On the science of Floods: Rainfall, Flooding, and Flood Control revisited in the Design of a Table-Top Model

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EPSRC-funded UK Network *Maths Foresees*
Thanks to Tiffany Aslam & Tom Kent; MPE Seminar June 1st 2016

1. Introduction

In a recent public event in Hebden Bridge, a town in Yorkshire that has seen a lot of sustained and flash floods over the last decade, I addressed the following questions:

- Is it going to rain more in the future?
- Can we define extreme precipitation & flooding events?
- How (well) can we predict heavy precipitation & floods?
- How (well) can we mitigate and control flooding?
- How can we elucidate the above in an interactive, conceptual table-top demonstration?
  - Hydro- & Meteorological Simulator

1. Introduction

I will elucidate the answers to the above and use these in the design and modelling of an interactive, conceptual table-top demonstration and experiment on rainfall, flooding as well as flood mitigation and control. The design will serve 2 purposes:

- as public demonstration of the concepts of flooding and
- simplified test environment for flood modeling:
  - reliable estimation of extreme flooding events
  - improve short-term forecasts using data assimilation
  - multi-scale modelling of street-sewer networks
  - rational mathematical strategies for active flood control

1. Introduction

I will identify mathematical elements involved to model:

- "random" rain supply, channel/river flow,
- the hydrology of "Hele-Shaw valleys (illustrating the effects porous moors, fast run-off from tarmac, and the function of reservoirs),
- flow in bypass canals, and
- control features such as weirs and sluice gates as well as
- data assimilation using a few measurement points.

Obviously, this is work in progress.

3. Is it going to rain more in the future?

Definitions:

- *Return period*: if 100 years of *daily rainfall* data were available, an event with a 1 in 20 year return period would be expected to occur 5 times in that data set.
- In any year such an event has on average a 5% chance to happen on one day per year.
- *Extreme events*: events with a (longer) return period of 1 in 5, 1 in 10, 1 in 20, 1 in 30, 1 in 50 or 1 in 100 years.

Is it going to rain more in the future?

IPCC report 2013 , measurements:

- No, there is low to medium confidence in increased average annual rainfall.
- But, for extreme precipitation Europe: “High confidence: likely increases in more regions than decreases but regional and seasonal variation”
- “. . . median reduction in 5-to 20-year return periods of 21% with an increase in moderate extremes”.

Is it going to rain more in the future?

Similarly, Michael Sanderson's UKCP09 report Met Office 2010 (data 1960–2006), for UK:

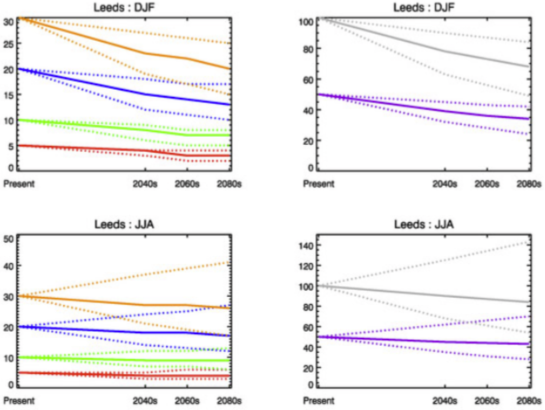
- No, annual mean daily rainfall has not increased significantly since 1766.
- But proportion of winter rainfall in heavy rainfall events has increased across UK in last 45 years.

- In summer rainfall has decreased except in NE England and Northern Scotland.
- "Although summer rainfall may decrease, it could be concentrated into a number of intense downpours from storms".

4 Can we define extreme precipitation & flooding events?

Caution:

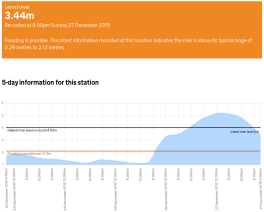
- The uncertainty in the return period for a 1 in 100 year event will be larger than in a 1 in 20 year event.
- Uncertainty will be smaller for larger data sets.



Extreme precipitation and flooding events?

Challenges and basic problem (Coles, 2001):

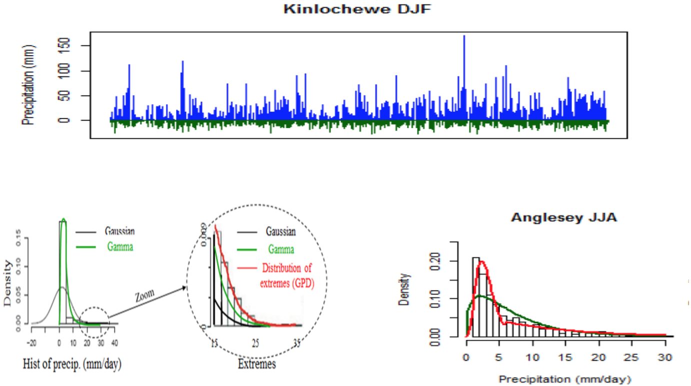
- There are very few observations “in the tail of the distribution”, i.e. for extremes with long return periods.
- Estimates required beyond largest observed data values.
- E.g., Aire River gauge at Armley/Leeds had highest record of 4.03m prior to 26-12-2015; how to estimate return period for the Boxing Day flood with 5.21m at this gauge?



- Standard statistical techniques work well when there are a lot of data, but don’t work well for extremes.

Modelling Extreme Rainfall

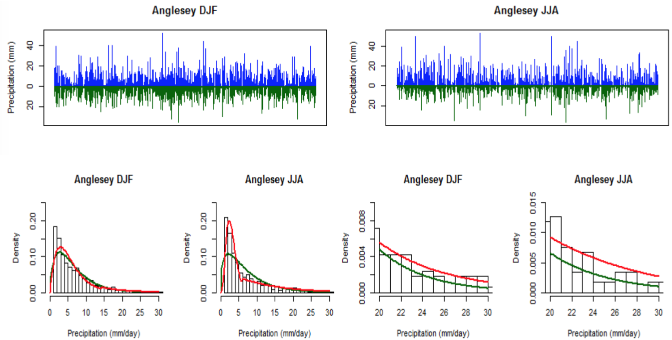
- Wong, Maraun, . . . Kent 2014, J. Climate; MSc Kent:



Modelling Extreme Rainfall

- Fit of Γ– & generalised Pareto distributions (Ziggurat):

$$h(r;\beta) = c_\beta\big((1 - w(r;m,\tau))f(r;\gamma,\lambda) + w(r;m,\tau)g(r;\xi,\sigma\mu = 0)\big) \qquad (1)$$



(a) The bulk of the distribution

(b) The extreme upper tail of the distribution

5. How (well) can we predict heavy precipitation and (overland) floods?

Distinguish between downpours and sustained rainfall:

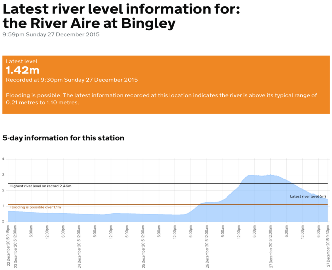
- Downpours (short ca. 1 hour) and flash floods, e.g., *Hebden Bridge July 2012* http://www.bbc.co.uk/news/uk-18778840

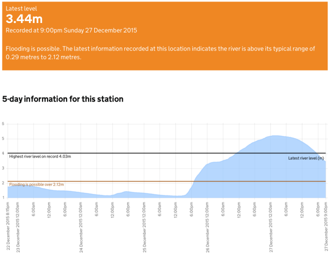
Hebden Bridge hit by flash floods

- Aquaduct as 2012 flood control point: https://www.youtube.com/watch?v=k4OPhlCYn4A (Sara Dee)

How (well) can we predict heavy precipitation and (overland) floods?

- Sustained rainfall and flooding, e.g.
  Boxing Day rainfall and floods 26-12-2015
  Bradford: 69mm/48hrs; *Bingley*: 94mm/48hrs; *Kildwick – Armley*
- *Aire River Flood* & *Aire River Kirkstall The Forge*





How (well) can we predict heavy precipitation and (overland) floods?

48hr UK RAINFALL TOTALS 9am 25 DEC - 9am 27 DEC 2015

SITE	AREA	RAINFALL TOTAL (MM)
CAPEL CURIG	GWYNEDD	210.6
STONYHURST	LANCASHIRE	100
PATELEY BRIDGE, RAVENS NEST	NORTH YORKSHIRE	97
BINGLEY	WEST YORKSHIRE	93.6
BAINBRIDGE	NORTH YORKSHIRE	89.8
BALA	GWYNEDD	89.4
SHAP	CUMBRIA	86.4
SPADEADAM	CUMBRIA	79.4
PRESTON, MOOR PARK	LANCASHIRE	73.2
MYERSCOUGH	LANCASHIRE	72.4
BRADFORD	WEST YORKSHIRE	69.4
ROCHDALE	GREATER MANCHESTER	68.2
MORECAMBE	LANCASHIRE	65.8
MONA	ISLE OF ANGLESEY	63.6
KIELDER CASTLE	NORTHUMBERLAND	61.2
DISHFORTH AIRFIELD	NORTH YORKSHIRE	60.8

This wet spell has added to the heavy rainfall through the rest of the month to make December 2015 already the wettest on record in parts of the UK.

Figure 2: http://blog.metoffice.gov.uk Rainfall summary 25-27 December 2015.

How (well) can we predict heavy precipitation and (overland) floods?

- Downpours, their location and the amount of rain, are very difficult to predict.
- Numerical Weather Prediction (NWP is computer modelling from e.g. the Met Office) cannot handle those cases (well), due to lack of computer power and lack of insights in the physics of precipitation.
- Also, NWP uses/assimilates lots of data to bring the computer model back to reality.
- Sustained rainfall and river flooding is by and large well-predicted: the Met Office and Environment Agency do a reasonably good job e.g. Aire River floods.
- Prediction of localised surface water/brook flooding is more uncertain/less good, due to a lack of data.

6. How (well) can we mitigate flooding?

- Create multiple storage and buffers upstream.
  - Lowers peak values but broadens the flood peaks.
  - May work less well for large water volumes, including consecutive heavy rainfall events.
- Create more permeable surfaces, green soft-surface gardens and absorbing roofs in urban areas, wadis.
- Use bypass canals as overflow channels to lessen peaks. Does require maintenance and alteration to canal system.

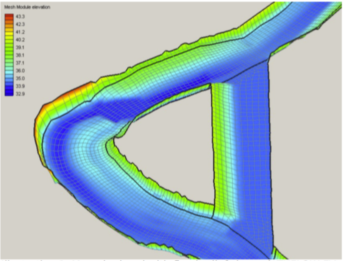


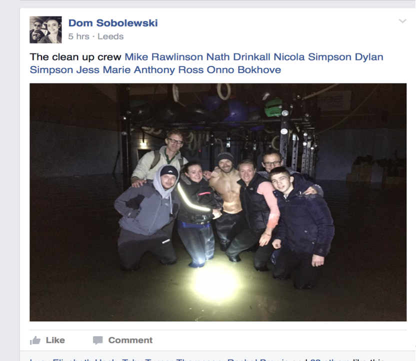
Illustration 1: Meander bend with flood relief channel, TUFLOW FV mesh

How (well) can we mitigate flooding?

- Sustainable flood plain management: designated flood plains (pay owners), houses on stilts, houses on mounts with surrounding canals/wadis.
- Legislation & "Waterschappen" (Water Governance).
- Create "Ruimte voor Water" (Space-for-Water), cf. The Netherlands & Germany 1995 & 1997 floods.

How (well) can we mitigate flooding?

- Active (mathematical) control of weirs, sluice gates, and buffer areas.
- Teach people how to make a local flood evacuation plan (goods/people), based on online river-level gauges, previous floods (CCTV/mobile phone data), and rainfall predictions, themselves.



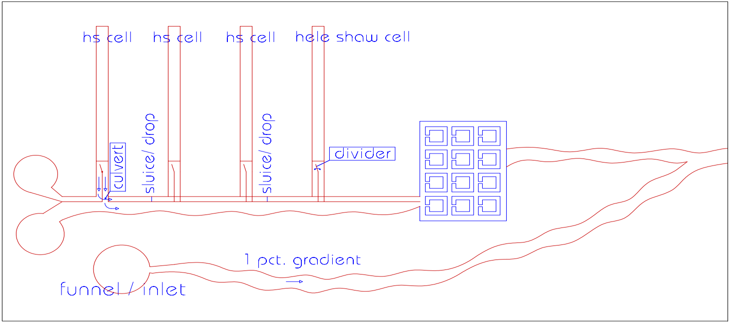
7. How can we elucidate the above in an interactive, conceptual table-top demo?

A conceptual demo to highlight the concepts of flooding, complementary to the experiments shown this afternoon:

- Use *vertical Hele-Shaw cells* as conceptual valleys:
  - spungy material=moor
  - hard surface=urban asphalt,
  - adjustable weirs for storage in reservoirs, etc.

How can we elucidate the above in an interactive, conceptual table-top experiment?

- In progress. Sketch plan-view:



- More demos and movies on fluid dynamics & flooding: `https://www.facebook.com/resurging.flows`

8. Design Hydro- & Meteorological Simulator

Prototype 0.0 with one river, one canal, one Hele-Shaw moor as valley, one Hele-Shaw reservoir as valley:

- River; modelled 1D, straight with depth $h_r(x,t)$ with $x \in [0, L_x]$, e.g., $L_x = 1.2$m
- River: simple kinematic model, bottom slope balances Manning friction
- Canal: modelled 1D, straight as compartments with depths $h_{2c}(t)$ for $x \in [0, L_{2c}]$ and $h_{1c}(t)$ for $x \in [L_{2c}, L_{1c}]$
- Canal: closed at $x = 0$, weir between canal 2 & 1 at $x = L_{2c}$, weir between canal 1 and river at $x = L_{1c}$.

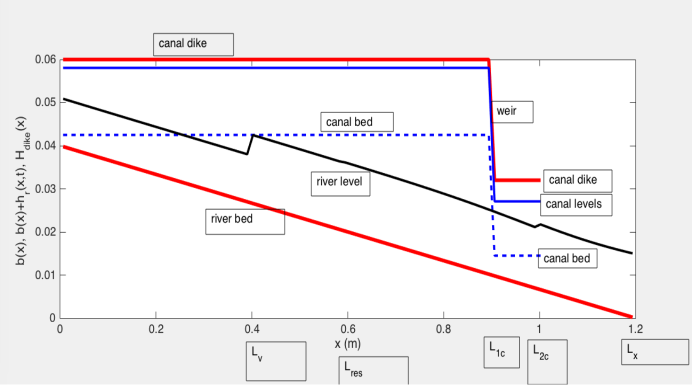
Design Hydro- & Meteorological Simulator

Prototype zero with one river, one canal, one Hele-Shaw moor as valley, one Hele-Shaw reservoir as valley:

- Hele-Shaw moor: porous cell with Darcy flow and ground water level $h_m(y,t)$ orthogonal to canal; $\gamma_m \in [0,1]$ flows via weir into canal and $1-\gamma_m$ into the river at $x = L_w < L_{1c}$
- Reservoir: Hele-Shaw cell with weir height $P_{wr}(t)$.
- Controls: splitter and weir heights $\gamma_m(t), P_{1w}(t), P_{2w}(t)$.

Design Hydro- & Meteorological Simulator

Side view (make sketch):



River Flow

- Cross section $A = A(x,t) = A(h)$ with depth $h = h(x,t)$ above bottom $b = b(x)$, e.g. $A = w_r h$, velocity $u(x,t)$:

$$\begin{aligned} \partial_t A + \partial_x (Au) =& F \\ \underline{\partial_t u + u\partial_x u + g\partial_x h} =& -g(\partial_x b + C_m u|u|/R^{4/3}) \qquad (2) \end{aligned}$$

- hydraulic radius $R(h) = \frac{\text{wet area}}{\text{wetted perimeter}} = \frac{w_r h}{2h + w_r}$
- Manning friction coefficient $C_m \in [0.01, 0.15]$
- Kinematic model for $u > 0$, upwind information speed $dQ(A)/dA > 0$ for flux $Q(A) = Au$ and inflow $A(h(0,t))$:

$$\begin{aligned} u =& R^{2/3}\sqrt{-\partial_x b/C_m} \\ \partial_t A + \partial_x (AR^{2/3}\sqrt{-\partial_x b/C_m}) =& F \\ \text{or} \quad \partial_t (w_r h) + \partial_x \left(w_r h R(h)^{2/3}\sqrt{-\partial_x b/C_m}\right) =& F. \qquad (3) \end{aligned}$$

River Flow

- Kinematic model or nonlinear conservation law in h for $u > 0$ with $A = w_r h$:

$$\partial_t(w_r h) + \partial_x\left(w_r h R(h)^{2/3}\sqrt{-\partial_x b/C_m}\right) = 0$$

- Upwind information speed $dQ(A)/dA > 0$ for flux $Q(A) = Au$ and inflow $A(h(0,t))$:
- F: influx water from moor $x = L_v \approx 0.4, y = 0$

$$Q_m(h_m) = (1-\gamma_m) m_{por}\sigma_e w_v g\alpha \frac{1}{2}\partial_y(h^2) \qquad (4)$$

- F inflows: canal section 1 water at $x = L_{1c}$, inflow reservoir water at $x = L_{res}$.

Groundwater Flow

- Depth-averaged ground water model with level $h_m(y,t)$ from Barenblatt (2002) in Hele-Shaw cell of width w_v, e.g. $w_v = 0.01$m.
- Nonlinear diffusion equation for ground water level $h_m(y,t)$ for $y \in [0, L_y]$:

$$\partial_t(w_v h_m) - \alpha g \partial_y\left(w_v h \partial_y h\right) = \frac{w_v R}{m_{por}\sigma_e} \tag{5}$$

- Rainfall $R(y,t)$, via wider funnel, into Hele-Shaw cell
- Porosity $m_{por} \in [0.1, 0.3]$, fraction $\sigma_e \in [0.5, 1]$ pores filled with water
- $\alpha = k/(\nu m_{por}\sigma_e)$ with permeability $k = 10^{-12} m^2$ and viscosity $\nu = 10^{-6}\mathrm{m}^2/\mathrm{s}$

Groundwater Flow

- Nonlinear diffusion equation in ground water level $h_m(y,t)$ for $y \in [0, L_y]$:

$$\partial_t(w_v h_m) - \alpha g \partial_y\big(w_v h \partial_y h\big) = \frac{w_v R}{m_{por}\sigma_e} \tag{6}$$

- Boundary conditions: no flux/wall at $y = L_y$
- Time-dependent canal level $h_{2c}(t)$ at $y = 0$.
- In summary: Hele-Shaw cell represent sub-catchment to conceptualise/visualise ground water flow.

Canal 2

Canal with two sections; canal 2 of width w_c and depth $h_{2c}(t)$:

- section 2 for $x \in [0, L_{2c}]$, e.g. $L_{2c} = 0.9$m:

$$w_c L_{2c} \frac{\mathrm{d}h_{2c}}{\mathrm{d}t} = \gamma_m m_{por} w_v \alpha g \frac{1}{2} \partial_y (h^2) - Q_{2c} \tag{7}$$

- Canal 2: berm at $z = 0.06$m & bottom at $z = 0.0425$m
- Weir outflow into canal 1 based on Bernoulli & flow criticality at $x = L_{2c}$, i.e. for subcritical downstream flow

$$V_c = \sqrt{g h_c} \quad \text{and} \tag{8}$$

$$g h_{2c} + \frac{1}{2} V_{2c}^2 = g(h_c + P_{2w}) + \frac{1}{2} V_c^2 = \frac{3}{2} g h_c + g P_{2w}$$

$$h_c = \frac{2}{3}(h_{2c} - P_{2w}) \quad \text{s.t.} \tag{9}$$

$$Q_{2c} = h_c V_c = \sqrt{g} h_c^{3/2} = C_f \sqrt{g} max(h_{2c} - P_{2w}, 0)^{3/2}.$$

Canal 1

- Section 1 for $x \in [L_{1c}, L_{2c}]$, e.g. $L_{1c} = 1.0$m, width w_c and depth $h_{1c}(t)$:

$$
\begin{aligned}
w_c L_{1c} \frac{\mathrm{d}h_{1c}}{\mathrm{d}t} =& Q_{2c} - Q_{1c} \\
Q_{1c} =& h_c V_c = \sqrt{g} h_c^{3/2} = C_f \sqrt{g} max(h_{1c} - P_{1w}, 0)^{3/2}
\end{aligned}
$$

- Canal 1 has berm at $z = 0.032$m and its bottom at $z = 0.0145$m
- Weir at $x = L_{1c}$ when flow into river subcritical, i.e. sufficient drop from canal 1 to river level.

Reservoir with Weir

- Reservoir with length $L_{yy} = 0.4$m, width $w_{res} = 0.01$m, level $h_{res}(t)$:

$$\begin{aligned}\frac{\mathrm{d}h_{res}}{\mathrm{d}t} =& R_{res}(t) - Q_w/(L_{res} w_{res})\\ Q_w =& C_f \sqrt{g} w_{res} max(h_{res} - P_{wr}, 0)^{3/2}\end{aligned}$$

- Weir located at $x = L_{res} = 0.6$m, where water flows into the river.

Rainfall

Discretise rainfall events (risk):

- Rainfall is discretized in mm/day. In our Simulator, we discretise rainfall further into two sets of categories: location & rain amount.
- Rain amount has four categories $(1, 2, 4, 8)r_0$mm/s, gauged such that there is no flooding for $(1, 2)r_0$ rainfall, with limited flooding for $4r_0$ depending on preceding days/saturation, and flooding for $8r_0$.
- It rains or is dry uniformly over one Time Unit $-TU$.
- Rain location has four categories: rain in reservoir, moor & reservoir, moor, or no rain in the catchment.

Rainfall

- The probabilities for rain amount are "binomially" distributed physically using a half/stepped 8-pins Galton board: $\frac{3}{16}, \frac{7}{16}, \frac{5}{16}, \frac{1}{16}$ (risk).
- The probabilities for rain location are determined using another 8-pins Galton board as $\frac{3}{16}, \frac{7}{16}, \frac{5}{16}, \frac{1}{16}$.
- There are therefore five rainfall outcomes possible per Time Unit with probabilities: no rain at $\frac{1}{16}$
- The rain amount $r_0 TU$ per event will be determined by gauging such that there is no flooding with rain amounts $(1, 2)r_0 TU$, moderate flooding when $4r_0 TU$ and massive flooding with $8r_0 TU$.

Rainfall

- Table of rainfall amount/TU versus rain location:

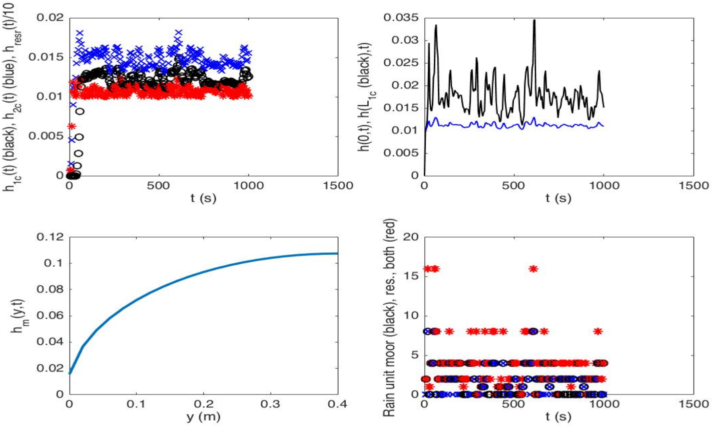
Table : Probability matrix times 256.

	r_0	$2r_0$	$4r_0$	$8r_0$
reservoir	9	21	15	3
both	21	49	35	7
moor	15	35	25	5
no rain	3	7	5	1

- Accumulated rainfall amounts (divided by Time Units) in catchment: $0 : 0.0625 = 1/16, r_0 : 0.0938 = 24/256, 2r_0 : 0.3008 = 77/256, 4r_0 : 0.3477 = 89/256, 8r_0 : 47/256 = 0.1836, 16r_0 : 0.0273$.

Sample Simulation

At $t = 0$: canals, moor and reservoir empty. $TU = 10s$, $r_0 = 1.75\mathrm{mm/s}$



Sample Simulation

Flood Demo

Onno Bokhove

Introduction

Is it going to rain more in the future?

Extreme precipitation and flooding events?

Can we predict heavy precipitation & floods?

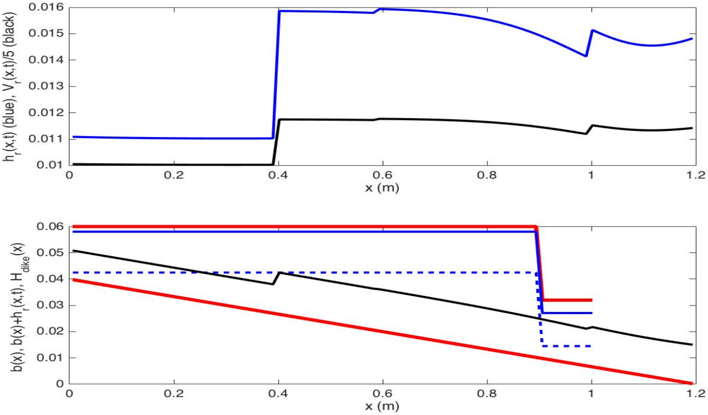
Can we mitigate flooding?

Conceptual flood demo?

Simulator

Discussion

At $t = 0$: canals, moor and reservoir empty. *Simulation*.



Flow Control

Control options adjustable parameters, to do's:

- Weir heights canal and reservoir: P_{1w}, P_{2w}, P_{wr}.
- Water distribution canal versus river via γ_w.
- Feedback loops and control on teh fly (Kerrigan et al.).

Data Assimilation

Parameter estimation and data assimilation, to do's:

- Measure river levels at $x = 0$ and $x = L_{1c}$.
- Measure rain fall at moor and reservoir.
- Estimate parameters Manning friction C_m and combined α.
- Use ensemble Kalman filtering methods . . . .

9. Discussion

- Physical construction Simulator 0.0: next week; routing of canals/river bed into foam.
- *Zeroth order 1D model* established; established some space and time scales.
- To do: control and data assimilation: measure river and canal levels & rainfall.
- More advanced: 2D shallow-water modelling, 3D Richard's equation for groundwater, ensemble Kalman DA, homogenisation of streets/sewers . . . (Firedrake).
- Add features: Hele-Shaw cells, city street grid, etc. to mathematical and physcial models.