

Correlated Equilibria & Learning

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PhD Defense

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Problem & Related work

Correlated Equilibria

- ▶ Defined in (Aumann, 1974) and (Aumann, 1987)
- ▶ A second proof of existence and a generalization to infinite games in (Hart & Schmeidler, 1989)
- ▶ Defined for the extensive form in (Von Stengel & Forges, 2008)
- ▶ Other extensions in (Forges, 2020), (Brandenburger & Dekel, 1992) and (Grant & Stauber, 2022)

Constraints & Generalized Equilibria

- ▶ (Debreu, 1952) defines the concept of generalized equilibrium
- ▶ (Arrow & Debreu, 1954) shows a proof of existence
- ▶ (Rosen, 1965) considers coupled constraints and shows the existence and uniqueness of equilibria in concave games
- ▶ Many other theoretical and applied contributions.

⇒ A solution concept combining **correlation and constraints** has **not yet been studied** in the literature

⇒ We consider this problem for **finite non-cooperative games** and **propose a solution**

Related work



While working on this topic, another paper dealing with the same issue came out
The author contacted us

Highlights of the Contributions

1. Arbitrary constraints and arbitrary correlation device

1.1 Characterizations of Equilibrium Strategies

- ▶ Definition derived from the generalized Nash equilibrium and correlated equilibrium
- ▶ Alternative characterizations of constrained correlated strategies

1.2 Properties and relationship to (unconstrained) Correlated Equilibria

- ▶ A feasible correlated equilibrium is a constrained correlated equilibrium
- ▶ The set of constrained correlated equilibrium distributions may not be convex
- ▶ There exist constrained correlated equilibrium distributions outside the set of correlated equilibrium distributions

2. Constraints on probability distributions

2.1 Characterization of constrained correlated equilibrium distributions

- ▶ Closed-form expression of the set of constrained correlated equilibrium distribution

2.2 Sufficient Conditions of Existence

- ▶ There does not always exist a constrained correlated equilibrium in finite games
- ▶ The existence of a constrained correlated equilibrium

2.3 Constrained correlated equilibrium distributions of the mixed extension of a game

- ▶ The set of constrained correlated equilibrium distribution of ΔG is included in the set of constrained correlated equilibrium of G

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State-of-the-art

X

Motivation

- ▶ Planning, on the fly, a path from a starting position such that the **robot covers every point in an initially unknown spatial environment**.
- ▶ Currently,
 - ▶ Finding an optimal path that visits every node in a graph exactly once is **NP-hard problem**.
 - ▶ **Approximate and heuristic solutions** are usually used for the complete coverage path planning task.
 - ▶ Most methods rely on the **a priori knowledge of the map of the environment** and cope with unknown obstacles detected by range sensors.
- ▶ Objectives:
 - ▶ **Partially or completely unknown environments** (i.e. exploration task).
 - ▶ Cover as close to 100% of the land as possible.
 - ▶ Avoid double coverage of areas.
 - ▶ Avoid obstacles and impassable areas.
 - ▶ Be as efficient as possible, i.e., keep costs to a minimum to prevent unnecessary, wastage of time and resources

Contributions

Background & Context

Correlated Equilibria

- ▶ A **generalization** of Nash equilibria (Aumann, 74)(Aumann, 87) resulting from Bayesian rationality
- ▶ A solution concept with appealing **computational properties**
- ▶ **Regret-learning** dynamics naturally lead to correlated equilibria (Hart & Mas-Colell, 00)
- ▶ Many other theoretical and applied contributions in engineering, economics, etc.
- ▶ Extensions to extensive form (Von Stengel & Forges, 08) and infinite games (Hart & Schmeidler, 89)
- ▶ Other generalizations in (Forges, 20), (Brandenburger & Dekel, 92) and (Grant & Stauber, 22)

Constraints in Games

- ▶ Generalized Nash equilibrium problem (Facchinei, et al., 07) (Fischer, et al., 14)
- ▶ (Debreu, 52) defines the concept of generalized equilibrium
- ▶ A proof of existence is presented in (Arrow & Debreu, 54)
- ▶ (Rosen, 65) shows existence and uniqueness of equilibria in concave games with coupled constraints
- ▶ (Bernasconi et al., 23) introduces the concept of constrained Phi-equilibrium

In this work,

- ▶ We **define** and **characterize** a new solution concept called **constrained correlated equilibrium**
- ▶ Several characterizations of equilibrium strategies and study their properties
 - ▶ sufficient **conditions** of **existence** in case of **constraints on probability distributions over action profiles**
 - ▶ characterization by **canonical correlation devices**
 - ▶ linearly constrained correlated equilibrium problem → **MILP**

Model & Definitions

Finite non-cooperative game $G = (\mathcal{N}, (\mathcal{A}_i)_{i \in \mathcal{N}}, (u_i)_{i \in \mathcal{N}})$

- ▶ Set of players \mathcal{N}
- ▶ Action set \mathcal{A}_i for each $i \in \mathcal{N}$
- ▶ Utility function $u_i : \prod_{i \in \mathcal{N}} \mathcal{A}_i = \mathcal{A} \rightarrow \mathbb{R}$

Correlation device $d = (\Omega, (\mathcal{P}_i)_{i \in \mathcal{N}}, \mathbf{q})$

- ▶ A sample space Ω
- ▶ Partition \mathcal{P}_i of Ω for each $i \in \mathcal{N}$
- ▶ Probability distribution \mathbf{q} over Ω

Player i 's **set of strategies** is $\mathcal{S}_{i,d} = \{f_i : \Omega \rightarrow \mathcal{A}_i \text{ s.t. } f_i \text{ is } \mathcal{P}_i\text{-measurable}\}$ and the **set of strategy profiles** is $\mathcal{S}_d = \mathcal{S}_{1,d} \times \dots \times \mathcal{S}_{n,d}$.

Definition 1 – Correlated equilibrium (Aumann, 1974)

A correlated equilibrium of G is a pair (d, α^*) where $\alpha^* : \Omega \rightarrow \mathcal{A}$ is a correlated strategy profile such that

$$\forall i \in \mathcal{N}, \forall \alpha'_i : \Omega \rightarrow \mathcal{A}_i \quad \sum_{\omega \in \Omega} \mathbf{q}(\omega) u_i(\alpha_i^*(\omega), \alpha_{-i}^*(\omega)) \geq \sum_{\omega \in \Omega} \mathbf{q}(\omega) u_i(\alpha'_i(\omega), \alpha_{-i}^*(\omega)) \quad (1)$$

Given a **constraint set** $\mathcal{R}_d \subseteq \mathcal{S}_d$, we define a constrained correlated equilibrium,

Definition 2 – Constrained correlated equilibrium (Boufous et al., 2024)

A constrained correlated equilibrium of G is a triplet $(d, \mathcal{R}_d, \alpha^*)$ where $\alpha^* : \Omega \rightarrow \mathcal{A}$ is a correlated strategy profile such that $\alpha^* \in \mathcal{R}_d$ and

$$\forall i \in \mathcal{N}, \forall \alpha'_i : \Omega \rightarrow \mathcal{A}_i \text{ s.t. } (\alpha'_i, \alpha_{-i}^*) \in \mathcal{R}_d \quad \sum_{\omega \in \Omega} \mathbf{q}(\omega) u_i(\alpha_i^*(\omega), \alpha_{-i}^*(\omega)) \geq \sum_{\omega \in \Omega} \mathbf{q}(\omega) u_i(\alpha'_i(\omega), \alpha_{-i}^*(\omega)) \quad (2)$$

Example : a 2-by-2 Matrix Game

Two-player game G

	P	A
P	8, 8	3, 10
A	10, 3	0, 0

Correlation device d

- ▶ $\Omega = \{H, M, L\}$
- ▶ $\mathcal{P}_1 = \{\{H\}, \{M, L\}\}$ and $\mathcal{P}_2 = \{\{H, M\}, \{L\}\}$
- ▶ $q(H) = q(M) = q(L) = 1/3$

Game G extended with correlation device d

	$L \mapsto P$ $L^c \mapsto P$	$L \mapsto A$ $L^c \mapsto A$	$L \mapsto A$ $L^c \mapsto P$	$L \mapsto P$ $L^c \mapsto A$
$H \mapsto P$ $H^c \mapsto P$	8, 8	3, 10	6.33, 8.67	4.67, 9.33
$H \mapsto A$ $H^c \mapsto A$	10, 3	0, 0	6.67, 2	3.33, 1
$H \mapsto A$ $H^c \mapsto P$	8.67, 6.33	2, 6.67	7, 7	3.67, 6
$H \mapsto P$ $H^c \mapsto A$	9.33, 4.67	1, 3.33	6, 3.67	4.33, 4.33

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Constrained extended game

	$L \mapsto P$ $L^c \mapsto P$	$L \mapsto A$ $L^c \mapsto A$	$L \mapsto A$ $L^c \mapsto P$	$L \mapsto P$ $L^c \mapsto A$
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Part 1 : Learning Correlated Equilibria

Motivation

Part 2 : Constrained Correlated Equilibria

Properties of Constrained Correlated Equilibrium Strategies

Consider a finite non-cooperative game $G = (\mathcal{N}, (\mathcal{A}_i)_{i \in \mathcal{N}}, (u_i)_{i \in \mathcal{N}})$, a correlation device $d = (\Omega, (\mathcal{P}_i)_{i \in \mathcal{N}}, q)$ and a constraint set $\mathcal{R}_d \subset \mathcal{S}_d$.

Proposition 1 : If (d, α^*) is a correlated equilibrium and $\alpha^* \in \mathcal{R}_d$, then $(d, \mathcal{R}_d, \alpha^*)$ is a constrained correlated equilibrium.

Proposition 2 : If $\alpha^* \in \mathcal{R}_d$ and for any $i \in \mathcal{N}$, for any α'_i s.t. $(\alpha'_i, \alpha_{-i}^*) \in \mathcal{R}_d$, for any $\omega \in \Omega$

$$\sum_{\omega' \in P_i(\omega)} q(\omega') \left[u_i(\alpha_i^*(\omega), \alpha_{-i}^*(\omega)) - u_i(\alpha'_i(\omega'), \alpha_{-i}^*(\omega')) \right] \geq 0 \quad (3)$$

then $(d, \mathcal{R}_d, \alpha^*)$ is a constrained correlated equilibrium.

Proposition 3 : The triplet $(d, \mathcal{R}_d, \alpha^*)$ is a constrained correlated equilibrium if and only if $\alpha^* \in \mathcal{R}_d$ and for any $i \in \mathcal{N}$, for any $\alpha'_i \in \mathcal{S}_{i,d}$,

$$\sum_{\omega \in \Omega} q(\omega) \left[u_i(\alpha_i^*(\omega), \alpha_{-i}^*(\omega)) - u_i(\alpha'_i(\omega), \alpha_{-i}^*(\omega)) \right] \geq 0 \quad \text{or} \quad (\alpha'_i, \alpha_{-i}^*) \notin \mathcal{R}_d \quad (4)$$

Motivation

Open Perspectives

X

Thank you!



Questions?

For more information:

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