Further understanding of that lack of potential is obtained by addressing the following questions:

- is the value of  $V_b$  used from the Devon beaver colony representative, and
- is the ratio ( $V_b/V_e$ ) of available flood-storage storage volume ( $V_b$ ) for a particular mitigation measure over the required FEV ( $V_e$ ) providing an adequate estimate to assess the effectiveness of a flood-mitigation measure?

The second question will be considered in chapter ???. The first question is analysed in several ways, next:

- The available flood-storage volume for a beaver-dam complex can either be found from two hydrographs, one taken before the beavers started building their dams and one thereafter during a heavy rainfall flooding period, or from one hydro-

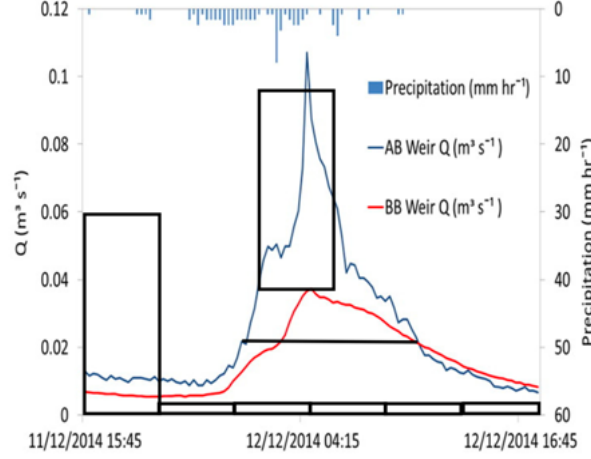
graph taken prior to a heavy rainfall and flooding period and during this period. Alternatively, the available flood-storage volume can be estimated by obtaining the average pond-level fluctuations times the pond area. Such area estimates can follow from aerial photographs. Estimates in the uncertainty of that volume can be estimated by considering beaver-dam complexes in different areas, in different landscapes and over time.

- The flood-storage volume over a unit river length can be estimated by taking into account a (hypothetical) density of beaver colonies along a main river and its tributaries.
- Given the FEV of a target or design flood, the above allows one to calculate how many beaver colonies would be needed to cover a certain fraction of the FEV. Additionally, the density estimates permit one to assess whether there is sufficient space in a river catchment for (an abundance of) beaver dams.

One caveat arises in that the scientific literature on beavers with adequate flood hydrographs appears to be limited, thereby restricting the following analysis.

The appropriateness of the value of  $V_b \approx 1100\text{m}^3$  is, furthermore, straightforward to check from one of Puttock et al.'s hydrographs, adapted in Fig. 2.8, showing a roughly estimated volume of circa  $900\text{m}^3$ . There is, however, a caveat in the above analysis, reported by [39]. The surface area of the beaver ponds is said to be circa  $A_s = 2000\text{m}^2$ , which also follows by estimation via [40]'s Fig. 2 (top left) as circa  $A_s \approx 25\text{m} \times 100\text{m} = 2500\text{m}^2$  (i.e., the valley width times accumulative length of the beaver ponds is the area covered by beaver ponds). The depth increase  $\Delta h$  in a flood event is seen to be small in Puttock et al.'s Fig. 2 (bottom [40]), circa  $0.1\text{m}$  on average except for a few events for one pond, where  $\Delta h \approx 0.5$  to  $0.8\text{m}$ . These adapted estimates yield an available flood-storage volume of  $V_b = A_s \Delta h \approx 200\text{m}^3$  which is circa a fifth of the reported value of  $V_b \approx 1100\text{m}^3$ . The origin of this difference is unclear: there may be evaporation losses  $E_v$ , unaccounted for groundwater and return flows  $Q_{gw}$ ,  $Q_{rf}$ , and/or the hydrograph measurements may contain larger errors than reported. In the next chapter, errors of various FEVs will be estimated to lie in the range of 15% to over 50%. In addition, for this event inspection of Fig. 3 in [40] shows that water volume may be missing since  $Q_{out}$  is not larger than  $Q_{in}$  after the peak event to account for the stored volume being released at later times.

The next beaver site considered is located on flat wetlands near Hudson Bay in Northern Ontario, Canada [48]. This region is extremely flat and numerous beaver ponds have been observed; Woo and Waddington [48] mention 60 beaver dams in various states of preservation in a  $1\text{km}^2$  area. Given its close proximity to the coast, their estimates of storage volume may be less relevant to river-flood cases considered in Table 2.2, so the resulting outcome should perhaps be taken as an upper limit. Dam density ranges from 5 to 19dams/km, with an average of 14.3dams/km. Woo and Waddington [48] note that this density is close to the 10.6dams/km in South-eastern Quebec but higher than the 2.5dams/km in Northern Minnesota [34]. The discharge reduction due to one beaver dam near Ekwan Point, North Ontario, can be calculated from the difference of the inflow hydrograph and the outflow hydrograph during a rainfall event on 21-06-1988, see Fig. 8 of [48]. Approximate integration by drawing in a rectangle of equivalent area to the actual area between the two hydrographs



**Fig. 2.8** Hydrographs at weirs upstream (AB, blue line) and downstream (BB, red line) of a beaver colony for a flood peak in a tributary of the River Tamar, in Devon, UK. Taken and adapted Fig. 3 (bottom-left) from [40] with the approximate (eye-ball) integration technique displayed. The rectangular box drawn in is roughly equivalent to the desired area of integration between the two hydrographs of the flood peak. Measurements of that rectangle ( $0.06\text{m}^3/\text{s}$  over  $23/6\text{hr}$ ) along the respective axes indicate that it concerns rectangle sides of  $\sim 0.06 \times (25/6) \times 60 \times 60\text{m}^3 = 900\text{m}^3$ . Note that more precision is not required for obtaining an estimate.

around the peak flow, i.e. by using eyeball measures and subsequent estimates of the rectangle's sides, yields an effective storage volume of  $V_b = (10/6.33) \times 24 \times 3600 \times 2 \times 10/7\text{m}^3 = 390\text{m}^3$  behind one dam, cf. Fig. 2.9. Whether there are significant other losses such as evaporation for this site is unknown. In addition, error estimates on the hydrographs are unknown and [48] seems to contain insufficient information to check the volume  $V_b$  as the multiplication of the pond areas times the height difference, i.e. as  $A_s \Delta h$ , although the authors state that they have used such estimates. Given the dam density, we have a high storage per kilometre of river network, i.e.

$$V_{Ek} = 14.3 \times 390\text{m}^3/\text{km} = 5570\text{m}^3/\text{km}. \quad (2.15)$$

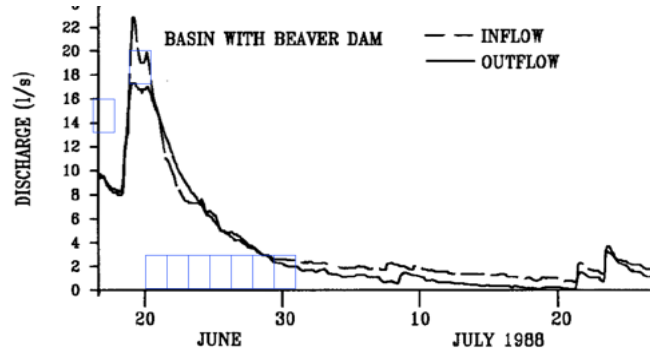
There may be water fluxes missing, since the measured outflow does not seem to be sufficiently larger than the inflow after the flood event upon inspection of the two hydrographs in Fig. 2.9 to account for the total mass balance. Evaporation losses could, however, be large and account for the difference.

Combining [42]'s reported colony density between 3km and 20km with [40], a dam density estimate for the Devon case lies between  $(13/20)\text{dams}/\text{km} = 0.65\text{dams}/\text{km}$  and  $(13/3)\text{dams}/\text{km} = 4.33\text{dams}/\text{km}$ . Given that there are 13 dams in Devon with a quoted effective volume of  $1100\text{m}^3$ , the volume stored per dam is  $(1100/13)\text{m}^3/\text{dam} = 85\text{m}^3/\text{dam}$ . Hence, by taking the best case, the storage volume

per kilometre in the Devon case becomes

$$V_{De} = 85 \times 4.33 \text{ m}^3/\text{km} = 367 \text{ m}^3/\text{km}, \quad (2.16)$$

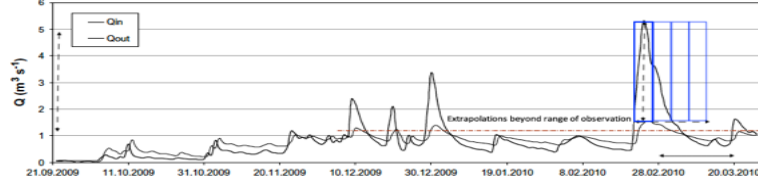
an order of magnitude less than estimate (2.15) in the Ekwon Point case. Given that [39] argues that the  $1100 \text{ m}^3$  is a factor five too large, a second estimate becomes  $V_{De2} \approx 75 \text{ m}^3/\text{km}$ .



**Fig. 2.9** Taken and adapted Fig. 8 from [48] with integration techniques displayed. The rectangular box drawn in is roughly equivalent to the desired area of integration between the two hydrographs of the flood peak. Measurements of that rectangle along the respective axes indicate that it concerns rectangle sides of  $\sim (10/6.33)$  days and  $\sim 2 \times (10/7) \text{ l/s}$  yielding  $V_b = 390 \text{ m}^3$ . Again note that more precision is not required for obtaining an estimate.

The review article [26] includes two hydrographs around 28-02-2010 with  $Q_{in}$  and (part of)  $Q_{out}$  from [35], shown and adapted here in Fig. 2.10, concerning a beaver colony along the Chevral River in the Ardennes in Belgium. What is immediately clear is that the storage volume  $V_b = 1.782 \text{ Mm}^3$  is not released later in the  $Q_{out}$  hydrograph, so there is water volume missing, either due to evaporation, groundwater flow or floodplain flow around the point of outflow measurement? As independent estimate, one observes that there are six beaver dams over a river stretch of 300m. Using Figs. 3 (taken at lower water levels than the peak on 28-02-2010) and 6 of [35], a generous estimate yields a water-covered area of  $A_s = 300 \times 50 \text{ m}^2 = 15000 \text{ m}^2$  and by taking a generous pond-height difference  $\Delta h = 2 \text{ m}$ , a volume estimate of  $V_B = A_s \Delta h = 30000 \text{ m}^3$  results, far off the difference between the two hydrographs. To be clear, the authors indicate “extrapolations beyond range of observation”. In their Table 4, the volume stored in the pond relative to that stored at 30-09-2009 is stated but those volumes do not seem to be the relevant available flood storage volumes. Given this spread in possible values of  $V_b$  and lack of clarity on the values for  $V_b$ , the results from [35] are discarded. In general, there is plenty of literature on beaver colonies with remarks on beaver ponds holding flood waters but it seems

much more difficult to find data verifying or defining what the relevant available flood storage volumes are.



**Fig. 2.10** Taken and adapted Fig. 12 from [35] with integration techniques displayed. The rectangular box drawn in is roughly equivalent to the desired area of integration between the two hydrographs of the flood peak. Measurements of that rectangle along the respective axes indicate that it concerns rectangle sides of  $\sim (20/4) \times 24 \times 3600\text{hr}$  and  $\sim 4\text{m}^3/\text{s}$  yielding  $V_b = 1.728\text{Mm}^3$ .

Chaubey and Ward [12] report hydrographs of inflow and outflow for two storm events in 1994 in Talladega Wetland, West Central Alabama, USA, one is consistent in showing that the outflow picks up after the event, releasing a volume similar in size to the estimated storage volume of  $V_b = 3600\text{m}^3$  with a lowered outflow peak delayed by about 3 to 4 hours, and another one which shows the outflow exceeding the inflow due to overflow of the beaver dam and a record in time displayed that is too short to assess consistency. There seem to be about five beaver dams over a length of 200m so the storage per beaver dam would be about  $600\text{m}^3$  and with the highest density of  $4.33\text{dam}/\text{km}$ , the storage per kilometre then becomes

$$V_{Tal} = 2598\text{m}^3/\text{km}. \quad (2.17)$$

This latter value is consistent with  $V_{Ek}$ , which larger value is used hereafter.

Given these three beaver-pond available flood-storage densities  $V_{Ek}$ ,  $V_{De}$ ,  $V_{De2}$ , a triplet range

$$R_{100} = [V_e/V_{Ek}, V_e/V_{De}, V_e/V_{De2}] \quad (2.18)$$

can be defined, yielding the hypothetical range of lengths for the respective river networks to mitigate the FEV for 100% under ideal circumstances. These lengths, as well as a tenth thereof, are compared with main river lengths in Table 2.3. It shows that using beavers as flood-mitigation strategy appears to be unrealistic. These lengths needed within one river catchment, including its tributaries, are too large despite all the favourable assumptions used. The analysis above shows that the estimates are surrounded by a lot of uncertainty. Other assumptions used are that the beaver dams are perfect, do not collapse and are all used to full capacity. Dams arranged in series could cause an accumulation of collapsing dam and debris flows during extreme floods. The analysis does show how the efficacy of a certain measure proposed, here beaver dams, can be readily assessed using FEV and hydrograph analysis. Finally, both the lead author of [39] and I are in favour of reintroducing more beavers in

the wild and are, furthermore, arguing in a quantifiable manner that there is both no need and no justification to promote beavers as actors in flood mitigation in order to justify this otherwise just cause of wildlife enhancement.

Finally, Piton in [39] suggests that beavers may have a different objective than flood mitigation: the beavers' goal is to minimise fluctuations in beaver ponds such that their beaver burrows are neither subject to pond-level decrease in droughts nor to pond-level flooding in floods. Beaver burrows, see for example Fig. 2.11, have an entrance under water to a drying chamber and a dry top chamber. The beaver's objective may possibly be to keep the entrance under water, to deter intruders, and keep both chambers dry, which requires the pond's water level to stay between two limits. Such different objectives by humans and beavers will be explored mathematically by exploring two different cost functions in the next section. The model will involve a porous dam which porosity can be varied and used as control in attempts to reach the respective objectives.

River -	flood date(s)	length km	$V_e$ Mm <sup>3</sup>	$R_{100}$ km	$0.1R_{100}$ km
Aire	26-12-2015	148	$9.34 \pm 1.50$	[838,12725,62333]	[168,2545,12640]
Calder	26-12-2015	72	$1.65 \pm 0.60$	[296,4496,22000]	[296,4496,2200]
Don	25/26-06-2007	142	$3.00 \pm 0.71$	[539,8174,40000]	[54,817,4000]
Tamar	23-12-2012	98	1.96	[352,5341,26133]	[35,534,2613]
Tamar	24-12-2013	98	3.65	[655,9946,48667]	[66,995,4867]
Finchingfield	2014	~ 11 (?)	0.053	[10,144,707]	[1,14,70]

**Table 2.3** Given  $V_{Ek} = 5570\text{m}^3/\text{km}$ ,  $V_{De} = 367\text{m}^3/\text{km}$  and  $V_{De2} = 75\text{m}^3/\text{km}$ , a range  $R_{100} = [V_e/V_{Ek}, V_e/V_{De}, V_e/V_{De2}]$  is defined, yielding the hypothetical range of lengths of the respective river networks to mitigate the FEV for 100% under ideal circumstances. In addition, 10% of these lengths, in the column indicated by  $0.1R_{100}$ , are calculated assuming that the remaining (fraction of the) FEV is either left unmitigated or is mitigated by other flood-mitigation measures. We have also given river lengths but note that Leeds, Mytholmroyd, Sheffield are lying in the upper half or third of the respective rivers, while Gunnislake is at the end of the non-tidal part of the River Tamar in which the Tamar catchment is the one with the Devon beaver colony [40]. FEVs concern floods in the cities or villages mentioned. The given cumulative length of the brooks upstream of the village of Finchingfield is a rough estimate based on inspection of a map; hence, the question mark.

## 2.8 Controlling water levels: human versus beaver engineering

To do using porous dam formulation in [18] but with controllable porosity; using smart flood control on the discharge  $Q$ , making matters linear for  $Q$ , except for the a posteriori relation to porosity and water level, cf. Willemsen et al. - Explore hypotheses. - Two cost functions for hydraulic control (human and beaver one). - Some exact maths and hydraulics examples using HESS article of Study Group



**Fig. 2.11** Beaver burrow in Sutherland Beck, North Yorkshire, UK, on the bank of the beck with an underwater entrance –guided tour to a beaver colony in North Yorkshire on 12-03-2020.

that can be modified into mini-control problems. - Phrase in terms of available flood-storage volume with some mathematics. (All new. Incompleted start below.)

Consider a porous dam of height  $H$  with (one-dimensional) porosity  $\phi$  such that  $\phi H$  is the accumulative length of holes in the dam. A straightforward model will be formulated of approximate hydraulic river flow upstream of a porous dam and downstream of a porous dam. The slope  $S$  of the river is taken constant. The model is inspired by the model with routed and branched flow between nodes as used in [18]. Here, only one such branch will be considered between an upstream and downstream node, with a village lying at the node downstream of the dam. A sketch of the situation is provided in Fig. ???. Subsequently, the fate of the river flow in the village under extreme rainfall and extreme drought upstream of the dam will be considered for two optimisation cases; a time-dependent porosity parameter  $\phi = \phi(t)$  will be used, as follows: (i) by humans in an attempt to keep the flow depth in the village below a flood threshold level, and (ii) by beavers in an attempt to keep their beaver burrow in the lake upstream of the dam both dry in flooding and the burrow entrance submerged under droughts.