Correlated Equilibria & Learning

Omar Boufous

Supervised by

Rachid El-Azouzi[2], Mikael Touati[1], Eitan Altman[2,3], Mustapha Bouhtou[1]

[1]Orange, Châtillon, France
[2]CERI/LIA, Université d'Avignon, Avignon, France
[3]INRIA Sophia Antipolis, France

December 20, 2024

PhD Thesis Defense

Presentation Outline

1. **Background & Preliminaries**

   1.1 Introduction
   1.2 Related Work
   1.3 Notations & Model

2. **Learning Correlated Equilibria**

   2.1 Online Learning with Regret
   2.2 Algorithm Description
   2.3 Stochastic Model
   2.4 Simulation Results
   2.5 Time-varying Game
   2.6 Application to a Congestion Game

3. **Constrained Correlated Equilibria**

   3.1 Definition & Example
   3.2 Properties of Constrained Correlated Equilibrium Strategies
   3.3 Canonical Representation and Existence of Constrained Correlated Equilibria
   3.4 Computation of Constrained Correlated Equilibria
   3.5 Simulation Results

4. **Conclusions & Possible Directions of Research**

Presentation Outline

1. **Background & Preliminaries**

   1.1 Introduction
   1.2 Related Work
   1.3 Notations & Model

2. **Learning Correlated Equilibria**

   2.1 Online Learning with Regret
   2.2 Algorithm Description
   2.3 Stochastic Model
   2.4 Simulation Results
   2.5 Time-varying Game
   2.6 Application to a Congestion Game

3. **Constrained Correlated Equilibria**

   3.1 Definition & Example
   3.2 Properties of Constrained Correlated Equilibrium Strategies
   3.3 Canonical Representation and Existence of Constrained Correlated Equilibria
   3.4 Computation of Constrained Correlated Equilibria
   3.5 Simulation Results

4. **Conclusions & Possible Directions of Research**

Presentation Outline

1. **Background & Preliminaries**
   - 1.1 Introduction
   - 1.2 Related Work
   - 1.3 Notations & Model
2. **Learning Correlated Equilibria**
   - 2.1 Online Learning with Regret
   - 2.2 Algorithm Description
   - 2.3 Stochastic Model
   - 2.4 Simulation Results
   - 2.5 Time-varying Game
   - 2.6 Application to a Congestion Game
3. **Constrained Correlated Equilibria**
   - 3.1 Definition & Example
   - 3.2 Properties of Constrained Correlated Equilibrium Strategies
   - 3.3 Canonical Representation and Existence of Constrained Correlated Equilibria
   - 3.4 Computation of Constrained Correlated Equilibria
   - 3.5 Simulation Results
4. **Conclusions & Possible Directions of Research**

Presentation Outline

1. **Background & Preliminaries**

   1.1 Introduction
   1.2 Related Work
   1.3 Notations & Model

2. **Learning Correlated Equilibria**

   2.1 Online Learning with Regret
   2.2 Algorithm Description
   2.3 Stochastic Model
   2.4 Simulation Results
   2.5 Time-varying Game
   2.6 Application to a Congestion Game

3. **Constrained Correlated Equilibria**

   3.1 Definition & Example
   3.2 Properties of Constrained Correlated Equilibrium Strategies
   3.3 Canonical Representation and Existence of Constrained Correlated Equilibria
   3.4 Computation of Constrained Correlated Equilibria
   3.5 Simulation Results

4. **Conclusions & Possible Directions of Research**

Presentation Outline

1. **Background & Preliminaries**

   1.1 Introduction
   1.2 Related Work
   1.3 Notations & Model

2. **Learning Correlated Equilibria**

   2.1 Online Learning with Regret
   2.2 Algorithm Description
   2.3 Stochastic Model
   2.4 Simulation Results
   2.5 Time-varying Game
   2.6 Application to a Congestion Game

3. **Constrained Correlated Equilibria**

   3.1 Definition & Example
   3.2 Properties of Constrained Correlated Equilibrium Strategies
   3.3 Canonical Representation and Existence of Constrained Correlated Equilibria
   3.4 Computation of Constrained Correlated Equilibria
   3.5 Simulation Results

4. **Conclusions & Possible Directions of Research**

Presentation Outline

1. **Background & Preliminaries**
   - 1.1 Introduction
   - 1.2 Related Work
   - 1.3 Notations & Model
2. **Learning Correlated Equilibria**
   - 2.1 Online Learning with Regret
   - 2.2 Algorithm Description
   - 2.3 Stochastic Model
   - 2.4 Simulation Results
   - 2.5 Time-varying Game
   - 2.6 Application to a Congestion Game
3. **Constrained Correlated Equilibria**
   - 3.1 Definition & Example
   - 3.2 Properties of Constrained Correlated Equilibrium Strategies
   - 3.3 Canonical Representation and Existence of Constrained Correlated Equilibria
   - 3.4 Computation of Constrained Correlated Equilibria
   - 3.5 Simulation Results
4. Conclusions & Possible Directions of Research

Presentation Outline

1. **Background & Preliminaries**
   - 1.1 Introduction
   - 1.2 Related Work
   - 1.3 Notations & Model
2. **Learning Correlated Equilibria**
   - 2.1 Online Learning with Regret
   - 2.2 Algorithm Description
   - 2.3 Stochastic Model
   - 2.4 Simulation Results
   - 2.5 Time-varying Game
   - 2.6 Application to a Congestion Game
3. **Constrained Correlated Equilibria**
   - 3.1 Definition & Example
   - 3.2 Properties of Constrained Correlated Equilibrium Strategies
   - 3.3 Canonical Representation and Existence of Constrained Correlated Equilibria
   - 3.4 Computation of Constrained Correlated Equilibria
   - 3.5 Simulation Results
4. **Conclusions & Possible Directions of Research**

Introduction

- A **generalization** of Nash equilibria with strong connections to Bayesian rationality introduced by Aumann
- A solution concept with appealing **computational properties**
- Basic **learning dynamics** naturally lead to correlated equilibria
- **Canonical** form characterizing the set of correlated equilibrium distributions
- Simple **implementation** using a correlation device
- Many **applications** in engineering, economics, etc.

Introduction

- A **generalization** of Nash equilibria with strong connections to Bayesian rationality introduced by Aumann
- A solution concept with appealing **computational properties**
- Basic **learning dynamics** naturally lead to correlated equilibria
- **Canonical** form characterizing the set of correlated equilibrium distributions
- Simple **implementation** using a correlation device
- Many **applications** in engineering, economics, etc.

Figure 1: Robert John Aumann

Introduction

- A **generalization** of Nash equilibria with strong connections to Bayesian rationality introduced by Aumann
- A solution concept with appealing **computational properties**
- Basic **learning dynamics** naturally lead to correlated equilibria
- **Canonical** form characterizing the set of correlated equilibrium distributions
- Simple **implementation** using a correlation device
- Many **applications** in engineering, economics, etc.

Figure 1: Robert John Aumann

Introduction

- A **generalization** of Nash equilibria with strong connections to Bayesian rationality introduced by Aumann
- A solution concept with appealing **computational properties**
- Basic **learning dynamics** naturally lead to correlated equilibria
- **Canonical** form characterizing the set of correlated equilibrium distributions
- Simple **implementation** using a correlation device
- Many **applications** in engineering, economics, etc.

Figure 1: Robert John Aumann

Introduction

- A **generalization** of Nash equilibria with strong connections to Bayesian rationality introduced by Aumann
- A solution concept with appealing **computational properties**
- Basic **learning dynamics** naturally lead to correlated equilibria
- **Canonical** form characterizing the set of correlated equilibrium distributions
- Simple **implementation** using a correlation device
- Many **applications** in engineering, economics, etc.

Figure 1: Robert John Aumann

Introduction

- A **generalization** of Nash equilibria with strong connections to Bayesian rationality introduced by Aumann
- A solution concept with appealing **computational properties**
- Basic **learning dynamics** naturally lead to correlated equilibria
- **Canonical** form characterizing the set of correlated equilibrium distributions
- Simple **implementation** using a correlation device
- Many **applications** in engineering, economics, etc.

Figure 1: Robert John Aumann

Introduction

- A **generalization** of Nash equilibria with strong connections to Bayesian rationality introduced by Aumann
- A solution concept with appealing **computational properties**
- Basic **learning dynamics** naturally lead to correlated equilibria
- **Canonical** form characterizing the set of correlated equilibrium distributions
- Simple **implementation** using a correlation device
- Many **applications** in engineering, economics, etc.

Figure 1: Robert John Aumann

Introduction

- A **generalization** of Nash equilibria with strong connections to Bayesian rationality introduced by Aumann
- A solution concept with appealing **computational properties**
- Basic **learning dynamics** naturally lead to correlated equilibria
- **Canonical** form characterizing the set of correlated equilibrium distributions
- Simple **implementation** using a correlation device
- Many **applications** in engineering, economics, etc.

Figure 1: Robert John Aumann

Related Work

Correlated Equilibria

- Defined in (Aumann, 1974) and (Aumann, 1987)
- A proof of existence and a generalization to infinite games in (Hart & Schmeidler, 1989)
- Extensive form correlated equilibrium defined in (Von Stengel & Forges, 2008)
- Other extensions in (Forges, 2020), (Brandenburger & Dekel, 1992) and (Grant & Stauber, 2022)

Learning Correlated Equilibria

- Regret-matching (Hart & Mas-Colell, 2000) implying the convergence to the set of correlated equilibria
- Correlated Q-learning for selection of specific correlated equilibrium points (Greenwald & Hall, 2003)
- Convergence to the social welfare maximizing correlated equilibrium (Borowski et al., 2015)
- Learning algorithms for specific applications e.g., resource allocation game (Cigler & Faltings, 2011)

Constraints & Correlated Equilibria

- (Debreu, 1952) defines the concept of generalized equilibrium
- (Arrow & Debreu, 1954) shows a proof of existence
- (Rosen, 1965) shows existence and uniqueness of equilibria in concave games with coupled constraints
- (Bernasconi et al., 23) introduces the concept of constrained Phi-equilibrium

Many Other Theoretical and Applied Contributions

- Applications in energy systems and smart grids (Kulkarni, 2017), resource allocation problems (Pang et al., 2008) (Nowak et al., 2018)

Related Work

Correlated Equilibria

- Defined in (Aumann, 1974) and (Aumann, 1987)
- A proof of existence and a generalization to infinite games in (Hart & Schmeidler, 1989)
- Extensive form correlated equilibrium defined in (Von Stengel & Forges, 2008)
- Other extensions in (Forges, 2020), (Brandenburger & Dekel, 1992) and (Grant & Stauber, 2022)

Learning Correlated Equilibria

- Regret-matching (Hart & Mas-Colell, 2000) implying the convergence to the set of correlated equilibria
- Correlated Q-learning for selection of specific correlated equilibrium points (Greenwald & Hall, 2003)
- Convergence to the social welfare maximizing correlated equilibrium (Borowski et al., 2015)
- Learning algorithms for specific applications e.g., resource allocation game (Cigler & Faltings, 2011)

Constraints & Correlated Equilibria

- (Debreu, 1952) defines the concept of generalized equilibrium
- (Arrow & Debreu, 1954) shows a proof of existence
- (Rosen, 1965) shows existence and uniqueness of equilibria in concave games with coupled constraints
- (Bernasconi et al., 23) introduces the concept of constrained Phi-equilibrium

Many Other Theoretical and Applied Contributions

- Applications in energy systems and smart grids (Kulkarni, 2017), resource allocation problems (Pang et al., 2008) (Nowak et al., 2018)

Related Work

Correlated Equilibria

- Defined in (Aumann, 1974) and (Aumann, 1987)
- A proof of existence and a generalization to infinite games in (Hart & Schmeidler, 1989)
- Extensive form correlated equilibrium defined in (Von Stengel & Forges, 2008)
- Other extensions in (Forges, 2020), (Brandenburger & Dekel, 1992) and (Grant & Stauber, 2022)

Learning Correlated Equilibria

- Regret-matching (Hart & Mas-Colell, 2000) implying the convergence to the set of correlated equilibria
- Correlated Q-learning for selection of specific correlated equilibrium points (Greenwald & Hall, 2003)
- Convergence to the social welfare maximizing correlated equilibrium (Borowski et al., 2015)
- Learning algorithms for specific applications e.g., resource allocation game (Cigler & Faltings, 2011)

Constraints & Correlated Equilibria

- (Debreu, 1952) defines the concept of generalized equilibrium
- (Arrow & Debreu, 1954) shows a proof of existence
- (Rosen, 1965) shows existence and uniqueness of equilibria in concave games with coupled constraints
- (Bernasconi et al., 23) introduces the concept of constrained Phi-equilibrium

Many Other Theoretical and Applied Contributions

- Applications in energy systems and smart grids (Kulkarni, 2017), resource allocation problems (Pang et al., 2008) (Nowak et al., 2018)

Related Work

Correlated Equilibria

- Defined in (Aumann, 1974) and (Aumann, 1987)
- A proof of existence and a generalization to infinite games in (Hart & Schmeidler, 1989)
- Extensive form correlated equilibrium defined in (Von Stengel & Forges, 2008)
- Other extensions in (Forges, 2020), (Brandenburger & Dekel, 1992) and (Grant & Stauber, 2022)

Learning Correlated Equilibria

- Regret-matching (Hart & Mas-Colell, 2000) implying the convergence to the set of correlated equilibria
- Correlated Q-learning for selection of specific correlated equilibrium points (Greenwald & Hall, 2003)
- Convergence to the social welfare maximizing correlated equilibrium (Borowski et al., 2015)
- Learning algorithms for specific applications e.g., resource allocation game (Cigler & Faltings, 2011)

Constraints & Correlated Equilibria

- (Debreu, 1952) defines the concept of generalized equilibrium
- (Arrow & Debreu, 1954) shows a proof of existence
- (Rosen, 1965) shows existence and uniqueness of equilibria in concave games with coupled constraints
- (Bernasconi et al., 23) introduces the concept of constrained Phi-equilibrium

Many Other Theoretical and Applied Contributions

- Applications in energy systems and smart grids (Kulkarni, 2017), resource allocation problems (Pang et al., 2008) (Nowak et al., 2018)

Related Work

Correlated Equilibria

- Defined in (Aumann, 1974) and (Aumann, 1987)
- A proof of existence and a generalization to infinite games in (Hart & Schmeidler, 1989)
- Extensive form correlated equilibrium defined in (Von Stengel & Forges, 2008)
- Other extensions in (Forges, 2020), (Brandenburger & Dekel, 1992) and (Grant & Stauber, 2022)

Learning Correlated Equilibria

- Regret-matching (Hart & Mas-Colell, 2000) implying the convergence to the set of correlated equilibria
- Correlated Q-learning for selection of specific correlated equilibrium points (Greenwald & Hall, 2003)
- Convergence to the social welfare maximizing correlated equilibrium (Borowski et al., 2015)
- Learning algorithms for specific applications e.g., resource allocation game (Cigler & Faltings, 2011)

Constraints & Correlated Equilibria

- (Debreu, 1952) defines the concept of generalized equilibrium
- (Arrow & Debreu, 1954) shows a proof of existence
- (Rosen, 1965) shows existence and uniqueness of equilibria in concave games with coupled constraints
- (Bernasconi et al., 23) introduces the concept of constrained Phi-equilibrium

Many Other Theoretical and Applied Contributions

- Applications in energy systems and smart grids (Kulkarni, 2017), resource allocation problems (Pang et al., 2008) (Nowak et al., 2018)

Related Work

Correlated Equilibria

- Defined in (Aumann, 1974) and (Aumann, 1987)
- A proof of existence and a generalization to infinite games in (Hart & Schmeidler, 1989)
- Extensive form correlated equilibrium defined in (Von Stengel & Forges, 2008)
- Other extensions in (Forges, 2020), (Brandenburger & Dekel, 1992) and (Grant & Stauber, 2022)

Learning Correlated Equilibria

- Regret-matching (Hart & Mas-Colell, 2000) implying the convergence to the set of correlated equilibria
- Correlated Q-learning for selection of specific correlated equilibrium points (Greenwald & Hall, 2003)
- Convergence to the social welfare maximizing correlated equilibrium (Borowski et al., 2015)
- Learning algorithms for specific applications e.g., resource allocation game (Cigler & Faltings, 2011)

Constraints & Correlated Equilibria

- (Debreu, 1952) defines the concept of generalized equilibrium
- (Arrow & Debreu, 1954) shows a proof of existence
- (Rosen, 1965) shows existence and uniqueness of equilibria in concave games with coupled constraints
- (Bernasconi et al., 23) introduces the concept of constrained Phi-equilibrium

Many Other Theoretical and Applied Contributions

- Applications in energy systems and smart grids (Kulkarni, 2017), resource allocation problems (Pang et al., 2008) (Nowak et al., 2018)

Related Work

Correlated Equilibria

- Defined in (Aumann, 1974) and (Aumann, 1987)
- A proof of existence and a generalization to infinite games in (Hart & Schmeidler, 1989)
- Extensive form correlated equilibrium defined in (Von Stengel & Forges, 2008)
- Other extensions in (Forges, 2020), (Brandenburger & Dekel, 1992) and (Grant & Stauber, 2022)

Learning Correlated Equilibria

- Regret-matching (Hart & Mas-Colell, 2000) implying the convergence to the set of correlated equilibria
- Correlated Q-learning for selection of specific correlated equilibrium points (Greenwald & Hall, 2003)
- Convergence to the social welfare maximizing correlated equilibrium (Borowski et al., 2015)
- Learning algorithms for specific applications e.g., resource allocation game (Cigler & Faltings, 2011)

Constraints & Correlated Equilibria

- (Debreu, 1952) defines the concept of generalized equilibrium
- (Arrow & Debreu, 1954) shows a proof of existence
- (Rosen, 1965) shows existence and uniqueness of equilibria in concave games with coupled constraints
- (Bernasconi et al., 23) introduces the concept of constrained Phi-equilibrium

Many Other Theoretical and Applied Contributions

- Applications in energy systems and smart grids (Kulkarni, 2017), resource allocation problems (Pang et al., 2008) (Nowak et al., 2018)

Related Work

Correlated Equilibria

- Defined in (Aumann, 1974) and (Aumann, 1987)
- A proof of existence and a generalization to infinite games in (Hart & Schmeidler, 1989)
- Extensive form correlated equilibrium defined in (Von Stengel & Forges, 2008)
- Other extensions in (Forges, 2020), (Brandenburger & Dekel, 1992) and (Grant & Stauber, 2022)

Learning Correlated Equilibria

- Regret-matching (Hart & Mas-Colell, 2000) implying the convergence to the set of correlated equilibria
- Correlated Q-learning for selection of specific correlated equilibrium points (Greenwald & Hall, 2003)
- Convergence to the social welfare maximizing correlated equilibrium (Borowski et al., 2015)
- Learning algorithms for specific applications e.g., resource allocation game (Cigler & Faltings, 2011)

Constraints & Correlated Equilibria

- (Debreu, 1952) defines the concept of generalized equilibrium
- (Arrow & Debreu, 1954) shows a proof of existence
- (Rosen, 1965) shows existence and uniqueness of equilibria in concave games with coupled constraints
- (Bernasconi et al., 23) introduces the concept of constrained Phi-equilibrium

Many Other Theoretical and Applied Contributions

- Applications in energy systems and smart grids (Kulkarni, 2017), resource allocation problems (Pang et al., 2008) (Nowak et al., 2018)

Related Work

Correlated Equilibria

- Defined in (Aumann, 1974) and (Aumann, 1987)
- A proof of existence and a generalization to infinite games in (Hart & Schmeidler, 1989)
- Extensive form correlated equilibrium defined in (Von Stengel & Forges, 2008)
- Other extensions in (Forges, 2020), (Brandenburger & Dekel, 1992) and (Grant & Stauber, 2022)

Learning Correlated Equilibria

- Regret-matching (Hart & Mas-Colell, 2000) implying the convergence to the set of correlated equilibria
- Correlated Q-learning for selection of specific correlated equilibrium points (Greenwald & Hall, 2003)
- Convergence to the social welfare maximizing correlated equilibrium (Borowski et al., 2015)
- Learning algorithms for specific applications e.g., resource allocation game (Cigler & Faltings, 2011)

Constraints & Correlated Equilibria

- (Debreu, 1952) defines the concept of generalized equilibrium
- (Arrow & Debreu, 1954) shows a proof of existence
- (Rosen, 1965) shows existence and uniqueness of equilibria in concave games with coupled constraints
- (Bernasconi et al., 23) introduces the concept of constrained Phi-equilibrium

Many Other Theoretical and Applied Contributions

- Applications in energy systems and smart grids (Kulkarni, 2017), resource allocation problems (Pang et al., 2008) (Nowak et al., 2018)

Related Work

Correlated Equilibria

- Defined in (Aumann, 1974) and (Aumann, 1987)
- A proof of existence and a generalization to infinite games in (Hart & Schmeidler, 1989)
- Extensive form correlated equilibrium defined in (Von Stengel & Forges, 2008)
- Other extensions in (Forges, 2020), (Brandenburger & Dekel, 1992) and (Grant & Stauber, 2022)

Learning Correlated Equilibria

- Regret-matching (Hart & Mas-Colell, 2000) implying the convergence to the set of correlated equilibria
- Correlated Q-learning for selection of specific correlated equilibrium points (Greenwald & Hall, 2003)
- Convergence to the social welfare maximizing correlated equilibrium (Borowski et al., 2015)
- Learning algorithms for specific applications e.g., resource allocation game (Cigler & Faltings, 2011)

Constraints & Correlated Equilibria

- (Debreu, 1952) defines the concept of generalized equilibrium
- (Arrow & Debreu, 1954) shows a proof of existence
- (Rosen, 1965) shows existence and uniqueness of equilibria in concave games with coupled constraints
- (Bernasconi et al., 23) introduces the concept of constrained Phi-equilibrium

Many Other Theoretical and Applied Contributions

- Applications in energy systems and smart grids (Kulkarni, 2017), resource allocation problems (Pang et al., 2008) (Nowak et al., 2018)

Related Work

Correlated Equilibria

- Defined in (Aumann, 1974) and (Aumann, 1987)
- A proof of existence and a generalization to infinite games in (Hart & Schmeidler, 1989)
- Extensive form correlated equilibrium defined in (Von Stengel & Forges, 2008)
- Other extensions in (Forges, 2020), (Brandenburger & Dekel, 1992) and (Grant & Stauber, 2022)

Learning Correlated Equilibria

- Regret-matching (Hart & Mas-Colell, 2000) implying the convergence to the set of correlated equilibria
- Correlated Q-learning for selection of specific correlated equilibrium points (Greenwald & Hall, 2003)
- Convergence to the social welfare maximizing correlated equilibrium (Borowski et al., 2015)
- Learning algorithms for specific applications e.g., resource allocation game (Cigler & Faltings, 2011)

Constraints & Correlated Equilibria

- (Debreu, 1952) defines the concept of generalized equilibrium
- (Arrow & Debreu, 1954) shows a proof of existence
- (Rosen, 1965) shows existence and uniqueness of equilibria in concave games with coupled constraints
- (Bernasconi et al., 23) introduces the concept of constrained Phi-equilibrium

Many Other Theoretical and Applied Contributions

- Applications in energy systems and smart grids (Kulkarni, 2017), resource allocation problems (Pang et al., 2008) (Nowak et al., 2018)

Related Work

Correlated Equilibria

- Defined in (Aumann, 1974) and (Aumann, 1987)
- A proof of existence and a generalization to infinite games in (Hart & Schmeidler, 1989)
- Extensive form correlated equilibrium defined in (Von Stengel & Forges, 2008)
- Other extensions in (Forges, 2020), (Brandenburger & Dekel, 1992) and (Grant & Stauber, 2022)

Learning Correlated Equilibria

- Regret-matching (Hart & Mas-Colell, 2000) implying the convergence to the set of correlated equilibria
- Correlated Q-learning for selection of specific correlated equilibrium points (Greenwald & Hall, 2003)
- Convergence to the social welfare maximizing correlated equilibrium (Borowski et al., 2015)
- Learning algorithms for specific applications e.g., resource allocation game (Cigler & Faltings, 2011)

Constraints & Correlated Equilibria

- (Debreu, 1952) defines the concept of generalized equilibrium
- (Arrow & Debreu, 1954) shows a proof of existence
- (Rosen, 1965) shows existence and uniqueness of equilibria in concave games with coupled constraints
- (Bernasconi et al., 23) introduces the concept of constrained Phi-equilibrium

Many Other Theoretical and Applied Contributions

- Applications in energy systems and smart grids (Kulkarni, 2017), resource allocation problems (Pang et al., 2008) (Nowak et al., 2018)

Related Work

Correlated Equilibria

- Defined in (Aumann, 1974) and (Aumann, 1987)
- A proof of existence and a generalization to infinite games in (Hart & Schmeidler, 1989)
- Extensive form correlated equilibrium defined in (Von Stengel & Forges, 2008)
- Other extensions in (Forges, 2020), (Brandenburger & Dekel, 1992) and (Grant & Stauber, 2022)

Learning Correlated Equilibria

- Regret-matching (Hart & Mas-Colell, 2000) implying the convergence to the set of correlated equilibria
- Correlated Q-learning for selection of specific correlated equilibrium points (Greenwald & Hall, 2003)
- Convergence to the social welfare maximizing correlated equilibrium (Borowski et al., 2015)
- Learning algorithms for specific applications e.g., resource allocation game (Cigler & Faltings, 2011)

Constraints & Correlated Equilibria

- (Debreu, 1952) defines the concept of generalized equilibrium
- (Arrow & Debreu, 1954) shows a proof of existence
- (Rosen, 1965) shows existence and uniqueness of equilibria in concave games with coupled constraints
- (Bernasconi et al., 23) introduces the concept of constrained Phi-equilibrium

Many Other Theoretical and Applied Contributions

- Applications in energy systems and smart grids (Kulkarni, 2017), resource allocation problems (Pang et al., 2008) (Nowak et al., 2018)

Related Work

Correlated Equilibria

- Defined in (Aumann, 1974) and (Aumann, 1987)
- A proof of existence and a generalization to infinite games in (Hart & Schmeidler, 1989)
- Extensive form correlated equilibrium defined in (Von Stengel & Forges, 2008)
- Other extensions in (Forges, 2020), (Brandenburger & Dekel, 1992) and (Grant & Stauber, 2022)

Learning Correlated Equilibria

- Regret-matching (Hart & Mas-Colell, 2000) implying the convergence to the set of correlated equilibria
- Correlated Q-learning for selection of specific correlated equilibrium points (Greenwald & Hall, 2003)
- Convergence to the social welfare maximizing correlated equilibrium (Borowski et al., 2015)
- Learning algorithms for specific applications e.g., resource allocation game (Cigler & Faltings, 2011)

Constraints & Correlated Equilibria

- (Debreu, 1952) defines the concept of generalized equilibrium
- (Arrow & Debreu, 1954) shows a proof of existence
- (Rosen, 1965) shows existence and uniqueness of equilibria in concave games with coupled constraints
- (Bernasconi et al., 23) introduces the concept of constrained Phi-equilibrium

Many Other Theoretical and Applied Contributions

- Applications in energy systems and smart grids (Kulkarni, 2017), resource allocation problems (Pang et al., 2008) (Nowak et al., 2018)

Related Work

Correlated Equilibria

- Defined in (Aumann, 1974) and (Aumann, 1987)
- A proof of existence and a generalization to infinite games in (Hart & Schmeidler, 1989)
- Extensive form correlated equilibrium defined in (Von Stengel & Forges, 2008)
- Other extensions in (Forges, 2020), (Brandenburger & Dekel, 1992) and (Grant & Stauber, 2022)

Learning Correlated Equilibria

- Regret-matching (Hart & Mas-Colell, 2000) implying the convergence to the set of correlated equilibria
- Correlated Q-learning for selection of specific correlated equilibrium points (Greenwald & Hall, 2003)
- Convergence to the social welfare maximizing correlated equilibrium (Borowski et al., 2015)
- Learning algorithms for specific applications e.g., resource allocation game (Cigler & Faltings, 2011)

Constraints & Correlated Equilibria

- (Debreu, 1952) defines the concept of generalized equilibrium
- (Arrow & Debreu, 1954) shows a proof of existence
- (Rosen, 1965) shows existence and uniqueness of equilibria in concave games with coupled constraints
- (Bernasconi et al., 23) introduces the concept of constrained Phi-equilibrium

Many Other Theoretical and Applied Contributions

- Applications in energy systems and smart grids (Kulkarni, 2017), resource allocation problems (Pang et al., 2008) (Nowak et al., 2018)

Related Work

Correlated Equilibria

- Defined in (Aumann, 1974) and (Aumann, 1987)
- A proof of existence and a generalization to infinite games in (Hart & Schmeidler, 1989)
- Extensive form correlated equilibrium defined in (Von Stengel & Forges, 2008)
- Other extensions in (Forges, 2020), (Brandenburger & Dekel, 1992) and (Grant & Stauber, 2022)

Learning Correlated Equilibria

- Regret-matching (Hart & Mas-Colell, 2000) implying the convergence to the set of correlated equilibria
- Correlated Q-learning for selection of specific correlated equilibrium points (Greenwald & Hall, 2003)
- Convergence to the social welfare maximizing correlated equilibrium (Borowski et al., 2015)
- Learning algorithms for specific applications e.g., resource allocation game (Cigler & Faltings, 2011)

Constraints & Correlated Equilibria

- (Debreu, 1952) defines the concept of generalized equilibrium
- (Arrow & Debreu, 1954) shows a proof of existence
- (Rosen, 1965) shows existence and uniqueness of equilibria in concave games with coupled constraints
- (Bernasconi et al., 23) introduces the concept of constrained Phi-equilibrium

Many Other Theoretical and Applied Contributions

- Applications in energy systems and smart grids (Kulkarni, 2017), resource allocation problems (Pang et al., 2008) (Nowak et al., 2018)

Related Work

Correlated Equilibria

- Defined in (Aumann, 1974) and (Aumann, 1987)
- A proof of existence and a generalization to infinite games in (Hart & Schmeidler, 1989)
- Extensive form correlated equilibrium defined in (Von Stengel & Forges, 2008)
- Other extensions in (Forges, 2020), (Brandenburger & Dekel, 1992) and (Grant & Stauber, 2022)

Learning Correlated Equilibria

- Regret-matching (Hart & Mas-Colell, 2000) implying the convergence to the set of correlated equilibria
- Correlated Q-learning for selection of specific correlated equilibrium points (Greenwald & Hall, 2003)
- Convergence to the social welfare maximizing correlated equilibrium (Borowski et al., 2015)
- Learning algorithms for specific applications e.g., resource allocation game (Cigler & Faltings, 2011)

Constraints & Correlated Equilibria

- (Debreu, 1952) defines the concept of generalized equilibrium
- (Arrow & Debreu, 1954) shows a proof of existence
- (Rosen, 1965) shows existence and uniqueness of equilibria in concave games with coupled constraints
- (Bernasconi et al., 23) introduces the concept of constrained Phi-equilibrium

Many Other Theoretical and Applied Contributions

- Applications in energy systems and smart grids (Kulkarni, 2017), resource allocation problems (Pang et al., 2008) (Nowak et al., 2018)

Model & Definitions

Finite non-cooperative game $G = (\mathcal{N}, (\mathcal{A}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$

- Set of players $\mathcal{N}$
- Action set $\mathcal{A}_i$ for each $i \in \mathcal{N}$
- Utility function $u_i : \prod_{i\in\mathcal{N}} \mathcal{A}_i = \mathcal{A} \to \mathbb{R}$

Correlation device $d = (\Omega, (\mathcal{P}_i)_{i\in\mathcal{N}}, \boldsymbol{q})$

- A sample space Ω
- Partition $\mathcal{P}_i$ of Ω for each $i \in \mathcal{N}$
- Probability distribution $\boldsymbol{q}$ over Ω

Player i**'s set of strategies** is $\mathcal{S}_{i,d} = \{f_i : \Omega \to \mathcal{A}_i \text{ s.t. } f_i \text{ is } \mathcal{P}_i\text{-measurable}\}$ and the **set of strategy profiles** is $\mathcal{S}_d = \mathcal{S}_{1,d} \times ... \times \mathcal{S}_{n,d}$.

Definition 1 – Correlated equilibrium (Aumann, 1974)

A correlated equilibrium of G is a pair $(d, \boldsymbol{\alpha}^*)$ where $\boldsymbol{\alpha}^* : \Omega \to \mathcal{A}$ is a correlated strategy profile such that

$$\forall i \in \mathcal{N}, \forall \alpha_i' \in \mathcal{S}_{i,d} \quad \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\boldsymbol{\alpha}_i^*(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \geq \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\alpha_i'(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \tag{1}$$

A correlated equilibrium distribution of G (Forges, 2020) is a probability distribution $\boldsymbol{p} \in \Delta(\mathcal{A})$ such that,

$$\forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i \quad \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}_i, \boldsymbol{a}_{-i}) \geq \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \tag{2}$$

Model & Definitions

Finite non-cooperative game $G = (\mathcal{N}, (\mathcal{A}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$

- Set of players $\mathcal{N}$
- Action set $\mathcal{A}_i$ for each $i \in \mathcal{N}$
- Utility function $u_i : \prod_{i\in\mathcal{N}} \mathcal{A}_i = \mathcal{A} \to \mathbb{R}$

Correlation device $d = (\Omega, (\mathcal{P}_i)_{i\in\mathcal{N}}, \boldsymbol{q})$

- A sample space Ω
- Partition $\mathcal{P}_i$ of Ω for each $i \in \mathcal{N}$
- Probability distribution $\boldsymbol{q}$ over Ω

Player i**'s set of strategies** is $\mathcal{S}_{i,d} = \{f_i : \Omega \to \mathcal{A}_i \text{ s.t. } f_i \text{ is } \mathcal{P}_i\text{-measurable}\}$ and the **set of strategy profiles** is $\mathcal{S}_d = \mathcal{S}_{1,d} \times ... \times \mathcal{S}_{n,d}$.

Definition 1 – Correlated equilibrium (Aumann, 1974)

A correlated equilibrium of G is a pair $(d, \boldsymbol{\alpha}^*)$ where $\boldsymbol{\alpha}^* : \Omega \to \mathcal{A}$ is a correlated strategy profile such that

$$\forall i \in \mathcal{N}, \forall \alpha_i' \in \mathcal{S}_{i,d} \quad \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\boldsymbol{\alpha}_i^*(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \geq \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\alpha_i'(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \tag{1}$$

A correlated equilibrium distribution of G (Forges, 2020) is a probability distribution $\boldsymbol{p} \in \Delta(\mathcal{A})$ such that,

$$\forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i \quad \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}_i, \boldsymbol{a}_{-i}) \geq \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \tag{2}$$

Model & Definitions

Finite non-cooperative game $G = (\mathcal{N}, (\mathcal{A}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$

- Set of players $\mathcal{N}$
- Action set $\mathcal{A}_i$ for each $i \in \mathcal{N}$
- Utility function $u_i : \prod_{i\in\mathcal{N}} \mathcal{A}_i = \mathcal{A} \to \mathbb{R}$

Correlation device $d = (\Omega, (\mathcal{P}_i)_{i\in\mathcal{N}}, \boldsymbol{q})$

- A sample space Ω
- Partition $\mathcal{P}_i$ of Ω for each $i \in \mathcal{N}$
- Probability distribution $\boldsymbol{q}$ over Ω

Player i**'s set of strategies** is $\mathcal{S}_{i,d} = \{f_i : \Omega \to \mathcal{A}_i \text{ s.t. } f_i \text{ is } \mathcal{P}_i\text{-measurable}\}$ and the **set of strategy profiles** is $\mathcal{S}_d = \mathcal{S}_{1,d} \times ... \times \mathcal{S}_{n,d}$.

Definition 1 – Correlated equilibrium (Aumann, 1974)

A correlated equilibrium of G is a pair $(d, \boldsymbol{\alpha}^*)$ where $\boldsymbol{\alpha}^* : \Omega \to \mathcal{A}$ is a correlated strategy profile such that

$$\forall i \in \mathcal{N}, \forall \alpha_i' \in \mathcal{S}_{i,d} \quad \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\boldsymbol{\alpha}_i^*(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \geq \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\alpha_i'(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \tag{1}$$

A correlated equilibrium distribution of G (Forges, 2020) is a probability distribution $\boldsymbol{p} \in \Delta(\mathcal{A})$ such that,

$$\forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i \quad \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}_i, \boldsymbol{a}_{-i}) \geq \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \tag{2}$$

Model & Definitions

Finite non-cooperative game $G = (\mathcal{N}, (\mathcal{A}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$

- Set of players $\mathcal{N}$
- Action set $\mathcal{A}_i$ for each $i \in \mathcal{N}$
- Utility function $u_i : \prod_{i\in\mathcal{N}} \mathcal{A}_i = \mathcal{A} \to \mathbb{R}$

Correlation device $d = (\Omega, (\mathcal{P}_i)_{i\in\mathcal{N}}, \boldsymbol{q})$

- A sample space Ω
- Partition $\mathcal{P}_i$ of Ω for each $i \in \mathcal{N}$
- Probability distribution $\boldsymbol{q}$ over Ω

Player i**'s set of strategies** is $\mathcal{S}_{i,d} = \{f_i : \Omega \to \mathcal{A}_i \text{ s.t. } f_i \text{ is } \mathcal{P}_i\text{-measurable}\}$ and the **set of strategy profiles** is $\mathcal{S}_d = \mathcal{S}_{1,d} \times ... \times \mathcal{S}_{n,d}$.

Definition 1 – Correlated equilibrium (Aumann, 1974)

A correlated equilibrium of G is a pair $(d, \boldsymbol{\alpha}^*)$ where $\boldsymbol{\alpha}^* : \Omega \to \mathcal{A}$ is a correlated strategy profile such that

$$\forall i \in \mathcal{N}, \forall \alpha_i' \in \mathcal{S}_{i,d} \quad \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\boldsymbol{\alpha}_i^*(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \geq \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\alpha_i'(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \tag{1}$$

A correlated equilibrium distribution of G (Forges, 2020) is a probability distribution $\boldsymbol{p} \in \Delta(\mathcal{A})$ such that,

$$\forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i \quad \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}_i, \boldsymbol{a}_{-i}) \geq \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \tag{2}$$

Model & Definitions

Finite non-cooperative game $G = (\mathcal{N}, (\mathcal{A}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$

- Set of players $\mathcal{N}$
- Action set $\mathcal{A}_i$ for each $i \in \mathcal{N}$
- Utility function $u_i : \prod_{i\in\mathcal{N}} \mathcal{A}_i = \mathcal{A} \to \mathbb{R}$

Correlation device $d = (\Omega, (\mathcal{P}_i)_{i\in\mathcal{N}}, \boldsymbol{q})$

- A sample space Ω
- Partition $\mathcal{P}_i$ of Ω for each $i \in \mathcal{N}$
- Probability distribution $\boldsymbol{q}$ over Ω

Player i**'s set of strategies** is $\mathcal{S}_{i,d} = \{f_i : \Omega \to \mathcal{A}_i \text{ s.t. } f_i \text{ is } \mathcal{P}_i\text{-measurable}\}$ and the **set of strategy profiles** is $\mathcal{S}_d = \mathcal{S}_{1,d} \times ... \times \mathcal{S}_{n,d}$.

Definition 1 – Correlated equilibrium (Aumann, 1974)

A correlated equilibrium of G is a pair $(d, \boldsymbol{\alpha}^*)$ where $\boldsymbol{\alpha}^* : \Omega \to \mathcal{A}$ is a correlated strategy profile such that

$$\forall i \in \mathcal{N}, \forall \alpha_i' \in \mathcal{S}_{i,d} \quad \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\boldsymbol{\alpha}_i^*(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \geq \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\alpha_i'(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \tag{1}$$

A correlated equilibrium distribution of G (Forges, 2020) is a probability distribution $\boldsymbol{p} \in \Delta(\mathcal{A})$ such that,

$$\forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i \quad \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}_i, \boldsymbol{a}_{-i}) \geq \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \tag{2}$$

Model & Definitions

Finite non-cooperative game $G = (\mathcal{N}, (\mathcal{A}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$

- Set of players $\mathcal{N}$
- Action set $\mathcal{A}_i$ for each $i \in \mathcal{N}$
- Utility function $u_i : \prod_{i\in\mathcal{N}} \mathcal{A}_i = \mathcal{A} \to \mathbb{R}$

Correlation device $d = (\Omega, (\mathcal{P}_i)_{i\in\mathcal{N}}, \boldsymbol{q})$

- A sample space Ω
- Partition $\mathcal{P}_i$ of Ω for each $i \in \mathcal{N}$
- Probability distribution $\boldsymbol{q}$ over Ω

Player i**'s set of strategies** is $\mathcal{S}_{i,d} = \{f_i : \Omega \to \mathcal{A}_i \text{ s.t. } f_i \text{ is } \mathcal{P}_i\text{-measurable}\}$ and the **set of strategy profiles** is $\mathcal{S}_d = \mathcal{S}_{1,d} \times ... \times \mathcal{S}_{n,d}$.

Definition 1 – Correlated equilibrium (Aumann, 1974)

A correlated equilibrium of G is a pair $(d, \boldsymbol{\alpha}^*)$ where $\boldsymbol{\alpha}^* : \Omega \to \mathcal{A}$ is a correlated strategy profile such that

$$\forall i \in \mathcal{N}, \forall \alpha_i' \in \mathcal{S}_{i,d} \quad \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\boldsymbol{\alpha}_i^*(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \geq \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\alpha_i'(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \tag{1}$$

A correlated equilibrium distribution of G (Forges, 2020) is a probability distribution $\boldsymbol{p} \in \Delta(\mathcal{A})$ such that,

$$\forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i \quad \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}_i, \boldsymbol{a}_{-i}) \geq \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \tag{2}$$

Model & Definitions

Finite non-cooperative game $G = (\mathcal{N}, (\mathcal{A}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$

- Set of players $\mathcal{N}$
- Action set $\mathcal{A}_i$ for each $i \in \mathcal{N}$
- Utility function $u_i : \prod_{i\in\mathcal{N}} \mathcal{A}_i = \mathcal{A} \to \mathbb{R}$

Correlation device $d = (\Omega, (\mathcal{P}_i)_{i\in\mathcal{N}}, \boldsymbol{q})$

- A sample space Ω
- Partition $\mathcal{P}_i$ of Ω for each $i \in \mathcal{N}$
- Probability distribution $\boldsymbol{q}$ over Ω

Player i**'s set of strategies** is $\mathcal{S}_{i,d} = \{f_i : \Omega \to \mathcal{A}_i \text{ s.t. } f_i \text{ is } \mathcal{P}_i\text{-measurable}\}$ and the **set of strategy profiles** is $\mathcal{S}_d = \mathcal{S}_{1,d} \times ... \times \mathcal{S}_{n,d}$.

Definition 1 – Correlated equilibrium (Aumann, 1974)

A correlated equilibrium of G is a pair $(d, \boldsymbol{\alpha}^*)$ where $\boldsymbol{\alpha}^* : \Omega \to \mathcal{A}$ is a correlated strategy profile such that

$$\forall i \in \mathcal{N}, \forall \alpha_i' \in \mathcal{S}_{i,d} \quad \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\boldsymbol{\alpha}_i^*(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \geq \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\alpha_i'(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \tag{1}$$

A correlated equilibrium distribution of G (Forges, 2020) is a probability distribution $\boldsymbol{p} \in \Delta(\mathcal{A})$ such that,

$$\forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i \quad \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}_i, \boldsymbol{a}_{-i}) \geq \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \tag{2}$$

Model & Definitions

Finite non-cooperative game $G = (\mathcal{N}, (\mathcal{A}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$

- Set of players $\mathcal{N}$
- Action set $\mathcal{A}_i$ for each $i \in \mathcal{N}$
- Utility function $u_i : \prod_{i\in\mathcal{N}} \mathcal{A}_i = \mathcal{A} \to \mathbb{R}$

Correlation device $d = (\Omega, (\mathcal{P}_i)_{i\in\mathcal{N}}, \boldsymbol{q})$

- A sample space Ω
- Partition $\mathcal{P}_i$ of Ω for each $i \in \mathcal{N}$
- Probability distribution $\boldsymbol{q}$ over Ω

Player i**'s set of strategies** is $\mathcal{S}_{i,d} = \{f_i : \Omega \to \mathcal{A}_i \text{ s.t. } f_i \text{ is } \mathcal{P}_i\text{-measurable}\}$ and the **set of strategy profiles** is $\mathcal{S}_d = \mathcal{S}_{1,d} \times ... \times \mathcal{S}_{n,d}$.

Definition 1 – Correlated equilibrium (Aumann, 1974)

A correlated equilibrium of G is a pair $(d, \boldsymbol{\alpha}^*)$ where $\boldsymbol{\alpha}^* : \Omega \to \mathcal{A}$ is a correlated strategy profile such that

$$\forall i \in \mathcal{N}, \forall \alpha_i' \in \mathcal{S}_{i,d} \quad \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\boldsymbol{\alpha}_i^*(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \geq \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\alpha_i'(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \tag{1}$$

A correlated equilibrium distribution of G (Forges, 2020) is a probability distribution $\boldsymbol{p} \in \Delta(\mathcal{A})$ such that,

$$\forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i \quad \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}_i, \boldsymbol{a}_{-i}) \geq \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \tag{2}$$

Model & Definitions

Finite non-cooperative game $G = (\mathcal{N}, (\mathcal{A}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$

- Set of players $\mathcal{N}$
- Action set $\mathcal{A}_i$ for each $i \in \mathcal{N}$
- Utility function $u_i : \prod_{i\in\mathcal{N}} \mathcal{A}_i = \mathcal{A} \to \mathbb{R}$

Correlation device $d = (\Omega, (\mathcal{P}_i)_{i\in\mathcal{N}}, \boldsymbol{q})$

- A sample space Ω
- Partition $\mathcal{P}_i$ of Ω for each $i \in \mathcal{N}$
- Probability distribution $\boldsymbol{q}$ over Ω

Player i**'s set of strategies** is $\mathcal{S}_{i,d} = \{f_i : \Omega \to \mathcal{A}_i \text{ s.t. } f_i \text{ is } \mathcal{P}_i\text{-measurable}\}$ and the **set of strategy profiles** is $\mathcal{S}_d = \mathcal{S}_{1,d} \times ... \times \mathcal{S}_{n,d}$.

Definition 1 – Correlated equilibrium (Aumann, 1974)

A correlated equilibrium of G is a pair $(d, \boldsymbol{\alpha}^*)$ where $\boldsymbol{\alpha}^* : \Omega \to \mathcal{A}$ is a correlated strategy profile such that

$$\forall i \in \mathcal{N}, \forall \alpha_i' \in \mathcal{S}_{i,d} \quad \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\boldsymbol{\alpha}_i^*(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \geq \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\alpha_i'(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \tag{1}$$

A correlated equilibrium distribution of G (Forges, 2020) is a probability distribution $\boldsymbol{p} \in \Delta(\mathcal{A})$ such that,

$$\forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i \quad \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}_i, \boldsymbol{a}_{-i}) \geq \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \tag{2}$$

Model & Definitions

Finite non-cooperative game $G = (\mathcal{N}, (\mathcal{A}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$

- Set of players $\mathcal{N}$
- Action set $\mathcal{A}_i$ for each $i \in \mathcal{N}$
- Utility function $u_i : \prod_{i\in\mathcal{N}} \mathcal{A}_i = \mathcal{A} \to \mathbb{R}$

Correlation device $d = (\Omega, (\mathcal{P}_i)_{i\in\mathcal{N}}, \boldsymbol{q})$

- A sample space Ω
- Partition $\mathcal{P}_i$ of Ω for each $i \in \mathcal{N}$
- Probability distribution $\boldsymbol{q}$ over Ω

Player i**'s set of strategies** is $\mathcal{S}_{i,d} = \{f_i : \Omega \to \mathcal{A}_i \text{ s.t. } f_i \text{ is } \mathcal{P}_i\text{-measurable}\}$ and the **set of strategy profiles** is $\mathcal{S}_d = \mathcal{S}_{1,d} \times ... \times \mathcal{S}_{n,d}$.

Definition 1 – Correlated equilibrium (Aumann, 1974)

A correlated equilibrium of G is a pair $(d, \boldsymbol{\alpha}^*)$ where $\boldsymbol{\alpha}^* : \Omega \to \mathcal{A}$ is a correlated strategy profile such that

$$\forall i \in \mathcal{N}, \forall \alpha_i' \in \mathcal{S}_{i,d} \quad \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\boldsymbol{\alpha}_i^*(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \geq \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\alpha_i'(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \tag{1}$$

A correlated equilibrium distribution of G (Forges, 2020) is a probability distribution $\boldsymbol{p} \in \Delta(\mathcal{A})$ such that,

$$\forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i \quad \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}_i, \boldsymbol{a}_{-i}) \geq \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \tag{2}$$

Model & Definitions

Finite non-cooperative game $G = (\mathcal{N}, (\mathcal{A}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$

- Set of players $\mathcal{N}$
- Action set $\mathcal{A}_i$ for each $i \in \mathcal{N}$
- Utility function $u_i : \prod_{i\in\mathcal{N}} \mathcal{A}_i = \mathcal{A} \to \mathbb{R}$

Correlation device $d = (\Omega, (\mathcal{P}_i)_{i\in\mathcal{N}}, \boldsymbol{q})$

- A sample space Ω
- Partition $\mathcal{P}_i$ of Ω for each $i \in \mathcal{N}$
- Probability distribution $\boldsymbol{q}$ over Ω

Player i**'s set of strategies** is $\mathcal{S}_{i,d} = \{f_i : \Omega \to \mathcal{A}_i \text{ s.t. } f_i \text{ is } \mathcal{P}_i\text{-measurable}\}$ and the **set of strategy profiles** is $\mathcal{S}_d = \mathcal{S}_{1,d} \times ... \times \mathcal{S}_{n,d}$.

Definition 1 – Correlated equilibrium (Aumann, 1974)

A correlated equilibrium of G is a pair $(d, \boldsymbol{\alpha}^*)$ where $\boldsymbol{\alpha}^* : \Omega \to \mathcal{A}$ is a correlated strategy profile such that

$$\forall i \in \mathcal{N}, \forall \alpha_i' \in \mathcal{S}_{i,d} \quad \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\boldsymbol{\alpha}_i^*(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \geq \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\alpha_i'(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \tag{1}$$

A correlated equilibrium distribution of G (Forges, 2020) is a probability distribution $\boldsymbol{p} \in \Delta(\mathcal{A})$ such that,

$$\forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i \quad \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}_i, \boldsymbol{a}_{-i}) \geq \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \tag{2}$$

Model & Definitions

Finite non-cooperative game $G = (\mathcal{N}, (\mathcal{A}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$

- Set of players $\mathcal{N}$
- Action set $\mathcal{A}_i$ for each $i \in \mathcal{N}$
- Utility function $u_i : \prod_{i\in\mathcal{N}} \mathcal{A}_i = \mathcal{A} \to \mathbb{R}$

Correlation device $d = (\Omega, (\mathcal{P}_i)_{i\in\mathcal{N}}, \boldsymbol{q})$

- A sample space Ω
- Partition $\mathcal{P}_i$ of Ω for each $i \in \mathcal{N}$
- Probability distribution $\boldsymbol{q}$ over Ω

Player i**'s set of strategies** is $\mathcal{S}_{i,d} = \{f_i : \Omega \to \mathcal{A}_i \text{ s.t. } f_i \text{ is } \mathcal{P}_i\text{-measurable}\}$ and the **set of strategy profiles** is $\mathcal{S}_d = \mathcal{S}_{1,d} \times ... \times \mathcal{S}_{n,d}$.

Definition 1 – Correlated equilibrium (Aumann, 1974)

A correlated equilibrium of G is a pair $(d, \boldsymbol{\alpha}^*)$ where $\boldsymbol{\alpha}^* : \Omega \to \mathcal{A}$ is a correlated strategy profile such that

$$\forall i \in \mathcal{N}, \forall \alpha'_i \in \mathcal{S}_{i,d} \quad \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\boldsymbol{\alpha}^*_i(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) \geq \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\alpha'_i(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) \tag{1}$$

A correlated equilibrium distribution of G (Forges, 2020) is a probability distribution $p \in \Delta(\mathcal{A})$ such that,

$$\forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i \quad \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}_i, \boldsymbol{a}_{-i}) \geq \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \tag{2}$$

Model & Definitions

Finite non-cooperative game $G = (\mathcal{N}, (\mathcal{A}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$

- Set of players $\mathcal{N}$
- Action set $\mathcal{A}_i$ for each $i \in \mathcal{N}$
- Utility function $u_i : \prod_{i\in\mathcal{N}} \mathcal{A}_i = \mathcal{A} \to \mathbb{R}$

Correlation device $d = (\Omega, (\mathcal{P}_i)_{i\in\mathcal{N}}, \boldsymbol{q})$

- A sample space Ω
- Partition $\mathcal{P}_i$ of Ω for each $i \in \mathcal{N}$
- Probability distribution $\boldsymbol{q}$ over Ω

Player i**'s set of strategies** is $\mathcal{S}_{i,d} = \{f_i : \Omega \to \mathcal{A}_i \text{ s.t. } f_i \text{ is } \mathcal{P}_i\text{-measurable}\}$ and the **set of strategy profiles** is $\mathcal{S}_d = \mathcal{S}_{1,d} \times ... \times \mathcal{S}_{n,d}$.

Definition 1 – Correlated equilibrium (Aumann, 1974)

A correlated equilibrium of G is a pair $(d, \boldsymbol{\alpha}^*)$ where $\boldsymbol{\alpha}^* : \Omega \to \mathcal{A}$ is a correlated strategy profile such that

$$\forall i \in \mathcal{N}, \forall \alpha_i' \in \mathcal{S}_{i,d} \quad \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\boldsymbol{\alpha}_i^*(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \geq \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\alpha_i'(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \tag{1}$$

A correlated equilibrium distribution of G (Forges, 2020) is a probability distribution $\boldsymbol{p} \in \Delta(\mathcal{A})$ such that,

$$\forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i \quad \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}_i, \boldsymbol{a}_{-i}) \geq \sum_{\boldsymbol{a}\in\mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \tag{2}$$

Part 1 : Learning Correlated Equilibria

Online Learning with Regret

The game G is played repeatedly through time $t = 1, 2, 3, ...$ Each step t, action profile $\boldsymbol{a}^t = (a_1^t, ..., a_n^)$ is played

Definition 3 – Regret (Hart & Mas-Colell, 2000)

The regret player i experiences at time t for any pair of actions $j, k \in \mathcal{A}_i$ is

$$R_i^t(j,k) = max\left(0, \frac{1}{t} \sum_{\tau \leq t : a_i^\tau = j} \left[u_i\left(k, \boldsymbol{a}_{-i}^\tau\right) - u_i(\boldsymbol{a}^\tau)\right]\right) = max\left(0, D_i^t(j,k)\right) \tag{3}$$

Regret-Matching algorithm (Hart & Mas-Colell, 2000)

$$\begin{cases} p_i^{t+1}(k) = \dfrac{1}{\mu} R_i^t(j,k), \\ p_i^{t+1}(j) = 1 - \displaystyle\sum_{k \in \mathcal{A}_i : k \neq j} p_i^{t+1}(k). \end{cases}$$

Proposition 1 (Hart & Mas-Colell, 2000)

Let $(a^t)_{t=1,2,...}$ be a sequence of plays (i.e., $\boldsymbol{a}^t \in \mathcal{A}$ for all t) and let $\varepsilon \geq 0$. Then: $\limsup_{t \to \infty} R_i^t(j,k) \leq \varepsilon$ for every $i \in \mathcal{N}$ and every $j, k \in \mathcal{A}_i$ with $j \neq k$, if and only if the sequence of empirical distributions $\boldsymbol{q}^t$ converges to the set of correlated ε-equilibria.

Online Learning with Regret

The game G is played repeatedly through time $t = 1, 2, 3, ...$ Each step t, action profile $\boldsymbol{a}^t = (a_1^t, ..., a_n^)$ is played

Definition 3 – Regret (Hart & Mas-Colell, 2000)

The regret player i experiences at time t for any pair of actions $j, k \in \mathcal{A}_i$ is

$$R_i^t(j,k) = max\left(0, \frac{1}{t}\sum_{\tau \leq t : a_i^\tau = j}\left[u_i\left(k, \boldsymbol{a}_{-i}^\tau\right) - u_i(\boldsymbol{a}^\tau)\right]\right) = max\left(0, D_i^t(j,k)\right) \tag{3}$$

Regret-Matching algorithm (Hart & Mas-Colell, 2000)

$$\begin{cases} p_i^{t+1}(k) = \dfrac{1}{\mu} R_i^t(j,k), \\ p_i^{t+1}(j) = 1 - \displaystyle\sum_{k \in \mathcal{A}_i : k \neq j} p_i^{t+1}(k). \end{cases}$$

Proposition 1 (Hart & Mas-Colell, 2000)

Let $(a^t)_{t=1,2,...}$ be a sequence of plays (i.e., $\boldsymbol{a}^t \in \mathcal{A}$ for all t) and let $\varepsilon \geq 0$. Then: $\limsup_{t \to \infty} R_i^t(j,k) \leq \varepsilon$ for every $i \in \mathcal{N}$ and every $j, k \in \mathcal{A}_i$ with $j \neq k$, if and only if the sequence of empirical distributions $\boldsymbol{q}^t$ converges to the set of correlated ε-equilibria.

Online Learning with Regret

The game G is played repeatedly through time $t = 1, 2, 3, ...$ Each step t, action profile $\boldsymbol{a}^t = (a_1^t, ..., a_n^)$ is played

Definition 3 – Regret (Hart & Mas-Colell, 2000)

The regret player i experiences at time t for any pair of actions $j, k \in \mathcal{A}_i$ is

$$R_i^t(j,k) = max\left(0, \frac{1}{t} \sum_{\tau \leq t : a_i^\tau = j} \left[u_i\left(k, \boldsymbol{a}_{-i}^\tau\right) - u_i(\boldsymbol{a}^\tau)\right]\right) = max\left(0, D_i^t(j,k)\right) \tag{3}$$

Regret-Matching algorithm (Hart & Mas-Colell, 2000)

$$\begin{cases} p_i^{t+1}(k) = \dfrac{1}{\mu} R_i^t(j,k), \\ p_i^{t+1}(j) = 1 - \displaystyle\sum_{k \in \mathcal{A}_i : k \neq j} p_i^{t+1}(k). \end{cases}$$

Proposition 1 (Hart & Mas-Colell, 2000)

Let $(a^t)_{t=1,2,...}$ be a sequence of plays (i.e., $\boldsymbol{a}^t \in \mathcal{A}$ for all t) and let $\varepsilon \geq 0$. Then: $\limsup_{t \to \infty} R_i^t(j,k) \leq \varepsilon$ for every $i \in \mathcal{N}$ and every $j, k \in \mathcal{A}_i$ with $j \neq k$, if and only if the sequence of empirical distributions $\boldsymbol{q}^t$ converges to the set of correlated ε-equilibria.

Online Learning with Regret

The game G is played repeatedly through time $t = 1, 2, 3, ...$ Each step t, action profile $\boldsymbol{a}^t = (a_1^t, ..., a_n^)$ is played

Definition 3 – Regret (Hart & Mas-Colell, 2000)

The regret player i experiences at time t for any pair of actions $j, k \in \mathcal{A}_i$ is

$$R_i^t(j,k) = max\left(0, \frac{1}{t} \sum_{\tau \leq t : a_i^\tau = j} \left[u_i\left(k, \boldsymbol{a}_{-i}^\tau\right) - u_i(\boldsymbol{a}^\tau)\right]\right) = max\left(0, D_i^t(j,k)\right) \tag{3}$$

Regret-Matching algorithm (Hart & Mas-Colell, 2000)

$$\begin{cases} p_i^{t+1}(k) = \dfrac{1}{\mu} R_i^t(j,k), \\ p_i^{t+1}(j) = 1 - \displaystyle\sum_{k \in \mathcal{A}_i : k \neq j} p_i^{t+1}(k). \end{cases}$$

Proposition 1 (Hart & Mas-Colell, 2000)

Let $(a^t)_{t=1,2,...}$ be a sequence of plays (i.e., $\boldsymbol{a}^t \in \mathcal{A}$ for all t) and let $\varepsilon \geq 0$. Then: $\limsup_{t \to \infty} R_i^t(j,k) \leq \varepsilon$ for every $i \in \mathcal{N}$ and every $j, k \in \mathcal{A}_i$ with $j \neq k$, if and only if the sequence of empirical distributions $\boldsymbol{q}^t$ converges to the set of correlated ε-equilibria.

Online Learning with Regret

The game G is played repeatedly through time $t = 1, 2, 3, ...$ Each step t, action profile $\boldsymbol{a}^t = (a_1^t, ..., a_n^)$ is played

Definition 3 – Regret (Hart & Mas-Colell, 2000)

The regret player i experiences at time t for any pair of actions $j, k \in \mathcal{A}_i$ is

$$R_i^t(j,k) = max\left(0, \frac{1}{t} \sum_{\tau \leq t : a_i^\tau = j} \left[u_i\left(k, \boldsymbol{a}_{-i}^\tau\right) - u_i(\boldsymbol{a}^\tau)\right]\right) = max\left(0, D_i^t(j,k)\right) \tag{3}$$

Regret-Matching algorithm (Hart & Mas-Colell, 2000)

$$\begin{cases} p_i^{t+1}(k) = \dfrac{1}{\mu} R_i^t(j,k), \\ p_i^{t+1}(j) = 1 - \displaystyle\sum_{k \in \mathcal{A}_i : k \neq j} p_i^{t+1}(k). \end{cases}$$

Proposition 1 (Hart & Mas-Colell, 2000)

Let $(a^t)_{t=1,2,...}$ be a sequence of plays (i.e., $a^t \in \mathcal{A}$ for all t) and let $\varepsilon \geq 0$. Then: $\limsup_{t \to \infty} R_i^t(j,k) \leq \varepsilon$ for every $i \in \mathcal{N}$ and every $j, k \in \mathcal{A}_i$ with $j \neq k$, if and only if the sequence of empirical distributions $\boldsymbol{q}^t$ converges to the set of correlated ε-equilibria.

Online Learning with Regret

The game G is played repeatedly through time $t = 1, 2, 3, ...$ Each step t, action profile $\boldsymbol{a}^t = (a_1^t, ..., a_n^)$ is played

Definition 3 – Regret (Hart & Mas-Colell, 2000)

The regret player i experiences at time t for any pair of actions $j, k \in \mathcal{A}_i$ is

$$R_i^t(j,k) = max\left(0, \frac{1}{t} \sum_{\tau \leq t : a_i^\tau = j} \left[u_i\left(k, \boldsymbol{a}_{-i}^\tau\right) - u_i(\boldsymbol{a}^\tau)\right]\right) = max\left(0, D_i^t(j,k)\right) \tag{3}$$

Regret-Matching algorithm (Hart & Mas-Colell, 2000)

$$\begin{cases} p_i^{t+1}(k) = \dfrac{1}{\mu} R_i^t(j,k), \\ p_i^{t+1}(j) = 1 - \displaystyle\sum_{k \in \mathcal{A}_i : k \neq j} p_i^{t+1}(k). \end{cases}$$

Proposition 1 (Hart & Mas-Colell, 2000)

Let $(\boldsymbol{a}^t)_{t=1,2,...}$ be a sequence of plays (i.e., $\boldsymbol{a}^t \in \mathcal{A}$ for all t) and let $\varepsilon \geq 0$. Then: $\limsup_{t \to \infty} R_i^t(j,k) \leq \varepsilon$ for every $i \in \mathcal{N}$ and every $j, k \in \mathcal{A}_i$ with $j \neq k$, if and only if the sequence of empirical distributions $\boldsymbol{q}^t$ converges to the set of correlated ε-equilibria.

Online Learning with Regret

Regret-Matching algorithm (Hart & Mas-Colell, 2000)

$$\begin{cases} p_i^{t+1}(k) = \dfrac{1}{\mu} R_i^t(j,k), \\ p_i^{t+1}(j) = 1 - \displaystyle\sum_{k \in \mathcal{A}_i : k \neq j} p_i^{t+1}(k). \end{cases}$$

Regret-Matching algorithm applied to a 3-by-2 game

	D	E
A	(2, 29)	(16, 7)
B	(4, 7)	(6, 13)
C	(4, 4)	(6, 6)

Table 1: Utility matrix.

- The empirical distribution does not seem to converge towards a correlated equilibrium point
- We propose a new dynamic called Correlated Perturbed Regret Minimization (CPRM)

Online Learning with Regret

Regret-Matching algorithm (Hart & Mas-Colell, 2000)

$$\begin{cases} p_i^{t+1}(k) = \dfrac{1}{\mu} R_i^t(j,k), \\ p_i^{t+1}(j) = 1 - \displaystyle\sum_{k \in \mathcal{A}_i : k \neq j} p_i^{t+1}(k). \end{cases}$$

Regret-Matching algorithm applied to a 3-by-2 game

	D	E
A	(2, 29)	(16, 7)
B	(4, 7)	(6, 13)
C	(4, 4)	(6, 6)

Table 1: Utility matrix.

- The empirical distribution does not seem to converge towards a correlated equilibrium point
- We propose a new dynamic called Correlated Perturbed Regret Minimization (CPRM)

Online Learning with Regret

Regret-Matching algorithm (Hart & Mas-Colell, 2000)

$$\begin{cases} p_i^{t+1}(k) = \dfrac{1}{\mu} R_i^t(j,k), \\ p_i^{t+1}(j) = 1 - \displaystyle\sum_{k \in \mathcal{A}_i : k \neq j} p_i^{t+1}(k). \end{cases}$$

Regret-Matching algorithm applied to a 3-by-2 game

	D	E
A	(2, 29)	(16, 7)
B	(4, 7)	(6, 13)
C	(4, 4)	(6, 6)

Table 1: Utility matrix.

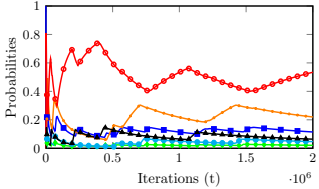


Figure 2: Evolution of the empirical probability distribution.

- The empirical distribution does not seem to converge towards a correlated equilibrium point
- We propose a new dynamic called Correlated Perturbed Regret Minimization (CPRM)

Online Learning with Regret

Regret-Matching algorithm (Hart & Mas-Colell, 2000)

$$\begin{cases} p_i^{t+1}(k) = \dfrac{1}{\mu} R_i^t(j,k), \\ p_i^{t+1}(j) = 1 - \displaystyle\sum_{k \in \mathcal{A}_i : k \neq j} p_i^{t+1}(k). \end{cases}$$

Regret-Matching algorithm applied to a 3-by-2 game

	D	E
A	(2, 29)	(16, 7)
B	(4, 7)	(6, 13)
C	(4, 4)	(6, 6)

Table 1: Utility matrix.

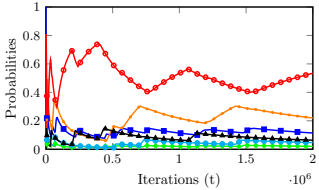


Figure 2: Evolution of the empirical probability distribution.

- The empirical distribution does not seem to converge towards a correlated equilibrium point
- We propose a new dynamic called Correlated Perturbed Regret Minimization (CPRM)

Online Learning with Regret

Regret-Matching algorithm (Hart & Mas-Colell, 2000)

$$\begin{cases} p_i^{t+1}(k) = \dfrac{1}{\mu} R_i^t(j,k), \\ p_i^{t+1}(j) = 1 - \displaystyle\sum_{k \in \mathcal{A}_i : k \neq j} p_i^{t+1}(k). \end{cases}$$

Regret-Matching algorithm applied to a 3-by-2 game

	D	E
A	(2, 29)	(16, 7)
B	(4, 7)	(6, 13)
C	(4, 4)	(6, 6)

Table 1: Utility matrix.

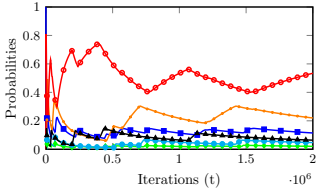


Figure 2: Evolution of the empirical probability distribution.

- The empirical distribution does not seem to converge towards a correlated equilibrium point
- We propose a new dynamic called Correlated Perturbed Regret Minimization (CPRM)

CPRM Algorithm

Inspired from the learning algorithm introduced in (Young, H. P., 2009) for learning pure Nash equilibria.

At time t, each player i is characterized by a mood $m_i^t \in \{syn, asyn\}$

asynchronous player

synchronous player

CPRM Algorithm

Inspired from the learning algorithm introduced in (Young, H. P., 2009) for learning pure Nash equilibria.

At time t, each player i is characterized by a mood $m_i^t \in \{syn, asyn\}$

asynchronous player

synchronous player

CPRM Algorithm

Inspired from the learning algorithm introduced in (Young, H. P., 2009) for learning pure Nash equilibria.

At time t, each player i is characterized by a mood $m_i^t \in \{syn,\ asyn\}$

asynchronous player

synchronous player

CPRM Algorithm

Inspired from the learning algorithm introduced in (Young, H. P., 2009) for learning pure Nash equilibria.

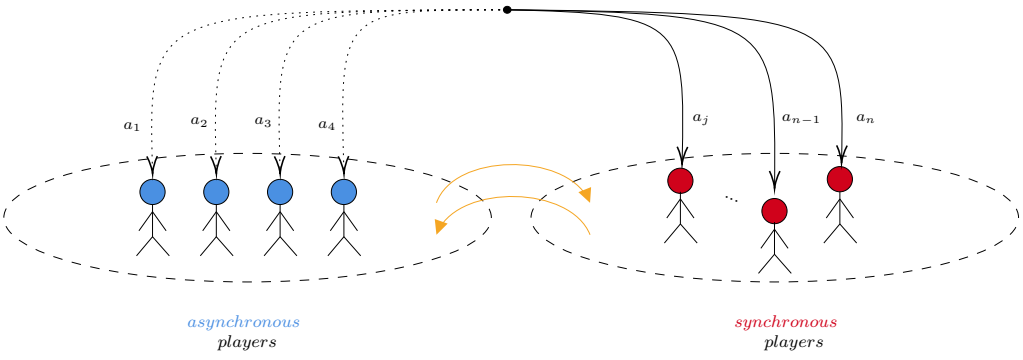
At time t, each player i is characterized by a mood $m_i^t \in \{syn,\ asyn\}$

asynchronous player

synchronous player

CPRM Algorithm

$$\boldsymbol{a} = (a_1, a_2, a_3, \ldots, a_n) \quad drawn\ from\ \boldsymbol{q}^t$$

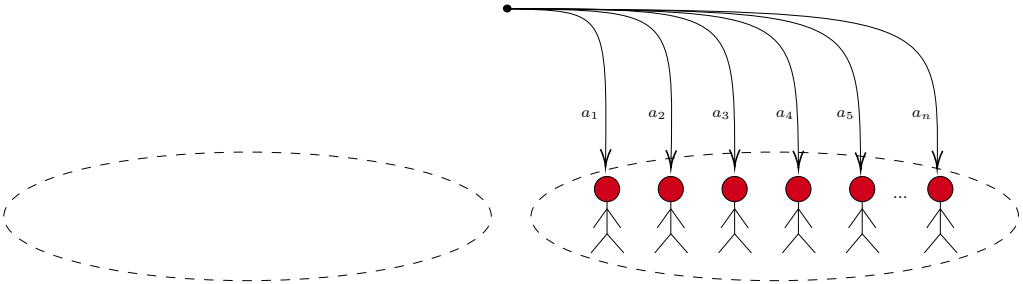


Players implementing a regret-minimizing strategy

Players play the actions drawn from the empirical distribution

CPRM Algorithm

$\boldsymbol{a} = (a_1, a_2, a_3, ..., a_n)$
$drawn\ from\ \boldsymbol{q}^t$

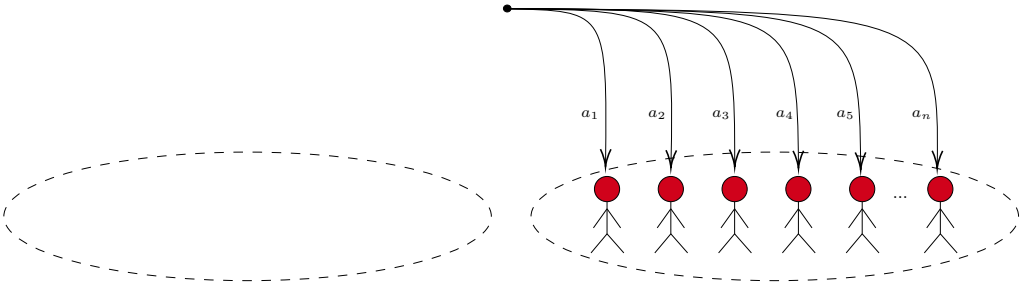


In the long run

↪ We expect players to play the action profiles drawn from the current empirical distributions to stabilize the play at an equilibrium point.

CPRM Algorithm

$\boldsymbol{a} = (a_1, a_2, a_3, ..., a_n)$
$drawn\ from\ \boldsymbol{q}^t$

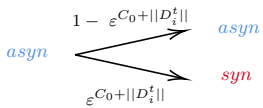


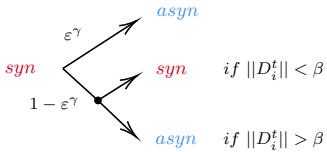
In the long run

$\hookrightarrow$ We expect players to play the action profiles drawn from the current empirical distributions to stabilize the play at an equilibrium point.

Algorithm Analysis

Mood dynamics



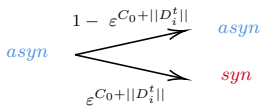


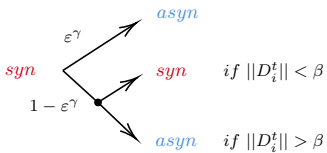
Characteristics of the stochastic process

- Transition probabilities depend on the regrets and a perturbation parameter $\varepsilon > 0$
- The stochastic process $X^t = (X_1^t, ..., X_n^t)$ describing players' states is a perturbed non-homogeneous Markov chain
- The Markov chain is defined over a infinitely countable state spaces
- For a regularly perturbed homogenous Markov chain, the process spends almost all time in the stochastically stable states when $\varepsilon \to 0$

Algorithm Analysis

Mood dynamics



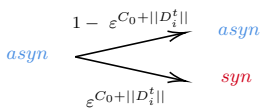


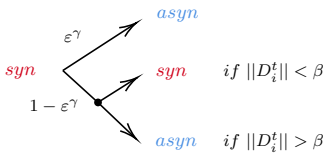
Characteristics of the stochastic process

- Transition probabilities depend on the regrets and a perturbation parameter $\varepsilon > 0$
- The stochastic process $X^t = (X_1^t, ..., X_n^t)$ describing players' states is a perturbed non-homogeneous Markov chain
- The Markov chain is defined over a infinitely countable state spaces
- For a regularly perturbed homogenous Markov chain, the process spends almost all time in the stochastically stable states when $\varepsilon \to 0$

Algorithm Analysis

Mood dynamics



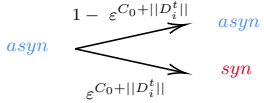


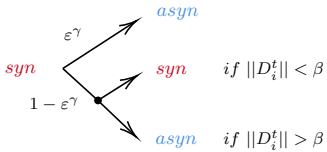
Characteristics of the stochastic process

- Transition probabilities depend on the regrets and a perturbation parameter $\varepsilon > 0$
- The stochastic process $X^t = (X_1^t, ..., X_n^t)$ describing players' states is a perturbed non-homogeneous Markov chain
- The Markov chain is defined over a infinitely countable state spaces
- For a regularly perturbed homogenous Markov chain, the process spends almost all time in the stochastically stable states when $\varepsilon \to 0$

Algorithm Analysis

Mood dynamics



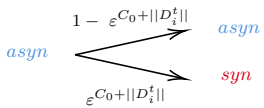


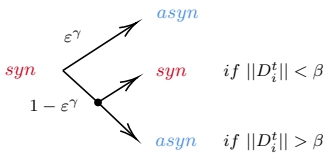
Characteristics of the stochastic process

- Transition probabilities depend on the regrets and a perturbation parameter $\varepsilon > 0$
- The stochastic process $\boldsymbol{X}^t = (\boldsymbol{X}_1^t, ..., \boldsymbol{X}_n^t)$ describing players' states is a perturbed non-homogeneous Markov chain
- The Markov chain is defined over a infinitely countable state spaces
- For a regularly perturbed homogenous Markov chain, the process spends almost all time in the stochastically stable states when $\varepsilon \to 0$

Algorithm Analysis

Mood dynamics



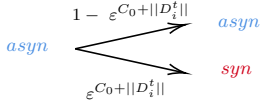


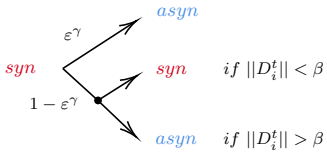
Characteristics of the stochastic process

- Transition probabilities depend on the regrets and a perturbation parameter $\varepsilon > 0$
- The stochastic process $\boldsymbol{X}^t = (\boldsymbol{X}_1^t, ..., \boldsymbol{X}_n^t)$ describing players' states is a perturbed non-homogeneous Markov chain
- The Markov chain is defined over a infinitely countable state spaces
- For a regularly perturbed homogenous Markov chain, the process spends almost all time in the stochastically stable states when $\varepsilon \to 0$

Algorithm Analysis

Mood dynamics





Characteristics of the stochastic process

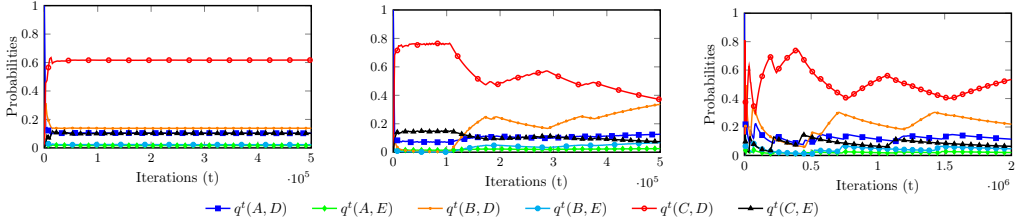
- Transition probabilities depend on the regrets and a perturbation parameter $\varepsilon > 0$
- The stochastic process $\boldsymbol{X}^t = (\boldsymbol{X}_1^t, ..., \boldsymbol{X}_n^t)$ describing players' states is a perturbed non-homogeneous Markov chain
- The Markov chain is defined over a infinitely countable state spaces
- For a regularly perturbed homogenous Markov chain, the process spends almost all time in the stochastically stable states when $\varepsilon \to 0$

Simulation Results

	D	E
A	(2, 29)	(16, 7)
B	(4, 7)	(6, 13)
C	(4, 4)	(6, 6)

Table 2: Utility matrix.

Simulation Results



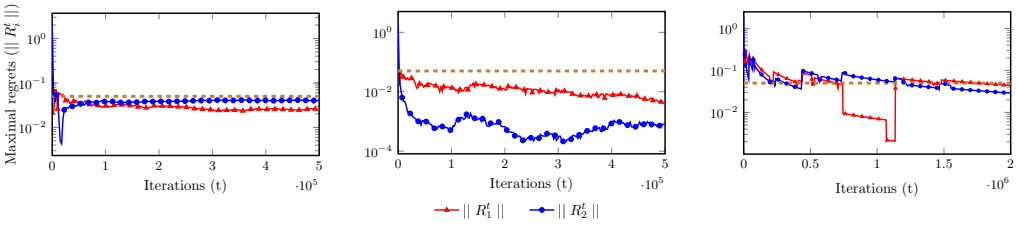
(a) CPRM algorithm

(b) Regret minimizing strategy

(c) Regret-matching

Figure 3: Evolution of the empirical distribution over action profiles.

Simulation Results



(a) CPRM algorithm.

(b) Regret minimizing strategy.

(c) Regret-matching.

Figure 4: Evolution of players' regrets for the three algorithms.

Time-varying Game

	D	E
A	(2, 29)	(16, 7)
B	(4, 7)	(6, 13)
C	(4, 4)	(6, 6)

↓

	D	E
A	(2, 29, 2)	(16, 7, 8)
B	(4, 7, 2)	(6, 13, 0)
C	(4, 4, 1)	(6, 6, 5)

X

	D	E
A	(9, 4, 0)	(4, 1, 4)
B	(8, 0, 1)	(6, 7, 2)
C	(11, 9, 3)	(2, 0, 4)

Y

↓

	D	E
A	(2, 29)	(16, 7)
B	(4, 7)	(6, 13)
C	(4, 4)	(6, 6)

↓

	D	E
A	(2, 29, 2)	(16, 7, 8)
B	(4, 7, 2)	(6, 13, 0)
C	(4, 4, 1)	(6, 6, 5)

X

	D	E
A	(9, 4, 0)	(4, 1, 4)
B	(8, 0, 1)	(6, 7, 2)
C	(11, 9, 3)	(2, 0, 4)

Y

Figure 5: Sequence of stage games in the dynamic case.

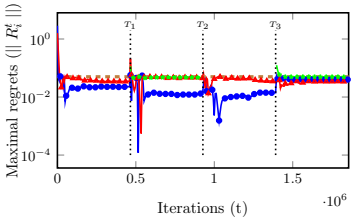


Figure 6: Evolution of maximal regrets

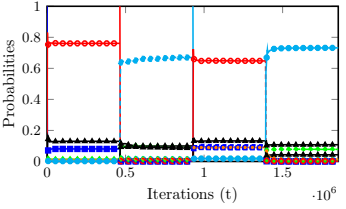


Figure 7: Evolution of the empirical distribution

Application to a Congestion Game

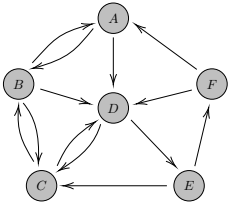


Figure 8: Network graph

Player	Source node	Destination node
1	B	F
2	B	E
3	D	B
4	F	E

Figure 9: Source-destination nodes for each player

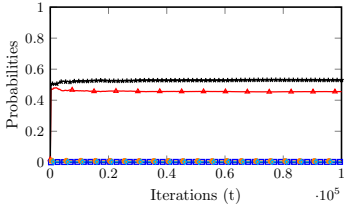


Figure 10: Evolution of the empirical distribution

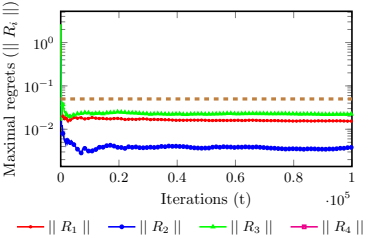


Figure 11: Evolution of maximal regrets

Part 2 : Constrained Correlated Equilibria

Constrained Correlated Equilibrium Strategies

Definition 1 – Correlated equilibrium (Aumann, 1974)

A correlated equilibrium of G is a pair $(d, \boldsymbol{\alpha}^*)$ where $\boldsymbol{\alpha}^* : \Omega \to \mathcal{A}$ is a correlated strategy profile such that

$$\forall i \in \mathcal{N}, \forall \alpha'_i \in \mathcal{S}_{i,d} \quad \sum_{\omega \in \Omega} \boldsymbol{q}(\omega) u_i(\boldsymbol{\alpha}^*_i(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) \geq \sum_{\omega \in \Omega} \boldsymbol{q}(\omega) u_i(\alpha'_i(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) \tag{4}$$

	P	A
P	8, 8	3, 10
A	10, 3	0, 0

Table 3: Utility matrix of the game of Chicken.

- $\Omega = \{H, M, L\}$
- $\mathcal{P}_1 = \{\{H\}, \{M, L\}\}$ and $\mathcal{P}_2 = \{\{H, M\}, \{L\}\}$
- $\boldsymbol{q}(H) = \boldsymbol{q}(M) = \boldsymbol{q}(L) = 1/3$

Constrained Correlated Equilibrium Strategies

Definition 1 – Correlated equilibrium (Aumann, 1974)

A correlated equilibrium of G is a pair $(d, \boldsymbol{\alpha}^*)$ where $\boldsymbol{\alpha}^* : \Omega \to \mathcal{A}$ is a correlated strategy profile such that

$$\forall i \in \mathcal{N}, \forall \alpha'_i \in \mathcal{S}_{i,d} \quad \sum_{\omega \in \Omega} \boldsymbol{q}(\omega) u_i(\boldsymbol{\alpha}^*_i(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) \geq \sum_{\omega \in \Omega} \boldsymbol{q}(\omega) u_i(\alpha'_i(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) \tag{4}$$

	P	A
P	8, 8	3, 10
A	10, 3	0, 0

Table 3: Utility matrix of the game of Chicken.

- $\Omega = \{H, M, L\}$
- $\mathcal{P}_1 = \{\{H\}, \{M, L\}\}$ and $\mathcal{P}_2 = \{\{H, M\}, \{L\}\}$
- $\boldsymbol{q}(H) = \boldsymbol{q}(M) = \boldsymbol{q}(L) = 1/3$

Constrained Correlated Equilibrium Strategies

	P	A
P	8, 8	3, 10
A	10, 3	0, 0

Table 4: Utility matrix of the game of Chicken.

- $\Omega = \{H, M, L\}$
- $\mathcal{P}_1 = \{\{H\}, \{M, L\}\}$ and $\mathcal{P}_2 = \{\{H, M\}, \{L\}\}$
- $\boldsymbol{q}(H) = \boldsymbol{q}(M) = \boldsymbol{q}(L) = 1/3$

	$L \mapsto P$ $L^c \mapsto P$	$L \mapsto A$ $L^c \mapsto A$	$L \mapsto A$ $L^c \mapsto P$	$L \mapsto P$ $L^c \mapsto A$
$H \mapsto P$ $H^c \mapsto P$	$8, 8$	$3, 10$	$6.33, 8.67$	$4.67, 9.33$
$H \mapsto A$ $H^c \mapsto A$	$10, 3$	$0, 0$	$6.67, 2$	$3.33, 1$
$H \mapsto A$ $H^c \mapsto P$	$8.67, 6.33$	$2, 6.67$	$7, 7$	$3.67, 6$
$H \mapsto P$ $H^c \mapsto A$	$9.33, 4.67$	$1, 3.33$	$6, 3.67$	$4.33, 4.33$

Figure 12: Extension of the game of Chicken.

	$L \mapsto P$ $L^c \mapsto P$	$L \mapsto A$ $L^c \mapsto A$	$L \mapsto A$ $L^c \mapsto P$	$L \mapsto P$ $L^c \mapsto A$
$H \mapsto P$ $H^c \mapsto P$	$8, 8$	$3, 10$	~~$6.33, 8.67$~~	~~$4.67, 9.33$~~
$H \mapsto A$ $H^c \mapsto A$	~~$10, 3$~~	$0, 0$	$6.67, 2$	$3.33, 1$
$H \mapsto A$ $H^c \mapsto P$	~~$8.67, 6.33$~~	~~$2, 6.67$~~	$7, 7$	$3.67, 6$
$H \mapsto P$ $H^c \mapsto A$	~~$9.33, 4.67$~~	$1, 3.33$	~~$6, 3.67$~~	$4.33, 4.33$

Table 2: Constrained extension of the game of Chicken.

Constrained Correlated Equilibrium Strategies

	P	A
P	8, 8	3, 10
A	10, 3	0, 0

Table 4: Utility matrix of the game of Chicken.

- $\Omega = \{H, M, L\}$
- $\mathcal{P}_1 = \{\{H\}, \{M, L\}\}$ and $\mathcal{P}_2 = \{\{H, M\}, \{L\}\}$
- $\boldsymbol{q}(H) = \boldsymbol{q}(M) = \boldsymbol{q}(L) = 1/3$

	$L \mapsto P$ $L^c \mapsto P$	$L \mapsto A$ $L^c \mapsto A$	$L \mapsto A$ $L^c \mapsto P$	$L \mapsto P$ $L^c \mapsto A$
$H \mapsto P$ $H^c \mapsto P$	$8, 8$	$3, 10$	$6.33, 8.67$	$4.67, 9.33$
$H \mapsto A$ $H^c \mapsto A$	$10, 3$	$0, 0$	$6.67, 2$	$3.33, 1$
$H \mapsto A$ $H^c \mapsto P$	$8.67, 6.33$	$2, 6.67$	$7, 7$	$3.67, 6$
$H \mapsto P$ $H^c \mapsto A$	$9.33, 4.67$	$1, 3.33$	$6, 3.67$	$4.33, 4.33$

Figure 12: Extension of the game of Chicken.

	$L \mapsto P$ $L^c \mapsto P$	$L \mapsto A$ $L^c \mapsto A$	$L \mapsto A$ $L^c \mapsto P$	$L \mapsto P$ $L^c \mapsto A$
$H \mapsto P$ $H^c \mapsto P$	$8, 8$	$3, 10$	~~$6.33, 8.67$~~	~~$4.67, 9.33$~~
$H \mapsto A$ $H^c \mapsto A$	~~$10, 3$~~	$0, 0$	$6.67, 2$	$3.33, 1$
$H \mapsto A$ $H^c \mapsto P$	~~$8.67, 6.33$~~	~~$2, 6.67$~~	$7, 7$	$3.67, 6$
$H \mapsto P$ $H^c \mapsto A$	~~$9.33, 4.67$~~	$1, 3.33$	~~$6, 3.67$~~	$4.33, 4.33$

Table 2: Constrained extension of the game of Chicken.

Constrained Correlated Equilibrium Strategies

	$L \mapsto P$ $L^c \mapsto P$	$L \mapsto A$ $L^c \mapsto A$	$L \mapsto A$ $L^c \mapsto P$	$L \mapsto P$ $L^c \mapsto A$
$H \mapsto P$ $H^c \mapsto P$	$8, 8$	$3, 10$	$6.33, 8.67$	$4.67, 9.33$
$H \mapsto A$ $H^c \mapsto A$	$10, 3$	$0, 0$	$6.67, 2$	$3.33, 1$
$H \mapsto A$ $H^c \mapsto P$	$8.67, 6.33$	$2, 6.67$	$7, 7$	$3.67, 6$
$H \mapsto P$ $H^c \mapsto A$	$9.33, 4.67$	$1, 3.33$	$6, 3.67$	$4.33, 4.33$

Figure 13: Extension of the game of Chicken.

	$L \mapsto P$ $L^c \mapsto P$	$L \mapsto A$ $L^c \mapsto A$	$L \mapsto A$ $L^c \mapsto P$	$L \mapsto P$ $L^c \mapsto A$
$H \mapsto P$ $H^c \mapsto P$	$8, 8$	$3, 10$	~~$6.33, 8.67$~~	~~$4.67, 9.33$~~
$H \mapsto A$ $H^c \mapsto A$	~~$10, 3$~~	$0, 0$	$6.67, 2$	$3.33, 1$
$H \mapsto A$ $H^c \mapsto P$	~~$8.67, 6.33$~~	~~$2, 6.67$~~	$7, 7$	$3.67, 6$
$H \mapsto P$ $H^c \mapsto A$	~~$9.33, 4.67$~~	$1, 3.33$	~~$6, 3.67$~~	$4.33, 4.33$

Figure 14: Constrained extension of the game of Chicken.

Definition 4 – Constrained Correlated equilibrium (Boufous et al., 2024)
A constrained correlated equilibrium of G is a triplet $(d, \mathcal{R}_d, \boldsymbol{\alpha}^*)$ where d is a correlation device, $\mathcal{R}_d$ is a coupled constraint set in G_d and $\boldsymbol{\alpha}^*$ a correlated strategy profile such that, $\boldsymbol{\alpha}^* \in \mathcal{R}_d$ and for any $i \in \mathcal{N}$, for any $\alpha'_i \in \mathcal{S}_{i,d}$ such that $(\alpha'_i, \boldsymbol{\alpha}^*_{-i}) \in \mathcal{R}_d$,

$$\sum_{\omega \in \Omega} \boldsymbol{q}(\omega) \left[u_i(\alpha^*_i(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) - u_i(\alpha'_i(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) \right] \geq 0 \tag{5}$$

Constrained Correlated Equilibrium Strategies

	$L \mapsto P$ $L^c \mapsto P$	$L \mapsto A$ $L^c \mapsto A$	$L \mapsto A$ $L^c \mapsto P$	$L \mapsto P$ $L^c \mapsto A$
$H \mapsto P$ $H^c \mapsto P$	$8, 8$	$3, 10$	$6.33, 8.67$	$4.67, 9.33$
$H \mapsto A$ $H^c \mapsto A$	$10, 3$	$0, 0$	$6.67, 2$	$3.33, 1$
$H \mapsto A$ $H^c \mapsto P$	$8.67, 6.33$	$2, 6.67$	$7, 7$	$3.67, 6$
$H \mapsto P$ $H^c \mapsto A$	$9.33, 4.67$	$1, 3.33$	$6, 3.67$	$4.33, 4.33$

Figure 13: Extension of the game of Chicken.

	$L \mapsto P$ $L^c \mapsto P$	$L \mapsto A$ $L^c \mapsto A$	$L \mapsto A$ $L^c \mapsto P$	$L \mapsto P$ $L^c \mapsto A$
$H \mapsto P$ $H^c \mapsto P$	$8, 8$	$3, 10$	~~$6.33, 8.67$~~	~~$4.67, 9.33$~~
$H \mapsto A$ $H^c \mapsto A$	~~$10, 3$~~	$0, 0$	$6.67, 2$	$3.33, 1$
$H \mapsto A$ $H^c \mapsto P$	~~$8.67, 6.33$~~	~~$2, 6.67$~~	$7, 7$	$3.67, 6$
$H \mapsto P$ $H^c \mapsto A$	~~$9.33, 4.67$~~	$1, 3.33$	~~$6, 3.67$~~	$4.33, 4.33$

Figure 14: Constrained extension of the game of Chicken.

Definition 4 – Constrained Correlated equilibrium (Boufous et al., 2024)

A constrained correlated equilibrium of G is a triplet $(d, \mathcal{R}_d, \boldsymbol{\alpha}^*)$ where d is a correlation device, $\mathcal{R}_d$ is a coupled constraint set in G_d and $\boldsymbol{\alpha}^*$ a correlated strategy profile such that, $\boldsymbol{\alpha}^* \in \mathcal{R}_d$ and for any $i \in \mathcal{N}$, for any $\alpha'_i \in \mathcal{S}_{i,d}$ such that $(\alpha'_i, \boldsymbol{\alpha}^*_{-i}) \in \mathcal{R}_d$,

$$\sum_{\omega \in \Omega} \boldsymbol{q}(\omega) \left[u_i(\alpha^*_i(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) - u_i(\alpha'_i(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) \right] \geq 0 \tag{5}$$

Properties of Constrained Correlated Equilibrium Strategies

Consider a finite non-cooperative game $G = (\mathcal{N}, (\mathcal{A}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$, a correlation device $d = (\Omega, (\mathcal{P}_i)_{i\in\mathcal{N}}, \boldsymbol{q})$ and a constraint set $\mathcal{R}_d \subset \mathcal{S}_d$.

Proposition 1 : If $(d, \boldsymbol{\alpha}^*)$ is a correlated equilibrium and $\boldsymbol{\alpha}^* \in \mathcal{R}_d$, then $(d, \mathcal{R}_d, \boldsymbol{\alpha}^*)$ is a constrained correlated equilibrium.

Proposition 2 : If $\boldsymbol{\alpha}^* \in \mathcal{R}_d$ and for any $i \in \mathcal{N}$, for any α_i' s.t. $(\alpha_i', \boldsymbol{\alpha}_{-i}) \in \mathcal{R}_d$, for any $\omega \in \Omega$

$$\sum_{\omega' \in P_i(\omega)} \boldsymbol{q}(\omega') \left[u_i(\alpha_i^*(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) - u_i(\alpha_i'(\omega'), \boldsymbol{\alpha}^*_{-i}(\omega')) \right] \geq 0 \tag{6}$$

then $(d, \mathcal{R}_d, \boldsymbol{\alpha}^*)$ is a constrained correlated equilibrium.

Proposition 3 : The triplet $(d, \mathcal{R}_d, \boldsymbol{\alpha}^*)$ is a constrained correlated equilibrium if and only if $\boldsymbol{\alpha}^* \in \mathcal{R}_d$ and for any $i \in \mathcal{N}$, for any $\alpha_i' \in \mathcal{S}_{i,d}$,

$$\sum_{\omega \in \Omega} \boldsymbol{q}(\omega) \left[u_i(\alpha_i^*(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) - u_i(\alpha_i'(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) \right] \geq 0 \quad \text{or} \quad (\alpha_i', \boldsymbol{\alpha}^*_{-i}) \notin \mathcal{R}_d \tag{7}$$

Properties of Constrained Correlated Equilibrium Strategies

Consider a finite non-cooperative game $G = (\mathcal{N}, (\mathcal{A}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$, a correlation device $d = (\Omega, (\mathcal{P}_i)_{i\in\mathcal{N}}, \boldsymbol{q})$ and a constraint set $\mathcal{R}_d \subset \mathcal{S}_d$.

Proposition 1 : If $(d, \boldsymbol{\alpha}^*)$ is a correlated equilibrium and $\boldsymbol{\alpha}^* \in \mathcal{R}_d$, then $(d, \mathcal{R}_d, \boldsymbol{\alpha}^*)$ is a constrained correlated equilibrium.

Proposition 2 : If $\boldsymbol{\alpha}^* \in \mathcal{R}_d$ and for any $i \in \mathcal{N}$, for any α_i' s.t. $(\alpha_i', \boldsymbol{\alpha}_{-i}) \in \mathcal{R}_d$, for any $\omega \in \Omega$

$$\sum_{\omega' \in P_i(\omega)} \boldsymbol{q}(\omega') \left[u_i(\alpha_i^*(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) - u_i(\alpha_i'(\omega'), \boldsymbol{\alpha}^*_{-i}(\omega')) \right] \geq 0 \tag{6}$$

then $(d, \mathcal{R}_d, \boldsymbol{\alpha}^*)$ is a constrained correlated equilibrium.

Proposition 3 : The triplet $(d, \mathcal{R}_d, \boldsymbol{\alpha}^*)$ is a constrained correlated equilibrium if and only if $\boldsymbol{\alpha}^* \in \mathcal{R}_d$ and for any $i \in \mathcal{N}$, for any $\alpha_i' \in \mathcal{S}_{i,d}$,

$$\sum_{\omega \in \Omega} \boldsymbol{q}(\omega) \left[u_i(\alpha_i^*(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) - u_i(\alpha_i'(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) \right] \geq 0 \quad \text{or} \quad (\alpha_i', \boldsymbol{\alpha}^*_{-i}) \notin \mathcal{R}_d \tag{7}$$

Properties of Constrained Correlated Equilibrium Strategies

Consider a finite non-cooperative game $G = (\mathcal{N}, (\mathcal{A}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$, a correlation device $d = (\Omega, (\mathcal{P}_i)_{i\in\mathcal{N}}, \boldsymbol{q})$ and a constraint set $\mathcal{R}_d \subset \mathcal{S}_d$.

Proposition 1 : If $(d, \boldsymbol{\alpha}^*)$ is a correlated equilibrium and $\boldsymbol{\alpha}^* \in \mathcal{R}_d$, then $(d, \mathcal{R}_d, \boldsymbol{\alpha}^*)$ is a constrained correlated equilibrium.

Proposition 2 : If $\boldsymbol{\alpha}^* \in \mathcal{R}_d$ and for any $i \in \mathcal{N}$, for any α_i' s.t. $(\alpha_i', \boldsymbol{\alpha}_{-i}) \in \mathcal{R}_d$, for any $\omega \in \Omega$

$$\sum_{\omega' \in P_i(\omega)} \boldsymbol{q}(\omega') \left[u_i(\alpha_i^*(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) - u_i(\alpha_i'(\omega'), \boldsymbol{\alpha}^*_{-i}(\omega')) \right] \geq 0 \tag{6}$$

then $(d, \mathcal{R}_d, \boldsymbol{\alpha}^*)$ is a constrained correlated equilibrium.

Proposition 3 : The triplet $(d, \mathcal{R}_d, \boldsymbol{\alpha}^*)$ is a constrained correlated equilibrium if and only if $\boldsymbol{\alpha}^* \in \mathcal{R}_d$ and for any $i \in \mathcal{N}$, for any $\alpha_i' \in \mathcal{S}_{i,d}$,

$$\sum_{\omega \in \Omega} \boldsymbol{q}(\omega) \left[u_i(\alpha_i^*(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) - u_i(\alpha_i'(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) \right] \geq 0 \quad \text{or} \quad (\alpha_i', \boldsymbol{\alpha}^*_{-i}) \notin \mathcal{R}_d \tag{7}$$

Properties of Constrained Correlated Equilibrium Strategies

Consider a finite non-cooperative game $G = (\mathcal{N}, (\mathcal{A}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$, a correlation device $d = (\Omega, (\mathcal{P}_i)_{i\in\mathcal{N}}, \boldsymbol{q})$ and a constraint set $\mathcal{R}_d \subset \mathcal{S}_d$.

Proposition 1 : If $(d, \boldsymbol{\alpha}^*)$ is a correlated equilibrium and $\boldsymbol{\alpha}^* \in \mathcal{R}_d$, then $(d, \mathcal{R}_d, \boldsymbol{\alpha}^*)$ is a constrained correlated equilibrium.

Proposition 2 : If $\boldsymbol{\alpha}^* \in \mathcal{R}_d$ and for any $i \in \mathcal{N}$, for any α_i' s.t. $(\alpha_i', \boldsymbol{\alpha}_{-i}) \in \mathcal{R}_d$, for any $\omega \in \Omega$

$$\sum_{\omega' \in P_i(\omega)} \boldsymbol{q}(\omega') \left[u_i(\alpha_i^*(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) - u_i(\alpha_i'(\omega'), \boldsymbol{\alpha}^*_{-i}(\omega')) \right] \geq 0 \tag{6}$$

then $(d, \mathcal{R}_d, \boldsymbol{\alpha}^*)$ is a constrained correlated equilibrium.

Proposition 3 : The triplet $(d, \mathcal{R}_d, \boldsymbol{\alpha}^*)$ is a constrained correlated equilibrium if and only if $\boldsymbol{\alpha}^* \in \mathcal{R}_d$ and for any $i \in \mathcal{N}$, for any $\alpha_i' \in \mathcal{S}_{i,d}$,

$$\sum_{\omega \in \Omega} \boldsymbol{q}(\omega) \left[u_i(\alpha_i^*(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) - u_i(\alpha_i'(\omega), \boldsymbol{\alpha}^*_{-i}(\omega)) \right] \geq 0 \quad \text{or} \quad (\alpha_i', \boldsymbol{\alpha}^*_{-i}) \notin \mathcal{R}_d \tag{7}$$

Properties of Constrained Correlated Equilibrium Strategies

Consider a finite non-cooperative game $G = (\mathcal{N}, (\mathcal{A}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$, a correlation device $d = (\Omega, (\mathcal{P}_i)_{i\in\mathcal{N}}, \boldsymbol{q})$ and a constraint set $\mathcal{R}_d \subset \mathcal{S}_d$.

Proposition 1 : If $(d, \boldsymbol{\alpha}^*)$ is a correlated equilibrium and $\boldsymbol{\alpha}^* \in \mathcal{R}_d$, then $(d, \mathcal{R}_d, \boldsymbol{\alpha}^*)$ is a constrained correlated equilibrium.

Proposition 2 : If $\boldsymbol{\alpha}^* \in \mathcal{R}_d$ and for any $i \in \mathcal{N}$, for any α_i' s.t. $(\alpha_i', \boldsymbol{\alpha}_{-i}) \in \mathcal{R}_d$, for any $\omega \in \Omega$

$$\sum_{\omega' \in P_i(\omega)} \boldsymbol{q}(\omega') \left[u_i(\alpha_i^*(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) - u_i(\alpha_i'(\omega'), \boldsymbol{\alpha}_{-i}^*(\omega')) \right] \geq 0 \tag{6}$$

then $(d, \mathcal{R}_d, \boldsymbol{\alpha}^*)$ is a constrained correlated equilibrium.

Proposition 3 : The triplet $(d, \mathcal{R}_d, \boldsymbol{\alpha}^*)$ is a constrained correlated equilibrium if and only if $\boldsymbol{\alpha}^* \in \mathcal{R}_d$ and for any $i \in \mathcal{N}$, for any $\alpha_i' \in \mathcal{S}_{i,d}$,

$$\sum_{\omega \in \Omega} \boldsymbol{q}(\omega) \left[u_i(\alpha_i^*(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) - u_i(\alpha_i'(\omega), \boldsymbol{\alpha}_{-i}^*(\omega)) \right] \geq 0 \quad \text{or} \quad (\alpha_i', \boldsymbol{\alpha}_{-i}^*) \notin \mathcal{R}_d \tag{7}$$

Constraints induced by a set of feasible probability distributions

- Let $\mathcal{C} \subseteq \Delta(\mathcal{A})$ be a set of probability distributions. For each correlation device d,

$$\mathcal{R}_d = \{\boldsymbol{\alpha} \in \mathcal{S}_d \mid \boldsymbol{p}_{\boldsymbol{\alpha}} \in \mathcal{C}\} \tag{8}$$

- Performance measures in many applications can be expressed in terms of probability distribution over action profiles. Examples include applications in smart grids, wireless networks, etc.
  - Constraints on the social welfare : $\sum_{i \in \mathcal{N}} \sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}) \geq D_1$
  - Constraints on the Nash product : $\prod_{i \in \mathcal{N}} \left(\sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}) \right) \geq D_2$

Theorem 1 – Characterization of the Set of Constrained Correlated Equilibrium Distributions
Let G be a finite non-cooperative game and $\mathcal{C}$ a set of feasible probability distributions. The distribution $\boldsymbol{p} \in \Delta(\mathcal{A})$ is a constrained correlated equilibrium distribution if and only if for any player $i \in \mathcal{N}$, for any strategy $\beta_i : \mathcal{A}_i \to \mathcal{A}_i$, if $\boldsymbol{z}_{\beta_i, \boldsymbol{p}} \in \mathcal{C}$, then

$$\sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) \left[u_i(a_i, \boldsymbol{a}_{-i}) - u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \right] \geq 0 \tag{9}$$

where $\boldsymbol{z}_{\beta_i, \boldsymbol{p}}(\boldsymbol{a}) = \Sigma_{b_i \in \mathcal{A}_i} \boldsymbol{p}(b_i, \boldsymbol{a}_{-i}) \mathbb{1}_{\beta_i(b_i) = a_i}$ for any $\boldsymbol{a} \in \mathcal{A}$ is the distribution resulting from player i's unilateral deviation β_i.

Constraints induced by a set of feasible probability distributions

- Let $\mathcal{C} \subseteq \Delta(\mathcal{A})$ be a set of probability distributions. For each correlation device d,

$$\mathcal{R}_d = \{\boldsymbol{\alpha} \in \mathcal{S}_d \mid \boldsymbol{p}_{\boldsymbol{\alpha}} \in \mathcal{C}\} \tag{8}$$

- Performance measures in many applications can be expressed in terms of probability distribution over action profiles. Examples include applications in smart grids, wireless networks, etc.
  - Constraints on the social welfare : $\sum_{i \in \mathcal{N}} \sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}) \geq D_1$
  - Constraints on the Nash product : $\prod_{i \in \mathcal{N}} \left(\sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}) \right) \geq D_2$

Theorem 1 – Characterization of the Set of Constrained Correlated Equilibrium Distributions
Let G be a finite non-cooperative game and $\mathcal{C}$ a set of feasible probability distributions. The distribution $\boldsymbol{p} \in \Delta(\mathcal{A})$ is a constrained correlated equilibrium distribution if and only if for any player $i \in \mathcal{N}$, for any strategy $\beta_i : \mathcal{A}_i \to \mathcal{A}_i$, if $\boldsymbol{z}_{\beta_i, \boldsymbol{p}} \in \mathcal{C}$, then

$$\sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) \left[u_i(a_i, \boldsymbol{a}_{-i}) - u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \right] \geq 0 \tag{9}$$

where $\boldsymbol{z}_{\beta_i, \boldsymbol{p}}(\boldsymbol{a}) = \Sigma_{b_i \in \mathcal{A}_i} \boldsymbol{p}(b_i, \boldsymbol{a}_{-i}) \mathbb{1}_{\beta_i(b_i) = a_i}$ for any $\boldsymbol{a} \in \mathcal{A}$ is the distribution resulting from player i's unilateral deviation β_i.

Constraints induced by a set of feasible probability distributions

- Let $\mathcal{C} \subseteq \Delta(\mathcal{A})$ be a set of probability distributions. For each correlation device d,

$$\mathcal{R}_d = \{\boldsymbol{\alpha} \in \mathcal{S}_d \mid \boldsymbol{p}_{\boldsymbol{\alpha}} \in \mathcal{C}\} \tag{8}$$

- Performance measures in many applications can be expressed in terms of probability distribution over action profiles. Examples include applications in smart grids, wireless networks, etc.
  - Constraints on the social welfare : $\sum_{i \in \mathcal{N}} \sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}) \geq D_1$
  - Constraints on the Nash product : $\prod_{i \in \mathcal{N}} \left(\sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}) \right) \geq D_2$

Theorem 1 – Characterization of the Set of Constrained Correlated Equilibrium Distributions
Let G be a finite non-cooperative game and $\mathcal{C}$ a set of feasible probability distributions. The distribution $\boldsymbol{p} \in \Delta(\mathcal{A})$ is a constrained correlated equilibrium distribution if and only if for any player $i \in \mathcal{N}$, for any strategy $\beta_i : \mathcal{A}_i \to \mathcal{A}_i$, if $\boldsymbol{z}_{\beta_i, \boldsymbol{p}} \in \mathcal{C}$, then

$$\sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) \left[u_i(a_i, \boldsymbol{a}_{-i}) - u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \right] \geq 0 \tag{9}$$

where $\boldsymbol{z}_{\beta_i, \boldsymbol{p}}(\boldsymbol{a}) = \Sigma_{b_i \in \mathcal{A}_i} \boldsymbol{p}(b_i, \boldsymbol{a}_{-i}) \mathbb{1}_{\beta_i(b_i) = a_i}$ for any $\boldsymbol{a} \in \mathcal{A}$ is the distribution resulting from player i's unilateral deviation β_i.

Constraints induced by a set of feasible probability distributions

- Let $\mathcal{C} \subseteq \Delta(\mathcal{A})$ be a set of probability distributions. For each correlation device d,

$$\mathcal{R}_d = \{\boldsymbol{\alpha} \in \mathcal{S}_d \mid \boldsymbol{p}_{\boldsymbol{\alpha}} \in \mathcal{C}\} \tag{8}$$

- Performance measures in many applications can be expressed in terms of probability distribution over action profiles. Examples include applications in smart grids, wireless networks, etc.
  - Constraints on the social welfare : $\sum_{i \in \mathcal{N}} \sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}) \geq D_1$
  - Constraints on the Nash product : $\prod_{i \in \mathcal{N}} \left(\sum_{a \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}) \right) \geq D_2$

Theorem 1 – Characterization of the Set of Constrained Correlated Equilibrium Distributions
Let G be a finite non-cooperative game and $\mathcal{C}$ a set of feasible probability distributions. The distribution $\boldsymbol{p} \in \Delta(\mathcal{A})$ is a constrained correlated equilibrium distribution if and only if for any player $i \in \mathcal{N}$, for any strategy $\beta_i : \mathcal{A}_i \to \mathcal{A}_i$, if $\boldsymbol{z}_{\beta_i, \boldsymbol{p}} \in \mathcal{C}$, then

$$\sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) \left[u_i(a_i, \boldsymbol{a}_{-i}) - u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \right] \geq 0 \tag{9}$$

where $\boldsymbol{z}_{\beta_i, \boldsymbol{p}}(\boldsymbol{a}) = \Sigma_{b_i \in \mathcal{A}_i} \boldsymbol{p}(b_i, \boldsymbol{a}_{-i}) \mathbb{1}_{\beta_i(b_i) = a_i}$ for any $\boldsymbol{a} \in \mathcal{A}$ is the distribution resulting from player i's unilateral deviation β_i.

Constraints induced by a set of feasible probability distributions

- Let $\mathcal{C} \subseteq \Delta(\mathcal{A})$ be a set of probability distributions. For each correlation device d,

$$\mathcal{R}_d = \{\boldsymbol{\alpha} \in \mathcal{S}_d \mid \boldsymbol{p}_{\boldsymbol{\alpha}} \in \mathcal{C}\} \tag{8}$$

- Performance measures in many applications can be expressed in terms of probability distribution over action profiles. Examples include applications in smart grids, wireless networks, etc.
  - Constraints on the social welfare : $\sum_{i \in \mathcal{N}} \sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}) \geq D_1$
  - Constraints on the Nash product : $\prod_{i \in \mathcal{N}} \left(\sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}) \right) \geq D_2$

Theorem 1 – Characterization of the Set of Constrained Correlated Equilibrium Distributions
Let G be a finite non-cooperative game and $\mathcal{C}$ a set of feasible probability distributions. The distribution $\boldsymbol{p} \in \Delta(\mathcal{A})$ is a constrained correlated equilibrium distribution if and only if for any player $i \in \mathcal{N}$, for any strategy $\beta_i : \mathcal{A}_i \to \mathcal{A}_i$, if $\boldsymbol{z}_{\beta_i, \boldsymbol{p}} \in \mathcal{C}$, then

$$\sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) \left[u_i(a_i, \boldsymbol{a}_{-i}) - u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \right] \geq 0 \tag{9}$$

where $\boldsymbol{z}_{\beta_i, \boldsymbol{p}}(\boldsymbol{a}) = \Sigma_{b_i \in \mathcal{A}_i} \boldsymbol{p}(b_i, \boldsymbol{a}_{-i}) \mathbb{1}_{\beta_i(b_i) = a_i}$ for any $\boldsymbol{a} \in \mathcal{A}$ is the distribution resulting from player i's unilateral deviation β_i.

Constraints induced by a set of feasible probability distributions

- Let $\mathcal{C} \subseteq \Delta(\mathcal{A})$ be a set of probability distributions. For each correlation device d,

$$\mathcal{R}_d = \{\boldsymbol{\alpha} \in \mathcal{S}_d \mid \boldsymbol{p}_{\boldsymbol{\alpha}} \in \mathcal{C}\} \tag{8}$$

- Performance measures in many applications can be expressed in terms of probability distribution over action profiles. Examples include applications in smart grids, wireless networks, etc.
  - Constraints on the social welfare : $\sum_{i \in \mathcal{N}} \sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}) \geq D_1$
  - Constraints on the Nash product : $\prod_{i \in \mathcal{N}} \left(\sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}) \right) \geq D_2$

Theorem 1 – Characterization of the Set of Constrained Correlated Equilibrium Distributions

Let G be a finite non-cooperative game and $\mathcal{C}$ a set of feasible probability distributions. The distribution $\boldsymbol{p} \in \Delta(\mathcal{A})$ is a constrained correlated equilibrium distribution if and only if for any player $i \in \mathcal{N}$, for any strategy $\beta_i : \mathcal{A}_i \to \mathcal{A}_i$, if $\boldsymbol{z}_{\beta_i, \boldsymbol{p}} \in \mathcal{C}$, then

$$\sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) \left[u_i(a_i, \boldsymbol{a}_{-i}) - u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \right] \geq 0 \tag{9}$$

where $\boldsymbol{z}_{\beta_i, \boldsymbol{p}}(\boldsymbol{a}) = \Sigma_{b_i \in \mathcal{A}_i} \boldsymbol{p}(b_i, \boldsymbol{a}_{-i}) \mathbb{1}_{\beta_i(b_i) = a_i}$ for any $\boldsymbol{a} \in \mathcal{A}$ is the distribution resulting from player i's unilateral deviation β_i.

Constraints induced by a set of feasible probability distributions

- Let $\mathcal{C} \subseteq \Delta(\mathcal{A})$ be a set of probability distributions. For each correlation device d,

$$\mathcal{R}_d = \{\boldsymbol{\alpha} \in \mathcal{S}_d \mid \boldsymbol{p}_{\boldsymbol{\alpha}} \in \mathcal{C}\} \tag{8}$$

- Performance measures in many applications can be expressed in terms of probability distribution over action profiles. Examples include applications in smart grids, wireless networks, etc.
  - Constraints on the social welfare : $\sum_{i \in \mathcal{N}} \sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}) \geq D_1$
  - Constraints on the Nash product : $\prod_{i \in \mathcal{N}} \left(\sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) u_i(\boldsymbol{a}) \right) \geq D_2$

Theorem 1 – Characterization of the Set of Constrained Correlated Equilibrium Distributions

Let G be a finite non-cooperative game and $\mathcal{C}$ a set of feasible probability distributions. The distribution $\boldsymbol{p} \in \Delta(\mathcal{A})$ is a constrained correlated equilibrium distribution if and only if for any player $i \in \mathcal{N}$, for any strategy $\beta_i : \mathcal{A}_i \to \mathcal{A}_i$, if $\boldsymbol{z}_{\beta_i, \boldsymbol{p}} \in \mathcal{C}$, then

$$\sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) \left[u_i(a_i, \boldsymbol{a}_{-i}) - u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \right] \geq 0 \tag{9}$$

where $\boldsymbol{z}_{\beta_i, \boldsymbol{p}}(\boldsymbol{a}) = \Sigma_{b_i \in \mathcal{A}_i} \boldsymbol{p}(b_i, \boldsymbol{a}_{-i}) \mathbb{1}_{\beta_i(b_i) = a_i}$ for any $\boldsymbol{a} \in \mathcal{A}$ is the distribution resulting from player i's unilateral deviation β_i.

Existence of Constrained Correlated Equilibria

Consider the following two-player game:

	L	R
U	$(2, 2)$	$(1, 1)$
D	$(3, 0)$	$(0, 5)$

Figure 13: Two-player game in matrix form.

Let $\mathcal{C} \subset \Delta(\mathcal{A})$ be a feasible set of probability distributions such that,

$$\mathcal{C} = \{\boldsymbol{p} \in \Delta(\mathcal{A}) \mid \boldsymbol{p}(U, L) = 1 \text{ or } \boldsymbol{p}(U, R) = 1 \text{ or } \boldsymbol{p}(D, L) = 1 \text{ or } \boldsymbol{p}(D, R) = 1\} \tag{10}$$

- The set of feasible strategies is the set of pure action profiles
- The players must play a correlated strategy profile inducing a pure action profile in G
- There does not exist a constrained correlated equilibrium for this game

Theorem 2 - Existence of Constrained Correlated Equilibria

Let G be a finite non-cooperative game and $\mathcal{C}$ a feasible set of probability distributions. If the intersection of $\mathcal{C}$ with the set of correlated equilibrium distributions of the game G is non-empty, then a constrained correlated equilibrium of G exists.

Existence of Constrained Correlated Equilibria

Consider the following two-player game:

	L	R
U	$(2, 2)$	$(1, 1)$
D	$(3, 0)$	$(0, 5)$

Figure 13: Two-player game in matrix form.

Let $\mathcal{C} \subset \Delta(\mathcal{A})$ be a feasible set of probability distributions such that,

$$\mathcal{C} = \{\boldsymbol{p} \in \Delta(\mathcal{A}) \mid \boldsymbol{p}(U, L) = 1 \text{ or } \boldsymbol{p}(U, R) = 1 \text{ or } \boldsymbol{p}(D, L) = 1 \text{ or } \boldsymbol{p}(D, R) = 1\} \tag{10}$$

- The set of feasible strategies is the set of pure action profiles
- The players must play a correlated strategy profile inducing a pure action profile in G
- There does not exist a constrained correlated equilibrium for this game

Theorem 2 - Existence of Constrained Correlated Equilibria

Let G be a finite non-cooperative game and $\mathcal{C}$ a feasible set of probability distributions. If the intersection of $\mathcal{C}$ with the set of correlated equilibrium distributions of the game G is non-empty, then a constrained correlated equilibrium of G exists.

Existence of Constrained Correlated Equilibria

Consider the following two-player game:

	L	R
U	$(2, 2)$	$(1, 1)$
D	$(3, 0)$	$(0, 5)$

Figure 13: Two-player game in matrix form.

Let $\mathcal{C} \subset \Delta(\mathcal{A})$ be a feasible set of probability distributions such that,

$$\mathcal{C} = \{\boldsymbol{p} \in \Delta(\mathcal{A}) \mid \boldsymbol{p}(U, L) = 1 \text{ or } \boldsymbol{p}(U, R) = 1 \text{ or } \boldsymbol{p}(D, L) = 1 \text{ or } \boldsymbol{p}(D, R) = 1\} \tag{10}$$

- The set of feasible strategies is the set of pure action profiles
- The players must play a correlated strategy profile inducing a pure action profile in G
- There does not exist a constrained correlated equilibrium for this game

Theorem 2 - Existence of Constrained Correlated Equilibria

Let G be a finite non-cooperative game and $\mathcal{C}$ a feasible set of probability distributions. If the intersection of $\mathcal{C}$ with the set of correlated equilibrium distributions of the game G is non-empty, then a constrained correlated equilibrium of G exists.

Existence of Constrained Correlated Equilibria

Consider the following two-player game:

	L	R
U	$(2, 2)$	$(1, 1)$
D	$(3, 0)$	$(0, 5)$

Figure 13: Two-player game in matrix form.

Let $\mathcal{C} \subset \Delta(\mathcal{A})$ be a feasible set of probability distributions such that,

$$\mathcal{C} = \{\boldsymbol{p} \in \Delta(\mathcal{A}) \mid \boldsymbol{p}(U, L) = 1 \text{ or } \boldsymbol{p}(U, R) = 1 \text{ or } \boldsymbol{p}(D, L) = 1 \text{ or } \boldsymbol{p}(D, R) = 1\} \tag{10}$$

- The set of feasible strategies is the set of pure action profiles
- The players must play a correlated strategy profile inducing a pure action profile in G
- There does not exist a constrained correlated equilibrium for this game

Theorem 2 - Existence of Constrained Correlated Equilibria

Let G be a finite non-cooperative game and $\mathcal{C}$ a feasible set of probability distributions. If the intersection of $\mathcal{C}$ with the set of correlated equilibrium distributions of the game G is non-empty, then a constrained correlated equilibrium of G exists.

Existence of Constrained Correlated Equilibria

Consider the following two-player game:

	L	R
U	$(2, 2)$	$(1, 1)$
D	$(3, 0)$	$(0, 5)$

Figure 13: Two-player game in matrix form.

Let $\mathcal{C} \subset \Delta(\mathcal{A})$ be a feasible set of probability distributions such that,

$$\mathcal{C} = \{\boldsymbol{p} \in \Delta(\mathcal{A}) \mid \boldsymbol{p}(U, L) = 1 \text{ or } \boldsymbol{p}(U, R) = 1 \text{ or } \boldsymbol{p}(D, L) = 1 \text{ or } \boldsymbol{p}(D, R) = 1\} \tag{10}$$

- The set of feasible strategies is the set of pure action profiles
- The players must play a correlated strategy profile inducing a pure action profile in G
- There does not exist a constrained correlated equilibrium for this game

Theorem 2 - Existence of Constrained Correlated Equilibria

Let G be a finite non-cooperative game and $\mathcal{C}$ a feasible set of probability distributions. If the intersection of $\mathcal{C}$ with the set of correlated equilibrium distributions of the game G is non-empty, then a constrained correlated equilibrium of G exists.

Existence of Constrained Correlated Equilibria

Consider the following two-player game:

	L	R
U	$(2, 2)$	$(1, 1)$
D	$(3, 0)$	$(0, 5)$

Figure 13: Two-player game in matrix form.

Let $\mathcal{C} \subset \Delta(\mathcal{A})$ be a feasible set of probability distributions such that,

$$\mathcal{C} = \{\boldsymbol{p} \in \Delta(\mathcal{A}) \mid \boldsymbol{p}(U, L) = 1 \text{ or } \boldsymbol{p}(U, R) = 1 \text{ or } \boldsymbol{p}(D, L) = 1 \text{ or } \boldsymbol{p}(D, R) = 1\} \tag{10}$$

- The set of feasible strategies is the set of pure action profiles
- The players must play a correlated strategy profile inducing a pure action profile in G
- There does not exist a constrained correlated equilibrium for this game

Theorem 2 - Existence of Constrained Correlated Equilibria

Let G be a finite non-cooperative game and $\mathcal{C}$ a feasible set of probability distributions. If the intersection of $\mathcal{C}$ with the set of correlated equilibrium distributions of the game G is non-empty, then a constrained correlated equilibrium of G exists.

Existence of Constrained Correlated Equilibria

Consider the following two-player game:

	L	R
U	$(2, 2)$	$(1, 1)$
D	$(3, 0)$	$(0, 5)$

Figure 13: Two-player game in matrix form.

Let $\mathcal{C} \subset \Delta(\mathcal{A})$ be a feasible set of probability distributions such that,

$$\mathcal{C} = \{\boldsymbol{p} \in \Delta(\mathcal{A}) \mid \boldsymbol{p}(U, L) = 1 \text{ or } \boldsymbol{p}(U, R) = 1 \text{ or } \boldsymbol{p}(D, L) = 1 \text{ or } \boldsymbol{p}(D, R) = 1\} \tag{10}$$

- The set of feasible strategies is the set of pure action profiles
- The players must play a correlated strategy profile inducing a pure action profile in G
- There does not exist a constrained correlated equilibrium for this game

Theorem 2 - Existence of Constrained Correlated Equilibria

Let G be a finite non-cooperative game and $\mathcal{C}$ a feasible set of probability distributions. If the intersection of $\mathcal{C}$ with the set of correlated equilibrium distributions of the game G is non-empty, then a constrained correlated equilibrium of G exists.

Existence of Constrained Correlated Equilibria

Consider the following two-player game:

	L	R
U	$(2, 2)$	$(1, 1)$
D	$(3, 0)$	$(0, 5)$

Figure 13: Two-player game in matrix form.

Let $\mathcal{C} \subset \Delta(\mathcal{A})$ be a feasible set of probability distributions such that,

$$\mathcal{C} = \{\boldsymbol{p} \in \Delta(\mathcal{A}) \mid \boldsymbol{p}(U, L) = 1 \text{ or } \boldsymbol{p}(U, R) = 1 \text{ or } \boldsymbol{p}(D, L) = 1 \text{ or } \boldsymbol{p}(D, R) = 1\} \tag{10}$$

- The set of feasible strategies is the set of pure action profiles
- The players must play a correlated strategy profile inducing a pure action profile in G
- There does not exist a constrained correlated equilibrium for this game

Theorem 2 - Existence of Constrained Correlated Equilibria

Let G be a finite non-cooperative game and $\mathcal{C}$ a feasible set of probability distributions. If the intersection of $\mathcal{C}$ with the set of correlated equilibrium distributions of the game G is non-empty, then a constrained correlated equilibrium of G exists.

Existence of Constrained Correlated Equilibria

Consider the following two-player game:

	L	R
U	$(2, 2)$	$(1, 1)$
D	$(3, 0)$	$(0, 5)$

Figure 13: Two-player game in matrix form.

Let $\mathcal{C} \subset \Delta(\mathcal{A})$ be a feasible set of probability distributions such that,

$$\mathcal{C} = \{\boldsymbol{p} \in \Delta(\mathcal{A}) \mid \boldsymbol{p}(U, L) = 1 \text{ or } \boldsymbol{p}(U, R) = 1 \text{ or } \boldsymbol{p}(D, L) = 1 \text{ or } \boldsymbol{p}(D, R) = 1\} \tag{10}$$

- The set of feasible strategies is the set of pure action profiles
- The players must play a correlated strategy profile inducing a pure action profile in G
- There does not exist a constrained correlated equilibrium for this game

Theorem 2 - Existence of Constrained Correlated Equilibria

Let G be a finite non-cooperative game and $\mathcal{C}$ a feasible set of probability distributions. If the intersection of $\mathcal{C}$ with the set of correlated equilibrium distributions of the game G is non-empty, then a constrained correlated equilibrium of G exists.

Constrained Correlated Equilibria of the Mixed Extension

Define the mixed extension of G by the game $\Delta G = (\mathcal{N}, (\Delta(\mathcal{A}_i))_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$ where $\Delta(\mathcal{A}_i) = \{\boldsymbol{p} \in \mathbb{R}_+^{|\mathcal{A}_i|} \mid \sum_{a_i\in\mathcal{A}_i} \boldsymbol{p}(a_i) = 1\}$ is the set of probability distributions on $\mathcal{A}_i$.

Let d be a correlation device and ΔG_d be the extension of ΔG by d. The set of strategies of player i in ΔG_d is

$$\tilde{\mathcal{S}}_{i,d} = \{\gamma_i : \Omega \to \Delta(\mathcal{A}_i) \mid \gamma_i \text{ is } \mathcal{P}_i\text{-measurable}\} \tag{11}$$

For any correlated strategy profile $\boldsymbol{\gamma} \in \tilde{\mathcal{S}}_d = \times_{i\in\mathcal{N}} \tilde{\mathcal{S}}_{i,d}$, the utility function of player i is $\tilde{u}_i : \tilde{\mathcal{S}}_d \to \mathbb{R}$ such that,

$$\tilde{u}_i(\gamma_i, \boldsymbol{\gamma}_{-i}) = \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\gamma_i(\omega), \boldsymbol{\alpha}_{-i}(\omega)) \tag{12}$$

Theorem 2 : Let G be a finite non-cooperative game, $\mathcal{C}$ a feasible set of probability distributions, d a correlation device and $\boldsymbol{\gamma}^* \in \tilde{\mathcal{S}}_d$ a correlated strategy profile. If $(d, \mathcal{R}_d, \boldsymbol{\gamma}^*)$ is a constrained correlated equilibrium of ΔG, then it exists a constrained correlated equilibrium $(d', \mathcal{R}_{d'}, \boldsymbol{\alpha}^*)$ of G such that $\boldsymbol{p}_{\boldsymbol{\gamma}^*} = \boldsymbol{p}_{\boldsymbol{\alpha}^*}$.

$\rightarrow$ The set of constrained correlated equilibrium distributions of the game ΔG is included in the set of constrained correlated equilibrium distributions of G.

Constrained Correlated Equilibria of the Mixed Extension

Define the mixed extension of G by the game $\Delta G = (\mathcal{N}, (\Delta(\mathcal{A}_i))_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$ where $\Delta(\mathcal{A}_i) = \{\boldsymbol{p} \in \mathbb{R}_+^{|\mathcal{A}_i|} \mid \sum_{a_i\in\mathcal{A}_i} \boldsymbol{p}(a_i) = 1\}$ is the set of probability distributions on $\mathcal{A}_i$.

Let d be a correlation device and ΔG_d be the extension of ΔG by d. The set of strategies of player i in ΔG_d is

$$\tilde{\mathcal{S}}_{i,d} = \{\gamma_i : \Omega \to \Delta(\mathcal{A}_i) \mid \gamma_i \text{ is } \mathcal{P}_i\text{-measurable}\} \tag{11}$$

For any correlated strategy profile $\gamma \in \tilde{\mathcal{S}}_d = \times_{i\in\mathcal{N}}\tilde{\mathcal{S}}_{i,d}$, the utility function of player i is $\tilde{u}_i : \tilde{\mathcal{S}}_d \to \mathbb{R}$ such that,

$$\tilde{u}_i(\gamma_i, \gamma_{-i}) = \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\gamma_i(\omega), \boldsymbol{\alpha}_{-i}(\omega)) \tag{12}$$

Theorem 2 : Let G be a finite non-cooperative game, $\mathcal{C}$ a feasible set of probability distributions, d a correlation device and $\gamma^* \in \tilde{\mathcal{S}}_d$ a correlated strategy profile. If $(d, \mathcal{R}_d, \gamma^*)$ is a constrained correlated equilibrium of ΔG, then it exists a constrained correlated equilibrium $(d', \mathcal{R}_{d'}, \boldsymbol{\alpha}^*)$ of G such that $\boldsymbol{p}_{\gamma^*} = \boldsymbol{p}_{\boldsymbol{\alpha}^*}$.

$\rightarrow$ The set of constrained correlated equilibrium distributions of the game ΔG is included in the set of constrained correlated equilibrium distributions of G.

Constrained Correlated Equilibria of the Mixed Extension

Define the mixed extension of G by the game $\Delta G = (\mathcal{N}, (\Delta(\mathcal{A}_i))_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$ where $\Delta(\mathcal{A}_i) = \{\boldsymbol{p} \in \mathbb{R}_+^{|\mathcal{A}_i|} \mid \sum_{a_i\in\mathcal{A}_i} \boldsymbol{p}(a_i) = 1\}$ is the set of probability distributions on $\mathcal{A}_i$.

Let d be a correlation device and ΔG_d be the extension of ΔG by d. The set of strategies of player i in ΔG_d is

$$\tilde{\mathcal{S}}_{i,d} = \{\gamma_i : \Omega \to \Delta(\mathcal{A}_i) \mid \gamma_i \text{ is } \mathcal{P}_i\text{-measurable}\} \tag{11}$$

For any correlated strategy profile $\gamma \in \tilde{\mathcal{S}}_d = \times_{i\in\mathcal{N}}\tilde{\mathcal{S}}_{i,d}$, the utility function of player i is $\tilde{u}_i : \tilde{\mathcal{S}}_d \to \mathbb{R}$ such that,

$$\tilde{u}_i(\gamma_i, \gamma_{-i}) = \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\gamma_i(\omega), \boldsymbol{\alpha}_{-i}(\omega)) \tag{12}$$

Theorem 2 : Let G be a finite non-cooperative game, $\mathcal{C}$ a feasible set of probability distributions, d a correlation device and $\gamma^* \in \tilde{\mathcal{S}}_d$ a correlated strategy profile. If $(d, \mathcal{R}_d, \gamma^*)$ is a constrained correlated equilibrium of ΔG, then it exists a constrained correlated equilibrium $(d', \mathcal{R}_{d'}, \boldsymbol{\alpha}^*)$ of G such that $\boldsymbol{p}_{\gamma^*} = \boldsymbol{p}_{\boldsymbol{\alpha}^*}$.

$\rightarrow$ The set of constrained correlated equilibrium distributions of the game ΔG is included in the set of constrained correlated equilibrium distributions of G.

Constrained Correlated Equilibria of the Mixed Extension

Define the mixed extension of G by the game $\Delta G = (\mathcal{N}, (\Delta(\mathcal{A}_i))_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$ where $\Delta(\mathcal{A}_i) = \{\boldsymbol{p} \in \mathbb{R}_+^{|\mathcal{A}_i|} \mid \sum_{a_i\in\mathcal{A}_i} \boldsymbol{p}(a_i) = 1\}$ is the set of probability distributions on $\mathcal{A}_i$.

Let d be a correlation device and ΔG_d be the extension of ΔG by d. The set of strategies of player i in ΔG_d is

$$\tilde{\mathcal{S}}_{i,d} = \{\gamma_i : \Omega \to \Delta(\mathcal{A}_i) \mid \gamma_i \text{ is } \mathcal{P}_i\text{-measurable}\} \tag{11}$$

For any correlated strategy profile $\boldsymbol{\gamma} \in \tilde{\mathcal{S}}_d = \times_{i\in\mathcal{N}}\tilde{\mathcal{S}}_{i,d}$, the utility function of player i is $\tilde{u}_i : \tilde{\mathcal{S}}_d \to \mathbb{R}$ such that,

$$\tilde{u}_i(\gamma_i, \boldsymbol{\gamma}_{-i}) = \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\gamma_i(\omega), \boldsymbol{\alpha}_{-i}(\omega)) \tag{12}$$

Theorem 2 : Let G be a finite non-cooperative game, $\mathcal{C}$ a feasible set of probability distributions, d a correlation device and $\gamma^* \in \tilde{\mathcal{S}}_d$ a correlated strategy profile. If $(d, \mathcal{R}_d, \gamma^*)$ is a constrained correlated equilibrium of ΔG, then it exists a constrained correlated equilibrium $(d', \mathcal{R}_{d'}, \boldsymbol{\alpha}^*)$ of G such that $\boldsymbol{p}_{\gamma^*} = \boldsymbol{p}_{\boldsymbol{\alpha}^*}$.

$\rightarrow$ The set of constrained correlated equilibrium distributions of the game ΔG is included in the set of constrained correlated equilibrium distributions of G.

Constrained Correlated Equilibria of the Mixed Extension

Define the mixed extension of G by the game $\Delta G = (\mathcal{N}, (\Delta(\mathcal{A}_i))_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$ where $\Delta(\mathcal{A}_i) = \{\boldsymbol{p} \in \mathbb{R}_+^{|\mathcal{A}_i|} \mid \sum_{a_i\in\mathcal{A}_i} \boldsymbol{p}(a_i) = 1\}$ is the set of probability distributions on $\mathcal{A}_i$.

Let d be a correlation device and ΔG_d be the extension of ΔG by d. The set of strategies of player i in ΔG_d is

$$\tilde{\mathcal{S}}_{i,d} = \{\gamma_i : \Omega \to \Delta(\mathcal{A}_i) \mid \gamma_i \text{ is } \mathcal{P}_i\text{-measurable}\} \tag{11}$$

For any correlated strategy profile $\boldsymbol{\gamma} \in \tilde{\mathcal{S}}_d = \times_{i\in\mathcal{N}}\tilde{\mathcal{S}}_{i,d}$, the utility function of player i is $\tilde{u}_i : \tilde{\mathcal{S}}_d \to \mathbb{R}$ such that,

$$\tilde{u}_i(\gamma_i, \boldsymbol{\gamma}_{-i}) = \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\gamma_i(\omega), \boldsymbol{\alpha}_{-i}(\omega)) \tag{12}$$

Theorem 2 : Let G be a finite non-cooperative game, $\mathcal{C}$ a feasible set of probability distributions, d a correlation device and $\boldsymbol{\gamma}^* \in \tilde{\mathcal{S}}_d$ a correlated strategy profile. If $(d, \tilde{\mathcal{R}}_d, \boldsymbol{\gamma}^*)$ is a constrained correlated equilibrium of ΔG, then it exists a constrained correlated equilibrium $(d', \mathcal{R}_{d'}, \boldsymbol{\alpha}^*)$ of G such that $\boldsymbol{p}_{\boldsymbol{\gamma}^*} = \boldsymbol{p}_{\boldsymbol{\alpha}^*}$.

$\rightarrow$ The set of constrained correlated equilibrium distributions of the game ΔG is included in the set of constrained correlated equilibrium distributions of G.

Constrained Correlated Equilibria of the Mixed Extension

Define the mixed extension of G by the game $\Delta G = (\mathcal{N}, (\Delta(\mathcal{A}_i))_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$ where $\Delta(\mathcal{A}_i) = \{\boldsymbol{p} \in \mathbb{R}_+^{|\mathcal{A}_i|} \mid \sum_{a_i\in\mathcal{A}_i} \boldsymbol{p}(a_i) = 1\}$ is the set of probability distributions on $\mathcal{A}_i$.

Let d be a correlation device and ΔG_d be the extension of ΔG by d. The set of strategies of player i in ΔG_d is

$$\tilde{\mathcal{S}}_{i,d} = \{\gamma_i : \Omega \to \Delta(\mathcal{A}_i) \mid \gamma_i \text{ is } \mathcal{P}_i\text{-measurable}\} \tag{11}$$

For any correlated strategy profile $\boldsymbol{\gamma} \in \tilde{\mathcal{S}}_d = \times_{i\in\mathcal{N}}\tilde{\mathcal{S}}_{i,d}$, the utility function of player i is $\tilde{u}_i : \tilde{\mathcal{S}}_d \to \mathbb{R}$ such that,

$$\tilde{u}_i(\gamma_i, \boldsymbol{\gamma}_{-i}) = \sum_{\omega\in\Omega} \boldsymbol{q}(\omega) u_i(\gamma_i(\omega), \boldsymbol{\alpha}_{-i}(\omega)) \tag{12}$$

Theorem 2 : Let G be a finite non-cooperative game, $\mathcal{C}$ a feasible set of probability distributions, d a correlation device and $\boldsymbol{\gamma}^* \in \tilde{\mathcal{S}}_d$ a correlated strategy profile. If $(d, \tilde{\mathcal{R}}_d, \boldsymbol{\gamma}^*)$ is a constrained correlated equilibrium of ΔG, then it exists a constrained correlated equilibrium $(d', \mathcal{R}_{d'}, \boldsymbol{\alpha}^*)$ of G such that $\boldsymbol{p}_{\boldsymbol{\gamma}^*} = \boldsymbol{p}_{\boldsymbol{\alpha}^*}$.

$\rightarrow$ The set of constrained correlated equilibrium distributions of the game ΔG is included in the set of constrained correlated equilibrium distributions of G.

Computation of Constrained Correlated Equilibria

maximize 0

$$\text{s.t. } \boldsymbol{p}(\boldsymbol{a}) \geq 0 \quad \forall \boldsymbol{a} \in \mathcal{A}, \quad \Sigma_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) = 1 \tag{13}$$

$$\forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i$$

$$\sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) \left[u_i(\boldsymbol{a}) - u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \right] \geq 0. \tag{14}$$

maximize 0

$$\text{s.t. } \boldsymbol{p}(\boldsymbol{a}) \geq 0 \quad \forall \boldsymbol{a} \in \mathcal{A}, \quad \Sigma_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) = 1 \tag{15}$$

$$\forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i \text{ s.t. } z_{\beta_i, \boldsymbol{p}} \in \mathcal{C}$$

$$\sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) \left[u_i(\boldsymbol{a}) - u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \right] \geq 0, \tag{16}$$

$$\boldsymbol{p} \in \mathcal{C} \tag{17}$$

Assuming linear constraints e.g., $\mathcal{C} = \{\boldsymbol{p} \in \Delta(\mathcal{A}) \mid F\boldsymbol{p} \leq 0\}$, we show that the problem of computing a constrained correlated equilibrium distribution **can be formulated as a Mixed-Integer Linear Program**.

Computation of Constrained Correlated Equilibria

maximize 0

s.t. $\boldsymbol{p}(\boldsymbol{a}) \geq 0 \quad \forall \boldsymbol{a} \in \mathcal{A}, \quad \Sigma_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) = 1$ (13)

$\forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i$

$$\sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) \left[u_i(\boldsymbol{a}) - u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \right] \geq 0. \quad (14)$$

maximize 0

s.t. $\boldsymbol{p}(\boldsymbol{a}) \geq 0 \quad \forall \boldsymbol{a} \in \mathcal{A}, \quad \Sigma_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) = 1$ (15)

$\forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i$ s.t. $\boldsymbol{z}_{\beta_i, \boldsymbol{p}} \in \mathcal{C}$

$$\sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) \left[u_i(\boldsymbol{a}) - u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \right] \geq 0, \quad (16)$$

$\boldsymbol{p} \in \mathcal{C}$ (17)

Assuming linear constraints e.g., $\mathcal{C} = \{p \in \Delta(\mathcal{A}) \mid Fp \leq 0\}$, we show that the problem of computing a constrained correlated equilibrium distribution **can be formulated as a Mixed-Integer Linear Program**.

Computation of Constrained Correlated Equilibria

maximize 0

s.t. $\boldsymbol{p}(\boldsymbol{a}) \geq 0 \quad \forall \boldsymbol{a} \in \mathcal{A}, \quad \Sigma_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) = 1$ (13)

$\forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i$

$$\sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) \left[u_i(\boldsymbol{a}) - u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \right] \geq 0. \quad (14)$$

maximize 0

s.t. $\boldsymbol{p}(\boldsymbol{a}) \geq 0 \quad \forall \boldsymbol{a} \in \mathcal{A}, \quad \Sigma_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) = 1$ (15)

$\forall i \in \mathcal{N}, \forall \beta_i : \mathcal{A}_i \to \mathcal{A}_i$ s.t. $\boldsymbol{z}_{\beta_i, \boldsymbol{p}} \in \mathcal{C}$

$$\sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) \left[u_i(\boldsymbol{a}) - u_i(\beta_i(a_i), \boldsymbol{a}_{-i}) \right] \geq 0, \quad (16)$$

$\boldsymbol{p} \in \mathcal{C}$ (17)

Assuming linear constraints e.g., $\mathcal{C} = \{\boldsymbol{p} \in \Delta(\mathcal{A}) \mid F\boldsymbol{p} \leq 0\}$, we show that the problem of computing a constrained correlated equilibrium distribution **can be formulated as a Mixed-Integer Linear Program**.

Simulation Example

	P	A
P	8, 8	3, 10
A	10, 3	0, 0

Table 3: Game of Chicken

$$SW(\boldsymbol{q}) = \sum_{i \in \mathcal{N}} u_i(\boldsymbol{q}) \qquad (18)$$

where $u_i(\boldsymbol{q}) = \sum_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{q}(\boldsymbol{a}) u_i(\boldsymbol{a})$

- Feasible utilities in green
- CE utilities in yellow
- **CCE** utilities in red

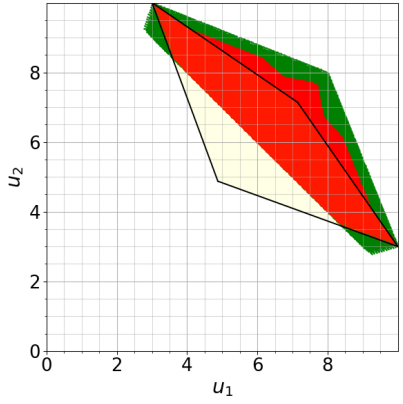


Figure 14: Sets of Utilities of the two players with $SW \geq 12$

Simulation Results

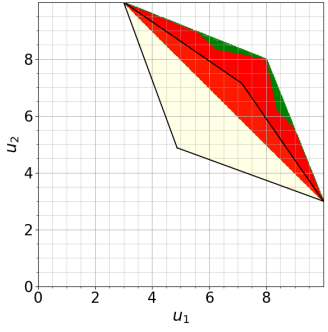


Figure 15: Sets of Utilities of the two players with $SW \geq 13$

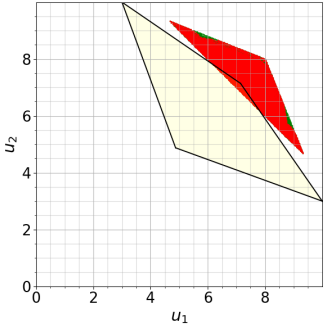


Figure 16: Sets of Utilities of the two players with $SW \geq 14$

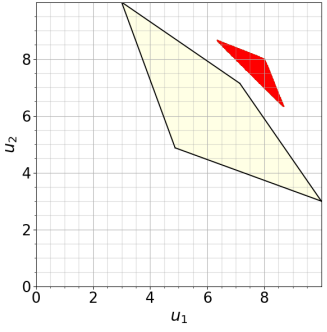


Figure 17: Sets of Utilities of the two players with $SW \geq 15$

- The set may not be convex
- There are constrained correlated equilibria outside the set of correlated equilibrium distributions

Simulation Results

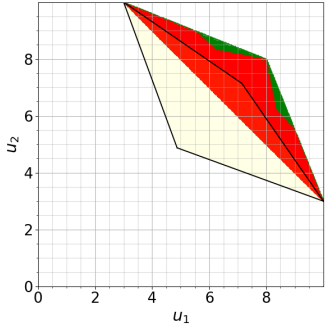


Figure 15: Sets of Utilities of the two players with $SW \geq 13$

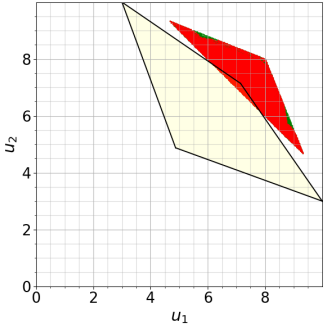


Figure 16: Sets of Utilities of the two players with $SW \geq 14$

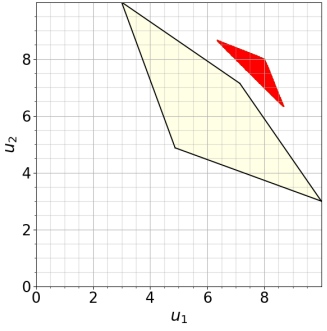


Figure 17: Sets of Utilities of the two players with $SW \geq 15$

- The set may not be convex
- There are constrained correlated equilibria outside the set of correlated equilibrium distributions

Simulation Results

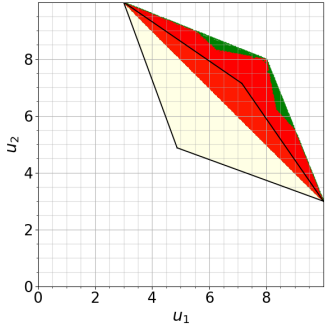


Figure 15: Sets of Utilities of the two players with $SW \geq 13$

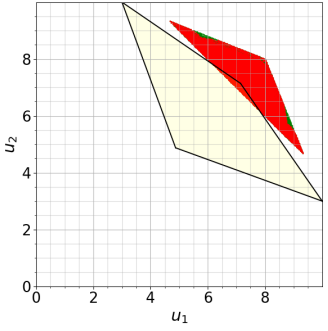


Figure 16: Sets of Utilities of the two players with $SW \geq 14$

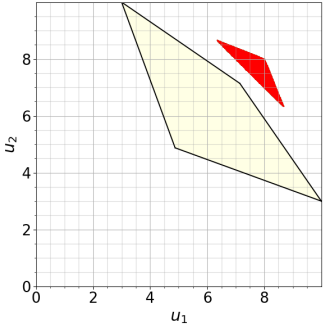


Figure 17: Sets of Utilities of the two players with $SW \geq 15$

- The set may not be convex
- There are constrained correlated equilibria outside the set of correlated equilibrium distributions

Conclusion & Highlight of Contributions

- **A learning rule to converge towards a correlated equilibrium distribution**
  - Empirical evidence of convergence of the distribution to a constrained correlated equilibrium
  - A Markov chain models the learning algorithm
  - The algorithm is adaptive to changes in games e.g., payoff functions or number of players
- **Arbitrary constraints and arbitrary correlation device**
  - **Characterizations of Equilibrium Strategies**
    - Definition through generalized Nash equilibrium and correlated equilibrium with alternative characterizations
  - **Properties and relationship to (unconstrained) Correlated Equilibria**
    - A feasible correlated equilibrium is a constrained correlated equilibrium
    - The set of constrained correlated equilibrium distributions may not be convex
    - There exist constrained correlated equilibrium distributions outside the set of correlated equilibrium distributions
- **Constraints on probability distributions**
  - **Characterization of constrained correlated equilibrium distributions**
    - Closed-form expression of the set of constrained correlated equilibrium distribution
  - **Conditions of Existence**
    - There does not always exist a constrained correlated equilibrium in finite games
  - **Constrained correlated equilibrium distributions of the mixed extension of a game**
    - Every constrained correlated equilibrium distribution of ΔG is a constrained correlated equilibrium distribution of G
  - **Computation of linearly constrained correlated equilibria**

Conclusion & Highlight of Contributions

- **A learning rule to converge towards a correlated equilibrium distribution**
  - Empirical evidence of convergence of the distribution to a constrained correlated equilibrium
  - A Markov chain models the learning algorithm
  - The algorithm is adaptive to changes in games e.g., payoff functions or number of players
- **Arbitrary constraints and arbitrary correlation device**
  - **Characterizations of Equilibrium Strategies**
    - Definition through generalized Nash equilibrium and correlated equilibrium with alternative characterizations
  - **Properties and relationship to (unconstrained) Correlated Equilibria**
    - A feasible correlated equilibrium is a constrained correlated equilibrium
    - The set of constrained correlated equilibrium distributions may not be convex
    - There exist constrained correlated equilibrium distributions outside the set of correlated equilibrium distributions
- **Constraints on probability distributions**
  - **Characterization of constrained correlated equilibrium distributions**
    - Closed-form expression of the set of constrained correlated equilibrium distribution
  - **Conditions of Existence**
    - There does not always exist a constrained correlated equilibrium in finite games
  - **Constrained correlated equilibrium distributions of the mixed extension of a game**
    - Every constrained correlated equilibrium distribution of ΔG is a constrained correlated equilibrium distribution of G
  - **Computation of linearly constrained correlated equilibria**

Conclusion & Highlight of Contributions

- **A learning rule to converge towards a correlated equilibrium distribution**
  - Empirical evidence of convergence of the distribution to a constrained correlated equilibrium
  - A Markov chain models the learning algorithm
  - The algorithm is adaptive to changes in games e.g., payoff functions or number of players
- **Arbitrary constraints and arbitrary correlation device**
  - **Characterizations of Equilibrium Strategies**
    - Definition through generalized Nash equilibrium and correlated equilibrium with alternative characterizations
  - **Properties and relationship to (unconstrained) Correlated Equilibria**
    - A feasible correlated equilibrium is a constrained correlated equilibrium
    - The set of constrained correlated equilibrium distributions may not be convex
    - There exist constrained correlated equilibrium distributions outside the set of correlated equilibrium distributions
- **Constraints on probability distributions**
  - **Characterization of constrained correlated equilibrium distributions**
    - Closed-form expression of the set of constrained correlated equilibrium distribution
  - **Conditions of Existence**
    - There does not always exist a constrained correlated equilibrium in finite games
  - **Constrained correlated equilibrium distributions of the mixed extension of a game**
    - Every constrained correlated equilibrium distribution of ΔG is a constrained correlated equilibrium distribution of G
  - **Computation of linearly constrained correlated equilibria**

Open Perspectives

- **Research Questions**
  - Are there any theoretical guarantees of convergence to the a correlated equilibrium distribution?
  - Connection to Bayesian rationality still holds in the presence of constraints ?
  - What happens in the case of infinite games ?
  - Existence problem should be studied for weaker or alternative assumptions
  - Study of the problem of computing constrained correlated equilibria (complexity, feasbility)
- **Some Unexplored Topics**
  - Learning rule leading to a specific correlated equilibrium (e.g., social welfare maximizing)
  - Learning with constraints
  - Subjective correlated equilibrium
- **Potential applications of constrained correlated equilibria and learning correlated equilibria in various fields**
  - Mobile Edge Computing Systems
  - Wireless Networks with constraints
  - Engineering and beyond

Open Perspectives

- **Research Questions**
  - Are there any theoretical guarantees of convergence to the a correlated equilibrium distribution?
  - Connection to Bayesian rationality still holds in the presence of constraints ?
  - What happens in the case of infinite games ?
  - Existence problem should be studied for weaker or alternative assumptions
  - Study of the problem of computing constrained correlated equilibria (complexity, feasbility)
- **Some Unexplored Topics**
  - Learning rule leading to a specific correlated equilibrium (e.g., social welfare maximizing)
  - Learning with constraints
  - Subjective correlated equilibrium
- **Potential applications of constrained correlated equilibria and learning correlated equilibria in various fields**
  - Mobile Edge Computing Systems
  - Wireless Networks with constraints
  - Engineering and beyond

Open Perspectives

- **Research Questions**
  - Are there any theoretical guarantees of convergence to the a correlated equilibrium distribution?
  - Connection to Bayesian rationality still holds in the presence of constraints ?
  - What happens in the case of infinite games ?
  - Existence problem should be studied for weaker or alternative assumptions
  - Study of the problem of computing constrained correlated equilibria (complexity, feasbility)
- **Some Unexplored Topics**
  - Learning rule leading to a specific correlated equilibrium (e.g., social welfare maximizing)
  - Learning with constraints
  - Subjective correlated equilibrium
- **Potential applications of constrained correlated equilibria and learning correlated equilibria in various fields**
  - Mobile Edge Computing Systems
  - Wireless Networks with constraints
  - Engineering and beyond

Open Perspectives

- **Research Questions**
  - Are there any theoretical guarantees of convergence to the a correlated equilibrium distribution?
  - Connection to Bayesian rationality still holds in the presence of constraints ?
  - What happens in the case of infinite games ?
  - Existence problem should be studied for weaker or alternative assumptions
  - Study of the problem of computing constrained correlated equilibria (complexity, feasbility)
- **Some Unexplored Topics**
  - Learning rule leading to a specific correlated equilibrium (e.g., social welfare maximizing)
  - Learning with constraints
  - Subjective correlated equilibrium
- **Potential applications of constrained correlated equilibria and learning correlated equilibria in various fields**
  - Mobile Edge Computing Systems
  - Wireless Networks with constraints
  - Engineering and beyond

Thank you!

Questions?

For more information:

omar.t.boufous@gmail.com

Bibliography

R. J. Aumann. *Subjectivity and correlation in randomized strategies*. Journal of Mathematical Economics, 1974.

R. J. Aumann. *Correlated equilibrium as an expression of bayesian rationality*. Econometrica, 1987.

S. Hart & D. Schmeidler. *Existence of correlated equilibria*. Mathematics of Operations Research, 1989.

B. von Stengel & F. Forges. *Extensive-form correlated equilibrium: Definition and computational complexity*. Mathematics of Operations Research, 2008.

F. Forges. *Correlated equilibria and communication in games*. Complex Social and Behavioral Systems: Game Theory and Agent-Based Models, 2020.

A. Brandenburger, E. Dekel, and John Geanakoplos. *Correlated equilibrium with generalized information structures*. Games and Economic Behavior, 1992.

S. Grant and R. Stauber. *Delegation and ambiguity in correlated equilibrium*. Games and Economic Behavior, 2022.

G. Debreu. *A social equilibrium existence theorem*. Proceedings of the National Academy of Sciences, 1952.

K. J. Arrow and G Debreu. *Existence of an equilibrium for a competitive economy*. Econometrica: Journal of the Econometric Society, 1954.

J. B. Rosen. *Existence and Uniqueness of Equilibrium Points for Concave N-Person Games*. Econometrica, 1965.

Young, H. P. (2009). *Learning by trial and error*. Games and economic behavior.

Bibliography

Ankur A Kulkarni. *Games and teams with shared constraints*. Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, 375(2100):20160302, 2017.

Jong-Shi Pang, Gesualdo Scutari, Francisco Facchinei, and Chaoxiong Wang. *Distributed power allocation with rate constraints in gaussian parallel interference channels*. IEEE Transactions on Information Theory, 54(8):3471–3489, 2008.

Daniel Nowak, Tobias Mahn, Hussein Al-Shatri, Alexandra Schwartz, and Anja Klein. *A generalized nash game for mobile edge computation offloading*. In 2018 6th IEEE International Conference on Mobile Cloud Computing, Services, and Engineering (MobileCloud), pages 95–102. IEEE, 2018.

Computation of Constrained Correlated Equilibria

Mixed-Integer Linear Program

$$\boldsymbol{p} \geq 0, \Sigma_{\boldsymbol{a} \in \mathcal{A}} \boldsymbol{p}(\boldsymbol{a}) = 1, F\boldsymbol{p} \leq 0 \tag{19}$$

$$\left[U_i - U_i B_{\beta_i}\right] \boldsymbol{p} \leq M_i \times \boldsymbol{y}_{\beta_i} \quad \forall i \in \mathcal{N}, \quad \forall \beta_i \tag{20}$$

$$F B_{\beta_i} \boldsymbol{p} \geq -K \left(1 - \boldsymbol{y}_{\beta_i}\right) + \delta \quad \forall i \in \mathcal{N}, \quad \forall \beta_i \tag{21}$$

$$F B_{\beta_i} \boldsymbol{p} \leq K y_{\beta_i} \quad \forall i \in \mathcal{N}, \quad \forall \beta_i \tag{22}$$

$$y_{\beta_i} \in \{0, 1\} \quad \forall i \in \mathcal{N}, \quad \forall \beta_i \tag{23}$$