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EECS 4215 - Mobile Communications
Lab 2 - Rayleigh and Rician Fading
March 1st 2019

1. The mean of a sum of two i.i.d. Gaussian random variables is equal to the sum of their means.
   $\mu_{|Z|} = \mu_X + \mu_Y = 0 + 0 = 0$

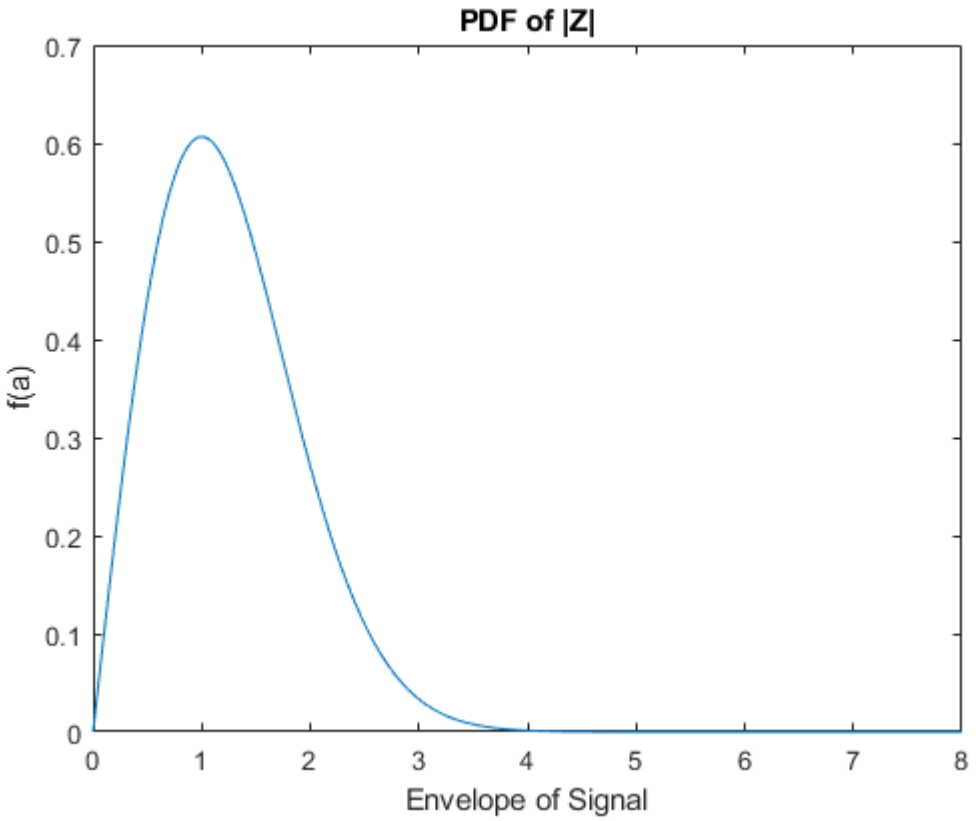
   The variance of a sum of two i.i.d. Gaussian random variables is equal to the sum of their variances.
   $\sigma^2_{|Z|} = \sigma^2_X + \sigma^2_Y = 1 + 1 = 2$

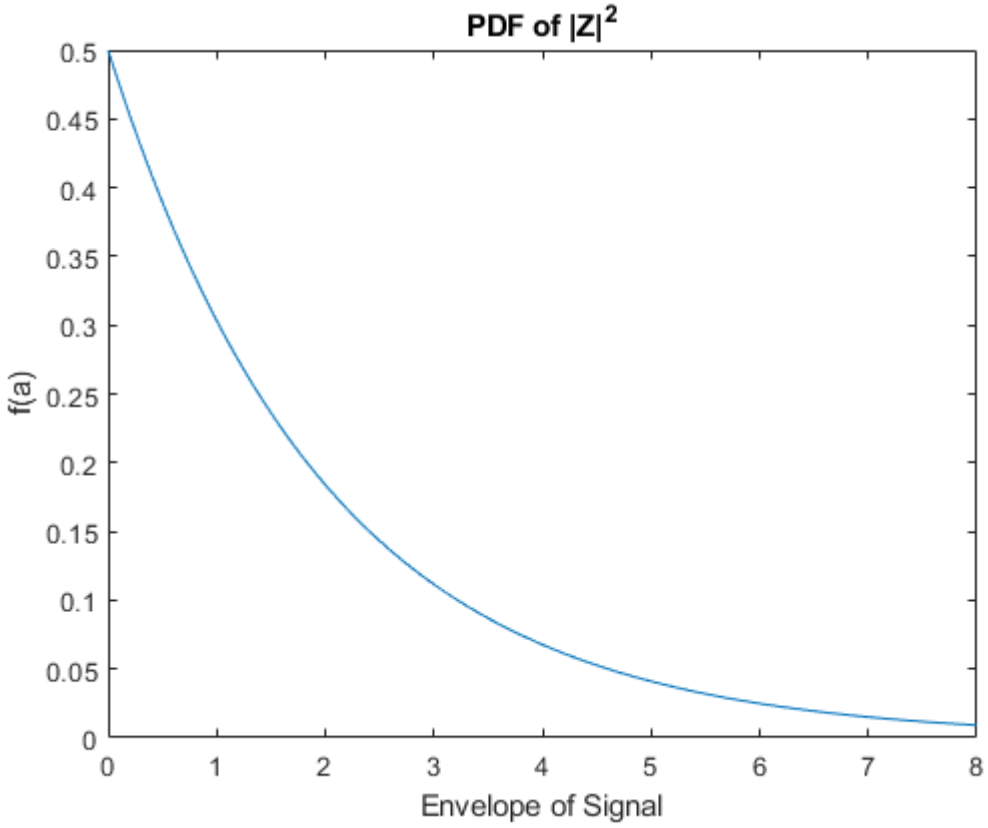
PDF of |Z|

$$f_A(a) = \frac{a}{\sigma^2} e^{\left(-\frac{a^2}{2\sigma^2}\right)}, \quad a \geq 0 \quad \sigma^2\text{: variance of the random variable } X \text{ (or } Y\text{)}$$

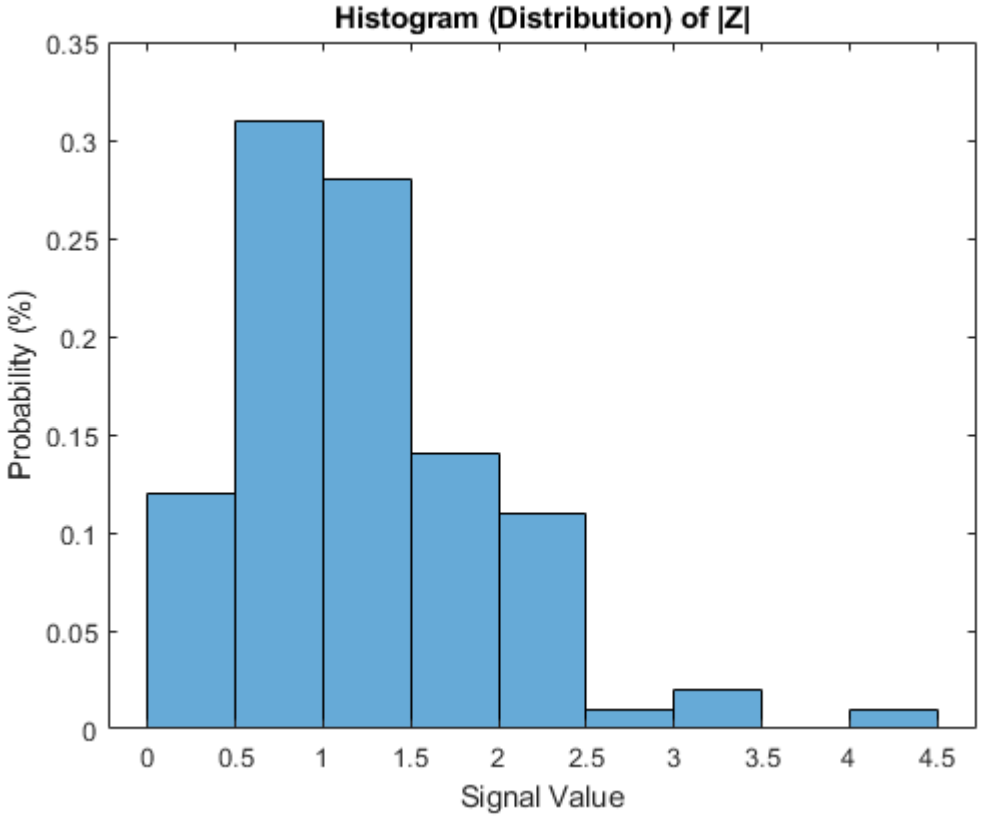
PDF of $|Z|^2$

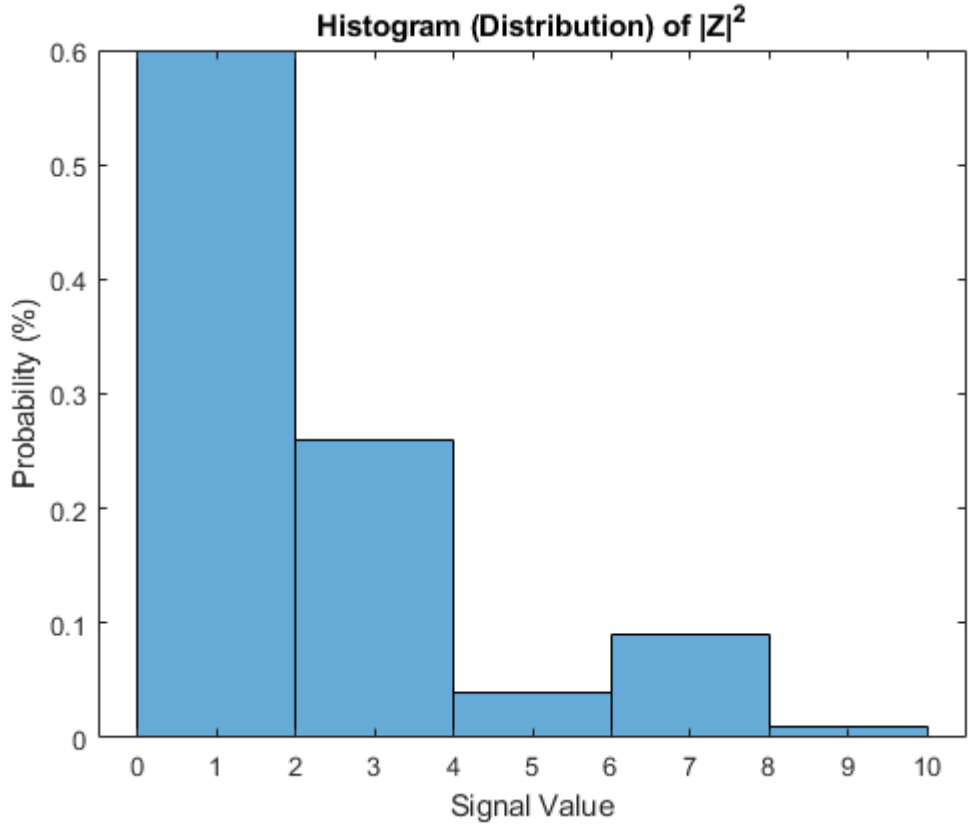
$$f_p(p) = \frac{1}{2\sigma^2} e^{\left(-\frac{p}{2\sigma^2}\right)}, \quad p \geq 0 \qquad \text{average power } \bar{p} = 2\sigma^2$$




PDF of |Z|2
f(a)
Envelope of Signal
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0
1
2
3
4
5
6
7
8

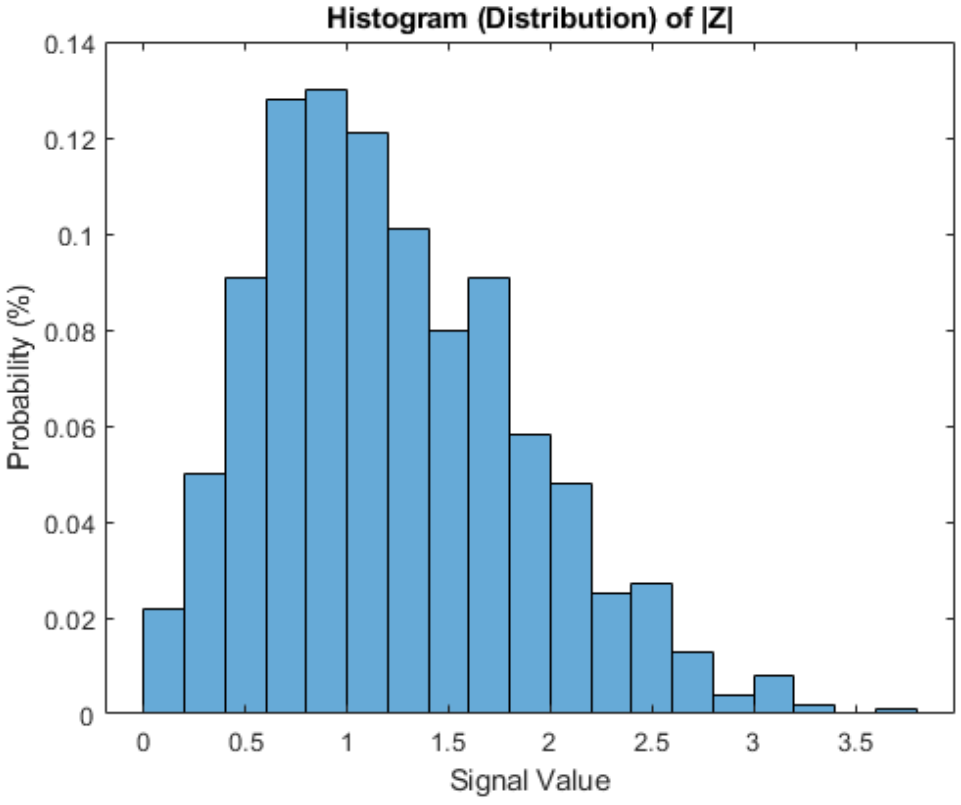
2.

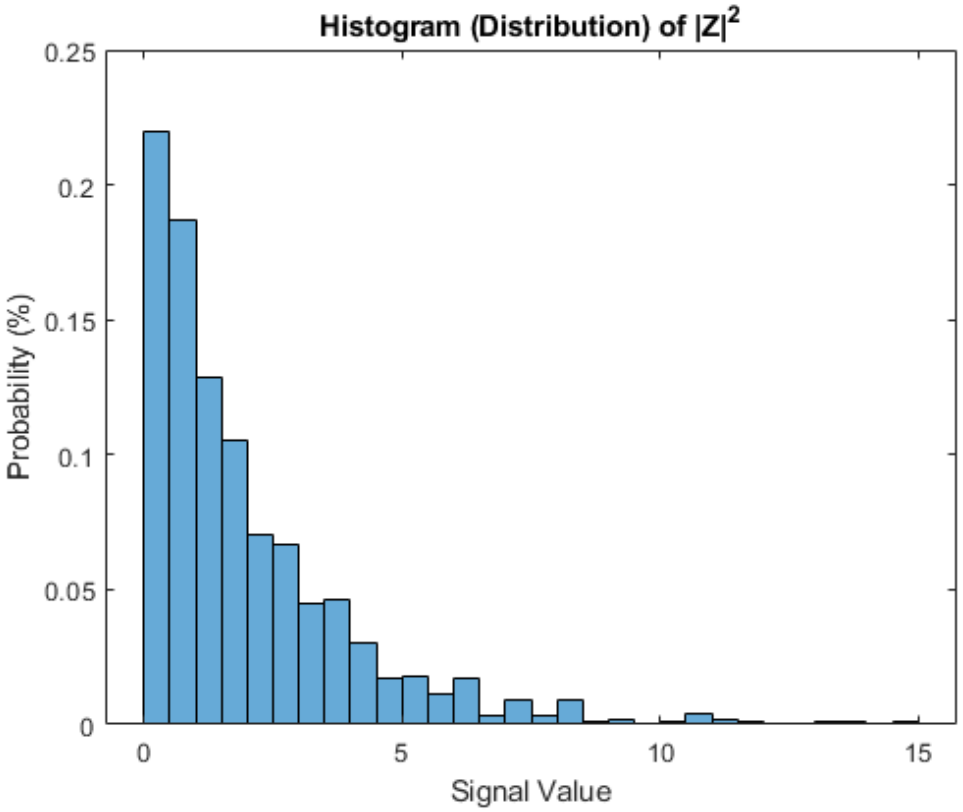

Histogram (Distribution) of |Z|
Probability (%)
Signal Value
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5



These two empirical distributions are very close to the expected Rayleigh and Exponential distributions in the slides and compared to examples I found on the internet. The Rayleigh distribution peaks around 1 and the exponential distribution peaks at 0. These histograms are normalized in order to make them empirical PDFs.

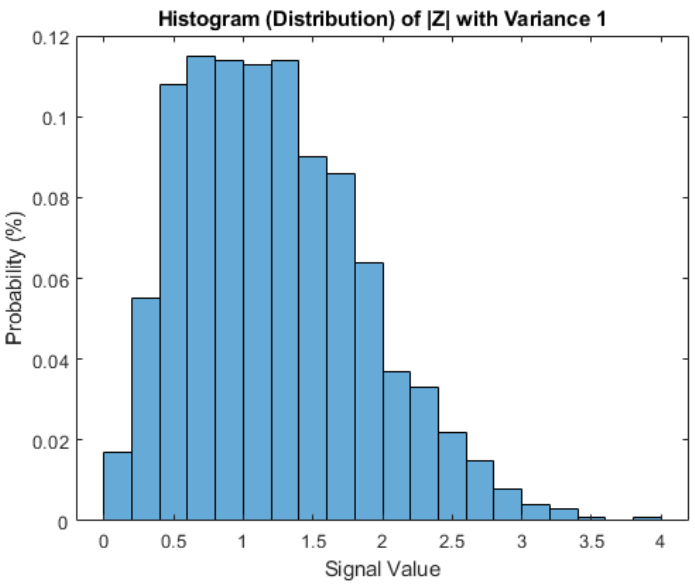
3.

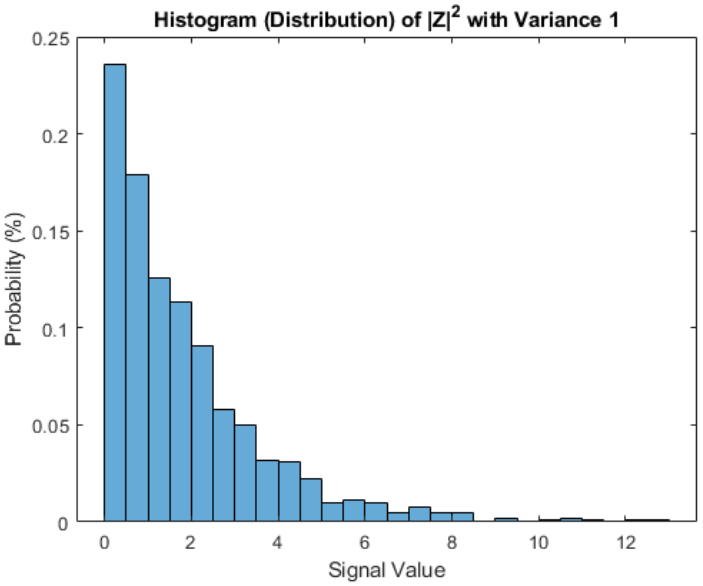




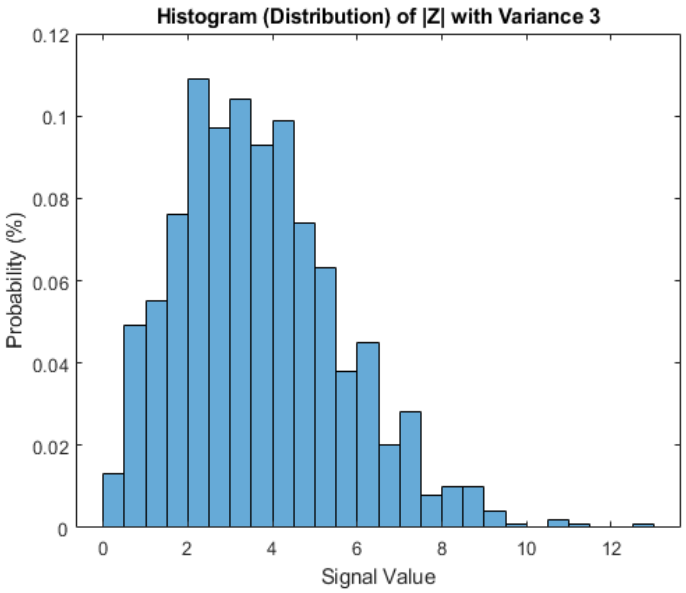
Increasing the number of samples in the random variable has created the need for many more divisions in the histogram, however the increased number of samples has no effect on the shape of the histograms. This increased sampling has caused the bars in the histogram to more properly represent the Rayleigh and Exponential distributions.

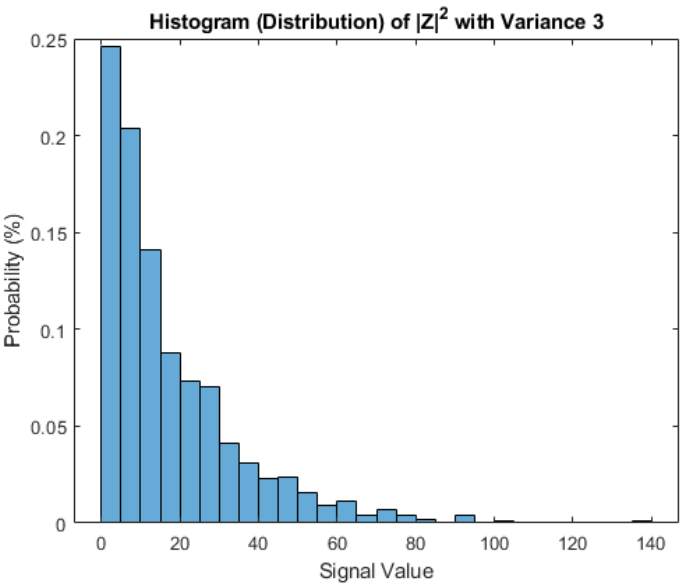
4. $\sigma^2 = 1$



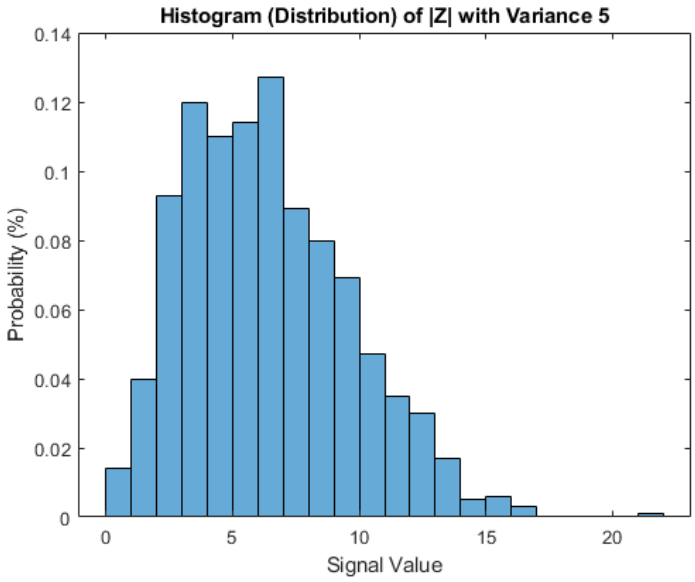


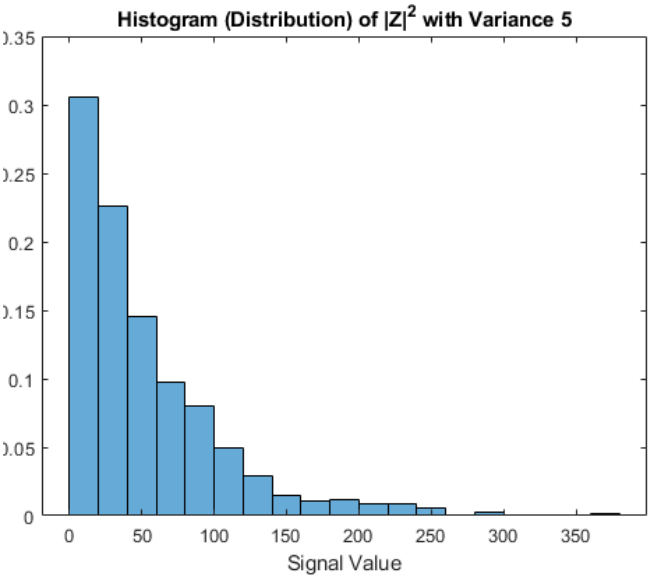
$\sigma^2 = 3$





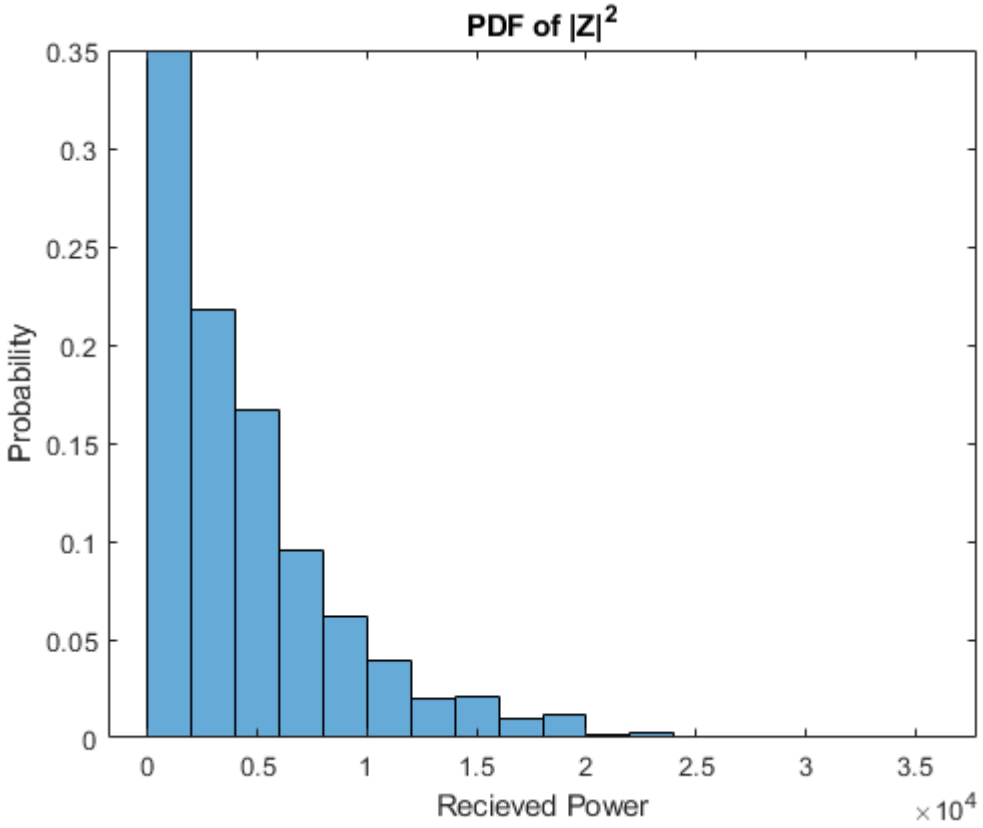
$\sigma^2 = 5$





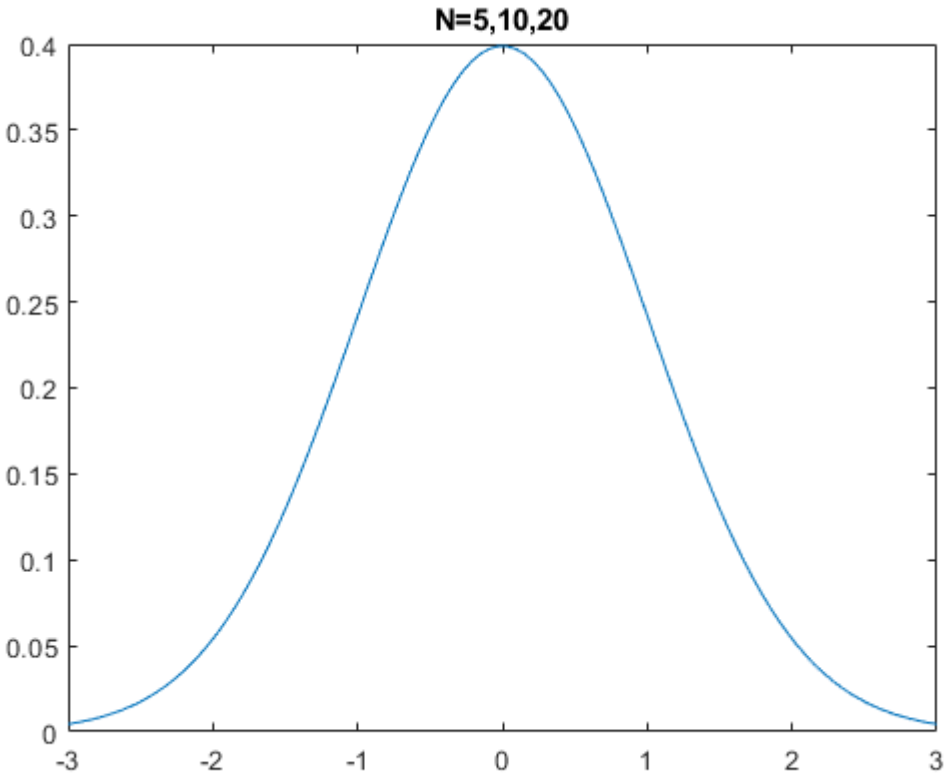
As you can see by the changing numbers on the x-axis of each pair of results, increasing the variance increases the range of signal values received. This should come as no surprise, since the higher the variance the higher the values move away from the mean. Changing the variance has no effect on the shape of the graph however, because the Rayleigh and Exponential shapes of each graph remains in tact with the changing variance.

5.



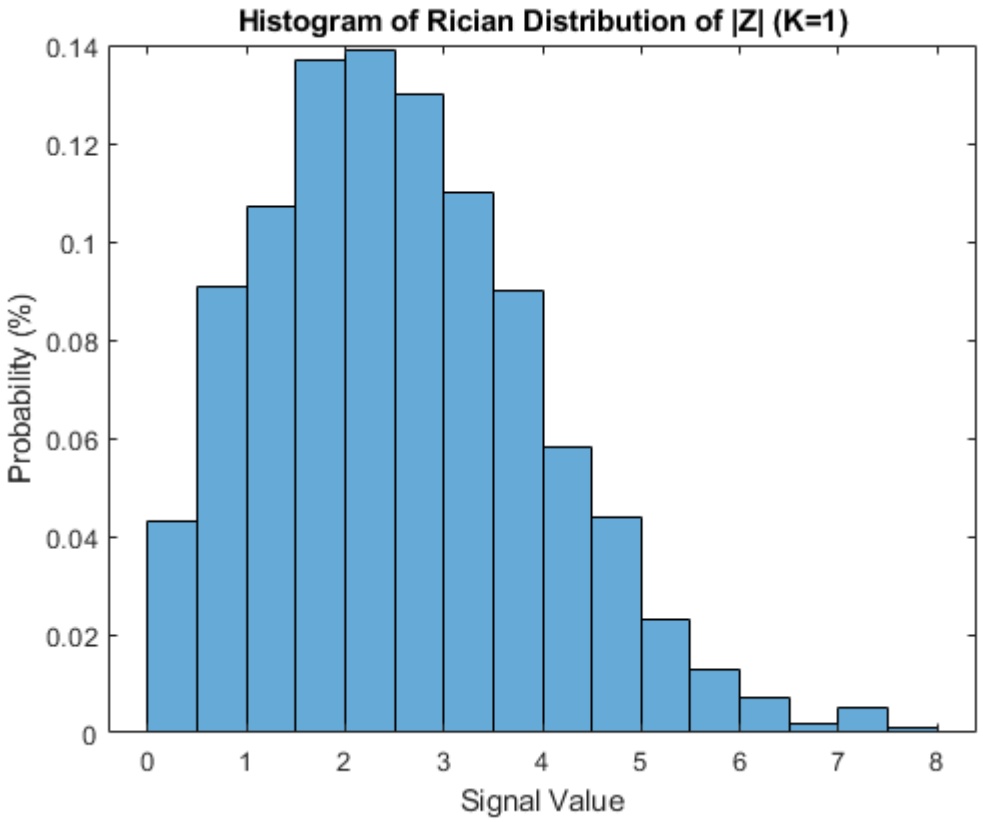
Both of these values calculated using |Z|
You can see how I calculated these numbers in my code Q5.m
Average of Samples = 63.6621
Comparing this number with the variance which is 50, we can see that this is an appropriate average considering there is a zero mean.
(# of Samples Below 50) / 1000 = 388 / 1000 = 0.388
The number given slide is 0.3935, so this result is very close to the expected, meaning that they agree.

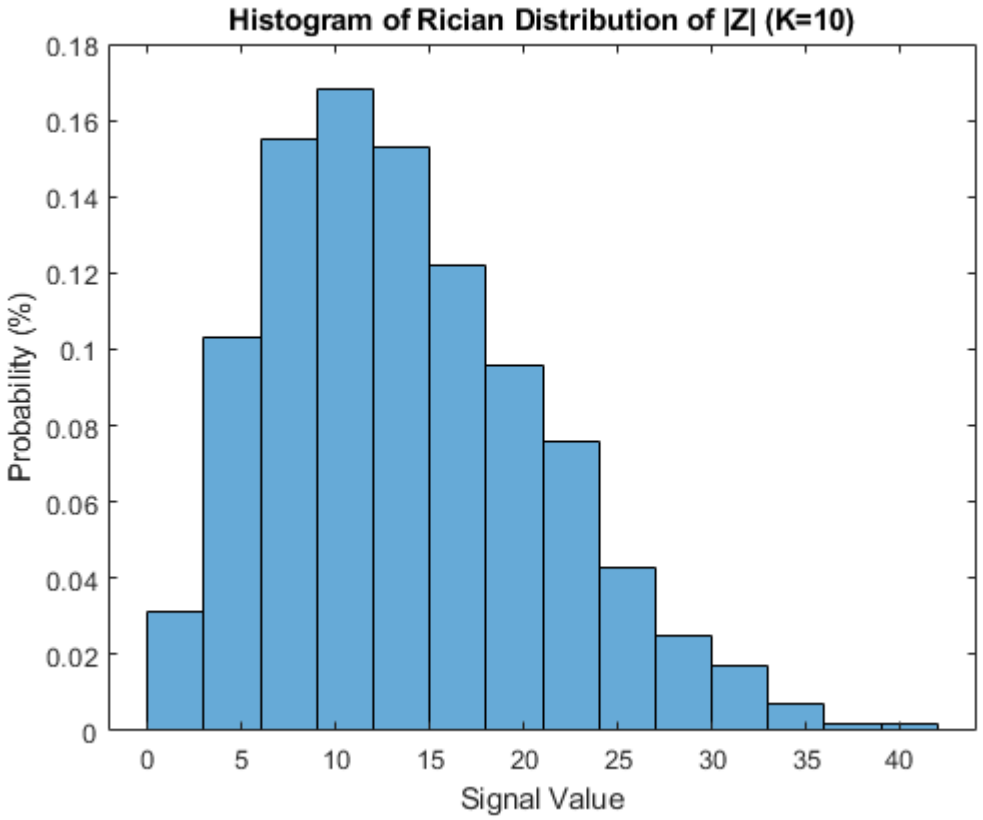
6. In my code I represented the theoretical derivation of the Rayleigh Distribution in MATLAB, I then plugged in the $e_r(t)$ values into the $f_p(p)$ equation and produced the distribution you see below. I was expecting some change in the graph with changing N but I could not produce it so I believe I have made some error along the way (unless that is the intention and it went over my head).

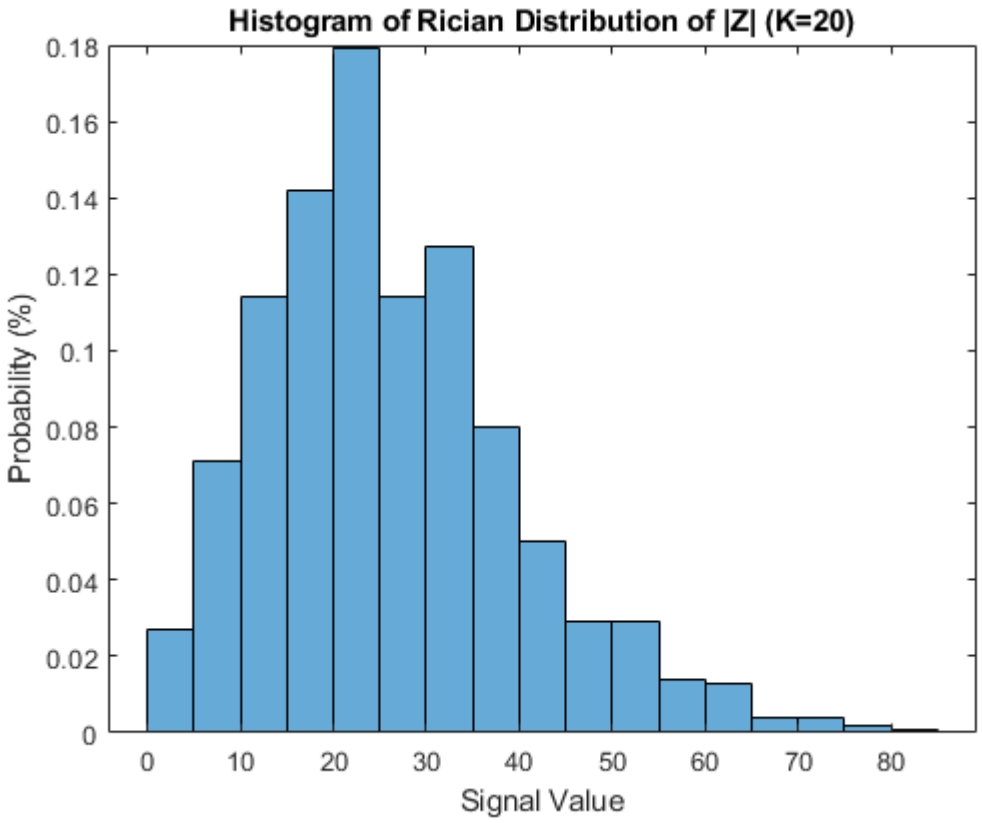


7. In order to change the relation to a Rician fading one, we have to change the mean of both gaussian variables. We had a zero mean, which is correct for the Rayleigh model, but for the Rician model, the mean of X must equal v*cosθ, and the mean of Y must equal v*sinθ. This is from the definition of the Rician distribution model. (There is no code included for this question.)

8.


Histogram of Rician Distribution of |Z| (K=1)
Probability (%)
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
1
2
3
4
5
6
7
8
Signal Value


Histogram of Rician Distribution of |Z| (K=10)
Probability (%)
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
5
10
15
20
25
30
35
40
Signal Value


Histogram of Rician Distribution of |Z| (K=20)
Probability (%)
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
10
20
30
40
50
60
70
80
Signal Value