COMP 353 Databases

Design Theory for Relational Databases

Functional Dependencies

Schema Refinement (Decomposition)

Normal Forms

Functional Dependencies (FDs)

- A **functional dependency** (FD) is a kind of **constraint**
- Suppose **R** is a relation schema and $\mathbf{X,Y} \subseteq \mathbf{R}$.

  A FD on **R** is a statement of the form $\mathbf{X} \rightarrow \mathbf{Y}$, which asserts: "For every "legal/valid" instance **r** of **R**, and for all pairs of tuples t1 and t2 in **r**, if t1 and t2 **agree** on the values in **X**, then t1 and t2 **agree** also on the values in **Y**."

  In symbols: $\forall$ t1,t2 $\in$ **r**: t1[**X**] = t2[**X**] ➔ t1[**Y**] = t2[**Y**].
- We read $\mathbf{X} \rightarrow \mathbf{Y}$ as:

  X (functionally) determines Y (or Y is determined by X)
- We say that the FD: $\mathbf{X} \rightarrow \mathbf{Y}$ is *relevant to* **R** if $\mathbf{X \cup Y} \subseteq \mathbf{R}$.

Functional Dependencies

- Consider the relation schema:

  Star (<u>name</u>, SIN, street, city, postalCode, phone)

- Since we know the semantics of this relation from the design phase, we can answer the following question:
  - What are the functional dependencies on **Star**?
- Note that in general, FDs on a relation R may not be determined based on a given instance of R!

Functional Dependencies

- Consider the relation:

  Movie (title, year, length, filmType)

- What are the FD's on the **Movie** relation?

  We use the semantics of this relation to answer.

Keys

- The concept of FD generalizez the concept of key. How?
  - Let **X** $\subseteq$ **R**. Then **X** is a key of **R** iff **X** $\rightarrow$ **R**
- **X** is a (**candidate) key** of **R** (or a key, for short) if
  1. **X** $\rightarrow$ **R**. That is, attributes in **X** functionally determine **all** the attributes of **R**
  2. No proper subset of **X** is key, i.e., a candidate key must be **minimal**
- Is {title, year, filmType} a key for relation **Movie**?
- A set of attributes that contains a key is called a **superkey (that is, a superset of a key)**
  - Note that every key is a **superkey**, but not vice versa

Functional Dependencies

- **X** $\rightarrow$ **Y** is called a **functional dependency** because, in principle, there is a function that takes a list of values, one for each attribute in **X,** and returns at most one value (i.e., a *unique* value or no value at all) for the attributes in **Y**

Functional Dependencies

- Consider the relation:

  Movie (title, year, length, filmType, studioName, starName)

- What are the functional dependencies?

  {title, year} $\rightarrow$ length

  {title, year} $\rightarrow$ filmType

  {title, year} $\rightarrow$ studioName

  ➔ {title, year} $\rightarrow$ {length, filmType, studioName}

  Note: {title, year} $\rightarrow$ starName does not hold

- What is the key of the **Movie** relation?

Trivial FD's

- An FD $\mathbf{X} \rightarrow \mathbf{Y}$ is said to be **trivial** if $\mathbf{Y} \subseteq \mathbf{X}$.
  - For example: {**title, year**} $\rightarrow$ **title** is a trivial FD
- Otherwise, the FD is called **nontrivial**
  - For example: {**title, year**} $\rightarrow$ **length** is a nontrivial FD

Functional Dependencies

- Why are we interested in functional dependencies?

Redundancy Problem

- Redundancy – a “piece” of information is unnecessarily repeated in different tuples in a relation
- Recall that ***redundancy*** is the main source of problems:
  - Storage waste
    - Some information stored repeatedly
  - Update anomalies
    - If a copy of such information is updated, an inconsistency may arise unless all its copies are updated
  - Insertion anomalies
    - Unless we allow nulls, it may not be possible to store some information unless we have all the information to store
  - Deletion anomalies
    - Deleting some information may results in loosing some other information (which we don’t want to loose)

Is this a good design for relation R?

Name	SSN	Phone
Fred	123-321-99	(201) 555-1234
Fred	123-321-99	(201) 572-4312
Joe	909-438-44	(908) 464-0028
Mary	938-401-54	(201) 555-1234

The only FD on R is: **SSN$\rightarrow$Name**

Therefore, the only key of R is: **{SSN, Phone}**

What about this design, replacing R with R1 and R2?

R1

SSN	**Name**
123-321-99	Fred
909-438-44	Joe
938-401-54	Mary

R2

SSN	**Phone**
123-321-99	(201) 555-1234
123-321-99	(201) 572- 4312
909-438-44	(908) 464-0028
938-401-54	(201) 555-1234

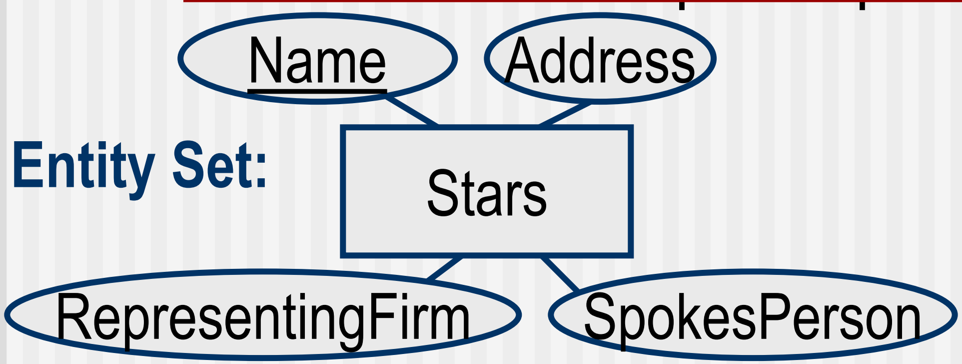
D = {R1(<u>SSN</u>, Name),
R2(<u>SSN, Phone</u>)}

FD's on R1 and R2?

Another Example

- Suppose each star has a representing *firm* and each firm has one spokes person)

Entity Set:



Relation Schema:

Star (<u>Name</u>, Address, RepresentingFirm,SpokesPerson)

RepresentingFirm → SpokesPerson

Relation Instance:

Name	Address	RepresentingFirm	SpokesPerson
Carrie Fisher	123 Maple	Star One	Joe Smith
Harrison Ford	789 Palm rd.	Star One	Joe Smith
Mark Hamill	456 Oak rd.	Movies & Co	Mary Johns

Redundancy Problem

What is the role of FDs in detecting redundancy?

- Consider the relation scheme **R(A, B, C)**
  - Suppose no (nontrivial) FD holds on **R**
    - There is no redundancy in any instance **r** of **R**.
  - Now suppose FD: **A** $\rightarrow$ **B** holds on **R**
    - If several tuples have the same **A** value ➔ they must all have the same **B** value; otherwise this FD is violated
- Presence of some FDs in a relation suggests possibility of redundancy

Implications of FDs and Reasoning

- Consider relation **R(A, B,C)** with the set of FDs:
  F = {**A**$\rightarrow$**B**, **B**$\rightarrow$**C**}
- We can deduce from F that **A**$\rightarrow$**C** also holds on **R.**
  How? Apply the definition…
- To detect possible data redundancy, is it necessary to consider "all" the FDs (implicit and explicit)?
  - As shown above, there might be some additional hidden (nontrivial) FDs "**implied**" by a given set of FD's

Implications of FDs

- Defn: If a relation instance **r** satisfies every FD in a given set **F** of FD's, then we say that **r** satisfies **F**.
  - In this case, we also say that **r** is a *legal/valid instance.*
- Given <**R**,**F**>, we say that **F** implies a FD **X** $\rightarrow$ **Y,** if *every* instance **r** of **R** that satisfies **F** also satisfies **X** $\rightarrow$ **Y.**

Formally, we express this as: **F** $\models$ **X** $\rightarrow$ **Y**.

We may also say that **X** $\rightarrow$ **Y** follows from **F**.

- To show **F** $\not\models$ **X** $\rightarrow$ **Y**, we may give a counter-example, i.e., an instance **r** of **R** that satisfies **F** but not **X** $\rightarrow$ **Y**.

FDs Implication (Cont'd)

- Consider **R**(A_1,A_2,A_3,A_4,A_5) with FDs:

  F = $\{ A_1 \rightarrow A_2,\ A_2 \rightarrow A_3,\ A_2A_3 \rightarrow A_4,\ A_2A_3A_4 \rightarrow A_5 \}$

  Prove that **F** $\not\models A_5 \rightarrow A_1$

  Solution method: Provide a counter-example; give a relation instance **r** of R that satisfies every FD in **F** but not $A_5 \rightarrow A_1$

  A desired instance **r** of **R:**

	A_1	A_2	A_3	A_4	A_5
t1:	0	1	1	1	1
t2:	1	1	1	1	1

Closure of a set F of FDs

- Defn: The **closure of F**, denoted by $\mathbf{F^+}$, is the set of every FD: $\mathbf{X} \longrightarrow \mathbf{Y}$ that is implied by **F**.
- How can we determine $\mathbf{F^+}$?
  - Clearly, $\mathbf{F^+}$ includes **F** and possibly some more FDs
  - To answer the question we need to *reason* about FDs

Equivalence of two sets of FD's

- Let **R** be a relation schema, and **S, T** be sets of FDs on **R**.
- Defn: we say **S** ***covers*** **T** (**S** $\models$ **T)** if for every instance **r** of **R**, whenever **r** satisfies (every FD in) **S**, **r** also satisfies **T**.
- Defn: **T** and **S** are ***equivalent*** (**S** $\equiv$ **T**) iff **S** $\models$ **T** and **T** $\models$ **S**.
- Note: **F** and **F⁺** are equivalent.

Example: Suppose **R** = {A,B,C}, and

$$\mathbf{S} = \{A \rightarrow B,\ B \rightarrow C,\ A \rightarrow C\}$$

$$\mathbf{T} = \{A \rightarrow B,\ B \rightarrow C\}$$

We can show that **S** $\equiv$ **T**.

Armstrong's Axioms [1974]

- **R** is a relation schema, and **X**, **Y**, **Z** are subsets of **R**.
- **Reflexivity**
  - If **Y** $\subseteq$ **X**, then **X** $\rightarrow$ **Y** (trivial FDs)
- **Augmentation**
  - If **X** $\rightarrow$ **Y**, then **XZ** $\rightarrow$ **YZ,** for every **Z**
- **Transitivity**
  - If **X** $\rightarrow$ **Y** and **Y** $\rightarrow$ **Z**, then **X** $\rightarrow$ **Z**
- These are **sound** and **complete** inference rules for FDs

Additional rules / axioms

Other useful rules that follow from Armstrong Axioms:

Suppose **X, Y, Z,** and **W** are sets of attributes.

- **Union (Combining) Rule**
  - If **X** $\longrightarrow$ **Y** and **X** $\longrightarrow$ **Z**, then **X** $\longrightarrow$ **YZ**
- **Decomposition (Splitting) Rule**
  - If **X** $\longrightarrow$ **YZ**, then **X** $\longrightarrow$ **Y** and **X** $\longrightarrow$ **Z**
- **Pseudotransitivity Rule**
  - If **X** $\longrightarrow$ **Y** and **WY** $\longrightarrow$ **Z**, then **XW** $\longrightarrow$ **Z**

Example – Discovering hidden FD's

- Consider **R** = {**A**, **B**, **C**, **G**, **H**, **I**} with the FDs:

  $\mathbf{F} = \{\, \mathbf{A} \rightarrow \mathbf{B},\ \mathbf{A} \rightarrow \mathbf{C},\ \mathbf{CG} \rightarrow \mathbf{H},\ \mathbf{CG} \rightarrow \mathbf{I},\ \mathbf{B} \rightarrow \mathbf{H} \,\}$

- Using Armstrong's rules, we can derive more FDs
  - Since $\mathbf{A} \rightarrow \mathbf{B}$ and $\mathbf{B} \rightarrow \mathbf{H}$, then $\mathbf{A} \rightarrow \mathbf{H}$, by transitivity
  - Since $\mathbf{CG} \rightarrow \mathbf{H}$ and $\mathbf{CG} \rightarrow \mathbf{I}$, then $\mathbf{CG} \rightarrow \mathbf{HI}$, by union
  - Since $\mathbf{A} \rightarrow \mathbf{C}$ then $\mathbf{AG} \rightarrow \mathbf{CG}$, by augmentation

    Now, since $\mathbf{AG} \rightarrow \mathbf{CG}$ and $\mathbf{CG} \rightarrow \mathbf{I}$, then $\mathbf{AG} \rightarrow \mathbf{I}$, by transitivity (and in a similar way, we get $\mathbf{F} \models \mathbf{AG} \rightarrow \mathbf{H}$)
  - Many trivial dependencies can be derived(!) by augmentation

Implication Problem

- Given a set **F** of FDs, does **X** $\rightarrow$ **Y** follow from **F**?
- In other words: is **X** $\rightarrow$ **Y** in the closure of **F**?

(In symbols, does $\mathbf{F} \models \mathbf{X} \rightarrow \mathbf{Y}$ hold, or is

$\mathbf{X} \rightarrow \mathbf{Y} \in \mathbf{F}^+$ true?

- How to answer this question?
  - Compute the closure of **F** & check if it includes $\mathbf{X} \rightarrow \mathbf{Y}$
- What is the problem with this approach?
  - Computing $\mathbf{F}^+$ is *expensive*! Is there a better solution?

Closure of a Set of Attributes

- Given <**R, F**>; Let $\mathbf{X} \subseteq \mathbf{R}$.

  The closure of **X** under **F** is the set of all attributes **Y** in **R** that are determined by **X**. This yields $\mathbf{X} \to \mathbf{Y}$, i.e., every valid instance of **R** (that satisfies **F**) also satisfies $\mathbf{X} \to \mathbf{Y}$

  - We denote the closure of a set of attributes **X** under **F** by $\mathbf{X}^{+}_{\mathbf{F}}$
    - When **F** is known , we simply write $\mathbf{X}^{+}$ (and omit **F**)
    - Closure of $\{\mathbf{A1, A2, \ldots, A_n}\}$ is denoted $\{\mathbf{A1, A2, \ldots, A_n}\}^{+}$
  - Note that $\mathbf{X} \subseteq \mathbf{X}^{+}$, for any set **X** of attributes (because $\mathbf{X} \to \mathbf{X}$)

Computing the Closure of Attributes

- Given a set **F** of FD's and a set **X** of attributes, how to compute the closure of **X** w.r.t. **F**?
  - Starting with set $\mathbf{X^+=X}$, we repeatedly expand $\mathbf{X^+}$ by adding the RHS Z for every FD: $\mathbf{W \rightarrow Z}$ in **F**, if the LHD **W** is already in $\mathbf{X^+}$.
  - This process terminates when $\mathbf{X^+}$ could not be expanded further.
  - This process is expressed as an algorithm in the next slide.

An Algorithm to Compute X^+ under F

$\mathbf{X}^+ \leftarrow \mathbf{X}$ (initialization step)

repeat

for each FD $\mathbf{W} \rightarrow \mathbf{Z}$ in **F** *do*:

if $\mathbf{W} \subseteq \mathbf{X}^+$ *then*

$\mathbf{X}^+ \leftarrow \mathbf{X}^+ \cup \mathbf{Z}$ // add **Z** to the result

until $\mathbf{X}^+$ does not change

Complexity? In the worst case, how many times the "repeat" statement may be executed?

Examples

- Consider a relation schema **R** = { **A**, **B**, **C**, **D**, **E**, **H** } with the FD's **F** = { **AB** $\rightarrow$ **C**, **BC** $\rightarrow$ **AD**, **D** $\rightarrow$ **E**, **CH** $\rightarrow$ **B** }
- Suppose **X**={**A**,**B**}. Compute $\mathbf{X^+}$
- Execution result at each iteration:
  - Initially, $\mathbf{X^+}$ = {**A**, **B**}
  - Using **AB** $\rightarrow$ **C,** we get $\mathbf{X^+}$ = {**A**, **B**, **C**}
  - Using **BC** $\rightarrow$ **AD,** we get $\mathbf{X^+}$ = {**A**, **B**, **C**, **D**}
  - Using **D** $\rightarrow$ **E,** we get $\mathbf{X^+}$ = {**A**, **B**, **C**,**D**, **E**}
  - No more change to $\mathbf{X^+}$ is possible.

    ➔ $\mathbf{X^+}$ = {**A**, **B**}$^+$ = {**A**, **B**, **C**, **D**, **E**}
- Does the order in which FD's appear in **F** affects the computation?

Implication Problem Revisited

- Given a set of FD's **F**, does an FD: $\mathbf{X} \rightarrow \mathbf{Y}$ follow from **F**?
  - That is, is FD $\mathbf{X} \rightarrow \mathbf{Y}$ in $\mathbf{F}^+$?
- To answer this, we can compute $\mathbf{X}^+$ under **F**, and check if **Y** is in $\mathbf{X}^+$ or not
  - If yes, then the answer is positive! ($\mathbf{F} \models \mathbf{X} \rightarrow \mathbf{Y}$ ☺)
  - Otherwise, it is negative ($\mathbf{F} \not\models \mathbf{X} \rightarrow \mathbf{Y}$ ☹)

Example

- Consider <**R**,**F**> where **R** = { **A**, **B**, **C**, **D**, **E**, **H** } and

  F = { **AB** $\rightarrow$ **C**, **BC** $\rightarrow$ **AD**, **D** $\rightarrow$ **E**, **CH** $\rightarrow$ **B** }
- Does **AB** $\rightarrow$ **D follow from F**?
- Two steps:
  1. Compute {**A**, **B**}$^+$ = {**A**, **B**, **C**, **D**, **E**}
  2. Check if **D** $\in$ **{A,B}$^+$**
  - So, here we conclude that **AB** $\rightarrow$ **D** is implied by **F**

Example

- Consider a relation schema **R** = { **A**, **B**, **C**, **D**, **E**, **H** } with FDs: **F** = { **AB** $\rightarrow$ **C**, **BC** $\rightarrow$ **AD**, **D** $\rightarrow$ **E**, **CH** $\rightarrow$ **B** }
- True/False: Does **D** $\rightarrow$ **A** follow from **F**?
- Two steps:
  1. Compute $X^+ = \{D\}^+ = \{D, E\}$
  2. Check if $A \in X^+$
  - Since $A \notin \{D, E\}$, the answer is NO, i.e., $F \not\models D \rightarrow A$

Closures and Keys

- Consider a case where $\mathbf{X}^+$ includes **all** the attributes of a relation **R**
  - ➔ Clearly, **X** is a (super) **key** of **R**

➔To check if **X is a candidate key of R**, we should check 2 things:

1. If $\mathbf{X}^+$ is a superkey **R,** i.e., when $\mathbf{X}^+ = \mathbf{R}$, and
2. If **no** proper subset of **X** is a key, i.e., $\forall A \in \mathbf{X}$: $(\mathbf{X} – \{A\})^+ \neq \mathbf{R}$

- To find the keys of a relation, we can use the algorithm on slide 26
- This would be exponential in the number of attributes! Can do better?
- Knowledge about keys is essential to understand "Normal forms."