

MATH2640 Introduction to Optimisation

Semester: 1; No. of credits: 10; Level: 2

Prerequisites: MATH1010 or (MATH1050 & MATH1060) or (MATH1050 & MATH1331) or equivalent.

Lecturer: Prof. Onno Bokhove, School of Mathematics, Rm. 8.04; E-mail: o.bokhove@leeds.ac.uk

Minerva VLE Web Page:

Course information, some past papers and example sheets as well as summary lecture notes will be posted (as the course progresses) on the University of Leeds Minerva VLE Blackboard. To access this, select the portal from the front page of the University web site, and then choose VLE from menu on the MATH2640 module. The Minerva VLE webpages will be used from time to time to make announcements.

Aims:

To provide a collection of theoretical techniques for determining optimal extrema of functions of several variables, either with or without constraints, and to apply these in the context of maximising profit and/or revenue in financial scenarios.

Objectives:

By the end of this module, students should be able to: a) determine the definiteness of both unconstrained and constrained quadratic forms; b) determine exactly the extrema of functions of several variables, either with or without constraints, using a variety of techniques, namely: Lagrange multipliers, Bordered Hessians and Kuhn-Tucker theory; c) apply the theory in b) to profit- and/or revenue-maximisation strategies in economics.

Methods of teaching:

Hours: Lectures 20, Workshops 10, Revision (in week 11).

Monitoring of progress: Four example sheets. Note that assessed coursework contributes 15% to the final module mark, the remaining 85% from the exam.

Outline syllabus:

Several-variable calculus, quadratic forms, unconstrained optimisation, constrained optimisation via Lagrange multipliers, applications in economics.

Detailed syllabus:

Several-variable calculus, (6 lectures): Representing and visualising functions of two variables. Partial derivatives, total derivatives and chain rule. Gradient vectors and directional derivatives. Implicit differentiation. Several-variable Taylor series, Hessian matrix, stationary points.

Unconstrained optimisation (4 lectures): Quadratic forms and eigenvalues. Definiteness using principal minor tests. Stationary points, local extrema, unconstrained optimization, applications in economics. Cobb-Douglas production functions.

Constrained optimisation (10 lectures): Constrained maximisation with equality constraints, Jacobian derivative, first-order conditions, constraint qualifications, Lagrange multipliers, constrained quadratic forms, bordered Hessian, constrained maximisation with inequality constraints and mixed constraints, constrained minimization. Kuhn-Tucker theory.

Booklist, including additional exercises:

1. *C. P. Simon, L. Blume, *Mathematics for Economists*, W W Norton, 1994. (Highly recommended.)
2. Alpha C. Chiang, *Fundamental Methods of Mathematical Economics* (International Edn.), McGraw-Hill, 1984. (Very good.)
3. D. Bailey, *Mathematics in Economics*, McGraw-Hill, 1998. (Good, with lots of exercises.)

4. A. Ostaszewski, *Mathematics in Economics: Models Methods*, Blackwell, 1993.

Informal description:

Optimisation –the quest for the best– plays a major role in financial and economic theory, e.g. in maximising a company's profits or minimising its production costs. How to achieve such optimality is the concern of this course, which develops the theory and practice of maximising or minimising a function of many variables, either with or without both equality and inequality constraints. This course lays a solid foundation for progression onto more advanced topics, such as dynamic optimisation, which are central to the understanding of realistic economic and financial scenarios.

It should be observed from the above syllabus that neither gradient methods nor search methods are covered in this course, such methods being more suited to a course in programming. Here, only purely mathematical techniques are used, with a definite emphasis on applications in Mathematical Finance.

Assessment: 85% 2-hour written examination; 15% coursework, based on five assessed example sheets (part A of the example sheets contains the assessed questions; only the answers to these you need to hand in). The homework will have to be handed in on Wednesdays October 16th, October 30th, November 13th, November 27th and December 11th. Homework has to be handed in via the pigeon holes on level 8 of the School of Mathematics of the designated markers allocated to each student. This has to be done by 5 pm on the above collection dates. **No late homework will be accepted under any circumstance!**

Lectures: Mondays 10-11am in Roger Stevens LT20 and Wednesdays 12-13 in Roger Stevens LT20.

Workshops: There are workshops scheduled in different groups on Mondays and Tuesdays. Please check the appropriate time/venue for your workshop. These workshops are intended for students to work on the example sheets, (parts B of the example sheets are intended as workshop problems) and a member of staff will be present to give assistance. **Note that these workshops are compulsory, and attendance may be taken from time to time.**

Review session: A review session is scheduled for the Friday afternoon in week 11, from 3-4pm, in Chemistry West Block LT F (G.74).