Introduction to optimisation

Onno Bokhove

School of Mathematics, University of Leeds, UK
Room 8.04; E-mail: o.bokhove@leeds.ac.uk

Organisation

- See the handout online.
- Semester: 1; No. of credits: 10; Level: 2
- Prerequisites: MATH1010 or (MATH1050 & MATH1060) or (MATH1050 & MATH1331) or equivalent.
- All material will be available electronically via the MATH2640 Minerva pages.
- If you wish to have paper copies, then please email me, given our low-carbon use policy.

Introduction

- **What is optimisation**?
- Answer: Optimisation is about "the quest for the best", i.e. designing strategies for obtaining the best result. (Optimum is Latin for best.) In many cases of human endeavour we search for conditions under which we attain the optimum. So often, we are trying to find maxima (or minima).
- In economics, this can concern the highest profit, best rewards and maximum growth.
- In river or coastal flood-mitigation, this concerns mitigation or minimisation of flood damage, expressed in terms of staying below a specified water level at given locations, properties saved or economic damage saved, along a river or a coast.
- For wave-energy devices, this concerns maximising energy output given incoming waves.
- In real life, optimisation will include uncertainty, but we will not deal with the effects of uncertainty in this course.

Introduction

- In mathematics, optimisation is the subject of dealing with finding maxima or minima of functions.
- The definition of a function or functions: an expression or expressions in terms of variables $x, y, \ldots$ that can be altered, often also under additional conditons.
- Then we are tasked to find maxima or minima of the expression $f(x, y)$ (or expressions $f(x, y)$ and $g(x, y)$) subject to such additional conditions.
- Here the functions f, g can can describe the quantities we are interested in, such as profit, revenue, tax, energy output, flood damage, etc.
- We will (mainly) consider applications in economics.

Breakdown of the course

Chapter 1: Several variable calculus (~ 6 lectures)

- Develop techniques of multivariate functions, partial derivatives, total derivatives, gradients, directional derivatives, implicit differentiation, chain rule (in many variables), Taylor series, Hessian and stationary points.

Chapter 2: Unconstrained optimization (~ 4 lectures)

- Quadratic forms, eigenvalues, definiteness and principal minor test, stationary points, local extrema, applications to economics (Cobb-Douglas functions).

Chapter 3: Constrained optimization (~ 10 lectures)

- A) Equality constraints (conditions involving an equality $=$–symbol)
- B) Inequality constraints (conditions involving an inequality $<$–symbol)
- Lagrange multiplier formalism to find:
  - critical points and maximisers;
  - (bordered) Hessians to classify critical points;
  - Kuhn-Tucker theory; and,
  - applications to economics, etc.

Updates and feedback

- In 2018, the course structure was updated: e.g., 5 homework assignments.
- In 2019, some notes/assignment solutions will be typed out, with accompanying Python graphs.
- Every lecture, I will ask two volunteers to give me feedback immediately after the lecture (email/in person).
- There will be online questionnaires regarding comments and feedback, halfway and at the end of the course.
- Please feel free to ask questions.