Correlated Equilibria & Learning

Omar Boufous

Supervised by

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PhD Thesis Defense

Presentation Outline

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Introduction

- A **generalization** of Nash equilibria with strong connections to Bayesian rationality introduced by Aumann
- A solution concept with appealing **computational properties**
- Basic **learning dynamics** naturally lead to correlated equilibria
- **Canonical** form characterizing the set of correlated equilibrium distributions
- Simple **implementation** using a correlation device
- Many **applications** in engineering, economics, etc.

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Model & Definitions

Finite non-cooperative game $G = (\mathcal{N}, (\mathcal{A}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$

- Set of players $\mathcal{N}$
- Action set $\mathcal{A}_i$ for each $i \in \mathcal{N}$
- Utility function $u_i : \prod_{i\in\mathcal{N}} \mathcal{A}_i = \mathcal{A} \to \mathbb{R}$

Correlation device $d = (\Omega, (\mathcal{P}_i)_{i\in\mathcal{N}}, \boldsymbol{q})$

- A sample space Ω
- Partition $\mathcal{P}_i$ of Ω for each $i \in \mathcal{N}$
- Probability distribution $\boldsymbol{q}$ over Ω

Player i**'s set of strategies** is $\mathcal{S}_{i,d} = \{f_i : \Omega \to \mathcal{A}_i \text{ s.t. } f_i \text{ is } \mathcal{P}_i\text{-measurable}\}$ and the **set of strategy profiles** is $\mathcal{S}_d = \mathcal{S}_{1,d} \times \ldots \times \mathcal{S}_{n,d}$.

Definition 1 – Correlated equilibrium (Aumann, 1974)

A correlated equilibrium of G is a pair $(d, \boldsymbol{\alpha}^*)$ where $\boldsymbol{\alpha}^* : \Omega \to \mathcal{A}$ is a correlated strategy profile such that

$$\forall i \in \mathcal{N}, \forall \alpha'_i \in \mathcal{S}_{i,d} \quad \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\boldsymbol{\alpha}^*_i(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) \geq \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\alpha'_i(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) \tag{1}$$

A correlated equilibrium distribution of G (Forges, 2020) is a probability distribution $\boldsymbol{p} \in \Delta(\mathcal{A})$ such that,

$$\forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i \quad \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}_i, \boldsymbol{a}_{-i}) \geq \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\beta_i(\boldsymbol{a}_i), \boldsymbol{a}_{-i}) \tag{2}$$

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Part 1 : Learning Correlated Equilibria

Online Learning with Regret

The game G is played repeatedly through time $t = 1, 2, 3, \ldots$. Each step t, action profile $\boldsymbol{a}^t = (a_1^t, \ldots, a_n^)$ is played

Definition 3 – Regret (Hart & Mas-Colell, 2000)

The regret player i experiences at time t for any pair of actions $j, k \in \mathcal{A}_i$ is

$$R_i^t(j,k) = max\left(0, \frac{1}{t} \sum_{\tau \leq t : a_i^\tau = j} \left[u_i\left(k, \boldsymbol{a}_{-i}^\tau\right) - u_i(\boldsymbol{a}^\tau)\right]\right) = max\left(0, D_i^t(j,k)\right) \tag{3}$$

Regret-Matching algorithm (Hart & Mas-Colell, 2000)

$$\begin{cases} p_i^{t+1}(k) = \dfrac{1}{\mu} R_i^t(j,k), \\ p_i^{t+1}(j) = 1 - \displaystyle\sum_{k \in \mathcal{A}_i : k \neq j} p_i^{t+1}(k). \end{cases}$$

Proposition 1 (Hart & Mas-Colell, 2000)

Let $(\boldsymbol{a}^t)_{t=1,2,\ldots}$ be a sequence of plays (i.e., $\boldsymbol{a}^t \in \mathcal{A}$ for all t) and let $\varepsilon \geq 0$. Then: $\limsup_{t \to \infty} R_i^t(j,k) \leq \varepsilon$ for every $i \in \mathcal{N}$ and every $j, k \in \mathcal{A}_i$ with $j \neq k$, if and only if the sequence of empirical distributions $\boldsymbol{q}^t$ converges to the set of correlated ε-equilibria.

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$$\begin{cases} p_i^{t+1}(k) = \dfrac{1}{\mu} R_i^t(j,k), \\ p_i^{t+1}(j) = 1 - \displaystyle\sum_{k \in \mathcal{A}_i : k \neq j} p_i^{t+1}(k). \end{cases}$$

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The game G is played repeatedly through time $t = 1, 2, 3, \ldots$. Each step t, action profile $\boldsymbol{a}^t = (a_1^t, \ldots, a_n^)$ is played

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Regret-Matching algorithm applied to a 3-by-2 game

	D	E
A	(2, 29)	(16, 7)
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Table 1: Utility matrix.

- The empirical distribution does not seem to converge towards a correlated equilibrium point
- We propose a new dynamic called Correlated Perturbed Regret Minimization (CPRM)

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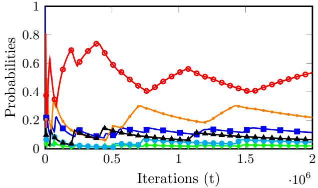

Figure 2: Evolution of the empirical probability distribution.

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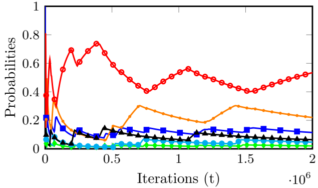

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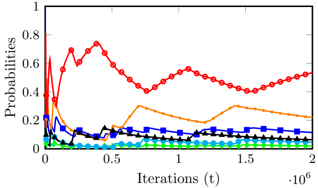

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CPRM Algorithm

Inspired from the learning algorithm introduced in (Young, H. P., 2009) for learning pure Nash equilibria.

At time t, each player i is characterized by a mood $m_i^t \in \{syn, asyn\}$

asynchronous
player

synchronous
player

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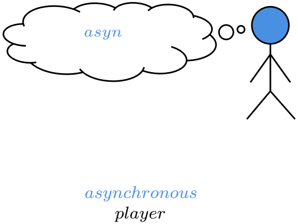

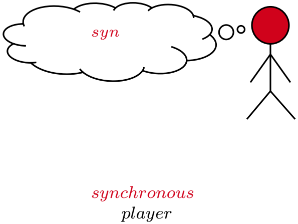

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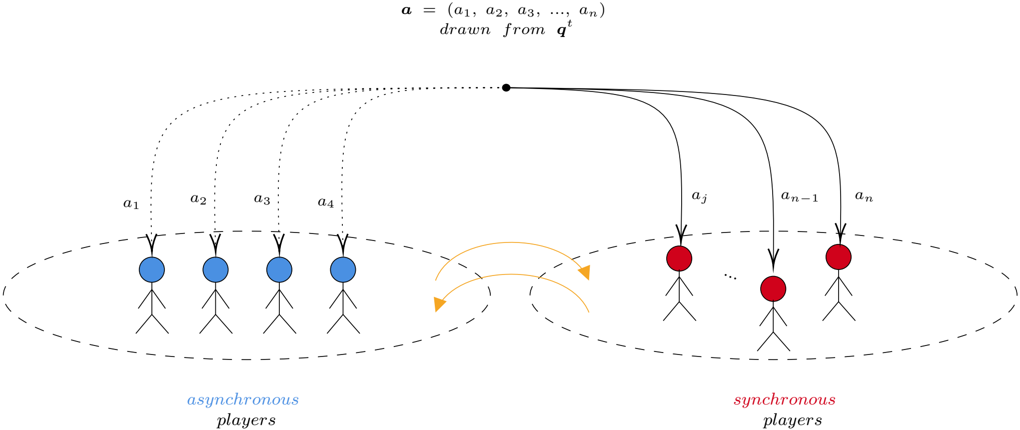
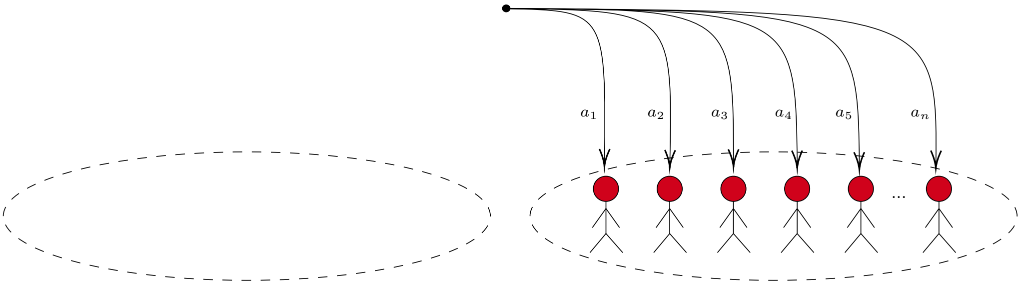


Players implementing a regret-minimizing strategy

Players play the actions drawn from the empirical distribution

CPRM Algorithm

$\boldsymbol{a} = (a_1, a_2, a_3, \ldots, a_n)$
$drawn \ from \ \boldsymbol{q}^t$



asynchronous
players

synchronous
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In the long run
↪ We expect players to play the action profiles drawn from the current empirical distributions to stabilize the play at an equilibrium point.

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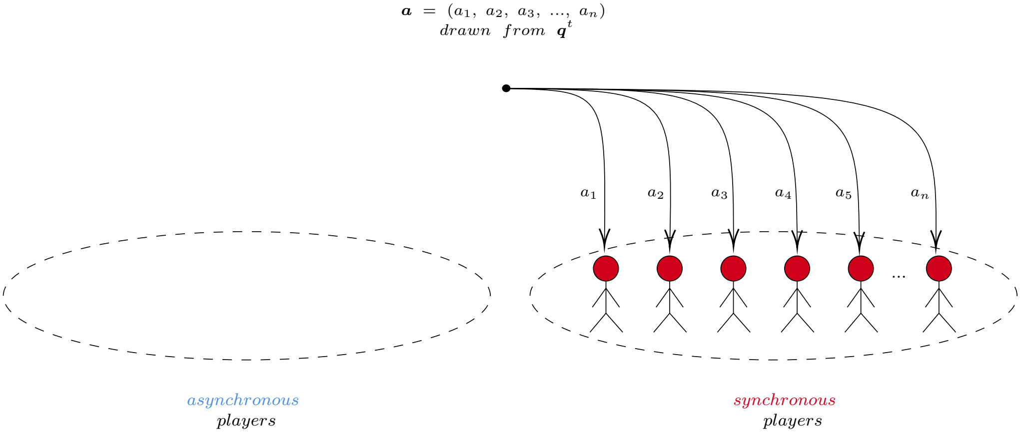

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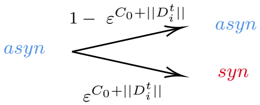
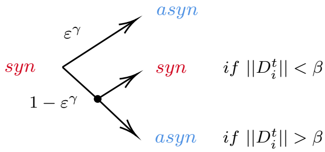
Algorithm Analysis

Mood dynamics





Characteristics of the stochastic process

- Transition probabilities depend on the regrets and a perturbation parameter $\varepsilon > 0$
- The stochastic process $\boldsymbol{X}^t = (\boldsymbol{X}_1^t, ..., \boldsymbol{X}_n^t)$ describing players' states is a perturbed non-homogeneous Markov chain
- The Markov chain is defined over a infinitely countable state spaces
- For a regularly perturbed homogenous Markov chain, the process spends almost all time in the stochastically stable states when $\varepsilon \to 0$

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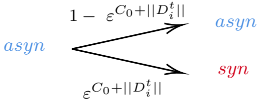

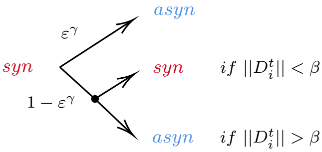

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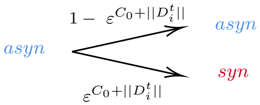

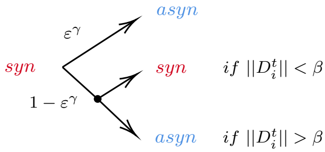

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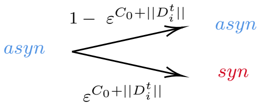

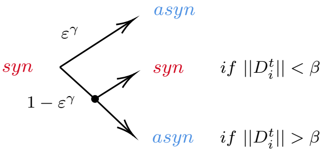

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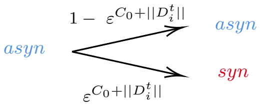

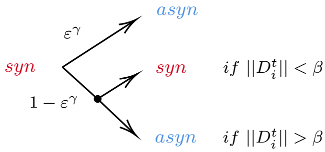

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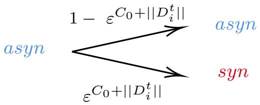

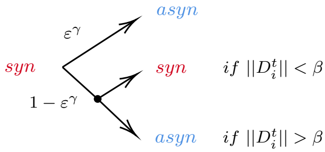

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Simulation Results

	D	E
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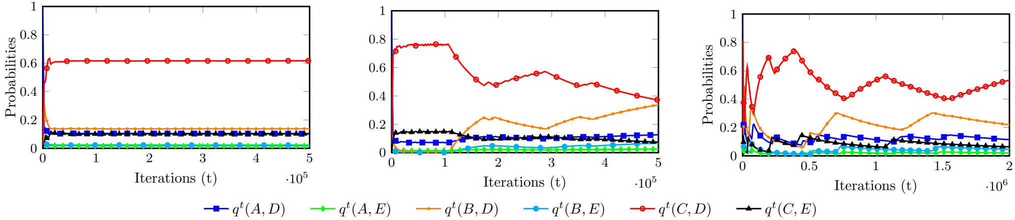
Table 2: Utility matrix.

Simulation Results



(a) CPRM algorithm (b) Regret minimizing strategy (c) Regret-matching

Figure 3: Evolution of the empirical distribution over action profiles.

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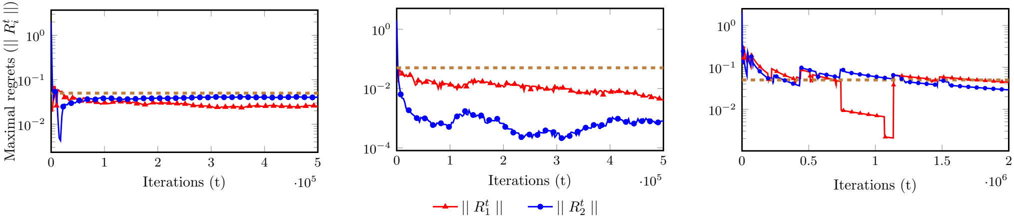

(a) CPRM algorithm.

(b) Regret minimizing strategy.

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Figure 4: Evolution of players' regrets for the three algorithms.

Time-varying Game

	D	E
A	(2, 29)	(16, 7)
B	(4, 7)	(6, 13)
C	(4, 4)	(6, 6)

↓

	D	E
A	(2, 29, 2)	(16, 7, 8)
B	(4, 7, 2)	(6, 13, 0)
C	(4, 4, 1)	(6, 6, 5)

X

	D	E
A	(9, 4, 0)	(4, 1, 4)
B	(8, 0, 1)	(6, 7, 2)
C	(11, 9, 3)	(2, 0, 4)

Y

↓

	D	E
A	(2, 29)	(16, 7)
B	(4, 7)	(6, 13)
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↓

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Y

Figure 5: Sequence of stage games in the dynamic case.

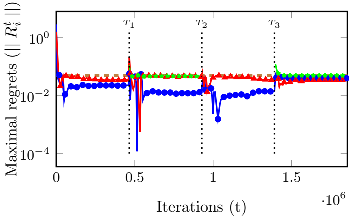

Figure 6: Evolution of maximal regrets

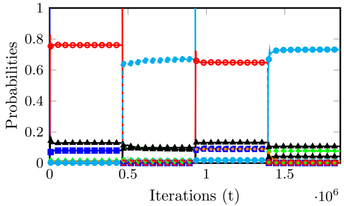

Figure 7: Evolution of the empirical distribution

Application to a Congestion Game

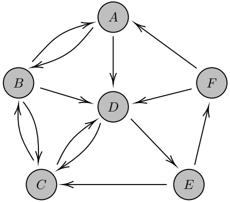

Figure 8: Network graph

Player	Source node	Destination node
1	B	F
2	B	E
3	D	B
4	F	E

Figure 9: Source-destination nodes for each player

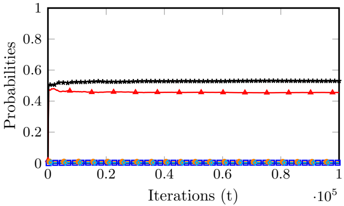

Figure 10: Evolution of the empirical distribution

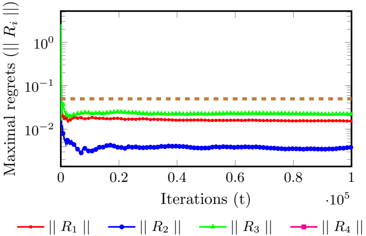

Figure 11: Evolution of maximal regrets

Part 2 : Constrained Correlated Equilibria

Constrained Correlated Equilibrium Strategies

Definition 1 – Correlated equilibrium (Aumann, 1974)

A correlated equilibrium of G is a pair $(d, \boldsymbol{\alpha}^*)$ where $\boldsymbol{\alpha}^* : \Omega \to \mathcal{A}$ is a correlated strategy profile such that

$$\forall i \in \mathcal{N}, \forall \alpha_i' \in \mathcal{S}_{i,d} \quad \sum_{\omega \in \Omega} \boldsymbol{q}(\omega) u_i(\boldsymbol{\alpha}_i^*(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \geq \sum_{\omega \in \Omega} \boldsymbol{q}(\omega) u_i(\alpha_i'(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \tag{4}$$

	P	A
P	8, 8	3, 10
A	10, 3	0, 0

Table 3: Utility matrix of the game of Chicken.

- $\Omega = \{H, M, L\}$
- $\mathcal{P}_1 = \{\{H\}, \{M, L\}\}$ and $\mathcal{P}_2 = \{\{H, M\}, \{L\}\}$
- $\boldsymbol{q}(H) = \boldsymbol{q}(M) = \boldsymbol{q}(L) = 1/3$

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- $\boldsymbol{q}(H) = \boldsymbol{q}(M) = \boldsymbol{q}(L) = {}^1/_3$

Constrained Correlated Equilibrium Strategies

	P	A
P	8, 8	3, 10
A	10, 3	0, 0

Table 4: Utility matrix of the game of Chicken.

- $\Omega = \{H, M, L\}$
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	$L \mapsto P$ $L^c \mapsto P$	$L \mapsto A$ $L^c \mapsto A$	$L \mapsto A$ $L^c \mapsto P$	$L \mapsto P$ $L^c \mapsto A$
$H \mapsto P$ $H^c \mapsto P$	$8, 8$	$3, 10$	$6.33, 8.67$	$4.67, 9.33$
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Figure 12: Extension of the game of Chicken.

	$L \mapsto P$ $L^c \mapsto P$	$L \mapsto A$ $L^c \mapsto A$	$L \mapsto A$ $L^c \mapsto P$	$L \mapsto P$ $L^c \mapsto A$
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Figure 14: Constrained extension of the game of Chicken.

Definition 4 – Constrained Correlated equilibrium (Boufous et al., 2024)
A correlated equilibrium of G is a pair (d, α^*) where $\alpha^* : \Omega \to \mathcal{A}$ is a correlated strategy profile such that

$$\forall i \in \mathcal{N}, \forall \alpha_i' : \Omega \to \mathcal{A}_i \quad \sum_{\omega \in \Omega} q(\omega) u_i(\alpha_i^*(\omega), \alpha_{-i}^*(\omega)) \geq \sum_{\omega \in \Omega} q(\omega) u_i(\alpha_i'(\omega), \alpha_{-i}^*(\omega)) \tag{5}$$

Constrained Correlated Equilibrium Strategies

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Properties of Constrained Correlated Equilibrium Strategies

Consider a finite non-cooperative game $G = (\mathcal{N}, (\mathcal{A}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$, a correlation device $d = (\Omega, (\mathcal{P}_i)_{i\in\mathcal{N}}, \boldsymbol{q})$ and a constraint set $\mathcal{R}_d \subset \mathcal{S}_d$.

Proposition 1 : If $(d, \boldsymbol{\alpha}^*)$ is a correlated equilibrium and $\boldsymbol{\alpha}^* \in \mathcal{R}_d$, then $(d, \mathcal{R}_d, \boldsymbol{\alpha}^*)$ is a constrained correlated equilibrium.

Proposition 2 : If $\boldsymbol{\alpha}^* \in \mathcal{R}_d$ and for any $i \in \mathcal{N}$, for any α_i' s.t. $(\alpha_i', \boldsymbol{\alpha}_{-i}) \in \mathcal{R}_d$, for any $\omega \in \Omega$

$$\sum_{\omega' \in P_i(\omega)} \boldsymbol{q}(\omega') \left[u_i(\alpha_i^*(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) - u_i(\alpha_i'(\omega'), \boldsymbol{\alpha}_{-i}^*(\omega')) \right] \geq 0 \tag{6}$$

then $(d, \mathcal{R}_d, \boldsymbol{\alpha}^*)$ is a constrained correlated equilibrium.

Proposition 3 : The triplet $(d, \mathcal{R}_d, \boldsymbol{\alpha}^*)$ is a constrained correlated equilibrium if and only if $\boldsymbol{\alpha}^* \in \mathcal{R}_d$ and for any $i \in \mathcal{N}$, for any $\alpha_i' \in \mathcal{S}_{i,d}$,

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Constraints induced by a set of feasible probability distributions

- Let $\mathcal{C} \subseteq \Delta(\mathcal{A})$ be a set of probability distributions. For each correlation device d,

$$\mathcal{R}_d = \{\boldsymbol{\alpha} \in \mathcal{S}_d \mid \boldsymbol{p}_{\boldsymbol{\alpha}} \in \mathcal{C}\} \tag{8}$$

- Performance measures in many applications can be expressed in terms of probability distribution over action profiles. Examples include applications in smart grids, wireless networks, etc.
  - Constraints on the social welfare : $\sum_{i \in \mathcal{N}} \sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}) \geq D_1$
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Theorem 1 – Characterization of the Set of Constrained Correlated Equilibrium Distributions

Let G be a finite non-cooperative game and $\mathcal{C}$ a set of feasible probability distributions. The distribution $\boldsymbol{p} \in \Delta(\mathcal{A})$ is a constrained correlated equilibrium distribution if and only if for any player $i \in \mathcal{N}$, for any strategy $\beta_i : \mathcal{A}_i \to \mathcal{A}_i$, if $\boldsymbol{z}_{\beta_i, \boldsymbol{p}} \in \mathcal{C}$, then

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Constraints induced by a set of feasible probability distributions

- Let $\mathcal{C} \subseteq \Delta(\mathcal{A})$ be a set of probability distributions. For each correlation device d,

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- Performance measures in many applications can be expressed in terms of probability distribution over action profiles. Examples include applications in smart grids, wireless networks, etc.
  - Constraints on the social welfare : $\displaystyle\sum_{i \in \mathcal{N}} \sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}) \geq D_1$
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Theorem 1 – Characterization of the Set of Constrained Correlated Equilibrium Distributions

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Existence of Constrained Correlated Equilibria

Consider the following two-player game:

	L	R
U	$(2,2)$	$(1,1)$
D	$(3,0)$	$(0,5)$

Figure 13: Two-player game in matrix form.

Let $\mathcal{C} \subset \Delta(\mathcal{A})$ be a feasible set of probability distributions such that,

$$\mathcal{C} = \{\boldsymbol{p} \in \Delta(\mathcal{A}) \mid \boldsymbol{p}(U,L) = 1 \text{ or } \boldsymbol{p}(U,R) = 1 \text{ or } \boldsymbol{p}(D,L) = 1 \text{ or } \boldsymbol{p}(D,R) = 1\} \tag{10}$$

- The set of feasible strategies is the set of pure action profiles
- The players must play a correlated strategy profile inducing a pure action profile in G
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Theorem 2 - Existence of Constrained Correlated Equilibria

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Define the mixed extension of G by the game $\Delta G = (\mathcal{N}, (\Delta(\mathcal{A}_i))_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$ where $\Delta(\mathcal{A}_i) = \{\boldsymbol{p} \in \mathbb{R}_+^{|\mathcal{A}_i|} \mid \sum_{a_i\in\mathcal{A}_i} \boldsymbol{p}(a_i) = 1\}$ is the set of probability distributions on $\mathcal{A}_i$.

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Computation of Constrained Correlated Equilibria

$$\text{maximize } 0$$
$$\text{s.t. } \boldsymbol{p}(\boldsymbol{a}) \geq 0 \quad \forall \boldsymbol{a} \in \mathcal{A}, \quad \Sigma_{\boldsymbol{a}\in\mathcal{A}}\boldsymbol{p}(\boldsymbol{a}) = 1 \tag{13}$$
$$\forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i$$
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$$\forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i \text{ s.t. } \boldsymbol{z}_{\beta_i,\boldsymbol{p}} \in \mathcal{C}$$
$$\sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) \left[u_i(\boldsymbol{a}) - u_i(\beta_i(a_i), \boldsymbol{a}_{-i})\right] \geq 0, \tag{16}$$
$$\boldsymbol{p} \in \mathcal{C} \tag{17}$$

Assuming linear constraints e.g., $\mathcal{C} = \{p \in \Delta(\mathcal{A}) \mid Fp \leq 0\}$, we show that the problem of computing a constrained correlated equilibrium distribution **can be formulated as a Mixed-Integer Linear Program**.

Computation of Constrained Correlated Equilibria

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& \forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i \text{ s.t. } \boldsymbol{z}_{\beta_i, \boldsymbol{p}} \in \mathcal{C} \\
& \sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) \left[u_i(\boldsymbol{a}) - u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \right] \geq 0, && (16) \\
& \boldsymbol{p} \in \mathcal{C} && (17)
\end{aligned}
$$

Assuming linear constraints e.g., $\mathcal{C} = \{\boldsymbol{p} \in \Delta(\mathcal{A}) \mid F\boldsymbol{p} \leq 0\}$, we show that the problem of computing a constrained correlated equilibrium distribution **can be formulated as a Mixed-Integer Linear Program**.

Computation of Constrained Correlated Equilibria

$$\begin{aligned} & \text{maximize } 0 \\ \text{s.t. } & \boldsymbol{p}(\boldsymbol{a}) \geq 0 \quad \forall \boldsymbol{a} \in \mathcal{A}, \quad \Sigma_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) = 1 \qquad (13) \\ & \forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i \\ & \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) \left[u_i(\boldsymbol{a}) - u_i(\beta_i(a_i), \boldsymbol{a}_{-i})\right] \geq 0. \qquad (14) \end{aligned}$$

$$\begin{aligned} & \text{maximize } 0 \\ \text{s.t. } & \boldsymbol{p}(\boldsymbol{a}) \geq 0 \quad \forall \boldsymbol{a} \in \mathcal{A}, \quad \Sigma_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) = 1 \qquad (15) \\ & \forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i \text{ s.t. } \boldsymbol{z}_{\beta_i,\boldsymbol{p}} \in \mathcal{C} \\ & \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) \left[u_i(\boldsymbol{a}) - u_i(\beta_i(a_i), \boldsymbol{a}_{-i})\right] \geq 0, \qquad (16) \\ & \boldsymbol{p} \in \mathcal{C} \qquad (17) \end{aligned}$$

Assuming linear constraints e.g., $\mathcal{C} = \{\boldsymbol{p} \in \Delta(\mathcal{A}) \mid F\boldsymbol{p} \leq 0\}$, we show that the problem of computing a constrained correlated equilibrium distribution **can be formulated as a Mixed-Integer Linear Program**.

Simulation Example

	P	A
P	8, 8	3, 10
A	10, 3	0, 0

Table 3: Game of Chicken

$$SW(\boldsymbol{q}) = \sum_{i \in \mathcal{N}} u_i(\boldsymbol{q}) \qquad (18)$$

where $u_i(\boldsymbol{q}) = \sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{q}(\boldsymbol{a}) u_i(\boldsymbol{a})$

- Feasible utilities in green
- CE utilities in yellow
- **CCE** utilities in red

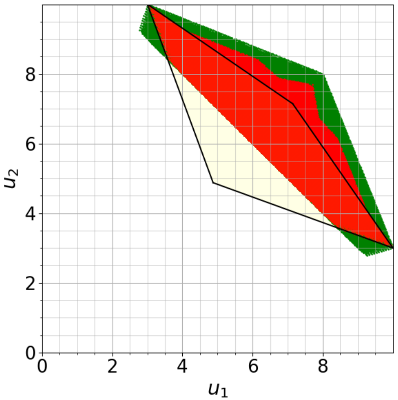


Figure 14: Sets of Utilities of the two players with $SW \geq 12$

Simulation Results

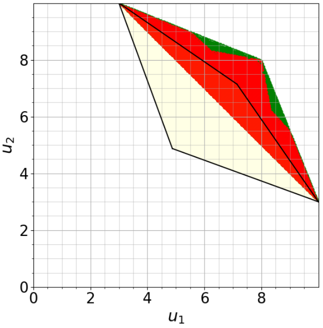


Figure 15: Sets of Utilities of the two players with $SW \geq 13$

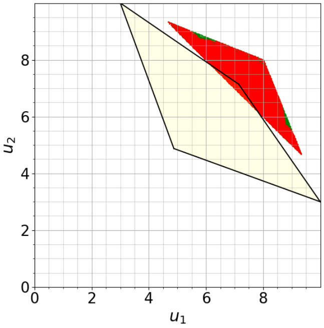


Figure 16: Sets of Utilities of the two players with $SW \geq 14$

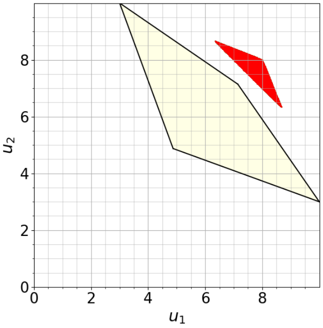


Figure 17: Sets of Utilities of the two players with $SW \geq 15$

- The set may not be convex
- There are constrained correlated equilibria outside the set of correlated equilibrium distributions

Simulation Results

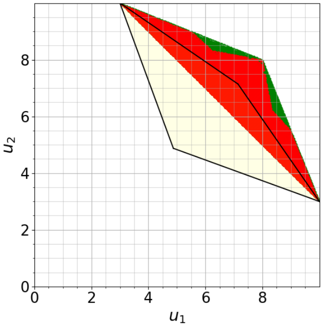
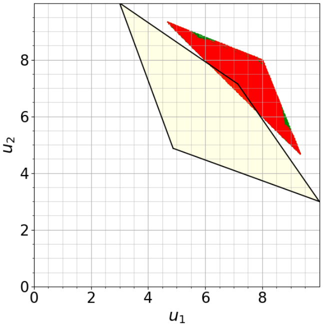


Figure 15: Sets of Utilities of the two players with $SW \geq 13$



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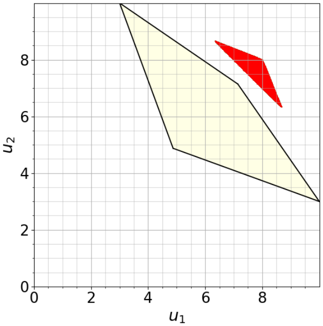

Figure 17: Sets of Utilities of the two players with $SW \geq 15$

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- There are constrained correlated equilibria outside the set of correlated equilibrium distributions

Simulation Results

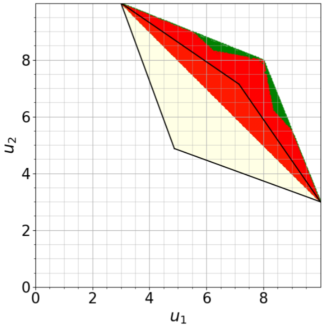
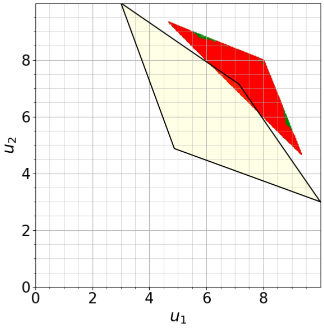


Figure 15: Sets of Utilities of the two players with $SW \geq 13$



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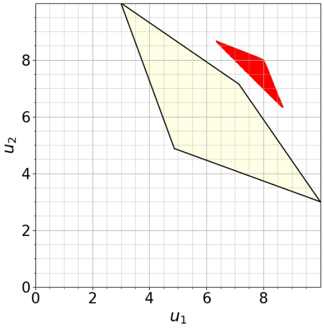

Figure 17: Sets of Utilities of the two players with $SW \geq 15$

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Conclusion & Highlight of Contributions

- **A learning rule to converge towards a correlated equilibrium distribution**
  - Empirical evidence of convergence of the distribution to a constrained correlated equilibrium
  - A Markov chain models the learning algorithm
  - The algorithm is adaptive to changes in games e.g., payoff functions or number of players
- **Arbitrary constraints and arbitrary correlation device**
  - **Characterizations of Equilibrium Strategies**
    - Definition through generalized Nash equilibrium and correlated equilibrium with alternative characterizations
  - **Properties and relationship to (unconstrained) Correlated Equilibria**
    - A feasible correlated equilibrium is a constrained correlated equilibrium
    - The set of constrained correlated equilibrium distributions may not be convex
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- **Constraints on probability distributions**
  - **Characterization of constrained correlated equilibrium distributions**
    - Closed-form expression of the set of constrained correlated equilibrium distribution
  - **Conditions of Existence**
    - There does not always exist a constrained correlated equilibrium in finite games
  - **Constrained correlated equilibrium distributions of the mixed extension of a game**
    - Every constrained correlated equilibrium distribution of ΔG is a constrained correlated equilibrium distribution of G
  - **Computation of linearly constrained correlated equilibria**

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Open Perspectives

- **Research Questions**
  - Are there any theoretical guarantees of convergence to the a correlated equilibrium distribution?
  - Connection to Bayesian rationality still holds in the presence of constraints ?
  - What happens in the case of infinite games ?
  - Existence problem should be studied for weaker or alternative assumptions
  - Study of the problem of computing constrained correlated equilibria (complexity, feasbility)
- **Some Unexplored Topics**
  - Learning rule leading to a specific correlated equilibrium (e.g., social welfare maximizing)
  - Learning with constraints
  - Subjective correlated equilibrium
- **Potential applications of constrained correlated equilibria and learning correlated equilibria in various fields**
  - Mobile Edge Computing Systems
  - Wireless Networks with constraints
  - Engineering and beyond

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Thank you!

Questions?

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Computation of Constrained Correlated Equilibria

Mixed-Integer Linear Program

$$\boldsymbol{p} \geq 0, \Sigma_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) = 1, F\boldsymbol{p} \leq 0 \tag{19}$$

$$\left[U_i - U_i B_{\beta_i}\right] \boldsymbol{p} \leq M_i \times \boldsymbol{y}_{\beta_i} \quad \forall i \in \mathcal{N}, \quad \forall \beta_i \tag{20}$$

$$F B_{\beta_i} \boldsymbol{p} \geq -K \left(1 - \boldsymbol{y}_{\beta_i}\right) + \delta \quad \forall i \in \mathcal{N}, \quad \forall \beta_i \tag{21}$$

$$F B_{\beta_i} \boldsymbol{p} \leq K y_{\beta_i} \quad \forall i \in \mathcal{N}, \quad \forall \beta_i \tag{22}$$

$$y_{\beta_i} \in \{0, 1\} \quad \forall i \in \mathcal{N}, \quad \forall \beta_i \tag{23}$$