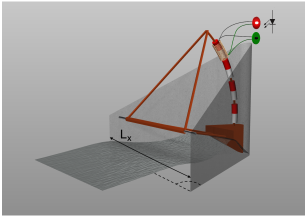
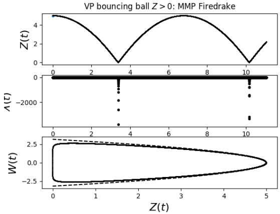
Firedrake/FEniCS meetings LIFD

LIFD (Onno Bokhove)

Leeds Institute for Fluid Dynamics, UK





"Agenda" Firedrake first meeting LIFD

- Introduce ourselves;
- circa 3-min-each slide presentation of what we (aim to) do with Firedrake (or FEniCS);
- discuss the goals of the meetings; e.g., GitHub page, examples' board, open questions, et cetera; and,
- (announcement of) Firedrake 2025 (in Leeds), September Mon-Wed 15, 16, 17.

My (team's) work in Firedrake

(2015/16: lost improved Taylor-Hood implementation of 2-phase flows Navier-Stokes-Cahn-Hilliard)

Use variational principles (VPs) or Hamiltonian dynamics to derive weaks forms or automated generation weak forms from VP in FD:

- 2016-onwards Dr Anna Kalogirou, 2DH Benney-Luke wave equations (B. & Kalogirou 2016, Gidel et al. 2017, Choi et al. 2022 & 2024); FD-example.
- Will Booker's (unfinished) work with Mark Walkley on 3D incompressible flows/chaotic attractors.
- Dr Floriane Gidel 3D potential flow water wave equations (Gidel 2018, B. 2022).
- Dr Tomasz Salwa (2018) linear FSI wave-mast system (Salwa et al. 2027); FD-example.

My (team's) work in Firedrake

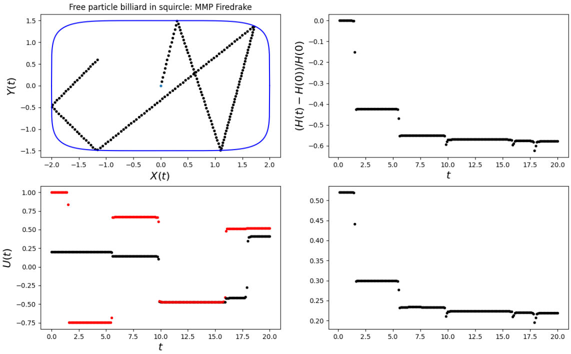
- Dr Choi et al. 2024, Lu et al. (2025): VP 3D potential flow implemented directly, with automated generation of (complicated 3D+1D) weak forms of the equations.
- Advantages: enormous reduction in development time, efficient, flexible, higher-order spectrally-accurate space discretisations plus (automatic) preservation of discrete forms of conservation properties.
- Time discetisation: second-order implicit modified-midpoint scheme (MMP).

Yang Lu's work in Firedrake

- Presentation on the Firedrake 23 Workshop; Firedrake implementation of the two variational approaches: https://www.youtube.com/watch?v=_GnuqSTqhAs
- Slides: https://github.com/luyanggeorge/3D_tank_VP/blob/main/Firedrake23_Yang.pdf

My work in Firedrake

- Bokhove, now: VP of ODE's (billiards), wave equations, wave-to-wire wave energy device with inequality constraints and FSI.



Grand VP of wave-to-wire model

Equations of motion follow from variational principle (red=waves, blue=buoy, green=EM-generator, coupling, B. et al. 2019):

$$0 = \delta \int_0^T \int_0^{L_x} \int_{R(t)}^{l_y(x)} \int_0^h -(\partial_t \phi + \frac{1}{2}|\nabla \phi|^2) \mathrm{d}z - gh(\frac{1}{2}h - H_0)$$

$$- \underline{\frac{1}{2\gamma}\Big(F_+(\gamma(h-h_b)-\lambda)^2 - \lambda^2\Big)}\mathrm{d}y\mathrm{d}x$$

$$MW\dot{Z} - \frac{1}{2}MW^2 - MgZ + (L_i I - \underline{K(Z)})\dot{Q} - \frac{1}{2}L_i I^2 \mathrm{d}t \qquad (1)$$

velocity $\mathrm{u} = \nabla \phi(x,y,z,t)$, depth $h(x,y,t)$, rest depth H_0, buoy $h_b(Z,y) = Z - K_h - \tan\theta(L_y - y)$, piston $R(t)$, coupling function $\gamma_m G(Z) = K'(Z)$, buoy mass M, keel height K_h, buoy coordinate $Z(t)$, buoy velocity $W(t) = \dot{Z}$, charge $Q(t)$, current $I(t) = \dot{Q}$.