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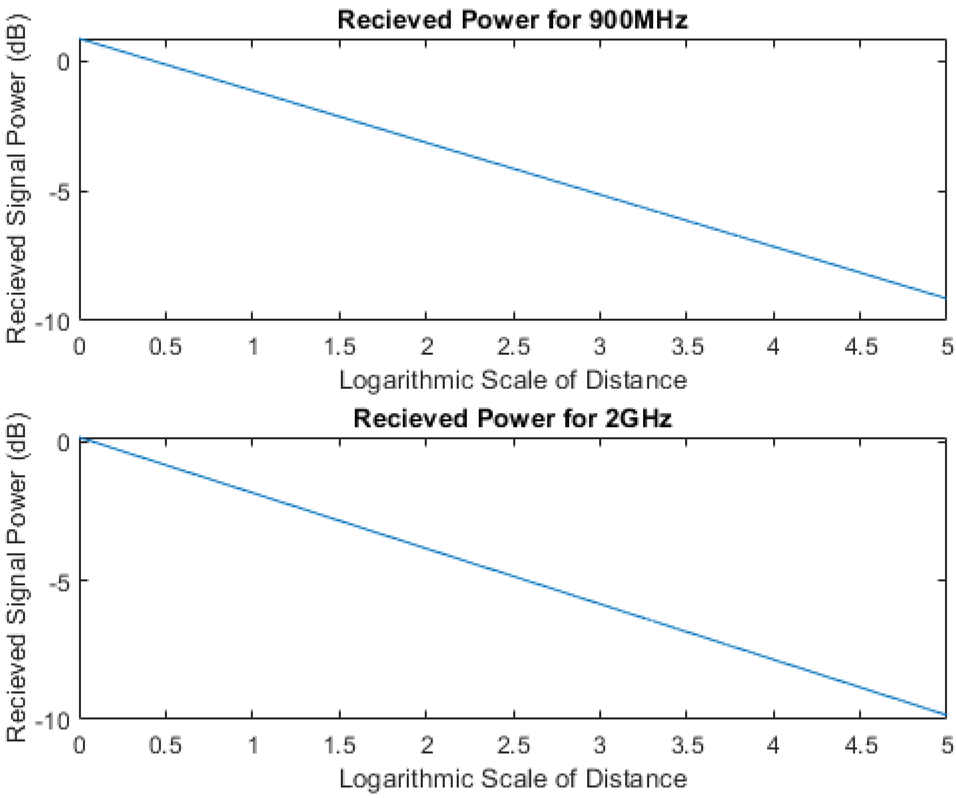
EECS 4215: Mobile Communications

Lab 1 - Large Scale Path Loss

Friday February 1st 2019

1.

The matlab function I wrote to produce these graphs is called freeSpace.m.



2. $d_1 = (d^2 + (h_t - h_r)^2)^{1/2}$

$d_2 = (d^2 + (h_t + h_r)^2)^{1/2}$

3.

$$r_1(t) = \sqrt{2 \cdot P_r(d_1)} \cdot \cos\left(2\pi \cdot f_c\left(t - \frac{d_1}{c}\right)\right)$$

$$P_r(d_1) = \left(\frac{\sqrt{G_t G_r}\,\lambda}{4\pi d_1}\right)^2 \cdot P_t = P_{r0}$$

LOS signal $\quad Re\left\{\sqrt{2P_{r0}} \cdot e^{j2\pi f_c\left(t - \frac{d_1}{c}\right)}\right\}$

Reflected path $\quad r_2(t) = \sqrt{2P_r(d_2)}\,\cos\left(2\pi f_c\left(t - \frac{d_2}{c}\right)\right)$

$$P_r(d_2) = \left(\frac{\sqrt{G_t G_r}\,\lambda}{4\pi d_2}\right)^2 \cdot P_t = P_{r0} \cdot \left(\frac{d_1}{d_2}\right)^2$$

$$Re\left\{\sqrt{2P_{r0}} \cdot \frac{d_1}{d_2} \cdot e^{j2\pi}\right.$$

$$r(t) = r_1(t) + R \cdot r_2(t)$$

reflection coefficient (-1 for reflection)

$$= r_1(t) - r_2(t)$$

$$r(t) = Re\left\{\sqrt{2P_{r0}}\; e^{j2\pi f_c t}\left(e^{j2\pi f_c\left(\frac{-d_1}{c}\right)} - \frac{d_1}{d_2} e^{j2\pi f_c\left(-\frac{d_2}{c}\right)}\right)\right.$$

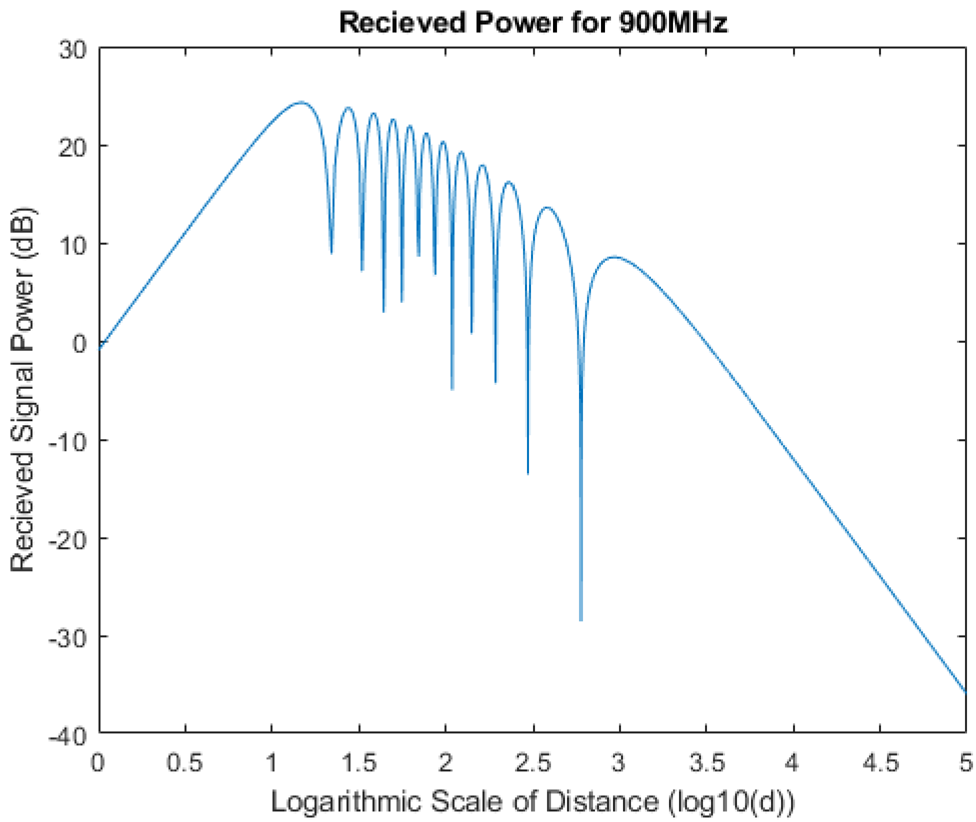
$$P_r = \frac{\left|\sqrt{2P_{r0}} \cdot e^{j2\pi f_c t}\left[e^{j2\pi f_c\left(\frac{-d_1}{c}\right)} - \frac{d_1}{d_2} e^{j2\pi f_c\left(\frac{-d_2}{c}\right)}\right]\right|^2}{2}$$

multiply inside square brackets by $\left|e^{j2\pi f \frac{d_1}{c}}\right|$

$$= \frac{\left|\sqrt{2P_{r0}} \cdot \left|1 - \frac{d_1}{d_2} e^{j2\pi f_c \frac{d_1 - d_2}{c}}\right|\right|^2}{2} = P_{r0}\left|1 - \frac{d_1}{d_2} e^{-j\Delta\phi}\right|^2$$

$$2\pi f_c \frac{d_2 - d_1}{c} = 2\pi \frac{d_2 - d_1}{\lambda} = \Delta\phi$$

The matlab function I wrote to produce this graph is called 2rayGR.m



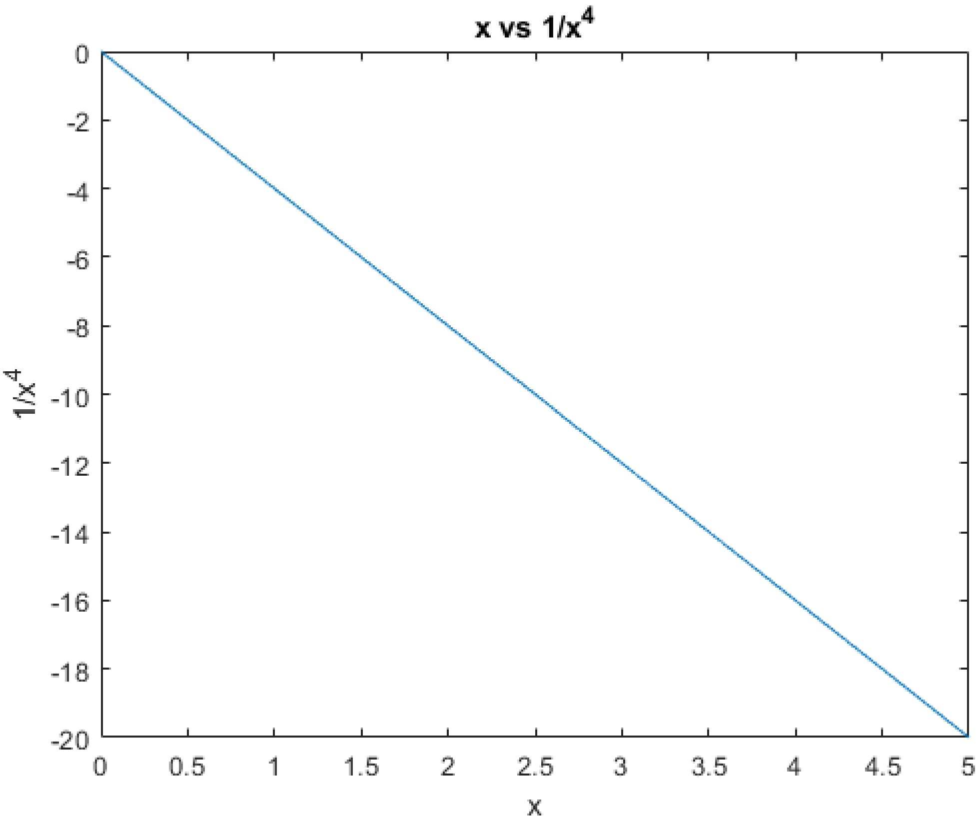
4.

The range that the deep fades can occur within is when the logarithm of the distance is greater than the height of the transmitter and smaller than the critical distance. Within this range the signal power is subjected to both constructive and destructive interference. The cause of these deep fades is the destructive interference being stronger than the constructive interference. The last one occurs at approximately x = log(d) = 3.08, and therefore d is approximately equal to 1202.26m. This is the last deep fade because after this point the signal moves from constructive and destructive interference to exclusively destructive interference.

5.

The falloff becomes proportional to d^{-4} at when the distance exceeds the critical distance, $d_c = 4 * h_t * h_r / \lambda = 4 * 50 * 2 / 0.333 = 1200$m. Since the x-axis is a log scale, x = log10(1200) = 3.079, you can look at the plot in question 3, and see that it corresponds with the graph of x vs. x^{-4} after x = 3.

The matlab script I wrote to produce this graph is called x4.m



6. $\Delta\phi = \dfrac{2\pi(x'+x-l)}{\lambda}$

$$x'+x-l = \sqrt{(h_t+h_r)^2+d^2} - \sqrt{(h_t-h_r)^2+d^2}$$

$$= d\left[\sqrt{\left(\frac{h_t+h_r}{d}\right)^2+1} - \sqrt{\left(\frac{h_t-h_r}{d}\right)^2+1}\right]$$

$d \gg h_t, h_r$ only need to keep first order terms

$$\sim d\left\{\left[\frac{1}{2}\sqrt{\left(\frac{h_t+h_r}{d}\right)^2+1}\right] - \left[\frac{1}{2}\sqrt{\left(\frac{h_t-h_r}{d}\right)^2+1}\right]\right\}$$

$$= \frac{2(h_t+h_r)}{d}$$

$$\Delta\phi \sim \frac{2\pi}{\lambda}\frac{2(h_t+h_r)}{d}$$

$$x+x' = \sqrt{(h_t+h_r)^2+d^2}$$

$$l = \sqrt{(h_t-h_r)^2+d^2}$$

7. $\Delta d = x+x'-l = d\left(\sqrt{\dfrac{(h_t+h_r)^2}{d^2}+1} - \sqrt{\dfrac{(h_t-h_r)^2}{d^2}+1}\right)$

using Taylor series of $\sqrt{1+x}$

$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \ldots$ and taking the first term only,

$$x+x'-l \approx \frac{d}{2}\cdot\left(\frac{(h_t+h_r)^2}{d^2} - \frac{(h_t-h_r)^2}{d^2}\right) = \frac{2h_th_r}{d}$$

$\Delta\phi \approx \dfrac{4\pi h_t h_r}{\lambda d}$

$$P_r \approx P_t\left(\frac{\lambda\sqrt{G}}{4\pi d}\right)^2 \times \left|1-e^{-j\Delta\phi}\right|^2$$

Expanding $e^{-j\Delta\phi}$ using Taylor Series + only retaining first two terms.

$$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \ldots$$

$$e^{-j\Delta\phi} \approx 1 - j\Delta\phi$$

when $d \gg (h_t+h_r)$

$$d \approx l \approx x+x', \quad \Gamma(\theta) \approx -1, \quad G_{los} \approx G_{gr} = G$$

$$P_r \approx P_t \left(\frac{\lambda\sqrt{G}}{4\pi d}\right)^2 \times |1-(1-j\Delta\phi)|^2$$

$$= P_t \left(\frac{\lambda\sqrt{G}}{4\pi d}\right)^2 \times \Delta\phi^2$$

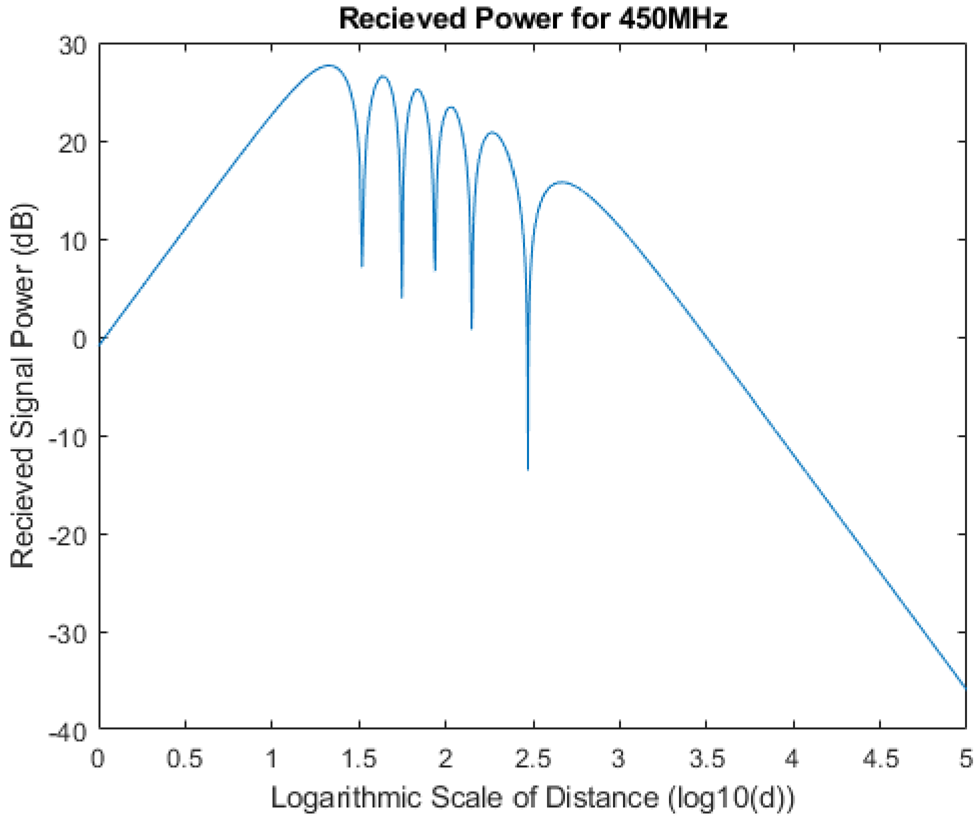
$$= P_t \left(\frac{\lambda\sqrt{G}}{4\pi d}\right)^2 \times \left(\frac{4\pi h_t h_r}{\lambda d}\right)^2$$

$$= \frac{P_t G h_t^2 h_r^2}{d^4}$$

$$= P_t \left[\frac{\sqrt{G_r G_t} \cdot h_r \cdot h_t}{d^2}\right]^2$$

8. From the results of 7 it is clear that in the case of a very large seperation between the transmitters, there is a direct correlation between the height of the transmitters and the recieved signal power. It is also worth noting that the result of 7 implies that the frequency has a diminished effect on the recieved signal power, implied by the fact that you can approximate P_r without considering f_c or λ. In the case when the transmitters are closer together, the strength of the signal is directly related to the strength of the free space model, which has an inverse correlation with frequency and direct correlation with height of the transmitters.

Repeating question 3 with a new frequency.



Repeating question 3 with a larger transmitter height.