MARKSCHEME

November 2001

MATHEMATICAL METHODS

Standard Level

Paper 1

1. (a) $u_1 = 7, d = 2.5$ *(M1)*

$u_{41} = u_1 + (n-1)d = 7 + (41-1)2.5$
$= 107$ *(A1)* *(C2)*

(b) $S_{101} = \frac{n}{2}[2u_1 + (n-1)d]$

$= \frac{101}{2}[2(7) + (101-1)2.5]$ *(M1)*

$= \frac{101(264)}{2}$

$= 13332$ *(A1)* *(C2)*

[4 marks]

2. Direction vector $= \binom{6}{5} - \binom{1}{3}$ *(M1)*

$= \binom{5}{2}$ *(A1)*

$\binom{x}{y} = \binom{1}{3} + t\binom{5}{2}$ *(A2)*

OR

$\binom{x}{y} = \binom{6}{5} + t\binom{5}{2}$ *(A2)* *(C4)*

[4 marks]

3.
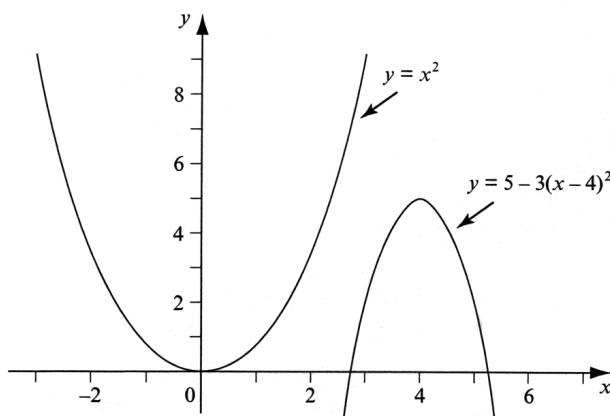
$q = 5$ *(A1)* *(C1)*
$k = 3, p = 4$ *(A3)* *(C3)*

[4 marks]

4. (a) Area $= \frac{1}{2}r^2\theta = \frac{1}{2}(15^2)(2)$ **(M1)**

$= 225 \,(\text{cm}^2)$ **(A1)** **(C2)**

(b) Area $\triangle OAB = \frac{1}{2}15^2 \sin 2 = 102.3$ **(A1)**

Area $= 225 - 102.3 = 122.7 \,(\text{cm}^2)$

$= 123 \;(3 \text{ s.f.})$ **(A1)** **(C2)**

[4 marks]

5. **METHOD 1**

$\log_9 81 + \log_9\left(\frac{1}{9}\right) + \log_9 3 = 2 - 1 + \frac{1}{2}$ **(M1)**

$\Rightarrow \frac{3}{2} = \log_9 x$ **(A1)**

$\Rightarrow x = 9^{\frac{3}{2}}$ **(M1)**

$\Rightarrow x = 27$ **(A1)** **(C4)**

METHOD 2

$\log_9 81 + \log_9\left(\frac{1}{9}\right) + \log_9 3 = \log_9\left[81\left(\frac{1}{9}\right)3\right]$ **(M2)**

$= \log_9 27$ **(A1)**

$\Rightarrow x = 27$ **(A1)** **(C4)**

[4 marks]

6. (a)
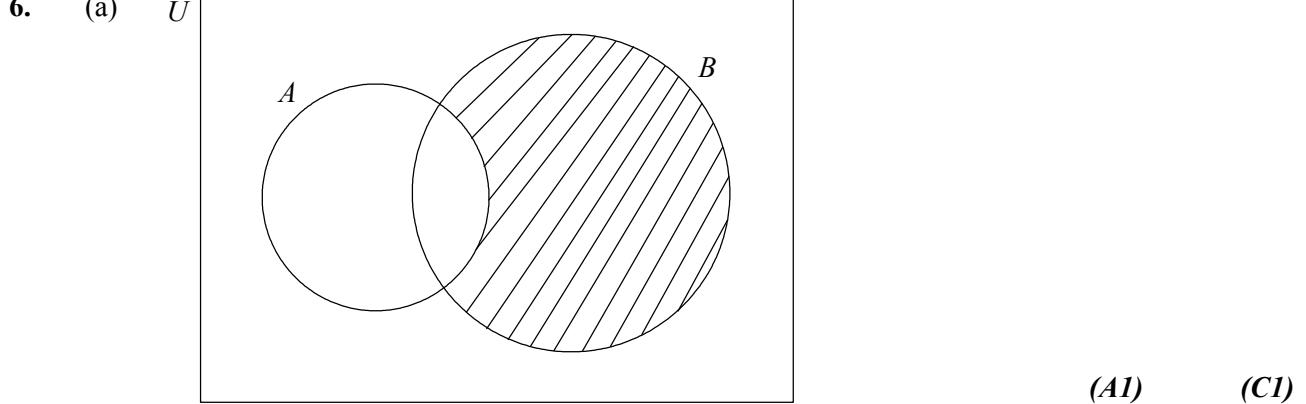
(A1) **(C1)**

(b) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$65 = 30 + 50 - n(A \cap B)$

$\Rightarrow n(A \cap B) = 15$ (may be on the diagram) **(M1)**

$n(B \cap A') = 50 - 15 = 35$ **(A1)** **(C2)**

(c) $P(B \cap A') = \dfrac{n(B \cap A')}{n(U)} = \dfrac{35}{100} = 0.35$ **(A1)** **(C1)**

[4 marks]

7. (a) $f'(x) = k\cos x + 3$ *(A1)(A1)* *(C2)*

(b) $k\cos\left(\dfrac{\pi}{3}\right) + 3 = 8$ *(M1)*

$\Rightarrow k\left(\dfrac{1}{2}\right) + 3 = 8$

$\Rightarrow k = 10$ *(A1)* *(C2)*

[4 marks]

8.
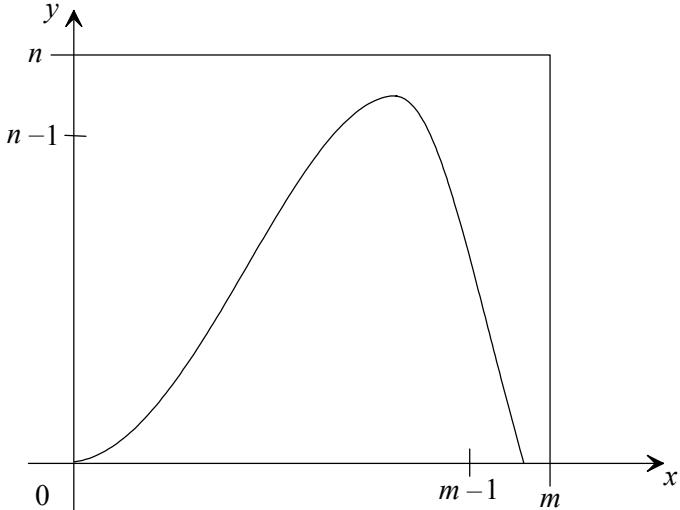
(a) $y = 0 \Rightarrow x = 0$ or $\sin\dfrac{x}{3} = 0$ *(M1)*

$\Rightarrow \dfrac{x}{3} = 0, \pi$

$\Rightarrow x = 0, 3\pi$

$m = 10$ *(A1)*

OR

From a graphic display calculator
$y = 0 \Rightarrow x = 9.43$ (or x between 9 and 10) *(M1)*

$\Rightarrow m = 10$ *(A1)* *(C2)*

(b) $y\max = 5.46$ (or between 5 and 6) *(M1)*

$\Rightarrow n = 6$ *(A1)* *(C2)*

[4 marks]

9. $f(x) = 2e^{3x}$. Let $x = 2e^{3y}$ *(M1)*

$\Rightarrow \dfrac{x}{2} = e^{3y}$ *(A1)*

$\Rightarrow \ln\left(\dfrac{x}{2}\right) = 3y$ *(A1)*

$\Rightarrow y = \dfrac{1}{3}\ln\left(\dfrac{x}{2}\right)$ *(A1)*

that is $f^{-1}(x) = \dfrac{1}{3}\ln\left(\dfrac{x}{2}\right)$ *(C4)*

[4 marks]

10. (a) $(1+1)^4 = 2^4 = 1 + \dbinom{4}{1}(1) + \dbinom{4}{2}(1^2) + \dbinom{4}{3}(1^3) + 1^4$ *(M1)*

$\Rightarrow \dbinom{4}{1} + \dbinom{4}{2} + \dbinom{4}{3} = 16 - 2$

$= 14$ *(A1)* *(C2)*

(b) $(1+1)^9 = 1 + \dbinom{9}{1} + \dbinom{9}{2} + \dbinom{9}{3} + \ldots + \dbinom{9}{8} + 1$ *(M1)*

$\Rightarrow \dbinom{9}{1} + \dbinom{9}{2} + \dbinom{9}{3} + \ldots + \dbinom{9}{8} = 2^9 - 2$

$= 510$ *(A1)* *(C2)*

[4 marks]

11. (a) $\begin{pmatrix} 2x \\ x-3 \end{pmatrix} \bullet \begin{pmatrix} x+1 \\ 5 \end{pmatrix} = 0$ *(M1)(M1)*

$\Rightarrow 2x(x+1) + (x-3)(5) = 0$ *(A1)*

$\Rightarrow 2x^2 + 7x - 15 = 0$ *(C3)*

(b) **METHOD 1**

$2x^2 + 7x - 15 = (2x-3)(x+5) = 0$

$\Rightarrow x = \dfrac{3}{2}$ or $x = -5$ *(A1)* *(C1)*

METHOD 2

$x = \dfrac{-7 \pm \sqrt{7^2 - 4(2)(-15)}}{2(2)}$

$\Rightarrow x = \dfrac{3}{2}$ or $x = -5$ *(A1)* *(C1)*

[4 marks]

12. (a)

$$\frac{\sin(\hat{ACB})}{20} = \frac{\sin 50°}{17}$$ *(M1)*

$$\Rightarrow \sin(\hat{ACB}) = \frac{20\sin 50°}{17} = 0.901$$

$$\hat{ACB} > 90° \Rightarrow \hat{ACB} = 180° - 64.3° = 115.7°$$

$$\hat{ACB} = 116° \text{ (3 s.f.)}$$ *(A1)*   *(C2)*

(b)   In Triangle 1, $\hat{ACB} = 64.3°$

$$\Rightarrow \hat{BAC} = 180° - (64.3° + 50°)$$

$$= 65.7°$$ *(A1)*

$$\text{Area} = \frac{1}{2}(20)(17)\sin 65.7° = 155 \text{ (cm}^2) \text{ (3 s.f.)}$$ *(A1)*   *(C2)*

[4 marks]

13. (a)
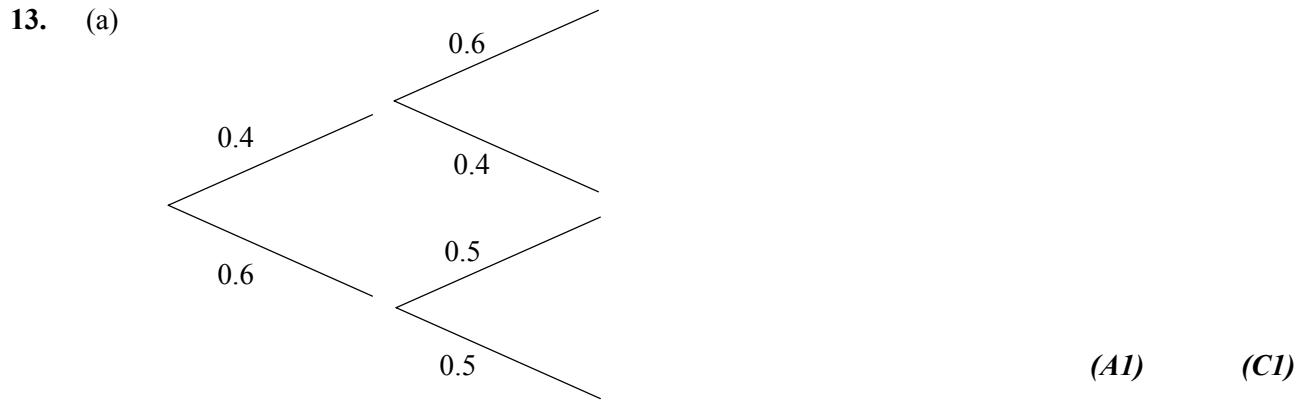
(A1)   *(C1)*

(b) $P(B) = 0.4(0.6) + 0.6(0.5) = 0.24 + 0.30$ *(M1)*

$$= 0.54$$ *(A1)*   *(C2)*

(c) $P(C|B) = \dfrac{P(B \cap C)}{P(B)} = \dfrac{0.24}{0.54} = \dfrac{4}{9} (= 0.444, \text{ 3 s.f.})$ *(A1)*   *(C1)*

[4 marks]

14. **METHOD 1**

The value of cosine varies between -1 and $+1$. Therefore:
$t = 0 \Rightarrow a + b = 14.3$
$t = 6 \Rightarrow a - b = 10.3$

$\quad\quad \Rightarrow 2a = 24.6 \quad \Rightarrow \quad a = 12.3$ *(A1)*   *(C1)*

$\quad\quad \Rightarrow 2b = 4.0 \quad \Rightarrow \quad b = 2$ *(A1)*   *(C1)*

Period = 12 hours $\quad \Rightarrow \quad \dfrac{2\pi(12)}{k} = 2\pi$ *(M1)*

$\quad\quad\quad\quad \Rightarrow \quad k = 12$ *(A1)*   *(C2)*

METHOD 2
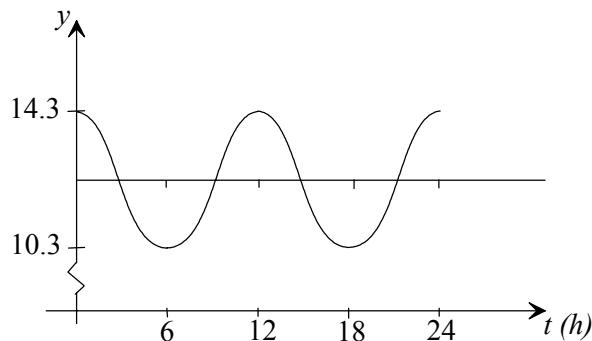
From consideration of graph: Midpoint $= a = 12.3$ *(A1)*   *(C1)*

$\quad\quad\quad\quad\quad\quad\quad\quad$ Amplitude $= b = 2$ *(A1)*   *(C1)*

$\quad\quad\quad\quad\quad\quad\quad$ Period $= \dfrac{2\pi}{\dfrac{2\pi}{k}} = 12$ *(M1)*

$\quad\quad\quad\quad\quad\quad\quad\quad \Rightarrow k = 12$ *(A1)*   *(C2)*

[4 marks]

15. (a) Domain of $\arcsin x = [-1, 1] \Rightarrow a = 1$ *(A1)*   *(C1)*

$\quad$ (b) $-1 \le 3x \le 1$ *(M1)*

$\quad\quad$ Domain $\left\{ x : -\dfrac{1}{3} \le x \le \dfrac{1}{3} \right\}$ *(A1)*   *(C2)*

$\quad$ (c) Range of arcsin function $= \left[\dfrac{-\pi}{2}, \dfrac{\pi}{2} \right]$

$\quad\quad\quad\quad\quad \Rightarrow b = \dfrac{\pi}{2} \, (\text{or } 90°)$ *(A1)*   *(C1)*