Wetropolis extreme rainfall and flood demonstrator: from mathematical design to outreach and research

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Abstract. Wetropolis is a transportable "table-top" demonstration model with extreme rainfall and flooding events. It is a conceptual model with random rainfall, river flow, a flood plain, an upland reservoir, a porous moor, representing the upper catchment and visualising groundwater flow, and a city which can flood following extreme rainfall. Its aim is to let the viewer experience extreme rainfall and flood events in a life model on reduced spatial and temporal scales. In addition, it conveys concepts of flood storage and control, via manual intervention. To guide the building of an operational Wetropolis, we have explored its spatial and temporal dimensions first in a simplified mathematical design. We will explain this mathematical model in detail since it was a crucial step in Wetropolis' design. The key novelty is the supply of rainfall every Wetropolis day (unit wd), variable in terms of both the amount of rain and the rainfall location. Rain amount times rain location are determined daily as one of 16 possible outcome from two asymmetric Galton boards, in which steel balls fall down every wd, with the most extreme rainfall event involving 90% rainfall in both moor and reservoir which with a probability of circa 3% can cause severe floods in the city. This randomised rainfall has a Wetropolis' return period of $6{:}06\mathrm{min}$, short enough to wait for but sufficiently extreme or long to get slightly irritated as viewer. While Wetropolis should be experienced life, here we provide a photographic overview. To date Wetropolis has been showcased to over 200 flood victims at workshops and exhibitions on recent UK floods, as well as to flood practitioners and scientists at various workshops. To enhance Wetropolis' reach, we will analyse how both a general public and professionals interacted with Wetropolis. To conclude, we will discuss some ongoing design changes, including how people can experience natural flood management in a revised Wetropolis' design. Finally, we will also highlight how the Wetropolis-experience can stimulate new approaches in hydrological modelling, flood mitigation and control in science, education and water management.

1 Introduction

The Boxing Day flood of 2015 caused widespread damage in Yorkshire, UK, due to extreme flooding of the River Aire in and around Leeds and the River Calder in and around Todmorden, Mytholmroyd and Hebden Bridge. Thankfully no fatalities occurred but the economic damage was severe and estimated to be around £500M (West Yorkshire, 2016). November 2015 was the third wettest mouth on record qua precipitation and December 2015 was the wettest on record (Environment Agency, 2016). The soil was already saturated when the 48hrs of extreme rainfall leading up to the Boxing Day floods of 2015 started. After the Boxing Day and other recent floods, many flood victims asked why extreme flood events, seemingly occurring more and more often, were causing such havoc in their communities?

To provide some mathematical background on modelling, mitigation and statistics of extreme flood events, we were asked to disseminate scientific background on "risk in the age of extremes" at the citizens' conference "Science of floods" in Hebden Bridge (Science of floods, 2016). In the first one-and-half decade of the 2000's, Hebden Bridge has been hit by both summer flash-floods and winter floods leading to concerns amongst flood victims that their lives and properties were insufficiently protected. It led to further and intense discussions with environmental agencies on the need for more and different types of flood defences. Important questions in this discussion are the following:

- Is it going to rain more in the future in the UK?
- Can we define extreme precipitation and flooding events?
- How (well) can we predict heavy precipitation and floods?
- Finally, how can we elucidate these questions, their answers and uncertainties, in an interactive table-top demonstration?

We will discuss some answers to these questions in turn.

Is it going to rain more in the future in the UK? Both the IPCC report (ICCP, 2013) and Sanderson's UKCP09 report of the Met Office (Sanderson, 2010) show that there is no increase of significance in average annual rainfall foreseen in climate projections, not across the globe on average and also not in the UK. There are geographical and seasonal variations foreseen: winter rainfall will generally increase and summer rainfall will thus decrease but with more intense downpours in the summer.

Can we define extreme precipitation and flooding events? Extreme events tend to be expressed as the chance that an extreme event occurs on one day in a year. If that chance is 1%, for example, then we can also say that that extreme event has a return period of $1:100$ years. Flooding events can be classified in terms of such return periods as $1:10, 1:20, 1:50$ and $1:100$ or $1:200$ year events with the latter two considered to be extreme. The uncertainty in an event with a $1:100$ year return period will be larger than one with a $1:20$ year return period. Given a record of data over a finite time, since more extreme events will occur less often in such a finite-time data set their uncertainty is larger, because there are fewer or no observations to establish the tail of the distribution. Events with return periods longer than the data record cannot be classified directly from that data record. Extreme-value theory comes here to the rescue. By assuming a suitable probability distribution function (pdf) one can use the data to fit the parameters of the pdf and subsequently generate data in the tail for events with return periods beyond the length of the data record. Extreme events have higher tails than classical pdfs, such as (half a) Gaussian distribution or a Gamma distribution, the latter pdfs which are often used to described rainfall pdfs. We can, however, combine a Gamma distribution, with its two fitting parameters, and a Generalised Pareto Distribution (GPD), also with two fitting parameters, and suitable for modelling the tail of the pdf, using yet two other fitting parameters to smoothly merge these Gamma and GPD distribution together, cf. Wong et al. (2014). Alternatively, one can only model the tail of the distribution, say with the GPD, given a sufficient amount of data in a rainfall or river level record to establish that tail.

How (well) can we predict heavy precipitation and floods? The Boxing Day floods of 2015 were caused by large-scale winter rainfall in which the 48 hours of consecutive rainfall were the wettest on record. In Bradford a record 69.4mm and in Bingley a record 93.6mm of rainfall were measured over 48 hours. It resulted in the flooding of the River Aire with river-level records reached in Leeds and elsewhere along the river. In Armley, Leeds, the gauge station measured a maximum river level of 5.21m while the previous electronic record was 4.03m in the Autumn of 2000 (Environment Agency, 2016). The river level during the 1866 flood was roughly around 4.5m, cf. Bokhove et al. (2018a) (their Fig. 2). Both rainfall and river levels were by and large well-predicted by the UK Met Office via Numerical Weather Prediction and by the Environment Agency (cf. a presentation by an EA–Yorkshire leader in Leeds). Predictions are generally quite good for large-scale winter rainfall and the resultant changes in river levels. Downpours, e.g. in the summer, tend to be more localised and are therefore much more difficult to predict in terms of both location and intensity. The same holds for resulting flashfloods and down-pour induced surface-water flooding. Hence, simply put, fluvial or river flooding in winter tends to be easier to predict than pluvial or surface-water flooding events in the summer.

Finally, how can we elucidate these questions, their answers and uncertainties, in an interactive table-top demonstration? The expression of extreme events in terms of return periods is often misunderstood, also by the public. The Boxing Day flood of 2015 was classified as an event with a $1:200^+$ year return period –including the unclear meaning of the plus sign in 200^+ (Environment Agency, 2016). That does –of course– not mean that it has to take another 200^+ years, so till after 2215, before the next Boxing-Day-type flood might occur in Leeds. It does, however, mean that the average time between events of similar magnitude will be 200^+ years, given a sufficiently long record of "stationary" statistical data. To let people experience such an extreme event in a table-top set-up they can of course not be asked to wait for 200 years on average, so our design for a flood demonstrator with rainfall must be scaled down both in size and duration. Miniature river flooding has been demonstrated in small-scale experiments (e.g., as in the Lego model of Pampaloni et al. (2018)) but these all tend to involve deterministically imposed extreme water input –with water inflow supplied and adjusted deterministically and/or manually. The key novelty in our design does lie in the way random rainfall is supplied to our table-top hydrodynamic set-up for both river and groundwater flow. We have modified a classical and symmetric Galton-board set-up, inspired by such a set-up used at Leeds' School of Mathematics open days. A typical Galton board has a tilted surface in which a (steel) ball falls down under gravity and encounters a series of symmetric pins or channel corners, each determining with a p and $(1-p)$ chance whether the ball continues or falls to the left or to the right. The design is usually such that $p \approx 1/2$ but small variations can occur in practice and after a series of n splittings a binomial distribution arises, given a sufficiently large number of trials. Moreover, for $n \to \infty$, a Gaussian distribution emerges. Such a Galton-board device is often used to visualise and demonstrate statistical distributions in real time during open days. To obtain an asymmetric discrete distribution with a discrete tail representing relatively-extreme events, the standard symmetric Galton board described above was modified as follows. For $p = 1/2$ and $n = 1$, the first and only split leads to a $(1,1)/2$–distribution. The first split of the second row for $n = 2$ is now eliminated while the second split is not, leading to a $(3,1)/4$–distribution. Continuing to the third row of splittings as usual, for $n = 3$, we obtain a $(3,4,1)/8$–

distribution. The last and fourth row for $n = 4$ yields the final $(3,7,5,1)/16$–distribution. An image of such a asymmetric Galton board is given in Fig. 1. Two of these Galton boards will be used to supply rain to our table-top river and ground-water flow model, one concerning the duration and amount $(0.1, 0.2, 0.4, 0.9)r_0$ of rainfall during a Wetropolis Day, with its unit wd, and another one concerning the location of the rainfall. Rain duration will be either $(10, 20, 40, 90)\%$ of the amount r_0 of rainfall on a Wetropolis Day and rain location will be either rainfall (i) in a reservoir with generally instant run-off into the river; (ii) in both a reservoir and a moor; (iii) in the moor with groundwater flow and its nonlinear, delayed release of water into the river; and, (iv) no rain, in the Wetropolis' catchment. Both duration and location are determined by the outcome of one trial through two Galton boards on each Wetropolis day, together yielding a 4 by 4–matrix with 16 possible outcomes, with the no-rain case having a-for-the-UK-rare chance of $1/16^{\mathrm{th}}$ comprised by four of those 16 outcomes. By design, an extreme event occurs when it rains 90% in both locations (i.e. in the reservoir and moor) with a chance of $7/256 \approx 0.0273 = 2.73\%$, which in our construction will by design lead to flooding of a city further downstream along a (winding) river in the set-up. A Boxing-Day-type event with two consecutive days of extreme 90%–rainfall then has a chance of $49/(256^2) \approx 0.000748 = 0.0075\%$. The next and crucial step in the design was to identify and determine the various unknowns in order to assess whether a feasible design was possible –at all.

Given a (winding) river of length L and curvilinear coordinate x along this river, these remaining key unknowns are as follows:

- the influx discharge Q_0 at the unstream boundary at $x = 0$,
- the locations x_{res} and x_m where the reservoir and moor enter into the river, with a section further downstream along the river comprising a city plain that is prone to (extreme) flooding,
- the rainfall amount or speed r_0, determining the strength of the pumps required and whether the pumping rates can be realistic at all,
- the length of a Wetropolis Day wd, in relation to the extreme rainfall and corresponding extreme flooding event, such that the viewer experiences some irritation in having to wait for a randomly-induced extreme event but on average will experience such an extreme event within a reasonable time, i.e. on average within a handful of minutes.

We chose x_{res}, x_m, Q_0 a priori and determined wd and r_0 by simulation of a simplified mathematical model. A plan view of a sketch of Wetropolis is given in Fig. 2.

Given the above introduction, our paper has the following outline. The above unknowns were determined mathematically by a simplified mathematical model before any design and construction of the table-top set-up were undertaken. This mathematical and numerical modelling is explained and used to determine the design unknowns in §2. The resulting table-top design of the Wetropolis flood demonstrator is disseminated in §3. Our experience in demonstrating Wetropolis to the general public and to flood practitioners is summarised in §4, including the a-priori surprising outcome that professionals in flood prediction and mitigation have also been inspired by Wetropolis, despite our original public-outreach aims. A discussion is found in §5, in which we also outline some directions for further scientific research, as gathered from such public-engagement sessions.

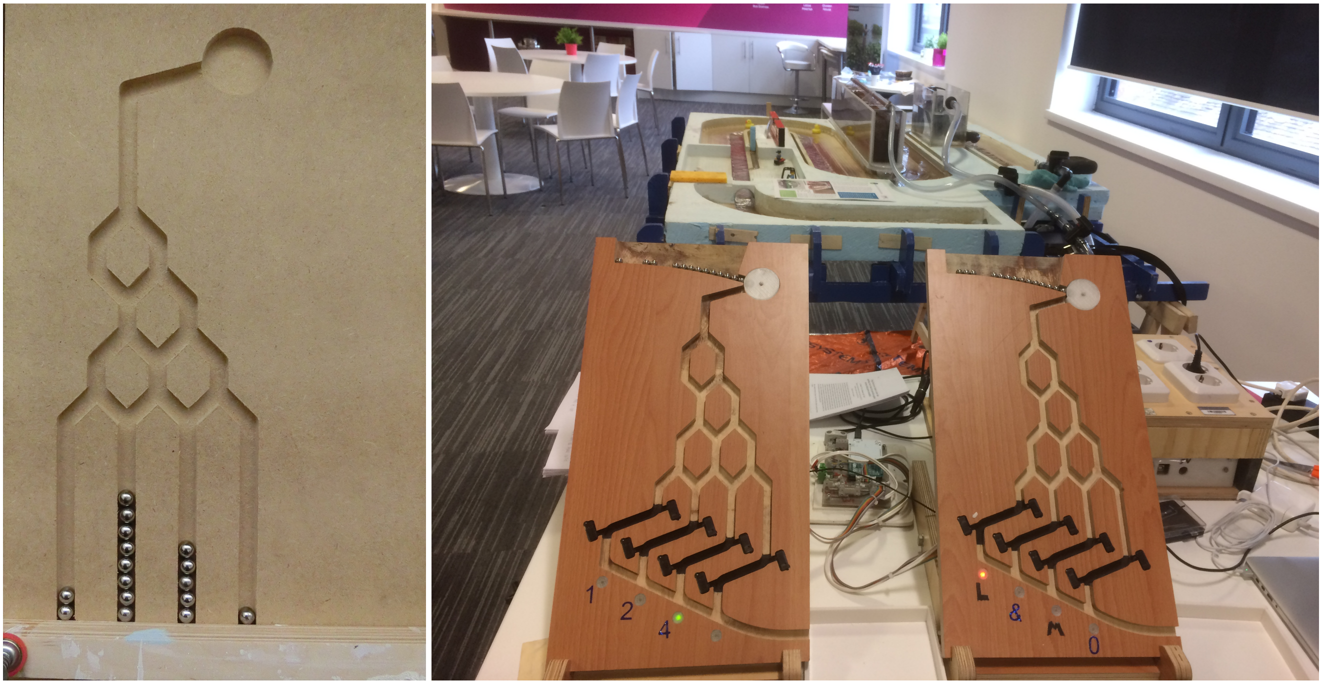

Figure 1. Photographs of asymmetric Galton boards. Test board on the left and a final set-up on the right. At every split the chance of a steel ball falling to the left or right is 50% for a well-balanced Galton board. When a sufficiently large number of steel balls falls through this Galton board, the discrete distribution becomes $(3,7,5,1)/16$. The 4×4 possible outcomes in two of such boards, registered in each by four electronic eyes (located in the black-painted areas along $2\times4=8$ channels), determine both the rainfall and its location(s) in Wetropolis. The outcome of the random draw shown by the lit-up lights will lead to $4\mathrm{s}$ of rain in the reservoir/lake. Photos: OB & WZ.

2 Mathematical design

We will construct a mathematical model of Wetropolis next. It will consist of random rainfall and space-time continuous hydraulic modelling of the interconnected river flow, reservoir- and canal-level changes as well as groundwater flow in the moor. Subsequently, we will establish a numerical discretisation of this space-time continuous model and use numerical simulations to determine the a-priori unknown parameters of rainfall amount r_0 and the length wd of the Wetropolis day. Meanwhile other parameters will be chosen based on reasonable guesses, in order to obtain a desirable size of the experimental set-up. The actual numerical discretisation has been relagated to an Appendix.

2.1 Statistical modelling of randomised rainfall

As discussed, rainfall is modelled statistically as risk by using the outcome of draws from two Galtonboards. In the mathematical model these outcomes are simulated, while in the physical flood demonstrator we have either used two actual Galtonboards with two steel balls or one Galtonboard with one steel ball running through two consecutive Galton-board channels. We have

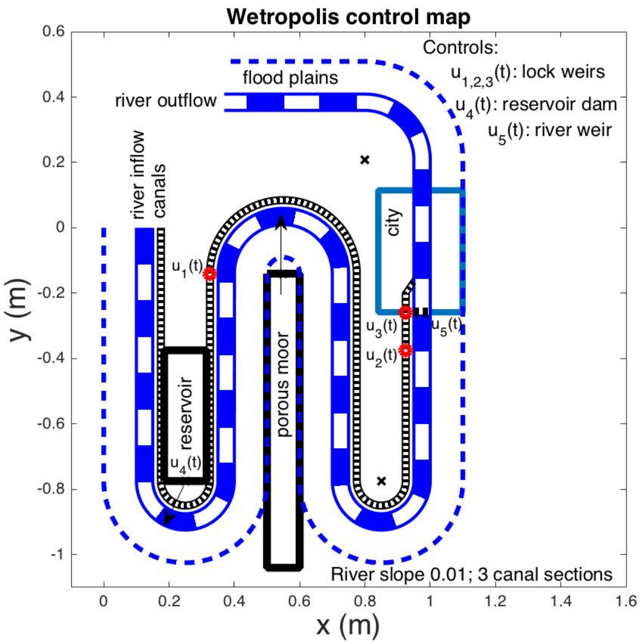

Figure 2. Plan view of the Wetropolis table-top experiment. The main river channel is indicated in white-blue blocks and its one-sided flood plain extent by a dashed line. A "Leeds-Liverpool" canal with lock-weirs flanks the $1:100$–sloped river, which has constant upstream inflow and gets fed by water from a reservoir as well as a porous moor filled with lava grains. In both locations, it can rain intermittently and randomly. Outflow is at the end of the river channel, after a city plain that can flood and where the canal flows into the river. Water falling in a full reservoir flows instantly with a manually adjustable fraction of $0 < \gamma \leq 1$ into the canal and the river, the latter with a fraction $(1-\gamma)$. The reservoir level can also be adjusted manually, which provides some flood control. This control can be used to demonstate the role of a holding reservoir to lessen flooding in cases of extreme rainfall.

discretised rainfall into two categories: location and rain amount, per Wetropolis day wd. Rain location has four outcomes: rain in reservoir, moor and reservoir, moor, or no rain in the catchment with a discrete distribution of $(3,7,5,1)/16$. Independently, rain amount has per location four outcomes $(1,2,4,9)r_0/\mathrm{wd}$ with again the discrete distribution $(3,7,5,1)/16$, in which r_0 will be gauged such that there is no flooding for $(1,2,4)r_0/\mathrm{wd}$ rainfall, with potentially limited flooding for $(8,9)r_0/\mathrm{wd}$ and generally major flooding in the city plain for $18r_0/\mathrm{wd}$ rain falling in both moor and reservoir. Hence, there are $4\times 4 = 16$ outcomes determined as a direct product of these two independent distributions, given in Table 1. The resulting, accumulated distribution of the rainfall per wd, in the reservoir and/or moor combined, therefore becomes: $0 : 0.0625 = 1/16, r_0 : 0.0938 = 24/256, 2r_0 : 0.3008 = 77/256, 4r_0 : 0.3477 = 89/256, 8r_0 : 35/256 = 0.1367, 9r_0 : 8/256 = 0.0312$ and the extreme case of $18r_0 : 7/256 = 0.0273$. A pdf of this discrete distribution for a computer trial over $500\mathrm{wd}$ is shown in Fig. 3. What the suitable values for r_0 and wd are, will have to established by further modelling, described next.

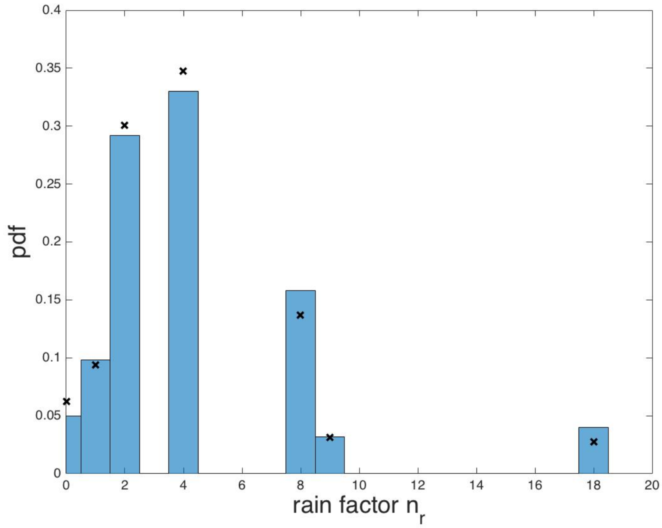

Figure 3. The pdf of random rainfall outcomes over 500wd's, displayed as a scaled histogram, is compared with the theoretical discrete pdf, denoted by the black crosses. Extreme cases with $18r_0$ rainfall are observed in moor and reservoir combined, here in this 500–day trial with occurrences just above the average (black cross) expected.

2.2 Mathematical modelling in space-time

Wetropolis contains a river channel with a one-sided flood plain, a groundwater moor, a reservoir, and canals with three segments seperated by lock-weirs. The canal flows into the river in the city plain which lies at the downstream end of the set-up. We refer to Fig. 2 for a plan view locating these elements in the actual table-top experiments. In the design model, the reservoir and moor were swapped and we used a shorter length river channel. Simplified mathematical submodels of these different elements will be provided next in isolation before they will be interconnected into one complete mathematical model by using reasonable boundary and interface conditions.

River dynamics: River flow is often modelled as one-dimensional flow in a channel with a cross-section $A = A(x,t)$ as function of space, with a horizontal curvilinear coordinate $x \in [0,L]$ along a winding river channel of length L, and time t. Both $A(x,t) = A(h;b(x))$ and the in-situ waterdepth $h = h(x,t)$, above a fixed river bottom $b = b(x)$, and the mean flow

Table 1. Probability matrix of the 16 outcomes of rainfall times 256 with underlined the extreme case of $7/256$.

location	r_0	$2r_0$	$4r_0$	$9r_0$
reservoir	9	21	15	3
both	21	49	35	$\underline{7}$
moor	15	35	25	5
no rain	3	7	5	1

velocity $u(x,t)$ are all averaged over the cross section of the river. E.g., for a rectangular river channel we have $A = w_r h$ with fixed width w_r but in general $A(h;x)$ can depend in a complicated fashion on the river depth h and directly on x –the latter via $b = b(x)$. The governing equations are the Saint-Venant equations (e.g., Bates et al (2010)) consisting of continuity and momentum equations, augmented with source terms at the locations x_m and x_{res} along the river where water from the moor and reservoir enters the river, as well as a parameterization of the channel friction, i.e.

$$\partial_t A + \partial_x(Au) = Q \equiv Q_m\delta(x - x_m) + Q_{res}\delta(x - x_{res}) \tag{1a}$$

$$\underline{\partial_t u + u\partial_x u + g\partial_x h} = -g(\partial_x b + C_m^2 u|u|/R^{4/3}) \tag{1b}$$

with the hydraulic radius $R(h) = \frac{\text{wet area}}{\text{wetted perimeter}}$ (in m), the phenomenological Manning friction coefficient $C_m \in [0.01, 0.15]\mathrm{m}^{1/6}$, cf. Munro et al. (2005), and the discharge rates Q_m and Q_{res} of the water flow from the moor and reservoir into the river. These discharge rates are modelled as point sources using delta functions $\delta(x - x_m)$ and $\delta(x - x_{res})$. In reality the same inflow rates will occur along finite-length, short strips along the river, centred around x_m and x_{res}. The two unknown fields are A and u with $h = h(A;x)$ an often implicit relation at every location x. For the above example with a river channel of rectangular cross-section, we find $R(h) = \frac{w_r h}{2h + w_r}$ and $h = A/w_r$. Initial conditions $A(x,0)$ and $u(x,0)$ have to be imposed at $t = 0$ as well as boundary conditions $A(0,t), u(0,t)$ and $A(L,t), u(L,t)$ at $x = 0$ and $x = L$. These latter boundary conditions are used (partially) according to the way the charactistics at $x = 0, L$ of the hyperbolic equations (1) determine whether the boundary data are flowing into the domain or not, cf. Toro (2001).

Even though the actual winding river channel with its one-sided flood plain and city plain has a varying cross-section, for the design calculations we made the simplification to model only a rectangular river channel. In addition, a zeroth-order kinematic model approximation to the St. Venant equations (1) has been used for the limiting case with positive velocity $u > 0$ and a constant slope $-\partial b/\partial x > 0$ of the river channel. To zeroth-order, we assume that the bed slope and friction are in balance such that we obtain

$$u = R(h)^{2/3}\sqrt{-\partial_x b}/C_m, \tag{2}$$

which is a classical approximation used in hydraulics (Munro et al., 2005), such that the river flow is modelled by a kinematic or scalar hyperbolic equations in A, as follows

$$\partial_t A + \partial_x(AR^{2/3}\sqrt{-\partial_x b}/C_m) \equiv \partial_t A + \partial_x Q_f(A) = Q_m\delta(x - x_m) + Q_{res}\delta(x - x_{res}), \tag{3}$$

with an upwind information speed $dQ_f(A)/dA > 0$ for flux $Q(A) = Au$ and inflow $A(h(0,t))$. Note that the flux $Q_f = Q_f(A)$ is a(n implicit) function of A since $h = h(A)$. For $u > 0$ with $A = w_r h$, it is a kinematic model or nonlinear, scalar conservation law in the waterdepth h given by

$$\partial_t(w_r h) + \partial_x\left(w_r h R(h)^{2/3}\sqrt{-\partial_x b}/C_m\right) \equiv \partial_t(w_r h) + \partial_x Q_f(h) = Q_m\delta(x - x_m) + Q_{res}\delta(x - x_{res}) \tag{4}$$

with the flux $Q_f = Q_f(h)$ rewritten in terms of h, initial condition $h(x,0)$ and upstream influx $Q_0(0,t) = w_r h(0,t)u(0,t)$ defining $h(0,t)$ since u is expressed in terms of h through (2). The Saint-Venant equations are more advanced than the above kinematic model and allow both sub- and supercritical flows. An interim and better model arises when we instead use the following balance

$$u = R(h)^{2/3}\sqrt{-\partial_x(b+h)}/C_m, \tag{5}$$

which after substitution into the continuity equation (1a) yields an advection-diffusion equation, cf. Bates et al (2010). Both these more advanced models are not required for the design estimates but when one wishes to perform accurate predictions of the hydrodynamics in Wetropolis then such advanced models may be required.

Groundwater flow: Groundwater levels after rainfall are made visible in a transparant and elongated rectangular box filled to a high level with porous small lava rocks. The box is open at the top and one side, and has walls at the remaining four sides. Rain falls uniformly along this box via a copper pipe with a series of holes. The groundwater dynamics is modelled to zeroth order by assuming that the surface of the grains is flat, the rainfall uniform per surface area, that there is no surface run-off and that the fallen rainwater infiltrates sufficiently fast to contribute instantly to the groundwater level $h_m(y,t)$ with coordinate y in a different direction, locally orthogonal to x at the location x_m where the groundwater flows into the river. A depth-averaged groundwater model with level $h_m(y,t)$ from barenblatt (1996) is used in a cell of width w_v and length L_y, e.g. $w_v = 0.095\mathrm{m}$ and coordinate $y \in [0, L_y = 0.932\mathrm{m}]$. The nonlinear diffusion equation for the groundwater level $h_m(y,t)$, taken to be uniform in the lateral direction, is

$$\partial_t(w_v h_m) - \alpha g \partial_y\big(w_v h \partial_y h\big) = \frac{w_v R_m(t)}{m_{por}\sigma_e} \tag{6}$$

with moor rainfall $R_m(y,t) = R_m(t)$, here taken to be uniform in y, porosity $m_{por} \in [0.1, 0.3]$, the fraction $\sigma_e \in [0.5, 1]$ of pores filled with water, $\alpha = k/(\nu m_{por}\sigma_e)$ with permeability $k = 10^{-8}m^2$ and viscosity $\nu = 10^{-6}\mathrm{m}^2/\mathrm{s}$. The boundary conditions are no flux through the wall at $y = L_y$ such that $\partial_y h|_{y=L_y} = 0$, while at $y = 0$ the moor is held at the level $h_3(t)$ of canal–3, the upsteam branch of the canal running in parallel to the river, i.e., this a time-dependent Dirichlet condition $h_m(0,t) = h_3(t)$. The mass flux of moor water running in the river occurs at $x = x_m = 2.038\mathrm{m}$ is

$$Q_m(t) = (1-\gamma)Q_{tm} \equiv (1-\gamma)\frac{1}{2}m_{por}\sigma_e w_v \alpha g(\partial_y h_m)^2|_{y=0} \tag{7}$$

with Q_{tm} the mass flux following from integration of (6) over the domain $y \in [0, L_y]$. The reason to multiply by $m_{por}\sigma_e$ is that the water volume in the matrix of particles in the moor comes free into free space, which has porosity unity.

Reservoir: The reservoir is a rectangular box of dimensions $h_{res} \times w_{res} \times L_{yy}$ with time-dependent water level $h_{res}(t)$, e.g. $L_{res} = 0.293\mathrm{m}$ and $w_{res} = 0.123\mathrm{m}$. The random rainfall enters either via a pipe or a long pipe with numerous holes visualising the rainfall and it leaves the reservoir via an overflow pipe, here modelled by a straight weir, cf. Munro et al. (2005). Overflow of the reservoir once it is overfilled is not modelled. The reservoir-level dynamics is governed by

$$w_{res}L_{res}\frac{\mathrm{d}h_{res}}{\mathrm{d}t} = w_{res}L_{res}R_{res}(t) - Q_{res} \quad \text{with} \quad Q_{res} = C_f\sqrt{g}\,w_{res}\max(h_{res} - P_{wr}, 0)^{3/2}, \tag{8}$$

in which $L_{res}w_{res}$ is the area of the reservoir such that $L_{res}w_{res}h_{res}$ is its time-dependent volume, P_{wr} is the overflow height of the weir, $R_{res}(t)$ the reservoir rainfall and Q_{res} the flux down into the river. Note that the coefficient C_f is dimensionless.

The weir is located at $x = x_{res} = 0.925\text{m}$, where water flows into the river, cf. the delta function in the continuity equation (1a) of the river.

Canal sections: One canal of uniform width w_c runs alongside the river, cf. the Leeds-Liverpool canal and the River Aire in Yorkshire, in which the three canal sections are separated by weirs and time-dependent depths $h_{1c}(t), h_{2c}(t), h_{3c}(t)$. We are thus ignoring currents and height changes along the canal sections. Each canal section has a certain depth and is separated from the river by a berm. Canal–3 is the highest and is blocked off on one end, at $x = 0$, and has a weir located at $x = L_{3c} = 1.724\text{m}$. Its level is modelled as the variation of its volume due to partial inflow from the moor and outflow of water via a weir in canal–2

$$w_c L_{3c} \frac{\mathrm{d}h_{3c}}{\mathrm{d}t} = \gamma Q_{tm} - Q_{c3} \quad \text{with} \quad Q_{3c} = C_f \sqrt{g}\, w_c \max(h_{3c} - P_{3w}, 0)^{3/2} \tag{9}$$

with weir height P_{3w}. Canal–2 resides from $x \in [L_{3c}, L_{2c}]$ with $x = L_{2c} = 3.608\text{m}$ and is modelled likewise but with inflow Q_{3c} from canal–3 and outflow Q_{2c} into canal–1:

$$w_c(L_{2c} - L_{3c}) \frac{\mathrm{d}h_{2c}}{\mathrm{d}t} = \gamma_m m_{por} w_v \alpha g \frac{1}{2} \partial_y (h^2)|_{y=0} - Q_{2c} \quad \text{with} \quad Q_{2c} = C_f \sqrt{g}\, w_c \max(h_{2c} - P_{2w}, 0)^{3/2}. \tag{10}$$

The section of canal–1 runs from $x \in [L_{2c}, L_{1c}]$ with $L_{1c} = 3.858\text{m}$, width w_c and depth $h_{1c}(t)$. It is modelled in the same manner with inflow from canal–2 and outflow Q_{1c} into the river, as follows

$$w_c(L_{1c} - L_{2c}) \frac{\mathrm{d}h_{1c}}{\mathrm{d}t} = Q_{2c} - Q_{1c} \qquad \text{with} \qquad Q_{1c} = h_c V_c = \sqrt{g} h_c^{3/2} = C_f \sqrt{g}\, w_c \max(h_{1c} - P_{1w}, 0)^{3/2}. \tag{11}$$

The weir at $x = L_{1c}$ where water flow into the river is assumed to be subcritical, i.e. we assume there is a sufficient drop from canal–1 to the river level. In terms of height levels, canal–3 has a berm at $z = 0.06\text{m}$ and its bottom resides at $z = 0.04\text{m}$; canal–2 has a berm at $z = 0.04\text{m}$ and its bottom resides at $z = 0.02\text{m}$, and canal–1 has a berm at $z = 0.021\text{m}$ and its bottom resides at $z = 0.001\text{m}$. To wit, the outflow at the two weirs into canal–2 and canal–1 is based on Bernoulli's relation and flow criticality, cf. Munro et al. (2005). At $x = L_{3c}$, e.g., for subcritical flow with flow depth h_{2c} and flow speeds $V_{2c} \approx 0$ upstream as well as critical flow V_c of height h_c over the weir, we therefore derive the following

$$\begin{aligned}
V_c &= \sqrt{g h_c} \quad \text{and} \quad g h_{2c} + \frac{1}{2} V_{2c}^2 = g(h_c + P_{2w}) + \frac{1}{2} V_c^2 = \frac{3}{2} g h_c + g P_{2w} \\
V_{2c} &\approx 0 \quad \text{s.t.} \quad h_c = (2/3)(h_{2c} - P_{2w}) \qquad \text{and therefore:} \\
Q_{2c} &= w_c h_c V_c = w_c \sqrt{g} h_c^{3/2} = C_f \sqrt{g} w_c \max(h_{2c} - P_{2w}, 0)^{3/2}
\end{aligned} \tag{12}$$

with $C_f = (2/3)^{3/2}$. Similar derivations with suitable adaptations of the quantities involved determine the fluxes Q_{1c}, Q_{3c}, Q_{res} over the other weirs.

When all of the above models for the individual components are combined we obtain the entire model, including its initial and boundary conditions, for the unknowns $h_(x,t), h_m(y,t), h_{res}(t), h_{1c}(t), h_{2c}(t)$ and $h_{3c}(t)$:

$$\partial_t(w_r h) + \partial_x\left(w_r h R(h)^{2/3}\sqrt{-\partial_x b}/C_m\right) = Q_m\delta(x - x_m) + Q_{res}\delta(x - x_{res})$$
$$\text{on} \quad x \in [0, L] \quad \text{with} \quad Q_f = w_r h R(h)^{2/3}\sqrt{-\partial_x b}/C_m|_{x=0} = Q_0(t), \quad h(x,0) = h_0(x) \tag{13a}$$

$$\partial_t(w_v h_m) - \alpha g \partial_y\left(w_v h \partial_y h\right) = \frac{w_v R_m(t)}{m_{por}\sigma_e}$$
$$\text{on} \quad y \in [0, L_y] \quad \text{with} \quad \partial_t h_m|_{y=L_y} = 0, h_m(0,t) = h_{3c}(t), h_m(y,0) = h_{m0}(y) \tag{13b}$$

$$w_{res} L_{res}\frac{\mathrm{d}h_{res}}{\mathrm{d}t} = w_{res} L_{res} R_{res}(t) - Q_{res} \quad \text{with} \quad h_{res}(0) = h_{r0} \tag{13c}$$

$$w_c(L_{1c} - L_{2c})\frac{\mathrm{d}h_{1c}}{\mathrm{d}t} = Q_{2c} - Q_{1c} \quad \text{with} \quad h_{1c}(0) = h_{10} \tag{13d}$$

$$w_c(L_{2c} - L_{3c})\frac{\mathrm{d}h_{2c}}{\mathrm{d}t} = \gamma_m m_{por} w_v \alpha g \frac{1}{2}\partial_y(h^2)|_{y=0} - Q_{2c} \quad \text{with} \quad h_{2c}(0) = h_{20} \tag{13e}$$

$$w_c L_{3c}\frac{\mathrm{d}h_{3c}}{\mathrm{d}t} = \gamma Q_{tm} - Q_{3c} \quad \text{with} \quad h_{3c}(0) = h_{30}, \tag{13f}$$

$$Q_{1c} = C_f\sqrt{g}\, w_c \max(h_{1c} - P_{1w}, 0)^{3/2} \quad \text{and} \quad Q_{2c} = C_f\sqrt{g}\, w_c \max(h_{2c} - P_{2w}, 0)^{3/2} \tag{13g}$$

$$Q_{3c} = C_f\sqrt{g}\, w_c \max(h_{3c} - P_{3w}, 0)^{3/2} \quad \text{and} \quad Q_m = (1-\gamma)Q_{tm} \equiv (1-\gamma)\frac{1}{2} m_{por}\sigma_e W_v \alpha g(\partial_y h_m)^2|_{y=0} \tag{13h}$$

$$Q_{res} = C_f\sqrt{g}\, w_{res} \max(h_{res} - P_{wr}, 0)^{3/2} \quad \text{and} \quad R(h) = w_r h/(2h + w_r), \tag{13i}$$

with time-dependent rainfall functions $R_{res}(t), R_m(t)$ and upstream inflow $Q_0(t)$. The remanining parameters are constants, which units and typical values used listed in Table 2. The rainfall functions are defined such that in the absence of other effects, e.g. unit porosity in the moor, they directly lead to a linear increase of the moor's ground water level and the reservoir depth. A space-time numerical discretisation of (13) is given in Appendix A. It involves a second-order finite-difference approximation of the ground water equation (13b) in y, a first-order finite-volume discretisation of the river equation (13a) in x, and straightfoward first-order forward-Euler time discretisations of the time derivatives involved.

The rainfall functions are constant during every Wetropolis day and generally vary from Wetropolis day-to-day. One a given Wetropolis day,

$$R_{res}(t) = n_r n_{res} r_0 \qquad \text{and} \qquad R_m(t) = n_r n_{moor} r_0, \tag{14}$$

in which $n_r = 1, 2, 4, 9$ is drawn daily with probablity $(3, 7, 5, 1)/16$ via one Galton board, while one of the combinatiions

$$(n_{res}, n_{moor}) = (1,0), (1,1), (0,1), (0,0)$$

is drawn daily with probablity $(3, 7, 5, 1)/16$ via the other Galton board, as explained in §2.1. The rainfall speed r_0 will be determined by trail-and-error and has the units of $\partial_t h_m$, i.e. $\mathrm{m/s}$. Hence, the volumetric rate of rainfall per Wetropolis day on the moor for unit $n_{rain} = 1$ can then be calculated, yielding

$$V_{rate} = L_y w_v r_0 \mathrm{wd}. \tag{15}$$

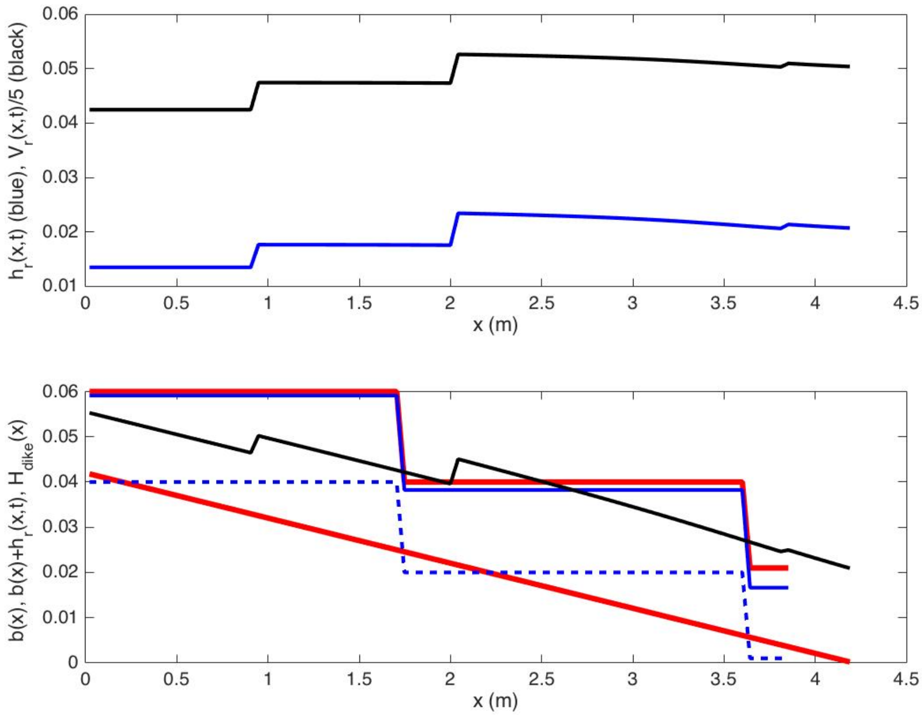

Figure 4. A two-panel figure in which: (i) the top panel displayes the river level $h_r(x,t)$ of the river (black) and the river velocity $V_r(x,t)$ as function of the along-river coordinate x at $t = 5000$s; and, the bottom compound panel contains the bottom of the bed (in red and it is fixed), the top of the berm or dike along river/canals in red (fixed); in dashed blue the bottom of the set of canals; in solid blue the canal levels; in black is the dynamic river level indicated above the bed; all as function of x at time $t = 5000$s. When the black line/river level lies above the red lines/berms there is flooding, because at $t = 5000$s the water level is seen to be high, cf. 5 bottom-left panel. The black line is seen to have three jumps at $x = x_{res} = 0.925\text{m}$, $x = x_m = 2.038\text{m}$ and a small one at $x = L_{c1} = 3.858\text{m}$ where water comes in from the reservoir, moor and canal. At $x = 0$ there is constant water influx. Flooding is just observed: in this simplified design model there is no actual water going out the river.

2.3 Numerical results

Given the choice of parameter values with (or near) the values given in Table 2, the goal is to determine a suitable rainfall speed r_0 and length of the Wetropolis day wd via trail-and-error through numerical simulation. As initial conditions we take $h(x,0) = 0.0135\text{m}$, $h_m(y,0) = 0$, zero canal and reservoir levels $h_{10} = h_{20} = h_{30} = h_{r0} = 0$, and an upstream influx of river water corresponding to the mass flux $Q_0 = Q_f\,(h(0,0))$ associated with $h(0,0)$. Rainfall will be varied dialy by changing the action of two pumps, which require a fraction of a second to change gear. For someone viewing Wetropolis, a length of day between 5s and 20s seems reasonable so we choose $\text{wd} = 10\text{s}$ as a first guess. In the design phase, values of r_0 have been

Table 2. Parameters: units and values used. Note that $\alpha = k/(m_{por}\nu\sigma_e)$. $\gamma \in [0,1]$.

Parameter	Units	Value	Parameter	Units	Value
g	$\mathrm{m/s^2}$	9.81	P_{1w}	m	0.01
L	m	4.211	P_{2w}	m	0.0125
C_f	-	$(2/3)^{3/2}$	P_{3w}	m	0.0125
C_m	$\mathrm{m}^{1/6}$	0.02	L_{1c}	m	3.858
$\mathrm{d}b/\mathrm{d}x$	-	0.01	L_{2c}	m	3.608
w_r	m	0.05	L_{3c}	m	1.724
w_v	m	0.095	x_{res}	m	0.932
L_y	m	0.925	w_{res}	m	0.123
m_{por}	-	0.3	L_{res}	m	0.293
σ_e	-	0.8	P_{wr}	m	0.1
k	m^2	10^{-8}	x_m	m	2.038
ν	$\mathrm{m^2/s}$	10^{-6}	γ	-	0.2
w_c	m	0.02			

chosen and tuned in simulations of 100 to 500wd's, i.e. 1000s to 5000s, which can be simulated in about a 10% of that real time.

To monitor whether r_0 roughly has the desired value during a simulation, major flooding is defined to occur when the river level significantly, i.e. by $0.01\mathrm{m}$ or more, exceeds the canal–1 berm along the strip of river bordering the city plain. This is monitored visually in dialy snapshots, one of which is given in the lower panel of Fig. 4. It contains a compound of levels to enable this flood monitoring, whcih requires some explanation. While the canal water enters the river in the city, for simplicity flood waters of the river are not modelled to enter the canal or city, which suffices for our design purposes. The information displayed in the lower panel of Fig. 4 is as follows. The vertical axis has units in m so the range across the length $L = 4.21\mathrm{m}$ of the set-up is about $0.06\mathrm{m}$ with the horizontal x–axis lying along the river and canal. The zero-level of the canals and rivers in the vertical is put at the river exit $(x,z) = (L,0)$ with vertical coordinate z. While realistically, river and canal vary slightly in length because they lie alongside each other, mathematically they have the same length as if they lie within each other. The lower, solid and thick red line displays the fixed, $1:100$-sloped bottom of the river. The thinner solid-black line displays the river level with the upstream input depth of $h(0,t) = h_0(t)$, which river depth is uniform because the river adjusts to steady state till the reservoir influx at $x_{res} = 0.932\mathrm{m}$, the moor influx at $x_m = 2.038\mathrm{m}$ and the canal–1 influx at $x = 3.858\mathrm{m}$ cause sudden increases in the river levels. These larger and smaller jumps are indeed visible and identifiable in the lower panel of Fig. 4. The flux into the river from canal 1 is small so the rise in the river level seen to be tiny, and much smaller than the time-varying influx of water from the reservoir and moor. The bottom of the three canal sections is displayed by the (stepped) thick-blue dashed line with the upper canal–3 level at $z = b_3 = 0.04\mathrm{m}$, the middle canal–2 level at $z = b_2 = 0.02\mathrm{m}$ and the lower canal–1 level at $z = b_1 = 0.0\mathrm{m}$ the three berm or dike height are $0.02\mathrm{m}$ higher at $\{d_3, d_2, d_1\} = \{0.0, 0.04, 0.02\}\mathrm{m}$. Canal berm or dike levels are displayed with a (stepped) thick solid-red line, while the three varying canal levels are displayed as the (stepped) solid-blue line. Steps in the berms occur where the weirs are placed and the jumps in the varying canal levels

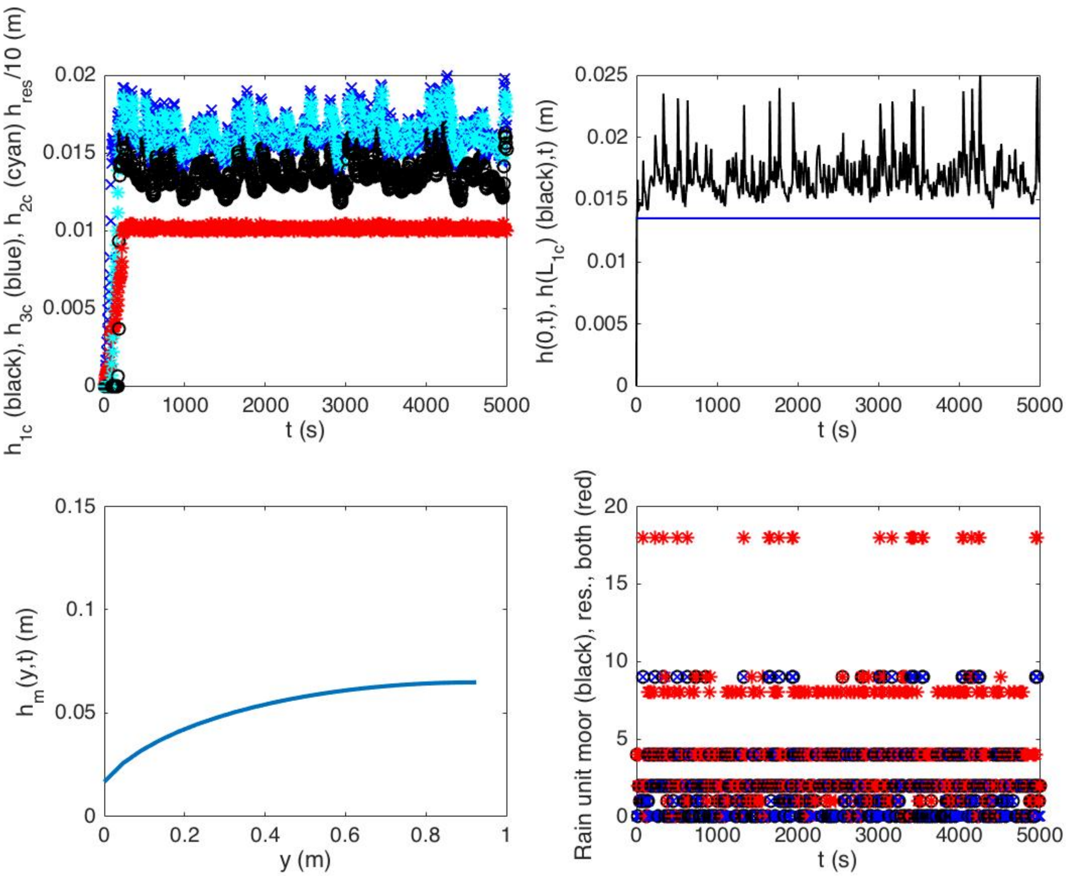

Figure 5. A four-panel figure in which: (i) the first top-left panel contains the three canal levels and the level of the reservoir as function of time t, all initialised at zero in this run; (ii) the second top-right panel displays the river level at $x = 0$ in blue and the river level at one point in the city in black as function of time; (iii) the bottom left panel shows moor groundwater level $h_m(y,t)$ as function of space y one snapshot at $t = 5000$s; and, (iv) panel four, bottom right, shows rainfall every Wetropolis day of $wd = 10s$ scaled with the magic factor r_0 versus time.

are determined by the hydraulic weir relations at these weirs. Some river flooding can occur when the river level, the (stepped) solid-black line, exceeds the canal–2 berm downstream of the second weir, as is visible as snapshot in the lower panel of Fig. 4. Major flooding is defined when the river levels exceeds the canal–1 berm in the city section, i.e. at $x = L_{1c} = 3.858\mathrm{m}$ the water depth $h_r(L_{1c}, t)$ signficantly exceeds $0.02\mathrm{m}$, visible as snapshot in the lower panel of Fig. 4, where the solid-black line of the river level is seen to exceed the solid-red canal–1 berm level downstream of the last weir around $x = 3.7\mathrm{m}$. Via visual optimisation while monitoring when major flooding ocurred in the city for the extreme or rare events of 90% rainfall in both the reservoir and moor, a suitable value of the rainfall speed is found to lie around the value

$$r_0 = 2.0510^{-4}\mathrm{m/s}. \tag{16}$$

The corresponding water volumes for the various Galton-board outputs required in the moor per wd are then

$$(1,2,4,8,18)V_{rate} = (0.18,0.36,0.72,1.44,3.24)\mathrm{l/wd} = (0.018,0.036,0.072,0.144,0.324)\mathrm{l/wd}. \tag{17}$$

Consequently, the pump supplying the rainfall in the moor should have a maximum discharge of about $324\mathrm{ml}/s$, which is a manageable amount from a design perspective. Such a discharge is feasible by using inexpensive off-the-shelf acquarium pumps, both for the supply of the upstream river influx Q_0 and the varying rainfall amounts in reservoir and moor.

An example of a simulation over $500\mathrm{wd}$ is summarised in Fig. 5. Since reservoir and canals are empty at $t = 0$, we observe that it takes about $25\mathrm{wd}$ before they are filled. During this time major flooding is lessened or prevented because reservoir and canal in essence act like flood-attenuation storage sites, supplying passive flood control. Extreme rainfall in this start-up period tends to be buffered such that city flooding is prevented. Reservoir and canal levels are displayed in the top-left panel of Fig. 5 versus time. The (constant) upstream river level $h_r(0,t)$ and city river level $h_r(L_{1c},t)$ are displayed as function of time t in the top-right panel of Fig. 5, in which extreme events with $n_r = 18$ are clearly identifiable as flood peaks at time t when $h_r(L_{1c},t) > 0.02\mathrm{m}$. A snapshot of the groundwater level $h_m(y,t)$ in the moor is displayed in the bottom-left panel of Fig. 5; it shows the no-flux upstream boundary condition at $y = L_y$ and the gradual decrease of the groundwater level towards its outflow location at $y = 0$. The rain units for the moor (being $1,2,4$ or 9), reservoir (being $1,2,4$ or 9) and their summation n_r, with the discrete values of $1,2,4,8,9$ or 18, are displayed in the bottom-right panel of Fig. 5. The peaks of extreme rainfall are, of course, seen to match the peaks in extreme flooding in the panels on the right except, possibly, during the first circa $25\mathrm{wd}$'s when the canals and reservoir tend to act as flood-attenuation buffers.

3 Table-top design

After the design calculations commenced on May 29^{th} 2016 and were completed at June 8^{th} 2016[1], the Wetropolis flood demonstrator was constructed and finalised between June 4^{th} and August 31^{st} by OB and WZ [2]. The final design was limited by the demand to transport it in the back of a car. We note that the reservoir and moor have been swapped in the actual set-up, relative to the mathematical design, and that the river-channel length has been increased to $5.2\mathrm{m}$.

Wetropolis consists of the following elements:

- The topographic landscape with a winding river channel, one-sided flood plain, canals and the city-plain has been routed out of two standard polysterene foam plates each of dimension $5 \times 60 \times 120\mathrm{cm}^3$ (plus a small extra foam plate) with an overlay to fit two plates together. A smaller third piece was added to extend the river length after the city which enhanced flooding in the city plain. An overview was given in Fig. 2 and a photograph is found in Fig. 6. Drawings have been made in the CAD programs Rhino/Grasshopper and used to steer the router. Routing precision is circa $0.8\mathrm{mm}$. After the routing, the river channel and its flood plain have been roughened by gluing fine sand to the base, after which it is varnishes with yacht varnish.

[1] The first design and complete model calculations were presented during a seminar at Imperial College London on June 1^{st} 2016. A week later an error in the use of the Manning coefficient C_m was fixed, leading to an increase of the river channel length L by a factor of four. Hence, the winding channel.

[2] See public postings in that period around 08-06-2016 and 31-08-2016 on https://www.facebook.com/resurging.flows

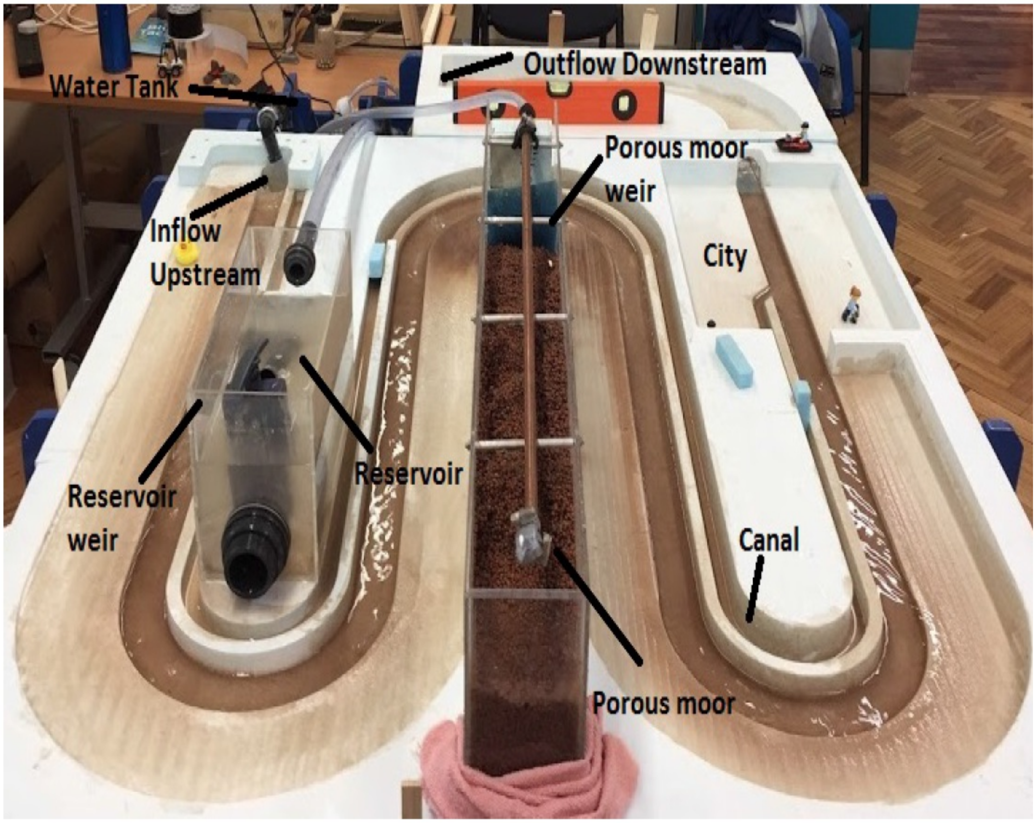

Figure 6. Overview of the Wetropolis flood demonstrator with its winding river channel of circa 5.2m and the slanted flood plains on one side of the river, a reservoir, the porous moor, the (constant) upstream inflow of water, the canal with weirs (the three small blue foam-wedges seen in the photograph), the higher city plain, and the outflow in the water tank/bucket with its three pumps. Two of these pumps switch on randomly for $(1,2,4)$ or 9s of the 10s for each "Wetropolis Day" (SI-unit: wd). Photo compilation: Luke Barber.

– A framework of wooden support slabs has been made that fits on four A-frames. This framework is put together with a bolt-nut system such that it can be disassembled for transport. Wooden wedges are used to squeeze and level the foam pieces within the slab-framework. The three foam pieces are squeezed together to limit leakage. Aluminum one-side-sticky tape is used locally to seal two sections of the river channel together and thus bridge two adjacent channels. Rather than sealing off all leakage, which in practice becomes impossible, an "acquifer" system of two interconected gutters underneath the seams of the foam pieces leads leaked water back to the holding resevoir with the three acquarium pumps. Hence, the water budget is closed in the absence of evaporation, the latter which is negligible on the time scale of operation, of typically one to a few hours.

– Three acquarium pumps[3] with a maximum pumping capacity of $0.375\mathrm{ml}/s$ are placed in a holding reservoir, a rectangular bucket with dimensions $\sim 0.3 \times 0.40 \times 0.22\mathrm{m}^3$, which bucket is hung under the wooden framework in a rectangular area adjacent to the upstream inflow point of river water. Plastic tubing with inner and outer diameters of circa $(1.8, 2.2)\mathrm{mm}$ leads the water from this reservoir to the upstream point and, depending on wether it rains or not, to the reservoir and the moor. Rainfall in the moor is spread out and visualised using a copper pipe with numerous downward-facing holes over the lava grit.

– The moor unit is made of acrylic and on the open side-face of the box a maze prevents the lava grit from avalanching into the river. The acrylic reservoir is open from the top and water can enter through a hole near the top edge. Outflow of water in the river is regulated via an internal pipe which outflow level can be manually adjusted. Hence, active flood control can be demonstrated by manually adjusting this outflow level. Via an adjustable valve, outflow into the canal can be arranged separately. Note that this is slightly different from the set-up in the mathematical and numerical design model, where the outflow of moor water was partitioned between the river and canal.

– The two draws from the discrete probability distribution are either computer generated or determined from the random paths of (a) steel ball(s) through the two asymmetric Galton boards. In the latter case, the steel ball triggers a signal by interupting optical sensors in one of the four channels on each Galton board, cf. Fig. 1. The signal subsequently steers either the reservoir pump, moor pump, both or none as arranged via the Arduino technology.

Further specifications, instructional photographs and design drawings of various components have been provided on a github site: https://github.com/obokhove/wetropolis20162020

4 Wetropolis illustrated and demonstrated

Illustrative images of Wetropolis in action are shown in the photographs of Figs. 7 and 8. It includes close-ups of excessively flooded river-bends and city plain, the reservoir and its outlets as well as the moor under heavy rainfall in Fig. 7. During extremely heavy 90% rainfall on a Wetropolis day after a relatively wet period, the moor becomes supersaturated and the groundwater level can rise through the lave grit and trigger fast surface run-off. In other situations the groundwater level is below the surface of the lava grit. Under varying rainfall the rising and falling groundwater level can be observed through the transparant acrylic walls. This visualisation was inspired by cartoons in hydrological textbooks, in which we often find similar cross-sections of the earth and its groundwater levels.

To date Wetropolis Flood Demonstrator has been showcased in public events to flood victims as well as to visitors of science fairs, to scientists in bespoke workshops on natural hazards and to flood professionals. Wetropolis has been shown: [4]:

[3]Syncra 1.5, 234–$240\mathrm{V}$, $50\mathrm{Hz}$, $23\mathrm{W}$, $Q\text{–}max = 1350\mathrm{l/h}$, $H\text{–}max = 1.8\mathrm{m}$.

[4]For movie footage see the posts dated 31-08-2016 [with an extremely rare Boxing-Day-2015-type flood after two consecutive days of extreme rainfall], 06-09-2016, 16-01-2017, 08-12-2016, and 07-04-2017 on https://www.facebook.com/resurging.flows

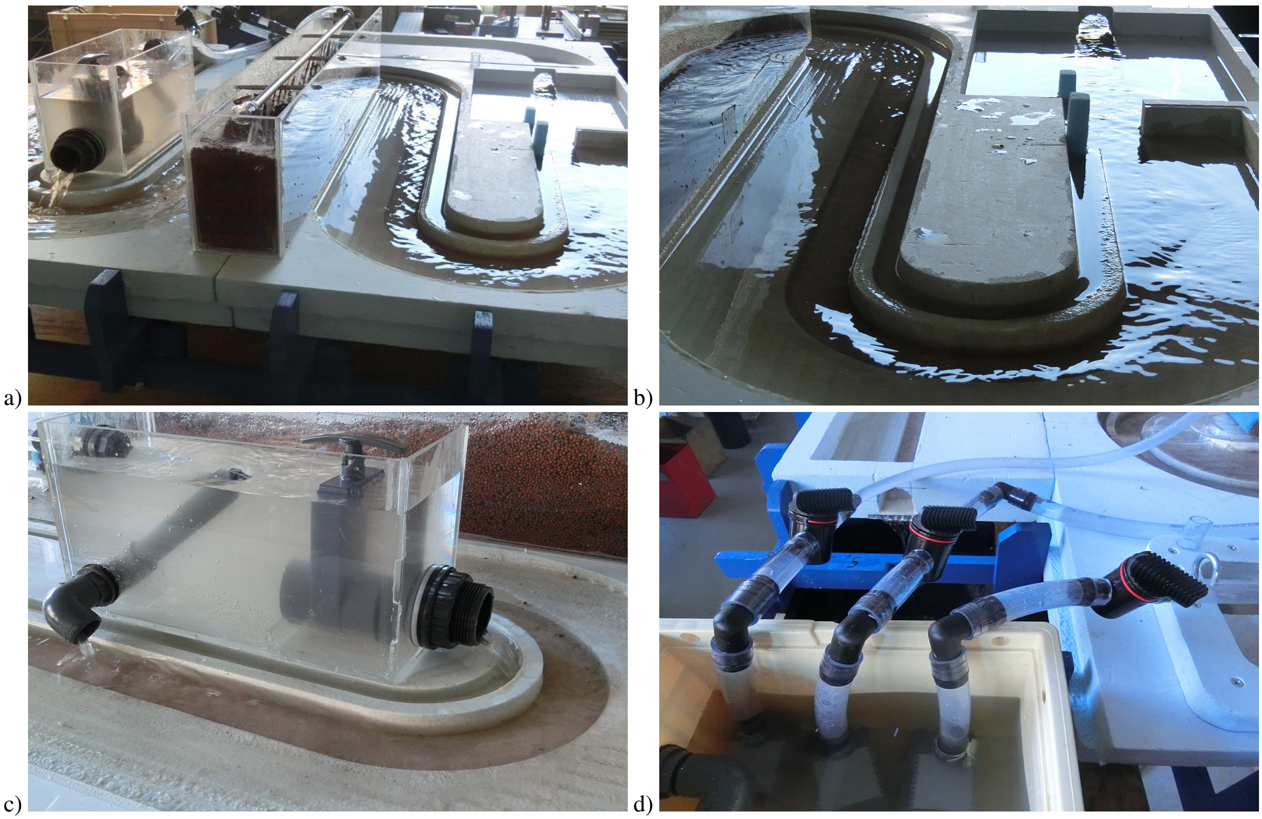

Figure 7. Action shots of Wetropolis: a) overview of overflowing reservoir on the left, the lave-grit filled moor under heavy rainfall in the middle and a flooded city in the background on the right during tests with massive flooding and 100% rainfall over several days; b) zoom-in of the final river bend and its one-sided flood plain and the canal before the city as well as a flooded city plain in the background on the right during massive flooding; c) zoom-in of the reservoir with water streaming through the manually adjustable outflow pipe into the river and the seperate valve-adjustable underflow into the canal on the right; and, d) zoom-in of the holding reservoir with the three acquarium pumps and tubing leading to the constant upstream inflow at the start of the river at $x = 0$ on the right and two other tubes leading to the reservoir and moor.

Figure 8. Photograph of the entire set-up at the Churchtown Flood Action Group workshop on 28-01-2017, with on the foreground the winding river channel, the city plain with a few smurfs, the groundwater moor, in the background on the left behind the moor the reservoir, and in the background on the right, the control table with the Arduino units and the two Galtonboards as well as a poster on Wetropolis.

- first at the general assembly of the EPSRC UK network Maths Foresees in Edinburgh on 06-09-2016 to an audience of scientists with expertise in environmental fluid dynamics and representatives from stakeholders such as the Met Office, JBA Trust and the Environment Agency; Wetropolis has been created as an outreach project within this Maths Foresees' network;

- at the Churchtown flood action group conference on 28-01-2017 in Lancashire to circa 140 flood victims and several experts on flooding, see Fig. 8;

- as part of two public exhibitions on the Boxing Day 2015 floods in Leeds' Armley Industrial Musuem (on 08-12-2016 and 26-03-2017);

- at a bespoke Canal and River Trust workshop in Liverpool in 2017 by students of Leeds' Centre for Doctoral Training in Fluid Dynamics;

- at the "Be Curious" science festival of the University of Leeds;

– at the Second Study Group of Maths Foresees in the Turing Gateway of Mathematics, Cambridge, which particular event focussed on solving mathematical challenges related to flooding, see https://gateway.newton.ac.uk/event/tgmw41; and,

– at the Yorkshire iCASP confluence (integrated catchment program) on 15-06-2017 for a range of scientists, flood professionals, stakeholders and politicians, see the right panel of Fig, 1.

The strength of Wetropolis is that it is a life visualisation of the probability of extreme rainfall and flooding events including actual and visual river hydraulics, groundwater level changes and interactive flow control. We rcall that the reservoir has valves such that the audience can store and release water interactively to control and possibly prevent flooding in the city. Wetropolis is, however, a conceptual model of flooding rather than a literal scale model of a specific catchment. It was, however, inspired by the Boxing Day Floods of 2015 of the River Aire, in and upstream of Leeds, UK. This is both a weakness and a strength because one needs to explain the translation of a $1:200$ year return period for a realistic extreme flooding and rainfall event such as the Boxing Day 2015 flood of the River Aire into one in Wetropolis with a $1:6:06$min return period, and one also needs to explain that the moor and reservoir are conceptual valleys where all the rain falls, since rain cannot fall everywhere in the Wetropolis catchment and in contrast with a real catchment such the River Aire one. This scaling and translation step is part of the conceptualisation, which the audience, whether public or scientific, needs to grasp. The visualisations of flooding in the city and the ground water level also involve learning steps. Hitherto, this conceptualisation step was either explained by the Wetropolis wardens in attendance at a demonstration, via our bespoke poster, or both. Alternatively, we aim to arrange bespoke audiovisual material.

Due to this learning curve required, the most receptive public audiences have been flood victims or people with friends or family who went through the unpleasant and traumatising experience of being flooded. We have perceived such audiences to be the most receptive, inquisitive and interactive because they have an intrinsic interest in flooding phenomena and wish particular questions to be addressed in order to gain more understanding as to what causes flood hazards, on how these hazards can be predicted and on how such floods can possibly be mitigated through flood management and mitigation. That is, they are exactly interested to obtain answers to the questions which were raised in the introduction. Wetropolis in particular aids in raising awareness of the probabilistic character and randomness of rainfall and flooding events, also in connection with the difficulties in predicting some of these extreme events. Combing showcasing Wetropolis with a general public lecture on the science of flooding has proven to be particularly succesful, cf. (**?**Potter, 2016), owing to such a presentation wetting the appetite to view a life scale model with rainfall and river flooding. While Wetropolis has been invented and intended as public outreach project, the reception by flood practitioners and scientists in environmental fluid dynamics has been surprisingly positive, which reception we will discuss later.

5 Summary and discussion

In summary, we have told the story of how Wetropolis Flood Demonstrator was constructed after an efficient mathematical and numerical design enabled us to estimate the characteristic components of the envisioned set-up. This efficient mathematical model was first presented as a coupled system of ordinary and partial differential equations which we subsequently solved numerically to define a near-optimal design. While that mathematical model is close to a prediction model for river and groundwater flows in Wetropolis, due to its relatively minimalist nature and purpose to facilitate the design, it is likely not quite sophisticated enough to make bonafide predictions. In a final modelling step we determined the reasonable rainfall and flow rates through numerical simulation, on which rates we based the actual design and construction of Wetropolis. Several improvements and extensions of Wetropolis are under exploration, including the following:

- the suggestion to accentuate an actual flooding event in the city more prominently e.g. by measuring the actual flood waters of each flood event via a separate drain into a measuring cup or by triggering flashlights in the city to go on by closing an electric circuit by the flood water and to go off when the circuit is broken; and,
- to visualise key principles of Natural Flood Management (NFM) or more broadly Nature Based Solutions (NBS) (e.g., Environment Agency (2017); Lane (2017); Potter (2016); Cabaneros et al. (2018); Bokhove et al. (2018b)) by visualising the effects of different riverbed roughness to slow down the flow in a cut-out river-bed segment via removeable river-channel inserts and by including a porous upland catchment with various small-scale river channels and flow-attentuation features to enhance water storage, which upland features can be manipulated by the audience.

5.1 Games?

One of the shortcomings of the current Wetropolis set-up is that it lacks to date bespoke educational material and games. The following game suggestions for the current Wetropolis set-up, that have arisen during various workshops, are as follows:

- The notion of pdfs can be developed and based on turfing histograms of actual Galton board outcomes and comparison of these outcomes against the ideal outcomes as well as games around blind-checks to determine whether the outcomes have been tampered with by human intervention, e.g., by triggering extreme flooding through tricking the electronic eyes by a finger or by purposely misaligning the Galton boards. While the audience generally likes massive flooding to occur more often in Wetropolis, by falsely triggering daily 90% rainfall in both moor and reservoir, turfing the outcomes would immediately reveal that such tampering is unrealistic in that it makes no rain and low to intermediate rainfall into rare rather than common events.
- Building a game on flood prevention in the city by controlling the valves on the reservoir, say over 30 to 100wd's, with the winning team having the least or zero amount of flooding in the city. This includes the discussion that the winning team can win by chance over a limited set of trials rather than by virtue of optimal flood control.
- The audience can play with the set-up. The set-up can namely be modified by interchanging the two-out-of-three locations where rainfall is random, changing the locations of the reservoir and moor, for example by bringing one unit in close proximity to the city, including an investigation as to what consequences these changes entail in observed spatio-temporal patters?

Each of these suggestions requires further development.

5.2 New approaches in science and water management?

Flood practitioners from various stakeholders have been quite positive about Wetropolis' novel way to visualise the probability of extreme rainfall and flooding events via the asymmetric Galton boards and how the outcomes from these boards directly lead to observable rainfall and river dynamics in the set-up. Stakeholders such as JBA Trust and the Environment Agency see Wetropolis as a potentially useful tool to trigger discussions about innovations in flood-mitigation and water management, as part of workshops and brainstorm sessions. To date, Wetropolis has triggered two innovations: one on the use of the revisited concept of flood-excess volume in devising a novel and graphical cost-effectiveness analysis to flood mitigation, in particular meant for decision makers, and one on education in water-engineering and watermanagement.

Flood-excess volume (FEV) concerns the volume of flood waters that caused flood damage. It is the flood volume of the river flow beyond a certain, chosen and relevant threshold water level. This FEV, expressed in cubic metres (m^3), or expressed more visually and comprehensively as a square lake of 2m in depth with a certain side length, is a useful measure to devise flood-mitigation strategies. It allows us to quantify what fraction of the FEV is mitigated by a certain strategy. Our cost-effectiveness analysis results in a series of square-lake graphs, one for each flood-mitigation scenario envisioned, which express both the flood volume mitigated by a particular flood-mitigation measure, its cost, its cost per percent mitigated, and the overall costs. When an accumulation of flood-mitigation measures captures the entire FEV, the FEV is essentially reduced to zero. Building higher flood-defence walls in a city at or just above the maximum river level to be mitigated does, for example, reduce the FEV to zero in one fell swoop. But building high walls around a river in a city is only one type of flood-mitigation scenario, one that may be undesirable, so in general flood-mitigation scenarios, expressed visually as square-lake graphs, will consist of an accumulation of measures such as river-bed widening, i.e. giving-room-to-the-river (GRR), active flood-storage plains, higher flood walls and NBS. Each measure cuts a certain fraction as a rectangular strip off the square flood lake, with accompanying costs displayed. The graphical cost-effectiveness analysis has been developed by us, in a series of papers Bokhove et al. (2018a, b, c), for several extreme river floods in the UK and France in order to facilitate and improve evidence-based decision-making by city councils and concerned citizens' groups.

The R&D consortium "Wetropolis, tangible models for education and watermanagement" is a regional EU EFRO-project[5] in the Dutch counties of Salland and Twente that has begun to develop strategy tools to support the dialogue on climate adapation to flooding and drought. It consists of three phases with an overarching theme:

–a museum exhibition on the water cycle in the local environment in the first phase;

–the development of educational tools for school that builds upon the Dutch GRR-programme in the second phase, cf. §5.1; and,

–development of do-it-yourself experiments with citizens to measure water processes and levels in urban areas in the third phase. The overarching theme of the project is to stimulate a climate-resilience dialogue on municipal levels.

A particular question pertaining a revised Wetropolis is how a combined flood-and-drought demonstrator can be created with a

[5]The Dutch wetropolis' project is led by Dr. Henk de Poot from Nobis, Enschede, The Netherlands, and involves a consortium of Dutch SMEs, schools and universities –https://www.wetropolis.nl

visual appeal similar to, or beyond, the one presented here, given the expectation that extremely dry European summers, such as those of 1976 and 2018, are likely to occur more often in the future.

Appendix A: Numerical discretisation of entire system

Allowing for irregular time steps $\Delta t_n = t^{n+1} - t^n$ with $t^0 = 0$, the entire system (13) has the following space-time discretisation, using regular finite differences for the groundwater equation, a first-order finite-volume method with upwinding for the river equation, and a first-order forward-Euler time discretisation for all differential equations involved, as follows:

$$\frac{(h_k^{n+1/2} - h_k^n)}{\Delta t_n} + (Q_{k+1/2}^n - Q_{k-1/2}^n) = \frac{Q_m}{w_r}\delta_{km} + \frac{Q_{res}}{w_r}\delta_{kr} \quad \text{for} \quad k = 1, \ldots, N_x$$

$$\text{with} \quad Q_{k+1/2}^n = h_k^n R(h_k^n)^{2/3}\frac{\sqrt{-\partial_x b}}{C_m}, Q_{1/2}^n = Q_0^n = Q_0(t^n), \quad h_k^0 = h_{0k} \tag{A1a}$$

$$\frac{(h_j^{n+1} - h_j^n)}{\Delta t_n} - \frac{\alpha g}{\Delta y^2}\left(h_{j+1/2}^n (h_{j+1}^n - h_j^n) - h_{j-1/2}^n (h_j^n - h_{j-1}^n\right) = \frac{R_m^n}{m_{por}\sigma_e} \quad \text{for} \quad j = 1, \ldots, N_y - 1$$

$$\text{on} \quad y_j = j\Delta y \quad \text{with} \quad (h_{N_y}^n - h_{(N_y-1)}^n) = 0, h_0^n = h_{3c}^n, h_j^0 = h_{0j} \tag{A1b}$$

$$w_{res} L_{res}\frac{(h_{res}^{n+1} - h_{res}^n)}{\Delta t_n} = w_{res} L_{res} R_{res}^n - Q_{res}^n \tag{A1c}$$

$$w_c(L_{1c} - L_{2c})\frac{(h_{1c}^{n+1} - h_{1c}^n)}{\Delta t_n} = Q_{2c}^n - Q_{1c}^n \tag{A1d}$$

$$w_c(L_{2c} - L_{3c})\frac{(h_{2c}^{n+1} - h_{2c}^n)}{\Delta t_n} = \gamma_m m_{por} w_v \alpha g \frac{1}{2}\left((h_1^n)^2 - h_{3c}^n\right) - Q_{2c}^n \tag{A1e}$$

$$w_c L_{3c}\frac{(h_{3c}^{n+1} - h_{3c}^n)}{\Delta t} = \gamma Q_{tm}^n - Q_{3c}^n \qquad \text{with} \tag{A1f}$$

$$Q_{1c}^n = C_f\sqrt{g}\, w_c \max(h_{1c}^n - P_{1w}, 0)^{3/2} \quad \text{and} \quad Q_{2c}^n = C_f\sqrt{g}\, w_c \max(h_{2c}^n - P_{2w}, 0)^{3/2} \tag{A1g}$$

$$Q_{3c}^n = C_f\sqrt{g}\, w_c \max(h_{3c}^n - P_{3w}, 0)^{3/2} \quad \text{and} \quad Q_m^n = (1-\gamma) Q_{tm} \equiv (1-\gamma) m_{por}\sigma_e w_v \alpha g \frac{1}{2}\left((h_1^n)^2 - (h_{3c}^n)^2\right)/\Delta y \tag{A1h}$$

$$Q_{res}^n = C_f\sqrt{g}\, w_{res} \max(h_{res}^n - P_{wr}, 0)^{3/2} \quad \text{and} \quad R(h) = w_r h/(2h + w_r), \tag{A1i}$$

with N_y the number of regular grid points across L_y, $\Delta y = L_y/N_y$ and $h_j^n = h_m(j\Delta y, t^n)$ and $h_{j+1/2} = (h_{j+1} + h_j)/2$; N_x for $k = 1, \ldots, N_x$ the number of finite-volume cells $\Delta x_k = x_{k+1/2} - x_{k-1/2}$ with cell faces $x_{k\pm 1/2}$ and cell average

$$\Delta x_k h_k^n = \int_{x_{k-1/2}}^{x_{k+1/2}} h(x, t^n)\,\mathrm{d}x, \tag{A2}$$

in which the Kronecker delta stymbol $\delta_{km} = 1$ for the cell k in which x_m resides and is zero elsewere, and likewise $\delta_{kr} = 1$ in the cell in whihch x_{res} resides, etc.

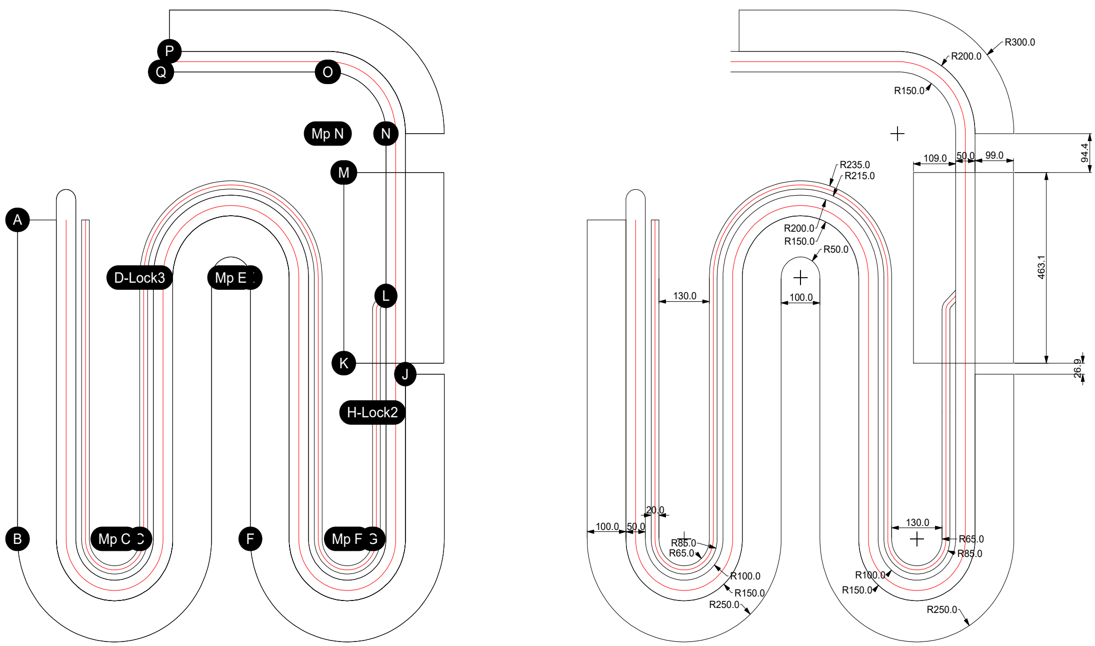

Figure A1. Drawings of the basic topographic landscape of Wetropolis on the right with letter indications matching coordinates in a corresponding excel file on the right.

Appendix B: Wetropolis' design details

A Github site contains all material https://github.com/obokhove/wetropolis20162020. Some design tools and materials are briefly outlined below:

– Blue foam plates from Gamma were used "Isolatieplaat polystyreen XPS" of dimensions $120 \times 60 \times 5\text{cm}^3$: https://www.gamma.nl/assortiment/isolatieplaat-polystyreen-xps-120x60x5-cm-4-stuks/p/B133540 Yacht varnish was mixed with fine calcinated sand and shells sieved with sieve (with holes of 0.9m and wire thickness of 0.1m). The following CAD programs were used: Solidworks for the designs, saved as a Step file for import in Rhino (V5); plugin in Rhino for routing: Rhinocam 5, which generates routing/freesfiles (NC files); foams were routed on a BZT 1400 PF router/frees with Winpc-nc driver.

– Acquarium pumps are used: Syncra 1.5, 234–240V, 50Hz, 23W, $Q\text{–}max = 1350\text{l/h}$, $H\text{–}max = 1.8\text{m}$.

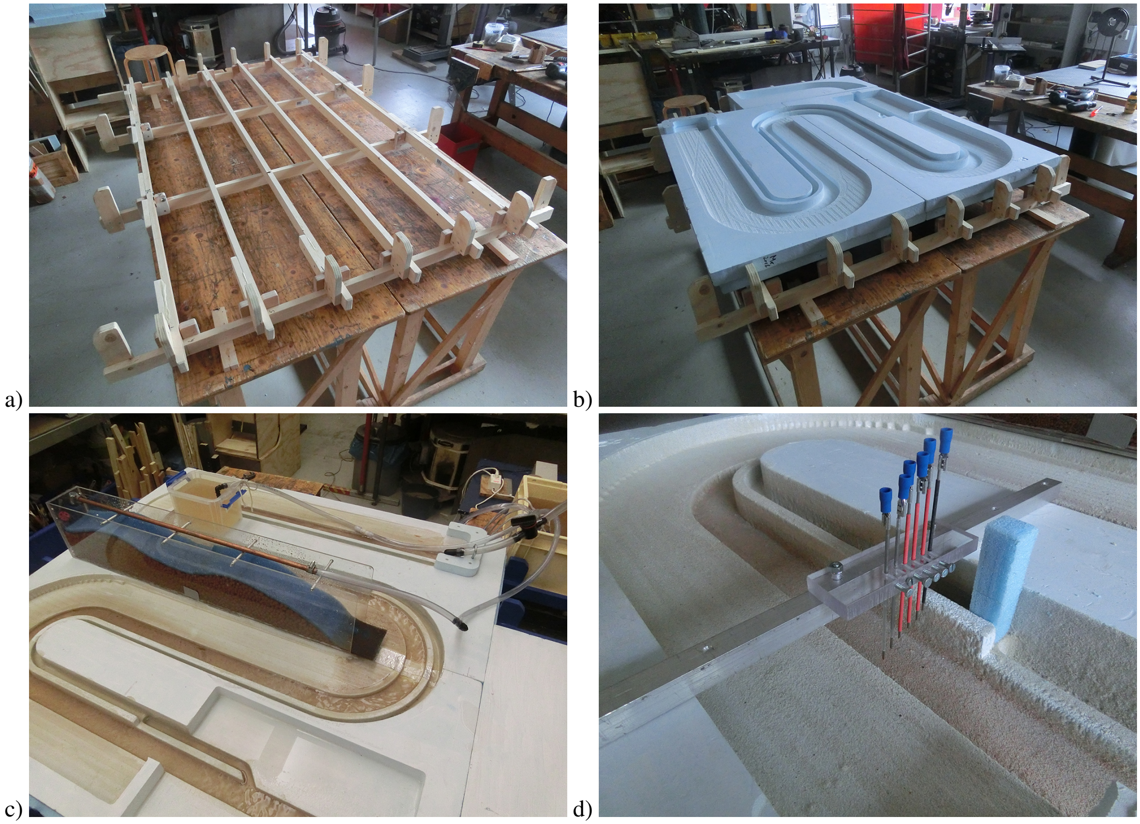

Figure B1. a,b) The making of the wooden support frame with its bolt-nut system. c) Overview with moor and the first reservoir; notice the aluminum tape sealing two foam plates. d) Detail of canal and sluice gate as well as a water-level measurement device involving Arduino technology.

– Design drawings first foam plates are given in Fig. A1 and photos ofthe wooden support system in Fig. B1.

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