Exercise 1

1a) Generate a 2 x 100 matrix with the first row containing even numbers and the second odd numbers.

1b) Find the mean of the odd numbers, even numbers and the full array.

1c) Find the sum of the odd numbers, even numbers and the full array.

1d) Display the transpose of the matrix/array.

Exercise 2

2a) Generate a N x N square matrix X using rand.

2b) Find the determinant.

2c) Find the inverse of the matrix.

2d) Multiple X by the inverse using matrix multiplication

2e) Multiple X by the inverse using element multiplication. Note the difference.

2f) Assign A=[1 2 3; 1 2 3; 2 3 4];

2g) Repeat steps (b) and (c). Note the problems.

Exercise 3

3a) Generate and plot the function $y(x) = x^2 - x - 2$ over the interval [-2 2]. Use different increments in x and see the difference.

3b) Using a small increment in x plot the function along with the derivative and integral. Plot the function and the derivative on the same subplot figure. Plot the function and the integral on a different subplot.

Exercise 4

4a) Generate random arrays of length 100, 1000, 10000, 100000.

4b) Find the mean and standard deviation of the arrays.

4c) Plot the histograms of the arrays on the same figure.

4d) Change the number of bins and replot the histogram

Exercise 5

5a) Write a function to find the roots of the quadratic equation $y(x) = ax^2 + bx + c$ using $x_{root} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Inputs should be a, b, c and outputs should be the two roots.

5b) Use the function to find the roots of $y(x) = x^2 - x - 2$ and $y(x) = x^2 - x + 2$

Exercise 6

6a) Use the expression below to calculate π for different values of N.

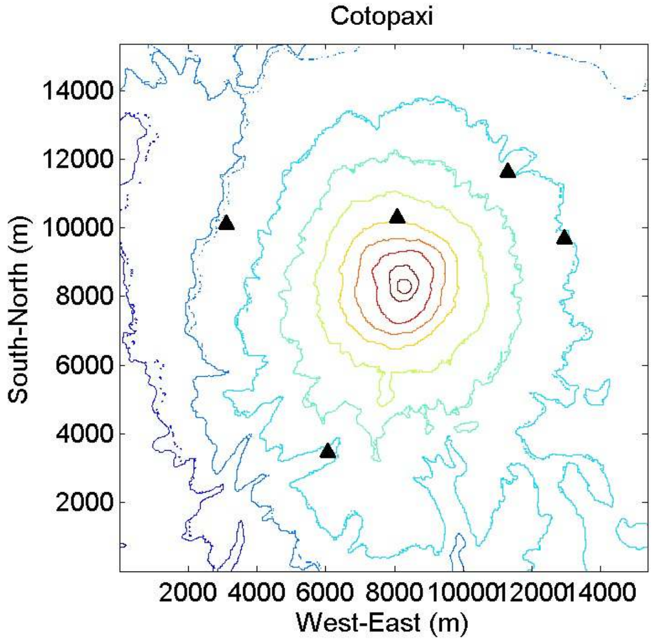
$$\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)}$$

6b) Examine the convergence of the expression as N tends to infinity.

Exercise 7

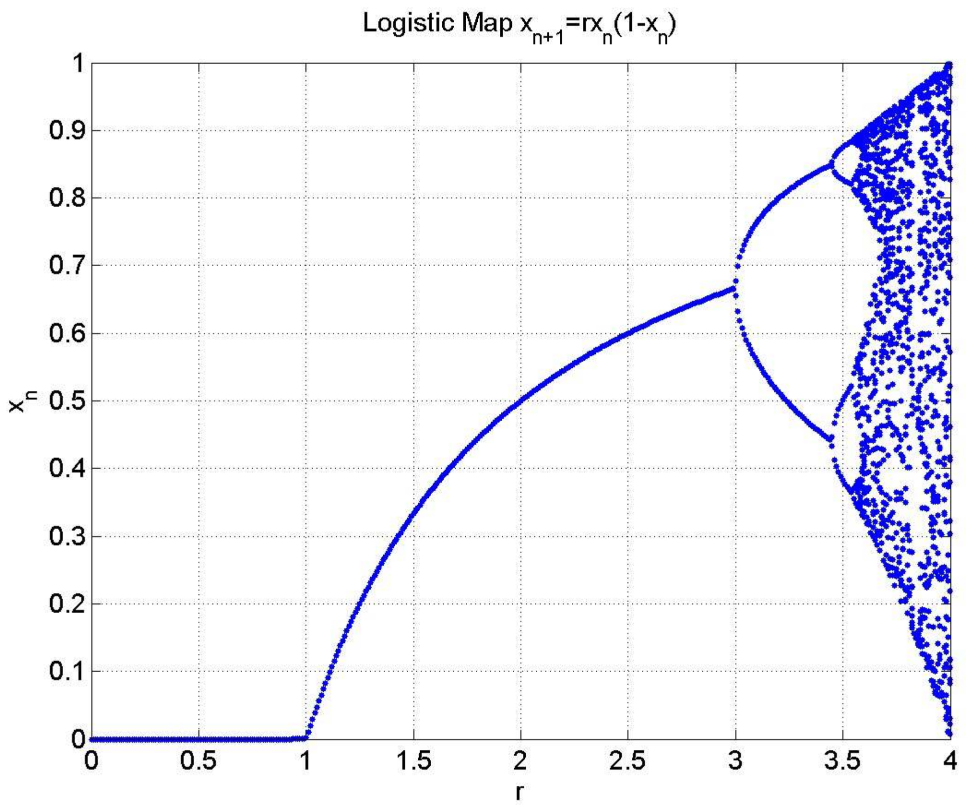
7a) Load in and plot the topography file cotopaxi1024x1024_15m. Format is Wes-East, North-South and altitude. Also plot the location of seismic recorders in the file recorders. Use pcolor and surf.

7b) Contour the topography instead of pcolor/surf. Use contour.



Exercise 8

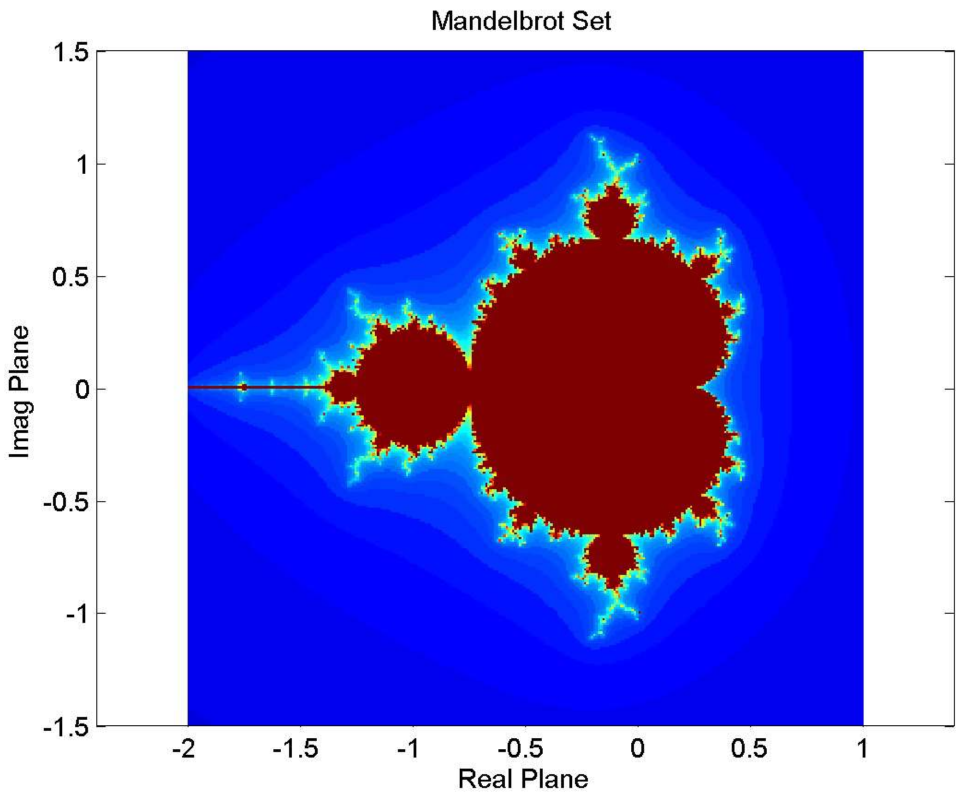
8) Plot the recurrence equation known as the logistic map $x_{n+1} = rx_n(1 - x_n)$ for r ranging from [0-4]. The logistic map is an analogue for population dynamics where r is the ratio of reproduction to starvation. X is the population at year n. The logistic map is an example of a simple equation leading to complex chaotic behaviour. This behaviour is commonly observed in Earth Sciences.



Exercise 9

9a) Generate the famous Mandelbrot set in the complex plane. The Mandelbrot set is based on the recurrence equation $z_{n+1} = z_n^2 + C$ where C is a point in the set. If z_n remains bounded then the point C is in the set, if $z_n \to \infty$ then the point C lies out side the set. The Mandelbrot set is a famous example of self-similar behaviour, i.e. see the same pattern at different scales. The field of interest is real=[-2 1], complex=[-1.5i 1.5i].

9b) Replot the set but zooming into a different region.



Exercise 10: Finite-Difference Solution to Pore Pressure Diffusion

Pore pressure in the subsurface is constantly changing in space and time leading to subsurface fluid flow. The pressure is driven by a multitude of different processes, e.g. compaction, chemical reactions, hydraulic heads and anthropological fluid migration. These anomalous fluid pore pressures will diffuse through the Earth's subsurface until it reaches equilibrium. This process is governed by the anisotropic pore pressure diffusion equation given below.

$$\frac{\partial p(\mathbf{x},t)}{\partial t} = \frac{\partial}{\partial x_\alpha}\left(D_{\alpha\beta} \frac{\partial p(\mathbf{x},t)}{\partial x_\beta} \right) \tag{1}$$

where p(**x**,t) is the pressure at **x** at time t and $D_{\alpha\beta}$ is the diffusion tensor which is related to the permeability tensor $K_{\alpha\beta}$ through

$$K_{\alpha\beta} = \phi\eta\beta D_{\alpha\beta} \tag{2}$$

Φ is the porosity, η the viscosity and β is the fluid compressibility. The fluid velocity is related to the pore pressure through Darcy's Law which is given by:

$$v = \frac{\Phi}{\eta} K_{\alpha\beta} \nabla p(\mathbf{x},t) \tag{3}$$

The aim of this exercise to solve a simplified version of the pore pressure diffusion equation using a finite-difference method in Matlab. We can simplify the equation by assuming that the diffusion tensor is symmetric and constant in space and time. This allows us to bring the diffusion tensor out from under the derivative and rewrite the equation in 2D as

$$\frac{\partial p(\mathbf{x},t)}{\partial t} = D_{xx}\frac{\partial^2 p(\mathbf{x},t)}{\partial x^2} + D_{yy}\frac{\partial^2 p(\mathbf{x},t)}{\partial y^2} + 2D_{xy}\frac{\partial^2 p(\mathbf{x},t)}{\partial y \partial x} \qquad (4)$$

To solve this equation we need to change the continuous variable p(**x**,t) and the continuous derivative to a discrete approximation we can implement on a digital computer. This is done by gridding the pressure on a Cartesian grid and using finite difference approximations for the derivatives.

Firstly, we grid the initial pressure field across a regular grid as in the figure below

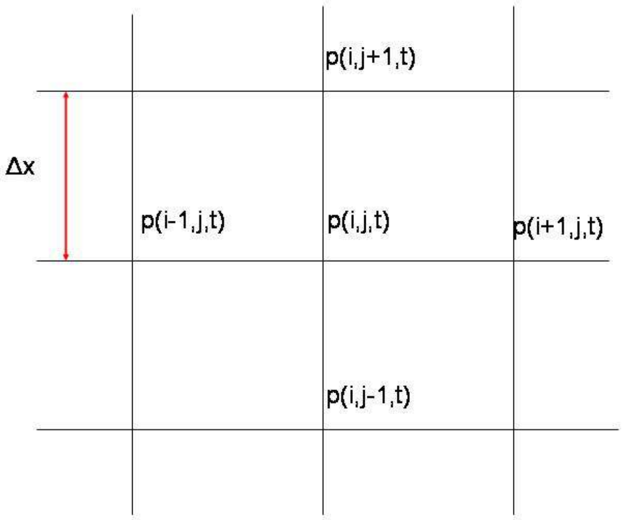


Figure 1: Discrete pore pressure field on a Cartesian grid at time t.

Next we use the finite difference approximations for the derivatives. The theory behind finite-difference methods is well established and can be found in most computational method texts, e.g. *W.H. Press*, *S.A. Teukolsky, W.T. Vetterling, and B.P. Flannery,* Numerical *Recipes: The Art of Scientific Computing, Third Edition*. The 2nd order time derivative finite-difference operator is given by

$$\frac{\partial p(i,j,t)}{\partial t} \approx \frac{p(i,j,t+\Delta t) - p(i,j,\Delta t)}{\Delta t} \qquad (5)$$

where Δt is the discrete time step. The 4th order spatial derivatives are given by

$$\frac{\partial^2 p(\mathbf{x},t)}{\partial x^2} \approx \frac{-p(i-2\Delta x,j,t) + 16p(i-\Delta x,j,t) - 30p(i,j,t) + 16p(i+\Delta x,j,t) - p(i+2\Delta x,j,t)}{12\Delta x^2}$$

$$\frac{\partial^2 p(\mathbf{x},t)}{\partial y^2} \approx \frac{-p(i,j-2\Delta x,t) + 16p(i,j-\Delta x,t) - 30p(i,j,t) + 16p(i,j+\Delta x,t) - p(i,j+2\Delta x,t)}{12\Delta x^2}$$

$$\frac{\partial p(\mathbf{x},t)}{\partial x} \approx \frac{-p(i-2\Delta x,j,t) - 8p(i-\Delta x,j,t) + 8p(i+\Delta x,j,t) - p(i+2\Delta x,j,t)}{12\Delta x} \qquad (6)$$

$$\frac{\partial p(\mathbf{x},t)}{\partial y} \approx \frac{p(i,j-2\Delta x,t) - 8p(i,j-\Delta x,t) + 8p(i,j+\Delta x,t) - p(i,j+2\Delta x,t)}{12\Delta x}$$

If we substitute these expressions in equation (4) we find that the pore pressure at coordinate (i,j) is evolved in time as a function of the previous time and its nearest neighbours.

Note: The transformation of the continuum differential equation into a discrete form is an approximation and as such, contains assumptions. If these assumptions are not met, then the solution will be unstable and incorrect. The analysis of these errors is a specialist field in computational physics and should be considered in any scientific application. In this example the scheme is stable once

$$\Delta t < \frac{\Delta x^2}{4D_{max}} \qquad (7)$$

Equation (7) simple says that no material can diffuse through the grid quicker than the physically allowable rate determined by the diffusion constant.

Exercise:

a) Solve equation (4) using a 6th order in space and second order in time finite-difference solution as discussed above. Use the web to find the 6th order operators.

b) Determine the pressure field after 5 seconds in an area 90 m x 60 m where the diffusion tensor is $D_{xx}=2$, $D_{yy}=1$, and $D_{xy}=0.5$. See figure 2 for the solution.

c) Calculate the velocity field using Darcy's law and a second order finite difference operator. The solution to this section is not given in the example code.

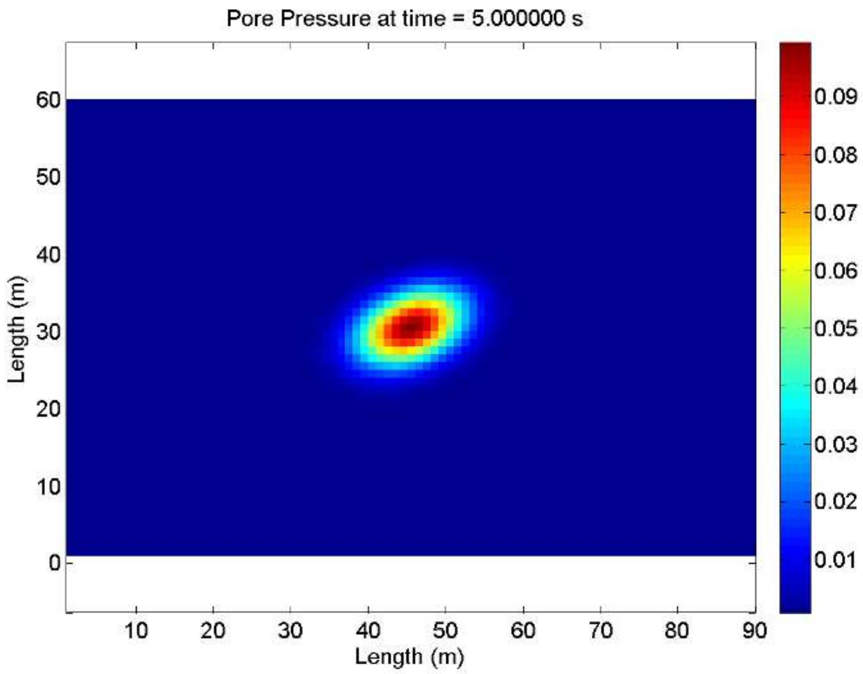


Figure: Solution to equation (4) as discussed in the exercise.

Exercise 11: Time Series Analysis

Exercise 1:

d) Generate the time series for 7 days with a sampling rate of 1 minute using the equation below

$$s(t) = A_1 \mathrm{Sin}(2\pi f_1 t) + A_2 \mathrm{Sin}(2\pi f_2 t) + A_3 \mathrm{Sin}(2\pi f_3 t) + A_4 \mathrm{Sin}(2\pi f_4 t)$$

where
$$\begin{array}{ll} A_1 = 1.00 & f_1 = 1/1\mathrm{hour} \\ A_2 = 1.25 & f_1 = 1/6\mathrm{hour} \\ A_3 = 0.50 & f_1 = 1/\mathrm{day} \\ A_4 = 0.00 & f_1 = 1/\mathrm{month} \end{array}$$

e) Plot the time series.

f) Calculate the spectrum using fft. Note that the absolute value of the output of fft is the power spectrum.

g) Using filtfilt, isolate each of the individual frequencies in the signal. Plot the results on the same figure.

h) Change A_4 to 1.0 and repeat the steps. What changes and why?

Exercise 2:

a) Load in the rainfall data in the file England_precipitation_monthly.dat. The format is given by

Monthly Central England precipitation (mm). Daily automated values used after 1996.
Wigley & Jones (J.Climatol.,1987), Gregory et al. (Int.J.Clim.,1991)
Jones & Conway (Int.J.Climatol.,1997), Alexander & Jones (ASL,2001). Values may change after QC.
YEAR JAN FEB MAR APR MAY JUN JUL AUG SEP OCT NOV DEC ANN

b) Rearrange the file and plot as a time series.

c) Calculate and plot the spectrum using fft.

d) What is observed and why?