

Visualising return periods of extreme flooding events & cost-effectiveness of flood-mitigation measures

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Outline

In the EPSRC network “Maths Foresees”, the Environment Agency (EA) posed and stated two challenges:

- ▶ **I.** How can we **visualise return periods** of extreme (flooding) events to a general audience in a fluid-dynamical set-up?
As opposed to
- ▶ This challenge is posed because people (often) mistakenly think that the time between extreme events of a certain magnitude expressed by a return period is (more or less) fixed, e.g. “I am safe for ~ 100 years after a 1 : 100 year flood”.
(BBC interview 2019)
- ▶ *Answer to challenge-I: the visualisation of return periods in the **Wetropolis flood investigator** (B et al. (2020, 2024)).*

Outline-continued

In the EPSRC network “Maths Foresees”, the Environment Agency posed and stated two challenges:

- ▶ ...
- ▶ **II.** To apply mathematics to flood mitigation with tools that are comprehensible to decision-makers. As opposed to
- ▶ *Answer to challenge-II:* a **graphical cost-effectiveness tool** to visualise flood-mitigation scenarios.



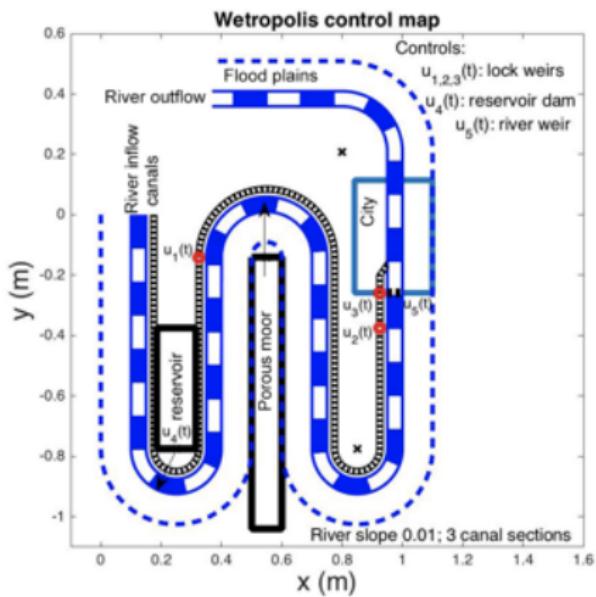
(Challenge-I) The weather machine: ingredients

The basic ideas and ingredients are the following:

- ▶ There is a **conceptual river catchment** with a river, a (one-sided) floodplain, a porous moor, a reservoir and downstream a city.
- ▶ Instead of 1:100 year extreme events in a 1000km^2 river catchment, say, **time and spatial scales need to be reduced**.
- ▶ There are only Wetropolis days (wd) of length T_d .
- ▶ It **rains in two locations**, in the moor and/or reservoir, or not: so there are 4 choices.
- ▶ It **rains** (f_1, f_2, f_3, f_4) T_d of a day with fractions $0 < f_1 < f_2 < f_3 < f_4 < 1$: so there are ≥ 4 "daily" rain amounts possible.
- ▶ If $f_1 = 0.1$ then the rain rate during a day is a "unit" r_0 .
- ▶ The river length L_r at slope 1 : 100 is $L_r \in [1, 5]\text{m}$ (-).
- ▶ Remaining design unknowns are therefore the **day length and "unit" rainfall rate** T_d, r_0 .

The weather machine: map of catchment

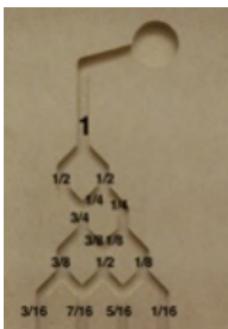
The basic ideas and ingredients on a map of “Wetropolis-I” (with LL-canal):



Inspiration for Wetropolis:
Boxing Day 2015 floods of
the River Aire in Leeds.

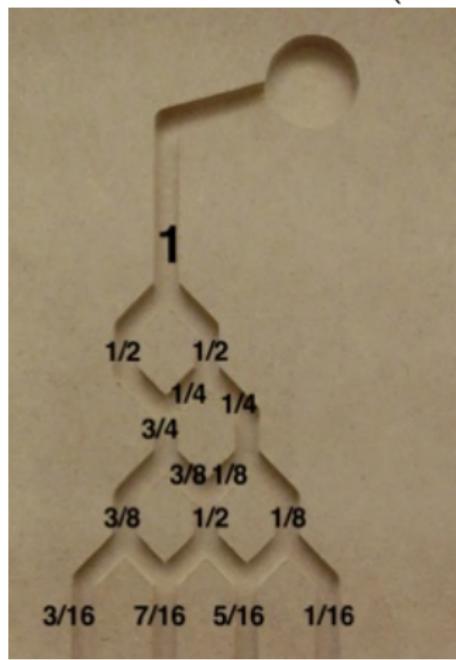
The weather machine: determine 16 outcomes

- ▶ It can rain in two **locations**, in the moor and/or reservoir, or not: 4 choices.
- ▶ It **rains an amount** (f_1, f_2, f_3, f_4) T_d of a day with fractions $0 < f_1 < f_2 < f_3 < f_4 < 1$: so there are ≥ 4 “daily” rain amounts.
- ▶ Hence, there are $4 \times 4 = 16$ or rather 13 outcomes.
- ▶ (On the back of an envelope on some train ride:) Use visual draws from two discrete probability distributions each with four outcomes and a tail.
- ▶ The tail represents a rare event.



The weather machine: skewed Galton board

- ▶ Use **visual draws from two discrete probability distributions** each with four outcomes and a tail. Modified **Galton board** with 4 (or 5) rows:



The weather machine: skewed Galton boards (2016-2023)

- ▶ Use visual draws from two discrete probability distributions each with four outcomes and a tail. **Two modified Galton boards** each with 4 rows:



Rain duration (left: (1, 2, 4, 9)s) and rain location (right).

Wetropolis-I weather: probability and statistics

- ▶ X, Q : probabilities p_i rainfall duration/wd versus q_j rain location:
- ▶ p_i, q_j with $i, j = 1, 2, 3, 4$ and $\sum p_i = 1, \sum q_j = 1$.
- ▶ With $p_1 = q_1 = 3/16, p_2 = q_2 = 7/16, q_3 = p_3 = 5/16, q_4 = p_4 = 1/16$:

Table: Probability matrix $P_{ij} = p_i q_j$ times 256. Rain/location (2016-2023).

	1s	2s	4s	9s
	p_1	p_2	p_3	p_4
reservoir q_1	9	21	15	3
both q_2	21	49	35	7
moor q_3	15	35	25	5
no rain q_4	3	7	5	1

The weather machine: discussion

Question: Is it unusual for a mathematician to build or propose fluid-dynamical devices and demonstrations? 몰라요:

- ▶ The inventor of the Galton board “[Sir Francis Galton](#) was a British poly-math . . . ” (and mathematician).
- ▶ The [innovation of Wetropolis](#) lies in the coupling between the weather or rain machine with its skew-Galton boards and the conceptual river catchment.
- ▶ Underlying Wetropolis is a mathematical and numerical [design model](#) of PDEs, ODEs and diagnostic relations linking the equations for various components.
- ▶ Wetropolis is one member in a suite of fluid-dynamical demonstrations created with designer [Wout Zweers](#).

... Is it unusual for a mathematician to propose fluid-dynamical devices?

That question came from a KAIST (Daejeon, Korea) member on 15-08-2024. 몰라요:

- ▶ Wetropolis is **one member in a suite** of my fluid-dynamical demonstrations, often based on mathematical and numerical design models.
- ▶ Note that a (PDE and ODE-based) design model aims to **accommodate a design** and is generally not a suitable or detailed predictive model (B et al., HESS, 2020).
- ▶ The design model with **HI-optimisation** suggested a Wetropolis day length of $T_d = 10\text{s} = 1\text{wd}$ and unit daily rainfall rate of $r_0 \approx 0.18\text{l}/\text{wd}$ (B et al., HESS, 2020).

Return period of floods: geometric distribution

- ▶ Rain amount per $T_d = 10\text{s} = 1\text{wd}$ determined by **design**: no to minor flooding for $(0, 1, 2, 4)$ & $(8, 9)$, **flooding** for 18 units r_0 .
- ▶ **Return period** T_r of extreme flooding at $t_n = nT_d$ determined by geometric distribution with here $p_n = (1 - p_e)^{n-1} p_e$ where $p_e = P_{24} = q_2 p_4 = 7/256$, s.t.

$$T_r = \mathbb{E}(t_n) = \sum_{n=1}^{\infty} T_d n (1 - p_e)^{n-1} p_e = \frac{T_d}{p_e} \approx 365.7\text{s} \approx \mathbf{6 : 06\text{min.}}$$

- ▶ Standard deviation σ_r (thanks to Daan C & Jason F):

$$\begin{aligned}\sigma_r^2 &= \mathbb{E}((t_n - \mathbb{E}(t_n))^2) = (1 - p_e) \frac{T_d^2}{p_e^2} \\ &= (1 - p_e) T_r^2 \implies \sigma_r = 36.07\text{wd} = 360.7\text{s} \approx 6\text{min.}\end{aligned}$$

Super- and megafloods: geometric distribution of order k

- ▶ Two consecutive “2015 Boxing Days” extreme rainfall WEP
 $p_e^2 = (7/256)^2$ s.t.

$$T_r^{(2)} \approx \frac{T_d}{p_e^2} = (256/7)^2 \times 10\text{s} \approx 223\text{min} \approx 3 : 43\text{hr.}$$

- ▶ $T_r^{(2)}$ & $\sigma_r^{(2)}$ follow from **geometric distribution of order $k = 2$**
(Viveros & Balakrishnan 1993, Koutras & Eryilmaz 2017):

$$\frac{T_r^{(k)}}{T_d} = \frac{(1 - p_e^k)}{(1 - p_e)p_e^k}, \quad \frac{\sigma_r^{(k)}}{T_d} = \frac{\sqrt{1 - (2k + 1)(1 - p_e)p_e^k - p_e^{2k+1}}}{(1 - p_e)p_e^k}.$$

Super- and megafloods: Wetropolis-II revisited design

- ▶ For floods on two consecutive days with old $p_e = 7/256$:

$$T_r^{(2)} = T_d \frac{(1 + p_e)}{p_e^2} = 1374 \text{wd} = 13740 \text{s} = 3.8 \text{hr},$$

$$\sigma_r^{(2)} = T_d \frac{\sqrt{1 - 5(1 - p_e)p_e^3 - p_e^5}}{(1 - p_e)p_e^3} = 3.8 \text{hr}.$$

- ▶ Long waiting times suggest *redesign*, e.g. take Galton board outcome $p_e = p_2 q_2 = 49/256 \approx 1/5$ for 9s rainfall in moor & reservoir, yielding **return periods for $k = 2, 3$ -day floodings:**

$$T_r = 5.2 \text{wd} = 52 \text{s}, T_r^{(2)} = 32.5 \text{wd} = 5 : 25 \text{min},$$

$$T_r^{(3)} = 175 \text{wd} = 29 : 11 \text{min}, \sigma_r^{(k)} \approx T_r^{(k)}, k = 1, 2, 3.$$

Wetropolis-II weather: revisited (2023-...)

- ▶ X, Q : probabilities p_i rainfall duration/wd versus q_j rain location.
- ▶ p_i, q_j with $i, j = 1, 2, 3, 4$ and $\sum p_i = 1, \sum q_j = 1$.
- ▶ With $p_1 = q_1 = 3/16, p_2 = q_2 = 7/16, q_3 = p_3 = 5/16, q_4 = p_4 = 1/16$:

Table: Probability matrix $P_{ij} = p_i q_j$ times 256. Current case.

	1s	7s	4s	2s
p_1	p_1	p_2	p_3	p_4
reservoir q_1	9	21	15	3
both q_2	21	49	35	7
moor q_3	15	35	25	5
no rain q_4	3	7	5	1

Video of Wetropolis-II: visualising extreme events

Goal: visualising return period/Annual Exceedance Probability (request EA & JBA Trust). <https://www.youtube.com/watch?v=yUjYfg2SfY0>



Wetropolis: few remarks

- ▶ Rainfall in Wetropolis is **spatio-temporal**, so the occurrence/distribution of flooding events is more complicated than the imposed random rainfall distribution. TBD.
- ▶ **Climate change** has been implemented via a switch activating rainfall in an additional upstream extra lake/reservoir that is in sync with the random rainfall in the moor. It adds on average $\sim 20\%$ more rain to Wetropolis.
- ▶ A Galton board yields a normal distribution in the infinite-row limit. What skew-Galton-board specification would lead to other (known) skew- or **extreme-value probability distributions?**
- ▶ By using an **LED-board with visualised “Galton-board” channels** various computer-generated discrete distributions can be implemented (Robin Furze).

Visualising flood-mitigation scenarios for decision-makers

Research triggered by:

- ▶ Challenge-II stated by EA in “Maths Foresees” network 2015-2018.
- ▶ Calling a **flood-evacuation** of a Leeds’ Crossfit-gym in the 2015 Boxing-Day floods (saving £20k, see ICS-REF2021):



How (well) can we mitigate flooding?

Flood-mitigation measures, but which ones to choose?

- ▶ Higher walls (HW)
- ▶ Flood-plain storage (FPS): dynamic using weirs and optimal control (underdeveloped)
- ▶ Giving-room-to-the-river (GRR)
- ▶ Natural _{Flood} Management (NFM): tree planting, peat land, leaky dams
- ▶ Beaver colonies
- ▶ Sustainable urban drainage systems (SuDS)
- ▶ Dredging
- ▶ Resilience?

How (well) can we mitigate flooding?

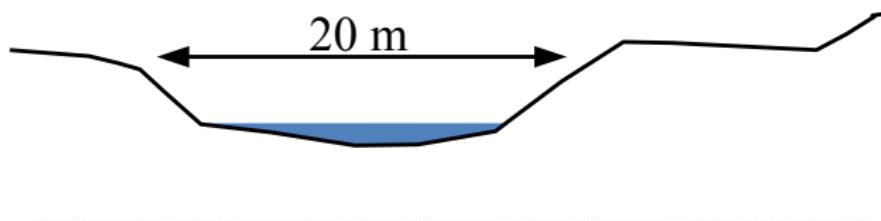
Higher flood defence walls – HW ($\sim 2\text{m}$ high ones in Leeds):



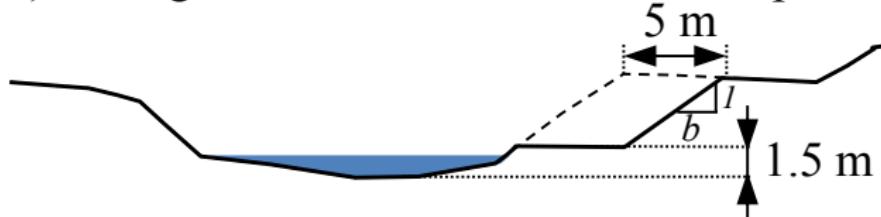
How (well) can we mitigate flooding?

Giving-room-to-the-river – GRR:

a) Current transverse profile



b) Giving-room-to-the-river transverse profile



How (well) can we mitigate flooding?

Giving-room-to-the-river – GRR, extra channel in River Aire at *Aire River at Kirkstall The Forge* (Leeds):

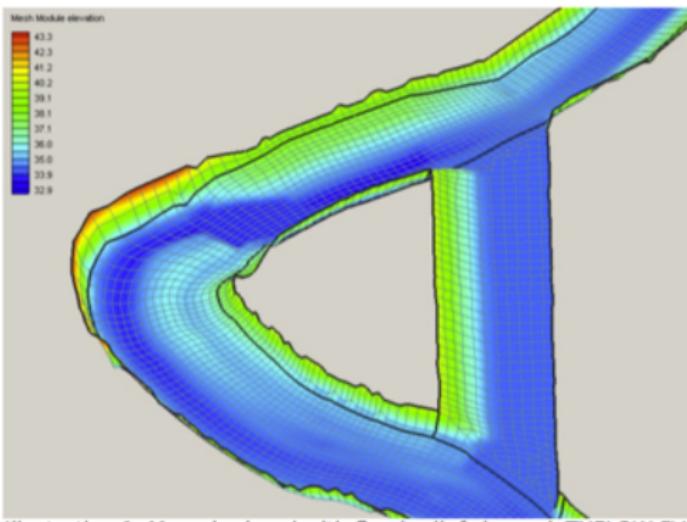


Illustration 1: Meander bend with flood relief channel, TUFLOW FV mesh

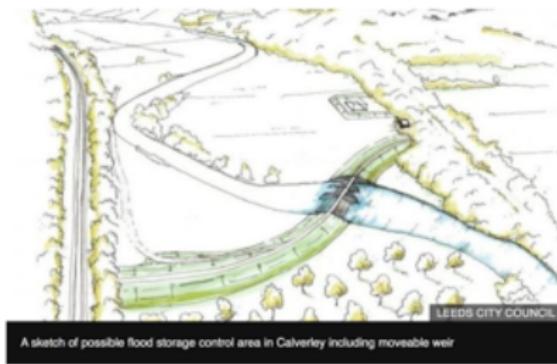
How (well) can we mitigate flooding?

Giving-room-to-the-river – GRR, extra channel in River Waal/Rhine Nijmegen (NL):



How (well) can we mitigate flooding?

Flood-plain storage –FPS & dynamic weir control:



How (well) can we mitigate flooding?

Extra storage –FPS active flooding of certain areas (Merwede, Storm Ciara, NL, 20Mm³):



How (well) can we mitigate flooding?

Natural flood management – NFM 1300 leaky dams & trees (public engagement & co-benefits, e.g. carbon sequestration)



Central part of one of the two experimental timber bunds in the River Seven catchment

How (well) can we mitigate flooding?

Imagine your home is flooded. Lots of **beaver colonies** then? Extra water storage behind dams: $\sim 1100\text{m}^3 = 1.1\text{Mlitres}$ (or $1/5^{\text{th}}$).

How beavers can help stop homes from flooding

© 17 Feb 2020 Last updated at 11:08

not



PA MEDIA

Beavers can play an important role in helping to keep our homes from being flooded.

That's according to scientists at Exeter University, who have carried out a five year study of wild animals living in Devon.

They found the animals helped to [redacted] reduce pollution and boost wildlife population.

How (well) can we mitigate flooding?

SuDS –Sustainable Drainage Systems:



How (well) can we mitigate flooding?

Dredging –Wainfleet Flood Action Group (flood June 2019, 67 homes & lots of farmland flooded):



How (well) can we mitigate flooding?

Resilience: raising of new houses now mandatory in Wainfleet:

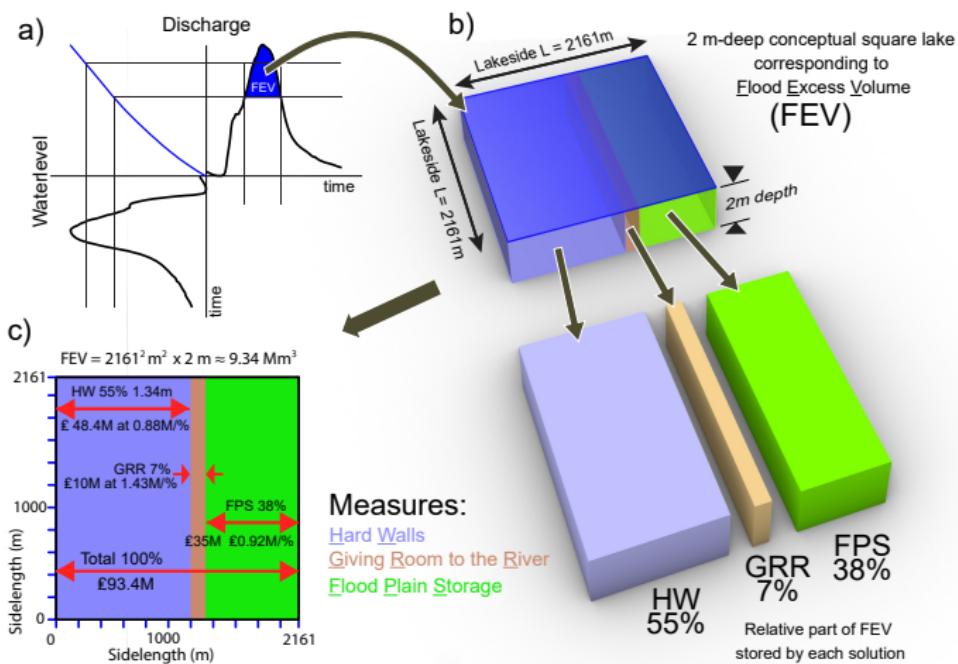


How (well) can we mitigate flooding?

Resilience: responsible flood-plain development (**zero-sum or negative volume rule**), Rhine valley:



Graphical cost-effectiveness tool for flood mitigation



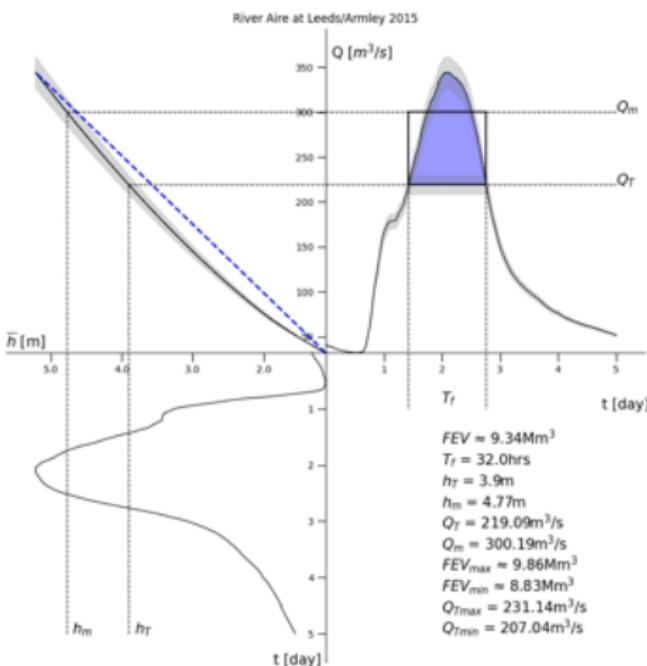
Graphical cost-effectiveness tool: three-panel graphs

Motivated by Boxing Day 2015 floods:

Flood-excess volume (*FEV*) is defined as volume of flood water one wishes to mitigate (i.e., reduce to zero) by cumulative effect of flood-mitigation measures.

Right:

River Aire gauge data of Jan. 2015 floods. **Bottom left:** 15min water-stage time series. **Top left:** longer-time rating curve. **Top right:** resulting discharge time series.



Graphical cost-effectiveness tool: square lake (1 : 200yr design flood)

B et al. 2020 Water:

Scenario S1 (of 4) in a square-lake cost-effectiveness analysis of flood-alleviation plans using flood-excess volume (FEV); each mitigation measure is represented by a colour, and an overall cost analysis is displayed.

HW: higher walls,

GRR:

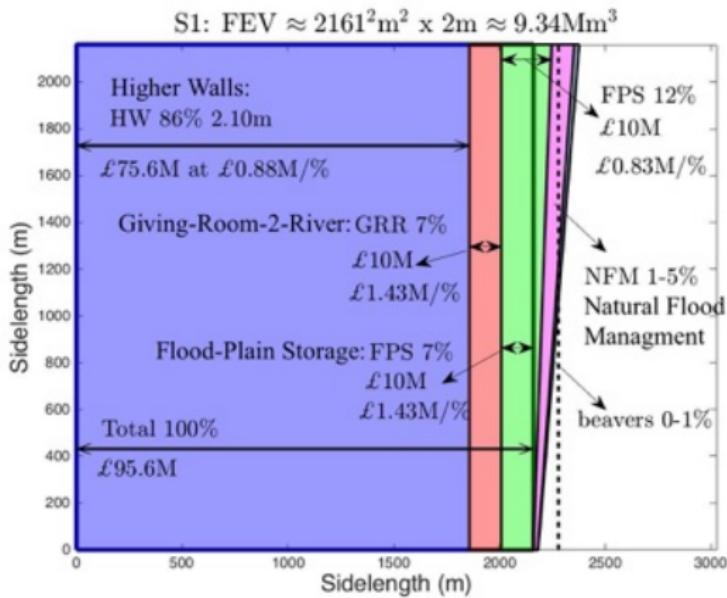
giving-room-to-the-river,

FPS: flood plain storage,

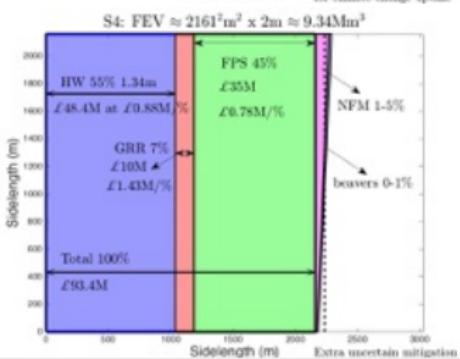
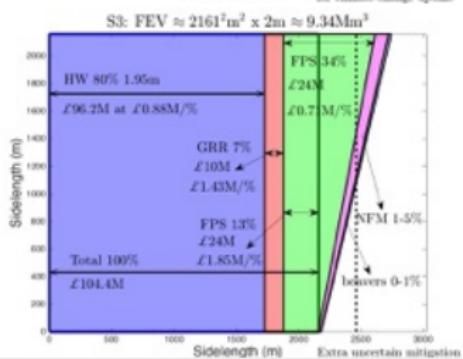
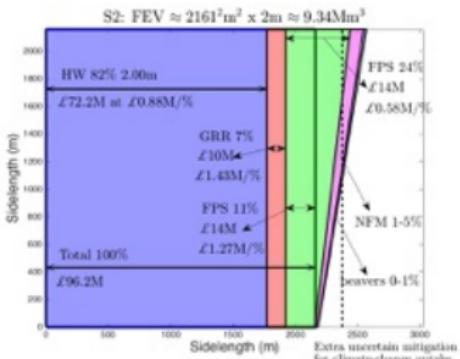
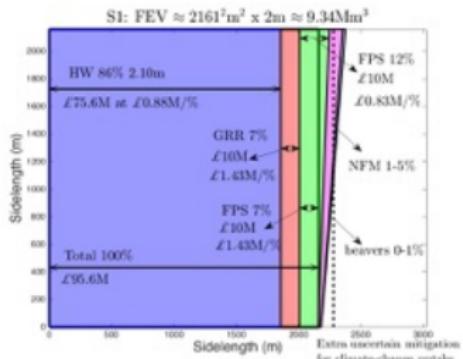
NFM: Natural flood

management,

beavers: 85 beaver colonies.



Graphical cost-effectiveness tool: square lake scenarios (1 : 200yr design flood)

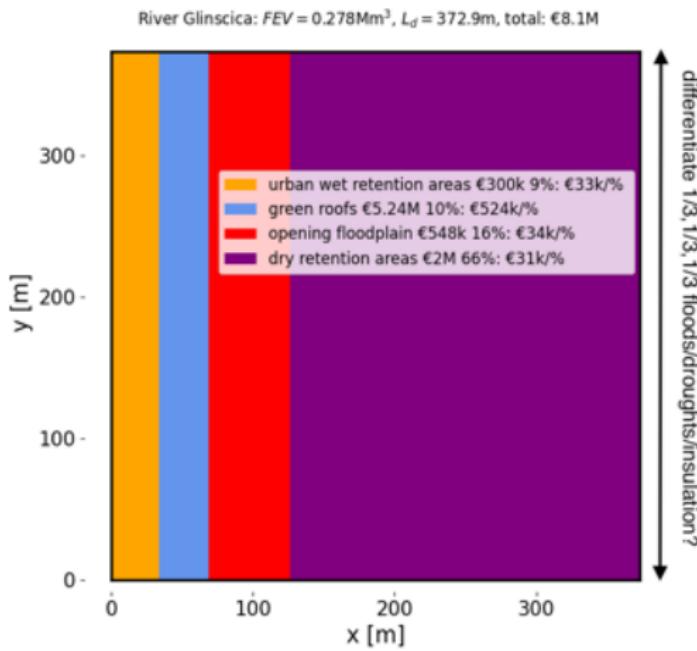


A priori FEV analysis: NBS for River Glinčica (1:100yrs)

Pengal et al. 2021 (EU project NAIADE):

A demonstration of **participatory catchment management with stakeholders** was undertaken for NBS as most suitable solution to reach these primary goals.

4 Nature Based Solutions (NBS) considered:
urban wet retention areas 9%
green roofs (floods/droughts/insulation) 10%
opening of flood plains 16%
dry retention areas 66% with round-off.

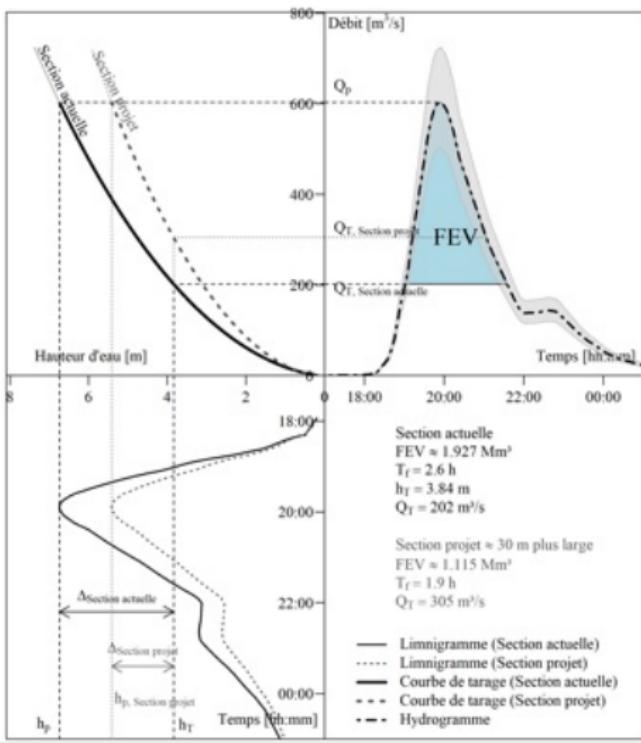


A posteriori FEV analysis: River Brague, France (1:500yrs)

Piton & Tacnet 2020
(NAIAD) after River Brague:

Based on data of hydrographs across the catchment following hydraulic simulations, FEV was calculated.

Three-panel graph of the 2015 flood of the River Brague, France, solid-line curves, as well as a GRR-modified case, dashed curves.

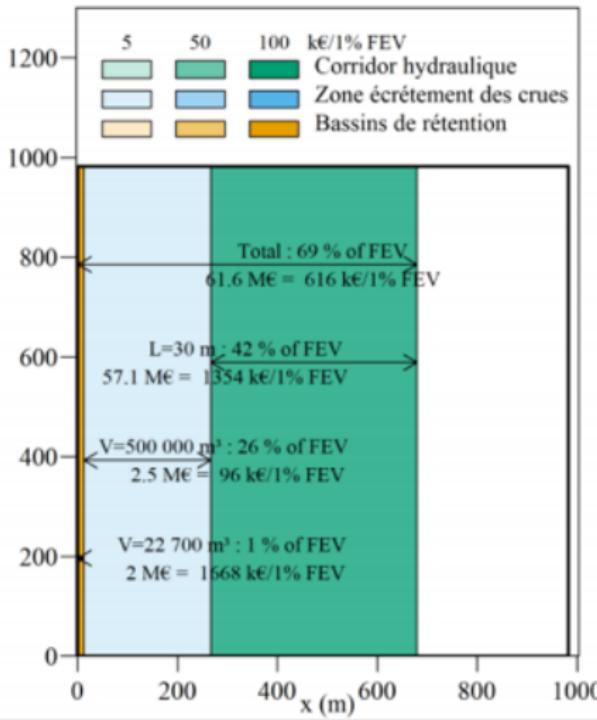


A posteriori FEV analysis: River Brague (1:500yrs)

Piton & Tacnet 2020:

Three measures cover 69% of the FEV: with **concrete basins** at 1% represented by the thin sliver, **natural retention areas** at 26% being the cheapest **per percent** and GRR at 42%.

Remaining 31% **unprotected FEV** requires additional measures for the worst-case design event of 1:500yrs or AEP = 0.2%.



Graphical tool and its ability to find inconsistencies

FEV and square-lake analysis-tool **uncovered inconsistencies** in a public [REDACTED] report (B et al. 2018, 2020):

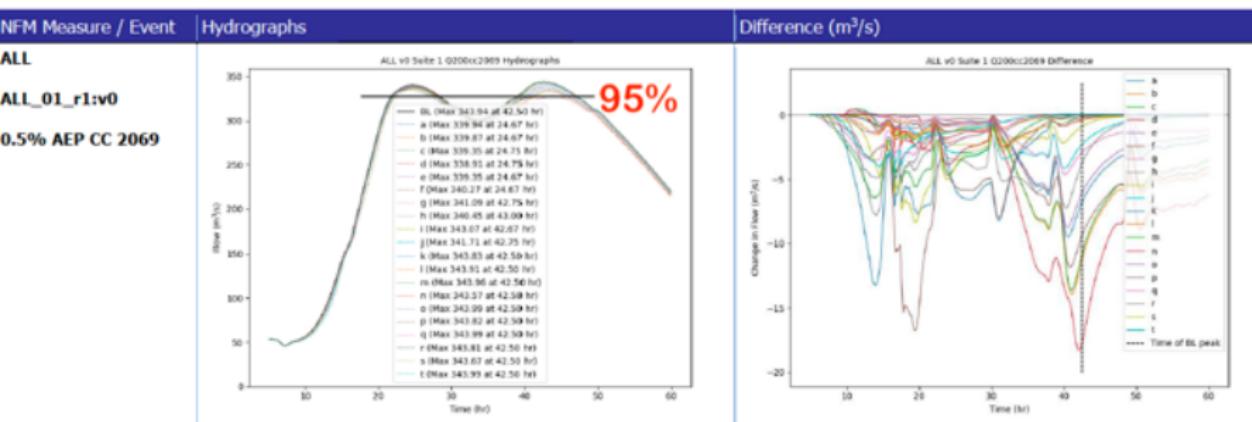
- ▶ Analysis showed that efficacy of Natural Flood Management (NFM) low [1, 5]% and has been (grossly) overstated;
- ▶ two vastly different flood-plain storage volumes emerge from this report leading us to define the novel concept of *available flood-storage volume*; and,
- ▶ the locations of weirs for the proposed dynamic flood-plain storage are suboptimal.



Graphical tool and its ability to find errors (02/07-2024)

FEV and square-lake analysis-tool uncovered (apparent) **errors** in private-public plans:

- ▶ A [REDACTED] Company's (RC) claim of 5% flood reduction by NFM against Climate Change effects is seemingly not seen in graphs provided as evidence, e.g.:



Graphical tool and its ability to find errors (02/07-2024)

FEV and square-lake analysis-tool uncovered (apparent) errors in private-public plans:

- ▶ Response (lukewarm) by [REDACTED]:
“[REDACTED]”
- ▶ To date 20-09-2024: data sharing of relevant hydrographs has been refused by [REDACTED] and “*limited time for a peer review*”.
- ▶ Are the (potential) investors in the flood works proposed (£4.25M needed) by RC aware of this anomaly between the claimed 5% efficacy and the (hitherto apparent lack of) evidence? Evidence provided seems to show only ~ 2.5%?
- ▶ The missing ~ 2.5% was stated to come from landmanagement including soil aeration (**which is not NFM**) but corresponding evidence has hitherto been lacking.

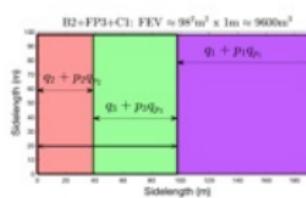
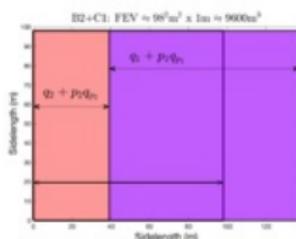
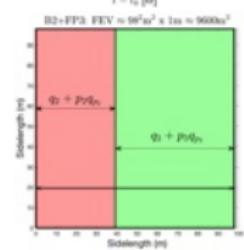
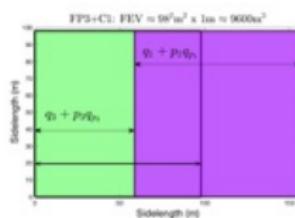
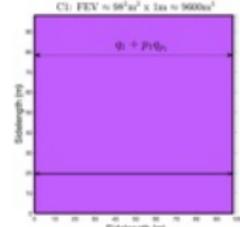
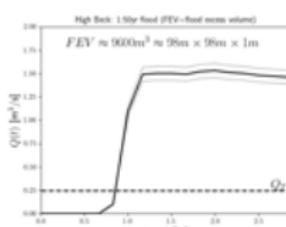
Graphical tool and its ability to find errors: remarks

Morgan and Henrion's advice (§7.8 "Uncertainty: ...", 1990) seems to apply:

- ▶ "*This means, however, that peer review should be more uniformly extended to policy focussed research and analysis than it has been in the past*".
- ▶ "... to develop institutions and traditions that *protect experts who participate in elicitation from subsequent legal or other entanglements*. ... has set an excellent example by providing partial anonymity to participating experts".
- ▶ Note that this **anonymity is in apparent conflict** with the UK academic and REF demands with associated funding to demonstrate impact.
- ▶ The central issue seems to be that **scrutiny of public spending**, here on flood-mitigation, by academics and especially by mathematicians, **is disliked**.
- ▶ I have flagged the (potential) issue with two contractors of the RC for further informal discussion.
- ▶ Such lack of scrutiny could be a point in a wider discussion?

High Beck flood-mitigation case study (1:10yrs)

- ▶ **Square-lake plots:** **size & costs** with flood-excess volume & mitigation measures.
- ▶ Base costs q_i , probability failure p_i , repair costs q_{p_i} , $i = 1, 2, 3$; **costs** $q_i + p_i q_{p_i}$.
- ▶ Combine Canal C1, bund B2, flood-plain-storage FP3 into 5 scenarios:



- ▶ **Utility functions:** $U_{23} = \sum_{j=1}^5 w_j C_j$, $U_1 = \sum_{j=1}^5 (w_j C_j - \sum_{k=1}^{N_j} \alpha_{kj} B_{jk})$ (co-benefits B_{jk} : e.g., droughts, extra CC, less pollution); if $B_{jk} = 0$: $U_1 > U_{23}$.
- ▶ If B_{jk} unknown, $U_1 = U_{23}$: insights on appreciating benefits w. **info-gap theory**. ↗ ↘ ↙ ↛ ↜

Discussion on visualising cost-effectiveness of flood mitigation

- ▶ FEV-analysis seemingly 0D but it captures a stretch of river, so becomes 1D.
- ▶ A priori investigation can be extended by using **ensemble forecasts** leading to an FEV cost-effectiveness analysis with **uncertainty**: see B et al. 2020 *Water* for a detailed roadmap.
- ▶ In-depth **Socratic-style dialogue** on critique, see B 2021 *ESREL2021*.
- ▶ In summary, the **FEV cost-effectiveness approach is an essential input in the whole chain**.
- ▶ It provides **valuable inputs in global approaches** dedicated to multifactorial analysis of flood protection measures' effectiveness.

- ▶ Note that our FEV tool is **by itself and alone not a proper safety** and reliability analysis approach.
- ▶ However, it is **an essential input in the whole chain**.

Wetropolis World: future work & proposal

How can a **Wetropolis laboratory set-up** and a “**Numerical Wetropolis Prediction**” model be used to understand:

- ▶ risk, extreme weather & flooding probability statistics –revisit **spatial-temporal rainfall** & change-point analysis;
- ▶ flood control –e.g., reservoirs in Wetropolis;
- ▶ data assimilation & parameter estimation –laboratory experiment as “**truth run**”?
- ▶ One Wetropolis World’s **goal**: to investigate “classical” PDE & Data Assimilation “**NWP**” model with ML predictions.
- ▶ **Proposal EPSRC-Fellowship⁺**: PDE vs. ML, info-gap theory on decision-making, **1/4 educational-version, board game, workshops**.

Thank you very much for your attention ...

- ▶ Knotters, B, Lamb, Poortvliet 2024: How to cope with uncertainty monsters in flood risk management? *Cambridge Prisms: Water 2.* <https://doi.org/10.1017/wat.2024.4> (Nominated paper.)
- ▶ B 2024: High Beck fluvial flood-mitigation case study. EGU Vienna: <https://obokhove.github.io/EGUBokhoveVienna2024.pdf>
- ▶ B, Kelmanson, Piton, Tacnet 2024: Visualising Flood Frequency, Flood Volume and Mitigation of Extreme Events. <https://obokhove.github.io/UKsuccessFEVpreprint23102023.pdf>
- ▶ B 2024/2022: Wetropolis videos for general public: <https://www.youtube.com/watch?v=yUjYfg2SfY0> & <https://www.youtube.com/watch?v=rNgEqWdafKk>
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